

A Change Detector for Polarimetric SAR Data Based on the Relaxed Wishart Distribution

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Abstract—In this paper, we present an unsupervised change detection method for polarimetric synthetic aperture radar (PolSAR) images based on the relaxed Wishart distribution. Most polarimetric change detectors assume the Gaussian-based complex Wishart model for multilook covariance matrices, which is only satisfied for homogeneous areas with fully developed speckle and no texture. Liu *et al.* recently proposed a new change detection algorithm under the multilook product model (MPM) to describe the heterogeneous clutters. The improvement has come at the expense of higher computational cost since the similarity measure is based on more advanced models accounting for texture, and they contain some mathematical special functions that is difficult to evaluate such similarity measures. In this paper, we will demonstrate the ability of the relaxed Wishart distribution for textured change detection analysis. Change results on simulated and real data demonstrate the effectiveness of the algorithm.

Index Terms—Synthetic aperture radar (SAR); polarimetry; local ENL estimation; similarity measure; relaxed Wishart distribution; multilook product model; unsupervised change detection.

I. INTRODUCTION

The use of synthetic aperture radar (SAR) sensors is attractive in temporal studies, because SAR sensors do not suffer from the limitations of cloud cover and solar incidence, contrary to optical sensors. Many studies have demonstrated the potential of SAR images in time series analysis, e.g., [1], [2]. This study is devoted to the multichannel polarimetric SAR (PolSAR) sensor, which potentially provides increased detection capability, as compared to single-polarization SAR.

The seminal work on test statistics for change detection in multilook PolSAR images was done by Conradsen *et al.*, who proposed a generalized likelihood ratio test (LRT) for equality of two complex covariance matrices and gave the asymptotic sampling distribution for the test statistic [3]. Akbari *et al.* proposed a new change detection statistic for multilook PolSAR images based on the complex-kind Hotelling-Lawley trace statistic [4]. Both approaches assume that the covariance matrices follow the Gaussian-based complex Wishart distribution. Non-Gaussian probability distributions provide better representation of the data for areas with pronounced heterogeneity, or high texture. Such distributions can be obtained using a matrix version of the product model [5]. Several statistical models have been proposed in the literature to account for texture, such as \mathcal{K}_d , \mathcal{G}_d^0 , or \mathcal{U}_d distributions. A change detection algorithm for non-Wishart PolSAR data described by the

multilook product model (MPM) was recently presented by Liu *et al.* [6]. Similarity measures based on more advanced distributions under the multilook product model allows for improved change detection capability for areas with high texture, but this comes at the cost of higher mathematical complexity. As an alternative to these complex models, we derive the similarity measure for the relaxed Wishart distribution proposed in [7], which has a simpler mathematical form. Here the equivalent number of looks (ENL) of the standard Wishart model, which is traditionally considered as an image constant, was allowed to vary between pixels.

The paper is organised as follows: Section II presents the data models for the PolSAR data. Section III details the theory of the proposed polarimetric change detection: the local ENL estimation and the similarity measure under the relaxed Wishart distribution. Experimental results with synthetic and real data are given in Section IV and conclusions in Section V.

II. DATA MODELS

One-look polarimetric SAR data are generally characterized by the scattering (or target) vector $\mathbf{k} \in \mathbb{C}^d$ which contains the complex scattering coefficients measured in d polarimetric channels. For fully developed speckle in a homogeneous (constant radar cross section) area, it is commonly assumed that the scattering vector elements of \mathbf{k} jointly follow a circular complex and multivariate Gaussian distribution [8], denoted as $\mathcal{N}_d^{\mathbb{C}}(\mathbf{0}, \Sigma)$, with zero mean vector, covariance matrix $\Sigma = E\{\mathbf{k}\mathbf{k}^H\}$, and dimension d . The probability density function (pdf) of \mathbf{k} is thus

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^d |\Sigma|} \exp(-\mathbf{k}^H \Sigma^{-1} \mathbf{k}), \quad (1)$$

where $(\cdot)^H$ and $|\cdot|$ mean the Hermitian transposition and determinant operators, respectively. The scattering vectors might also be transformed into multilooked sample covariance matrices, and the L -look covariance matrix is characterized by

$$\mathbf{C} = \frac{1}{L} \sum_{\ell=1}^L \mathbf{k}_{\ell} \cdot \mathbf{k}_{\ell}^H, \quad (2)$$

where L is the nominal number of looks used for averaging. Hence, after multilooking, each pixel in the image is a realization of the $d \times d$ stochastic matrix variable denoted \mathbf{C} , and the image is referred to as the multilook complex (MLC)

covariance image. It follows from the Gaussian assumption that if $L \geq d$ and the $\{\mathbf{k}_\ell\}_{\ell=1}^L$ are independent, then the sample covariance matrix \mathbf{C} follows a *scaled complex Wishart* distribution [9], denoted $s\mathcal{W}_d^{\mathbb{C}}(L, \Sigma)$, whose pdf is

$$p_{\mathbf{C}}(\mathbf{C}) = \frac{L^{Ld} |\mathbf{C}|^{L-d}}{\Gamma_d(L) |\Sigma|^L} \exp(-L \text{tr}(\Sigma^{-1} \mathbf{C})), \quad (3)$$

where

$$\Gamma_d(L) = \pi^{\frac{d(d-1)}{2}} \prod_{i=1}^d \Gamma(L - i + 1) \quad (4)$$

is the multivariate gamma function of the complex kind [9], while $\Gamma(\cdot)$ is the standard Euler gamma function. In reality, the scattering vectors \mathbf{k}_ℓ are correlated, and the nominal number of looks, L , is in practise substituted with an ENL, $\hat{L} < L$. This is estimated for the whole data set as a constant and global value. The relaxed Wishart distribution, denoted $\mathcal{RW}_d^{\mathbb{C}}(L_r, \Sigma)$ and introduced in [7] is the basis of our change detection algorithm. It has an identical functional form with the standard Wishart distribution in (3). The difference is that L_r is considered as a free parameter, which varies between multilook pixels and can be assumed to absorb effects of texture.

III. POLARIMETRIC CHANGE DETECTION

Let $\mathcal{A} = \{\mathbf{A}(i, j); 1 \leq i \leq I, 1 \leq j \leq J\}$ and $\mathcal{B} = \{\mathbf{B}(i, j); 1 \leq i \leq I, 1 \leq j \leq J\}$ be two equal-sized and coregistered MLC PolSAR images acquired over the same geographical area at times t_a and t_b , where I and J are the number of rows and columns of the images, respectively. For simplicity, we shall assume that images \mathcal{A} and \mathcal{B} have the same original resolution. Change detection is performed at the multilook level, i.e., the change map will have the same resolution as the MLC data. It is assumed that \mathcal{A} and \mathcal{B} are geometrically corrected, coregistered and radiometrically calibrated.

A. Local ENL Estimation

We here present how the ENL can be estimated from single look complex (SLC) data. The polarimetric whitening filter (PWF) from [10] is chosen to estimate the ENL from the scattering vectors within the multilook window. It is defined as

$$y = \mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}, \quad (5)$$

where \mathbf{C} denotes the sample covariance matrix from (2). The components of y have distributions:

$$y \sim [\mathcal{N}_d^{\mathbb{C}}(0, \Sigma)]^H \times [s\mathcal{W}_d^{\mathbb{C}}(L, \Sigma)]^{-1} \times [\mathcal{N}_d^{\mathbb{C}}(0, \Sigma)]. \quad (6)$$

The asymptotic distribution of y is known to be [11], [12]

$$y \sim \frac{Ld}{L-d+1} F_{2d, 2(L-d+1)}, \quad (7)$$

where $F_{a,b}$ denotes an F-distribution subscripted with its shape parameters a and b . The Fisher distribution defined in [13] has

been extended with a location parameter μ and uses shape parameters $\alpha = 2a$ and $\beta = 2b$, to be connected with intensities of complex variables in the PolSAR data. Therefore, the Fisher distribution is denoted as $\mathcal{F}(\mu, \alpha, \lambda)$, and we can write

$$y \sim \frac{Ld}{L-d+1} \mathcal{F}_{1,d,L-d+1}. \quad (8)$$

The ν th-order log-cumulants for the Fisher distribution are given by [13]

$$\begin{aligned} \kappa_1\{y\} &= \psi^{(0)}(d) - \psi^{(0)}(L-d+1) + \log\left(\frac{L-d+1}{d}\right) \\ \kappa_{\nu>1}\{y\} &= \psi^{(\nu-1)}(d) + (-1)^\nu \psi^{(\nu-1)}(L-d+1), \end{aligned} \quad (9)$$

where $\psi^{(\nu)}$ is the polygamma functions of order ν , L is the ENL, and d is the polarimetric dimension. The method of log-cumulant (MoLC) for SLC data consists of solving a set of log-cumulant equations defined by (9) for the unknown ENL value, with sample log-cumulants inserted for the population log-cumulants of y . The sample log-moments of order ν are computed from data as

$$\langle \mu_\nu\{y\} \rangle = \frac{1}{L} \sum_{\ell=1}^L (\log y_\ell)^\nu, \quad (10)$$

where the y_ℓ are computed from a sample of L scattering vectors $\{\mathbf{k}_\ell\}_{\ell=1}^L$, that are also used to compute the sample covariance matrix, \mathbf{C} , which goes into y_ℓ . The sample log-moments from (10) can be combined into the sample log-cumulants using the relations between log-moments and log-cumulants [9]

$$\begin{aligned} \kappa_1 &= \mu_1, \\ \kappa_2 &= \mu_2 - \mu_1^2, \\ \kappa_3 &= \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3. \end{aligned} \quad (11)$$

B. Similarity Measure for PolSAR Change Detection

In order to apply change detection to PolSAR data, we need to define an appropriate matrix distance measure. We want to measure the similarity between their corresponding MLC format measurements, i.e., $\mathbf{A}(i, j)$ and $\mathbf{B}(i, j)$. The sample covariance matrices are assumed to be statistically independent with dimension $d \times d$. They are defined on the cone of Hermitian and positive definite matrices. We further assume that the sample covariance matrices \mathbf{A} and \mathbf{B} follow relaxed Wishart distributions, potentially with different distribution parameters, which is denoted

$$\mathbf{A} \sim \mathcal{RW}_d^{\mathbb{C}}(L_a, \Sigma_a) \text{ and } \mathbf{B} \sim \mathcal{RW}_d^{\mathbb{C}}(L_b, \Sigma_b). \quad (12)$$

We propose a similarity measure under the relaxed Wishart distribution in which the ENL value is allowed to vary between multilook pixels. Each image is processed by first computing the sample covariance matrix in a sliding window of size $k \times k$ and then local ENL estimation from the technique in Section

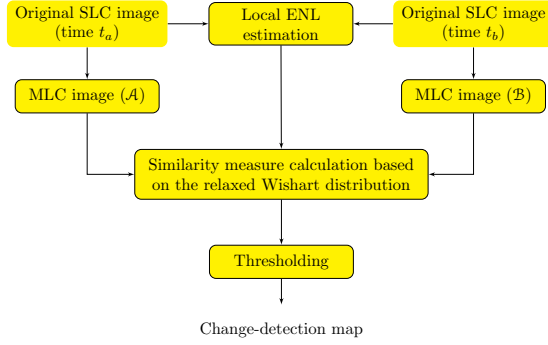


Fig. 1. General block diagram of the proposed change detection algorithm for PolSAR data.

III-A. Polarimetric change detection is therefore performed by choosing between the hypotheses:

$$\begin{aligned} H_0 &: \Sigma_a = \Sigma_b \text{ and } L_a = L_b, \\ H_1 &: \Sigma_a \neq \Sigma_b \text{ or/and } L_a \neq L_b. \end{aligned} \quad (13)$$

The null hypothesis (H_0) corresponds to *no-change* and the alternative hypothesis (H_1) to *change*. The generalized likelihood ratio test (GLRT) statistic is given by

$$Q = \frac{\max L_{H_0}(\Sigma, L | \mathbf{A}, \mathbf{B})}{\max L_{H_1}(\Sigma_a, \Sigma_b, L_a, L_b | \mathbf{A}, \mathbf{B})}, \quad (14)$$

The likelihood function under H_0 is

$$L_{H_0}(\Sigma, L | \mathbf{A}, \mathbf{B}) = p(\mathbf{A}; L, \Sigma) p(\mathbf{B}; L, \Sigma), \quad (15)$$

which is maximized by the maximum likelihood (ML) estimator $\hat{\Sigma} = (\mathbf{A} + \mathbf{B})/2$. The values of L_a and L_b under H_0 are replaced with an average of the estimated ENLs, i.e., $\hat{L} = (\hat{L}_a + \hat{L}_b)/2$. The likelihood function under H_1 is

$$L_{H_1}(\Sigma_a, \Sigma_b, L_a, L_b | \mathbf{A}, \mathbf{B}) = p(\mathbf{A}; L_a, \Sigma_a) p(\mathbf{B}; L_b, \Sigma_b), \quad (16)$$

which gains its maximum value for the ML estimators $\hat{\Sigma}_a = \mathbf{A}$ and $\hat{\Sigma}_b = \mathbf{B}$. The ENLs for pixels \mathbf{A} and \mathbf{B} are estimated from the estimation technique in Section III-A. The GLRT statistic thus becomes

$$\begin{aligned} Q &= \frac{p(\mathbf{A}; \hat{L}, \hat{\Sigma}) p(\mathbf{B}; \hat{L}, \hat{\Sigma})}{p(\mathbf{A}; \hat{L}_a, \hat{\Sigma}_a) p(\mathbf{B}; \hat{L}_b, \hat{\Sigma}_b)} \\ &= \frac{(\hat{L}_a + \hat{L}_b)^{(\hat{L}_a + \hat{L}_b)d}}{\hat{L}_a^{\hat{L}_a} \hat{L}_b^{\hat{L}_b}} \frac{|\mathbf{A}|^{\hat{L}_a} |\mathbf{B}|^{\hat{L}_b}}{|\mathbf{A} + \mathbf{B}|^{\hat{L}_a + \hat{L}_b}} \\ &\quad \cdot \frac{|\mathbf{A}\mathbf{B}|^{\frac{\hat{L}_a + \hat{L}_b}{2}d - d}}{|\mathbf{A}|^{\hat{L}_a - d} |\mathbf{B}|^{\hat{L}_b - d}} \frac{\Gamma_d(\hat{L}_a) \Gamma_d(\hat{L}_b)}{(\Gamma_d(\frac{\hat{L}_a + \hat{L}_b}{2}))^2}, \end{aligned} \quad (17)$$

If and if only $\hat{L} = \hat{L}_a = \hat{L}_b$,

$$\ln Q = \hat{L}(2d \ln 2 + \ln |\mathbf{A}| + \ln |\mathbf{B}| - 2 \ln |\mathbf{A} + \mathbf{B}|). \quad (18)$$

This distance measure is called the Bartlett distance in [14].

C. Summary

In summary, the block diagram of the proposed change detection approach in multipolarization SAR data is shown in Fig. 1 which is made up of three main steps: 1) local ENL estimation for each image in the pair of multitemporal PolSAR images; 2) calculation of similarity measure; and 3) distinguishing change from no-change through the application of a desired thresholding algorithm such as the Kittler and Illingworth (K&I) algorithm [15] to the test statistic.

IV. EXPERIMENTAL RESULTS

A. Simulated Data Set

We generated two multiclass multilook polarimetric K-distributed test images to validate the change detection algorithm. The simulated data was 25-look, quad-pol, with a range of texture, brightness and polarimetry values taken from real images [16]. The test images represent times t_a and t_b . They are shown in Fig. 2(a) and 2(b) as Pauli images. The second image is synthesized with three change areas [4]. The images are simulated with no spatial correlation, such that the locally estimated ENL values, obtained with the MoLC estimator and with window size of $k = 5$, are equal to the nominal number of looks for about 90% of the pixels.

Fig. 2(c) and 2(d) show the difference maps using the proposed similarity measure under the relaxed Wishart distribution and the MPM in [6], respectively. The distance measure based on the MPM provides smooth results and higher contrast between change and no-change classes, but it is sensitive to the window size. Some of estimation windows contain a mixture of pixels with different classes and boundary between change and no-change classes is blurred due to an effect of mixed classes, which increases with window size.

B. Real Data Example

Two quad-pol SAR images acquired by RADARSAT-2 captured over an urban area in Suzhou, East China¹ on 9 April, 2009 and 15 June, 2010, are used for the experiment with real SAR data. Fig. 3(a) and 3(b) present the corresponding Pauli RGB color composite image.

A window size of $k = 5$ is selected for calculating the sample covariance matrix and the ENL estimation. Fig. 3(c) and 3(d) show maps of the locally estimated ENL values for each image. The presence of multiple classes and texture within the estimation window leads to a lowered shape parameter. After generating the covariance matrix and ENL, the difference map can be extracted using the proposed similarity measure in (17), as shown in Fig. 3(e). We compare this difference map with the MPM-based, which is presented in 3(f), and the Wishart-based similarity measures in [3], which is not included in the paper due to the limited space. The change areas in Fig. 3(e) and 3(f) are bright on a background of dark no-change pixels. Both the wishart-based and MPM-based methods may lead to missed detection in the red box areas in Fig. 3(e).

¹The authors would like to thank Meng Liu, and the Chinese Academy of Sciences for providing the RADARSAT-2 data set. Copyright notice: the data are copyrighted by the MDA/CSA.



Fig. 2. Experiment with a pair of simulated 25-look quad-pol K-Wishart distributed images. (a) and (b) Pauli decomposition composite images with class labels at two different times with changes. (c) Theoretical result of change detection, where white is change and gray is no-change. (d) and (e) Difference maps of similarity measures under the relaxed Wishart and the MPM.

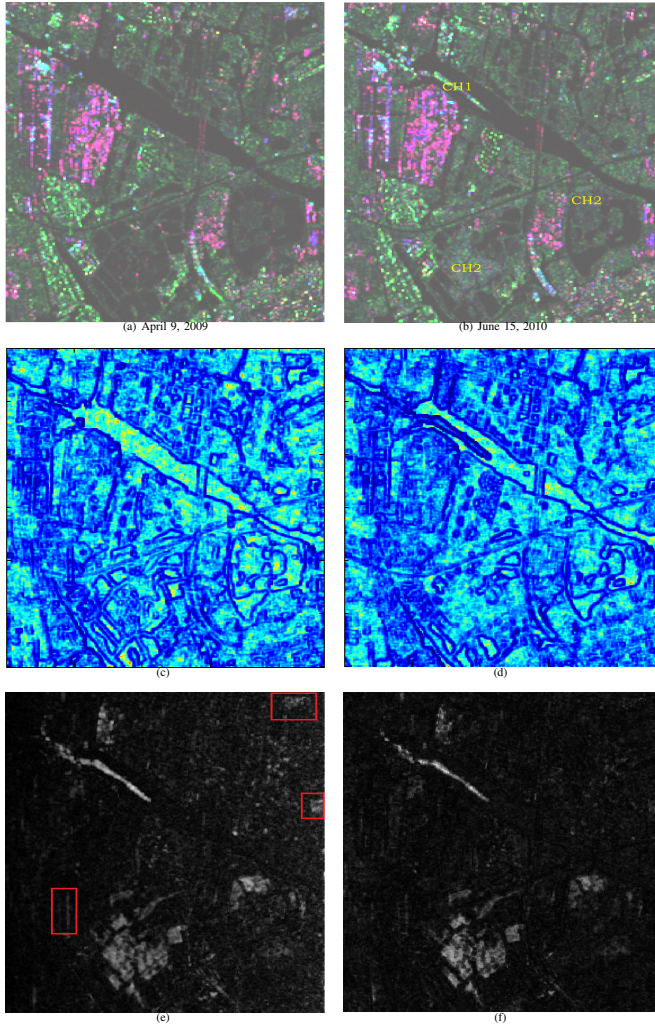


Fig. 3. Real experimental data set. (a) and (b) RGB Pauli composite images (R: [HH-VV]; G: 2[HV]; B: [HH+VV]) with RADARSAT-2 data captured on 9 April, 2009 and on 15 June, 2010, multilooked with window size of $k = 5$. (c) and (d) Local ENL estimates. (e) and (f) Difference maps from similarity measures under the relaxed Wishart and the MPM.

V. CONCLUSIONS

In this paper, we propose a novel method for SAR change detection based on the relaxed Wishart distribution. The local ENL estimation from the PWF implemented with scattering

vectors and sample covariance matrix is the key input to the proposed similarity measure. Application of the change detection algorithm was practically demonstrated on simulated and real PolSAR data sets.

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