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Swedish first-year engineering students' views of mathematics, self-efficacy and motivation and their effect on task performance

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ABSTRACT

We examine a group ($N = 88$) of Swedish first-year engineering students, their motivation, self-efficacy, and beliefs about the nature of mathematics, and how these relate to their task performance in mathematics. In our data, engineering students who emphasized the exact reasoning in their view of mathematics performed significantly better in a set of mathematical tasks than those who emphasized the applications of mathematics. Similarly, the higher self-efficacy and the intrinsic and utility values of mathematics relate to better performance in the tasks. In general, the students' task performance was quite modest in relation to the expressed self-efficacy and motivational values.

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Engineering student; motivation; self-efficacy; task performance; values

1. Introduction

Mathematics is widely understood to have a vital importance for studies for an engineering degree, at least, among the mathematics educators. Many of the professional subjects in engineering education require certain mathematical knowledge and concepts, and mastering mathematics is necessary for a deeper understanding of the engineering sciences. Nevertheless, mathematics is often not an engineering student's primary interest (Kümmerer, 2001) and the freshmen engineering students may not be aware of how mathematically demanding the education they have entered actually is (Harris, Black, Hernandez-Martinez, Pepin, & Williams, 2015). Further, becoming a freshman engineering student involves stepping into a new stage in life with new rules and expectations in a new educational setting. This transition affects both how a student perceives mathematics and his/her self-efficacy in mathematics (Bengmark, Thunberg, & Winberg, 2017; Jablonka, Ashjari, & Bergsten, 2017; Kouvela, Hernandez-Martinez, & Croft, 2018).

The transition from secondary to tertiary mathematics education has been considered in several studies. For example, recent studies (Bengmark et al., 2017; Jablonka et al., 2017; Kouvela et al., 2018) report on the success-factors in the transition to university mathematics, and how a right kind of communication helps students to accommodate themselves to

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a new environment, and on engineering students' problems with the changing criteria for what counts as a legitimate mathematical activity.

In this article, we examine a group of 88 Swedish first-year engineering students and their motivation in and views of the nature of mathematics in the beginning of their five-year study programme. Further, we investigate how their motivational values, self-efficacy, and views of mathematics are related to their performance in a set of mathematical tasks. A motivation for this study is to survey what kind of prerequisites for a successful transition from secondary to tertiary education the beginning engineering students have in mathematics and whether the variation in their mathematical skills at this stage can be, at least, partly explained by their views of the nature of mathematics and perceived motivation in mathematics.

This study is a part of a larger research project conducted in three Nordic countries: Finland, Norway, and Sweden. The same questionnaire, translated to the native languages, has been used in each country for the data collection. The national data may, however, reflect slightly different aspects because there are some fundamental differences between the participating universities. For example, the Finnish university is one of the biggest in Finland, whereas the Swedish and Norwegian universities are medium size and both situated in a region with a low density of population. A common feature of all participating universities is that they have both popular and less popular study programmes, yet the Finnish university gets on average remarkably more applications for a study place than the other two universities. Therefore, at the present stage of the project, we analyze the national data independently of one another and with slightly different focus areas.

2. Theoretical framework

Our theoretical approach is based on, on the one hand, a certain categorization of epistemological beliefs on the nature of mathematics and, on the other hand, on Expectancy–value theory.

The epistemological beliefs about mathematics concern the structure, quality, certainty, and the source of mathematical knowledge (Hofer & Pintrich, 1997). There are several studies on students' and teachers' beliefs about the nature of mathematics. Grigutsch, Raatz, and Törner (1998) studied German mathematics teachers' beliefs and they found four different categories or orientations to describe them. Their categorization has been used also for studying students' conceptions of mathematics, and Felbrich, Müller, and Blömeke (2008, p. 764) formulated them as follows.

- The formalism-related orientation: mathematics is viewed as an exact science that has an axiomatic basis and is developed by deduction (e.g. 'Mathematical thinking is determined by abstraction and logic'.)
- The scheme-related orientation: mathematics is regarded as a collection of terms, rules and formulae (e.g. 'Mathematics is a collection of procedures and rules which precisely determine how a task is solved'.)
- The process-related orientation: mathematics can be understood as a science which mainly consists of problem-solving processes and discovery of structure and regularities (e.g. 'If one comes to grip with mathematical problems, he/she can often discover something new (connections, rules and terms)'.)

- The application-related orientation: mathematics can be seen as a science which is relevant for society and life (e.g. 'Mathematics helps to solve daily tasks and problems'.).

The formalism- and scheme-related orientations emphasize the static nature of mathematics; mathematics is seen merely as a ready-made construction to be adopted and used, whereas the other two orientations emphasize the dynamic nature of mathematics; mathematics is essentially about developing and discovering processes and structures needed for problem-solving. A difference between the static orientations is, for example, whether the focus lies on the structure of mathematical knowledge itself or also on the tasks which the 'toolbox' of mathematics can be applied to. Similarly, the application-related orientation focuses on using mathematics to solve daily tasks and problems in society, whereas the process-related orientation appreciates also problems that are purely abstract and scientific, and interesting only in the context of mathematics itself.

The orientations are not exclusive, but a student or a teacher may simultaneously have beliefs that represent several orientations (Felbrich et al., 2008; Tossavainen, Viholainen, Asikainen, & Hirvonen, 2017). Therefore, it is more reasonable to study the distribution of orientations, both at the level of an individual and a group, yet a student can quite often express which of the orientations represents best his or her views of mathematics.

Expectancy–value theory is a model that aims at explaining, for example, an individual's achievement in studying a subject in terms of beliefs about how well an individual will do the related activity and the extent to which they value the activity. Here ability beliefs refer to an individual's perception of his or her current competence at a given activity.

Expectancy–value theory is one of the most widely used modern theories of motivation. It was formulated in the early 1980s, yet the role of values has been studied, at least, since the 1950s (Wigfield & Eccles, 2000). Expectancies and values are distinguished conceptually so that expectancies for success focus on the future aspect and values on the present abilities. Empirical studies have, however, showed that these constructs are highly related (Wigfield & Eccles, 2000, p. 70).

Within this theory, the individual's motivational values are distributed into four categories: intrinsic values, attainment values, utility values and cost. The intrinsic values in studying mathematics refer to the enjoyment of and interest in studying mathematics. The attainment values are related to the perceived importance of being good at mathematics, and the utility values to the perceived usefulness of knowing mathematics for short- and long-range goals. Cost stands for the cost of engaging in studying mathematics. For more details, see, e.g. Eccles et al. (1983), Eccles, Wigfield, Harold, and Blumenfeld (1993) and Wigfield and Eccles (2000).

Our questionnaire contains also some items measuring a student's self-efficacy in mathematics. Bandura (1982, p. 122) defined perceived self-efficacy as an individual's own judgement of how well one can execute courses of action required to deal with prospective situations. An implication of the theory is that a student who has a high self-efficacy in mathematics, probably sees a difficult mathematical task as something to overcome and master, whereas a student with a low self-efficacy in mathematics tries to avoid this kind of a task. Consequently, self-efficacy usually correlates with the learning outcomes (Bandura, 2012).

3. Review of literature

There is previous research showing that engineering students' relation to mathematics differs in some aspects from that of other students in mathematics. For example, Maull and Berry (2000, p. 916) studied a group of English mathematics and engineering students and they point out that 'the mathematical development of engineering students is different from that of mathematics students, particularly in the way in which they give engineering meaning to certain mathematical concepts'. Further, Flegg, Mallet, and Lupton (2012) found that a majority of Australian first-year building and design students recognize the relevance of mathematics to their future career and study, but there is a large variation in how and at what level the relevance is acknowledged.

This is related to what we have already mentioned, mathematics is usually not the primary focus of an engineering student (Kümmerer, 2001). Indeed, Harris et al. (2015) revealed that only few freshmen students are aware of how mathematically demanding their studies actually turned out to be, and that some of them would have chosen other studies if having known that. Further, engineering students seem to focus on 'use-value' of mathematics, but they simultaneously experience that the teaching of mathematics is decontextualized and does not offer the use-value-perspective. Decontextualization means here that mathematics is discussed without almost any references to engineering and the absence of relevant examples.

However, Harris et al. (2015) also pointed out that engineering students may change their view of mathematical knowledge during their studies. In school, good grades in mathematics were especially valuable for giving a student an entrance to an 'elite' university; this is a typical example of the exchange-value of mathematics. In university, the students may more often speak about mathematical knowledge in terms of use-value, mathematics as a valuable support to their engineering studies, than in terms of exchange-value, e.g. by referring to long-term prospects of getting well-paid jobs.

Bengmark et al. (2017) examined in a one-year longitudinal study different factors and their relative importance for Swedish mathematics students' performance during their first year at university. They noticed that self-efficacy, the type of motivation, study habits, and the view of mathematics are all important factors, although each of them alone explained less than five per cent of the variation of the students' grades in course exams. However, at the end of the first year, students' self-efficacy had become a strong predictor for their achievement.

Alves, Rodrigues, Rocha, and Coutinho (2016) studied a group of Portuguese undergraduate engineering students and the relations between students' self-efficacy, mathematics anxiety, and the perceived importance of mathematics. Their findings show that students rate the importance of mathematics high. Their self-efficacy was also relatively high and mathematics anxiety rather low. In this data, there were no significant differences between the genders but there were significant differences in perceived importance and anxiety between the study programmes.

Our study is naturally related to every study mentioned above but, especially, it can be seen as a complement to Bengmark et al. (2017) and Alves et al. (2016) as we are interested in the relations between the Swedish first-year engineering students' performance in mathematics, their views of mathematics, and their motivation and self-efficacy in mathematics.

4. Research questions

Our research questions are as follows:

(i) How students perform in seven mathematical tasks that measure basic calculation skills and represent different orientations to mathematics?
(ii) How students' orientations to mathematics are distributed and how their performance in the tasks depends on this distribution?
(iii) What kind of motivational values and self-efficacy students have in mathematics and how their performance in the tasks depends on them?

The formulation of the second question is related to the aforementioned fact that a student usually acknowledges several different aspects of mathematics and, therefore, it is more meaningful to speak of a distribution of the orientations to mathematics than a single orientation representing the student's view of mathematics. However, we will also study students' performance across groups that are based on a single metaphor (out of four given alternatives representing the four orientations), see Table 2. Such a metaphor can be interpreted in our theoretical framework to represent the primary or the dominating view of what mathematics is essentially about.

5. Method

5.1. Participants and data collection

The students who participated in the study are from a course in differential calculus which is arranged during their first semester at university. The course is a part of the basic course package in mathematics and compulsory for all engineering programmes. Our goal was to meet students at the early stage of the course before they are influenced by the new academic learning culture.

Altogether 479 students from various engineering programmes had signed up for this course and all of them were invited to participate on the voluntary basis during the second and third week of the course. Due to the large number of students, the purpose of the study was shortly explained in the end of a lecture and then each student was given the questionnaire with the instruction to return it in the beginning of the next lecture.

Unfortunately, only 88 students returned the questionnaire. The distribution of genders is such that 68 expressed being male and seventeen female, and three students chose the alternative 'other/I do not want to answer'. Since the number of female participants is so low, we exclude the gender perspective from this study.

The most probable reason for the low participation frequency is that the first-year students had a lot of other things going on at the same time and, since the participation neither was compulsory nor gave any extra points to the course exam, most of them chose to focus on other tasks. This naturally affects the reliability and validity of our results. We assume from experience that those who participated may be somewhat more engaged with studying and, hence, have a little higher motivation and self-efficacy also in mathematics than students in general. On the other hand, there are several reasons to believe that our data is representative and not too biased. For example, the gender distribution within the data corresponds well with the distribution within the whole course. The same applies rather well also for the distribution of study programmes. Further, a large variation in all measured variables is in itself a strong indication for the representativeness of data.

5.2. Questionnaire

The questionnaire consists of three sections surveying a student's educational background, views of the nature of mathematics and motivation in mathematics, and performance in seven mathematical tasks. The items in the questionnaire were developed in several stages. Initial statements about the views of the nature of mathematics or, shortly, orientations were inspired by previous research (Felbrich et al., 2008; Tossavainen et al., 2017). In these studies, the statements were designed for studying the views of student teachers and engineering students. Since the present investigation focuses exclusively on engineering students, some adjustments of statements were needed in order to better fit the respondents. An example of such statements is 'I am motivated to study mathematics mostly because it is useful to my other studies'.

As we wanted to investigate relations between the orientations and actual task performance, four mathematical tasks each emphasizing one orientation were included in the questionnaire. We also arranged a meeting with experienced university lecturers of mathematics courses in engineering education in order to get their feedback on the suitability of items. Based on their feedback, further simplifications of statements and tasks followed. Additionally, three straightforward mathematical tasks (4.1–4.3) involving percentages and the simplifications of symbolic expressions were included to serve both as 'warm-ups' and to measure their calculation skills.

Tasks 4.4–4.7 in the questionnaire were designed to embrace the different orientations. We comment now shortly how each of them stands for a selected orientation and what was expected in a student's solution, cf. Appendix.

Task 4.4 emphasizes the scheme-related orientation, it is to be solved by either providing a counter example and an explanation why the given rule is not sufficient to guarantee that (a_n) is increasing. Rules for solving tasks like this include also a need to consider the initial conditions. In order to get a full score, students have to consider the case $a_1 < 0$.

Task 4.5 is a task where one has to apply mathematical knowledge about increasing and decreasing functions in order to determine whether a medicine is effective or not, i.e. in order to solve a problem that is relevant for the society. It represents hence the application-related orientation. A detailed proof is not required, but full score presumes a solution that explicitly states that the number of bacteria tends to zero when the medicine is applied long enough.

Task 4.6 focuses on the definition of a decreasing functions, i.e. it stands for the formalism-related orientation. For a full score, a student solution needs to communicate a relevant definition and analyze the given function correctly using the definition.

Task 4.7 emphasizes the process-related orientation as it concerns the construction of a function satisfying the given conditions. Both verbal and graphical solutions are possible as long as a sufficient explanation is given. An example of applicable functions is the constant function $f(x) = -0.5$.

As for the overall design of the questionnaire, some questions about the student's educational background were added, but the answering was made anonymous in order to encourage a student to respond freely. Further, the questionnaire was translated to the student's mother tongue in order to avoid language difficulties. The questionnaire in its final form, in English, is to be found in Appendix.

5.3. Analyses

Every student's responses to 4.1–4.7 were independently evaluated by two researchers on scale 0 = 'No answer/foolish or almost completely wrong answer', 1 = 'A relevant strategy but major mistakes or deficiencies/a correct answer without justification', 2 = 'A relevant strategy but at most two minor mistakes or deficiencies in the argument', 3 = 'A correct answer with a sufficient justification'. The final scores were computed as the means of individual evaluations.

We analyzed our data using SPSS Statistics version 25. In addition to computing standard descriptive measures for the concerned variables, the following methods were applied: Student's t-tests, one-way analysis of variance (One-way ANOVA) and post hoc tests, and Pearson correlation analysis. Whenever it was necessary, we also controlled whether the analyzed variables were normally distributed. For that purpose, we used Kolmogorov–Smirnov test (with Lilliefors significance correction) and the test based on the skewness, kurtosis and their standard errors. More precisely, if the Kolmogorov–Smirnov test rejected the null hypothesis for a variable, we applied the following standard convention: if the absolute value of the quotient of the coefficient of skewness (and, respectively, kurtosis) and its standard error is less than two, then the variable's distribution is symmetric enough and the variable is applicable to a parametric test that assumes a normal distribution. We recall that, especially, Student's t-tests are quite robust and applicable also to variables with non-symmetric distribution whenever the number of elements per a group is large enough.

6. Results

We begin by summarizing how the participating students performed in seven mathematical tasks (Table 1). Then we shall consider how the performance relates to the expressed beliefs about the nature of mathematics (Tables 2–5) and motivation and self-efficacy in mathematics (Tables 6–7).

For the content of tasks 4.1–4.7, see Appendix. Recalling that the scale in each item is 0–3, Table 1 shows that the participants are quite good with percentages (4.1) and powers (4.3) but already have problems with simplifying rational expressions (4.2). Further, a half of them can apply mathematics to solve a concrete problem (4.5) if a detailed explanation is not assumed but only the correct conclusion is required. The most of them could not define a decreasing function (4.6) or determine and correctly test criteria for

Table 1. Performance in exercises 4.1–7 ($N = 88$).

	4.1	4.2	4.3	4.4	4.5	4.6	4.7	Sum
Mean (total)	1.94	0.92	1.42	0.47	0.94	0.47	0.27	6.38
Std. dev. (total)	1.23	1.18	1.17	0.83	1.02	0.80	0.72	4.60
Percentage of null answers	19	46	32	69	40	64	85	–
Mean (null set removed)	2.34	1.11	1.71	0.51	1.13	0.56	0.33	7.69

Table 2. The metaphor of mathematics and success in the tasks.

	N	Mean	Std. dev.	Min	Max
Toolbox	23	6.65	3.92	0	14
Applications	10	5.10	3.55	0	10.5
Problem-solving	32	7.30	4.44	0	17
Exact reasoning	8	10.69	5.70	2	19
Total	73	7.16	4.47	0	19

Note: $F(3,69) = 2.65, p = 0.06 < 0.10$.

Table 3. The dynamic–static dimension and success in the tasks.

	N	Mean	Std. dev.	Min	Max
Dynamic	43	7.21	4.70	0	19
Neutral	22	6.16	4.34	0	17
Static	21	5.50	4.41	0	15.5

Note: $F(2,83) = 1.09, p = 0.34 > 0.05$.

Table 4. Descriptive statistics on students' views of the nature of mathematics.

	N	Mean	Std. dev.
3.1. Mathematics is about describing the real world (A)	86	4.09	0.79
3.2. It is not mathematics if it cannot be proved theoretically (F)	86	3.47	0.84
3.3. Mathematics is a collection of formulas and concept (S)	85	3.39	0.83
3.4. Mathematics is solving problem (*P*)	86	4.06	0.76
3.5. The purpose of mathematics is to maintain functionality in society (A)	86	3.24	0.91
3.6. Mathematics is about discovering structures and regularities (*P*)	86	3.74	0.77
3.7. The main task of mathematics is to give correct rules for calculations (S)	86	3.42	0.93
3.8. In mathematics, all concept must be defined in a precise and clear way (F)	86	3.83	0.89

Table 5. Performance in the tasks (4.1–4.7) and different orientations.

	N_{1-3}	$\bar{x}_{1-3}$	Std. dev.	N_{4-5}	$\bar{x}_{4-5}$	Std. dev.	t	p
3.1.	17	4.21	4.10	69	7.09	4.50	2.41	< 0.05
3.2.	46	6.50	4.63	40	6.55	4.52	0.96	> 0.05
3.3.	47	6.31	4.99	38	6.96	3.89	0.66	> 0.05
3.4.	16	8.78	5.53	70	6.01	4.17	−2.25	< 0.05
3.5.	55	6.22	4.94	31	7.07	3.78	0.83	> 0.05
3.6.	31	6.26	5.30	55	6.67	4.11	0.40	> 0.05
3.7.	49	7.15	4.96	37	5.69	3.84	−1.49	> 0.05
3.8.	25	5.76	4.73	61	6.84	4.47	1.00	> 0.05

an increasing/decreasing sequence (4.4). Only one student out of 88 was able to construct a function that satisfies given criteria.

The mean of the sum scores is 6.38 with the standard deviation being 4.60. This is quite low in relation to the possible maximum 21 points. There were altogether fifteen students who did not provide any solution to the tasks. Therefore, Table 1 shows also the summary of students' performance in the group where those fifteen students has been removed. Then, the mean of the sum scores is 7.69 and the standard deviation 3.92. The improvement is seemingly remarkable and also statistically significant when the mean difference is studied using one-sample t-test with the test value being the mean of the original sample ($t(72) = 2.84, p < 0.01$).

Table 6. Descriptive statistics on the self-efficacy and motivational factors (Scale: 1–5).

	N	Mean	Std. dev.
2.5. In school, I was good in mathematics (S)	88	3.72	0.93
2.6. In school, I was able to understand the most of mathematics (S)	88	4.08	0.85
2.7. I really like mathematics (I)	88	3.63	0.93
2.8. I am motivated to study maths because it is useful to other studies (U)	88	3.90	0.89
2.9. I want to succeed as well as possible (A)	88	4.75	0.46
2.10. I could suspend a hobby in order to succeed in math exam (C)	88	4.10	0.80
2.11. I could do extra exercise to guarantee that I succeed well (C)	87	4.10	0.76
2.12. I would study maths voluntarily because every engineer must know it (U)	87	3.91	1.02
2.13. If I get a low grade in mathematics I want to take the exam again (A)	87	3.13	1.09
2.14. Mathematics is full of interesting problems and results (I)	87	3.86	0.99

Table 7. Performance in the tasks (4.1–4.7), self-efficacy, and the motivational values.

	N_{1-3}	$\bar{x}_{1-3}$	Std. dev.	N_{4-5}	$\bar{x}_{4-5}$	Std. dev.	t	p
2.5.	34	4.99	3.58	54	7.25	4.97	2.30	< 0.05
2.6.	15	5.30	5.06	73	6.60	4.51	0.99	> 0.05
2.7.	39	4.50	4.06	49	7.87	4.49	3.64	< 0.001
2.8.	25	4.12	3.97	63	7.27	4.55	3.03	< 0.01
2.9.	1	11.50	–	87	6.32	4.59	–	–
2.10.	18	6.17	4.58	70	6.43	4.64	0.83	> 0.05
2.11.	17	5.88	5.02	70	6.59	4.49	0.57	> 0.05
2.12.	28	4.79	4.13	59	7.24	4.60	2.40	< 0.05
2.13.	57	6.53	4.96	30	6.30	3.81	−0.22	> 0.05
2.14.	31	4.69	3.64	56	7.42	4.78	2.76	< 0.01

Table 2 reports on the item in the questionnaire, where a student was asked to choose one and only one of the given four metaphors to represent in the best way what he or she thinks mathematics essentially is. The alternatives and the descriptive statistics for the students' performance within the corresponding groups are shown in the table. Since fifteen students did not choose any metaphor and, consequently, the size of data reduces from 88 to 73 responses, we have used the significance level 10% instead of more commonly used 5%. Then there are statistically significant mean differences between groups ($F(3, 69) = 2.65$, $p < 0.10$). Further, in Bonferroni's post hoc test, the mean of the group *Exact reasoning* differs significantly from that of the group *Applications* ($p < 0.05$).

For comparing the students' distributions of orientations, we added first the values of the dynamic (process- and application-related orientation) and static (formalism- and scheme-related orientations) sum variables, respectively. The scale for the both sum variables is thus 4–20. Then we compared whether these constructs have same value – which represents a neutral distribution – or not. In the latter case, the centre of the distribution is either on dynamic or static side. Table 3 shows the distribution of the students' distributions into these three categories and the students' mean scores.

Due to large standard deviations (with respect to the sample size), the mean differences in Table 3 are not statistically significant. Further, the mean differences become essentially smaller if we remove the students who gained zero points: $\bar{x}_D = 7.95$, $\bar{x}_N = 7.53$, and $\bar{x}_S = 7.22$. In other words, the influencing factor that explains to a high degree the observed mean differences is the proportion of students who did not provide any solutions to the tasks.

Table 4 summarizes how students perceive what mathematics essentially is according to their responses to eight given claims. The letter in the end of each claim represents the corresponding orientation to mathematics: A = the application-related orientation, F = the formalism-related orientation, P = the process-related orientation, and S = the scheme-related orientation. In order to increase the readability, the claims are not presented in Table 4 as they were stated in the questionnaire but in the condensed form, cf. Appendix.

Table 4 shows that, all orientations can be found from the students' beliefs – which is not surprising by the previous research (Felbrich et al., 2008; Grigutsch et al., 1998; Tossavainen et al., 2017) – but there is variation between them and also within a single orientation construct. For example, the means of scales standing for the scheme-related orientation differ significantly from the means of most other scales in paired samples t-test (e.g. for 3.3 and 3.4, $t(84) = -6.31, p < 0.001$), and the mean difference between the items standing for the application-related orientation (3.1 and 3.5) is statistically highly significant ($t(85) = 8.06$, $p < 0.001$). Further, students appreciate a dynamic view of mathematics and problem-solving more than a static view and proving (e.g. for 3.2 and 3.4, $t(85) = -5.10, p < 0.001$). On the other hand, the quite high mean in Item 3.8 shows that they also acknowledge the demand to define mathematical notions precisely. The standard deviations do not differ significantly across the items. However, the fact that the two smallest are related to 3.4 and 3.6 which both stand for the process-related views, indicates that students are most unanimous with respect to these views.

In order to estimate how well 3.1–3.8 measure the intended orientations, we computed Pearson correlation coefficients for each pair representing the same orientation, e.g. 3.1 and 3.5 for the application-related orientation. For each pair, the correlation is significant with $p < 0.01$ or $p < 0.001$ and the correlation coefficient varies between 0.29 and 0.40. Obviously, these correlations alone do not prove the validity of the scales but they suggest that the reliability of the measurement is, at least, relatively good.

In Tables 2 and 3, we have already answered how the participants' performance in the given tasks (4.1–4.7) depends on their view of the nature of mathematics if the performance is concerned across the groups that are defined by the chosen metaphors or the centre of the distribution of orientations. In order to gain a more detailed understanding about the relationship between the performance and the orientations, in each of the items 3.1–3.8, we divided the participants into two groups (the values 1–3 = 'represents a neutral or downplayed orientation' and 4–5 = 'represents an emphasized orientation') and studied the mean differences of the sum scores between the groups. The results are shown in Table 5.

There are two statistically significant mean differences in Table 5 (Items 3.1 and 3.4). The first of these shows that the better performing students associate mathematics more strongly with the view that mathematics can be used to describe real world than the weaker performing students. In the latter item, the groups are in the opposite order: the better performing student's emphasize to a lesser degree mathematics as solving problems. This can be interpreted so that the better performing students view mathematics as something more than only problem-solving, i.e. their view of mathematics is wider.

If the students who did not provide any answer to the tasks are excluded from the analysis, the mean differences in Table 5 became somewhat smaller but the difference in 3.4 remains significant. This outcome is due to the fact that the most of these students belong to the first group in each item, except for 3.4.

Concerning the last research question, Table 6 summarizes students' motivational values of studying mathematics and how they perceive themselves as learners of mathematics. The letter at the end of each claim refers to the latent variable that is operationalized by the claim. They are S = self-efficacy, I = the intrinsic value, U = the utility value, A = the attainment value, and C = cost. Observe that, in order to increase the readability of Table 6, the claims below are not presented exactly as they are in the questionnaire but in the condensed form, cf. Appendix.

The means for Items 2.4 and 2.5 indicate that the students' self-efficacy in mathematics is quite high; they think that they have been quite good in mathematics in school. Further, students express very clearly that they want to succeed as well as possible (2.9) in mathematics but, on the other hand, this does not mean that they surely would take the exam again if they fail to get a high grade (2.13). So, their attainment value is somewhat ambiguous. Their intrinsic and utility values are rather high (2.7 and 2.14, 2.8 and 2.12, respectively) and cost value even higher (2.10 and 2.11). The lastly mentioned finding is somewhat unexpected and surprising compared with the responses in 2.13 but it parallels well with the students' responses in 2.9.

The Pearson correlation coefficients for the pairs representing self-efficacy and the motivational values are all significant with $p < 0.001$ and the correlation coefficient varies between 0.36 and 0.60, except for the attainment value (2.9 and 2.13) for which $r = 0.18$ and $p > 0.05$. The unexpected low correlation can be explained by the fact that the latter item (2.13) concerns taking an exam again if a student gets a low grade. Probably, only a few students would really do so although almost every student expressed that he or she wants to succeed as well as possible (2.9).

To complete our answer to the third research question, we again divided the participants into two groups according to their answers to the items 2.5–2.14 and analyzed the mean differences of the sum scores. In each item, the first group consists of those students whose answer lie in the interval 1–3 and the second group of those whose answers are 4 or 5. The results are shown in Table 7.

In 2.5, the first one of two self-efficacy items, the mean difference is statistically significant. The result is natural: those who clearly considered that they have been good in school mathematics also performed better in the tasks than those who were not as sure about their capacity in school mathematics.

The mean difference is statistically significant for four items related to the motivational values. Out of these, items 2.7 and 2.14 are related to the intrinsic value. Both of them support the same conclusion: the higher intrinsic value, the better performance in the tasks. Similarly, both 2.8 and 2.12 are related to the utility value, and the higher utility value is, the better performance in the tasks is.

In 2.9, it was not possible to perform the independent samples t-test because there is only one participant in the first group. Interestingly, this student, who agreed less than other students about that he or she wants to succeed in mathematics as well as possible, performed clearly better than the other students.

If those fifteen students who did not provide any answer to 4.1–4.7 are excluded, the mean differences in Table 7 become smaller similarly as in Table 5.

7. Discussion and conclusions

In general, students' success in the tasks was, at most, satisfactory (Table 1). The mean of the sum scores is only a little above 30% of the maximum. Also the fact that fifteen out of 88 students did not provide any solution to the tasks is worrying. The most probable causes are an inability to solve the tasks and the lack of motivation; in most cases, we doubt that the explanation lies in the motivation and self-efficacy. On the other hand, these students answered properly in the other section of the questionnaire.

Table 2 shows that there is a relation between the view of the nature of mathematics and task performance. The students who appreciate the exact reasoning seem to perform better in mathematical tasks. On the other hand, dynamic views are often related in the literature to the better performance, e.g. (Felbrich et al., 2008; Grigutsch et al., 1998; Tossavainen et al., 2017). In Table 3, the difference in the task performance between the students with more dynamic and the students with more static views is statistically insignificant.

Tables 4 and 5 show results that both support and add to the findings of previous research. For example, similar distributions of orientations have been reported in (Tossavainen et al., 2017). A new finding is that (Table 4, 3.1 and 3.5), in spite of that the participating engineering students clearly express that an important feature of mathematics is that it can be used to describe the real world, they are remarkably less sure about that mathematics should be used to improve people's life and the functionality in the society. This may be due to the decontextualization problem that Harris et al. (2015) have reported on.

Another interesting finding is that the scheme-related orientation is weaker than the other orientations in our data. This may be related to the previously mentioned fact that mathematics is not the primary interest of engineering students (Harris et al., 2015; Kümmerer, 2001). In (Tossavainen et al., 2017), the process-related orientation was somewhat weaker than other orientations among all mathematics students, but it was the most and the only significant orientation to predict that a student had chosen an engineering study programme (and not, e.g. a subject teacher programme).

Tables 6 and 7 rapport from students' motivation and self-efficacy. In general, the Swedish engineering students express quite high motivation and self-efficacy in mathematics, yet their task performance can be interpreted to speak for another kind of conclusion. Namely, self-efficacy correlates often with perseverance and persistence (Pajares, 1996). In designing of the tasks, it was carefully confirmed that they are solvable with the aid of the mathematical content knowledge that is covered in the national core curriculum. Especially, it should have been easy to take nine points already in 4.1–4.3.

Although we decided to exclude the analyses of data from the gender perspective due to the small number of female participants, we mention that the male students overall expressed stronger opinions than the female students when they were asked about their motivation and self-efficacy. This is well known among younger learners, e.g. (Eccles et al., 1993; Tossavainen & Juvonen, 2015), but it seems to be typical of adult Swedish students, too.

In (Alves et al., 2016), the authors found that the Portuguese engineering students have rather high self-efficacy in mathematics and rate the importance of mathematics high. Consequently, they proposed a study focusing on the relationship between these measures and students' grades. The study by Bengmark, Thunberg, and Winberg (Bengmark et al., 2017) is a kind of answer to that demand, but also our study (Table 7) verifies that the higher self-efficacy (2.5), the utility values (2.8 and 2.12) and the intrinsic values (2.7 and 2.14) correlate with the higher task performance. Another interesting and important finding here is that items standing for the cost values (2.10 and 2.11) do not correlate in our data. These outcomes support the conclusion which previous research has already verified several times: the inner motivation is fundamental for a successful performance in mathematics.

Due to relatively small sample size, it is not reasonable to make too far-reaching or general conclusions about the findings from the present data, but it seems that the Swedish first-year engineering students' self-efficacy and real performance in mathematics are not completely compatible. It is, however, neither reasonable nor productive to blame the beginning students for thinking too positively about their mathematical knowledge. Instead, the teachers of first-year engineering mathematics courses should build on their high self-efficacy and a relatively positive view of mathematics, and pay much attention to supporting students in accommodating the new mathematical culture. Like Kouvela et al. (2018) and Jablonka et al. (2017) point out, the challenges are essentially related to communication. If students get a feeling that their problems during the transition from secondary to tertiary mathematics education are acknowledged and respectably taken into account by their teachers, then their self-efficacy may improve on a more realistic basis in this process.

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No potential conflict of interest was reported by the authors.

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Appendix – the questionnaire (condensed)

Engineering students' mathematical views and performance in mathematics

1. Introduction

This questionnaire is related to a study to be carried out at X, Y, and Z, and it aims at surveying engineering students' views of mathematics and how they are related to their performance in solving mathematical problems.

It is important that you try to answer the following questions as thoroughly as possible. Including this introduction, the questionnaire contains four different sections on four pages. In each section, we ask you kindly to read the given instructions carefully. For further information, please do not hesitate to contact us.

Thank you very much for participating in our study!

Timo Tossavainen, Ragnhild Johanne Rensaa, and Monica Johansson

2. Background information

2.1. Your gender: ☐ Female ☐ Male ☐ Other/I do not want to answer

2.2. How many years is it since you last studied mathematics? ___________

2.3. THIS QUESTION IS ONLY FOR NORWEGIAN STUDENTS: Is your mathematical background from secondary school (R1+R2) or from a precourse in mathematics?

☐ Secondary school ☐ Precourse

2.4. What is your study programme?

____

Indicate how much you agree with the following statements.

	1=strongly disagree	2=disagree	3=not disagree, not agree	4= agree	5=strongly agree
2.5. In school, I was good in mathematics.					
2.6. In school, I was able to understand the most of it what was expected from us in mathematics.					
2.7. I really like studying mathematics.					
2.8. I am motivated to study mathematics mostly because it is useful to my other studies.					
2.9. I want to succeed as well as possible in my mathematics studies.					
2.10. I would be ready to suspend my hobbies in order to have enough time to prepare myself for exams in mathematics.					
2.11. I could do extra exercises to guarantee that I succeed well in mathematics exam.					
2.12. Even if it was not compulsory I would study mathematics because every engineer must know some mathematics.					
2.13. If I pass a mathematics course with a low grade, I want to take the exam again.					
2.14. Mathematics is full of interesting problems and results.					

3. Your views of what mathematics is

Spend a minute on reflecting what is essential in mathematics, or how the essence of mathematics could be described in short, and then answer how much you agree with the following statements.

	1=strongly disagree	2=disagree	3=not disagree, not agree	4=agree	5=strongly agree
3.1. A very important feature of mathematics is that it can be used to describe real world.					
3.2 It is not mathematics if it cannot be proved theoretically in an exact way.					
3.3. Mathematics is a collection of formulas and concepts.					
3.4. Mathematics is solving problems.					
3.5. The purpose of mathematics is to maintain functionality in the society and improve people's life.					
3.6. Mathematics is discovering structures and regularities.					
3.7. The main task of mathematics is to give the correct rules for calculations.					
3.8. A very important feature of mathematics is that all concepts are defined in a precise and clear way.					

3.9. Below there is four different metaphors to describe what mathematics is. Choose one (and only one) that fits best with your ideas.

Toolbox	Applications	Problem-solving	Exact reasoning

4. Some exercises

4.1: A store has a 40% off sale on headphones. With this discount, the price of one pair of headphones is NOK/SEK 360. What is the original price of the pair of headphones?

4.2: Simplify $\frac{3x^2-5x+2}{x^2-1}$.

4.3: Simplify $(2a^3)^3 - 3(a^2)^3$.

4.4. Suppose that sequence (a_n) satisfies the rule $a_{n+1} = 2a_n + 1$ (i.e. the next element equals two times the previous element plus one) for all $n = 1, 2, 3, \cdots$. Is it increasing? Explain why.

4.5. In a medical test, a researcher studies a sample of a fluid daily and finds out that a number N of bacteria in a nanolitre of the fluid follows the formula

$$N(x) = \frac{4x^3}{2^x},$$

where x is the number of the days the medicine has been applied on the fluid. Explain whether the medicine is effective against the bacteria or not.

4.6. Define what it means if a given function is decreasing. Explain why the function

$$f(x) = -3\mathrm{x}^2 - 1$$

is decreasing. Here $D_f = \mathbb{R}^+$ (i.e. the domain of f is the set of positive numbers).

4.7. Is it possible to find an increasing or decreasing function f such that its values (i.e. values of $f(x)$) are between -1 and 0 whenever the value of the variable x is an odd number? Explain your answer.