Solution of Pure Scattering Radiation Transport Equation using Finite Difference Method

H. Khawaja

UIT The Arctic University of Norway, Tromsø, Norway

Abstract

Radiative transfer is the physical phenomenon of energy transfer in the form of electromagnetic radiation. The propagation of radiation through a medium is affected by absorption, emission, and scattering processes. Equations of radiative transfer have application in a wide variety of subjects including optics, astrophysics, atmospheric science, and remote sensing. Analytic solutions to the radiative transfer equation (RTE) exist for simple cases but for more realistic media, with complex multiple scattering effects, numerical methods are required.

In the RTE, six different independent variables define the radiance at any spatial and temporal point. By making appropriate assumptions about the behaviour of photons in a scattering medium, the number of independent variables can be reduced. These assumptions lead to the diffusion theory (or diffusion equation) for photon transport. In this work, diffusive form of RTE will be discretized using Finite Difference Method Forward Time Central Space (FTCS) method and solved in MATLAB®. The results reveal that the penetration intensity of the photons and validate the inverse-square law.

Radiation Transport Equation

Radiation Transport Equation

$$\frac{\partial I_v(\boldsymbol{r},\hat{\boldsymbol{n}},t)}{c\partial t} + \hat{\Omega}.\nabla I_v(\boldsymbol{r},\hat{\boldsymbol{n}},t) + \left(k_{v,s} + k_{v,a}\right) I_v(\boldsymbol{r},\hat{\boldsymbol{n}},t) = j_v(\boldsymbol{r},t) + \frac{1}{4\pi} k_{v,s} \int_{\Omega} I_v(\boldsymbol{r},\hat{\boldsymbol{n}},t)\, d\Omega$$

I_v is spectral radiance of electromagnetic waves

c is speed of light

$\hat{\Omega}$ is solid angle

$k_{v,s}$ is the scattering opacity of the medium

$k_{v,a}$ is the absorption opacity of the medium

j_v is the emission coefficient of the medium

t is time variable

Pure Scattering

Radiation Transport Equation

$$\frac{\partial \phi_v(\boldsymbol{r},t)}{c\partial t} = D\nabla^2 \phi_v(\boldsymbol{r},t) = D\left(\frac{\partial^2 \phi_v(\boldsymbol{r},t)}{\partial x^2} + \frac{\partial^2 \phi_v(\boldsymbol{r},t)}{\partial y^2} + \frac{\partial^2 \phi_v(\boldsymbol{r},t)}{\partial z^2}\right)$$

$\phi_v(\boldsymbol{r},t)$ is radiance intensity

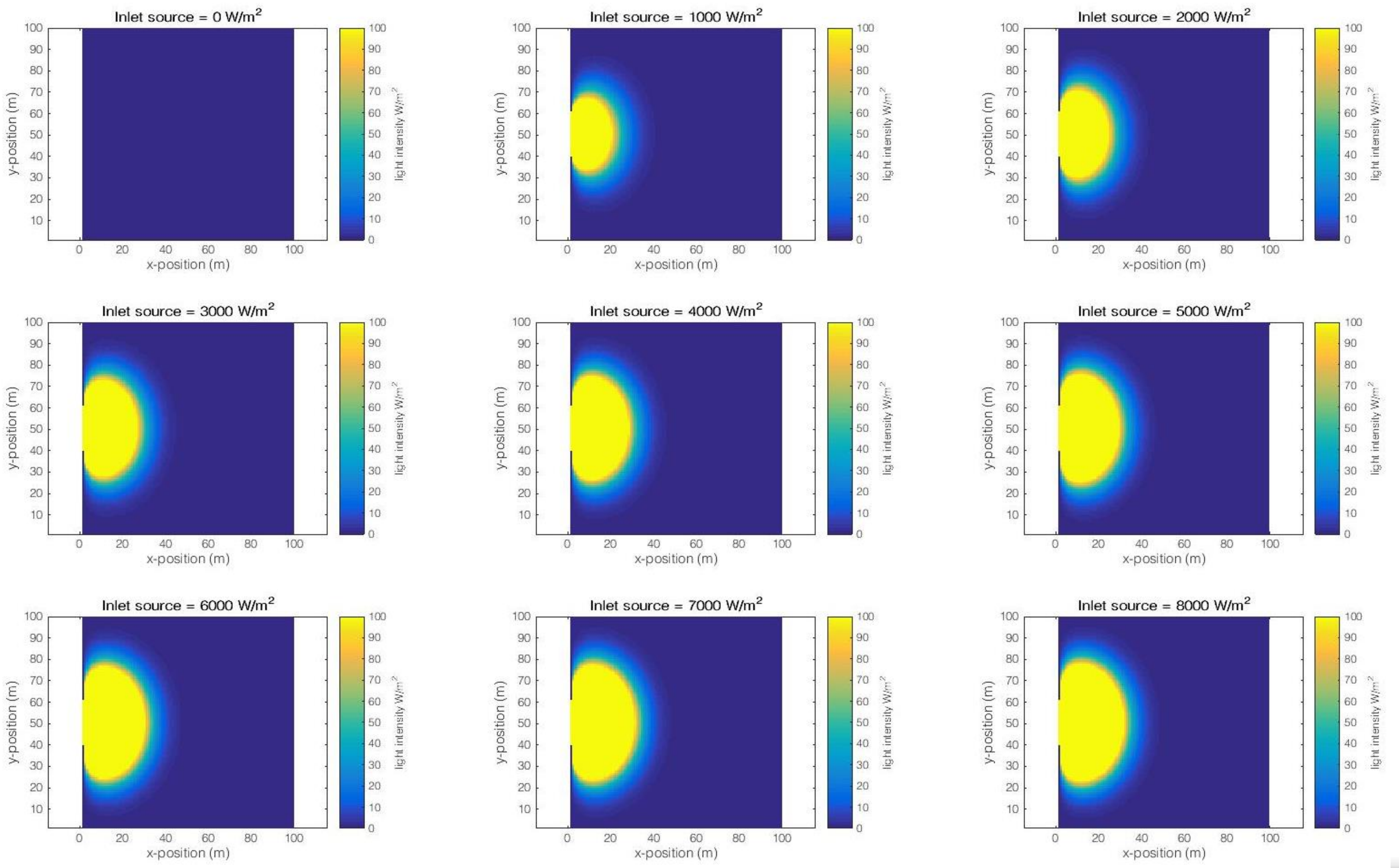
c is speed of light

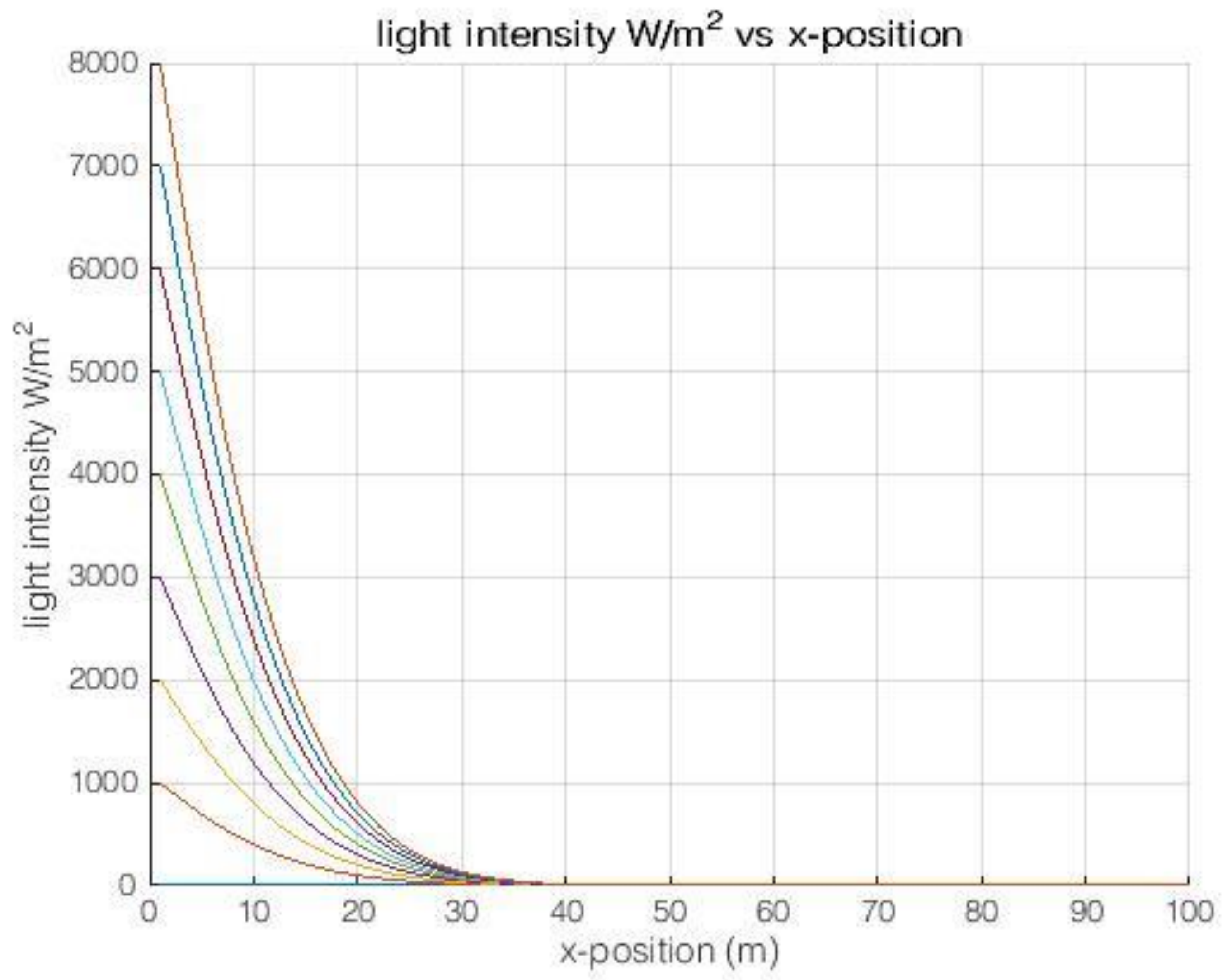
x, y, z are the space dimensions

D is diffusion coefficient

t is time variable

Numerical Results





Conclusion

The pure diffusive form of radiation transport equation (RTE) is solved using finite difference forward-time central-space (FTCS) method in MATLAB®. Results shows that light radiance follows inverse square law.

Recommendation

It is recommended to conduct the experiment and compare with the given results. This will validate the presented numerical modelling methodology and the underlying principles of radiation transport equation.

Contact

H. Khawaja

Associate Professor,
UiT The Arctic University of Norway, Tromsø, Norway

E-mail:
hassan.a.khawaja@uit.no