Power-law scaling of uncorrelated plasma bursts

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Self-organized criticality (SOC) is a well-known paradigm for explaining power law probability distributions and frequency spectra in astrophysical, space and laboratory plasmas [1, 2]. Some examples are presented in Figs. 1 and 2. By contrast, in the scrape-off layer (SOL) of magnetically confined fusion plasmas and other turbulent systems, probability distributions with exponential tails and Lorentzian frequency spectra are observed, see Figs. 3 and 4 [3, 4]. These observations are well explained by a stochastic model consisting of a superposition of exponential pulses, arriving according to a stationary Poisson process, called the filtered Poisson process (FPP) [3, 4]. Connections between SOC and the FPP were made as early as one of the original SOC publications [5], where power-law distributed event durations and power-law frequency spectra were explained based on viewing a SOC time series as a sequence of uncorrelated pulses.

In this contribution, we investigate power-law behavior in the FPP. By allowing pulse durations, pulse decay or pulse amplitudes to follow a power-law, different power-law scalings emerge in the power spectral density and in the distributions of process amplitude, avalanche durations and avalanche sizes. The findings are applied to example time series from a tokamak SOL.

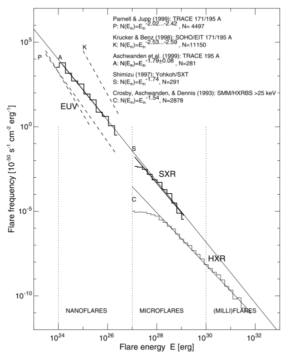


Figure 1: Solar flare energy distribution [1].

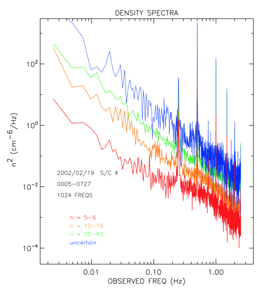


Figure 2: Power spectra of solar wind [6].

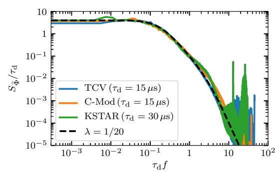


Figure 3: Power spectra from the SOL [7]. The black dashed line gives the standard FPP.

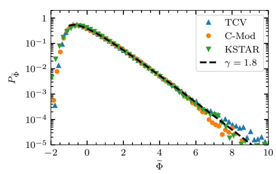


Figure 4: Probability densities from the SOL [7]. The black dashed line gives the standard FPP.

The filtered Poisson process

The filtered Poisson process (FPP) is given by

$$\Phi(t) = \sum_{k=0}^{K}(T) A_k \phi\left(\frac{t-t_k}{\tau_k}\right), \tag{1}$$

where Φ is defined on $t \in [0,T]$. The pulse amplitudes are denoted by A_k, the pulse shape is ϕ, the number of pulses in $[0,T]$ is given by $K(T)$, t_k denotes pulse arrival times and τ_k denotes pulse duration times. The pulse shape is normalized to $\int_{-\infty}^{\infty}\phi(\theta)\mathrm{d}\theta = 1$. All random variables are assumed independent, and $K(T)$ is Poisson distributed with intensity $\langle K\rangle = T/\tau_\mathrm{w}$.

We denote the special case of degenerately distributed duration times, exponentially decaying pulse shape and exponentially distributed amplitudes by the *standard* FPP. For $\langle\tau\rangle/\tau_\mathrm{w} \to 0$, there is practically no pulse overlap and each pulse can be considered separately. This is called the *intermittent* limit. For $\langle\tau\rangle/\tau_\mathrm{w} \to \infty$, infinitely many pulses arrive in the decay time of a single pulse. As long as all distributions have finite moments the normalized FPP $\widetilde{\Phi} = (\Phi - \langle\Phi\rangle)/\Phi_\mathrm{rms}$ approaches a normally distributed process. This is therefore the *normal* limit.

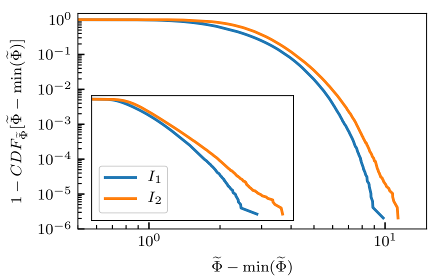
Power-law scaling in the FPP

We investigate power-law behavior in the following statistical properties:

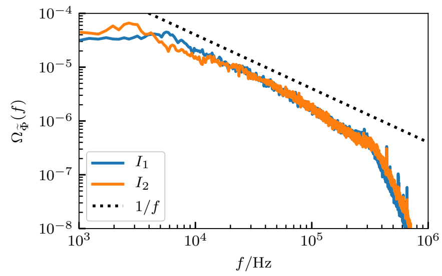
- Process amplitude probability distribution function, $p_\Phi(\Phi)$.
- Power spectral density, $\Omega_\Phi(\omega)$.
- Probability distribution of time above the mean value (avalanche duration), $p_{\triangle T}(\triangle T)$.
- Probability distribution of integral above mean value (avalanche size), $p_S(S)$.

Separately considering the cases of power-law distributed pulse duration times, power-law pulse decay and power-law distributed pulse amplitudes gives the following table. For derivations and further explanations, see [8].

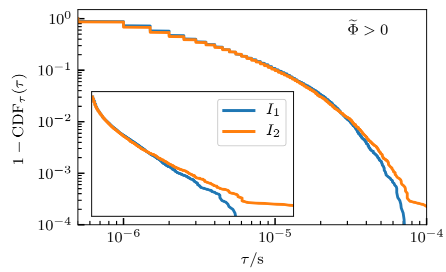
	Standard	$p_\tau(\tau) \sim \tau^{-\alpha}$	$\phi(\theta) \sim \theta^{-\alpha}$	$p_A(A) \sim A^{-\alpha}$
regime		$1 < \alpha < 3$	$0 < \alpha < 1$	
$p_\Phi(\Phi) \sim \Phi^{-s}$	None	None	None	Present
$\Omega_\Phi(\omega) \sim \omega^{-\beta}$	None	$\beta = 3-\alpha$	$\beta = 2(1-\alpha)$	None
Intermittent, $p_{\triangle T}(\triangle T) \sim \triangle T^{-\nu}$	None	$\nu = \alpha$	None	None
Normal, $p_{\triangle T}(\triangle T) \sim \triangle T^{-\nu}$	$\nu = 3/2$	$\nu = \alpha/2+1$	$\nu = \alpha+3/2$	$\nu = 3/2$
Intermittent, $p_S(S) \sim S^{-\chi}$	None	$\chi = \alpha$	None	$\chi = \alpha$
Normal, $p_S(S) \sim S^{-\chi}$	$\chi = 4/3$	$\chi = 4/(4-\alpha)$	$\chi = 4/(3-2\alpha)$	$\chi = 4/3$



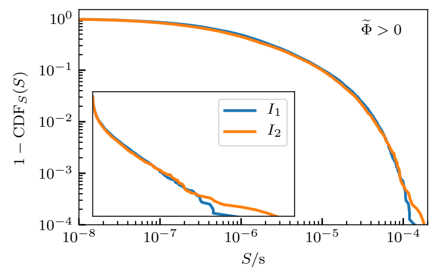
(a) Ion saturation current survival function.



(b) Ion saturation current power spectral density.



(c) Survival function of avalanche durations.



(d) Survival function of avalanche sizes.

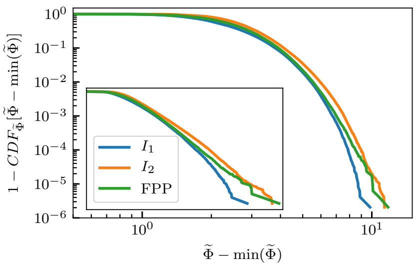
Figure 5: Example time series from the tokamak scrape-off layer.

Example: Ion saturation current in the tokamak scrape-off layer

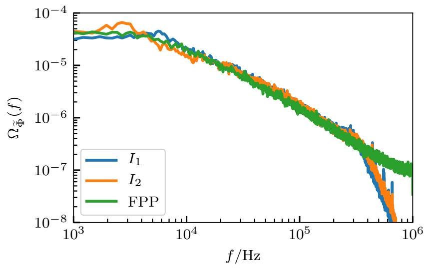
Here, we consider example time seres of the ion saturation current from the SOL of a tokamak. The ion saturation current time series have been detrended by removing a running mean and dividing by a running standard deviation. In Fig. 5, the statistical properties of the signals is presented. It is seen that only the power spectral density displays power-law behavior, indicating the presence of pulses decaying as a power law. To investigate this, we generate synthetic realizations of the FPP with power-law pulses with $\alpha = 1/2$ and an exponential cutoff at $35\mu s$. The results, which are consistent with the experimental time series, is presented in Fig. 6.

Conclusions and future work

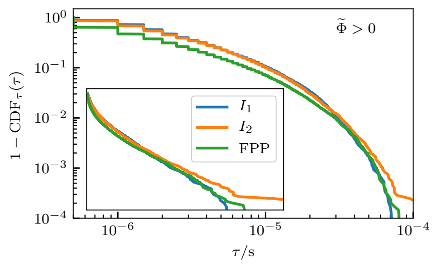
In conclusion, different assumptions in the inputs lead to different, separable scalings in the FPP. We found no evidence of duration time distributions or power-law amplitudes in example time series, while the FPP with power-law pulses was consistent with the example time series. In the future, we seek to extend the results to non-Poisson arrival times and to avalanche duration and size distributions for intermediate intermittency. This will be completed by an investigation of canonical SOC systems.



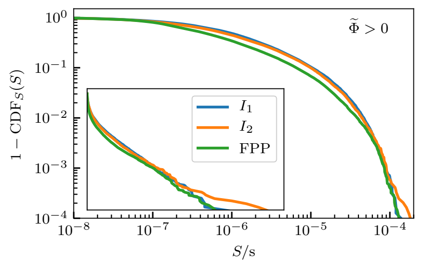
(a) Ion saturation current survival function.



(b) Ion saturation current power spectral density.



(c) Survival function of avalanche durations.



(d) Survival function of avalanche sizes.

Figure 6: Example time series from the tokamak scrape-off layer. The green line gives the realization of the FPP.

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