Solution of Pure Scattering Radiation Transport Equation using Finite Difference Method

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Abstract
Radiative transfer is the physical phenomenon of energy transfer in the form of electromagnetic radiation. The propagation of radiation through a medium is affected by absorption, emission, and scattering processes. Equations of radiative transfer have application in a wide variety of subjects including optics, astrophysics, atmospheric science, and remote sensing. Analytic solutions to the radiative transfer equation (RTE) exist for simple cases but for more realistic media, with complex multiple scattering effects, numerical methods are required.

In the RTE, six different independent variables define the radiance at any spatial and temporal point. By making appropriate assumptions about the behaviour of photons in a scattering medium, the number of independent variables can be reduced. These assumptions lead to the diffusion theory (or diffusion equation) for photon transport. In this work, diffusive form of RTE will be discretized using Finite Difference Method Forward Time Central Space (FTCS) method and solved in MATLAB®. The results reveal that the penetration intensity of the photons and validate the inverse-square law.

Radiation Transport Equation

\[
\frac{\partial I(\mathbf{r}, \hat{n}, t)}{c \partial t} + \nabla \cdot I(\mathbf{r}, \hat{n}, t) + \left(k_{sA} + k_{sA} \right) I(\mathbf{r}, \hat{n}, t) = j_s(\mathbf{r}, t) + \frac{1}{4\pi} \int I(v, \hat{n}, t) d\Omega
\]

\(c\) is speed of light
\(\nabla\) is solid angle
\(k_{sA}\) is the scattering opacity of the medium
\(k_{a}\) is the absorption opacity of the medium
\(j_s\) is the emission coefficient of the medium
\(t\) is time variable

Pure Scattering

\[
\frac{\partial \phi_v(r, t)}{c \partial t} = D \left( \frac{\partial^2 \phi_v(r, t)}{\partial r^2} + \frac{\partial^2 \phi_v(r, t)}{\partial y^2} + \frac{\partial^2 \phi_v(r, t)}{\partial z^2} \right)
\]

\(\phi_v(r, t)\) is radiance intensity
\(c\) is speed of light
\(D\) is diffusion coefficient
\(r, y, z\) are the space dimensions

Numerical Results

Conclusion
The pure diffusive form of radiation transport equation (RTE) is solved using finite difference forward-time central-space (FTCS) method in MATLAB®. Results shows that light radiance follows inverse square law.

Recommendation
It is recommended to conduct the experiment and compare with the given results. This will validate the presented numerical modelling methodology and the underlying principles of radiation transport equation.

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