Faculty of Engineering Science and Technology
Institute of Industrial Engineering

Research on Optimization of Freeform Surface Operation

Dingjun Liu
Master thesis in Industrial Engineering - May 2017
<table>
<thead>
<tr>
<th><strong>Title:</strong></th>
<th>Research on Optimization of Freeform Surface Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Date:</strong></td>
<td>01/06/2017</td>
</tr>
<tr>
<td><strong>Classification:</strong></td>
<td>* OPEN *)</td>
</tr>
<tr>
<td><strong>Author:</strong></td>
<td>Dingjun Liu</td>
</tr>
<tr>
<td><strong>Student no:</strong></td>
<td>540661</td>
</tr>
<tr>
<td><strong>Number of Pages:</strong></td>
<td>89 pages</td>
</tr>
<tr>
<td><strong>Number of Attachments:</strong></td>
<td>6 NC files</td>
</tr>
<tr>
<td><strong>Subject Name:</strong></td>
<td>Master’s Thesis</td>
</tr>
<tr>
<td><strong>Subject Code:</strong></td>
<td>SHO6266</td>
</tr>
<tr>
<td><strong>Department:</strong></td>
<td>Faculty of Engineering Science and Technology</td>
</tr>
<tr>
<td><strong>Master Program:</strong></td>
<td>Industrial Engineering</td>
</tr>
<tr>
<td><strong>Supervisor:</strong></td>
<td>Gabor Sziebig</td>
</tr>
<tr>
<td><strong>Co-supervisor:</strong></td>
<td>Sibul Lazar</td>
</tr>
<tr>
<td><strong>External Organization/Company:</strong></td>
<td></td>
</tr>
<tr>
<td><strong>External Organization’s/Company’s Liaison:</strong></td>
<td></td>
</tr>
</tbody>
</table>
Abstract (max 150 words):

Nowadays, many manufacturing companies need to face competition both in domestic and international level. Due to this circumstance, manufacturers recognize that one useful method to enhance their competence is reducing the lead-time of manufacturing.

There are many researchers focused on minimize the time of actual cutting, tool path optimization and energy consumption optimization etc. However, few researchers have investigated the operation optimization of CNC machine that integrating multiple ways to reduce the operation time of freeform surface cutting. For example, integrating tool path calculation optimization and machining parameters optimization together for the sake of providing an optimization solution package to enhance the efficiency of manufacturing.

This master thesis will investigate the main optimization methods of tool path (Iso parametric, Iso planar and Iso scallop), as well as prediction of operation time and energy consumption optimization. Then we will provide several approaches for machining a surface on a metal rectangle to research the effect of different tool paths generation such as Iso parametric and Iso scallop etc. depend on NC code on machining time and quality.
Table of Contents

Preface .................................................................................................................. 1

Abstract ............................................................................................................... 2

1 Introduction ........................................................................................................ 3
  1.1 Background ........................................................................................................ 4
    1.1.1 Three Algorithms in Machining Surface ...................................................... 5
    1.1.2 Prediction of Part Machining Times ............................................................. 7
    1.1.3 Tool Path Generation Regard to Energy Consumption ......................... 8

2 Research of Optimization Methods .................................................................... 10
  2.1 Tool Path Algorithm Optimization .................................................................. 10
    2.1.1 Definition of the free form surface ............................................................... 12
    2.1.2 CC path scheduling algorithm and cutting tool offsetting ...................... 13
      2.1.2.1 CC path scheduling algorithm ............................................................... 13
      2.1.2.2 Tool offsetting ....................................................................................... 13
    2.1.3 Three algorithms of CC path scheduling ............................................... 14
      2.1.3.1 Iso parametric machining ................................................................. 14
      2.1.3.2 Iso scallop machining ................................................................. 16
      2.1.3.3 Iso planar machining ....................................................................... 21
  2.2 Prediction of Part Machining Times ................................................................. 23
    2.2.1 Trajectory Generation ............................................................................. 23
  2.3 Tool Path Generation Consider to Energy Consumption ............................. 27
    2.3.1 Energy Potential Field ............................................................................. 29
      2.3.1.1 Determination of tool orientation ....................................................... 29
      2.3.1.2 The model of energy consumption ...................................................... 31
      2.3.1.3 Energy consumption based on tool path generation ....................... 35
      2.3.1.4 Feed direction optimization .............................................................. 35
      2.3.1.5 Principle curve generation ............................................................... 37
      2.3.1.6 Expansion algorithm according to Iso-scallop height ...................... 38

3 Case Study ........................................................................................................ 41
  3.1 Analysis of Default Generation Path Based on G codes .............................. 42
  3.2 Approaches to the Case Based on G codes (simulation) ............................ 43
    3.2.1 Surface Machining Optimization Method 1: Linear Path only ............... 43
    3.2.2 Surface Machining Optimization Method 2: Iso Scallop Method .......... 46
    3.2.3 Surface Machining Optimization: Optimization of Method 2 ............... 47
    3.2.4 Surface Machining Optimization: Optimization of Method 1 ............... 48
  3.3 Comparison of the Different Approaches & Final Proposal ......................... 50
  3.4 Experimental Results & Conclusion ............................................................... 51
    3.4.1 Experimental Process & Results .............................................................. 51
    3.4.2 Conclusion ................................................................................................. 54

Reference ............................................................................................................. 55

Appendix: Notes of Iso-Parametric Machining ..................................................... 59

Appendix: Notes of Iso-Planar Machining ............................................................. 60
List of Tables

Table 1 - Nomenclature ............................................................................................................................................ 11
Table 2 - Nomenclature ............................................................................................................................................ 27
Table 3 – Comparison of the five simulations ...................................................................................................... 50
Table 4 – Comparison of the two real CNC machining .......................................................................................... 53

List of Figures

Figure 1 – Manufacturing process: CAD-CAM-PP-CNC ....................................................................................... 3
Figure 2 – Four-axis CNC machine, Three-axis CNC machine and CNC lathe .................................................... 4
Figure 3 – Iso parametric machining path ............................................................................................................. 5
Figure 4 – Iso planar machining path ................................................................................................................... 6
Figure 5 – Iso scallop machining path ................................................................................................................ 6
Figure 6 – Action order processing in CNC systems ............................................................................................ 7
Figure 7 – Illustration of CC path, CL path, Tool axis vector and Normal vector. .................................................. 11
Figure 8 – Iso parametric machining paths ......................................................................................................... 12
Figure 9 – Convex and concave surface .............................................................................................................. 15
Figure 10 – Side step distance Δl and parametric side interval Δv ......................................................................... 16
Figure 11 – Iso scallop machining path ................................................................................................................ 17
Figure 12 – A comparison of the existing and the proposed methods ..................................................................... 18
Figure 13 – Schematic description for determining tkδ ........................................................................................ 20
Figure 14 – Iso planar machining paths .............................................................................................................. 21
Figure 15 – Jerk continuous trajectory command generation profile ...................................................................... 23
Figure 16 – Exponential feed generation profile .................................................................................................. 24
Figure 17 – 3-axis corner smoothing of sharp corner ............................................................................................. 26
Figure 18 – Four subsequent NC blocks [60] ......................................................................................................... 26
Figure 19 – Time shifting of motion blocks [60] ..................................................................................................... 27
Figure 20 – Fixed feed profiles for continuous block transitions [60] ................................................................. 27
Figure 21 – Parameters define the local frame ....................................................................................................... 29
Figure 22 – Side and rear gouging considering a flat-end cutter ............................................................................ 30
Figure 23 – The effective cutting shape (ellipse) .................................................................................................... 30
Figure 24 – Three adjacent cutter postures with the chord error ......................................................................... 32
Figure 25 – The two different cases considering of effective cutting width of flat-end milling .......................... 34
Figure 26 – Discrete the machine based energy potential field in the uv domain .................................................. 37
Figure 27 – The way to generate the first expanded curve according to the traditional iso-scallop height expansion rules .................................................................................................................. 39
Figure 28 – Three expansion groups of cutter contact curves that mantle the whole surface domain .................. 40
Figure 29 – The four views of the target stock and surface ................................................................................... 41
Figure 30 – The tool path illustration of default solution from EdgeCAM ............................................................ 42
Figure 31 – Tool path simulation figure of the default case .................................................................................... 43
Figure 32 – Rough milling illustration of linear path only optimization solution (RM module) ............................ 44
Figure 33 – Profile milling illustration of linear path only optimization solution (PM module) ............................ 44
Figure 34 – Machining illustration of linear path only optimization solution based on left view of the stock.........................................................................................................................45
Figure 35 – Tool path simulation figure of the linear only optimization case........................................45
Figure 36 – Tool path illustration of iso scallop optimization solution (CM module)......................46
Figure 37 – Tool path simulation figure of the scallop optimization case ........................................46
Figure 38 – Tool path illustration of combination of RM module & CM module (iso scallop) ..........47
Figure 39 – Tool path simulation figure of combination of RM module & CM module (iso scallop) case ............................................................................................................................................................................48
Figure 40 – Tool path illustration of optimization of method 1 (rough milling module optimization). .49
Figure 41 – Tool path simulation figure of optimized method 1 case .................................................50
Figure 42 – Linear method surface ..................................................................................................51
Figure 43 – Linear +3D method surface ..........................................................................................52
Figure 44 – Three different NC programs ......................................................................................53
Preface

Since the industrial revolution, many manufacturers need to enhance their production efficiency in order to increase their competitiveness. Because CNC machine is the Irreplaceable production tool of the manufacturing, so improving the production efficiency of CNC machine is the main way to improve the competitiveness of the entire manufacturing company.

This master thesis will focus on three-axis CNC machine and the research direction is free surface operation. The three-axis machine is widely applied in machining free form surface parts. Many researchers have done several researches on free form surface operation, such as tool path optimization methods, energy consumption optimization, prediction methods etc. In the second part of this master thesis, there will be a detailed research report about three different main types of tool path optimization algorithm, which are iso parametric, iso planar and iso scallop, as well as prediction model and also including the optimization of energy consumption etc. In the case study part, a free form surface will be machined on a metal rectangle to test the effect of different tool path optimization methods.

In the process of making my master thesis, I received lots of help from different people. I would like to say thank you to my supervisor Gabor Sziebig and co-supervisor Sibul Lazar, I appreciate their selfless help and patient.

Dingjun Liu

------------------------------------------
Abstract

Nowadays, many manufacturing companies need to face competition both in domestic and international level. Due to this circumstance, manufacturers recognize that one useful method to enhance their competence is reducing the lead-time of manufacturing.

There are many researchers focused on minimize the time of actual cutting, tool path optimization and tool change time optimization etc. However, few researchers have investigated the operation optimization of CNC machine that integrating multiple ways to reduce the operation time of freeform surface cutting. For example, integrating tool path calculation optimization, prediction method and machining parameters optimization together for the sake of providing an optimization solution package to enhance the efficiency of manufacturing.

This master thesis will investigate the main optimization methods of tool path (Iso parametric, Iso planar and Iso scallop), as well as prediction of operation time and energy consumption optimization. Then we will provide several approaches for machining a surface on a metal rectangle to research the effect of different tool paths generation such as Iso parametric and Iso scallop etc. depend on NC code on machining time and quality.


1 Introduction

Three-axis machine is widely used in manufacturing parts within the world. The manufacturers are committed to achieve the highest production efficiency so as to enhance their competitiveness. To improve the CNC machines’ production efficiency, we need to first know what is CNC machine and understand its working principle.

Today CNC machines have replaced manual machines that all paths and movements can be programed and controlled by computer and codes, which is more convenience than by hand, as well as decrease the total operation time and avoiding human errors [1]. CNC machine can also significantly increase the productivity by integrating Computer Aided Design (CAD), Computer Aided Manufacturing (CAM) and Numerical Control (NC), which are described in the below Fig.1. Normally, the G code can be generated by CAD/CAM packages automatically from the specific software. However, the parameters such as tool path, feed rate and tool selection are obtained automatically from the program of the software that is not the optimized solution. Due to this circumstance, we will find some place to insert in our optimization program after analysis.

![Diagram of Manufacturing process: CAD-CAM-PP-CNC](image)

CNC machine (Computer Numerical Control) is a sort of production tool that encompasses different types of machines with variety of shapes, sizes and functions [2]. CNC machines can be divided into two distinct types, which are turning machines and milling machines. A turning machine is generally used to remove materials from the workpiece by spinning the workpiece at a high speed and then use the sharp edge of cutting tool to achieve the desired form [3]. A milling machine is a kind of machine that holds the workpiece with a clamp and then remove the materials to get the need shape with a special high-speed rotation cutting tool to spin and cut in many directions and move in three distinct directions along the x, y and z axis [4].
Fig. 2 shows the 4-axis CNC machine, three moving directions of a real CNC machine (3-axis) and CNC Lathe. The difference between those three types of CNC machine can be seen obviously. 3-axis CNC machine has no function of rotation in any axis. However, cutting tool can move in x, y and z directions to produce the required free-form surface. Three-axis CNC machine is not the most advanced machine nowadays, but it has variety of advantages such as high stability, simple operation etc. so that we decided to focus on 3-axis CNC machine.

Because at present there is an increasing demand of complex parts with aerodynamic shapes. Therefore, this master thesis will mainly focus on researching sculpture free-form surfaces with 3-axis Computer Numerical Control machine. In this master paper, an efficient methodology to calculate tool-moving path in order to minimize the total operation time will be conducted, as well as other methods that can contribute to optimize the operation will be investigated in the main chapter.

This master thesis will be organized as follow: chapter one will brief introduce the CNC machine, as well as the background of the research, which includes three algorithms in machining surface, prediction methods and tool path generation according to energy consumption. These three research fields will be discussed detailed in the chapter two with three distinct sub-chapters. Finally, some of the research content will be utilized in the case study part in the chapter three for the sake of researching the effects of different tool path on surface machining.

1.1 Background

In order to achieve the highest production efficiency, there are plenty of methods that can help manufacturers, such as optimizing the energy consumption, tool path generation methods and prediction of operation cycle time etc. This master thesis will mainly focus on sculpture free form surface machining. These surfaces are usually produced by three-axis CNC machine by using ball-end tools.

In this report we will use the sort of ball-end milling of surface machining and then discuss about plenty of possible methods that can effect the total operation time such as three algorithms (iso
parametric, iso planar and iso scallop) in machining free form surface, optimization of energy consumption and prediction method during the machining process.

1.1.1 Three Algorithms in Machining Surface

There is variety of algorithms for three-axis tool path generation that has been researched, among those methods the three most popular algorithms in machining free form surface adopted in practice are the iso planar algorithm [5-9], the iso parametric algorithm [10-12] and the iso scallop algorithm [13-21]. Each of these algorithms has its own calculation methods and characteristics.

If we discuss about the first two tool path algorithms, the iso parametric algorithm can only be used to parametric surface but the iso planar algorithm has no restriction as iso parametric method. Either of them is able to calculate a tool path that shows the good surface finishing performance. However, overlap always occurs between the machining areas of adjacent CC curves on the surface if applying these two algorithms to generate tool path, sometimes it will lead to cost more machining time when severe on complicated surfaces. Iso scallop algorithm can eliminate the overlap cause it will start from an initial CC curve and then create the CC curves continuously so that any two neighboring CC curves can be maintained. By using this method, the overlap can be reduced dramatically.

![Iso parametric machining path](image)

*Figure 3 – Iso parametric machining path*
Fig. 3, 4 and 5 are schematic illustrations of the iso parametric machining path, iso planar machining path and iso scallop machining path. It can be seen from these figures that different machining method has its own features and ways to generate the tool path. Iso parametric method is selecting one of the surface parameters as the forward direction (it is $u$ in Fig. 3) and the initial path will be another parameter $v$ [22], while iso planar method captures the intersection between the free form surface and a parallel vertical planes as the CC paths [23]. The process of generating the tool path with iso scallop method is more complicated than the other two algorithms. The CC path (cutter contact) performs a tangential trajectory of the ball end machining and the free form surface. In the case of 3D surface machining, it is necessary to generate an offsetting surface in the normal direction with a distance equal to the cutter radius so as to get the CL path that is shown in Fig. 5 [24].
In the chapter 2 there will be a more specific explanation of these three tool path generation algorithms with illustration and calculation.

1.1.2 Prediction of Part Machining Times

The purpose of digital engineering is to simulate the operation systems by researching the corresponding mathematical models based on physical principles. This prediction of part machining times model can predict the structural dynamic behaviour of machine tools by finite element and multibody dynamics methods [25]. The interaction between the structure and manufacturing processes is modelled by feeding back the resulting deflections to the process, predicting the process forces and applying them on the machine structure [26]. The process forces and optimal cutting conditions can be predicted in a virtual model of machining part operations ahead of pricy physical trials [27]. All the methods that mentioned above are important in designing better performance machines and manufacturing operations, the actual machining time of the part is essential in designing and selecting cutting tools to machine specific part geometry, specially in the aerospace industry that the physical test are prohibitive because of the high costs of the parts.

Figure 6 – Action order processing in CNC systems

The total operation time of the part in not only decided by the feeds commanded in the NC program but also by the CNC machine cutting tool’s ability. The machining cycle times are predicted by the NC programs that are never accurate due to the CAM systems do not consider the rigid body dynamics of the machining tool. In order to obtain the accurate prediction of cycle time, it should be processing the part’s NC tool path by the real CNC of the machine or its own simulation model. Despite of these, it is
impossible to copy the commercial CNC’s interpolation, smoothing, trajectory generation, compensation and control algorithms, which are hidden in the CNC software.

There are several essays about prediction of machining time from NC programs. The travel time of circular and linear paths are estimated by the path lengths [28] and transition directions between them with the deceleration and acceleration constants of the machine [29,30].

The reference [31] has developed a five-axis research of CNC system in real that is used to validate different smooth trajectory generations, interpolation, active vibration damping and high speed tracking control of feed drives. When the physical machine’s drives are changed by their closed loop transfer function blocks, the corresponding CNC changes to a Virtual CNC, which can predict the exact cycle time of a part [32].

In chapter 2 of this master thesis the cycle time prediction model, which is mainly determined by the trajectory module of the CNC that can be decided by acceleration, velocity and jerk elements and limits of the machine will be researched. The trajectory profiles can be obtained from the CNC manufacturer or simple linear motion test. The path is treated by the trajectory generation module, which contains kinematic configuration of the cutting tool. The discrete position orders are generated from the trajectory profiler through the path that determines the cycle time.

1.1.3 Tool Path Generation Regard to Energy Consumption

A typical three axis machining process includes three independent parts. In the computer-aided manufacturing (CAM), a tool path can be generated on different strategies in the workpiece coordinate system (WCS), which can setup virtually aligned and fixed on the operation table. Then at the computer numerical control (CNC) part, the tool path can be transformed into the machine coordinate system (MCS) by inverse kinematics transformation (IKT) and obtains a part program such as G code part program, in which the feedrate is adjusted per the machine’s kinematic capacity by the controller. At the final stage, the cutting stage, the part program is conducted and the energy is consumed.

In this paragraph, the energy consumption and related works will be introduced. In order to improve the energy efficiency for a given machining process, the investigation of relationship between the energy consumption and machining parameters. It is important to realize the main contributors to the energy among all the relevant parameters. In reference [33] has made an overall review of existing energy consumption models and found that the cutting process contributes the main energy consumption, which is highly related to the cutting parameters and material removal rate. Reference [34] made a complete evaluation for various machine tools and made a conclusion that the idle power can take about 50% of the total power, which consumes more energy than needed. Reference [35] suggested an empirical way to calibrate the energy consumption model according to their models and they found that material removal rate (MRR) will result in a significant energy saving and cutting in a dry condition is more efficient than in a wet condition. Reference [36] established a model efficiency and specific energy as a unary intention of various parameters and they found that a given set of parameters could decide the specific energy. Energy reduction was researched by [37], with the power pattern for the X, Y and Z axis that is got to be linearly to the feedrate in a certain proportion. In mention to the feedrate, reference [38] has compared the average energy consumption between distinct feedrate and they found that either small or large feedrate could cause high total energy consumption.
Finally, they suggested a medium feedrate that can save about 25% of the total energy cost. Furthermore, such as [39] focused on optimizing different machining parameters by using distinct cutting tool for the sake of reducing energy consumption. Reference [40] also introduced a prediction model that can provide more accurate result of energy consumption by analysing the effect of the feedrate, spindle speed and cutting depth. The connection between the particular power, cutting width and cutting height is studied in [41]. In order to reduce the plunging energy, the relationship between particular cutting energy and cutter swept angle is investigated in reference [42]. Recently, a research for the purpose of minimizing the energy consumption was conduct in [43]. Other investigations such as [44] and [45] have done various production planning methods for the sake of making the control process more efficient.

In the chapter 2 of this master thesis, more details about energy consumption optimization will be introduced in the third sub chapter and it will be organized as follows. Firstly, an energy potential field on the specific surface will be researched, and then an energy consumption model will be build in order to obtain the quotient of energy consumption over the swept area. Sequentially, two essential parameters that used to determine the amount of total energy consumption will be calculated as well. Finally, the optimal feed direction and principle curve generation will be mentioned for the sake of optimize the whole machining process.
2 Research of Optimization Methods

In this chapter, three parts such as tool path algorithm optimization, prediction of operation cycle time and minimizing of energy consumption that related to optimize the machining would be researched for the sake of better balancing between the surface operation performance and machining time. This second chapter is including three axis and five axis CNC machine with ball end and flat end cutting tool in order to cover all the processing situation as much as possible in this not long master thesis.

The three tool path algorithms focus on optimizing the tool path by calculation in mathematical way and try to obtain a theoretical value that can be executed in the real operation, which will be conduct in the case study part as well. The methods in this master thesis have some limit in the real application due to the optimization method is based on the G code. In the G code optimization process, we can only adjust the cutting spacing, which will be calculate by using distinct algorithms. Furthermore, the surface in the real operation at CNC machine is a plane rather than a free form surface. The part that this master thesis did not contain will be accomplished in the future.

The prediction of machining cycle time is the method to predict the total operation time of the part with action order processing steps that has been mentioned in Fig.6. The prediction model will utilize the trajectory generation and corner smoothing models to provide a high accuracy result. In this part, the trajectory generation profiles will be introduced as the key function of the CNC and 3-axis corner smoothing will be mentioned in this part as well.

The third part is about the energy consumption model to find out the most efficient energy cost way, which can also be a part of optimization solution for machining. It should be mentioned that this part research is based on 5 axis CNC machine with the flat end cutter in order to cover a more comprehensive range of research. This part will contain a detailed and exhaustive explanation such as pre determination of tool orientation, establishment of energy consumption model, optimization of feed direction, principle curve generation and expansion algorithm based on Iso-scallop height. At the end of this part there will be a brief conclusion of this method.

The tool path algorithm optimization including iso parametric, iso planar and iso scallop will be the main part of utilization in the real operation. The prediction part should be a theoretical tool for the purpose of predicting the total operation cycle time and the tool path generation regards to energy consumption optimization will be used as a theoretical basis in future research work.

2.1 Tool Path Algorithm Optimization

Producing a part with free form surface is one of the most important technologies that are widely utilized in CAD/CAM software. In order to cut the free form surfaces, ball end cutting tool is the most popular type of tool that is utilized in three axis CNC milling machines. In the current approach, the CAM software response for scheduling the CC (cutter contact) path over the free form surface, and then calculate out their offset curves, which is the CL (cutter location) path [46, 47]. Fig.7 shows the differences between CC (cutter contact) path and CL (cutter location) path, as well as the location and direction of normal vector, which will be introduced in the following part. As Fig.7 described, the CC
path represents the contact or intersection point between the cutter edge and the free form surface, while the CL path denotes the path that made up by a mount of consecutive linear sectors of the centre of the ball end cutter.

![Diagram](image)

*Figure 7 – Illustration of CC path, CL path, Tool axis vector and Normal vector.*

Table.1 is the nomenclature of calculation in the following section. It should be noted that some of the parameters in the table.1 are illustrated in Fig.7 as well for the sake of better understanding of the parameters.

**Table 1 - Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Unit vector in the side-step or path-interval direction</td>
</tr>
<tr>
<td>C</td>
<td>Cutter-contact path</td>
</tr>
<tr>
<td>h</td>
<td>Scallop-height limit</td>
</tr>
<tr>
<td>L</td>
<td>Cutter-location path</td>
</tr>
<tr>
<td>M</td>
<td>Unit normal vector to the planes in iso-planar machining</td>
</tr>
<tr>
<td>N</td>
<td>Unit normal vector to the surface</td>
</tr>
<tr>
<td>r</td>
<td>Radius of the ball-end cutter</td>
</tr>
<tr>
<td>S</td>
<td>Parametric surface</td>
</tr>
<tr>
<td>T</td>
<td>Unit tangent vector in the CC path direction</td>
</tr>
<tr>
<td>t</td>
<td>Spatial parameter along the CC path</td>
</tr>
</tbody>
</table>
### Table

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>u–v curve in the parametric domain</td>
</tr>
<tr>
<td>$u, v$</td>
<td>Surface parameters</td>
</tr>
<tr>
<td>$V$</td>
<td>Feedrate along the CC path</td>
</tr>
<tr>
<td>$\Delta l$</td>
<td>Distance of the side step</td>
</tr>
<tr>
<td>$\Delta m$</td>
<td>Step distance of the planes in iso-planar machining</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Radius of surface curvature in the side-step direction</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Sampling period</td>
</tr>
</tbody>
</table>

In order to generate the tool path, it needs to first define the free form surface, and then choose the algorithm to generate the parametric curve. After which the cutting tool will be offset depends on the surface geometry and the cutting tool’s radius. Finally the CC path will be calculated by three different algorithms, which are iso parametric, iso planar and iso scallop. The following sections will describe all the steps in a more detailed way.

#### 2.1.1 Definition of the free form surface

In this master thesis, the free form surface will be machined in the case of ball end milling that can be defined as:

$$ S = S(u, v), $$

where $u$ and $v$ are the surface parameters that are shown in Fig. 8, notice that the scope of the $u$ and $v$ domain ($u_{\min} \leq u \leq u_{\max}$ and $v_{\min} \leq v \leq v_{\max}$).

![Figure 8 – Iso parametric machining paths](image)

---

Page 12 of 89
2.1.2 CC path scheduling algorithm and cutting tool offsetting

The algorithm for the CC path interpolation and the cutting tool offsetting will be explained in this section and it should be mentioned that these algorithms are essentially derived from the current methods [48].

2.1.2.1 CC path scheduling algorithm

The CNC system has the sampling rate function that can create a sequence of CC dots in order to follow the CC path at a specific CC velocity and feedrate. At present there are many algorithms about the parametric curve generation have been researched. One of these researched algorithms will be used [49]. Before the calculation of CC path interpolation, a $C(t)$, where $t$ is the spatial parameter that denotes CC path on the free form surface should be introduced. It should be noted that $t = u$ or $v$ for iso parametric machining. Then let $t_{i+1}$ and $t_i$ to be the amounts of the path parameter $t$ at two continuous sampling instants, $(i + 1)\tau$ and $i\tau$, where $\tau$ is the sampling time. Then:

$$t_{i+1} = \phi t_{i+1}^* + (1 - \phi) t_i, \tag{2}$$

Where,

$$t_{i+1}^* = 2.5 t_i - 2 t_{i-1} + 0.5 t_{i-2}, \tag{3}$$

$$\phi = \frac{\tau V}{|C(t_{i+1}^*) - C(t_i)|}, \tag{4}$$

where $V$ is the specific CC velocity or feedrate. It should be mentioned that at the beginning of the CC path, $t_{-1}$ and $t_{-2}$ need to be calculated by:

$$t_{-1} = t_0 - \frac{\tau V}{|dC/dt|_{t=t_0}}, t_{-2} = t_{-1} - \frac{\tau V}{|dC/dt|_{t=t_{-1}}} \tag{5}$$

Then parameter $t$ can be calculated by using Eqs. (2)-(5), recursively, at each sampling rate in order to get the CC point.

2.1.2.2 Tool offsetting

The cutting tool’s radius and the surface geometry can decide the cutter offsetting. $L$ is defined as the centre of the ball end cutter at the location of cutter:

$$L = C + r \cdot N \cdot sign(N_z), \tag{6}$$
where \( r \) is the radius of the cutting tool, \( N \) is the unit normal vector at point \( C \) to the surface, \( N_z \) is the component of \( N \) at \( z \) axis and \( \text{sign}(N_z) \) is the sign function that can keep the tool offsetting on the top side of the free form surface all the time. Then unit normal of the free form surface can be calculated by:

\[
N = \frac{\frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v}}{\left| \frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v} \right|}.
\]  

(7)

### 2.1.3 Three algorithms of CC path scheduling

In this part the three different algorithms of CC path scheduling, which are iso parametric, iso planar and iso scallop machining methods, respectively will be discussed and then find out the best proposal for machining free form surface.

#### 2.1.3.1 Iso parametric machining

In the previous section, Fig.8 has described the iso parametric path and introduced the values that should be used in the calculation. In this part, attending to select one of the surface parameters \( u \) as the forward direction, therefore another boundary curves, which is \( v = v_{min} \) will be the initial CC path. Let \( k \)th CC path can be expressed by \( C_k(u) = S(u, v_k) \). It should be noted that the curve \( v = v_k \) in the domain that constitute by \( u \) and \( v \) according to the CC path \( C_k \) in the \( x\text{-}y\text{-}z \) domain (Cartesian domain). The value of the side parameter can be decided one by one,

\[
i.e., \quad v_{k+1} = v_k + \Delta v_k,
\]

where \( \Delta v_k \) (the parametric side interval between two neighboring CC paths) can be determined depends on the scallop height limit, \( h \) (generally from 0.001 mm to 0.01 mm).

In the general case, the CC path of iso parametric algorithm does not correspond to a constant scallop height \( h \) and \( \Delta v_k \). For this reason, the maximum scallop height on the CC path will not exceed \( h \).

The calculation of \( \Delta v_k \) can be executed on line the generation of the \( k \)th CC path. During this generation process, every sampled point, which is \( C_{i,k} = C_k(u_i) \), are evaluated in order to get a corresponding value, \( \Delta v_{i,k} = \Delta v_k(u_i) \). At the final point of the \( k \)th CC path, the minimum value of these corresponding values has chosen, i.e., \( \Delta v_k = \min(\Delta v_{i,k})'s \). According to the \( v_{k+1} = v_k + \Delta v_k \), the next CC path can be settled consequently. The formulas that can calculate the corresponding value, \( \Delta v_{i,k} \), are shown as follows.

Given a sample point on the parametric surface \( C_{i,k} = S(u_i, v_k) \), the radius of curvature in the side direction \( \rho \) need to find first, which can be calculated by [50]:

\[
\rho = \frac{|\frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v}|}{\left| \frac{\partial S}{\partial u} \right|}.
\]
\[ \rho = \frac{e\alpha^2 + 2f\alpha + g}{a\alpha^2 + 2b\alpha + c}, \]  

(8)

Where,

\[ a = \frac{\partial S}{\partial v} \frac{\partial S}{\partial u^2}, e = \frac{\partial S}{\partial u} \cdot \frac{\partial S}{\partial u}, f = \frac{\partial S}{\partial v} \cdot \frac{\partial S}{\partial v}, g = \frac{\partial S}{\partial u} \cdot \frac{\partial S}{\partial v}, \]

\[ a = \frac{\partial^2 S}{\partial u^2} \cdot N, b = \frac{\partial^2 S}{\partial u \partial v} \cdot N, c = \frac{\partial^2 S}{\partial v^2} \cdot N, \]

where \( N \) is the unit normal vector to the parametric surface and \( T \) is the unit tangent vector on the CC path direction. Then \( T \) will be obtained since the tool path is used in the \( u \) direction:

\[ T = \frac{\frac{\partial S}{\partial u}}{\left| \frac{\partial S}{\partial u} \right|}, \]

then, side step distance \( \Delta l \) can be calculated for each evaluated point [51]:

\[ \Delta l = \frac{8\rho rh}{\rho \pm r}, \]

(9)

where \( h \) is the scallop height that has introduced in the previous description, \( r \) is the cutting tool radius, the plus minus sign depends on the case of the surface shape is convex or concave that is illustrated in Fig.9.

![Convex and concave surface](image)

Figure 9 – Convex and concave surface

It should be noted that the CC path direction \( T \) and the surface normal \( N \) are orthogonal to the side step direction. Since \( \Delta l \) is in mm unit distance and it is generally not in the \( v \) direction, a transformation from \( \Delta l \) to the parametric side \( \Delta v \) is necessary.
This transformation process is described in Fig. 10. Depends on the geometrical relationship that illustrate in Fig. 10,

\[ \Delta l = B \cdot \left( \frac{\partial S}{\partial v} \right) \Delta v, \]

where,

\[ B = N \times T, \]

where B is a unit vector in the side direction. In the end part, the corresponding path interval \( \Delta v_{i,k} \) for the \( i \)th sampled point on the \( k \)th path can be obtained by:

\[ \Delta v_{i,k} = \frac{\Delta l}{(N \times T \cdot |\frac{\partial S}{\partial v}|)}, \]

By using the CC path scheduling algorithm and tool offsetting that have described above, as well as the iso parametric algorithm presented in the above section, the iso parametric algorithm can be implemented in a CNC machine tool. More details and the process of calculation are shown in appendix.

### 2.1.3.2 Iso scallop machining

The iso scallop machining path is shown in a schematic illustration in Fig. 11. The scallop height is produced by two neighboring CC path that equal to the assigned limit \( h \). It can be seen from the figure.
below that each CC path $C(t)$ has a corresponding specific curve $U(t)$ in the domain made up by $u$ and $v$. In the general case, the curve does not follow a constant $u$ and $v$.

Figure 11 – Iso scallop machining path

There is a existing method that generate the $(k+1)$th CL or CC path based on the curve fitting of the increment points $x_{k+1}, y_{k+1}, z_{k+1}$ from a set of chosen points $x_k, y_k, z_k$ on the $k$th path. The 3D curve is fitting [51]:

$$L(t) = (x(t), y(t), z(t)),$$

where $t$ is the time consuming of the path parameter. Furthermore, in order to obtain sufficient position accuracy, a lot of points need to be evaluated, as well as several spine segments to fit a machining path. Therefore a proposed method is suggested that makes the machining path by 2D curve $U(t) = (u(t), v(t))$ in the parameter domain. It is obvious that the 2D curve fitting for $U(t)$ is uncomplicated for calculation compares with $L(t)$. Fig.12 compares the existing and the proposed methods for the machining path generation.
In addition, the proposed method does not need tolerance or allowable error to fit $U(t)$. The reasons of this are:

1. Even though an inaccurate value is applied into the surface function, the CC point is still located on the surface because of 2D curve.

2. The parameter error can only result in the deviation in the scallop height, which is extremely less than the path interval.

As same as the iso parametric method, the surface parameter $u$ is chosen as the forward machining direction. Therefore another boundary curves, which is $v = v_{min}$ will be the initial CC path and the initial curve is $U_0(t_0) = (u(t_0), v_{min})$ in the domain that consist of $u$ and $v$. In order to explain it in a more simplicity way, the initial path parameter need to be defined as $t_0 = u$, which can also represented as $U_0(t_0) = U_0(u)$. So that $U_k(t_k) = (u_k(t_k), v_k(t_k))$ means the $k$th curve in the $u - v$ domain and the two components $u_k(t_k)$ and $v_k(t_k)$ express $t_k$ through curve fitting as polynomials. Then in consequent, the $k$th CC path becomes to $C_k(t_k) = S_k(U_k(t_k))$ and a cubic spline is used to fit $U(t)$ for a exact CC path because a tight curve fitting for $U(t)$ is not required.

As mentioned in the above section and the illustration in Fig.12 (b), $U_{k+1}(t_{k+1})$ is obtained based on $U_k(t_k)$. In the existing method, four set of $(U_{k+1}, t_{k+1})$ should be calculated to fit the curve $U_{k+1}$. While the proposed method will finish this process by generating the $k$th and the $(k+1)$th CC path. In addition, the four sets path parameter are selected as:

$$t_k = t_k^e + j(t_k^e - t_k^f)/3, \ (j=0,1,2,3),$$
where $t_k^e$ and $t_k^s$ are the end and the start of the path parameter for $U_k$. For each of the four sets, they have:

$$U_{k+1} = U_k + \Delta U_k,$$

where $\Delta U_k = (\Delta u_k, \Delta v_k)$.

Let $t_{k+1} = t_k$ to define the path parameter for the subsequent path. Then the key issues can be focused to calculated in order to determine an iso scallop machining path, which are the increment of the surface parameters $(\Delta u_k, \Delta v_k)$, as well as the end and the start of the path parameter $t_k^e$ and $t_k^s$.

After the explanation, the proposed algorithm can be started. Given a set of $(U_k, t_k)$, the first step is to calculate $N$ (the unit surface normal) and $T$ (the tangent vector). $N$ can be obtained by using Eq. (7) and $T$ can be calculated by:

$$T = \frac{\frac{dS}{dt}}{|\frac{dS}{dt}|} = \frac{\frac{\partial S}{\partial u} \frac{du}{dt} + \frac{\partial S}{\partial v} \frac{dv}{dt}}{\left| \frac{\partial S}{\partial u} \frac{du}{dt} + \frac{\partial S}{\partial v} \frac{dv}{dt} \right|},$$

(11)

Then $\Delta l$ (the side step distance) can be calculated by utilizing a given scallop height $h$ based on Eqs. (8) and (9). The transformation from $\Delta l$ to the increment of the surface parameters $(\Delta u_k, \Delta v_k)$ can be obtained by:

$$\Delta l B = \frac{\partial S}{\partial u} \Delta u_k + \frac{\partial S}{\partial v} \Delta v_k,$$

(12)

where $B = N \times T$ is the unit vector in the side step direction.

In the calculation process, the end and the start path parameter $t_k^e$ and $t_k^s$ for $U_k$ are correlating to the intersections of the curve $U_k$ and $u = u_{\min}, u = u_{\max}, v = v_{\min}$ and/or $v = v_{\max}$ (the boundaries of the parametric domain). There are some numerical methods that have been researched to have these intersection points [52]. In this master thesis, a more advance and fast algorithm will be researched for determining $t_k^e$ and $t_k^s$ and the proposed algorithm is introduced in the following.
Figure 13 – Schematic description for determining $t^s_k$

Fig. 13 is the schematic description for determining $t^s_k$. As the explanations above, $U_k(t_k)$ is an iso-scallop increment curve that is obtained from $U_{k-1}(t_{k-1})$. The initial parameter set $U_{k-1}(t^s_{k-1})$ is defined that has a corresponding increment parameter set $(u^*_k, v^*_k)$. As shown in the Fig. 13 (a), $t^s_k$ is corresponding to the intersection of $U_k$ and $u = u_{min}$. Therefore, $(u^*_k, v^*_k)$ and $(u_{min}, v^s_k)$ are two adjacent point on $U_k$. Then $t^s_k$ can be calculated by:

$$t^s_k = t^*_k - \frac{u^*_k - u_{min}}{d u_k / d t_k (t_k = t^*_k)},$$

(13)

where $t^*_k = t^s_{k-1}$. Eq. (13) gives us a good method to find approximation value for $t^s_k$, it can be approached to the real solution additionally by changing $t^*_k$ by the presently calculated $t^s_k$ and repeating the calculation through Eq. (13). It should be mentioned that Fig. 13 (b) illustrate the value $t^s_k$ may be corresponding to the junction of $U_k$ and $v = v_{max}$. In this circumstance, Eq. (13) should be changed by:

$$t^s_k = t^*_k - \frac{v^*_k - v_{min}}{d v_k / d t_k (t_k = t^*_k)},$$

(14)

The algorithm for obtaining the end of the path parameter $t^s_k$ is as same as $t^s_k$. 

---

Page 20 of 89
By using the CC path scheduling algorithm and tool offsetting that have described above, as well as the iso scallop algorithm presented in the above section, the iso scallop algorithm can be implemented in a CNC machine tool. More details and the process of calculation are shown in appendix.

### 2.1.3.3 Iso planar machining

The iso planar machining path is shown in a schematic illustration in Fig.14. It can be seen from the figure that the CC path are obtained form the intersections of a series of parallel vertical planes and the parametric surface.

![Iso planar machining](image)

**Figure 14 – Iso planar machining paths**

In this master report, the unit normal vector perpendicular to the vertical planes that can be denoted by \( M = (m_x, m_y, 0) \), and the distance between two close parallel planes is represented by \( \Delta m \) as shown in Fig.14 as well. The proposed algorithm for the iso planar is as same as the iso scallop method, which can be seen from Fig.10 and Fig.14 that each CC path \( C(t) \) corresponds to a unique curve \( U(t) \) in the parametric domain. For instance, \( C(t) = S(U(t)) \).

In the processing, \( U(t) \) can be obtained recursively by:

\[
U_{k+1} = U_k + \Delta U_k,
\]

where \( \Delta U_k = (\Delta u_k, \Delta v_k) \).

The main difference between the iso scallop approach and iso planar algorithm is the method to calculate the parameter increment, which is \( (\Delta u_k, \Delta v_k) \).

The \( k \)th curve is defined as \( U_k(t_k) \), and then the iso planar increment curve can be obtained:

\[
U_{k+1}(t_{k+1}),
\]
where \( t_{k+1} = t_k \).

For two adjacent points on \( U_k \) and \( U_{k+1} \), the related CC points \( C_k = S(U_k) \) and \( C_{k+1} = S(U_{k+1}) \) are both placed on the surface. The different vector between these two points, which is \((C_{k+1} - C_k)\) can be obtained approximately by:

\[
\frac{\partial S}{\partial u} \Delta u_k + \frac{\partial S}{\partial v} \Delta v_k ,
\]

In the geometrical consideration, this distinction vector is placed on a cross section that is developed by the side vector \( M \) and the tool axis vector \( Z \). Therefore,

\[
\frac{\partial S}{\partial u} \Delta u_k + \frac{\partial S}{\partial v} \Delta v_k = \Delta m M + \Delta z Z ,
\]

(15)

Depends on the components \( x \) and \( y \), \( \Delta u_k \) and \( \Delta v_k \) can be solved, and then \( U_{k+1} = (u_k + \Delta u_k, v_k + \Delta v_k) \) can be obtained.

The same as the above section and the illustration in Fig.12 (b), \( U_{k+1}(t_{k+1}) \) is obtained based on \( U_k(t_k) \). In the existing method, four set of \((U_{k+1}, t_{k+1})\) need to be calculated to fit the curve \( U_{k+1} \). While the proposed method will finish this process by generating the \( k \)th and the \((k + 1)\)th CC path. In addition, the four sets path parameter are selected as:

\[
t_k = t_k^5 + j(t_k^e - t_k^5)/3, \quad (j=0,1,2,3),
\]

where \( t_k^e \) and \( t_k^5 \) are the end and the start of the path parameter for \( U_k \). For each of the four sets, they have:

\[
U_{k+1} = U_k + \Delta U_k ,
\]

where \( \Delta U_k = (\Delta u_k, \Delta v_k) \).

An initial curve \( U_0 \) should be created first in the case of this value does not correspond to a boundary of the \( u - v \) domain for the iso planar scheduling. There is an uncomplicated method that can get \( U_0 \). Lets say there exist four representative points on four directions from the left bottom corner \((u_{min}, v_{min})\) on \( U_0 \), and the four directions are 0\(^\circ\), 30\(^\circ\), 60\(^\circ\) and 90\(^\circ\), respectively. The original CC path, \( C_0 = S(U_0) \), is placed on a vertical plane and deviate from the surface corner \( S(u_{min}, v_{min}) \) by the distance of \( \Delta m M \). Correspondingly, all the points on \( U_0 = (u_0, v_0) \) should satisfy:

\[
\left[ \frac{\partial S}{\partial u}(u_0 - u_{min}) + \frac{\partial S}{\partial v}(v_0 - v_{min}) \right] \cdot M = \Delta m ,
\]

(16)

Therefore, \((u_0, v_0) = (\lambda, 0), [(\sqrt{3}/2)\lambda, (1/2)\lambda], [(1/2)\lambda, (\sqrt{3}/2)\lambda] \) and \((0, \lambda) \) can be inserted into Eq. (16) in order to obtain the four solutions.

By using the CC path scheduling algorithm and tool offsetting that have described above, as well as the iso planar algorithm presented in the above section, the iso planar algorithm can be implemented in a CNC machine tool. More details and the process of calculation are shown in appendix.
2.2 Prediction of Part Machining Times

The prediction model is introduced in this sub chapter based on researching of trajectory generation profiles and corner smoothing algorithm, which can determine the total cycle time. The trajectory module will be divided into acceleration, velocity and jerk in order to better explain the influence of these parameters. This prediction model will give us the theoretical basis of prediction field in the future utilization.

2.2.1 Trajectory Generation

The action order processing in a CNC system is mentioned before in Fig.6. The G code, which is the main optimization part in this master thesis that belongs to the NC program, can be parsed into linear, circular and spline path section. The total travel distance \( L \) for every path section can be calculated and divided into acceleration, constant feed and deceleration area that are shown in Fig. 15. The discrete displacement will be calculated through the path, which is a function of the trajectory profile at continuous interpolation time intervals \( T_{\text{int}} \) and then dissolve the constant interpolation time intervals into axis position orders. The function is implemented by the interpolator functions and sent the information to drive servo controllers over the cutting tool’s inverse kinematic module.

![Figure 15 – Jerk continuous trajectory command generation profile](image)
There is a replacement of a fifth order polynomial function of time when the CNC system attend to maintain the continuous velocity, acceleration and jerk profiles that is shown in Fig. 15. The less proportion content can be obtained is the acceleration and jerk are smooth and this will decrease the vibrations during high-speed contour operation [31].

The most CNC machine system has a double exponential feed profile that is illustrated in Fig.16. The three time zones including acceleration, constant feedrate and deceleration can be expressed in the mathematical way as following:

\[
\begin{align*}
\frac{f_s - f_c}{T_1} &= T_2 (T_1 e^{-(t/T_1)} - T_2 e^{-(t/T_2)}) + f_c , & t \in [0, t_1) \\
\frac{f_c - f_e}{T_1} &= T_2 (T_1 e^{-(t-t_1-t_2)/T_1} - T_2 e^{-(t-t_1-t_2)/T_2}) + f_e , & t \in [t_1 + t_2, t_1 + t_2 + t_3] \\
\end{align*}
\]

where \( t = kT_{\text{int}}, k = 1, 2, ..., N \) and \( T_1, T_2 \) are specified time constants.

![Exponential feed generation profile](image)

Figure 16 – Exponential feed generation profile

\( T_1, T_2 \) are specified time constants that obtained from a series of linear travel commands, which are conducted on each drive, as well as the real velocities are obtained and measured by using the CNC’s internal data storage part over the application of programming interface.

The time constants are determined by a non-linear least squared identification method in this research. So that the corresponding travel length \( l(t) \) can be obtained by integrating the feed from Eq. (17) along the path.
where

\[
\frac{f_e - f_c}{T_1 - T_2}(T_1^2(1 - e^{-(t/T_1)}) - T_2^2(1 - e^{-(t/T_2)})) + f_e t + l(0), \quad t \in [0, t_1)
\]

\[
l(t) = f_c(t - t_1) + l(t_1), \quad t \in [t_1, t_1 + t_2)
\]

\[
\frac{f_e - f_c}{T_1 - T_2}T_1^2 - T_2^2 + T_1^2 e^{-(t - t_1 - t_2)/T_1} + T_2^2 e^{-(t_1 - t_2)/T_2}
\]

+ \frac{f_e}{T_1} (t - t_1 - t_2) + l(t_1 + t_2), \quad t \in [t_1 + t_2, t_1 + t_2 + t_3]
\]

(18)

It should be mentioned that the time \( t \) is discretised as \( t = kT_{\text{int}} \) at the interpolation interval time \( T_{\text{int}} \). The part machining cycle time can be predicted by the machining time if assuming each path segment length is finished, for instance \( L = l(t_1 + t_2 + t_3) \).

The CNC systems have several trajectory generation modules, they are infinite and constant profiles, as well as continuous and exponential jerk profiles as shown in Fig. 15 and Fig. 16.

In order to evaluate the machining time, the main influencing factors should be known first. From reference [31] and [53] the operation time is influenced by the feed speed transitions between the smoothing feed and NC blocks to avoid high frequency jitters that may cause inertial vibrations. Nonetheless, the trajectory generation profile and corner smoothing calculations are the main factors that affect the machining time.

Fig. 17 is the three-axis corner smooth algorithm in order to increase the accuracy of prediction. The initial path has a sharp corner regarding to a tool tip coordinate of \( p_2 \) for a three-axis machining application. For the sake of avoiding the dimensional rightness, the corner path can be adjusted. In another hand, the CNC can be adjusted as well to stop at the end of each action so as to achieve zero error. A five order micro spline can be fitted by locating 7 points, which are \( P_0, P_1, P_2, P_3, \ldots, P_6 \) through the path sectors when maintaining the part tolerance \( \varepsilon_{\text{pos}} \) at the corner that can be seen in Fig. 17.

The tool path through the corner spline can be calculated by the following equations:

\[
P(u) = P_0 \sum_{n=0}^{5} C_{0n} u^n + \ldots + P_5 \sum_{n=5}^{5} C_{5n} u^n + P_6 \cdot 0, \quad 0 \leq u \leq 0.5
\]

\[
P_0 \cdot 0 + P_5 \sum_{n=1}^{5} D_{5n} u^n + \ldots + P_6 \sum_{n=1}^{5} D_{6n} u^n + P_6, \quad 0.5 \leq u \leq 1
\]

(19)

where \( u = 0.5 \) according to the corner point.

In order to make sure the jerk and acceleration continuity at the union points \( P_0(u = 0) \) and \( P_6(u = 1) \), the parameters of the spline and the locations of the control points should be defined.
It should be mentioned that some of the CNC systems start executing the next block earlier than while the machine reaches at the corner. It will be explained in the following example:

The example NC Program is:

N010 G01 X4 F1000  
N020 X2 F500      
N030 X8 F2000  
N040 X-4 F1000

The Fig. 18 [60] is the commanded feedrate in four subsequent NC blocks. The next NC block will start moving when the previous block is finished.

The time shifting of action blocks of CNC can be obtained and it can be seen from Fig. 19 that when the linear command begin to decelerate, the next NC block, for instance linear motion command, is shifted advanced of its schedule time.
This scheme mixes the sharp corners in a smooth way but the disadvantage of this is ignoring the constraint of the path error with the tolerance of the machining part. The block shifting strategy can be executed in the virtual CNC by mixing the block transitions that is shown in Fig. 20.

In this part of study, the two main factors have been researched that can affect the total machining cycle time in three-axis CNC system, which is not the limitation of the virtual CNC system. More descriptions are in the appendix. The virtual CNC system cannot only simulate three-axis tool path but also all range systems from one to five axis. All the servo states can be simulated including torque, position and acceleration etc., as well as total machining time and contouring errors. Despite that, the two main factors that have been studied still affect the total cycle time most.

## 2.3 Tool Path Generation Consider to Energy Consumption

This part of research is based on a five-axis CNC system for the purpose of making this master thesis study a wider range of coverage, then some useful part will be researched to utilize in the case study part. The case study part will only focus on three-axis surface machining optimization and the five-axis machining will be researched in the future work.

Table 2 is the nomenclature for the calculation in this section. The factors are listed according to the sequence of appearance.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Lead angle (rad)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Tilt angle (rad)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$r_f$</td>
<td>Surface radius of curvature through feed direction (mm)</td>
</tr>
<tr>
<td>$r_e$</td>
<td>Effective cutting radius of a flat-end mill (mm)</td>
</tr>
<tr>
<td>$r_k$</td>
<td>Surface radius of curvature that perpendicular to feed direction (mm)</td>
</tr>
<tr>
<td>$f$</td>
<td>Feedrate (mm/min)</td>
</tr>
<tr>
<td>$S$</td>
<td>Spindle speed (rad/s)</td>
</tr>
<tr>
<td>$P_{idle}$</td>
<td>Idle power (W)</td>
</tr>
<tr>
<td>$P_T$</td>
<td>Cutting power (W)</td>
</tr>
<tr>
<td>$P_D$</td>
<td>Driving power (W)</td>
</tr>
<tr>
<td>$J_k$</td>
<td>Inertia of each axis</td>
</tr>
<tr>
<td>$F_k$</td>
<td>Viscosity friction coefficient of each axis</td>
</tr>
<tr>
<td>$\mu_k$</td>
<td>Friction coefficient of each axis</td>
</tr>
<tr>
<td>$v$</td>
<td>Axis velocity</td>
</tr>
<tr>
<td>$a$</td>
<td>Axis acceleration</td>
</tr>
<tr>
<td>$E_T$</td>
<td>Energy consumption due to cutting power (J)</td>
</tr>
<tr>
<td>$E_D$</td>
<td>Energy consumption due to driving power (J)</td>
</tr>
<tr>
<td>$w_i$</td>
<td>Effective cutting width (mm)</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Swept area ($mm^2$)</td>
</tr>
<tr>
<td>$U$</td>
<td>Specific energy ($J/mm^2$)</td>
</tr>
<tr>
<td>$d_f$</td>
<td>Forward step (mm)</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Side step (mm)</td>
</tr>
</tbody>
</table>

The five-axis milling is widely used in machining complicated surfaces with high accuracy needs. It is implemented in many industries such as aerospace and shipbuilding etc. This part will study the optimization method regarding to energy consumption by the following sectors:

1. Build an energy potential field on the known surface that includes the distinct power demand through any feed directions at random cutter contact point.
   
   a. Determinate the tool orientation before establishing the energy consumption model
   
   b. Make a energy consumption model to find out the parameters that can determine the energy consumption

2. Find an optimal tool path generation that fits to the minimum directions of the field and meet the maximum scallop height needs at the same time.
2.3.1 Energy Potential Field

2.3.1.1 Determination of tool orientation

Generally, in a five-axis machining, a tool path is consisting of cutter location curve (CL), which is the trajectory of the tool tip and a tool orientation $T$. Sequentially, the surface normal $n$ can be used and feed direction $f$, as well as the cross product $k$ corresponding to $n$ and $f$ to determine a local reference frame.

Where, $k = f \times n$.

It is obvious to see in the Fig. 21 that the tool orientation $T$ can be defined by two angles $\alpha$ (lead angle) and $\beta$ (tilt angle).

In this part, the flat-end cutter will be considered to use because it has close tool surface contact and large cutting width. Furthermore, the flat-end cutter is intrinsically related to orientation of the tool and the work of analysis will be complex, specially considering the side and rear gouging that is illustrated in Fig. 22.

![Figure 21 – Parameters define the local frame](image)

If CC point $p$ is given, the lead angle $\alpha$ should be positive so as to protect the rear gouging when the surface radius of curvature $r_f$ through the feed direction $f$ is negative. The relationship of these parameters is shown in Fig. 22 as well.
The lead angle $\alpha$ should be:

$$\alpha \geq \sin^{-1}\left(\frac{r}{r_f}\right),$$

(20)

Figure 22 – Side and rear gouging considering a flat-end cutter

The real cutting profile of a flat-end cutter is a kind of ellipse and its effective cutting radius $r_e$ (as shown in schematic figure 23) can be calculated refer to reference [54]:

$$r_e = t^2 r^2 \left(\frac{1 + \tan^2 \theta}{t^2 + r^2 \tan^2 \theta}\right)^{3/2},$$

(21)

where $t = rsin\alpha \cdot \cos\beta = \theta = \tan^{-1}(\tan\alpha \cdot \sin\beta)$.

Figure 23 – The effective cutting shape (ellipse)

In order to meet the side gouging free needs, the lead angle $\alpha$ and the tilt angle $\beta$ need to be optimized by the following equation:
\[ r_k(\alpha, \beta) = \frac{1}{\text{Max}(-\frac{1}{r_k}, 0)} , \]  

(22)

where \( r_k \) is the radius of curvature on the surface at point \( p \) through \( k \).

Then the concave case can be known, in which the lead angle \( \alpha \) is positive when \( r_k \) is negative in order to protect the local gouging. On the contrary, the lead angle \( \alpha \) can be infinitely closed to \( 0 \) for the sake of having the largest cutting width.

In the case that the global collision has been maintained, the tilt angle \( \beta \) is set to \( 0 \) so as to have a more regular cutting strip. Then the pre-determined the tool orientation can be implemented by using the following equation:

\[ (\alpha, \beta) = (\text{Max}(0, \sin^{-1}(\frac{r}{r_f}), \sin^{-1}(\frac{r}{r_k})), 0) , \]  

(23)

In the workpiece coordinate system, the tool orientation can be expressed as:

\[ T_i = (a_i, b_i, c_i) = n \cos \alpha + f \sin \alpha \cos \beta - k \sin \alpha \sin \beta , \]  

(24)

The tool orientation can be a unary function of feed direction \( f \) because of the different feed directions can cause the changing of \( r_f \) and \( r_k \), which means the known CC curve can define a unique tool orientation of the flat-end cutter.

If we consider the ball-end tool that will be utilized for our case study, Eq. (24) will be simplified if a positive and fixed lead angle \( \alpha \) has been take care as invariant when the tilt angle \( \beta \) becomes to zero.

### 2.3.1.2 The model of energy consumption

We know that the feedrate is the tool tip’s speed when the tool moves through the tool path. This speed is usually changed in process by the controller of the machine so as to avoid from exceeding the machine’s kinematic or dynamic constraints when transforms the tool tip’s speed to the speed of the machine’s axis.

In order to make such similar system, a reference constant feedrate \( f \) should be assigned to simulate the machine in verse kinematic and then the machine axis’s velocity and acceleration can be calculated in advance to ensure whether the given feedrate is conservative, and then decrease it if the virtual numerical controller is unaccepted. When the above work is finished, the energy consumption model can be build with both the feedrate and tool orientation specified.

In general, there are three main contributing factors to the power demand that are power demand \( P_{dle} \), cutter power demand \( P_T \) and driving power demand \( P_D \), respectively according to reference [38] and [55].

If assuming the tool moves an infinitesimal distance from CC point \( P_i \) to \( P_{i+1} \) by the specified feedrate \( f \), then the total consumed energy of the cutting movement can be calculated:
\[
E = (P_{idle} + P_T + P_D) \times \frac{||p_{i+1} - p_i||}{f},
\]

where \(\frac{||p_{i+1} - p_i||}{f}\) is the time consuming between \(P_i\) and \(P_{i+1}\), to be denoted as \(\Delta t\).

Each machine tools have their own intrinsic characteristic that can be calibrated. For example the inertia of each axis \(J_k\), the idle running power \(P_{idle}\) and the viscosity friction coefficient \(\mu_k\). The energy requirement according to cutting power demand \(P_T\) for compensating the cutting force is:

\[
E = P_T \times \frac{||p_{i+1} - p_i||}{f} = \int_0^{\Delta t} \int_0^{h_0} (F_t(h, t)rS)dh \ dt,
\]

where \(F_t(h, t)\) is the intensity of the tangential force at a given height \(h\) and a given time \(t\). \(S\) is the spindle speed and \(r\) is the tool radius. Therefore, the part \(\int_0^{h_0} (F_t(h, t)rS)dh\) of the equation is the cutting power at time \(t\).

Each axis’s power demand \(P_D\) should be independent to each other, so other variable power demand in the machine coordinate system needs to be investigated. The internal friction force and the torque in the machine coordinate system require extra energy for acceleration. Therefore, the velocity \(v_k\) and acceleration \(a_k\) should be calculated of each axis at the first step. Suppose that the chord error \(e\) is known, and then the two adjacent CC points can be found out due to the feed direction \(f\).

Fig. 24 is the illustration of the parameters that are needed to the calculation. Assume that the cutter posture at point \(p_i\) in the workpiece coordinate system can be expressed as:

\[
(x_i, y_i, z_i, a_i, b_i, c_i),
\]

where \(T_i = (a_i, b_i, c_i)\) is the tool orientation that determined previously.

![Figure 24 – Three adjacent cutter postures with the chord error](image)

Then the cutter postures \(p_{i-1}, p_i\) and \(p_{i+1}\) can be converted into their corresponding machine coordinates by utilizing the inverse kinematics transformation:
\begin{equation}
(m_{i,1}, m_{i,2}, m_{i,3}, m_{i,4}, m_{i,5}) = IKT(x_i, y_i, z_i, a_i, b_i, c_i),
\end{equation}

For more information about IKT is given in the appendix. Then the velocity \( v_k \) and acceleration \( a_k \) of each axis can be calculated by following:

\begin{equation}
v_k = (m_{i+1,k} - m_{i,k})/\Delta t,
\end{equation}

\begin{equation}
a_k = (m_{i+1,k} - 2m_{i,k} + m_{i-1,k})/\Delta t^2, \quad k = 1, 2, 3, 4, 5,
\end{equation}

According to the Eq. (28) and Eq. (29), the total energy cost of the driving power demand can be obtained by:

\begin{equation}
E_D = P_D \Delta t = \sum_{k=1}^{5} (\mu_k v_k \Delta t + F_k v_k^2 \Delta t + \frac{1}{2} J_k ((v_k + a_k \Delta t)^2 - v_k^2), if \ a_k > 0 \\
0, if \ a_k \leq 0)
\end{equation}

It should be mentioned that the part kinetic energy demand \( \frac{1}{2} J_k ((v_k + a_k \Delta t)^2 - v_k^2) \) is 0 for any axis when \( a_k \leq 0 \) is following the laws of energy conservation. In the contrary, the motor has to provide more energy to accelerate the whole inertia that related with the axis.

Until now the three main factors of energy consumption have been calculated. In order to obtain the infinitesimal energy consumption of any neighboring two CC points, the three main factors, which are Eq. (25), Eq. (26) and Eq. (30) can be simply summed up as follows:

\begin{equation}
E = P_{idle} \Delta t + E_T + E_D,
\end{equation}

There is a new factor named the area swept \( A_i \) by the cutter from \( p_i \) to \( p_{i+1} \) that is used to better evaluate the energy efficiency. The area swept value \( A_i \) can be determined by the equation as following:

\begin{equation}
A_i = \frac{1}{2} (w_i + w_{i+1}) \| p_{i+1} - p_i \|,
\end{equation}

where \( w_i \) is the effective cutting width on the surface at point \( p_i \) that is determined by the length of the effective cutting chip from the nominal surface \( S \) to the tolerance surface \( S' \). Fig. 25 illustrates the two distinct cases according to the effective cutting width.
It is obvious to distinguish the difference between the concave and convex considering of the effective cutting width of flat-end milling. The distance of cutting width of concave case is defined between the two intersection points from the ellipse shape to the local arc of the tolerance surface $S'$ that can be determined by solving the two parameters $\theta$ and $\varphi$ of the following equations:

\[
\begin{align*}
  r_k - (r_k - h)\cos \theta &= rsin - rsin\cos \varphi , \\
  r_k \sin \theta &= rsin \varphi ,
\end{align*}
\]

The effective cutting width $w_i$ of the concave case can be simply calculated as follows and the illustration of the relationship between $r$ and $\varphi$ can be seen in appendix:

\[
w_i = 2rsin \varphi ,
\]

Similarly, it is obvious to see from the Fig. 25 that the distance of cutting width of convex case is degenerated into the cord length between the nominal surface $S$ and the tolerance surface $S'$. Then the effective cutting width $w_i$ of the convex case can be calculated by the following formula:

\[
w_i = \min(2r, \sqrt{8rh}) ,
\]

After determination of the effective cutting width $w_i$, the specific energy can be obtained at a random CC point and through any feed direction is then the quotient of the energy consumption over the swept area:

\[
U = \frac{E}{A} = \frac{P_{idle}\Delta t + E_p + E_D}{\frac{1}{2}(w_i + w_{i+1})p_{i+1} - p_{i}} ,
\]
2.3.1.3 Energy consumption based on tool path generation

In a five-axis machining, the tool path is composed of a series of discrete CL curves that dominate the tool tip position and the relevant tool orientation in the workpiece coordinate system. In general, the CC curves are composed of groups of discrete CC points. Usually, the CC curves are decided at first on the nominal surface, and then a tool orientation is decided subsequently, and then calculates the CL curves according to the CC curves, the corresponding tool orientation and the nominal surface.

According to the requirements of machining accuracy, two important constraints should be defined while planning a tool path:

1. The maximum value of scallop height should be kept below a threshold.
2. The chord error $e$ of any two neighboring CC points cannot exceed a known tolerance.

Fig. 26 shows the definition of the side step and forward step. There are two important factors $d_s$ and $d_f$ in this figure require to explain: the side step $d_s$ is the distance between any two adjacent CC points and the forward step $d_f$ is the maximum distance between any two subsequent CC points. Both the side step $d_s$ and the forward step $d_f$ are depending on the local surface curvature. In Eq. (35) the effective cutting width $w_i$ is being the side step when calculating $d_s$. The forward step $d_f$ can be calculated by the following formula [56]:

$$d_f = \min \left( 8e|r_f| - 4e^2, d_{f0} \right),$$

where $r_f$ is the curvature radius of the surface along the feed direction and $d_{f0}$ is a constant that can be set to bound the forward step in the situation $r_f$ is infinite.

As mentioned and explanations above, the feed direction, which is the direction orthogonal to the CC curve at the CC point, can determine the tool orientation at any CC point. The target of this subchapter is not only satisfying $d_s$ and $d_f$ at each CC point, but also to attend to optimize the total energy consumption according to the Eq. (36). The final solution should be the iteration of the principle curve generation and iso scallop height that will be introduced in the next part.

2.3.1.4 Feed direction optimization

The specific energy term is a pure quantity which changes continuously versus different feed directions, this means a vector field about the energy cost efficiency embedded on the entire surface, which is extremely dependent on the machine's configuration. It can be called as the machine based energy potential field. There is a distinct feed direction $f_i$ through the specific energy cost is the lowest between all the others for each CC point $p_i$. Furthermore, this feed direction can be named as the optimal specific energy direction.

There are two properties of the potential field that is described as below:
1. For any point on the surface, it has two opposite optimal directions. However, if the specific energy $U$ in Eq. (36) for arbitrary point on the surface is a constant, there is an exception for the case of flat surface.

2. The flow lines on the surface are continuous and will never interact to itself.

When the potential field is defined, the tool path should be defined in a way that the feed directions are as close as possible to the individual optimal directional flow lines of the machine based energy potential field in order to minimize the energy consumption. There is some of the research such as reference [57] and [58] that attended to fit a tool path into a known vector field. A better way to balance between the best fitting to the machine based energy potential field and the demand regular patterns of CC curves can be found.

If a free-form surface is defined as:

$$S(u, v) = (X(u, v), Y(u, v), Z(u, v),$$

a discrete $N \times N$ grid of the machine based energy potential field is created upon the parametric $uv$ domain $[0,1] \times [0,1]$ of the surface.

For each point $\mu_{ij} = (u, v)$ on the grid, the specific energy $U$ that is calculated in Eq. (36) at point $S(u,v)$ can be obtained for every $k$ radian of feed direction from 0 to $2\pi$, where

$$k = \frac{\pi}{180},$$

this can be recorded as a value of vector $U_{ij}$ that is shown in Fig. 26.

the machine based energy potential field is required to constantly calculate at an random $S(u, v)$ in the process of computation of CC curves. This is better than directly calculating it for the case of a grid point $\mu_{ij}$. To be more clear, for an arbitrary node:

$$p = S(u, v),$$

then let:

$$U_i, \ i = 1, 2, 3, 4,$$

which is the machine based energy potential field of the four neighboring grid nodes of $(u, v)$. Then the machine based energy potential field vector $u$ of $S(u, v)$ can be calculated by:

$$u = \frac{1}{\sum_{i=1}^{4} \frac{1}{||p - \mu_i||^2}} \sum_{i=1}^{4} \frac{U_i}{||p - \mu_i||^2},$$

(38)
The optimal feed direction $f$ at the arbitrary $S(u, v)$ is directly obtained if using the minimum of the $2\pi/k$ values once $u$ is calculated. By using this method, a principle CC curve can be obtained, which will be introduced in the next part.

### 2.3.1.5 Principle curve generation

Generally, the principle cutter contact curve is planned foremost to direct the general tendency of the subsequent cutter contact curve, which is expanded based on the distinct criteria that have been introduced in the sub-chapter 2.1 such as iso-parametric, iso-planar and iso-scallop.

The original principle curve is generated in the parametric $uv$ domain of the known surface on which the machine based energy potential field is embedded. It can be started with a random initial point $p_i$, which related vector $u_i$ and the optimal feed direction $f_i$ of the machine based energy potential field on the sampled $N \times N$ grid are already known if this node can be obtained by Eq. (38). By continuous the steps from $f_i$ with a forward step $d_f$ in the workpiece coordinate system, the following CC point $p_{i+1}$ can be calculated as shown:

$$p_{i+1} = p_i + \frac{d_f}{E(f_i \cdot \vec{u})^2 + 2F(f_i \cdot \vec{v})(f_i \cdot \vec{v}) + G(f_i \cdot \vec{v})^2} f_i,$$  

(39)
where \( \vec{u} \) and \( \vec{v} \) are the fundamental unit vectors of the \( uv \) domain, as well as E, F, G are the modulus of the first basic form in differential geometry \[59\].

The principle curve can be generated in the forward step until reach the \( uv \) domain’s boundary. Then it can be started the similar process from \( p_i \) to generate the backward step through the opposite direction of \( f_i \). Finally, the whole principle curve can be fully generated by cascading the forward and backward part.

### 2.3.1.6 Expansion algorithm according to Iso-scallop height

After generating the principle curve, cutter contact curves can be expanded to its both sides in order to filling the whole surface. According to the iso-scallop height needs, the side step between the neighboring CC curves should be enlarged as more as possible, as much as possible the maximum scallop height.

If the tangent direction determines with the optimal feed direction at each CC point on the principle curve, the expansion of the CC curve is impossible. Fig. 27 shows how to generate the first expanded curve according to the traditional iso-scallop height expansion rules: shift a side step \( d_s = w_i \) that perpendicular to the feed direction \( f_i \) for each CC point \( p_i \) on the principle curve.

Reference \[56\] provides the equation for the sake of calculating \( p_i \) as shown below:

\[
|p_i p_i'| = w_i , \tag{40}
\]

\[
(p_i - p_i') (\frac{\partial S}{\partial u} dt + \frac{\partial S}{\partial v} dt) = 0 , \tag{41}
\]

where the differential form \( \frac{\partial S}{\partial u} dt + \frac{\partial S}{\partial v} dt \) is the feed direction \( f_i \) at \( p_i \) in theoretically.

The Taylor expansion can eliminate higher order terms of the value \( p_i' \):

\[
p_i' = p_i + \frac{\partial S}{\partial u} \Delta u + \frac{\partial S}{\partial v} \Delta v , \tag{42}
\]
By utilization of the Eqs. (40)-(42), the parametric increment of $p_i'$ can be calculated in order to match the key equations:

$$\Delta u = \pm \frac{w_i (F \frac{du}{dt} + G \frac{dv}{dt})}{\sqrt{(EG - F^2)(E \frac{du}{dt})^2 + 2F \frac{du}{dt} \frac{dv}{dt} + G \frac{dv}{dt})^2}}$$ \hspace{1cm} (43)$$

$$\Delta v = \pm \frac{w_i (E \frac{du}{dt} + F \frac{dv}{dt})}{\sqrt{(EG - F^2)(E \frac{du}{dt})^2 + 2F \frac{du}{dt} \frac{dv}{dt} + G \frac{dv}{dt})^2}}$$ \hspace{1cm} (44)$$

where $E = \frac{\partial s}{\partial u} \cdot \frac{\partial s}{\partial u}$, $F = \frac{\partial s}{\partial u} \cdot \frac{\partial s}{\partial v}$, and $G = \frac{\partial s}{\partial v} \cdot \frac{\partial s}{\partial v}$ are the factors of the first basic form in differential geometry.
The bias between the optimal feed direction and the tangent direction of the expand curve will be extremely large if the expansion step have been finished and have the certain expanded curves. Then a concept of quality evaluation should be introduced, which will terminated the expansion when the deviation is exceeding a threshold that is expressed as the ratio:

$$\frac{\sum u_i - \sum u_{i0}}{\sum u_i},$$

where $\sum u_i$ is the sum of the machine based energy potential field value through the real feed direction at all the cutter contact points on the expanded curve, as well as $\sum u_{i0}$ is the sum of the optimal machine based energy potential field value at each CC point on the curve. It should be noted that the ratio should be 0 for a principle curve.

As Fig. 28 shown, three expansion groups with the principle curve coloured in blue can cover the entire surface domain. This sub-chapter has introduced a path generation considering the energy consumption, which will be selective utilized to the case study and the rest will be used to future work of research.
3 Case Study

The purpose of the case study section is to find the most optimized solution of the target shape by researching and adjusting the G codes of the machining. Generally, the machining simulation software such as EdgeCAM will generate a default tool path that can process the target shape according to the blueprint. However, the default solution cannot meet our specific requirements in instance, for example surface smoothness requirements and machining time optimization needs etc.. In this master thesis, we will choose a surface shape that is shown in Fig. 29, which is chosen for the sake of easier observing and comparing intuitively.

![Figure 29 – The four views of the target stock and surface](image)

We use EdgeCAM to plan the tool path, simulate and generate the G codes in this thesis. The G codes of all the methods and approaches are attached in the appendix part at the end of this paper. The illustration and investigation of the G codes will be conduct by the diagrammatic sketch in order to explain the codes in a simple and intuitive way.

In this chapter, we will first generate different types of tool path approaches and compare with them in the simulation software, and then the solution will be obtained by combining the optimized machining method. At the end of this chapter the final optimized solution will be conduct in the CNC machine and will be compared with the default case depends on the operation time and smoothness of working surface.
3.1 Analysis of Default Generation Path Based on G codes

As the above description, the tool path of the default solution can be read in the G code file and illustrated in the Fig. 30. The main tool path of the default one is moving in the y-axis direction (top figure of Fig. 30) and cutting layer by layer through the z-axis.

![Figure 30 – The tool path illustration of default solution from EdgeCAM](image)

Fig. 31 shows the tool path simulation by G code simulation software. The general shape of the target surface can be seen in this figure by the route of tool movement. The machining time is about 16 minutes and the surface is rough, which cannot meet our requirements in both smoothness and operation time. Therefore, the optimized solutions and approaches are proposed in the following sub chapters.
3.2 Approaches to the Case Based on G codes (simulation)

It is obvious that the default solution mentioned above is not a satisfactory program. Therefore, several approaches in this section will be mentioned to optimize the surface machining. It should be mentioned that this section’s research and comparison of the different approaches are based on the simulation software in order to save time and money, as well as comparison of different tool path’s processing performance. There are four optimization approaches that will be investigate by analysing the G codes, which are attached in the appendix part and in the attachment as well.

In this thesis, we will propose a module method to optimize the tool path movement. The main module can be divided into two different machining ways:

1. Linear tool path movement module
2. 3D tool path movement module

Then we will optimize each module to produce the optimized module, and then combine the optimized module together to provide the final proposal of tool path optimization, which will be execute in the CNC machine.

3.2.1 Surface Machining Optimization Method 1: Linear Path only

Fig. 32 and Fig 33 show the tool path illustration by investigating of the G codes. The Rough milling will be the first step to mill the target stock in the x-axis, afterwards the profile milling will be conducted in order to fulfil the smoothness requirements.
It should be mentioned that Fig. 32 and Fig. 33 are described the tool path movement in the top view of the stock. The interval between the adjacent tool paths can be seen from the G codes and illustrated in the Fig. 32 as well. However, in the profile milling stage, the neighboring tool paths are not in the same axis. In another word, each tool path is in the distinct depth, which is illustrated in the Fig. 34 (left view of the stock). The principle of the machining of surface is: through iterations of several small processing paths in the x-axis direction, the target surface will be formed by an infinite approximation of the shape of the surface.
Fig. 35 is generated from the same G code simulation software. It can be seen from this figure that the surface smoothness is better than the default case. Nonetheless, the machining time is longer than the default case. We will compare the operation time of all the simulation approaches in the comparison section.

Method 1 is the original method to conduct the surface machining in linear way as we introduced in the previous section. In the next section we will propose a 3D machining method that has different tool path compare to the linear operation.
3.2.2 Surface Machining Optimization Method 2: Iso Scallop Method

This section will experiment the feasibility of the 3D machining method. Fig. 36 is the tool path illustration of the iso scallop method. The tool path is through the y-axis direction if we consider the tool route in the top view of the stock. The actual path is increment in the z-axis direction and moved in the y-axis, which is the reason we use the short line on the arrow to indicate the incremental change on the other axis.

![Tool path illustration of iso scallop optimization solution (CM module)](image)

Figure 36 – Tool path illustration of iso scallop optimization solution (CM module)

There is a serious problem in the machining of iso scallop method. We can read the start point of the tool path through the G codes. The path is from the bottom to the top of the surface. However, the vertical distance of the surface is 30mm, which is longer than the tool length. Moreover, the ball-end cutting tool cannot be perpendicular to the target plane in the real machining. Therefore, we need to
purpose an optimization solution of this 3D machining method, which will be carried out in the next section.

3.2.3 Surface Machining Optimization: Optimization of Method 2

Due to the tool crash problem that has mentioned in the above section, we will add a rough milling step in the front of the method 2 in order to avoid tool damage.

![Figure 38 – Tool path illustration of combination of RM module & CM module (iso scallop)](image)

Fig. 38 describes the tool movement of iso scallop method. It needs the rough milling before iso scallop machining. Like a double-edged sword, the iso scallop method provides better smoothness while also consuming more processing time due to the small step over in our simple surface case. The iso scallop method might has a better balance performance in the processing time and processing accuracy in the more complex cases. Due to this reason, we will make a more complex surface to test the different algorithms on five-axis CNC machine in the future works.
It can be seen in Fig.39 that the optimization of method 2 is avoiding the damage to the cutting tool and increasing the accuracy of processing at the same time, which means the 3D machining method is an optimized solution that can be considered in the real CNC machining (adjusting of the parameters such as feedrate and spindle speed will be conducted in the real processing in the following chapter).

3.2.4 Surface Machining Optimization: Optimization of Method 1

Method 1 is an approach that makes the tool travel in a linear way in x-axis direction. As the illustration Fig. 32 shown, the cutting tool needs to travel back to the previous x-axis coordinates so as to start the new cutting step. This travel time will probably waste the energy and total machining time. In order to optimize the linear method, the best solution is to make the tool path moving without the travelling time.

Fig. 40 shows the tool path of the optimization of method 1. The main difference between the optimized one and the origin one is the rough milling’s tool path. The following machining can be profile milling or small step over rough milling so as to achieve the surface as close as possible to the final surface we want to process.
We can see from the Fig. 41 that the tool path is changed in the rough milling step. It should be mentioned that the path interval is not small enough in both the linear cutting and 3D cutting simulations. All the simulations are using the same parameters such as feedrate, path interval, cutting depth etc. to make the reasonable comparison, which will be made in the next section.

In the next part, the comparison through machining time, tool distance and program line will be made according to the four optimized approaches and the default case. Then the two real program proposals will be proposed so as to test the result of optimization method.
3.3 Comparison of the Different Approaches & Final Proposal

The data of Table 3 from left to right is arranged in the order in which they appear in the previous section and the resources of these data are attached in the appendix part.

Table 3 – Comparison of the five simulations

<table>
<thead>
<tr>
<th></th>
<th>DEFAULT CASE</th>
<th>LINEAR CASE</th>
<th>3D CASE</th>
<th>LINEAR+3D</th>
<th>OPT. LINEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machining Time</td>
<td>16m00s</td>
<td>24m07s</td>
<td>07m01s</td>
<td>18m12s</td>
<td>20m53s</td>
</tr>
<tr>
<td>Path Distance</td>
<td>18561.22</td>
<td>32474.83</td>
<td>12619.95</td>
<td>25449.27</td>
<td>28749.98</td>
</tr>
<tr>
<td>Feedrate</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Speed (RPM)</td>
<td>5730</td>
<td>6366</td>
<td>6366</td>
<td>6366</td>
<td>6366</td>
</tr>
<tr>
<td>Program Line</td>
<td>1288</td>
<td>710</td>
<td>2262</td>
<td>2382</td>
<td>588</td>
</tr>
<tr>
<td>Machining Technology</td>
<td>Rough</td>
<td>Smooth</td>
<td>Tool Damage</td>
<td>Smooth</td>
<td>Smooth</td>
</tr>
</tbody>
</table>

There are four methods from the chart that can be considered to the comparison such as default case, linear case, linear + 3D case and optimized linear case except the tool damage approach (3D case). Then we ruled out rough case for machined faces (default case) and the most time consuming case.
(linear case). At the final stage, we have two optimized options (blue column) that need to implement in the real CNC machining so as to test the performance of this two methods.

As the comparison chart shows to us, although the simulation above is the most resource saving method and the results of the simulation can help us to choose what kind of program should be implement in the real case, but its limitations are obvious, that is, it can not be intuitive to reflect the specific performance of specific program. In order to make our research more practical, the experiment of comparing these two methods will be conducted in the next section. It should be noted that the machining parameters would be modified for the sake of achieving the highest performance of each method.

### 3.4 Experimental Results & Conclusion

#### 3.4.1 Experimental Process & Results

Fig. 42 is the linear method surface, which will be the reference material compare with the linear + 3D method. In order to increase the contrast, our linear processing program is only used in the simulation part of the selected processing methods, which means the specific parameters are not optimized. The total operation time is about 22 minutes and the surface quality of this method is not satisfactory as shown in the figure.

![Figure 42 – Linear method surface](image)

Then we plan to machine the surface by using linear + 3D approach. As we mentioned before, this method is using the iso scallop method (the path interval is 0.4mm) for the profile milling step and the parameters such as feedrate and spindle speed are calculated by following:

\[ N = \frac{V_c}{\pi \cdot D}, \]  

(45)

where \( N \) is the spindle speed, \( V_c \) is the specific speed that is selected depends on different materials and \( D \) is the diameter of the cutting tool.
\[ f_r = N \cdot n \cdot f_z , \]  

(46)

where \( f_r \) is the feedrate of each machining, \( n \) is the number of teeth and \( f_z \) is the ERC feed depends on different materials.

We also need the formula to calculate the material removal rate so as to compare the performance of distinct machining:

\[ MRR = a_e \cdot a_p \cdot f_r , \]  

(47)

where \( a_e \) is the width of cut and \( a_p \) is the depth of cut.

Then we can utilize Eq. (45) and Eq. (46) to calculate the parameters for our specific milling method (linear + 3D machining).

For the machining of first rough milling by using 12mm Flute End Mill:

\[
N = \frac{V_c}{\pi \cdot D} = \frac{350}{\pi \cdot 0.12} = 9288 \text{ RPM}
\]

\[
f_r = N \cdot n \cdot f_z = 9288 \cdot 3 \cdot 0.1 = 2786 \text{ mm/min}
\]

For the machining of second rough milling by using 12mm Flute End Mill:

\[
f_r = N \cdot n \cdot f_z = 9288 \cdot 3 \cdot 0.14 = 3900 \text{ mm/min}
\]

where \( f_z \) for each step is adjusted by different correction factor depends on distinct \( a_e \).

For the machining of profile milling by using 10mm Ball Nose End Mill:

\[
N = \frac{V_c}{\pi \cdot D} = \frac{440}{\pi \cdot 0.01} = 14012 \text{ RPM}
\]

\[
f_r = N \cdot n \cdot f_z = 14012 \cdot 2 \cdot 0.057 = 1597 \text{ mm/min}
\]
After calculation of the specific parameters, we execute the machining by using these numbers. Then we obtain a smoother surface than the contrast surface as shown in Fig. 43 and the total machining time is about 30 minutes.

Table 4 – Comparison of the two real CNC machining

<table>
<thead>
<tr>
<th></th>
<th>LINEAR CASE</th>
<th>LINEAR + 3D CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MACHING TIME</td>
<td>22m</td>
<td>30m</td>
</tr>
<tr>
<td>FIRST STEP ROUGH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feedrate (mm/min)</td>
<td>2250</td>
<td>2786</td>
</tr>
<tr>
<td>Spindle Speed (RPM)</td>
<td>8000</td>
<td>9288</td>
</tr>
<tr>
<td>MRR (mm³/min)</td>
<td>32400</td>
<td>25074</td>
</tr>
<tr>
<td>SECOND STEP PROFILE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feedrate (mm/min)</td>
<td>995</td>
<td>3900</td>
</tr>
<tr>
<td>Spindle Speed (RPM)</td>
<td>9550</td>
<td>9288</td>
</tr>
<tr>
<td>MRR (mm³/min)</td>
<td>746</td>
<td>2808</td>
</tr>
<tr>
<td>THIRD STEP FINISH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feedrate (mm/min)</td>
<td>N/A</td>
<td>1597</td>
</tr>
<tr>
<td>Spindle Speed (RPM)</td>
<td>N/A</td>
<td>14012</td>
</tr>
<tr>
<td>MRR (mm³/min)</td>
<td>N/A</td>
<td>638</td>
</tr>
</tbody>
</table>

It is obvious that the optimized linear + 3D method can process more smoother surface without consuming too much time. More comparison of the two methods can be seen in table 4.

Figure 44 – Three different NC programs
However, the surface is not smooth enough in the horizontal direction cause the output method of NC program. There are three types of NC program that we can select for the surface machining and these three programs are shown in Fig. 44. In the end we chose the none type NC program, which is the linear moving of the cutting tool. The chosen surface is not made with a circular radius so that it cannot be implemented as a linear arc NC program, as well as the spline NC program cannot be used causes limitation of the CNC machine.

3.4.2 Conclusion

According to the experimental results, the optimized method (linear + 3D solution) can get more than the previous surface several times the smoothness, while with only an increase of 36% of the processing time. Thus, through the whole research in the previous chapters and the case study, we can draw the following conclusions:

1. Only the path optimization tool path method that combines the machining parameters with the optimized calculation can achieve maximum optimization performance.

2. 3D machining methods such as iso parametric, iso scallop and iso planar methods might cost more processing time than the linear methods, but they can achieve a higher degree of smoothness. So in the surface processing methods, 3D processing methods have better overall performance than linear ones.

3. For the 3D machining step, the type of spline NC program should have the best performance and smoothness than the other two options.

According to the research previously, in the 3D machining methods, iso scallop might have the best performance in machining more complex free form surface. In the future work, we will investigate the application of iso scallop method in five axis CNC machine for machining more complicated free form surface compare with other optimization methods. Moreover, the future work will be an increased optimization level. We will focus on the overall energy consumption of the machining that is mentioned in the before chapter, to optimize the energy consumption of the entire process so as to achieve the ultimate goal of the optimization.
Reference


[54] Lo, C. C., "Efficient cutter-path planning for five-axis surface machining with a flat-end cutter", Computer-Aided Des. 31, pp. 557-566, 1999


Appendix: Notes of Iso-Parametric Machining
Appendix: Notes of Iso-Planar Machining

The initial CC path \( \mathbf{c}(u) = (u-S(u)) \) is located on a vertical plane that is oriented from the surface vertex. If the point on the surface vertex \( S(u_0,0) \) must satisfy:

\[
\begin{bmatrix}
\frac{\partial S}{\partial u}
\end{bmatrix}
\begin{bmatrix}
(u_0,0)
\end{bmatrix}
= (0,0)
\]

Solve

\[
\begin{bmatrix}
\frac{\partial S}{\partial u}
\end{bmatrix}
\begin{bmatrix}
(u_0,0)
\end{bmatrix}
= (0,0)
\]
Appendix: Notes of Iso-Scallop Machining
Appendix: Notes of Prediction model page 1
APPENDIX: Notes of Prediction model page 2

Introduction

1.1 Methods about Energy Scaling

Two deficiencies:

(i) only adjustable at the second or third stage
(e.g., feedrate, spindle speed have almost no room to adjust
due to various constraints) but the tool path is fixed (modified)

1.2 Tool path generation algorithm (Introduction of other methods)

Structure:

2. Establish of energy potential field
3. Tool path generation to find best fitting directions
4. Physical cutting experiments & comparison
Appendix: Notes of Tool path generation regard to energy consumption page 1

2.1 Pre-determination of tool orientation

Conclusion on page 3: According to Fig. 1, 2, 3, local angle α should be positive if milliseconds is negative, or in as small as 0 to achieve largest cutting width.

Tool orientation

\[ T_i = (a_i, b_i, c_i) = \text{Reference} + f \sin(\alpha) \cos(\beta) - k \sin(\beta) \]

2.2 Energy consumption model

Reference constant feed rate \( f \).

\[ E = (P_d + P_r + P_d) \frac{\text{infinite distance}}{f} \]

Infeed power demand (infeed power demand) cutting power demand driving power demand

% calibrated

\[ E_p = P_f \frac{||P_{pm+1}||}{f} \]

\[ (F_r, t, t) \text{ where } \int dt \]

Cutting power out

Torque moment intensity

AC- type tilting table with only two rotary axes: (Appendix)

\[ (w_1, w_2, \ldots, w_5) = \text{IFT} (x_1, y_1, z_1, \alpha_1, \beta_1) \]

\[ V_k = \frac{c_k (w_1 \cdot w_k - w_1 \cdot w_k)}{\partial t} \]

\[ A_k = \frac{c_k (w_1 \cdot w_k - w_1 \cdot w_k + \text{const})}{\partial t}, k = 1, 2, 3, 4, 5 \]
Appendix: Notes of Tool path generation regard to energy consumption page 2

\[ E_0 = P_{\text{pot}} + \sum_{k=1}^{n} \left( M_k \dot{x}_k \dot{x}_k + F_k \dot{z}_k \dot{z}_k + \left\{ \frac{1}{2} \lambda_k \left( (\dot{x}_k + \dot{z}_k)^2 - \dot{x}_k^2 \right) \right\} \right) \]

\( \text{last term of the plastic energy demand} \)

\( \text{due to the law of energy conservation} \)

\[ E = P_{\text{elec}} \dot{z} + E_t + E_0 \]

The new swept by cutter from \( p_i \rightarrow P_{i+1} \):

\[ A_i = \frac{1}{2} (w_i + w_{i+1}) ||P_{i+1} - P_i|| \]

The quantity of energy consumed over the swept area:

\[ U = \frac{E}{A} = \frac{P_{\text{elec}} \dot{z} + E_t + E_0}{\frac{1}{2} (w_i + w_{i+1}) ||P_{i+1} - P_i||} \]

Machine-based energy potential field (MBEPP)

**Two properties:**

**Property 1:** Each point has two opposite optimal directions.

**Property 2:** Flow lines are continuous and never self-intersect.

Tool path should be as close as possible to the individual optimal directions of MBEPP.

3. Consumption energy based pure-axis tool path generation

- CC curves \( \rightarrow \) tool orientation in sequence for each CC point
- CL curves \( \rightarrow \) associated tool orientations
Appendix: Notes of Tool path generation regard to energy consumption page 3

Due to making accuracy, two constraints:

1. The maximum scallop height on the finish surface must stay below a threshold.

2. The chord error $e$ of any two adjacent C.C points must not exceed a given tolerance.

\[ d_s: \quad W_i = \min (x_i, 2\sqrt{r_i}) \]

\[ d_f: \quad W_i = \min \left( \sqrt{\frac{1}{8}E_{total} - 4e^2} \right) \]

Surface roots of curvature along the feed direction.

Satisfy: $d_s, d_f$ and minimize total energy consumption $E_{Total}$.

3.1 Optimal Feed Direction

\[ U = \frac{1}{\sum_{i=1}^{n} \left( \frac{1}{WP - MB^2} \right) U_i} \]

3.2 Principle Curve Generation

Next C.C point $P_{n+1} = P_f + \frac{df}{\sqrt{f(f, f) + \frac{df}{f(f)} \left( \frac{df}{f} \right)}}$

3.3 Iso-scallop height based expansion algorithm

\[ P_i \text{ calculation:} \]

\[ (P_i - P_i') \left( \frac{dx}{df} \frac{dx}{df} + \frac{dy}{df} \frac{dy}{df} \right) = 0 \]

Theoretically, the feed direction $f_i$ at $P_i$. 

Page 66 of 89
Appendix: Notes of Tool path generation regard to energy consumption page 4

Ray-Taylor equation:

\[ P_i' = P_i + \frac{\partial P}{\partial v} \Delta v + \frac{\partial P}{\partial w} \Delta w \]

Calculate \( \Delta v \), \( \Delta w \) using equation (24-22).

Triangular ratio \( \frac{Z_{u+} - Z_{u+}}{Z_u} \), which should be 0 on principle curve.

Not larger than \( u_{i+} \) on expanded curve, terminated because ratio < \( u_{i+} \).

New current increased only inside unused portion.

3. DBSEs in Fire-over welding.

Direction based Scalar Fields

3.1. The minimum cutting strip width

3.2. The workpiece feed rate

\[ \text{Fig. 4: } P_w \rightarrow PW \rightarrow \text{Workpiece coordinate system} \]

\[ \text{Meshing: } E_{w1} \rightarrow E_{w1} \rightarrow \text{Machine coordinate system} \]
Appendix: Notes of Tool path generation regard to energy consumption page 5
Appendix: Notes of Tool path generation regard to energy consumption page 6

To unify the different cutting force in a same working frame, the three components should be transformed back to the Tcs.

\[
\begin{bmatrix}
\frac{dF_u}{dt} \\
\frac{dF_v}{dt} \\
\frac{dF_w}{dt}
\end{bmatrix} =
\begin{bmatrix}
-sin(\psi) & -cos(\psi) & 0 \\
-cos(\psi) & sin(\psi) & 0 \\
0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\frac{dF_R}{dt} \\
\frac{dF_t}{dt} \\
\frac{dF_n}{dt}
\end{bmatrix}
\]

Inverse kinematics transformation of five-axis machine with a tilting table.

After the Tcs moves into a new position \( (x, y, z, a, b, c) \).

It is necessary to calculate the relative movement \( (\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2) \).

We should solve two rotation angles about axes \( X_w \) and \( Z_w \) to determine the \( \Delta \).

\( \text{\textit{Wm}} \)
Appendix: Notes of Tool path generation regard to energy consumption page 7

\[ \begin{bmatrix} a_i, b_i, c_i, 1 \end{bmatrix}^T = \text{Rot} (2w - m_i; \gamma_i) \cdot \text{Rot} (Xw - m_i, \omega_i) \cdot \begin{bmatrix} 0, 0, 0, 1 \end{bmatrix}^T \]

Expanding to:

\[ \begin{bmatrix} a_i \sin \omega_i \sin \gamma_i \sin m_i; \gamma_i \cos \omega_i \sin \gamma_i \cos m_i; \gamma_i \cos \omega_i \sin m_i; \gamma_i \end{bmatrix}^T \]

With the two rotational coordinates obtained, the translational movement \((m_{i1}, m_{i2}, m_{i3})\) of the Ics in terms of a rotated frame can be expressed as:

\[ \begin{bmatrix} m_{i1}, m_{i2}, m_{i3}, 1 \end{bmatrix}^T = \text{Rot} (Xw, \omega_i; \gamma_i) \cdot \text{Rot} (2w, m_i; \gamma_i) \cdot \begin{bmatrix} x_i, y_i, z_i, 1 \end{bmatrix}^T \]

Finally, the \([x, y, z] \times [x_i, y_i, z_i, 1]^T\) is obtained.

3. Singularity in inverse kinematics transformation
Appendix: G codes simple definition page 1

G-Codes simple definition
G00 Rapid traverse
G01 Linear interpolation with feedrate
G02 Circular interpolation (clockwise)
G03 Circular interpolation (counter clockwise)
G2/G3 Helical interpolation
G04 Dwell time in milliseconds
G05 Spline definition
G06 Spline interpolation
G07 Tangential circular interpolation / Helix interpolation / Polygon interpolation /
Feedrate interpolation
G08 Ramping function at block transition / Look ahead "off"
G09 No ramping function at block transition / Look ahead "on"
G10 Stop dynamic block preprocessing
G11 Stop interpolation during block preprocessing
G12 Circular interpolation (cw) with radius
G13 Circular interpolation (cw) with radius
G14 Polar coordinate programming, absolute
G15 Polar coordinate programming, relative
G16 Definition of the pole point of the polar coordinate system
G17 Selection of the X, Y plane
G18 Selection of the Z, X plane
G19 Selection of the Y, Z plane
G20 Selection of a freely definable plane
G21 Parallel axes "on"
G22 Parallel axes "off"
G24 Safe zone programming; lower limit values
G25 Safe zone programming; upper limit values
G26 Safe zone programming "off"
G27 Safe zone programming "on"
G33 Thread cutting with constant pitch
G34 Thread cutting with dynamic pitch
G35 Oscillation configuration
G38 Mirror imaging "on"
G39 Mirror imaging "off"
G40 Path compensations "off"
G41 Path compensation left of the work piece contour
G42 Path compensation right of the work piece contour
G43 Path compensation left of the work piece contour with altered approach
G44 Path compensation right of the work piece contour with altered approach
G50 Scaling
G51 Part rotation; programming in degrees
G52 Part rotation; programming in radians
G53 Zero offset off
Appendix: G codes simple definition page 2

G64 Zero offset #1
G65 Zero offset #2
G66 Zero offset #3
G67 Zero offset #4
G68 Zero offset #5
G69 Zero offset #6
G63 Feed / spindle override not active
G66 Feed / spindle override active
G70 Inch format active
G71 Metric format active
G72 Interpolation with precision stop "off"
G73 Interpolation with precision stop "on"
G74 Move to home position
G75 Curvature function activation
G76 Curvature acceleration limit
G78 Normalcy function "on" (rotational axis orientation)
G79 Normalcy function "off"
G80 - G89 for milling applications:
G80 Canned cycle "off"
G81 Drilling to final depth canned cycle
G82 Spot facing with dwell time canned cycle
G83 Deep hole drilling canned cycle
G84 Tapping or Thread cutting with balanced chuck canned cycle
G85 Reaming canned cycle
G86 Boring canned cycle
G87 Reaming with measuring stop canned cycle
G88 Boring with spindle stop canned cycle
G89 Boring with intermediate stop canned cycle
G81 - G88 for cylindrical grinding applications:
G81 Reciprocation without plunge
G82 Incremental face grinding
G83 Incremental plunge grinding
G84 Multi-pass face grinding
G85 Multi-pass diameter grinding
G86 Shoulder grinding
G87 Shoulder grinding with face plunge
G88 Shoulder grinding with diameter plunge
G90 Absolute programming
G91 Incremental programming
G92 Position preset
G93 Constant tool circumference velocity "on" (grinding wheel)
G94 Feed in mm / min (or inch / min)
G95 Feed per revolution (mm / rev or inch / rev)
G96 Constant cutting speed "on"
Note that some of the above G-codes are not standard. Specific control features, such as laser power control, enable those optional codes.

**M codes simple definition**

- **M00** Unconditional stop
- **M01** Conditional stop
- **M02** End of program
- **M03** Spindle clockwise
- **M04** Spindle counterclockwise
- **M05** Spindle stop
- **M06** Tool change (see Note below)
- **M19** Spindle orientation
- **M20** Start oscillation (configured by G35)
- **M21** End oscillation
- **M30** End of program
- **M40** Automatic spindle gear range selection
- **M41** Spindle gear transmission step 1
- **M42** Spindle gear transmission step 2
- **M43** Spindle gear transmission step 3
- **M44** Spindle gear transmission step 4
- **M45** Spindle gear transmission step 5
- **M46** Spindle gear transmission step 6
- **M70** Spline definition, beginning and end curve 0
- **M71** Spline definition, beginning tangential, end curve 0
- **M72** Spline definition, beginning curve 0, end tangential
- **M73** Spline definition, beginning and end tangential
- **M80** Delete rest of distance using probe function, from axis measuring input
- **M81** Drive On application block (resynchronize axis position via PLC signal during the block)
- **M101-M108** Turn off fast output byte bit 1 (to 8)
- **M109** Turn off all (8) bits in the fast output byte
- **M111-M118** Turn on fast output byte bit 1 (to 8)
- **M121-M128** Pulsate (on/off) fast output byte bit 1 (to 8)
- **M140** Distance regulation “on” (configured by G265)
- **M141** Distance regulation “off”
- **M150** Delete rest of distance using probe function, for a probe input (one of 16, M151-M168)
- **M151-M158** Digital input byte 1 bit 1 (to bit 8) is the active probe input
- **M159** PLC cannot define the bit mask for the probe inputs
- **M160** PLC can define the bit mask for the probe inputs (up to 16)
- **M161-M168** Digital input byte 2 bit 1 (to bit 8) is the active probe input
Appendix: G codes of default solution page 1
Appendix: G codes of default solution page 2
Appendix: G codes of linear path only page 1
Appendix: G codes of linear path only page 2
Appendix: G codes of linear path only page 3
Appendix: G codes of linear path only page 4
Appendix: G codes of linear path only page 5
Appendix: G codes of linear path only page 6
Appendix: G codes of iso scallop crash page 1
Appendix: G codes of iso scallop crash page 2
Appendix: G codes of linear path + iso scallop page 1

Appendix: G codes of linear path + iso scallop page 2
Appendix: G codes of optimized method 1 page 1
Appendix: G codes of optimized method 1 page 2
Appendix: G codes of optimized method 1 page 3
Appendix: Simulation data of five methods

Result of default case

Result of linear case

Result of 3D crash case

Result of linear+3D case

Result of optimized linear case