

Can Sámi Braiding Constitute a Basis for Teaching Discrete Mathematics? Teachers and Researchers' Investigations

Anne Birgitte Fyhn
Associate Professor
UiT – The Arctic University of Norway
anne.fyhn@uit.no

Ylva Jannok Nutti
Associate Professor Sámi University College, Norway

Maja Dunfjeld
Dr art., Harran, Norway

Ellen J. Sara Eira
Principal. Kautokeino Middle School, Norway

Ann Synnøve Steinfjell
Assistant Professor. Sámi University College, Norway

Tove Børresen
Teacher. Kautokeino Middle School, Norway

Ole Einar Hætta
Teacher. Kautokeino Middle School, Norway

Svein Ole Sandvik
Teacher. Kautokeino Middle School, Norway

Abstract

A group of Sámi middle school mathematics teachers cooperated with researchers over a period of three years in investigating *ruvden* (Sámi braiding). The aim was to find possibilities for teaching discrete mathematics based on *ruvden*. The Sámi are an Indigenous people of the Arctic and their braidings are intertwined with Sámi traditional knowledge. The teachers presented two different approaches to the *ruvden* procedure. One researcher presented a third approach and later, two students came up with a fourth. The analysis reveal that a) the four approaches reflect different aspects of Sámi traditional knowledge and b) investigations of *ruvden* may lead to two aspects of discrete mathematics; transitions from numbers to variables and combinatorics.

Introduction

This paper focuses on *ruvden*, a round-shaped Sámi braiding. The Sámi are an Indigenous people of the Arctic who live in the northern part of Scandinavia and on the Kola Peninsula of Russia. The traditional Sámi livelihoods are reindeer herding and combinations of smallholdings and fishery. However, during the last 50 years, approximately one third of the rural Sámi population moved to more urban areas (Sørli & Broderstad, 2011). The Sámi use a variety of braidings in different colors and the different braiding techniques are named after the cords. The numerous Sámi braided cords are used for different purposes and in addition, each cord communicates a message about the wearer's gender, family relations and regional belonging. The cords in Figure 1 are in red, white and blue, the traditional colors in the Guovdageaidnu area. Figure 2 shows examples of cords in different colors. This paper presents a study of relationships between *ruvden* and mathematics. The study is part of a larger research project from 2010 – 2014, creating mathematics teaching based on Sámi culture. The mathematics teachers at Guovdageaidnu middle school¹ in Norway cooperated with researchers in investigating *ruvden*², with a focus on how to perform the braiding procedure. Our data are from the first nine months of this project.

¹ The Norwegian 'Ungdomsskole' is for grades 8 – 10.

² The North Sámi verb *ruvdet* means to perform the *ruvden* braiding procedure, while the noun *ruvden* is the name of the particular braiding procedure. The outcome of *ruvdet* is *ruvdebáttit*, *ruvdet* cords.



Figure 1 Sámi fur shoes from Guovdageaidnu, Norway. The *ruvden* cord is connected to a woven band. (Photo: Ellen J. Sara Eira)



Figure 2 Four *ruvdebatit*, *ruvdet* cords, made by the teenage girls Ann-Kristina and Ronja. These cords are *ruvdet* with four threads

Bishop (1988) treats mathematics as a cultural product that has developed from six various activities: *counting, locating, measuring, designing, playing and explaining*. These activities, which occur in every cultural group, are both necessary and sufficient for the development of mathematical knowledge. *Counting* here refers to “[t]he use of a systematic way

to compare and order discrete phenomena” (p. 182). For instance the systematics in ordering and re-ordering of threads when braiding. Thus, the mathematical activity *counting* results in a variety of discrete mathematics. *Locating* includes position, change in position, and reflection, as well as orientation, journey, rotation, and angles. *Measuring* includes quantifying qualities for comparison and ordering, and measure-words. *Designing* means creating a shape or design for an object, making the object itself, or symbolizing it in some conventional way. For instance *designing* the different patterns of the cords in Figure 2. *Playing* means devising or engaging in games and leisurely activities with more or less formalized rules, like the rules for how to braid the different cords in Figure 2. *Explaining* includes finding ways to account for the existence of a phenomenon. The activities takes place on the culture’s premises, not on the premises of Western mathematics; explaining *ruvden* means to explain how and why you use one particular *ruvden* cord for one particular purpose as well as how you perform the braiding.

The study of mathematics based on *ruvden* is an example of research within Indigenous culture. The challenge is to deal with tensions between Western traditional mathematics and Sámi cultural practice. In order to face these tensions, our first step is to investigate *ruvden* as a Sámi traditional practice. During the first nine months of the research, four different approaches to the *ruvden* braiding procedure were uncovered; two teachers and one researcher, who are Sámi women, of different age and from different parts of Sápmi, provide three different approaches to *ruvden*, while two Sámi teenage girls provide the fourth approach. We analyze these approaches with respect to Sámi traditional knowledge and to Bishop’s six activities. The first research question is, “How do four different approaches to *ruvden* relate to Sámi traditional knowledge and to Bishop’s six basic activities?” The purpose of the paper is to provide insight into how different approaches to *ruvden* together contribute to a cultural product that may

function as a basis for teaching discrete mathematics. The second research question is, “How does the outcome of Bishop’s activities applied to *ruvden* relate to the learning goals of discrete mathematics in the mathematics curriculum?”

According to the research questions, our paper first presents Sámi traditional knowledge and relations between mathematics and culture. Then follows a presentation of the Sámi mathematics curriculum. The first part of the analysis reveals how the four approaches reflect Sámi traditional knowledge. The second part reveals how the outcome of the four approaches relates to Bishop’s (1988) six activities; activities that are structured differently than the Western categorization into algebra, calculus and geometry. The final analysis reflects on *ruvden* from a mathematics education point of view.

Knowledge and Knowledge Transfer

The local cultural context is part of the teaching of *ruvden* for Sámi students, because *ruvden* is intertwined with Sámi traditional knowledge. Mathematics, on the other hand, is a different kind of knowledge; it is an invention by Western societies. Culture is ideas, values, rules, norms, codes and symbols that a person takes over from the previous generation, something that most likely is changed when one tries to transfer it to the next generation (Klausen, 1992). Culture is therefore tied to tradition; more precisely, how it exists in the human experience. Culture is the experience that puts us in the intersection between past and future, between individuality and group community (Eriksen, 2001). Consequently, this paper is about how a cultural context can be respectfully considered in relation to mathematics.

Sámi traditional knowledge

Sámi traditional knowledge is about how to use nature and its resources and how to adapt and transform purchased materials for use in the local community (Sara, 2004). People who

ruvdet, or do handicraft, work with cultural expressions that arise out of the culture's traditional knowledge (Guttorm, 2007). Earlier, mainly women made clothes and the younger women learned how to sew, braid and weave from the elders (Porsbo, 1988). Traditional knowledge "... has been passed down from generation to generation both orally and through work and practical experiences" (Porsanger & Guttorm, 2011, p. 18).

Sámi traditional clothing varies between different areas and continues to be made of reindeer fur, leather, wool and homespun yarn. Today, other fabrics also are used. The braided and weaved bands and cords incorporated into the costumes are often made with woolen yarn (Porsbo, 1988). The braids have different names, related to the cord's application (Guttorm & Labba, 2008). *Ruvden* cords have a round shape and have practical, decorative and symbolic functions (Porsbo, 1988; Dunfjeld, 2001/2006). For example, the *ruvdet* cords can function as part of a shoelace like in Figure 1. The use of colors and patterns depends on what the clothing is used for and by whom it is used. There are different colors and patterns for men and women and specific colors are associated with a region.

Ornamentations and visual patterns complement and replace verbal language. A premise for understanding this knowledge is knowing the culture, code and underlying phenomena behind the ornaments, symbols and patterns (Dunfjeld, 2001/2006). Dunfjeld warns against situations where ornaments are treated just as decorations and not as reservoirs of meaning and hidden knowledge. The Swedish song diva Carola Häggkvist believed she was cool when she entered Kiruna in Northern Sweden wearing a Sámi cap. She did not know that the cap she had chosen was a cap for elderly men and that her behavior thus was disrespectful (Färsjö, Hegeval & Johansson, 2008).

Sámi cultural practice includes the use of several bodily units of measure like *goartil*, the distance between the tips of stretched thumb and stretched forefinger, *lávki*, the distance of one step, and *salla*, fathom (Jannok Nutti, 2010; Fyhn, Eira & Sriraman, 2011). These bodily measure units differ from person to person.

The main goal of Sámi child-rearing is to develop independent and responsible individuals who can master their lives under given conditions in a given society (Balto, 2005). The focus is on the learning process and less on teaching, so evidently experience-oriented learning is favored. Trial and error, trying out things when you are expected to fail as well as to succeed, is an important part of the learning process. This is important in order to develop independent individuals. In Sámi homes children are allowed to experiment and fail; they are expected to learn by doing. “One mother who brings up her children in the traditional way says that you should never hide sharp knives or protect children from the hot oven; sheltering them too much will not give them the training they need to manage their everyday life” (Buljo, 1999, in Balto, 2005, p. 102). According to Balto, Sámi children also learn by observing grown-ups and imitating their activities. Guttorm (2011) distinguishes between two kinds of knowledge, *diehtit* (knowledge of an action / to know something) and *máhttit* (the ability to perform the action). Observation and imitation are one way of achieving *máhttit*, for instance when a child learns how to braid.

Storytelling is important for passing on knowledge from one generation to another (Balto, 2005; Nergård, 2006; Jannok Nutti, 2007). Such approaches are also common in other Indigenous communities. The Navajo have a long standing tradition of learning through storytelling (Pinxten, 1994) and Yup’ik elders tell stories to pass on norms and values indirectly to the younger generations (Lipka, Andrew-Ihrke & Yanez, 2011). Stories may report concrete

experiences and contain practical knowledge and can therefore provide good advice. Gaski (1998) points out that Sámi stories often follows a pattern. The Sámi word *girji* has three meanings: *book*, *letter*, and *pattern* (Nielsen, 1932/1979). According to Gaski, the word *girji* can be developed into *girjálášvuohhta*, literature. Directly translated, the Sámi word for literature means “something that follows a pattern.”

Through stories, Sámi children indirectly learn norms and values. Sámi stories present rules for individuals’ relations with other humans, animals and nature, and express cultural knowledge about life and survival in vulnerable situations (Nergård, 2006; Balto, 2005; Pollan, 1997). For that reason, Sámi stories and storytelling serve different purposes on different occasions.

Mathematics Related to Culture

According to Bishop (1990), Western mathematics is one of the most powerful weapons for the imposition of Western culture on non-Western cultures. It had the status of culturally neutral phenomenon, and this was conventional wisdom until the 1970s. Mathematical ideas, however, like other ideas, are human constructions, constructed within a cultural context with a history. According to Averill et al. (2009), one key goal of culturally responsive teaching is the development of cultural competence. Teachers and teacher educators also need tools to ensure that culturally responsive teaching moves beyond specific exemplars. This paper focuses on teachers’ investigations of a braiding that belongs to Sámi traditional knowledge, investigations that develop their cultural competence. Because Sámi traditional knowledge is not a culturally neutral phenomenon, there are tensions between Sámi traditional knowledge and Western mathematics.

Pattern is one outcome of Bishop's (1988) activity *designing*. From a Western perspective, Devlin (1998) claims that mathematics is the science of patterns, it is a way of looking at the world. Zazkis and Liljedahl (2002) indicate that patterns are the heart and soul of mathematics, but the exploration of patterns does not always stand on its own as a curricular topic or activity.

Just as each cultural group generates its own language and religious beliefs, each cultural group is capable of generating its own mathematics (Bishop, 1988). Perspectives from D'Ambrosio (1999), Barton (1999) and Bishop (1988) consider mathematics from a cultural point of view. Opposed to D'Ambrosio and Barton, Bishop has comparison and ordering of discrete phenomena as one category and therefore, Bishop's is most appropriate for describing the relationship between *ruvden* and mathematics. According to D'Ambrosio (1999), each culture has developed its own ways, styles and techniques for doing and responding to the search for explanations, understanding and learning. These systems of knowledge use inference, quantification, comparison, classification, representation and measuring. Western mathematics can be considered as just one such system. Barton (1999) describes mathematics as a system that is used to make meaning of quantities, relations, and space, a "QRS system". In *ruvden*, the quantity (Q) has to be a multiple of four, while each step in the braiding procedure may be described as relations (R) between the threads, while the threads are moved in different directions in a three-dimensional space (S). The vocabulary here refers to quantity not as an exact number; relations between the threads constitute the organization of the threads in the braiding procedure; and space is expressed through the words "up"/"down", "above"/"below" and "left"/"right".

The Sámi Mathematics Curriculum

Indigenous teachers and parents wanting, on the one hand, for Indigenous children to grow up with a strong Indigenous identity and, on the other, for them to be successful at school and later in society, and have the opportunity to obtain well-paid jobs (D'Ambrosio, 2001; Meaney, 2001; Jannok Nutti, 2010, 2013). Sámi teachers and parents fear that a Sámi mathematics education has to omit some important topics from the curriculum, in order to have time to focus on cultural issues. Therefore, the Sámi mathematics education has to restore the cultural dignity of the pupils, and include the learning goals of the national mathematics curriculum.

Norway is currently the only country with a Sámi curriculum (Norwegian Directorate for Education and Training, 2007). This curriculum underlines that the teaching shall be based on Sámi culture, language and values. Norway has developed an overarching general Sámi curriculum in addition to Sámi curricula for most subjects. The Sámi mathematics curriculum, however, is a mere translation of the national one, but in addition, the overarching goals of the Sámi curriculum have to be considered. In their evaluation of Norway's 1997 curriculum, Hirvonen and Keskitalo (2004) point out that there is a need for a curriculum change in order for Sámi culture to become the basis and premise for the teaching rather than just an appendix. The 2006 Sámi curriculum points out that the Sámi School shall facilitate quality teaching based on Sámi language, culture and society (Norwegian Directorate for Education and Training, 2007).

The lack of a Sámi mathematics curriculum means that the national textbooks in mathematics are translated into Sámi languages; the curriculum's overarching goals are not considered in this subject. Thus, it is up to each mathematics teacher to involve cultural knowledge in his or her teaching. Generally, this results in no cultural implementation in

mathematics (Jannok Nutti, 2010; 2013). Jannok Nutti (2007) investigated Sámi handcrafters' and reindeer herders' knowledge of how to count, locate, measure and design, and used Bishop's (1988) six activities as an analytic tool.

There are some examples of successful combination of the two components traditional knowledge and learning goals of the national curriculum. In Alaska, Lipka and Adams (2004) show that Indigenous primary school students could reach the mainstream learning goals of mathematics with the integration of their everyday activities. Lipka, Mohatt and the Ciulistet Group's (1998) work in Alaska, inspired the teachers in Jannok Nutti's (2010; 2013) study. In Lipka et al.'s project, Yup'ik Inuit elders, teachers, mathematicians and mathematics educators worked together by means of collaborative research to transform the culturally based mathematics curriculum.

Method

Tensions between Sámi traditional knowledge and Western mathematics are brought to surface with *ruvden* as an example. Sámi traditional knowledge is intertwined with cultural context while Western mathematics has been treated as culture free and independent of context. One goal of this paper is to provide an example of why it is important to be aware of such tensions. A second goal is to provide insight into the variety of thinking embedded in *ruvden*.

Indigenous Methodologies

Kuokkanen (2008) points at a need in Sámi research to embark the path of transforming the previously asymmetrical and often exploitative colonial relations of research into more reciprocal, respectful and responsible relationship. The objective of Indigenous research ethics guidelines is to ensure that Indigenous peoples are no longer exploited, whether intellectually, materially, culturally or otherwise in the name of knowledge, science, or individual careers. She

adds that Indigenous individuals and communities must have a say in research involving them. One way to consider Kuokkanen's guidelines is to start from the cooperation between Indigenous teachers and researchers. The cooperation focuses on investigations of a culture-based practice, *ruvden*, as such, in order to gain insight into thinking that is embedded in the practice.

Method is considered as the way knowledge is acquired, invented or discovered and as a way to know what is real and trustworthy. Smith (1999/2006) points out, that social science fields of inquiry are dependent on the way society is viewed, and the body of knowledge, which legitimates that viewpoint.

The question of whose knowledge was being extended by research was of little consequence, as early ethnographers, educational researchers and occasional 'travellers' described, explained and recorded their accounts of various aspects of Maori society...

While this type of research was validated by 'scientific method' and 'colonial affirmation', it did little to extend the knowledge of Maori people. (p. 170)

Smith points out, that in a cross-cultural context, researchers need to ask questions like, "For whom is this study worthy and relevant?" and "Who says so?" The participating teachers' principal is an experienced Sámi mathematics teacher. When she claims that this study is important and that she wants it published, there are reasons to believe that the study could be worthy and relevant for Sámi mathematics education. Method is considered as the way knowledge is acquired, invented or discovered and as a way to know what is real and trustworthy. According to Smith, social science fields of inquiry are dependent on the way society is viewed as well as on the body of knowledge, which legitimates that viewpoint.

According to Battiste (2000), Indigenous culture-based teaching aims to rebuild Indigenous "peoples, communities, and selves by restoring Indigenous ecologies, consciousness, and languages and by creating bridges between Indigenous and Eurocentric knowledge" (p. xvii). Educational principles and working methods can be based on Indigenous people's culture and

traditions and be developed in cooperation between the institutional education and the Indigenous people's community. This will provide an education that is linked with every area of life, including the wellbeing of the students, environment, and land (King & Schiermann, 2004).

The Wisdom That Underlies a Process

Simpson (2014) presents a narrative that shows how knowledge about making maple sugar is transferred to Nishnaabeg children in Canada. She points out the importance of re-creating the conditions within which the traditional learning occurs, not merely to re-create the content of the practice itself.

Settlers easily appropriate and reproduce the content of the story every year, within the context of capitalism, when they make commercial maple syrup; but they completely miss the wisdom that underlies the entire process because they deterritorialize the mechanics of maple syrup production from Nishnaabeg intelligence and from aki.³ (p. 9)
Thus, in order not to miss the wisdom that underlies the entire *ruvden* practice, the

braiding procedure as such has to be taught as a Sámi cultural knowledge. In other words, the *ruvden* procedure and teaching is intertwined with cultural context. Figure 3 shows an example of how context for a *ruvdet* cord is intertwined with a description of the *ruvden* procedure in a book illustration by Dunfjeld (2001/2006).

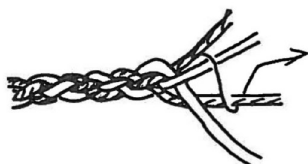


Figure 3 *Ulpieh*, a wearing edge in the bottom line of a South Sámi female dress (Dunfjeld, 2001/2006, p. 162). *Ulpieh* is *ruvdet* in a different color than the edge of the dress.

Meaney (2002) warns those who choose activities from experiences of Indigenous students in mathematics education. Such choices may result in the original purpose of the

³ 'Aki', the land, is both context and process.

activity becoming lost or denigrated through the embedding of the Western mathematical idea. Doolittle (2006) warns against oversimplifications where objects are treated as being independent of context. He refers to the example of viewing the tipi as a cone; the tipi is not a cone because its surface bulges and sinks, it has holes for people and smoke, it has various smells and sounds, and it has a body of tradition and ceremony attached. The reason for his warning is that “[s]tudents may, implicitly or explicitly, come to question the motives of the teachers who lead them away from the true complexities of their cultures” (Doolittle, 2006, p. 20). A narrow focus on structures in the *ruvden* braiding procedure probably a) causes that the original purpose of the braiding is lost or denigrated or b) leads to a simplification of *ruvden*. This paper considers Meaney’s and Doolittle’s warnings and intends to reveal some of the complexities that are embedded in *ruvden*. The braiding procedure needs to be focused and taught, embedded in a Sámi cultural context.

Four Different Approaches

The analyses concern four different approaches to *ruvden*, approaches that appeared during the first nine months of the project. Averill et al. (2009) claim, that teams with competences in mathematics as well as in culture are best placed to make useful links between the two forms of knowledge. Our team consists of Sámi mathematics teachers cooperating with researchers from the three fields: mathematics education, Sámi culture pedagogy, and Sámi handicraft, *duedie*⁴. The four approaches to *ruvden* are presented by women of different ages. Elle was educated as a teacher before Ann Synnøve was born and Riistiina’s age is somewhere

⁴ *Duedtie* is a South Sámi term, used because the researcher Dunfjeld has a South Sámi background. The North Sámi term is *duodji*, while the Lule Sámi term is *doudje*. These three Sámi languages are included in the curriculum in Norway.

between Elle's and Ann Synnøve's⁵. The two schoolgirls are teenagers. Elle and the schoolgirls were raised in Guovdageaidnu in Northern Norway and Riistina was raised in the Swedish part of Sápmi. Ann Synnøve was raised in Guovdageaidnu, but her family background is a mix of south Sámi and North Sámi. In Guovdageaidnu, people use *ruvden* cords made by four and eight cords, while in Northern Sweden it is also common to *ruvdet* with 12 and 16 threads.

The context and form of the four presentations differ. Elle explained in words what she did, step by step. Ann Synnøve used a storytelling approach, which is described further by Steinfjell (2013). The researcher Riistiina had worked with *ruvden* at teacher training school in the northern Swedish part of Sápmi, about 20 years earlier. She presented outcomes from her schoolwork: braided cords and drawings assisted by some text. The students Ronja and Ann-Kristina showed and explained *ruvdet* to their male mathematics teacher and they brought some examples of different braided cords. Their presentation has been incorporated into a video (Fyhn et al., 2014).

At the first meeting between teachers and researchers, the project leader asked the six mathematics teachers if they were familiar with *ruvden*. Some of them did not remember how to perform the braiding. Two of them, Elle and Ann Synnøve, volunteered to show their colleges. It turned out that Elle and Ann Synnøve had different approaches to *ruvden*. The researchers were not prepared for that. As this was the first meeting between the different people, no data were collected. The two researchers from the university had to meet the teachers face to face and start creating an atmosphere of cooperation before any data collection started. One month later, the teachers answered some questions about the presentations. Elle and her colleges then wrote down some words about what she had done. Some months later, the researcher Riistiina showed the

⁵ Elle, Riistiina and Ann Synnøve are authors of this paper. Elle's Norwegian name is Ellen. Riistiina is Ylva's second name.

outcome of her previous schoolwork to the project leader and together they realized that her work represented one more approach. Nine months after the first meeting, the project arranged a workshop at the school. Here the two students Ronja and Ann-Kristina were asked to present *ruvden* to their mathematics teacher. The girls' presentation turned out to be a fourth approach to *ruvden*.

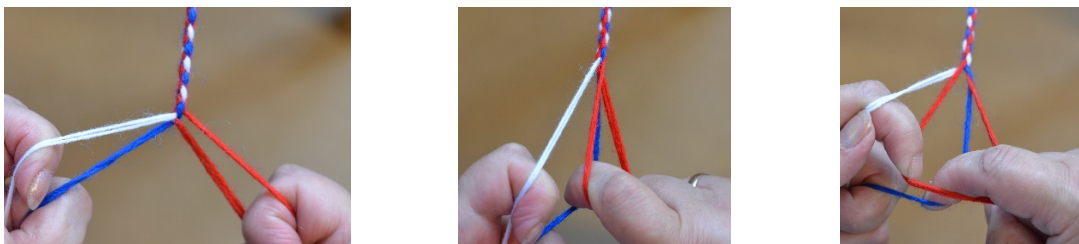
The data in our study are:

- a) Teacher Elle and her colleagues' description of her oral presentation
- b) Teacher Ann Synnøve's written version of her narrative presentation
- c) Researcher Riistiina's schoolwork; drawings and braidings from when she was a student
- d) Video recordings of the two students Ronja and Ann-Kristina's presentations.

The description of teacher Elle's presentation is low quality data because the project did not plan to analyze the presentations a) and b). This is a weakness of this study.

Analysis of Four Different Approaches to *Ruvden*

This section provides analyses of how the four approaches relate to Sámi traditional knowledge and analyses of how the four approaches relate to Bishop's (1988) six fundamental activities. Figure 4 provides teacher Elle's step-by-step presentation of thread moves in the *ruvden* procedure. This presentation was performed three years after the project started.



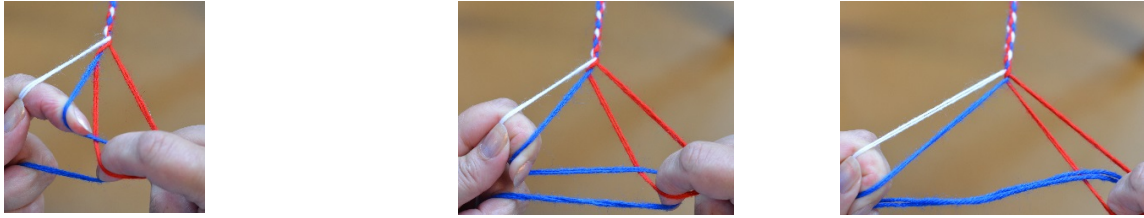


Figure 4. Six snapshots that show the right hand's move when *ruvden* with four threads (Braider: teacher Elle. Photos: Ellen Margrethe Skum)

The braiding procedure in Figure 4 can be explained as follows. You start out with the same number of threads in each hand. Then you lift the right hand's outermost thread and move it left across other threads and to the mid position of the left hand's threads. Then the thread is moved down and below the other threads, and backwards to the far left position of the right hand. The outcome is that you have reorganized the right hand's threads into new positions. Next step is to perform a reflected move with the left hand's outermost thread. You alternate between the outermost thread in the right and the left hand throughout the braiding procedure. The systematics in the step-by-step procedure together with the countable number of threads involved led to the idea of investigating the relationship between *ruvden* and discrete mathematics.

Relations to Traditional Sámi Knowledge Transfer

Teacher Elle's presentation of the *ruvden* procedure was systematic and detailed; which thread to move, how to move it, and where to move it. She described each move systematically. Elle introduced her presentation by telling that she had learned the procedure on her own, by observing her mother and trying it out by herself. This is in line with Sámi child-rearing; watching, experiencing and trying out yourself or together with others. Elle has experienced how to *ruvden* on her own and is able to perform the braiding independently of her mother. Elle pointed out all the important aspects to which the learners had to pay attention. She also commented on how they could tighten each thread in order to avoid slack in one or more of the

threads. Unlike her own learning, her presentation was an example of what Balto described as the ‘school way’ of teaching: the teacher decides what to teach and how the learning will take place. However, Balto (p. 104) points out that “[t]hese methods have their advantages.” Elle’s approach was a slow and clear explanation of how to perform the *ruvden* procedure with the yarn she could find at the moment.

In contrast, teacher Ann Synnøve told a story about behavior rules for visiting your neighbor’s *lávvu*⁶ as an analogy for how she made the braid. Each hand represented a *lávvu*, while each woolen thread represented one family member.

You come in there; you pass over one of the family members and sit down in between them. Then you go home again (behind the others), since you were not moving in at their place. When you arrive back home, you take your place nearest the campfire. (The outermost thread in the left hand is moved above two threads towards the right, and under one thread backwards towards the left.) Then it is the next person from the neighboring lávvu whose turn it is to visit you, because (s)he has waited so long. (The outermost thread from the right hand moves above two threads towards the left, and backwards under one thread towards the right.) The neighbor has to come to your place, before it finally is your turn to go for a visit again. (The threads are in the starting position again.)

Ann Synnøve’s story serves more than one purpose; it combines rules for behavior that courteous people are expected to follow, with practical insights into performing a stepwise procedure. Her story follows a pattern, like Sámi stories often do. Ann Synnøve’s story also expresses cultural knowledge about life, and it takes care of experiences and practical insights,

⁶ A *lávvu* is a Sámi tent with a fireplace in the middle. The tent looks like a Native American tipi.

like other Sámi stories. The story may assist the learner in understanding and memorizing *ruvdet*, because the narrative is a representation of the procedure in terms of social behavior.



Figure 5. Four different *ruddebáttit*, *ruvdet* cords, braided with four, eight, twelve and sixteen threads, respectively

Researcher Riistiina's schoolwork in Figures 5 and 6 belongs to a North Sámi cultural context in Sweden. The application of each cord is intertwined with a detailed description of the *ruvden* procedure with four, eight, twelve and sixteen threads⁷, respectively, Figure 6 provides examples of these. Example 1) shows *ruvdet* with 4 threads (2 reds, 1 yellow, 1 blue): The outermost thread goes under two and back again above one. Right, left, right, etcetera. Used as a

⁷ According to Guttorm and Labba (2008) *ruvden* is a braiding procedure for making round-shaped cords with 4 or 8 threads.

cord in mosquito nets, knotted dress cords, the part of the shoe band that is closest to the shoe.

Example 2) *Ruvdet* with 8 threads (4 reds, 2 yellows, 1 green, 1 blue). The outermost thread goes above 5 threads, back again below 2 threads. Right, left, right, etcetera. The fur cord. Example 3) *Ruvdet* with 12 threads (6 reds, 3 yellows, 3 greens). Fur cord (look at 8 threads). The outermost thread; above 8 threads, back again below 3 threads. Example 4) *Ruvdet* with 16 threads (8 reds, 8 yellows). A cord for women and children from Karesuando. The long cord for the shoe bands. The outermost thread; above 12 threads, back again below 4 threads. Right, left, right (look at 8 threads)

The illustrations show a trial-and-error approach, which is common in Sámi traditional child-rearing. The trial and error leads to a systematic ordering of the threads: The first illustration in Figure 6 starts with one thread moving *below* some of the other threads, while in the three following illustrations, the moving thread contrastingly goes *above* some of the other threads. This neat and tidy schoolwork shows that trial-and-error is fully acceptable. In a Norwegian mathematics classroom, this student would most likely have been encouraged to fix the first illustration, so that the moving thread in each figure consequently moves *above* some other threads.

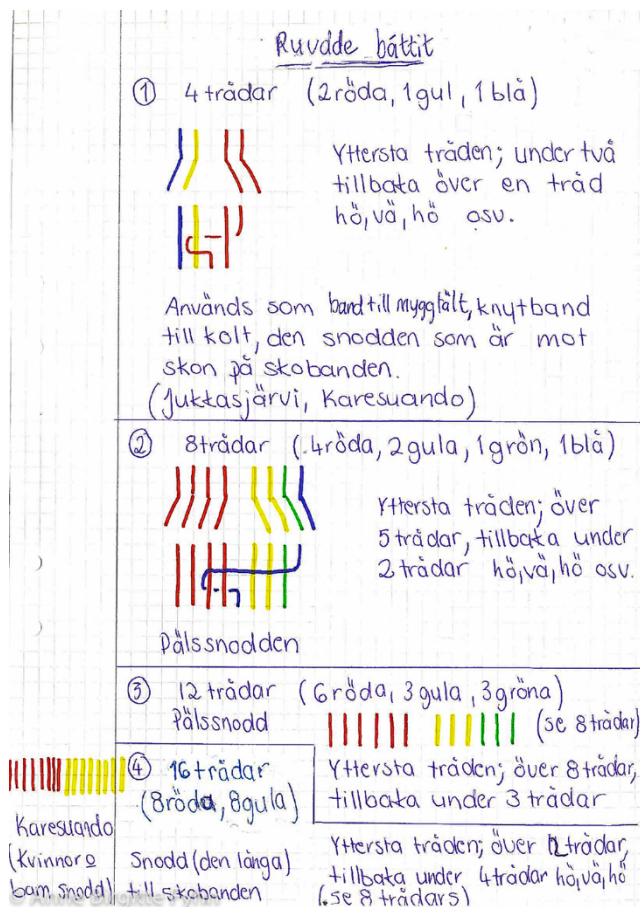


Figure 6. The *ruvden* procedure's unit of repeat, four examples.

Riistiina's work belongs to a school setting, but it is culture-based in the sense that each drawing shows the local use of colors and numbers of threads, and the written descriptions also explain the local application of each cord. These cords have practical, decorative and symbolic functions, as Sámi braidings have; the presentation of the cords is in line with Sámi traditional knowledge. Riistiina's braided cords also represent local Sámi culture-based norms, codes, and symbols, developed over generations; which colors to use for which context and how many threads for what context. This schoolwork shows the transfer of knowledge from an older generation to Riistiina's generation. She did not write that while you are braiding, half of the threads belong to the left hand, while the rest of the threads belong to the right hand. The other approaches explicitly emphasized this point.

The two girls, Ronja and Ann-Kristina, are skilled in performing *ruvdet* and other Sámi braidings. Their mathematics teacher asked them to participate in a video about *ruvden*, where the first recordings would focus on the girls explaining to him how to *ruvdet*. The girls brought five cords for this recording session, the four cords in Figure 2 and one cord that was not *ruvden*. The use of examples that do not work is important in Sámi traditional knowledge transfer. At first, the students presented these five cords. When presenting how to *ruvdet*, one of the students started out braiding without saying anything. She let the teacher watch in silence. This is *bagadallat*, which is common in traditional Sámi knowledge transfer. The girl's instruction is in the form of replies to questions from her teacher. She also slowed down her braiding speed for a while, in order to make each action observable.

Relations to Bishop's Six Activities

Based on Bishop's (1988) activities, the four presentations of *ruvden* reveal different outcomes. Some outcomes are issues from discrete mathematics, while other outcomes are phenomena that appear as discrete in this case. Table 1 presents a list of the variety of outcomes. Tallying is one outcome of *counting*. The four presentations focus on *counting* the number of threads you pass back and forth in each step in Figure 4. When *ruvdet* with four threads, you use a systematic way to order the threads. You pass above two threads and below one thread alternately, acquiring the pattern 2 – 1 – 2 - 1 – 2 etcetera. In Ann Synnøve's narrative, you similarly enter your neighbor's *lávvu* by passing in front of two persons and you leave by passing behind one person. The stepwise *ruvden* procedure repeats itself. Iterations is one outcome of *counting*, while numbers and number patterns are other outcomes. When performing *ruvden*, it is important to understand how you systematically order, compare, and re-order the threads, all of the time. All four approaches focused on this.

	Discrete mathematics	Phenomena that appear as discrete in the <i>ruvden</i> case
Counting	Tallying, numbers, number patterns, iterations, combinatorics	Representation
Locating	Dichotomous variables like above/ below, behind/in front of, right/left	Position, change in position, reflection, conceptualizing and symbolizing the environment with diagrams, models, words, drawings
Measuring	Ordering	Comparison, error, measuring instruments, length, thickness, estimation
Designing	Alternation	Making the object, making a mental template, shape, symbolizing with diagrams, models, words, drawings, pattern
Playing	Procedures, playing with changing colors	Devising, engaging in and considering formalized rules
Explaining		Considering the existence of a phenomenon, generalizations, conventions

Table 1. The outcome of Bishop's (1988) six activities applied to *ruvden*. The left column shows examples of discrete mathematics, while the right column shows phenomena that emerge as discrete in the case of *ruvden*.

The two students' presentation started by showing the cords in Figure 2, different cords *ruvdet* by four threads. The rightmost cord is made by two threads in one color and two threads in a contrasting color. The upper half of this cord represents the same color of threads in each hand, while the lower half represents threads in different colors in each hand. When *ruvden* with four threads, you have these two possible outcomes for two threads in each color. The girls' knowledge about possible combinations of colors is part of combinatorics, one outcome of *counting*. Representation is another outcome of *counting*; Elle and the two students Ronja and Ann-Kristina represented one thread by the word *árpui*, which means thread in Sámi, while one thread in Riistiina's work is represented by a drawn line or curve in the same color as the yarn. In Ann Synnøve's narrative, each thread was represented by a person.

Position as well as change in position are outcomes of *locating*. Both Elle, Ann Synnøve, and the two girls point out that a moving thread always ends up in the hand it departs from. A thread departs from one position and ends up with a change in position even though it returns to the hand where it belongs. In Figure 2, the two cords to the left are made by yarn in four different

colors. In the leftmost cord, the upper half shows a different positioning of the threads than the lower half. Riistiina's drawing also shows how the moving thread ends up in a new position. In Elle's and in the girls' presentations, the thread moves alternately above, in one direction, and below, in the opposite direction. In Ann Synnøve's narrative, the 'person' enters the *lávvu* in front of the 'people' on one side in the *lávvu* and leaves behind them. Elle and the girls say that you pick the outermost thread in alternately the right and the left hand. The dichotomous variables above/ below, in front of/ behind and left/right are examples of *locating*. Riistiina's drawing shows that you pick the outermost thread. Ann Synnøve's narrative underlined this, explaining that the next 'person' in turn was the outermost, the one who had waited the longest for visiting the neighbor.

Reflection is part of *locating*; when *ruvdet*, each hand always is reflecting the last moves of the opposite hand. All four approaches focused on this aspect of the procedure. *Locating* also includes conceptualizing and symbolizing the environment with models, diagrams, drawings, words or other means. Ann Synnøve's presentation used the Sámi tent, *lávvu*, as a model, while Riistiina's diagram was a model of the procedure. Elle and the girls symbolized each thread with the word *árpu*. Ann Synnøve conceptualized the environment by a narrative for rules of behavior, where each thread was a person, while each thread is symbolized by its color in Riistiina's drawing. Although the two students also represented the moves by drawings similar to Riistiina's, they claimed not to be familiar with this form of representation.

Measuring is an issue in *ruvden*, when it comes to comparison and ordering. Ordering includes observing the new order of the threads for each step and comparing the new order of threads with the rest of the growing pattern. If it is not too dark when braiding, the pattern informs you immediately if an error occurs and it provides you possibilities for correcting them.

Error is another part of *measuring*. When *ruvden*, you know the purpose of the cord and therefore you know how long it needs to be. For fur shoes like those in Figure 1, the length of the *ruvdet* cord is one *salla*, fathom. The thickness of the cord relates to the number of threads involved, Riistiina's presentation shows this. Finally, estimation and approximation are outcomes of *measuring*.

Making a *ruvden* cord or a 'mental template' of a *ruvden* cord is one outcome of *designing*. All four approaches included the making of *ruvden* cords. Riistiina's drawing is an example of a template. She added text that symbolizes the alternating moves in the *design*, "left – right – left – right, etcetera". Ann Synnøve made a 'mental template' by imagining the moves of how neighbors visit each other. Symbolizing by drawings, models, diagrams, and words is a common aspect for the two activities *designing* and *locating*. Elle pointed at how to tighten each thread in order to get an even and round shaped cord. One property of *ruvden* cords is their round shape; the Swedish word for *ruvden* means, "round braided cord". Shape, properties of objects, and pattern are other outcomes of *designing*. The colors in a *ruvdet* cord shape a repeating pattern. When braiding, you move the threads in accordance with a repeating pattern. Ann Synnøve's narrative is a story that follows a pattern.

Rules, strategies, and procedures are outcomes of *playing*. Before you *ruvdet* a cord, you have to devise, consider and engage in the formalized rules for what colors to use and how many threads to use, depending on the purpose for the final product. When exercising how to *ruvdet*, you work out a strategy that works. Exercising *ruvdet* means devising and engaging in the rules for how to braid the threads. When braiding, you engage in these rules. Before the presentation for their teacher, the two students had been *playing* with changing colors. They presented different cords that were made by the rules for how to *ruvdet* with four threads. The girls

informed that nowadays the rules regarding use of colors are not as strict as earlier; nowadays it is possible to devise new rules, for the use of colors. The four approaches to *ruvden* can be considered as outcomes of *playing*, because they relate to rules from Sámi traditional knowledge. Development of strategies and rules includes abstract thinking, in this case intertwined with context.

Explaining can be exemplified through considering the existence of the phenomenon *ruvden*. Conventions and generalizations are examples of outcomes of *explaining*. There exists many Sámi braidings, which have different names (Guttorm & Labba, 2008). However, round-shaped braidings with four, eight, twelve or sixteen threads are all classified as *ruvden*, as Riistiina describes in Figures 5 and 6. The Figures also provide information about one region's conventional use of *ruvdet* cords with different number of threads. The students Ronja and Ann-Kristina referred to conventions about different use of colors in different regions. They presented how to *ruvdet* with four and eight threads. The generalization of the name *ruvden* becomes apparent, as *ruvdet* with different number of threads follow the same procedure. Teacher Ann Synnøve's story telling is an outcome of *explaining*, it is an analogy where each thread in the braiding is represented by a person. Her narrative provides an *explanation* of well-known rules for behavior. The approach to *ruvden* through Ann Synnøve's narrative was not chosen in the teachers' final plan for a teaching experiment (Fyhn et al., 2015). The teachers appreciated her approach, but found it more appropriate for primary school students.

Summing up, the outcome of the six activities related to *ruvden* constitutes the cultural product presented in Table 1. The product includes a long list of mathematical terms: numbers, number patterns, iterations, representation, combinatorics, positioning, change in position, reflections, symbolizing, model, diagram, rules, comparison, ordering, pattern, error,

approximation, template, shape, properties, procedure, classification, convention, and generalization.

Relations to the Learning Goals of the Mathematics Curriculum

The second research question is about how the outcome of Bishop's activities relates to the learning goals in the Norwegian national (and the Sámi) mathematics curriculum. The discrete mathematics in Table 1 is included in the learning goals in two main subject areas in the curriculum (Norwegian Directorate for Education and Training, 2010). These goals are to enable the students to:

-discuss, elaborate on and solve simple combinatorics problems (subject area statistics, probability and combinatorics)

-use numbers and variables in exploration, experimentation, practical and theoretical problem-solving (subject area numbers and variables).

Regarding combinatorics, *ruvden* can be performed with yarn in different numbers of colors. If only one color is used, then there is only one way to make the braid⁸. If there are two threads in each color⁹, there are two possibilities for the braid, as the students showed in the rightmost cord in Figure 2. When the number of colors is three, you can make six possible cords, two of them are shown in Figure 7. You have two possibilities for organization of the threads¹⁰: The two equal threads are either in the same hand or in different hands. Different positions of the threads will not necessarily provide different cords, because of the cyclic repetition of colors.

⁸ 4 red threads may be used to braid the short cord of the shoe band (in the northern Swedish part of Sápmi).

⁹ 2 red and 2 blue threads may be used to braid the short cord of the shoe band (in the northern Norwegian part of Sápmi).

¹⁰ 2 red, 1 yellow, and 1 blue/green may be used to braid the short shoe band in the northern Swedish part of Sápmi. In the northern Norwegian part of Sápmi one may use 2 red, 1 white, and 1 blue.

When braiding the cord to the right in Figure 7, the red threads alternate and shape a red stripe, while the green and the white alternate and create a segment of green and white dots. In the left cord, you have one red thread in each hand. Then the red threads look like a kind of spiral that turns around the cord. The spiral may go sunwise or counter-sunwise, but both are considered to be the same main pattern. Thus, in this situation you can make just one cord with one red thread in each hand. From a Western mathematics perspective, however, there are more possibilities, but according to the *duodji* tradition, these are considered as the same.



Figure 7. *Ruvden* with two red, one white and one green thread. Left: One red thread in each hand. Right: The red threads belong to the left hand (Photo and braiding: Ellen Margrethe Skum)

When the number of colors is four – for instance red, yellow, white and blue – it is possible to *ruvdet* three different cords: The red thread can belong to the same hand as the white, the blue, or the yellow one. In the leftmost cord in Figure 2, red and white belong to the same hand in the bottom half, while red and yellow belong to the same hand in the upper half. These are examples of simple combinatorics problems, which are the outcome of the girls' *playing* with changing colors when exploring *ruvden*.

The transition from numbers to algebra, concerns providing insight into how numbers and processing numbers are part of systems and patterns. In Figure 5, Riistiina's work shows that

ruvden can be done with four, eight, twelve or sixteen threads; the number of threads is a multiple of four. In Table 2, we take this further by playing with the idea of what *ruvden*, with more than 16 threads, would look like. When playing this way, we do not consider whether such cords would suit any purpose. When *ruvden*, with whatever number of threads, half of the threads belong in the left hand and the other half in the right hand. Therefore, the number of threads has to be divisible by two. In order to split the threads in each hand into two equal sets, the number of threads involved has to be divisible by four. This is an example of how an integer's property, 'divisible by four', can be an outcome of exploring *ruvden*. Based on Riistiina's work the researchers created a table with relations between the number of threads involved in each step. During the project workshops, the teachers developed a similar table, under guidance of researchers. The outcome is a tool for describing relations between the number of threads involved in each step of the *ruvden* procedure, as shown in Table 2.

Total number of threads	Number of threads in each hand	Above how many threads do you move			Going back: Below how many threads
		- in the starting hand	- in the other hand	- total	
4	2	1	1	$1 + 1 = 2$	1
8	4	3	2	$3 + 2 = 5$	2
12	6	5	3	$5 + 3 = 8$	3
16	8	7	4	$7 + 4 = 11$	4
20	10	9	5	$9 + 5 = 14$	5
24	12	11	6	$11 + 6 = 17$	6
28	14	13	7	$13 + 7 = 20$	7
$4n$	$2n$	$2n - 1$	n	$2n - 1 + n = 3n - 1$	n

Table 2. The number of threads involved in each step of the *ruvden* procedure. Each row represents *ruvden* with different numbers of threads

Here $4n$ represents the total number of threads involved, while $2n$ represents the number of threads in each hand. When *ruvden*, you start with the outermost thread in the right hand. This

thread moves across the rest of the threads in that hand. Hence, the thread passes across $2n - 1$ threads in the right hand. The ‘minus one’ in this expression represents the thread that moves; the teachers highlighted that the thread could not pass across itself. During the development of Table 2, the teachers used numbers and variables in exploring a practical procedure. This presents one possible approach to the transition from numbers to variables. The two students Ronja and Ann-Kristina present this approach to their mathematics teacher in the video by Fyhn et al. (2014).

Discussion

One aim for a traditional Sámi child-rearing is to create independent individuals (Balto, 2005). Three aspects of this child-rearing are relevant for our study, a) children’s trial and error is treated as an important part of their learning process, b) children are provided examples that do not work and c) children are provided possibilities for finding appropriate strategies to their work. These aspects of Sámi traditional child-rearing are also highly relevant for mathematical problem solving, but they are not in line with the traditional deductive approach in Norwegian school mathematics.

One outcome of *ruvden* is an iterative procedure or a repeating pattern that follows rules for ordering and changing positions of discrete phenomena. Threlfall (1999/2005) describes three ways of generating a repeating sequence: (1) a procedure that relates items to adjacent items by remembering all the relationships and then constructing the sequence; (2) a memory of the unit of repeat; and (3) a rhythm or counter system, e.g. a chant with emphasis. Of these, only (2) may be thought of as a process for the creation and application of a rule. The first requires many conditions without applying any rules, while the rhythmic approach does not depend on any rules. The potential for mathematical development of repeating patterns is not fully realized if the

patterns are merely recognized and copied, the unit of repeat is crucial in this generation of a sequence.

The oral descriptions made by Elle and the two girls showed the unit of repeat as a thread's journey over to the other hand and back home again. On this journey, each thread has to follow strict rules. Figure 4 shows Elle's unit of repeat. Ann Synnøve's narrative showed the unit of repeat through the persons' movement from their *lávvu* over to the neighbor and back home again. Her narrative includes that all the other persons have visited their neighbor before it is your turn again. Each visit follows the same rules and the two neighbors alternate between who is the visitor and who is visited. Riistiina's drawings in Figure 6 explicitly identify a unit of repeat for *ruvdet*, as the moves of one thread above and below other threads and her drawing provides clear information about the thread's change in position. Therefore, Riistiina's diagrams can be considered examples of what Fyhn (2008) calls analytical drawings, ones that extract something from the context by focusing on details. After recognition and familiarization with the unit of repeat as moves of some of the threads, students can show and explain how they braid and then translate their descriptions to expressions, made up by numbers and variables. This is how the teachers planned their teaching experiment (Fyhn et al., 2015).

In a television interview (Sainte-Marie, 2015), the legendary Indigenous artist Buffy Sainte-Marie pointed out that play is generally underestimated. As soon as children begin school, they all of a sudden stop playing and start working. "We need to encourage the kids and adults to take time off to play. The way you do on Sundays when there is nobody around and you have nothing to do. That is really valuable." When the girls, Ronja and Ann-Kristina, were asked to present how to *ruvdet* for their teacher, they brought four different *ruvden* cords with them. These cords are outcomes of the girls playing with combinations of colors in *ruvden*. The two

girls claimed that nowadays the rules for using colors are not as strict as earlier. Their presentation of different cords with four threads reveals a thinking that in turn leads to combinatorics. The workshops with teachers and researchers, however, did not consider any combinatorics. After two-and-a-half years of preparations in workshops and meetings, one of the teachers carried out a teaching experiment where investigations of *ruvden* was the basis and mathematics teaching was the outcome (Fyhn et al., 2015). This experiment did not include any combinatorics.

Conclusion

Our paper presents four different approaches to the Sámi braiding procedure *ruvden*. The approaches are in line with different aspects of Sámi traditional knowledge transfer. Thus, there is not just one “correct” way of teaching *ruvden*. The analysis reveals relations between *ruvden* and Bishop’s (1988) six activities and the outcomes of these activities can be described as discrete mathematics. Combinatorics as well as the use of numbers and variables are explicit learning goals in the national (and the Sámi) mathematics curriculum in Norway (Norwegian Directorate for Education and Training, 2010). Therefore, *ruvden* may function as a basis for teaching discrete mathematics in the Sámi middle school since the teaching will be based on investigations of a cultural practice and the learning goals are in line with the national curriculum. We also conclude that Bishop’s (1988) six activities form an appropriate tool for teachers’ planning, but a future teaching plan needs to consider combinatorics as an outcome of *playing* with *ruvden* as well as relations between numbers and algebra.

Bishop (1990) encouraged raising the question of whether an alternative mathematical system exists. The variety of discrete mathematics revealed by *ruvden* indicates a need for further research into a Sámi mathematical system that is integrated with the culture. Barton,

Fairhall and Trinick (1998) claim that in order to have a truly Māori mathematics curriculum, it is necessary to search for the thought patterns that the Māori language allows and encourages. These thought patterns then need to be used in mathematics teaching. One aim of studying how *ruvden* relates to discrete mathematics is to investigate the basis for bringing thought patterns from *ruvdet* into school mathematics. Our study intends to contribute to the discussion of how to improve mathematics teaching for Sámi students.

Students' exploration of *ruvden* includes exploration of the purposes of different cords, the properties of different cords, the development of material for making the yarn and cultural traditions for the use of different cords. Exploration of patterns is just one part of these explorations and it must be integrated with the other explorations. Thus, exploration of *ruvden* can stand on its own as a curricular activity for students who follow the Sámi curriculum. Teachers with no Sámi students in their class may introduce the students to Sámi culture through *ruvden*: "While multicultural math activities are important, they should not be the final goal. Experiences of multicultural activities of people coming up from other cultural environments, may serve to develop the respect for the different." (D'Ambrosio, 2001, p. 68).

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