

BIOECONOMICS OF SUSTAINABLE HARVEST OF COMPETING SPECIES *

by

Ola Flaaten

The Norwegian College of Fishery Science
University of Tromsø
N-9001 Tromsø, Norway

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1 Introduction

Market failures in resource industries can often be classified as "problems of the commons", which means that a scarce natural resource is not imputed a rent. Analysis of open access and optimal management of common property resources already has a major place in advanced textbooks in resource economics [1], [2] and [3]. Renewable marine resources such as fish, seals and whales are, in addition to their common property nature, difficult to assess and their population dynamics are poorly known. Future management of renewable natural resources will probably place greater emphasis on species interactions. This increases the need for theoretical and applied interdisciplinary work. Theoretical exploration of the relationship between concepts and objectives in biology and economics and implications of these for management will prove valuable for interdisciplinary empirical resource management.

The theory of optimal management of renewable resources is fairly well developed within the single species context; see e.g. Munro and Scott [4] for a review of the fisheries economics literature. The policy implications of this analysis depends, to some degree, on the optimization criteria used. In single species models biological criteria such as maximum sustainable yield (MSY) and maximum yield per recruit, in general give solutions embodied in the solutions based on economic criteria, such as maximum sustainable economic yield (MEY) and maximum present value of rent (MPV) [1].

The problem of managing two or more interacting species is far more complex. Several authors have extended single species analysis to include multispecies interactions and

harvesting, for example:

- a) Theoretical analysis of management of competing species and predator-prey systems, e.g. Clark [1] ch. 9, Hannesson [5], May et al. [6], Silvert and Smith [7], Flaaten [13] and Getz [8]. The latter is especially concerned about the optimal path towards the long run steady state harvesting regime.
- b) Analysis of harvesting interactions, e.g. Clark [1] ch. 9, Clark [2] ch. 5, Lipton and Strand [9] and European Communities [10]. The latter is especially concerned about particular fisheries and of assessment methods.
- c) Applied studies on interacting species, e.g. Conrad and Adu-Asamoah [11] on competing species of tuna and Flaaten [12] on plankton feeders–fish–sea mammals interactions.

This article analyses steady state solutions in groups a) and c). Biological and economic optimization criteria are set in a common two species framework to analyse similarities and differences of their solutions. The implications of these solutions for management are compared both between them and with solutions from single species analysis.

The Gause-model [14] of two competing species is reviewed in section 2. Section 3 demonstrates how to derive the maximum sustainable yield frontier (MSF) and the locus of MSF-stocks. Section 4 shows that maximizing economic yield (rent) with positive harvesting costs and zero discount rate implies optimal stock levels above the locus of

MSF-stocks. Isoent lines prove to be ellipses in the state plane in this model. Section 5 shows that maximizing the present value of economic rent under costless harvesting with a positive discount rate implies optimal stock levels below the locus of MSF-stocks. With positive costs optimal harvesting might imply harvesting one of the species at a loss. Hence, open access harvesting of a system of competing species, using single product technology, may imply economic underexploitation of a common property resource. Section 5 also shows how the optimal stock levels are altered by changes in the exogenous social rate of discount, harvest prices and harvesting costs. Section 6 concludes the article.

2 Biological Competition

Most two species models analysed in the literature can be shown to have either a stable point or a stable limit cycle [15], p. 81. This is also the case for the Gause-model used in this paper.

Let W_1 and W_2 denote the stocks of two competing species. The model is specified as

$$\dot{W}_1 = dW_1/dt = r_1 W_1 (1 - W_1/L_1) - a_1 W_1 W_2 \quad (1)$$

$$\dot{W}_2 = dW_2/dt = r_2 W_2 (1 - W_2/L_2) - a_2 W_1 W_2, \quad (2)$$

where r_1 and r_2 are the intrinsic growth rates of the respective species.¹ L_1 and L_2 are the single species carrying capacities at which species one and two, respectively, will settle in the absence of the other species and harvest. In case of no competition, the per capita growth rate of species i , \dot{W}_i/W_i , decreases from r_i for stock levels close to zero, to zero for

stock levels equal to its own carrying capacity. The presence of a competitor reduces the per capita growth rate in proportion to the biomass of the competitor. The competitor coefficients, a_1 and a_2 , tell how much the per capita growth rate of species one and two, respectively, are reduced per unit of the other species.

To simplify the model rewrite the variables W_1 and W_2 into a dimensionless form. Defining $X_1 = W_1/L_1$ and $X_2 = W_2/L_2$ equations (1) and (2) can be rewritten as

$$\dot{X}_1 = dX_1/dt = G_1(X_1, X_2) = r_1 X_1 (1 - X_1 - \alpha_1 X_2) \quad (3)$$

$$\dot{X}_2 = dX_2/dt = G_2(X_1, X_2) = r_2 X_2 (1 - X_2 - \alpha_2 X_1), \quad (4)$$

where $\alpha_1 = a_1 L_2 / r_1$ and $\alpha_2 = a_2 L_1 / r_2$ are the dimensionless competitor parameters ($\alpha_1, \alpha_2 > 0$). These parameters tell, in a dimensionless form, how severe the competition from the other species is upon species one and two, respectively.

Harvesting is introduced in a simple way in the model by assuming that the resource stocks are harvested independently of each other, with constant effort per unit of time, F_i ($i = 1, 2$). The effort is scaled such that $F_i = 1$ corresponds to constant catchability coefficients equal to r_i . The normalized catch rates will be

$$y_1 = r_1 F_1 X_1 \quad (5)$$

$$y_2 = r_2 F_2 X_2. \quad (6)$$

The growth rates in equations (3) and (4) now will be changed to

$$\dot{X}_1 = r_1 X_1 (1 - X_1 - \alpha_1 X_2 - F_1) \quad (7)$$

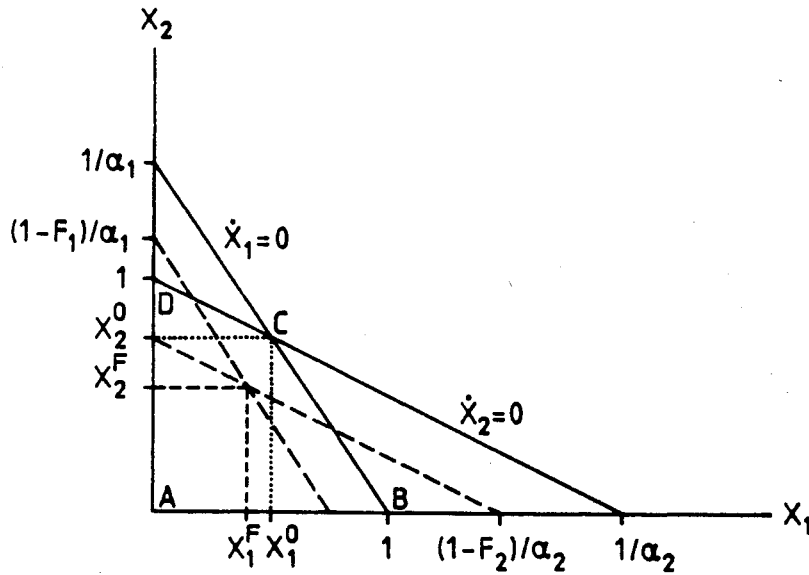


Figure 1: The phase diagram of the model of competing species.

$$\dot{X}_2 = r_2 X_2 (1 - X_2 - \alpha_2 X_1 - F_2). \quad (8)$$

The equilibrium properties of this ecological system depend only on the harvest efforts, F_1 and F_2 , and the competition parameters, α_1 and α_2 . The intrinsic growth rates, r_1 and r_2 , affect the dynamics of the system outside equilibrium. The isoclines are found from equations (7) and (8) by setting $\dot{X}_1 = 0$ and $\dot{X}_2 = 0$. This gives

$$X_2 = (1/\alpha_1)(1 - X_1 - F_1) \quad \text{for} \quad \dot{X}_1 = 0 \quad (9)$$

$$X_2 = (1 - \alpha_2 X_1 - F_2) \quad \text{for} \quad \dot{X}_2 = 0. \quad (10)$$

The phase diagram for the system with a stable node, C , is shown in Figure 1. The isoclines for the pristine system without harvesting ($F_i = 0$; $i = 1, 2$) are shown as solid

lines whereas the isoclines for the harvesting case are the dotted lines.² The pre-harvesting levels of the stocks are denoted X_i^0 whereas the post-harvesting levels are denoted X_i^F . Even if there is a competitive coexistence equilibrium in the pristine system, harvesting may extinct one of the species.

If positive, stable equilibrium levels of X_1 and X_2 , denoted X_1^F and X_2^F , exist simultaneously, they are found where the isoclines intersect. From equations (9) and (10) follow

$$X_1^F = \frac{1 - \alpha_1 - F_1 + \alpha_1 F_2}{1 - \alpha_1 \alpha_2} \quad (11)$$

$$X_2^F = \frac{1 - \alpha_2 - F_2 + \alpha_2 F_1}{1 - \alpha_1 \alpha_2}. \quad (12)$$

To have an interior, stable equilibrium with both stocks positive without harvesting, it is necessary to have

$$\alpha_1 < 1 \quad \text{and} \quad \alpha_2 < 1. \quad (13)$$

The species which has the lowest competitor parameter, α_i , will have the highest stock level, X_i^0 , at the pristine equilibrium, i.e. the equilibrium without harvesting. To have an equilibrium with both stocks being positive when just species i is harvested, it is necessary that the effort rates do not exceed the following levels

$$F_i < 1 - \alpha_i \quad \text{and} \quad F_j = 0, \quad (i = 1 \text{ or } 2, i \neq j). \quad (14)$$

Equations (11) and (12) show that the equilibrium level of each stock is negatively affected by the own effort rate, and positively affected by the effort rate of the competitor. This means i.a. that it is possible to increase each of the stocks above its

pristine level by harvesting only the competitor. The sustainable yield area in Figure 1 is the quadrangle *ABCD*. Combinations of stock levels outside this quadrangle are not possible to maintain.³ The sustainable yield area is of great importance as a biological restriction on the economic utilization of the competing species. This may be compared with the biomass axis in the yield-biomass diagram of single species models. To obtain a positive sustainable yield in a single species model it is necessary that the stock remains between its minimum viable level and the carrying capacity of the system. This compares to a bounded area in the biomass plane of two species models, e.g. the quadrangle *ABCD* in Figure 1. Within the boundaries mentioned above each stock level in a single species model may produce a given sustainable yield. Likewise, in a two species model a given combination of the two stock levels within the sustainable yield area, produce a combination of sustainable yields of the two species. The concept of maximum sustainable yield (MSY), and the corresponding stock level, in single species models are of great biological importance, as well as of economic importance as a reference point for optimal management of the stock. The corresponding concept to MSY for the two species model of competition will be introduced in the next section.

3 The Maximum Sustainable Yield Frontier (MSF)

The importance of the MSY concept in biological and bioeconomic single species analysis is well known. The concept of maximum sustainable yield frontier (MSF) will be shown to be of the same importance for the two species analysis as MSY is in the single species framework. The MSF is derived⁴ by maximizing the sustainable yield of one species for

a constant sustainable yield of the other. This problem is equivalent to that of welfare economics: deriving the production possibility frontier by maximizing the output of one good for a specified amount of output of the other, given a fixed amount of factors of production. In a two species model of competition the limited amount of factors of production are embodied in the carrying capacities and the intrinsic growth rates. In the real world the limited factor of production can be e.g. the zooplankton communities of the sea in the case of competing fish species, and the grass of the plain in the case of grazing animals.

From equations (5)–(8) the following equilibrium harvest rates are derived, i.e. when $\dot{X}_1 = \dot{X}_2 = 0$,

$$y_1 = r_1 X_1 (1 - X_1 - \alpha_1 X_2) \quad (15)$$

$$y_2 = r_2 X_2 (1 - X_2 - \alpha_2 X_1). \quad (16)$$

The problem of maximizing y_2 subject to the constraint $y_1 = \text{constant}$, can be done using the Lagrange method. First we introduce the Lagrangian expression

$$L = r_2 X_2 (1 - X_2 - \alpha_2 X_1) - \mu (r_1 X_1 (1 - X_1 - \alpha_1 X_2) - y_1). \quad (17)$$

From the necessary conditions for optimality the following quadratic equation, which implicitly gives X_2 as a function of X_1 , is derived:

$$2\alpha_2 X_1^2 + 4X_1 X_2 + 2\alpha_1 X_2^2 - (2 + \alpha_2)X_1 - (2 + \alpha_1)X_2 + 1 = 0, \quad (18)$$

when $X_1 > 0$. For $X_1 = 0$ it follows immediately from maximization of y_2 in equation

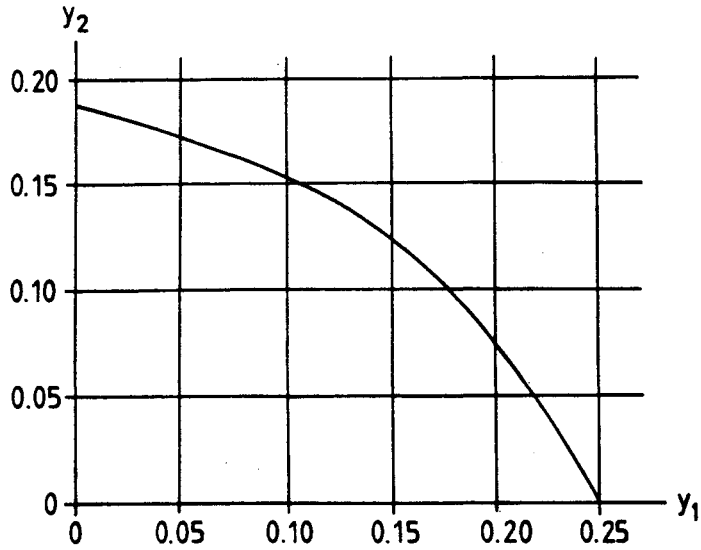


Figure 2: The maximum sustainable yield frontier (MSF) for two competing species, for parameters given in the text.

(16) that

$$X_2 = \frac{1}{2}, \quad \text{when } X_1 = 0. \quad (19)$$

Correspondingly it is easy to see that

$$X_1 = \frac{1}{2}, \quad \text{when } X_2 = 0. \quad (20)$$

For each level of X_1 we compute X_2 from (18) and the resulting yields, y_1 and y_2 , are given by (15) and (16). The locus combining the yields of the two species is shown in Figure 2 for parameters $\alpha_1 = 2/3$, $\alpha_2 = 1/2$, $r_1 = 1$, $r_2 = 3/4$. In this example species one has the greatest reproductive potential, biologically speaking, as seen from the MSF in Figure 2.

The curve depicted by $X_2 = X_2^{MSF}(X_1)$, implicitly given by equations (18)–(20) we shall call the X^{MSF} -locus. The MSF will be affected by changes in r_1 and r_2 , and so will the X^{MSF} -locus be, as seen from equation (18) recalling that $\alpha_i = a_i L_j / r_i$ ($i, j = 1, 2$; $i \neq j$).

Equation (18) is an equation of a conic section of the form

$$AX_1^2 + 2BX_1X_2 + CX_2^2 + DX_1 + EX_2 + F = 0. \quad (21)$$

In this case $4AC < (2B)^2$, since $\alpha_1\alpha_2 < 1$ and $B = 2$, and the X^{MSF} -locus given by equation (18) is a hyperbola. Only the branch giving $0 < X_i^{MSF} < 1$ ($i = 1, 2$) make sense, biologically speaking. The other branch is therefore excluded from the analysis.

In the $X_1 - X_2$ -plane the curve of $X_2 = X_2^{MSF}(X_1)$ implicitly given by (18) will always be downward sloping since

$$\frac{dX_2}{dX_1} = -\frac{4\alpha_2 X_1 + 4X_2 - (2 + \alpha_2)}{4\alpha_1 X_2 + 4X_1 - (2 + \alpha_1)} < 0 \quad \text{along } X_2 = X_2^{MSF}(X_1) \quad (22)$$

for $X_1, X_2 \in [0, 0.5]$.

Since $\lim_{X_1^{MSF} \rightarrow 1/2} X_2^{MSF} = 0$ we have

$$\lim_{X_1 \rightarrow 1/2} \left(\frac{dX_2}{dX_1} \right)^{MSF} = \frac{\alpha_2 - 2}{\alpha_1} < 0, \quad (23)$$

and, since $\lim_{X_1^{MSF} \rightarrow 0} X_2^{MSF} = 1/2$

$$\lim_{X_1 \rightarrow 0} \left(\frac{dX_2}{dX_1} \right)^{MSF} = \frac{\alpha_2}{\alpha_1 - 2} < 0. \quad (24)$$

Equations (22)–(24) show that the slope of the X^{MSF} -locus will depend on the parameters α_1 and α_2 , i.e. on the slopes of the isoclines.⁵

From a biological point of view one could say that the X^{MSY} -stock level in single species models is the optimal one. The corresponding biological optimal harvesting in a two species model would be one of the innumerable combinations of stocks generating the MSF. Which point on the MSF, and the X^{MSF} -curve, is "best" is not possible to decide just from a biological reasoning. The inclusion of economics is necessary to make such a decision. This is the subject of the following two sections, and it will be shown that MSF-harvesting is the economic optimum only in special cases.

4 Maximum Sustainable Economic Yield

As demonstrated in the preceding section the competing species may be harvested on a sustainable yield basis for both of the species at innumerable combinations of the two stock levels. To see at which stock levels the system will settle under commercial harvesting, we shall study the two extreme harvesting regimes of open access harvesting and a profit maximizing sole owner. We shall mostly be concerned with solutions in the interior of the sustainable yield quadrangle.⁶ The dynamic behaviour of the biological and economic system will not be considered, we shall rather concentrate on analysing equilibrium points by means of comparative statics.

Given the Schaefer harvest function of equations (5) and (6), and assuming constant costs, c_i , per unit of rescaled effort, $E_i = r_i F_i$, the unit harvesting cost becomes

$$c_i(X_i) = c_i/X_i \quad (i = 1, 2). \quad (25)$$

Assuming the demand for each of the two species is independent of each other and

infinitely elastic with respect to its own price, p_1 and p_2 are the constant prices of species one and two, respectively. Under these assumptions the net profit per unit of harvest is

$$b_i(X_i) = p_i - c_i/X_i \quad (i = 1, 2), \quad (26)$$

and the total profit from the harvest of each of the two species is

$$\pi_i(X_i) = (p_i - c_i/X_i)y_i \quad (i = 1, 2). \quad (27)$$

Bioeconomic equilibrium requires we simultaneously have⁷

$$\pi_i(X_i) = 0 \quad \text{and} \quad \dot{X}_i = 0 \quad (i = 1, 2). \quad (28)$$

Denoting the open access stock level of the two species as X_1^∞ and X_2^∞ , it is seen from (27) that unless $y_i = 0$ the open access stock levels are

$$X_i^\infty = c_i/p_i \quad (i = 1, 2). \quad (29)$$

Equation (30) also implies the standard tragedy of the commons result that price equal average costs (Cornes and Sandler [17]; Dasgupta and Heal [3]). Given our assumptions the open access stock levels solely depend on economic parameters.⁸ However, the biological parameters will affect the harvest rates as seen by substituting from equations (5)–(6) and (28)–(29) into the growth equations (7) and (8).

In Figure 3 the open access equilibrium point, X^∞ , is (arbitrarily) outside the X^{MSF} -curve, for parameters $c_1 = 1$, $c_2 = 1$, $p_1 = 4$, $p_2 = 2$, which imply $X_1^\infty = 1/4$ and $X_2^\infty = 1/2$. Note that decreasing the stock levels somewhat, to move the equilibrium point in the SW-direction from X^∞ to the X^{MSF} , would increase biological yield from both species.

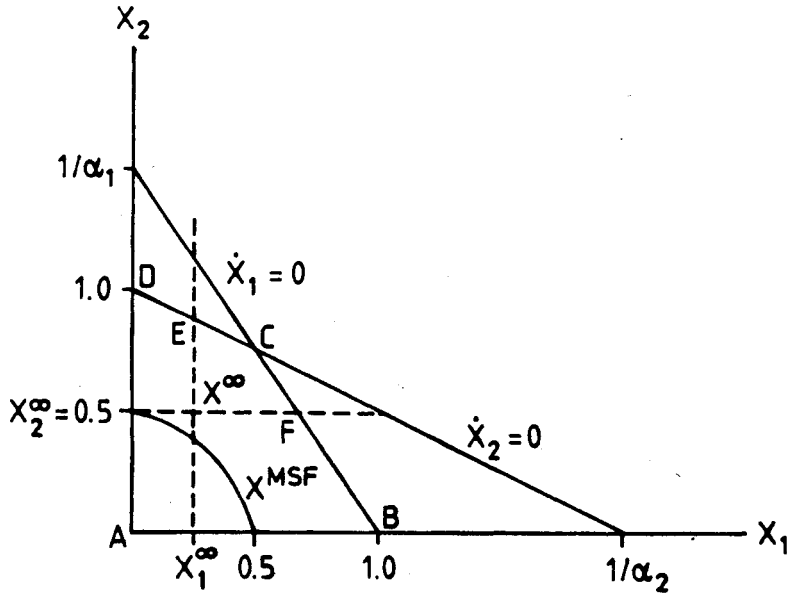


Figure 3: The areas of sustainable yield and positive profit, for parameters given in the text.

The total profit from harvesting the two species can be derived from equation (27):

$$\pi^*(X_1, X_2) = (p_1 - c_1/X_1)y_1 + (p_2 - c_2/X_2)y_2. \quad (30)$$

Equation (30) gives the standard industry profit function consisting of a profit per unit expression and a steady state harvest quantity for each of the two species. Substituting for y_i from (15) and (16), inserting X_i^∞ from (29) and normalizing profit by dividing by $p_1 r_1$ in (30) gives

$$\begin{aligned} \pi(X_1, X_2) = & (X_1 - X_1^\infty)(1 - X_1 - \alpha_1 X_2) \\ & + \beta(X_2 - X_2^\infty)(1 - X_2 - \alpha_2 X_1), \end{aligned} \quad (31)$$

where $\beta = p_2 r_2 / p_1 r_1$ is the bioprice ratio, and $\pi(X_1, X_2) = \pi^*(X_1, X_2) / p_1 r_1$.

Interpreting equation (31) it should be noticed that it consists of two terms, one for each of the two species, and each term is a product of two parts. The first part, $(X_i - X_i^\infty)$, takes care of the stock effect,⁹ i.e. the effect the size of the stock has on the net profit due to decreasing costs as the stock rises. The stock level must be above the open access level, X_i^∞ , to yield a positive profit. The second part is the expression from which the isocline is derived. Since the isoclines are the borders of sustainable yield of the respective species, the product of the two parts can be interpreted the following way. On the one hand it pays to stay below the isoclines because that enhances biological yield, however, on the other hand it pays to have stocks above the open access levels because that reduces harvesting costs thereby increasing the unit harvesting profit. All in all there must be a tradeoff between these two effects to maximize the total profit expressed by equation (31).

Before proceeding to the problem of finding the optimal combination of the two stocks, we shall have a closer look at which combinations of stocks in Figure 3 give positive total profit. The zero profit line obviously passes through the X^∞ -point, and there must also be zero profit at point C where the isoclines intersect, i.e. where the growth rates simultaneously equal zero. At point E in Figure 3 the term $(X_1 - X_1^\infty)$ equal zero and the yield from species two is zero because $(1 - X_2 - \alpha_2 X_1)$ equal zero. Both terms in equation (31) are therefore equal to zero, and so is of course the total profit. For the same reasons this also happens at point F . To see what the zero profit line looks like outside the four points X^∞ , F , C and E , we start with the more general case of what the isoprofit lines in general look like.

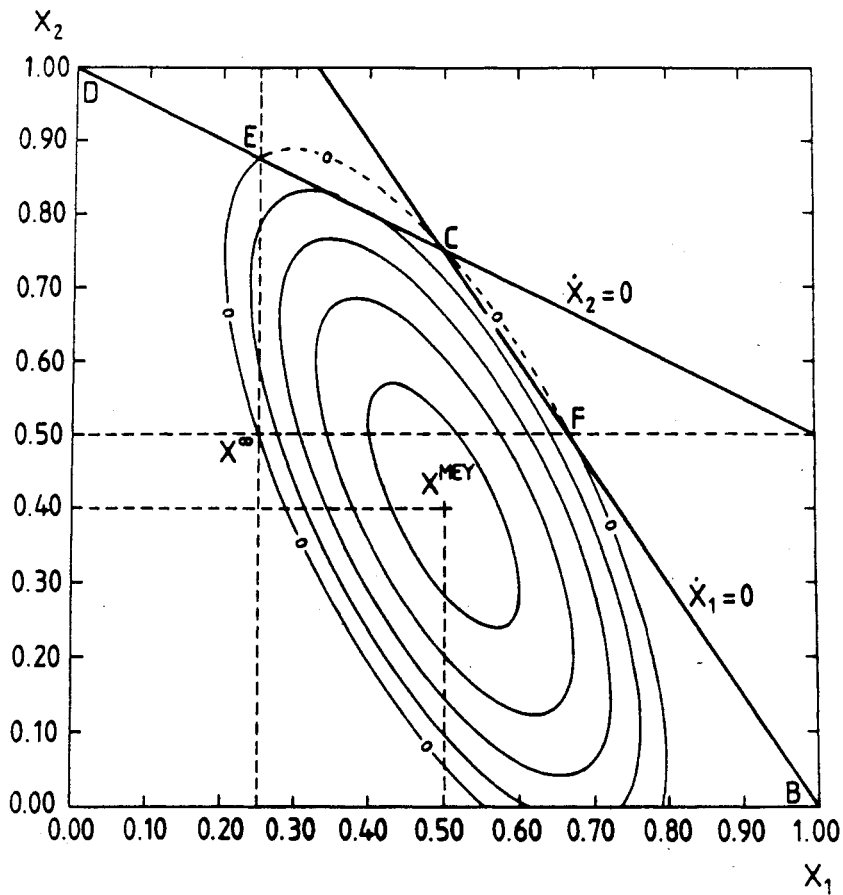


Figure 4: Isoprofit ellipses and the sustainable yield region.

Rewriting the net profit function in equation (31) gives

$$\begin{aligned} \pi(X_1, X_2) = & -X_1^2 - \beta X_2^2 - (\alpha_1 + \beta\alpha_2)X_1X_2 + (1 + X_1^\infty + \beta\alpha_2X_2^\infty)X_1 \quad (32) \\ & + (\beta + \beta X_2^\infty + \alpha_1 X_1^\infty)X_2 - (X_1^\infty + \beta X_2^\infty). \end{aligned}$$

For a given profit level, $\bar{\pi}$, this is an equation of a conic section, cfr. equation (21). When $4AC - (2B)^2 > 0$ and $B \neq 0$ this is the equation of an ellipse with axes not parallel to the abscissa and the ordinate axes.¹⁰ The isoprofit lines are therefore ellipses in the $X_1 - X_2$ -plane. Some isoprofit lines, with the zero profit line as the outermost, are shown in Figure 4, for biological parameters used in Figure 2 and economic parameters used in Figure 3. Combinations of stocks within the zero profit ellipse, inside the sustainable

yield quadrangle, give sustainable positive total profit. The size of the zero profit ellipse depends on the parameters of equation (32), i.e. on economic as well as on biological parameters. An increase in the open access stock levels, i.e. a change in point X^∞ towards point C in Figure 4, contracts the area of profitable harvesting. Such a change eventually results in a zero profit ellipse that does not intersect the abscissa (and/or the ordinate axis). Reduction of the intrinsic growth rates also reduces the area of profitable joint harvesting of the two species.

In the single species model stock levels between the open access level, X^∞ , and the carrying capacity are capable of giving positive sustainable profit. The lower bound, X^∞ , is solely determined by economic and harvest-technological factors, whereas the carrying capacity is a pure biological constraint. In this model of competing species the upper bound of the profit yielding stock levels is a pure biological constraint given by the isoclines intersection point, C , in Figure 4. Also for the lower bound is there a similarity between this two species model and the single species model, the open access stock levels are solely determined by economic and technological factors. The points E and F on the zero profit ellipse correspond to a mixture of economic, technological and biological factors. However, it should be noted that these points are uniquely determined by the open access stock level, X^∞ , and the pristine stock levels.

Assuming there is an interior solution, the necessary conditions for the maximization of the profit in equation (31) gives the following MEY-stocks:¹¹

$$X_1^{MEY} = \frac{2\beta(1 + \bar{X}^\infty) - \gamma(\beta + \hat{X}^\infty)}{4\beta - \gamma^2} \quad (33)$$

$$X_2^{MEY} = \frac{2(\beta + \hat{X}^\infty) - \gamma(1 + \bar{X}^\infty)}{4\beta - \gamma^2}, \quad (34)$$

where $\gamma = \alpha_1 + \alpha_2\beta$, $\bar{X}^\infty = X_1^\infty + \alpha_2\beta X_2^\infty$, $\hat{X}^\infty = \alpha_1 X_1^\infty + \beta X_2^\infty$, recalling that $\beta = p_2 r_2 / p_1 r_1$. X^{MEY} is the center of the isoprofit ellipses and the optimal long run stock levels in case of zero discount rate, and is shown in Figure 4.

Harvesting at stock levels either below X_1^∞ or below X_2^∞ can make sense in the case that one of the two species has a low economic value, expressed by p_i/c_i , and the other has a high value. The low valued species, species two in Figure 4, therefore should be harvested at a loss. To implement such a solution it is necessary to subsidize the harvest of species two to reduce this stock below its open access level. This amount of subsidy is more than offset by increased sustainable economic yield of species one. Hence, the total profit from joint harvesting of the two species increases.

From the analysis of single species biomass models is known that $X^{MEY} > X^{MSY}$ in case of stock dependent harvest costs. In the special case of costless harvesting, or no stock dependent costs, the MEY and the MSY stock levels coincides: $X^{MEY} = X^{MSY}$. In this two species context we shall show:

Theorem 1. In case of two competing species and costless harvesting the combinations of stocks giving the maximum economic yield (MEY) coincides with the combinations maximizing the yield of one species for a given yield of the other (MSF).

The result stated in Theorem 1 is being proved over all possible bioprice ratios, β . The X^{MEY} is a single point for a given set of bioprices.

In the case of costless harvesting, $c_i = X_i^\infty = 0$ ($i = 1, 2$), β may be eliminated from the necessary conditions for an interior solution of the maximization of sustainable economic yield in equation (31), to give

$$2\alpha_2 X_1^2 + 4X_1 X_2 + 2\alpha_1 X_2^2 - (2 + \alpha_2)X_1 - (2 + \alpha_1)X_2 + 1 = 0. \quad (35)$$

Equation (35) gives $X_2 = X_2^{MEY}(X_1)$ and is exactly the same as equation (18) which gave $X_2 = X_2^{MSF}(X_1)$. We have thereby shown Theorem 1.

Having studied the case of costless harvesting, now proceed to the case of positive harvesting costs for both species, i.e. $c_i > 0$ ($i = 1, 2$), to show, for all possible bioprice ratios,

Theorem 2. In case of two competing species that are harvested independently of each other at positive harvesting costs, the MEY combinations of stocks are greater than the MSF combinations.

To show this start by eliminating β from the necessary conditions for the maximization of sustainable economic yield in equation (31), to arrive at

$$\begin{aligned} 2\alpha_2 X_1^2 + 4X_1 X_2 + 2\alpha_1 X_2^2 - (2 + \alpha_2 + \alpha_2 X_1^\infty + (2 - \alpha)X_2^\infty)X_1 \\ - (2 + \alpha_1 + (2 - \alpha)X_1^\infty + \alpha_1 X_2^\infty)X_2 + (1 + X_1^\infty + X_2^\infty) = 0, \end{aligned} \quad (36)$$

where $\alpha = \alpha_1 \alpha_2$. This is an equation of a conic section. The constants of the two quadratic terms and that of the product term in equation (35) are the same as in the MSF-case given in equation (18), while the three others are different. This means that

also the MEY combinations of the two stocks are described by a hyperbola in the case of positive harvesting costs. Since the constants of the quadratic terms and the product term are equal in the MEY and the MSF cases, the asymptotes of the hyperbolas have the same slope in these two cases. Therefore, the MEY- and the MSF-hyperbolas can not intersect. In case of the MSF we have shown in equation (20) that¹²

$$X_1^{MSF} = 1/2 \quad \text{when} \quad X_2^{MSF} = 0. \quad (37)$$

To find the intersection point between the MEY-hyperbola and the abscissa, return to the profit equation (31). When $X_2 = 0$ the MEY stock of species one is found from

$$d\pi/dX_1 = 1 - 2X_1 - X_1^\infty = 0, \quad (38)$$

which gives, by use of (37),

$$X_1^{MEY} = X_1^{MSF} + X_1^\infty/2 \quad \text{when} \quad X_2^{MEY} = 0. \quad (39)$$

In other words, if species two is extinct the maximum economic yield of species one is obtained for a larger stock level than the one giving maximum sustainable yield. Since the MEY- and the MSF-hyperbolas can not intersect this implies

$$X^{MSF} < X^{MEY}, \quad \text{when} \quad c_i > 0 \quad (i = 1, 2), \quad (40)$$

and theorem 2 is proved.

Having focused on maximum sustainable economic yield in this section, we now proceed with the objective of maximizing present value of rent from the two competing species.

5 Maximum Present Value of Rent

Given joint management of the two competing species and assuming the objective of the management program is to maximize the present value of the rent from the resources, the objective functional is

$$PV = \int_0^{\infty} e^{-\delta t} \pi(X_1, X_2) dt, \quad (41)$$

where $\pi(X_1, X_2)$ is defined in (30) and δ is the social rate of discount. The social managers problem is to choose the harvest rates, the y 's, so as to maximize the present value, given the biological constraints imposed by the growth equations (7) and (8). Assuming the solution is an interior one, there are two joint equilibrium equations that must be satisfied at the optimum (see [1], ch. 9.3). They can be written as

$$b_1(X_1)G_{11} + b_2(X_2)G_{21} - c'_1(X_1)G_1(X_1, X_2) = \delta b_1(X_1) \quad (42)$$

$$b_1(X_1)G_{12} + b_2(X_2)G_{22} - c'_2(X_2)G_2(X_1, X_2) = \delta b_2(X_2), \quad (43)$$

where $b_i(X_i)$ are defined in (26), $c'_i(X_i) = dc_i(X_i)/dX_i$ and $G_{ij} = \partial G_i(X_1, X_2)/\partial X_j$ ($i, j = 1, 2$). The growth functions, $G_i(X_1, X_2)$ ($i = 1, 2$) are defined in equations (3) and (4). At the optimum the net profit from investing in the resource capital of species one, i.e. the l.h.s. of equation (42), should equal the net profit from possibly investing the current profit at the social opportunity cost of capital, δ .

Dividing equation (42) by the social rate of discount gives, on the l.h.s., the change in the present value of the infinite horizon sustainable economic yield per unit of change in the stock level. Expressed this way the l.h.s. is often referred to as the user cost of

the resource. This should equal the current profit from harvesting of one unit of the stock given on the r.h.s. Compared to the single species model the additional term on the l.h.s. of equation (42), $b_2(X_2)G_{21}$, reflects the addition to the marginal value product of species one afforded by means of species two. The interpretation of equation (43) is similar.

In single species models the optimal resource stock is above the open access stock level, whereas that need not be the case for the predator in predator-prey models (see [5] and [13]). To see that the latter result also applies to any one species in a model of competing species we rewrite equations (42)–(43) and, after having taken the derivatives, arrive at

$$(1 - X_1 - \alpha_1 X_2) - (X_1 - X_1^\infty) - \alpha_2 \beta (X_2 - X_2^\infty) = \frac{\delta}{X_1 r_1} (X_1 - X_1^\infty) \quad (44)$$

$$\beta(1 - X_2 - \alpha_2 X_1) - \beta(X_2 - X_2^\infty) - \alpha_1 (X_1 - X_1^\infty) = \frac{\delta \beta}{X_2 r_2} (X_2 - X_2^\infty), \quad (45)$$

where it is substituted for $X_i^\infty = c_i/p_i$ and $\beta = p_2 r_2 / p_1 r_1$. The long run optimal equilibrium stocks implicitly given by equations (44) and (45) shall be denoted X_1^{MPV} and X_2^{MPV} . The first term on the l.h.s. of equation (44) will always be positive within the sustainable yield area. Assuming the optimal stock level of species two is above its open access level implies the last term on the l.h.s. of (44) is negative, included the minus sign. Now it is possible that $X_1^{MPV} < X_1^\infty$ which implies that the harvest of species one should be subsidized at the optimum. This result is more likely if α_1 , p_1 and/or r_1 are low, or c_1 high, compared to the corresponding parameters of species two. The loss from the harvest of species one is more than offset by the increased profit from the harvest of

species two which is more bioeconomic valuable. This reasoning of course also holds for the opposite case with species two being harvested at a loss.

From single species models is well known that in the extreme case of costless harvesting the optimal stock level will always be below the MSY level when the discount rate is positive. This result from the single species models shall be used to prove

Theorem 3. For costless harvesting and a positive discount rate the combination of stocks giving the maximum present value (MPV) of harvesting will be inside the locus of MSF stocks.

To prove Theorem 3 insert $c_i = X_i^\infty = 0$ ($i = 1, 2$) into equations (44) and (45) and eliminate β . This gives the following equation

$$2\alpha_2 X_1^2 + 4X_1 X_2 + 2\alpha_1 X_2^2 - (2 + \alpha_2 - \alpha_2 \delta_1 - 2\delta_2)X_1 - (2 + \alpha_1 - \alpha_1 \delta_2 - 2\delta_1)X_2 + 1 + \delta_1 \delta_2 - \delta_1 - \delta_2 = 0, \quad (46)$$

where $\delta_i = \delta/r_i$ ($i = 1, 2$) are the bioeconomic growth ratios. This is an equation of the quadratic form and compared with the equation of MSF stocks in (18), it is noticed that the constants of the two quadratic terms and that of the product term are equal, while the three others are different. Hence, the X^{MPV} -locus is a branch of a hyperbola with asymptotes parallel to the asymptotes of the X^{MSF} hyperbola. Therefore, they do not intersect.

To find the terminal point of the $X_{c=0}^{MPV}$ -locus at the abscissa, insert $X_2 = 0$ into the

PV function in equation (41) to arrive at

$$PV = \int_0^{\infty} e^{-\delta t} X_1(1 - X_1)dt, \quad X_2 = 0, \quad X_i^{\infty} = 0 \quad (i = 1, 2). \quad (47)$$

The problem of maximizing PV is now reduced to that of a single species problem (known from Clark [1], ch. 2), and the solution is

$$X_1^{MPV} = (1 - \delta_1)/2, \quad X_2 = 0, \quad X_i^{\infty} = 0 \quad (i = 1, 2). \quad (48)$$

Since the terminal point of the $X_{c=0}^{MPV}$ locus at the abscissa is inside the corresponding point at the X^{MSF} -locus, the conclusion is that the entire $X_{c=0}^{MPV}$ -locus is inside the X^{MSF} -locus in the case of costless harvesting. Thereby Theorem 3 is proved.

From Theorems 2 and 3 and from equations (44)–(45) it now follows that in the general case of stock dependent costs and a positive social rate of discount the X^{MPV} -locus is between two borders. The inner border is determined by the stock levels, $X_{c=0}^{MPV}$, maximizing the present value of the resource rent for the zero cost case and a given (maximum) social rate of discount. The outer border is determined by the stock levels, X^{MEY} , maximizing the sustainable economic yield in the case of (maximum) stock dependent costs. For a given bioprice ratio, β , of the two species the long run optimal equilibrium stock levels are uniquely determined by a point in the $X_1 - X_2$ -plane at or between these two borders, depending on the size of the harvest costs and the social rate of discount.

In single species biomass models with positive harvest costs and a positive social rate of discount, the optimal stock, X^{MPV} , approaches the MEY stock level when $\delta \rightarrow 0$, and

approaches the open access stock level, X^∞ , when $\delta \rightarrow \infty$ (see [1], ch. 2). For this two species model of competing species we can now show

Theorem 4. In a model of two competing species the long run optimal equilibrium stock level for any one species may be below the open access stock level. When that is the case the optimal stock level of this particular species increases with an increase in the social rate of discount.

In this model equations (44)–(45) approaches the necessary conditions for the maximization of sustainable rent in equation (31) when $\delta \rightarrow 0$. This is to say that X^{MPV} approaches X^{MEY} when the social rate of discount approaches zero. It also follows from equations (44)–(45), after having divided by δ , that the r.h.s. must approach zero when $\delta \rightarrow \infty$, i.e. the X^{MPV} approaches the open access stock levels, X^∞ . Thereby it is demonstrated that like in the single species model we have

$$\lim_{\delta \rightarrow 0} X_i^{MPV} = X_i^{MEY} \quad (i = 1, 2) \quad (49)$$

and

$$\lim_{\delta \rightarrow \infty} X_i^{MPV} = X_i^\infty \quad (i = 1, 2). \quad (50)$$

This also implies

$$\frac{\partial X_i^{MPV}}{\partial \delta} > 0 \quad \text{if} \quad X_i^{PV} < X_i^\infty \quad (i = 1 \text{ or } 2), \quad (51)$$

and Theorem 4 is proved. The results in Theorem 4 is not possible to have in single species models, but it may happen for the predator in predator–prey models [13].

The long run optimal equilibrium stock levels are functions of biological and economic parameters. It is of interest to analyse how these stock levels are changed by marginal changes in the harvest prices and effort costs. Let us first prove

Theorem 5. For two competing species the effect of an own price increase on the long run optimal resource stock is negative, whereas an increase in the price of the other species increases the optimal stock level of the former species.

Start with the equilibrium equations (42) and (43) which may be written as

$$\frac{1}{\delta} \frac{\partial \pi(\cdot)}{\partial X_i} = b_i(\cdot) \quad (i = 1, 2). \quad (52)$$

With harvest price, p_i , and effort cost, c_i , as exogeneous variables, differentiating equation (26) gives

$$b_{ip} = \frac{\partial b_i(\cdot)}{\partial p_i} > 0, \quad b_{ic} = \frac{\partial b_i(\cdot)}{\partial c_i} < 0 \quad \text{and} \quad b_{ix} = \frac{\partial b_i(\cdot)}{\partial x_i} > 0, \quad (53)$$

which shall be used in the analysis. Differentiating equation (52) w.r.t. p_1 , rearranging somewhat and by using Cramer's rule we find

$$\frac{\partial X_1^{MPV}}{\partial p_1} = \frac{\delta b_{1p} \left(\frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x} \right)}{|D|} \quad (54)$$

$$\frac{\partial X_2^{MPV}}{\partial p_1} = \frac{-\delta b_{1p} \frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|}, \quad (55)$$

where

$$|D| = \begin{vmatrix} \frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x} & \frac{\partial^2 \pi}{\partial X_1 \partial X_2} \\ \frac{\partial^2 \pi}{\partial X_2 \partial X_1} & \frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x} \end{vmatrix}.$$

The second order conditions for the existence of an interior solution to the maximization problem in equation (41) are

$$\left| \begin{array}{cc} \frac{\partial^2 \pi}{\partial X_1^2} & \frac{\partial^2 \pi}{\partial X_1 \partial X_2} \\ \frac{\partial \pi}{\partial X_2 \partial X_1} & \frac{\partial^2 \pi}{\partial X_2^2} \end{array} \right| > 0 \quad \text{and} \quad \frac{\partial^2 \pi}{\partial X_1^2} < 0, \quad (56)$$

and from this follows that $\partial^2 \pi / \partial X_2^2 < 0$. The second order conditions imply $|D| > 0$ because of (53). Taking the second-order derivatives of the profit function (31) w.r.t. X_i we get

$$\frac{\partial^2 \pi}{\partial X_i^2} = -r_i p_i \quad (i = 1, 2) \quad (57)$$

and

$$\frac{\partial^2 \pi}{\partial X_i \partial X_j} = -\alpha_1 r_1 p_1 - \alpha_2 r_2 p_2 \quad (i \neq j). \quad (58)$$

The results in (57)–(58) imply that the second order conditions are met, and $|D| > 0$.

From (54) and (55) now follows that

$$\frac{\partial X_1^{MPV}}{\partial p_1} < 0 \quad \text{and} \quad \frac{\partial X_2^{MPV}}{\partial p_1} > 0. \quad (59)$$

Since the relationship between the two species in this model is symmetric it is obvious that differentiating w.r.t. p_2 in equation (52) give similar results as in (59), hence, Theorem 5 is proved.

Using the same method as to prove Theorem 5 it is straightforward to prove

Theorem 6. For two competing species the effect of an own effort cost increase on the long run optimal resource stock is positive, whereas an increase in the effort cost of the other species reduces the optimal stock level of the former species.

From Theorems 5 and 6 we learn that a price increase (cost decrease) for one of the two species initially augments the value of the resource capital of this particular species proportionally to the price increase (cost decrease). However, it pays to transform some of the increased wealth into capital in general, rewarding the society with the interest expressed by the discount rate, and some into the resource capital of the competing species. The reward to the society from the latter investment is increased revenues and reduced harvesting costs for this competing species.

6 Conclusion

The concept of sustainable development has been widely recognized by the release of the report of the United Nations' World Commission on Environment and Development [18]. However, the interpretation of this concept is not always clear. For renewable resources such as forests, wildlife and fish it seems evident that within a multispecies framework sustainable development must take place for combinations of stocks within the sustainable yield area. Harvesting at the maximum sustainable yield frontier (MSF) may at a first glance seem evident from a biological point of view. However, from an economic point of view MSF-harvesting is optimal only in special cases, such as when there is no discounting and harvest costs are zero.

Identifying the limits of sustainable yields in ecosystems is mainly an ecological task, whereas the utilization of such a system for sustainable economic development in addition involves aspects of economics, technology and social organization. Hence, successful sustainable economic exploitation of a complicated ecosystem is a complex problem which

requires a multi-disciplinary approach.

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Notes

¹The dynamics of the model was analysed in Clark [1], ch. 6.6. We assume that there is a competitive coexistence equilibrium solution of the model.

²The effect of one species on another is similar to the interaction of Cournot oligopolists. The isoclines in this model of competing species are identical to the reaction functions of the Cournot model with linear demand and costs.

³Essentially all deterministic two species models will have a bounded sustainable yield area, but not necessarily a quadrangle. The predator-prey model used in Flaaten [13] has a triangle as the sustainable yield area.

⁴The notion of MSF was used by Flaaten [12]. The method, however, was first described and used by Beddington and May [16].

⁵Since the X^{MSF} is a hyperbola and it is downward sloping at both of the terminal points of the first quadrant, it is concave to the origin in the sustainable yield area.

⁶Solutions at the boundaries of the sustainable yield quadrangle, with only one species being harvested, may often be the case in the real world. Flaaten [13] analysed such cases, however, it will not be done in this article.

⁷It is not immediately obvious what "open access" in general means in the context of multispecies harvesting. However, recalling the assumption of technically independent

harvesting of the two competing species, open access bioeconomic equilibrium in this case means that profit on each species must be zero.

⁸In general technological parameters also affect the open access stock levels. However, in this case with the Schaefer production function and rescaled fishing effort such that the catchability coefficient equal unity, the open access stock levels in equation (29) are seemingly independent of the harvest technology.

⁹The stock effect is usually interpreted as the effect a marginal increase in the stock level of one species has on its own harvesting costs.

¹⁰The angle, v , between the axes of the ellipse and the coordinate system can be found from $\tan 2v = 2B/(A - C)$, according to standard geometry. In the case of the profit ellipse of equation (32) v is found from $\tan 2v = (\alpha_1 + \beta\alpha_2)/(1 - \beta)$.

¹¹Graphically the MEY-stocks are found at the intersection of $X_1 = (1 + \bar{X}^\infty - \gamma X_2)/2$ and $X_2 = (\beta + \hat{X}^\infty - \gamma X_1)/2$ derived from equations (33) and (34).

¹²Solutions outside the sustainable yield area are not of biological or economic interest. Therefore, only the negative root is of interest in this case.

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