

Recent trends in operation modal analysis techniques and its application on a steel truss bridge

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Dedicated to 75 years anniversary of professor Lars-Erik Persson - The happy Mathematician

Abstract

Ageing of infrastructure causes many problems with great consequences, essentially economical. Operation modal analysis (OMA) is one of the most crucial techniques used for dynamic analysis of civil engineering structures (e.g. bridges, dams or tunnels). OMA uses various time and frequency domain methods to obtain the modal parameters. The analysis of OMA techniques can be used to detect, locate and quantify the damage in a structure. The major challenge for damage detection using OMA is the analysis of large amount of noisy data collected from sensors. New signal processing techniques and artificial intelligence can play an important role for future research in the area. In this article we present and discuss recent developments in OMA techniques and also give a concrete example on a steel truss bridge, where the most popular OMA techniques have been implemented and applied.

Keywords: Operation modal analysis, Structural health monitoring, Stochastic sub-space identification, Frequency domain decomposition, Damage detection, Bridge, Civil engineering structures, Wavelet, Hilbert transform, Artificial intelligence, Arctic conditions

1 Introduction

1 Ageing of the civil infrastructure around the world causes many serious problems, and new challenging
2 research techniques are needed to solve them. Governments and municipalities have to devote more
3 time and budget for maintenance, repairs or construction of new structures in place of deteriorated or
4 damaged ones to provide decent service to the citizens. Some reasons for the deterioration of buildings
5 and civil engineering structures are: environmentally induced degradation, poor understanding of
6 initial conditions or lack of maintenance.

7 For example in Germany, the value of constructed infrastructure is about 20 trillion Euro. If the
8 life of the infrastructure is assumed to be 100 years then the replacement rate is approximately 200
9 billion Euro per year (see [1]). Additionally, the American Society of Civil Engineers did a case study
10 on the infrastructure in USA. This study found out that an investment of over 2 trillion USD is needed
11 over the next 10 years to reduce the risks of ageing infrastructure (see [2]). The report also found
12 out that 9.1 % of bridges in USA were structurally deficient in 2016, and approximately 123 billion
13 USD would be needed for bridge rehabilitation (see [2]). The state of Michigan spent approximately 4
14 billion USD to address transportation needs in 2017 (see [3]). Furthermore, in Canada, the estimated
15 maintenance cost of public infrastructure in 2003 was close to 6 billion CAD. Maintenance costs are
16 on the rise as the infrastructure around the globe is approaching the end of its life cycle (see [4]).
17 Analysis of such problems is important e.g. in Scandinavia, as the impact of extreme arctic conditions
18 is quite intense. The infrastructure discussed above mainly concern bridges, dams and tunnels. In this
19 paper we will concentrate on bridges but the techniques we present are applicable in the two other
20 cases as well.

21 As per current trends, operators of bridges, dams and tunnels base their infrastructure assets man-
22 agement decisions on visual inspections, which could be aided by localized diagnosis techniques such as

23 the use of acoustic, ultrasonic or magnetic field non-destructive testing methodologies. Nevertheless,
 24 these testing methodologies have several limitations such as, inaccessibility to some parts of the struc-
 25 ture, inability to detect internal damage, location of the damage, and continuous monitoring cannot
 26 be carried out. With the advancement in technology, new techniques are under development for the
 27 monitoring of structures. These techniques are commonly called Structural Health Monitoring (SHM)
 28 techniques, where sensors distributed throughout the structure are used to estimate the conditions of
 29 the structures. In order to do damage detection and localization, the raw data generated by sensors
 30 is processed to find the key parameters: mode shapes, mode frequencies and mode damping. Once
 31 these parameters have been estimated, damage detection algorithms can be utilized to figure out the
 32 magnitude of damage occurred, if any.

33 Some of the commonly used techniques to calculate these modal parameters are Finite Element
 34 Model (FEM), FEM updating, Experimental Modal Analysis (EMA) and OMA. In FEM the modal
 35 parameters are computed by a software package whereas in FEM updating sensors are used to calibrate
 36 the FEM model and reduce the errors. For EMA, the structure is excited by an impulse that is given
 37 by an instrumented hammer and the vibration response is measured with accelerometers. In the case
 38 of OMA no artificial excitation is needed as it identifies the modal properties when the structure is
 39 in its operating conditions. These techniques are discussed in Section 2. The technique that we are
 40 going to focus in this article is OMA.

41 Taking into consideration all the parameters including safety and investments, it is very important
 42 to further develop more efficient OMA techniques that will contribute towards the development of
 43 SHM. This is the reason why the research around the world on this topic is so intense at the moment.
 44 See e.g. the books [1], [5], [6], [7], [8] and [9] and the references in these books and in this article. The
 45 aim of this paper, is to report on the recent developments in OMA techniques and present the results
 46 of an experiment, conducted over a steel truss bridge in Sweden. The bridge that has been analysed is
 47 located about 45 km west of the Piteå town in northern Sweden over Åby river. Moreover, the article
 48 will form the basis for further research in this area.

49 The paper is organised as follows: In Section 2 we present the three most powerful approaches for
 50 dynamic analysis. In Section 3 we shortly discuss various signal processing techniques used in SHM.
 51 This is followed by Section 4 that covers theoretical and mathematical aspects of this paper. Here we
 52 describe the most popular OMA techniques that are classified as time domain methods and frequency
 53 domain methods. Section 5 is the heart of the paper, where we give a concrete example how one of
 54 these methods is implemented and applied in one of the bridges in Sweden. Finally, in Section 6 we
 55 give some final remarks. One of the remarks is important for research of this type of questions in
 56 arctic region.

57 **2 Three approaches for dynamic analysis**

58 In order to do an assessment of the structures condition, three commonly used methodologies are well
 59 acknowledged. They are classified as FEM, FEM updating and OMA.

60 **2.1 Dynamic analysis of finite element model**

61 Structural dynamics is the analysis when a structure is subjected to dynamic loading that is time
 62 dependent. The structure is exposed to actions having high accelerations. For basic structures,
 63 dynamic analysis can be carried out manually, but in the case of complex structures, FEM is used to
 64 calculate mode shapes and frequencies. In the dynamic analysis, bending and strains of the structure
 65 are compared against the bending and strains of FEM. FEM is the most appropriate tool for modelling
 66 the structures. The differential equation that is used for the modelling of linear dynamical model is

$$M \frac{d^2 u(t)}{dt^2} + C_2 \frac{du}{dt} + K u(t) = B_2 f(t), \quad (1)$$

67 where M is the mass matrix, C_2 is the damping matrix, K is the stiffness matrix and B_2 is the selection
 68 matrix (input matrix), $f(t)$ is a vector with nodal forces and $u(t)$ is a vector with nodal displacements
 69 (see [10]).

70 Basically, FEM is a numerical method to solve engineering problems related to structures. The
71 analytic solution to these problems require the solution to boundary value problems for partial differ-
72 ential equations. The FEM method approximates the unknown function described over the domain.
73 In order to find a solution to the problem, a large system is divided into smaller parts known as finite
74 elements. The simple equations that could model these finite elements are put together into a large
75 system of equations that models the entire problem. Variational methods from the calculus of varia-
76 tion are used to approximate the solution of the problem. In-depth study and analysis of structures
77 using FEM can be referred from [11]. The FEM model of a Långforsen bridge over the Kalix river
78 has been developed to check the possibility to increase the axle load (see [12]). Långforsen bridge is
79 a reinforced concrete railway bridge situated between Kalix and Boden in northern Sweden. Fatigue
80 assessment of the bridge had been carried out in conjunction with moving load and moving spring
81 mass damper vehicle models to evaluate dynamic performance of the bridge (see [13]). The model
82 computes the dynamic properties of the structure. Geometry and material properties of the bridge
83 have been studied and analysed in a detailed report [14].

84 2.2 Finite element model updating

85 High accuracy is needed in the FEM for implementing structural control and SHM strategies. This
86 accuracy depends on the type of FEM used to represent the structural members as well as the proper-
87 ties assigned to these elements. The finite element model has uncertainties in deciding the boundary
88 conditions, geometry or material properties that change when the material deteriorates. Non-linearity
89 occurs due to material properties depending upon loading conditions. Thus, the FEM model needs to
90 be calibrated based on the information from real structure (see [15]). Numerical optimization tech-
91 nique known as FEM updating is used to calibrate the key parameters in the finite element model
92 of the structure that minimizes the smallest possible difference between measured vibrations and the
93 simulated vibrations.

94 The significant differences in the dynamic behavior of a FEM model have been discovered after up-
95 dating and the corresponding real structure. A difference of 17.4 % was discovered in the experimental
96 natural frequency and the frequency calculated by the initial FEM model of the Kap Shui Mun cable-
97 stayed bridge (see [15]). At the Pioneer bridge in western Singapore difference of 23 % was discovered
98 between the experimental dynamic characteristics and those of FEM (see [15]). FEM updating of
99 Safti link bridge has been explained along with FEM in [16]. Mode shapes ϕ_m and frequencies ξ_m of a
100 structure can be computed using different methods. FEM updating gives the improved knowledge of
101 boundary conditions and local changes of material properties. The most complicated part in the FEM
102 is to compute damping ratios of different parts of the structure. Simplified models can be used to
103 estimate the damping. In general, software like Abaqus and Brigade, ignore damping while computing
104 mode shapes and frequencies. Even though the damping is usually small enough to be neglected but
105 it is significant for estimating the dynamic response. With such considerations, the problem in terms
106 of homogeneous differential equation can be reformulated as an eigenvalue problem as follows:

$$(K - \lambda_m M)\phi_m = 0, \quad (2)$$

107 where K is the stiffness matrix, M is the mass matrix, ϕ_m is the m th mode shape eigenvector and λ_m
108 is the m th eigenvalue of the structure (see [17]). Every element of ϕ_m corresponds to group element in
109 the FEM and the m th solution describes a vibration mode $\phi_m \cos(2\pi\xi_m t)$, where the mode frequency
110 ξ_m correspond to the eigenvalue $\lambda_m = (2\pi\xi_m)^2$. FEM updating techniques can be grouped as:

- 111 1. Updating using modal data.
- 112 2. Updating using frequency response functions.
- 113 3. Updating using gradient and gradient free methods.

114 Multiple alternatives for FEM updating have been discussed in [15] where acceleration records
115 from the permanent instrumentation on the Bill Emerson Memorial Bridge are used to update the
116 model.

117 2.3 Operation modal analysis

118 In the structures, modal damping ratio is more sensitive to damages than mode frequencies. Forced
119 vibration tests with artificial excitation forces can be performed on large structures, but such tests
120 are costly and complicated. Moreover, other vibration sources such as wind and traffic are treated
121 as noise. Another approach known as OMA is gaining popularity where ambient vibrations from the
122 wind and traffic are considered as unknown input, and output-only analysis is done to determine the
123 resulting vibration modes. OMA techniques are used in our main application on steel truss bridge
124 (see section 5) and we will present and discuss these techniques in detail in section 4.

125 3 Some signal processing techniques for SHM

126 Most of the signals in structural damage detection methods are time based signals that are recorded by
127 the sensors. The vibrations in the structures can be due to input time signals, like earthquake, wind,
128 loading, or due to artificial excitations and the output signals can be recorded such as accelerations,
129 strains or displacements. These types of signals are non-stationary in nature, that is, they change
130 their characteristics with time (see e.g. [18]). The damage identification is more effective in the
131 frequency domain (see [9]) so the signals in the time domain are transformed to the frequency domain.
132 The signal processing methods like Fourier based transforms, Wavelet transforms, S transform and
133 Hilbert-Huang transforms are applied. In this short review we do not give more details here because
134 of restrictions of length of this paper, moreover these methods are not used in our main application
135 in section 5. However, these methods will be discussed and compared with the results in this paper
136 in our forthcoming article.

137 4 Operation modal analysis techniques

138 Damage in a structure affects its dynamic properties. The information from the vibration signals of
139 the structure can be used for damage detection. In order to detect damage using SHM, one of the
140 most important parameters that needs to have a good estimation is the modal damping ratio, since it
141 is more sensitive to damages in comparison to mode frequencies (see [19]). As discussed in sub-section
142 2.3, OMA can do a good estimation of modal parameters.

143 The work in the area of OMA started in the 1960s but it got more organized and systematized in the
144 last two decades. Earlier output-only modal identification was referred to as ambient vibration testing.
145 Initially, the applications of OMA were based on Power Spectral Density (PSD) and the identification
146 of Operational Deflection Shapes (ODS). ODS represents the deflection of a structure at a particular
147 frequency under a generic input and is the result of the contribution of various mode shapes. It was
148 later discovered that under certain assumptions ODS is a close estimate of the actual mode shapes.
149 The OMA techniques are based on the assumptions of linearity, stationarity and observability (see
150 [7]).

151 In OMA, the loading of the structure is not controlled. The environmental loads such as wind,
152 traffic, etc., are assumed as unknown forces that excite the structure. Under this scenario, the mea-
153 sured response can be interpreted as the output of the combined system made by the excitation system
154 and the structure under test. The combined system is illustrated in Figure 1.

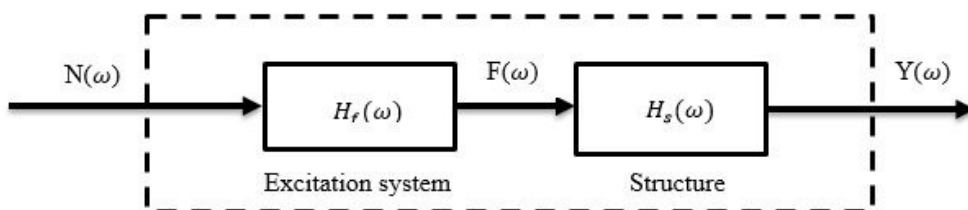


Figure 1: Combined system

155 As the excitation system and the structure are in series therefore the Frequency Response Function
 156 (FRF) of the combined system is the product of their respective FRFs (see [7], [8])

$$H_c(\omega) = H_f(\omega)H_s(\omega), \quad (3)$$

157 where $H_c(\omega)$, $H_f(\omega)$, and $H_s(\omega)$ are the FRFs of the combined system, the excitation system and
 158 the structure, respectively. For each subsystem the output is related by the following equations:

$$F(\omega) = H_f(\omega)N(\omega), \quad (4)$$

$$Y(\omega) = H_s(\omega)F(\omega), \quad (5)$$

160 where $N(\omega)$, $F(\omega)$ and $Y(\omega)$ represents the Fourier transforms of the white noise input to the
 161 excitation system, the excitation system output and the structure output, respectively. The measured
 162 response contains the information about the excitation system and the structure under test. The
 163 modal parameters of the structure are preserved and identifiable. The characteristics of the excitation
 164 have no influence on the accuracy of modal parameter estimation. It is possible to distinguish between
 165 structural modal properties and the properties of the excitation system because of the fact that the
 166 structural system has a narrow band response and time invariant properties, whereas, the excitation
 167 system has broadband response and can be either time variant or time invariant. The OMA methods
 168 can be classified as time domain methods and frequency domain methods. A detailed overview of
 169 these methods can be found in the books [7], [8] and the Ph.D. thesis [20]. Another class of OMA
 170 methods based on time-frequency analysis such as wavelets and Hilbert transform are discussed in
 171 Section 6. These time-frequency methods are under development phase. We also remark that most
 172 of the OMA techniques may be regarded as extensions of traditional Experimental Modal Analysis
 173 (EMA) techniques. Below, we shortly describe the most important OMA methods.

174 4.1 Time domain methods

175 The change in the dynamic properties of a structure due to the damage can be figured out from
 176 the change in statistical characteristics of the acceleration-time histories (see [1]). Damage detection
 177 can be performed on the information extracted from the vibration signals that are measured before
 178 and after the damage has occurred. Some examples of time domain methods are Autoregressive
 179 Moving Average (ARMA), Ibrahim Time Domain (ITD), Eigensystem Realization Algorithm (ERA),
 180 Stochastic Subspace Identification (SSI) and Second Order Blind Identification. Two of the most used
 181 time domain methods (ARMA and SSI) for extracting modal parameters are described below and in
 182 a little more general context.

183 4.1.1 Autoregressive family of methods

184 The simplest way to carry out OMA is to use Autoregressive (AR) models on the free decays of
 185 discrete time data. The concepts of AR and ARMA are described in all of the literature concerning
 186 sampled time varying signals. AR model is different from Poly Reference (PR) model in the sense
 187 that PR model uses impulse response functions whereas AR model uses correlation functions (see
 188 [8]). The motion of randomly excited linear time-invariant system can be described by a discrete-
 189 time ARMA vector by approximating the differential operator with finite differences over a finite time
 190 step Δt (see [7]). A detailed study of AR and the ARMA methods can be found in [21] and [22].
 191 ARMA models have been used to estimate the modal parameters of structures (see [23]) and are
 192 applicable for stationary process vibration. New techniques are needed to address the problem of non-
 193 stationary time series vibrations for linear systems, subjected to non-stationary ambient excitations.
 194 An extended time series algorithm for estimating the modal parameters of non-stationary time series
 195 vibrations is discussed in [24]. The ARMA model can be used to differentiate between the damaged
 196 and non-damaged state of the structure. In order to do so damage sensitive features are needed to be
 197 computed (see [1]). ARMA model did not get so popular because it requires excessive computational
 198 time and has convergence problems. If the structure is large, the system would have many outputs and
 199 many modes. Thus, estimation of parameters require huge computations (see e.g. [7]). Therefore, the
 200 stochastic state-space models gradually replaced ARMA models in the domain of modal identification.

201 4.1.2 Stochastic subspace identification based methods

202 The work presented by Overschee and Moor in the book [25] was obviously influenced by previous
 203 work of the Swedish professor Lennart Ljung and his collaborators. In 1993, professor Ljung claimed
 204 in a statement at the second European Research Network that “ *The development of Subspace Methods*
 205 *is the most exciting thing that has happened to system identification in the last 5 years or so...* ” . His
 206 work is a huge milestone in the development of SSI methods and techniques. Stochastic state space
 207 model is identified directly from the measured output data or output correlations. The model is a good
 208 representation of a vibrating structure that is excited by unknown forces, which are assumed to be
 209 white noise (see [26]) and it fits the discrete-time stochastic state space realizations (see [27]). In fact,
 210 today SSI is the most commonly used time domain technique for OMA. The SSI based model has been
 211 implemented e.g. when investigating the Confederation bridge in Canada (see [4]). SSI methods can be
 212 classified as covariance-driven and data based methods (see [20]). Covariance-driven SSI technique was
 213 inspired by the classical realization theory. By this method the problem of estimating the stochastic
 214 state space model from the output data can be resolved. The discrete-time deterministic model can
 215 be derived from equation (1) (see [7], [8] and [10]) and is represented by the following equations:

$$x_{k+1} = Ax_k + Bu_k, \quad (6)$$

$$y_k = Cx_k + Du_k, \quad (7)$$

216
 217 where x_k is the discrete time state vector that gives the sampled displacements and velocities, u_k is
 218 the sampled input and y_k is the sampled output. A is the discrete system matrix, B is the discrete
 219 input matrix, C is the discrete output matrix and D is the direct transmission matrix. The output-
 220 only analysis of the state space can be described without the measured input vector u_k . Thus, the
 221 discrete-time stochastic state-space model can be described as:

$$x_{k+1} = Ax_k + w_k, \quad (8)$$

$$y_k = Cx_k + v_k, \quad (9)$$

222
 223 where w_k is the process noise due to the disturbances and inaccuracies in the model and v_k is the
 224 measurement noise due to inaccuracies in the sensor (see [7]). As the dynamics of the system is
 225 described by the eigenvalues and eigenvectors of the matrix A , modal parameters can be obtained by
 226 the eigenvalue decomposition. The algorithm to determine modal parameters from the matrices A and
 227 C is discussed in [7], [8] and [27]. The covariance driven SSI can be implemented in three different ways
 228 namely: principal component method, canonical variant analysis method and unweighted principal
 229 component method. All the three methods have similar accuracy to determine the modal parameters
 230 (see [28] and [29]).

231 Data driven SSI algorithms are getting more popular in comparison to covariance driven SSI
 232 algorithms (see [7]). Data driven SSI is built on mathematical framework and robust linear algebra
 233 tools to identify the matrices A and C from the raw data. The data driven SSI algorithm is based
 234 on projecting the row space of the future outputs to the row space of the past outputs by means
 235 of QR decomposition of the data Hankel matrix. This process leads to the data reduction. System
 236 parameters are obtained by the Singular Value Decomposition (SVD) of the projection matrix and
 237 finally the modal parameters are obtained by the least square approach. More details of the data
 238 driven SSI algorithm can be found in [7] and [25].

239 In the ARMA model, noise is modeled due to which lots of spurious poles appear that are not
 240 related to the dynamics of the system under test. Therefore, the selection of the system poles become
 241 difficult and the presence of noise can effect the modal parameters as well (see [7]). Both the subspace
 242 methods, covariance driven SSI and data driven SSI have noise reduction mechanisms based on SVD. If
 243 the noise is present or the structure is poorly excited, the application of weighted matrices can improve
 244 the performance of the estimators. SSI methods perform equally well but data driven method is more
 245 efficient since it generates less data. These characteristics have made subspace methods more popular.
 246 An application of reference-based combined deterministic SSI for OMA has been verified and validated
 247 with experimental data on a bridge Z24 that overpasses the A1 highway between Bern and Zurich in
 248 Switzerland (see [10]).

249 4.2 Frequency domain methods

250 Time domain methods deal with the free responses that are present over the entire time span. However
 251 in the frequency domain each mode has a small frequency band where the mode dominates. Therefore,
 252 in frequency domain we have an advantage of natural modal decomposition by just considering the
 253 different frequency bands where different modes dominate (see [8]). This is the major advantage of
 254 this approach. In this sub-section we will present and discuss the Basic Frequency Domain (BFD)
 255 method, the Frequency Domain Decomposition (FDD) method and the Poly-Reference Least Square
 256 Complex Frequency method (p-LSCF).

257 4.2.1 The basic frequency domain method

258 The BFD method is one of the earliest methods for output-only modal parameter identification. The
 259 method is also known as peak picking method because of the fact that the modes are identified by
 260 picking the peaks in the PSD plot. The method is based on the computation of auto and cross spectra
 261 and is classified as a single degree of freedom method for OMA. It is assumed that at resonance
 262 only one mode is dominant. If the r -th mode is dominant, then the structural response $\{y(t)\}$ is
 263 approximately equal to the modal response as described in the relation below:

$$\{y(t)\} \approx \{\phi_r\}p_r(t), \quad (10)$$

264 where $p_r(t)$ is the modal coordinate associated to the r -th mode and ϕ_r is the r -th mode shape.
 265 Therefore the correlation function $[R_{yy}(\tau)]$ can be approximated as:

$$[R_{yy}(\tau)] = E[y(t+\tau)\{y(t)\}^T] = R_{p_r p_r}(\tau)\{\phi_r\}\{\phi_r\}^T, \quad (11)$$

266 where E is the expected value operator, $\{y(t)\}^T$ is the transpose of the structural response and $\{\phi_r\}^T$
 267 is the transpose of the r -th mode shape and

$$R_{p_r p_r}(\tau) = E[p_r(t+\tau)p_r(t)] \quad (12)$$

268 is the modal auto-correlation function and the spectral density matrix $[G_{YY}(\omega)]$ is given by:

$$[G_{YY}(\omega)] = G_{p_r p_r}(\omega)\{\phi_r\}\{\phi_r\}^H, \quad (13)$$

269 where $G_{p_r p_r}(\omega)$ is the auto- spectral density function of the modal coordinate and $\{\phi_r\}^H$ is the
 270 conjugate transpose of the r -th mode shape. From the equation above, it is understood that the
 271 rank of the PSD matrix is one. Therefore at resonance, any column of the PSD matrix could be
 272 considered as an estimate of the corresponding mode shape. The method is described in detail in [7].
 273 This technique was quite successful and it works especially well when the modes are well separated
 274 and have low damping. However, the method does not work well in situations where modes are not
 275 well separated, and the damping is moderate to heavy. Therefore, the identification of closely spaced
 276 modes is not possible by this technique.

277 4.2.2 Frequency domain decomposition method

278 FDD is one of the most popular techniques of OMA. FDD was introduced by professor Rune Brincker
 279 and his collaborators (see [30]) and it overcame the shortcomings of the peak picking method. In
 280 principle, FDD is similar to the Complex Mode Indicator Function (CMIF) algorithm. The FDD
 281 technique performs SVD on the output response power spectra matrix instead of Frequency Response
 282 Function (FRF) matrix. FRF and FRF matrix are described in detail in [8] and [30]. The PSD matrix
 283 is obtained by the Fourier transform of the correlation matrix of the responses (see [7]) and can be
 284 written as:

$$[G_{YY}(\omega)] = [\Phi_r]G_{p_r p_r}(\omega)[\Phi_r]^H \quad (14)$$

285 SVD of the PSD matrix at a certain frequency ω leads to the following factorization:

$$[G_{YY}(\omega)] = [U][\Sigma][V]^H, \quad (15)$$

286 where $[U]$ and $[V]$ represents the unitary matrices that are holding the left and right singular vectors,
 287 respectively, whereas $[\Sigma]$ is the matrix of singular values. Also, $[V]^H$ is the conjugate transpose of the
 288 matrix V . The PSD matrix is Hermitian and positive definite matrix, therefore $[U] = [V]$. Hence, the
 289 decomposition in the equation (15) can be written as:

$$[G_{YY}(\omega)] = [U][\Sigma][U]^H. \quad (16)$$

290 It is possible to establish one-to-one relationship between singular vectors and mode shapes by com-
 291 paring equations (14) and (16). Furthermore, it can be seen that singular values are related to the
 292 modal responses and can be used to define the spectra of the corresponding single degree of freedom
 293 systems that are characterized by the same modal parameters. As the number of non-zero elements
 294 in $[\Sigma]$ equals to the rank of the PSD matrix for the frequency under consideration, it helps to identify
 295 the closely spaced or coincident modes (see [7]). The equivalent single degree of freedom PSD function
 296 can be identified from the set of singular values, around the peak of the singular value plots that are
 297 characterized by similar singular vectors. In Enhanced Frequency Domain Decomposition (EFDD)
 298 the single degree of freedom PSD function is used to compute the modal damping ratio (see [7]). The
 299 method to compute the damping estimation is explained in a good way in [31].

300 4.2.3 Poly-reference least square complex frequency method

301 Maximum Likelihood (ML) estimators were developed to deal with noise in the signal. In the late
 302 1990s the Maximum Likelihood Frequency Domain (MLFD) method was proposed to use the FRF
 303 measurements for modal identification (see [28]). MLFD is a non-linear estimator that is implemented
 304 in an iteration process. Further, Least Square Complex Frequency (LSCF) method was incorporated
 305 to find the initial values for the iterative MLFD method (see [32]). The LSCF method can be studied
 306 in detail in [7]. The major advantage of the LSCF method is that it produces accurate enough modal
 307 parameters with much less computations. The major drawbacks of the LSCF method are:

- 308 1. It is difficult to obtain mode shapes and modal partition factor by reducing the residues to a
 309 rank-one matrix using SVD.
- 310 2. The poles that are closely spaced can be shown up as a single pole.

311 This led to the development of p-LSCF method (see [33]). This method removed the shortcomings
 312 of the LSCF method. The major advantage of p-LSCF lies in the fact that it is possible to have stable
 313 identification of the system poles and participation factors as a function of the specified system order.
 314 This resulted in much easier interpretation of stabilization diagrams. Therefore, the p-LSCF method
 315 has a potential to be applied on high-order and highly damped systems with large modal overlap. At
 316 the same time it is computationally efficient, for a detailed theoretical study of p-LSCF see e.g. [33].

317 5 Application of OMA on a steel truss bridge

318 We describe in this section some modal analysis results for a steel truss bridge over Åby River about
 319 45 km west of the town Piteå in northern Sweden (see Figure 2). The bridge was to be replaced by a
 320 new bridge in 2012. Vibration measurements were performed on the bridge while it was still in use.
 321 In addition to ambient vibrations excited by wind and the river, a train was running over the bridge
 322 before each measurement. In modal analysis with the ARTeMIS software, nine vibration modes were
 323 identified. In a comparison with a detailed FEM of the bridge, there was a 9.3 % difference for the
 324 first mode, 5.9 % for the fourth mode and between 0.4 % and 2.9 % for the others. Figure 3 shows
 325 a comparison of mode shapes and frequencies predicted by FEM with modal data computed using
 326 software ARTeMIS (see [34]). The difference between measured and predicted mode frequencies are
 327 below 3.4 % for all but two of the modes.

328 After replacing the bridge, the old bridge was placed on temporary supports, as shown in Figure
 329 4.

330 New measurements were performed by the Swedish team in cooperation with researchers from the
 331 Royal Institute of Technology in Stockholm lead by Dr. Andreas Andersson and by a Polish research



Figure 2: Steel truss bridge over Åby River, built 1955

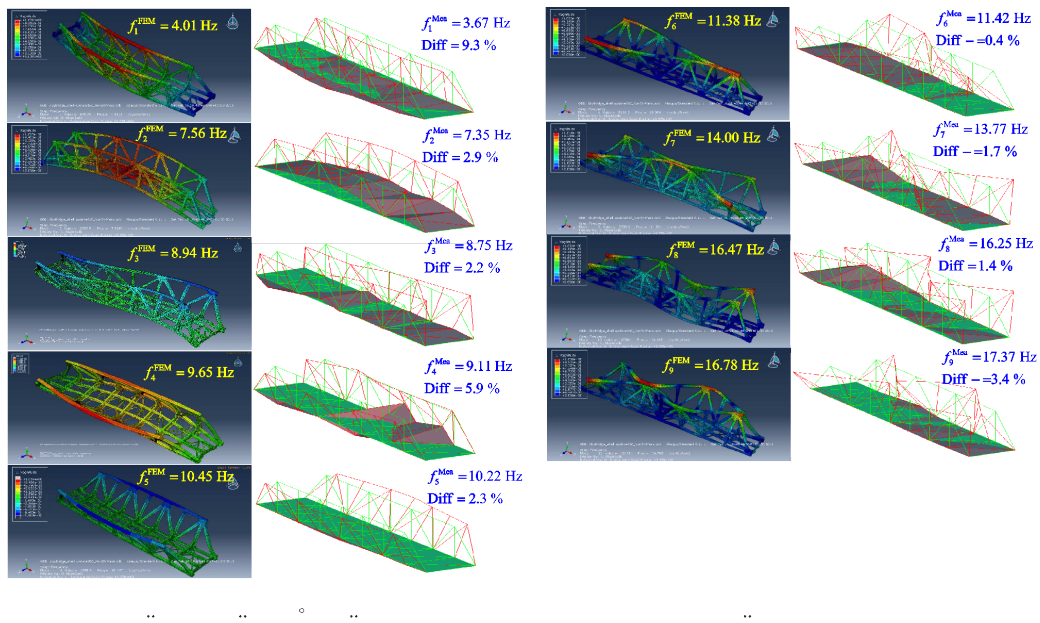


Figure 3: Comparison of mode shapes and frequencies

332 group lead by professor Jarosaw Zwolski at Wroclaw University of Science and Technology (KTH) in
 333 Poland. In addition to natural ambient vibration, the Polish team had a shaker that was used for
 334 exciting the bridge before each set of ambient vibration measurements.

335 For modal analysis of measurements while the shaker is in use, it is important that the shaker
 336 should be the dominating excitation. The excitation from the shaker is measured and can be used
 337 in computations of the bridge's frequency response (see [35]). Thus the rails were welded to the
 338 structure while some loose floor grating and small beams were removed. The vibration measurements
 339 were performed with tri-axial accelerometers from Luleå University of Technology and uni- and bi-axial
 340 accelerometers from KTH. The tri-axial accelerometers were calibrated with a method described in
 341 [36]. New vibration measurements were performed with the accelerometers placed in 41 measurement
 342 points as shown in Figure 5.

343 For each measurement setup, the shaker applied a cyclic excitation changing continuously from
 344 3 to 20 Hz during an interval of 25 minutes. A complete modal analysis is not computed for these
 345 measurements on the undamaged bridge, but the frequency response in two measurement points is
 346 shown in [34], Figure 24, showing modes roughly at 3.7 Hz, 7.4 Hz, 8.1 Hz, 8.7 Hz, 9.3 Hz, 11.4 Hz,
 347 16.2 Hz and 17.3 Hz. Estimated damping for the first and second mode is 0.4 % and 0.6 % respectively,
 348 based on the Half-Power Bandwidth method and curve fitting in the frequency domain.

349 The same measurements were repeated twice after introducing two minor damages. They had to



Figure 4: The bridge placed on new temporary supports.

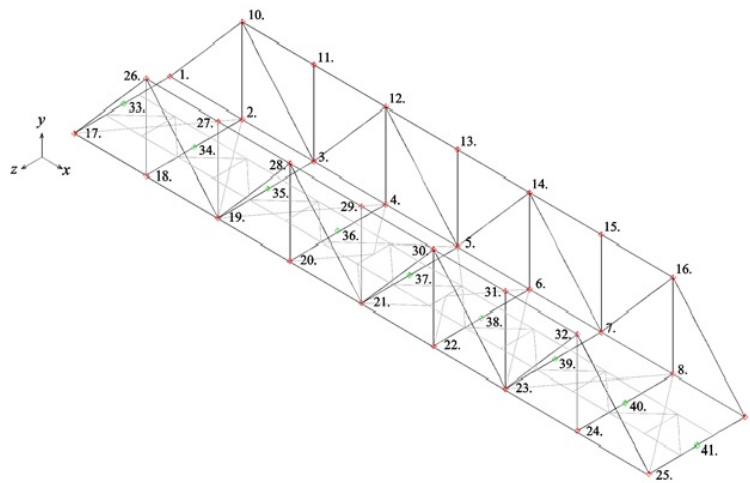


Figure 5: Measurement points for the Ambient vibration measurements.

350 be small enough to not interfere with other measurements on the bridge in any way. As a result we
 351 also expect them to be too small to be found with damage detection methods.

352 More interesting is a final set of ambient vibration measurements that were performed after loading
 353 the bridge to failure. The bridge was pulled downwards by two jacks that were anchored to the
 354 underlying bedrock with two injected cables as described in [37]. The bridge remained elastic up to
 355 about three times the original design load and the load could almost be doubled with substantial
 356 yielding deformations before a buckling failure appeared in the top girders for a load of 11 MN (1000
 357 short tons) for a midpoint deflection of approx. 0.2 m (8 inches).

358 The final ambient vibration measurements were done in 20 of the measurement points in Figure 5.
 359 No shaker was used, so this was pure ambient vibration measurements with excitation from the wind
 360 (reasonably strong in the morning but weaker in the evening) and by trains passing on the nearby
 361 railway. The vibrations caused by passing trains were strong enough for the measurement staff to feel
 362 the ground vibrating at the bridge, which was estimated to be a good additional source for ambient
 363 vibration measurements. Measurements were planned so that 2-3 trains were passing during each
 364 measurement.

365 For some measurement setups, we also tried manual excitation with random hammer blows at
 366 random times and points on the bridge during the last 10-15 minutes. These were later excluded
 367 from the measurements, since including those minutes gave less good modal analysis results, and the
 368 signal-to-noise ratio was also high enough with those minutes excluded.

369 Modal analysis with ARTeMIS gave mode shapes and frequencies shown in [34], Figure 28-29.
 370 Some further results and references for this bridge can be found in [14], including a preliminary run
 371 of some FEM updating based damage detection methods on the measurements of the damaged bridge
 372 (see Figure 6). Those methods do however need further development for smaller structures before
 373 expecting any good result on large complicated structures like a steel truss bridge.

374 In the damaged bridge, some of the beams were loaded enough for clearly visible plastic deformation,
 375 which, however neither changes the elastic modulus nor the cross section areas, so we do not
 376 expect the visible damages to change the dynamic properties of the bridge.

377 With FEM updating, we do instead hope to detect damages in the connections between the beams
 378 that are not visible to the eye. However, we suspect that updating only the elasticity modulus is
 379 not enough for good results when using both bending and torsion modes in the FEM updating (as in
 380 Section B.4 in [14], where we got best results by using only bending modes). Therefore one interesting
 381 next step could be to investigate how to adapt the FEM updating software to also update the shear
 382 modulus in different parts of the analyzed structure.

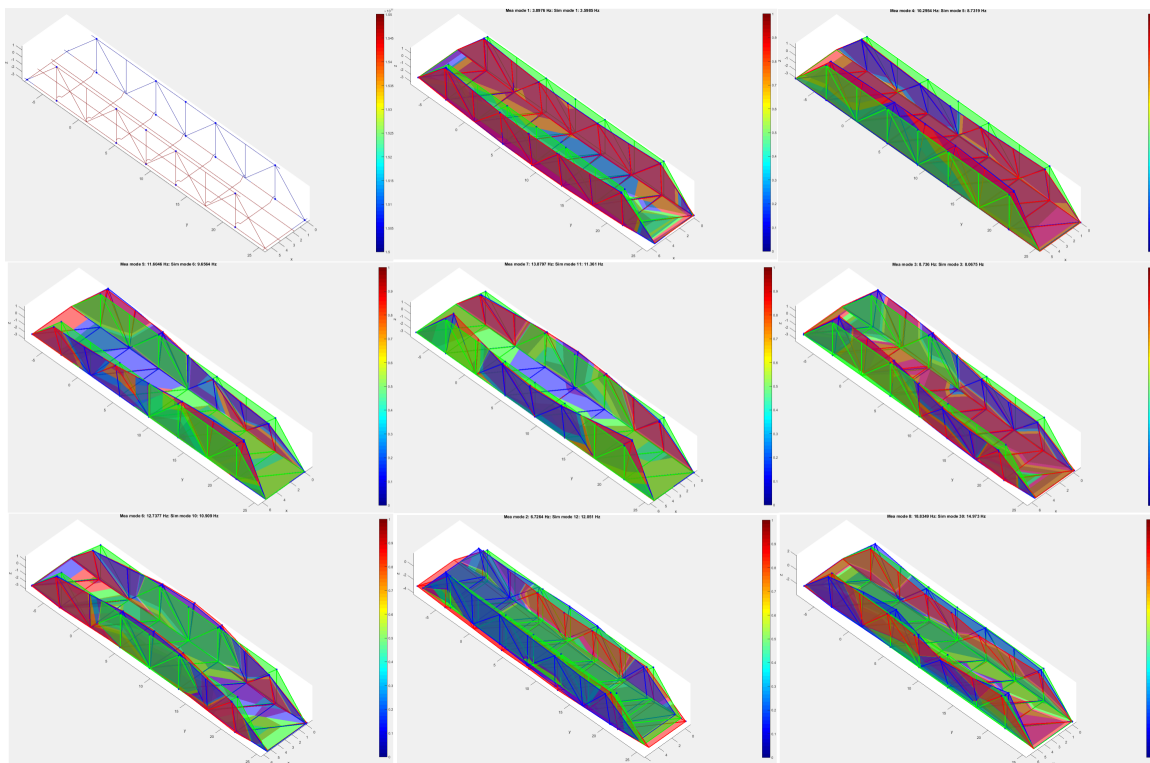


Figure 6: FEM Updating results for the steel truss bridge over Åby River.

383 6 Final remarks

384 OMA techniques have seen applications in many areas especially where the structures are difficult to
 385 excite. Research groups across the globe are working on exciting projects in the area of OMA, let
 386 us shortly mention some of them. Professor Carmelo Gentile and Anonella Saisi from Politecnico di
 387 Milano work on the implementation of OMA techniques to historical structures (see [38], [39], [40] and
 388 [41]). The project StormLamp focuses on wave loading and structural performance of rock lighthouses,
 389 as well as survivability assessment of lighthouses around the British Isles (see [42], [43] and [44]). At the
 390 University of Porto, researchers are working with the dynamic monitoring of dam structures (see [45],
 391 [46] and [47]), stadium buildings, onshore wind turbines, and bridges. In Sweden, at Luleå University
 392 of Technology OMA techniques has been tested and validated on bridges and high rise buildings (see
 393 [14]). OMA has a much wider scope and research groups around the world are exploring areas of civil
 394 engineering, mechanical engineering, aerospace engineering, offshore engineering with these techniques
 395 under different conditions.

396 **Remark 6.1** *Development of advanced state-of-the art technologies are supported by research grants.*
397 *But a few manage the transformation into a business. SHM using OMA has gone through a long period*
398 *of development and enhancement of different methodologies. Regardless of this not many business*
399 *application have been developed. The main reasons for such slower growth were due to the following*
400 *facts:*

- 401 1. *SHM of bridges is a very complex task.*
- 402 2. *There is a substantial void between the expectations of the bridge owner and the services that*
403 *can be provided with the current SHM techniques.*
- 404 3. *Development community is not able to terminate the problem of aging or damage in bridges, it*
405 *can just be identified.*
- 406 4. *Hardware involved for SHM is very costly and not robust w.r.t. the life expectancy of the bridge.*

407 **Remark 6.2** *FDD and SSI have become the most popular techniques for OMA in the past decade,*
408 *but in order to do the damage detection these techniques have some limitations as they are based on*
409 *FFT. Some of these limitations can be addressed by using advance time-frequency techniques. There*
410 *is a lot of research focused on these time-frequency techniques. Concrete examples of such techniques*
411 *and its applications based on wavelets (see [48], [49], [50], [51], [52] and [53]), S-transforms (see [54])*
412 *and Hilbert transforms (see [55], [56] and [57]) can be seen in the references. We will discuss these*
413 *techniques and compare with the techniques presented here in our forthcoming article. The research*
414 *in the area of machine learning and deep learning is a very hot topic nowadays. It can be seen that*
415 *researchers have started to implement neural networks, machine learning and artificial intelligence in*
416 *SHM (see e.g. [5] and [58]).*

417 **Remark 6.3** *The cost for centralized SHM has been high. This led to the development of decentralized*
418 *damage diagnosis where wireless structural health monitoring was sought as a solution. As a result,*
419 *damage diagnosis and prognosis could be performed at the sensing nodes itself. One of the first wireless*
420 *sensor network (WSN) based SHM was installed at Golden Gate Bridge in 2007 by researchers from*
421 *the University of California (see [59]). Technologies based on distributed wireless sensors is under*
422 *development and being tested in different parts of the world (see e.g. [55] and [59]). The advent*
423 *of 5G technology would give a huge boost to implement these new smart technologies to have better*
424 *monitoring of our infrastructure, ensure safety and help save billions of dollars.*

425 **Remark 6.4** *To work with this challenging and important problems under the arctic conditions we*
426 *are working with at UiT – The Arctic University of Norway and Luleå University of Technology in*
427 *Sweden causes additional difficulties and new research questions appear. This is one special reason for*
428 *our interest for future research and we hope to come in contact with research groups working under*
429 *similar arctic conditions.*

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