Stochastic modeling of scrape-off layer fluctuations

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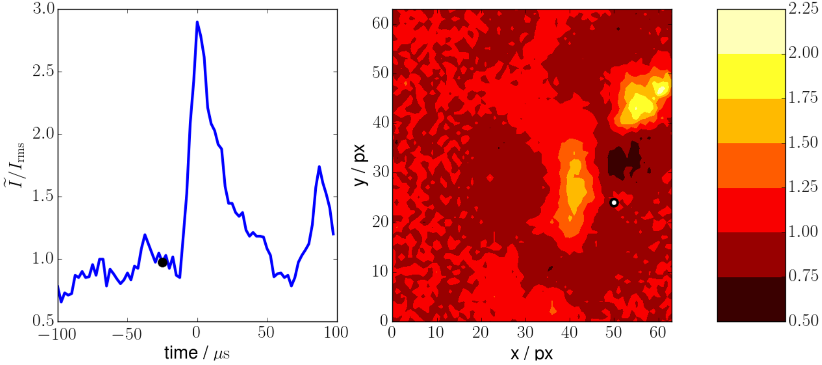
Bursts in single point measurements correspond to traversing blobs



Superpose uncorrelated pulses to model data time series

Superposition of K pulses in a time interval $[0 : T]$

$$\Phi_K(t) = \sum_{k=1}^{K(T)} A_k \phi\left(\frac{t - t_k}{\tau_\mathrm{d}}\right)$$

where k labels a pulse and

- A_k denotes the pulse amplitude
- t_k denotes pulse arrival time
- ϕ denotes a pulse shape
- τ_d denotes pulse duration time

Intermittency parameter: $\gamma = \tau_\mathrm{d}/\tau_\mathrm{w}$

Pulses arrive uncorrelated and form a Poisson process

Choose distribution for all random variables

- $P_K(K|T)$ gives the number of bursts in time interval $[0; T]$
- $P_A(A_k) \rightarrow$ distribution of pulse Amplitudes.
- $P_t(t_k) \rightarrow$ distribution of pulse arrival times.

Consider a Poisson process:

1. Pulses arrive uncorrelated: $P_t(t_k) = 1/T$
2. Avg. rate of pulse arrival is $1/\tau_\mathrm{w}$

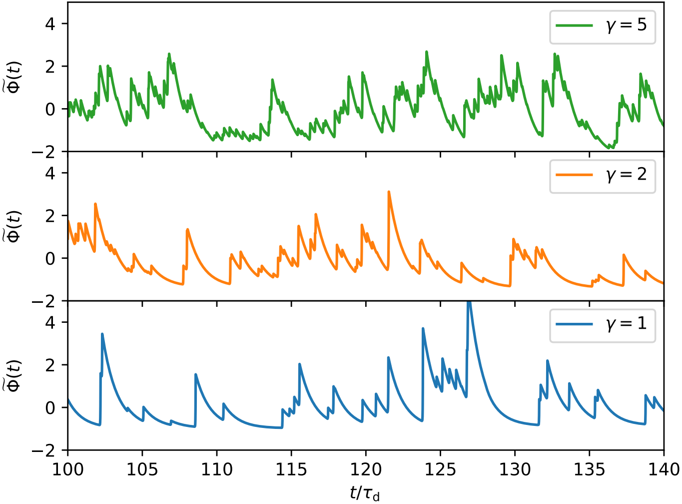
$$P_K(K|T) = \exp\left(\frac{-T}{\tau_\mathrm{w}}\right)\left(\frac{T}{\tau_\mathrm{w}}\right)^K \frac{1}{K!}$$

Exponentially distributed pulse amplitudes: $\langle A \rangle P_A(A_k) = \exp\left(A_k / \langle A \rangle\right)$

We often normalize the process as

$$\widetilde{\Phi} = \frac{\Phi - \langle \Phi \rangle}{\Phi_\mathrm{rms}}$$

Intermittency parameter governs pulse overlap



Model experimental data with double-exponential pulses

Experimental data is approximated by a double-exponential pulse shape

$$\phi(\theta) = \Theta\left(-\theta\right) \exp\left(\frac{\theta}{\lambda}\right) + \Theta\left(\theta\right) \exp\left(-\frac{\theta}{1-\lambda}\right)$$

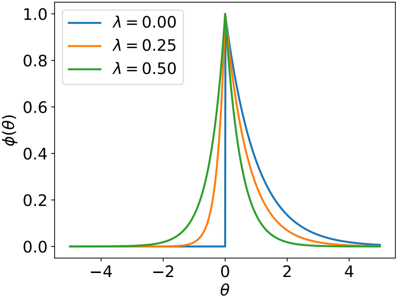
In physical units: $\theta = (t - t_k)/\tau_\mathrm{d}$, $\tau_\mathrm{d} \approx 10\mu\mathrm{s}$.

λ defines pulse asymmetry:

$$\tau_\mathrm{r} = \lambda \tau_\mathrm{d}$$
$$\tau_\mathrm{f} = (1-\lambda)\tau_\mathrm{d}$$

Notation: $I_n = \int\limits_{-\infty}^{\infty} \mathrm{d}\theta \left[\phi(\theta)\right]^n$

Normalization: $I_1 = 1$



Correlation and power spectral density depend on pulse asymmetry

Correlation function of the pulse shape is given by

$$\rho_\phi(\theta) = \frac{1}{I_2} \int\limits_{-\infty}^{\infty} \mathrm{d}\chi \phi(\chi)\phi(\chi + \theta)$$

$$= \frac{1}{1-2\lambda} \left[(1-\lambda) \exp\left(-\frac{|\theta|}{1-\lambda} \right) - \lambda \exp\left(-\frac{|\theta|}{\lambda} \right) \right]$$

Wiener-Khinchin theorem states that the power spectral density is the Fourier-transform of the autocorrelation function

$$\sigma_\phi(\omega) = \int\limits_{-\infty}^{\infty} \mathrm{d}\theta \rho_\phi(\theta) \exp\left(-i\omega\theta\right)$$

$$= \frac{2}{[1 + (1-\lambda)^2\omega^2][1 + \lambda^2\omega^2]}$$

O.E. Garcia and A. Theodorsen, Phys. Plasmas **24** 032309 (2017).

The mean of the process can be computed analytically

Averaging the process over all random variables and neglect finite box effects by extending time integration to the entire real axis:

$$\langle \Phi_K \rangle = \int\limits_{-\infty}^{\infty} \mathrm{d}A_1 P_A(A_1) \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}t_1}{T} \ldots \int\limits_{-\infty}^{\infty} \mathrm{d}A_K P_A(A_K) \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}t_K}{T} \sum_{k=1}^{K} A_k \phi \left(\frac{t - t_k}{\tau_\mathrm{d}} \right)$$
$$= \frac{K}{T} \tau_\mathrm{d} \langle A \rangle$$

Average over number of pulses K:

$$\langle \Phi \rangle = \frac{\tau_\mathrm{d}}{\tau_\mathrm{w}} \langle A \rangle$$

Mean value of the process increases with pulse overlap and average pulse amplitude.

The variance can be computed analytically

$$\langle \Phi_K^2 \rangle = \int\limits_{-\infty}^{\infty} \mathrm{d}A_1 P_A(A_1) \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}t_1}{T} \dots \int\limits_{-\infty}^{\infty} \mathrm{d}A_K P_A(A_K) \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}t_K}{T}$$

$$\sum_{k=1}^{K} A_k \phi\left(\frac{t - t_k}{\tau_\mathrm{d}}\right) \sum_{l=1}^{K} A_l \phi\left(\frac{t - t_l}{\tau_\mathrm{d}}\right)$$

This results in $K(K-1)$ terms with $k \neq l$, K terms with $k = l$.

$$\langle \Phi_K^2 \rangle = \tau_\mathrm{d} I_2 \langle A^2 \rangle \frac{K}{T} + \tau_\mathrm{d}^2 I_1^2 \langle A \rangle^2 \frac{K(K-1)}{T^2}$$

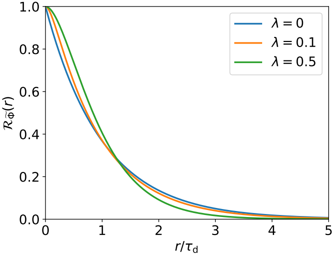
$$\Rightarrow \langle \Phi^2 \rangle = \frac{\tau_\mathrm{d}}{\tau_\mathrm{w}} I_2 \langle A^2 \rangle + \langle \Phi \rangle^2$$

where $\langle K(K-1) \rangle = \langle K \rangle^2$ has been used.

Auto-correlation is determined by the pulse shape

Auto-correlation function is computed from $\langle \Phi(t)\Phi(t+k) \rangle$

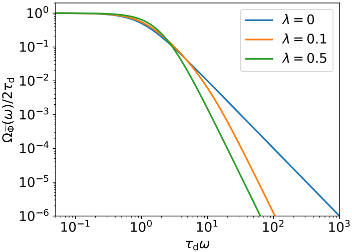
$$R_\Phi(r) = \langle \Phi \rangle^2 + \Phi_{\mathrm{rms}}^2 \rho_\phi \left(\frac{r}{\tau_\mathrm{d}} \right)$$
$$= \langle \Phi \rangle^2 + \frac{\Phi_{\mathrm{rms}}^2}{1-2\lambda} \left[(1-\lambda) \exp\left(-\frac{|r|}{(1-\lambda)\tau_\mathrm{d}} \right) - \lambda \exp\left(-\frac{|r|}{\tau_\mathrm{d}} \right) \right]$$



O.E. Garcia and A. Theodorsen, Phys. Plasmas **24** 032309 (2017).

Power spectral density

$$\Omega_\Phi(\omega) = 2\pi\langle\Phi\rangle^2\delta(\omega) + \Phi_{\mathrm{rms}}^2 \tau_{\mathrm{d}} \sigma_\phi(\tau_{\mathrm{d}}\omega)$$
$$= 2\pi\langle\Phi\rangle^2\delta(\omega) + 2\Phi_{\mathrm{rms}}^2 \frac{\tau_{\mathrm{d}}}{\left[1 + (1-\lambda)^2\, \tau_{\mathrm{d}}^2\omega^2\right]\left[1 + \lambda^2\tau_{\mathrm{d}}^2\omega^2\right]}$$



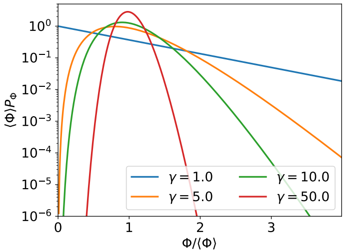
- $\lambda = 0$: Power law tail, $\sim \omega^{-2}$
- $\lambda = 1/2$: Power law tail, $\sim \omega^{-4}$
- Else: broken power law, curved spectrum.

O.E. Garcia and A. Theodorsen, Phys. Plasmas **24** 032309 (2017).

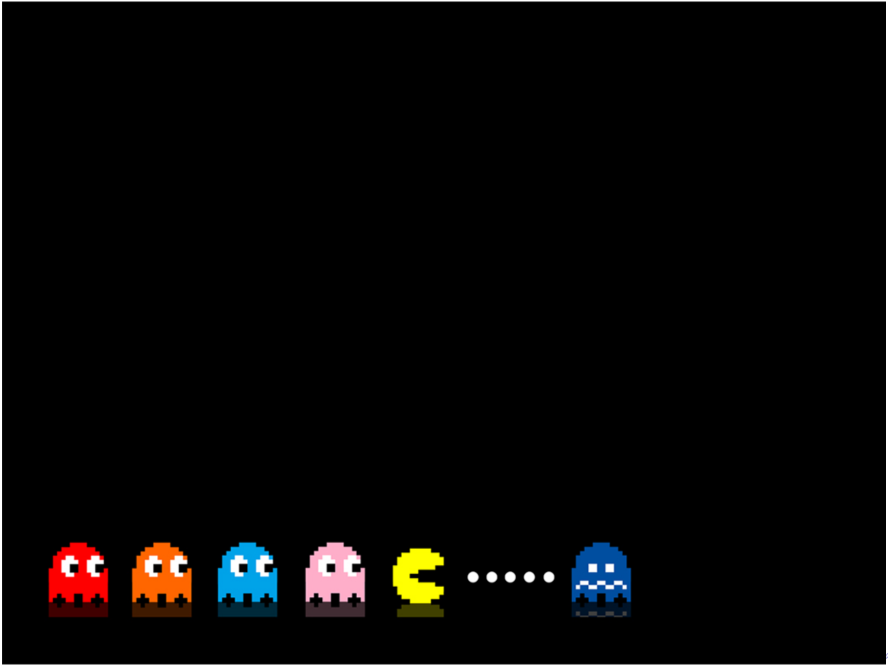
Probability distribution function

For exponentially distributed amplitudes and exponential wave forms is the process Gamma distributed:

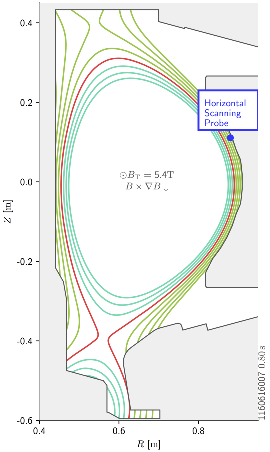
$$\langle\Phi\rangle P_\Phi(\Phi) = \frac{\gamma}{\Gamma(\gamma)} \left(\frac{\gamma\Phi}{\langle\Phi\rangle} \right)^{\gamma-1} \exp\left(-\frac{\gamma\Phi}{\langle\Phi\rangle} \right)$$



O.E. Garcia, Phys. Rev. Lett. **108** 265001 (2012).

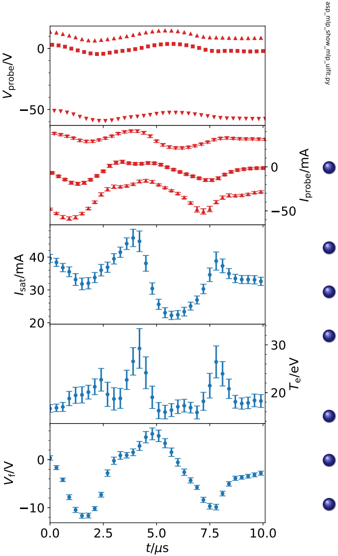


SOL fluctuations measured in a density scan



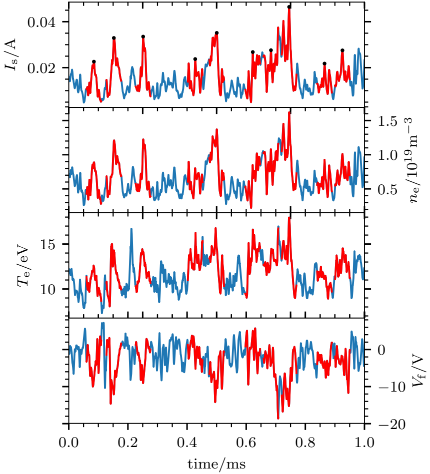
- Ohmic L-mode plasma
- Lower single-null magnetic geometry
- Density varied from $\overline{n}_e / n_G = 0.12..0.62$
- Probe head dwelled at the limiter radius
- 4 electrodes with Mirror Langmuir probes
- Approximately 1s long data time series in steady state

Mirror Langmuir Probe allows fast I_{s}, T_{e}, and V_{f} sampling



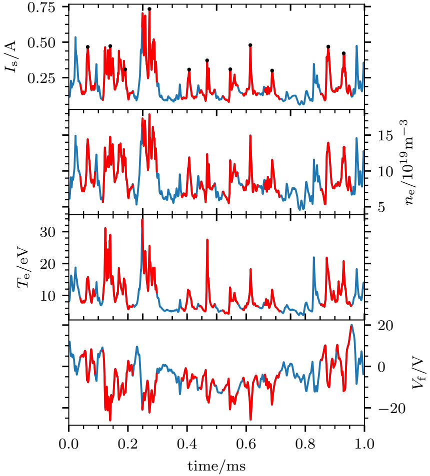
- MLP biases electrode to 3 voltages per microsecond.
- Voltage range is dynamically adjusted
- Probe current measured in each voltage state
- Fit input voltage and current is subject to 12pt smoothing (running average)
- Fit U-I characteristic on (U,I) samples
- Largest error on T_{e}.
- Resolves fluctuations on μs time scale

Low density discharge, $\overline{n}_\mathrm{e}/n_\mathrm{G} = 0.12$



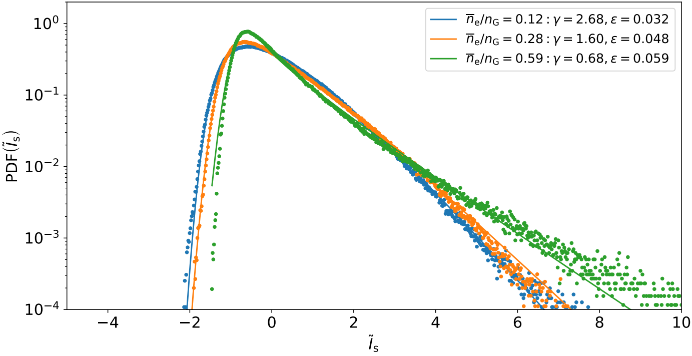
- Intermittent, large amplitude bursts in I_s.
- Bursts in n_e and T_e appear correlated
- Timescale approximately 25μs
- Irregular potential waveform

High density discharge, $\overline{n}_\mathrm{e}/n_\mathrm{G} = 0.62$



- Bursts appear more isolated
- Average density larger by factor of 10
- Average electron temperature approx. $8\mathrm{eV}$

Ion saturation current histograms are well described by a Gamma distribution



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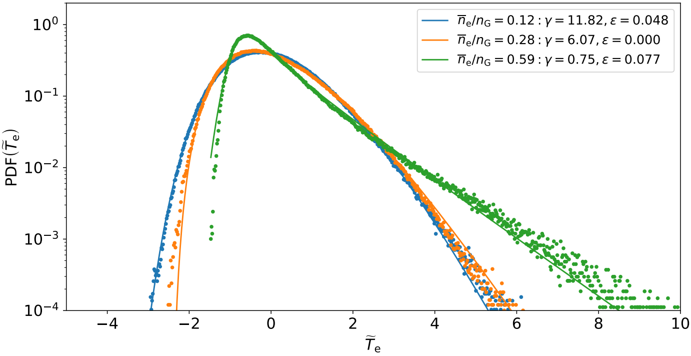
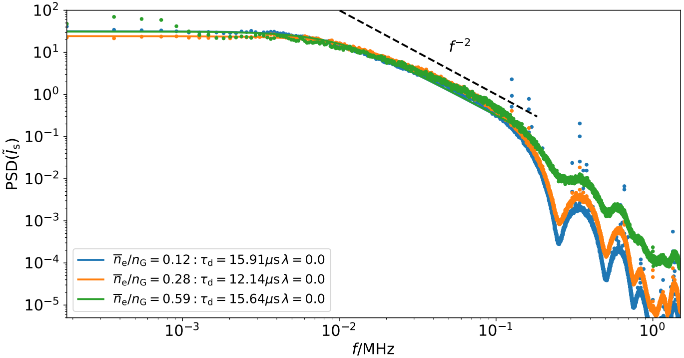
Theodorsen, O.E. Garcia, and M. Rypdal, Phys. Scr. **92** 054002 (2017)

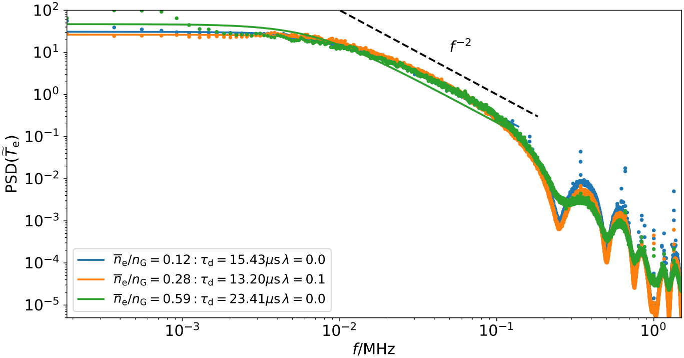
Electron temperature histograms are well described by a Gamma distribution



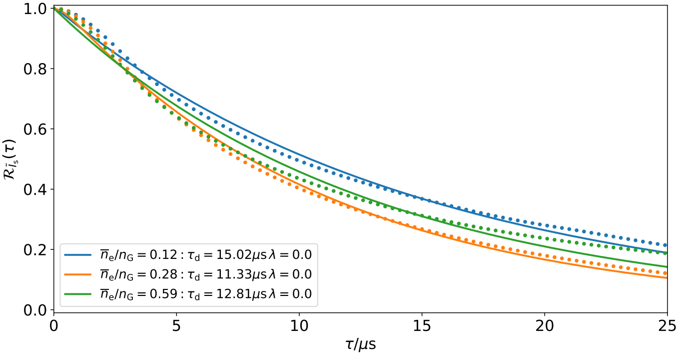
PSD of I_s shows broken power law



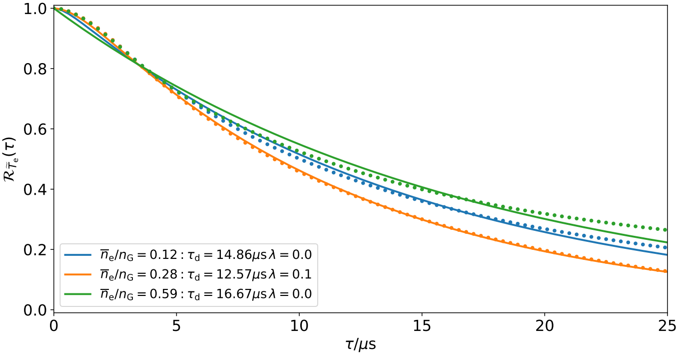
PSD of T_e shows broken power law



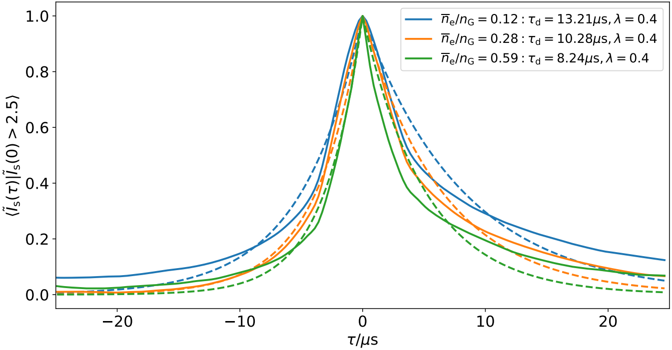
I_s shows exponential autocorrelation function



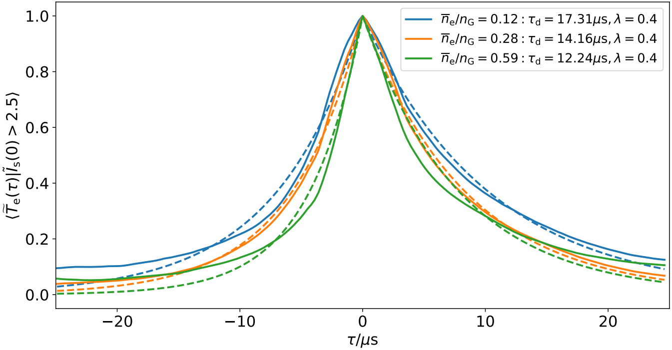
T_{e} shows exponential autocorrelation function



Bursts in I_s are approximated by double-exponential waveform

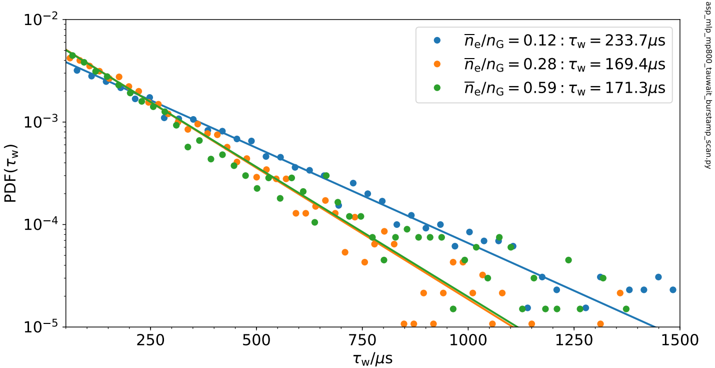


Bursts in T_e are approximated by double-exponential waveform

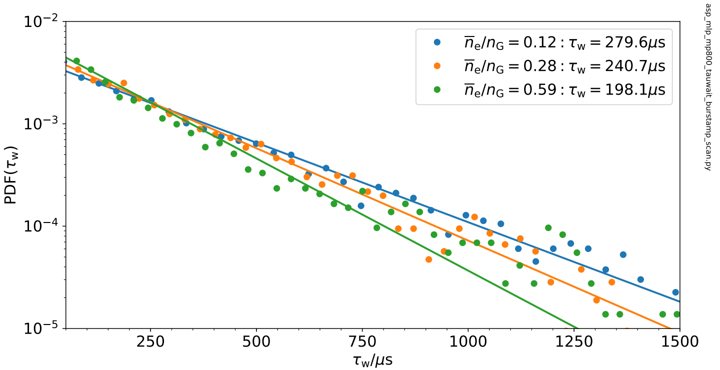


Time between bursts in I_s signal is exponentially distributed

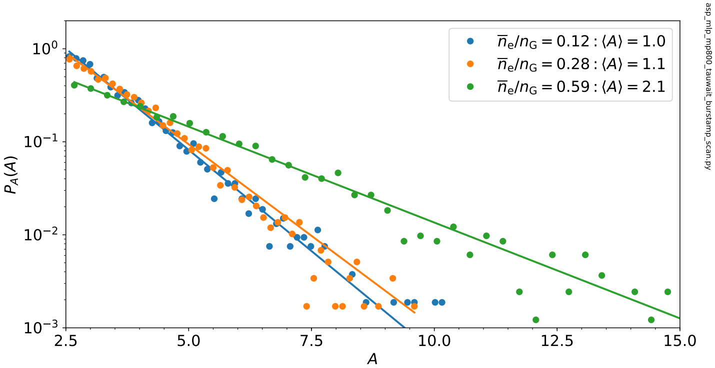
Exponential distribution describes the time between events in a Poisson process.



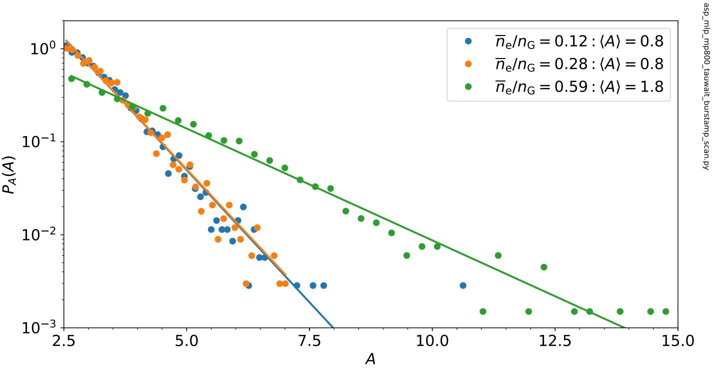
Time between bursts in T_e signal is exponentially distributed



Burst amplitude distribution - Isat



Burst amplitude distribution - Te



Conclusions

Overview of estimated parameters

	$\frac{\overline{n}_\mathrm{e}}{n_\mathrm{G}}$	γ (PDF)	$\gamma\left(\frac{\Phi_\mathrm{rms}}{\langle\Phi\rangle}\right)$	τ_d (PSD)	$\tau_\mathrm{d}, \mathcal{R}$	τ_d (CA)	τ_w	$\langle A\rangle$
I_s	0.12	2.68	8.0	15.0μs	15.0μs	13.2μs	234μs	1.0
I_s	0.28	1.60	5.7	12.1μs	11.3μs	10.3μs	169μs	1.1
I_s	0.59	0.68	4.4	15.6μs	12.8μs	8.24μs	171μs	2.1
T_e	0.12	11.82	25	15.4μs	14.9μs	17.3μs	280μs	0.8
T_e	0.28	6.07	13	13.2μs	12.6μs	14.2μs	241μs	0.8
T_e	0.59	0.75	4.6	23.4μs	16.7μs	12.2μs	198μs	1.8

Conclusions

Theory	Experimental data
Process is Gamma distributed	I_s and T_e time series are Gamma distributed
Pulses arrive uncorrelated	Waiting time between bursts in I_s and T_e is exponential distributed
Exponential distributed pulse amplitude	Burst amplitudes in I_s and T_e are expon. distributed
Double-exponential pulse shape	PSD, autocorrelation function and cond. avg. of I_s and T_e time series agree

- Less burst overlap at high densities
- Burst duration time changes little with $\overline{n}_e/n_G$.
- Burst amplitude increases with $\overline{n}_e/n_G$

Thank you for your attention.