UiT THE ARCTIC UNIVERSITY OF NORWAY

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Production And Logistics Systems Improvements – Biim Ultrasound AS

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Abstract

A crucial aspect of the supply chain network design process is deciding on optimal locations to situate new facilities. Facility location decisions rely on many factors, some of which might be conflicting with each other. The decision factors can be either quantitative or qualitative, thus a brute-force prioritization of one over another could be detrimental overall. To ensure the efficacy of the selection process, decision makers must consider both the quantitative and qualitative factors in tandem. Some of the common methods employed in the literature by organizations to facilitate their decision-making process include: optimization models and algorithms, decision support systems and computerized analytics tools. To this end, this thesis proposes a hybrid Multi-Criteria Decision Making (MCDM) model to aid the selection of an optimal location that suits the strategic fit of an organization. The proposed model integrates the Analytic Hierarchy Process (AHP) methodology for Multi-Attribute Decision Making (MADM) with Mixed Integer Programming (MIP). The solution is modeled and implemented with the AIMMS modeling language as well as the Gurobi Optimization tool in Python. This thesis work is based on a case study from Biim Ultrasound.

Keywords: Facility Location, Optimization, Multi-Criteria Decision-Making, AIMMS, Gurobi

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Introduction

In the era of the early industrial age, to remain profitable as a company, the development of a great product was often the priority. However, the competitive nature of modern-day production environments, such as the speed with which products are designed, manufactured and distributed, as well as the need for higher efficiency and lower operational costs, has motivated companies to continually search for ways to improve their operations. Optimization models and algorithms, decision support systems and computerized analytics tools are some of the methods companies are employing to improve their operational performance and remain competitive under the threat of increasing competition and rapidly evolving supply chains.

In modern business environments, where firms continually face stiff competition and reduced margins, increasing profits can be achieved through effective planning, design and management of the entire supply chain [1]. The motivation for an improved Supply Chain Management (SCM) is associated with new challenges and opportunities, such as the integration and coordination of inter-organizational efforts. Because of rapid globalization, the necessity of reducing lead times and cost pressures, and other factors make it a priority for firms to design and manage effective supply chain networks to meet customer requirements. Another result of the fast-paced globalization coupled with more stringent interconnections between different supply chain players have made firms for vulnerable to rising uncertainty [2].

Nowadays, companies are investing massively in planning and optimizing their supply chain network predominantly to stay a step ahead of competitors by providing the best products and services to their customers. Optimization tools and software solutions are a major area of investment for companies looking to have an edge over their competitors as well as remaining profitable.

1.1 Background

Biim Ultrasound (formerly Eyelife) started with a group of visionaries whose aim was to deliver ultrasound imaging to a larger medical market by providing a more cost effective and much simplified user interface. At Biim Ultrasound, they understand that the future of healthcare is smaller, faster and more accessible medical devices. The Biim probe connects easily with WiFi to a tablet to generate quality images [3]. The kind of ultrasound probe Biim provides is a high quality, ultra-portable, wireless and easy to use ultrasound imaging system that is affordable and attractive to the medical industry. This brilliant combination of portability and price is attractive to both existing and potential user groups. With the advent of tablet-based ultrasound scanning, this new breed of ultrasound probes has the potential of becoming the new visual stethoscope.

Biim intends to develop a prominent presence in both Europe and the USA, both being the largest potential markets for their products. The development of applications and software is done in Finland whiles the office in the USA is responsible for hardware development, production and distribution. As at November 2018, Biim Ultrasound has acquired both FDA clearance as well as the CE marking which allows them to market their products in both the USA and Europe respectively [3].

Figure 1.1: A Biim Ultrasound portable probe

Figure 1.2: A Biim Ultrasound portable probe in use

1.2 Problem Statement

Biim Ultrasound AS is seeking to improve their supply chain network with regards to the reduction of production and transportation costs which is in line with their desire to be consistently a competitive player in the world of portable ultrasound probes. Biim has a diversified customer group ranging from the military, public hospitals, schools, to individuals across different economic zones; the USA and EU being the most prominent. This level of diversification presents its own challenges with each customer group and economic zone requiring specific customization. Mass producing the probes is a challenge because each customer group has requirements coupled with the fact that current manufacturing agreements are not flexible enough to respond to customer orders. This phenomenon has made it difficult to locate customization and ordering points to efficiently serve the various stakeholders. Biim is currently interested in developing a model which can facilitate decision making in the face of the uncertainties posed by the stakeholders within their supply chain.

1.3 Project Scope

This thesis work will focus on several software solutions available to assist in decision making. To provide a better picture of how these software solutions help in decision making, a mathematical model will be developed and implemented in various software solutions. The advantages and performance of these software will be analyzed, and conclusions drawn.

The following are the main activities to be carried out as part of this project:

1. Review supply chain and logistics design methodologies
2. Review mathematical models for decision making.
3. Review various decision support systems.
4. Review software tools that enhance the process of decision making.
5. Review facility location problems
6. Develop a decision support framework to address the facility location problem.
7. Apply the framework to the case of Biim Ultrasound.

8. Conduct a performance and sensitivity analysis of results obtained.

This thesis work is organized into sections as follows.

Chapter 2 delves into the theoretical knowledge required in addressing the problem. This will involve reviewing existing literature in the areas of supply chain management, value stream mapping in a supply chain network, mathematical programming, decision support tools and decision making-methodologies.

Chapter 3 will deal with developing a framework to address the problem of facility location. The Analytic Hierarchy Process (AHP) will be combined with a mixed integer programming model to develop the framework. The mixed integer model developed will be implemented in the AIMMS modeling language as well as the Python programming language.

Chapter 4 covers the discussion of results from the implementation of the models. The performance of both will be analyzed and sensitivity analysis.

Chapter 5 concludes all the work done in previous chapters.

1.4 Limitations

The method of data collection and the quality of data available makes it a daunting task to develop a more realistic model. Biim Ultrasound is in a transitional phase of their life cycle and as such some generalized assumptions had to be made to complete this work. The following were the major assumptions made:

- Two products variants are assumed for building the mixed integer programming model.
- Three major customer zones were assumed.
- Only deterministic parameters are used.
- The Gurobi Optimization software license used is an academic version thus results from modeling in Gurobi cannot be published commercially.

Furthermore, the initial scope of this project changed mid-way through its execution because of a change in the administration of Biim. The original objective was to improve existing production systems however the focus shifted to redesigning their supply chain network. Due to this new development time is very constrained for the execution of the project.

2

Literature Review

2.1 Supply Chain Management

A supply chain is the cooperation among otherwise individualistic business units like manufacturers, distributors and suppliers towards the process of fulfilling customer orders or requirements. A supply chain is a network of facilities, distribution options, and procedure implemented towards the effective integration of suppliers, manufacturers, and distributors while executing the functions of procurement of materials, transformation of these materials into intermediate and finished products, and distribution of these products to customers in the right quantities, to the right locations, and at the right time in order to meet the required service level with minimal cost [4]. The aim of supply chain management is to interconnect and create a harmonious flow between upstream and downstream entities to provide a competitive advantage for companies by increasing flow performance and reducing operational cost [5]. Supply chain versus supply chain is the new mantra for contemporary competitive businesses. To reach the desired competitive advantage and sustainability, enterprises are acknowledging the importance of supply chain management [6].

2.1.1 Supply Chain Design

Supply chain design is an integral part of designing a new product and is one of the main if not the most important concern facing companies nowadays.

Supply chain can be designed in various ways based on who the end user is in the value chain and by considering the operating model of the business; whether the model is a business to customer (B2C) or a business to business (B2B) model [7]. To be able to deliver customer orders at the highest level of quality, while providing very short lead times at the lowest possible cost, the management and design of the supply chain must be effectively designed. Khaoula et al. [8] indicate in their paper that in designing the supply chain it is of great importance to consider the product life cycles. For the effective design of the supply chain, they present a multi-phase mathematical programming method for the consideration of the life cycle of products. The product life cycle refers to the main phases of a product in its development. They also emphasize that the priorities of peculiar products in terms of sourcing, manufacturing and distribution must be dynamic over the course of time to suit the strategic fit and objectives of the firm rather than emulating the status quo. To develop a model, a three-stage approach was used; the first stage involved the evaluation of sourcing options, manufacturers and distributors to ascertain their level of efficiency by combining the Ordered Weighted Averaging (OWA) and the Analytic Hierarchy Process (AHP) models to evaluate the performance. The second stage involved the use of integer programming for the selection of options for the supply chain network design in conjunction with the efficient methods determined in the first stage, requirements of demand and capacity as well as location constraints. The combination of the first and second stages help to determine the entities to be chosen for the network. The third and final phase provides optimization of the model to aid in decision making within the supply chain network.

In selecting a supply chain type, the product design must be taken into consideration. Taking a lean supply chain as an example, a new product design strategy must focus on maximizing the performance of the product while minimizing cost. Because an agile supply chain aims for a high degree of customization, strategies adopted for the design of new products must focus on meeting customer needs. A hybrid supply chain has some lean supply chain characteristics as well as some agile supply chain characteristics, a modular design that can postpone product differentiation is preferred, as shown in Table 2.1

Another factor which is playing a critical and strategic role in the decision-making process in supply chain design is green supply chain management. Managerial decisions on improving green performance and increasing green capacity has become even more complex [9]. This is because recently, environmental and social objectives based on energy cost, land use and construction cost, congestion, noise, quality of life, pollution, fossil fuel crisis and tourism are becoming customary [10].

Table 2.1: Product type and supply chain engagement [5]

Factors \ Supply Chain Type	Lean	Agile	Hybrid
Definition	Employ continuous improvement to focus on the elimination of waste or non-value-adding steps in the supply chain.	Build the capability to respond rapidly to changing and continually- fragmenting global markets.	Achieve some degree of customization in the back-end of the supply chain and "leanness" in the front-end.
Product Type	Standard Product	Innovative Product	Hybrid Product
Supplier Selection	Supplier attributes consist of low cost and high quality.	Supplier attributes consist of speed, flexibility and quality.	Supplier attributes consist of low cost and high quality, along with the capability for speed and flexibility when required.
Product Design Strategies	Maximize performance and minimize cost.	Design product to meet individual customer needs.	Apply modular design in order to postpone product differentiation for as long as possible.

2.1.2 Lean and Value Stream Mapping In Logistics

Lean logistics is related to the principle of lean thinking as a guide, driven by customer demand, through a series of lean methods to instigate continuous improvement, reduce waste to the barest minimum, and create value-added logistics activities throughout the value chain so that logistics can effectively and smoothly flow throughout the chain.[11] Lean production is one of the methods applied by many manufacturers to obtain the competitive advantages in the increasingly competitive global market. Value Stream Mapping (VSM) is one of the major lean tools to identify the opportunities for other lean method and for waste elimination in the production system. VSM illustrates all the activities, both value added and non-value added, that are required to take a product through the essential flows starting with raw material and ending with customer from raw materials stage into the acceptance of customers. The flow chart in figure 2.1 indicates a categorizing framework, by which the value stream mapping in a chain can be determined.

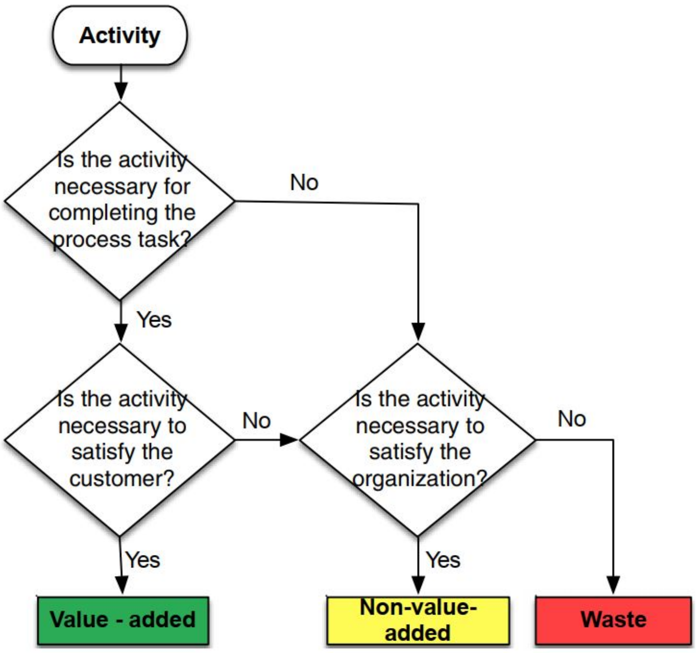


Figure 2.1: Flow chart guide to categorize activities into value- added activities, non-value-added activities and waste[12]

Value stream mapping (VSM) is an adequate introductory tool for the implementation of lean in labor-intensive small and medium-sized enterprises because VSM helps to completely redesign the entire production system [13] [14] [15]. The performance of the supply chain significantly influences all stakeholders and as such the implementation of lean thinking must be extended beyond the manufacturing plant to the entire value chain [16]. The work conducted in [17] shows the implementation of lean principles for performance improvement in a manufacturing firm. Value Stream Mapping and Waste Identification Diagrams (VSM+WID) are integrated to assess the level of currently existing waste and the overall current status of the manufacturing flow. The VSM+WID enables an increase in the awareness of relative waste distribution among different processes in the selected case study manufacturing unit. Classifying companies through their processes creates the possibility of gaining a well-established general understanding of the company. A study was made based on the daily operations of a small logistics company specialized in international transportation. Value Stream Mapping was performed in to propose improvements leading to reduced processing time. The Data Envelopment Analysis (DEA) based method was used to calculate the leanness score of the initial system and to estimate by how much the leanness changed after the implementation of proposed improvements. The results showed that waste produced by bad workplace layout and over-processing could be eliminated. Another suggested solution was to introduce standardized processes and to invest in technical instruments to automate production [12]. Value Stream mapping (VSM) attempts to identify the bottlenecks in the various stages of a value chain how its sole focus is on economic aspect thus other aspects of sustainability such as societal and environmental considerations are neglected. Sustainable Value Stream Mapping (SVSM) seeks to resolve this problem. SVSM involves three dimensions, namely economy, societal factors and environment. Managers and engineers are able to detect potential critical problems in the enterprise thanks to Radio Frequency Identification(RFID) system which enables the SVSM to run in real time [18].

2.1.3 Kanban in Logistics

Kanban, literally means "card", which is a way of labeling production lots to obtain improved control of in-progress inventory, raw materials and finished goods. The Kanban system is a material flow control mechanism, which controls the proper quantity and proper time of the production and/or delivery of required products and services. In [19] [20] a Kanban system was used for information sharing and coordination in to manage a multi-echelon inventory in a pharmaceutical supply chain. A multi-echelon supply chain is a chain with multiple points with probably many actors at each point. A multi-echelon supply chain is demonstrated in the Figure 2.2. This work focuses on the Kanban system to control the stock in a multi-echelon inventory system.

Kanban has been confirmed as an effective and promising software development method. However, using of Kanban still relies on the experiences of project managers. The balance between development and test is one of the critical problems in Kanban development process [21]. Information security risk management can be automated and connected with the processes in a software development company, using an Agile approach with Kanban [22]. The improvement activity in a manufacturing plant can been achieved by focusing on the effective Kanban size to achieve just-in-time (JIT) production system [23].

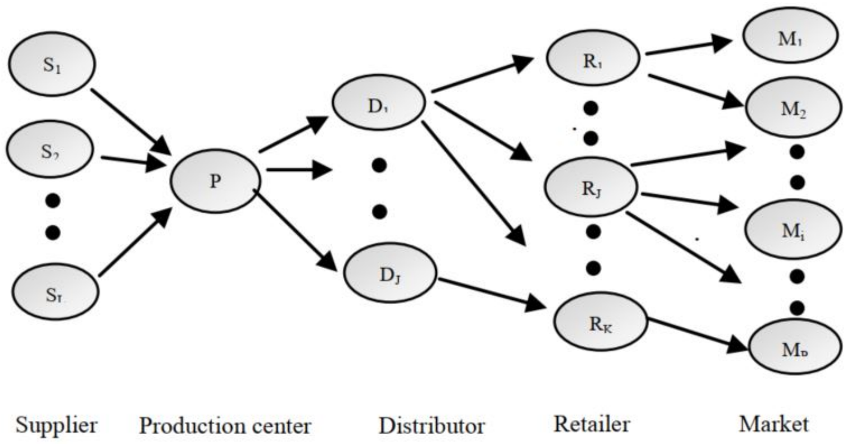


Figure 2.2: A multi-echelon supply chain configuration [19]

2.1.4 Ultrasound Devices and Regulations

Portable Ultrasound Devices

Ultrasound imaging uses sound waves to produce pictures of the inside of the body. It is used to help diagnose the causes of pain, swelling and infection in the body's internal organs and to examine a baby in pregnant women and the brain and hips in infants. It's also used to help guide biopsies, diagnose heart conditions, and assess damage after a heart attack. Ultrasound is safe, noninvasive, and does not use ionizing radiation.

Portable ultrasound scanners (PUS) have been shown to have advantages over other stationary scanners such as ease of use, accessibility, transportability and cost. However, PUS are severely restricted on size and power which directly limits the area and power consumption of the electronics [24]. There is a growing interest in a hand-held ultrasound (US) system for a point-of-care diagnosis. Portable ultrasound scanners are usually characterized by an interface (most often wireless) with a hand-held device. Below in Figure 2.3 and Figure 2.4 are schematics of sample portable ultrasound scanners.

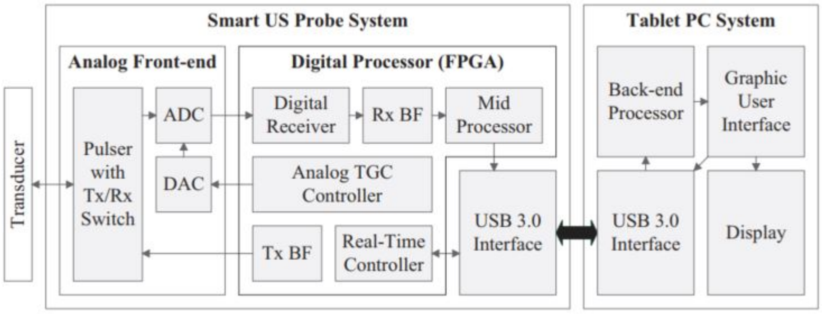


Figure 2.3: Overall block diagram of a developed hand-held Ultrasound imaging system composed of the smart ultrasound probe system and tablet PC [25]

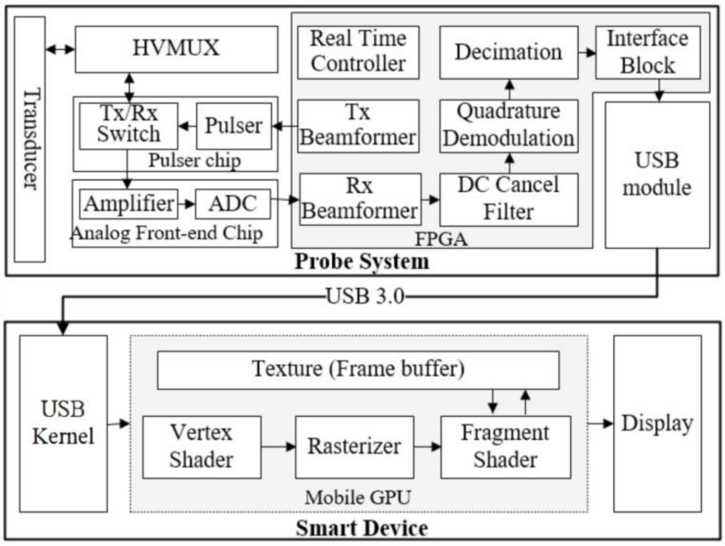


Figure 2.4: Overall block diagram of SMUS(Smart US imaging system) [26]

Certification

The CE mark, is a mandatory conformity mark for certain products that are sold within the European Economic Area since 1985. CE stands for Conformité Européenne, which means European conformity. With the CE mark a manufacturer proves compliance with EU health, safety and environmental protection legislation and confirms a product is compliant with relevant requirements. With a CE mark, your product can be sold in the EU, Iceland, Liechtenstein, Norway and Switzerland. The manufacturer or authorized representative must keep technical documentation for several years (depends on the product type) after the product has been released on the market. Unsafe products are shared in the EU by RAPEX. This is a Rapid Alert System which enables quick exchange of information between the 31 European countries. The size of the CE mark must be at least 5mm high. If the mark does not fit the product or cannot be placed on it, the marking must be affixed to the packaging. CE marking itself is not an evidence of compliance, your technical file must prove the conformity. Pre-testing or clever procurement of your product in an early development process can reduce cost and time to market [27].

Medical devices cannot be placed on the European market without conforming to the strict safety requirements of the European Union; one of these requirements is the affixation of the CE conformity mark. As a requirement, manufacturers to have proof in the form of documentation and an audit trail demonstrating that post-market surveillance is being performed. They also need to show that the data is being fed into appropriate national and international medical device monitoring system databases. Australia, Canada, EU, Japan, and USA are the five founding GHTF member states. Regulations for medical devices in these markets are well established and regularly reviewed and updated. The regulations adopted in these areas are often used as guides for regulations being introduced in other countries. In many cases, approval for a medical device by one of these regulatory bodies is the main requirement for registration in countries [27].

Table 2.2: Countries with EU conformity recognition agreements [28]

Country	*Area of regulation covered*	*Exclusions*
Australia	Conformity assessment	CE marked products manufactured outside the EU, Class 4 IVD devices
Canada		Not in operation for medical devices
Japan	Good laboratory practice	
New Zealand	Conformity assessment	CE marked products manufactured outside the EU
Switzerland	Conformity assessment	
Turkey	Conformity assessment	
USA	Quality system evaluations Product evaluations Post market vigilance reports	A modular agreement with some areas excluded

2.2 Theories of Optimization

Upon obtaining a tool to assess the nature and the performance of a system under study, it becomes necessary to optimize its functioning. This may be a single-criterion study, such as the minimization of cost or duration, or a multi-objective study in which we seek the best compromise between different performance indicators. The optimization problems that are encountered while analyzing logistics systems are often combinatorial, and are characterized by the fact that as the number of possible solutions (or combinations) increases, the size of the problem considerably (exponentially) increases. Consequently, the computation time required for the enumeration of all combinations is directly affected. A large number of these optimization problems are NP-hard. This means that if we wish to find the optimal solution to such a problem, it is necessary to list all the possible solutions. This enumeration requires a computation time which increases exponentially with the size of the problem. Hence, the decision support tool used to solve logistic system problems must give an answer quickly, and the best possible answer.

This leads to the classification of optimization methods into different categories: exact optimization methods, which are costly in computation time but guarantee the optimality of the solution, and approximate (heuristic and meta-heuristic) methods, which are faster, but do not guarantee the optimality of the obtained solution. The first step to be taken before developing or applying an optimization method is to model the problem mathematically (mathematical programming). Then, we must consider the size of the problems that are encountered in the industrial world and the decision level of the problems in question. It is reasonable to allow greater computation times for the resolution of tactical or strategic problems than for an operational management or real-time problem [29]

2.2.1 Mathematical Programming

In mathematical programming, the objective of the studied problem is expressed in the form of a function of some variable (known as decision variables). This objective function is subject to a set of constraints of equality or inequality, which delimit the solution space to be explored. When the decision variables are integers and the functions are linear, this is an integer linear programming model. In the case of real integer variables, the model lies in the field of mixed integer programming. Problems of binary programming and nonlinear programming are also encountered. Different tools exist for obtaining the optimal solution to the problem from the mathematical model. Examples include the simplex method, dedicated solvers such as CPLEX, XPRE, GAMS, Gurobi and EXCEL.

The idea of optimization is an established fundamental principle for the analysis of several complex decision-making problems. By applying the philosophy of optimization, one approaches a complex decision problem, involving the selection of values for a number of interrelated variables, by focusing attention on a single objective designed to quantify performance and measure the quality of the decision [30]. The objective is maximized or minimized, depending on the formulation subject to the constraints that may limit the selection of decision variable values. If a suitable single aspect of a problem can be isolated and characterized by an objective, be it profit or loss in a business setting, speed or distance in a physical problem, expected return in the environment of risky investments, or social welfare in the context of government planning, optimization may provide a suitable framework for analysis.

It is rarely feasible to fully represent all the complexities of variable interactions, constraints, and appropriate objectives when faced with a complex decision problem. With that said, a particular optimization formulation must only be seen as an approximation. A good technique in modeling, to capture the essential elements of a problem, and a good sense of judgment in the interpretation of results are a prerequisite to achieve a meaningful set of results. In view of this, Optimization should be seen as a mechanism of conceptualization and analysis rather than as a principle resulting a philosophically correct solution.

Technique and good judgment, with respect to problem formulation and interpretation of results, is enhanced through extensive practical experience and a painstaking understanding of pertinent theory. Problem formulation itself always involves a trade-off between the conflicting objectives of building a mathematical model sufficiently complex to accurately capture the problem description and building a model that is workable. The typical problem formulation types are Linear Programming, Unconstrained Problems, and Constrained Problems.

Linear Programming

Linear programming is among the most natural mechanism for formulating a wide range of problems with little effort. A linear programming problem is characterized, as the name implies, by linear functions of the unknowns; the objective is linear in the unknowns, and the constraints are linear equalities or linear inequalities in the unknowns. In terms of mathematical and computational properties, there are much broader classes of optimization problems than linear programming problems that have better and more efficient theories and for which effective algorithms are available. The popularity of linear programming lies primarily with the formulation phase of analysis rather than the solution phase. For one thing, a great number of constraints and objectives that arise in practice are indisputably linear. Another reason why linear forms for constraints and objectives are more popular in problem formulation is that they are often the least difficult to define. Thus, an objective function is not purely linear by virtue of its inherent definition, it is often far easier to define it as being linear than to decide on some other functional form and convince others that the more complex form is the best possible choice. Linearity, therefore, by virtue of its simplicity, often is selected as the easy way out or, when seeking generality, as the only functional form that will be equally applicable (or nonapplicable) in a class of similar problems. Of course, the theoretical and computational aspects do take on a somewhat special character for linear programming problems–the most significant development being the simplex method [30].

Unconstrained Problems

If the scope of a problem is broadened to the consideration of all relevant decision variables, there may then be no constraints. This is because constraints represent artificial delimitations of scope, thus they disappear when the scope is broadened. For example, a budget constraint is not characteristic of a meaningful problem formulation; because borrowing at an interest rate makes it possible to obtain additional funds, and hence rather than introducing a budget constraint, a term reflecting the cost of funds is incorporated into the objective. In the same light, constraints describing the availability of other resources which at some cost no matter how great could be supplemented.

Furthermore, many problems can be regarded as having no constraints if the constrained problems are sometimes easily converted to unconstrained problems. For instance, the sole effect of equality constraints is simply to limit the degrees of freedom, by essentially making some variables functions of others [30] [31].

Constrained Problems

Many problems met in practice are formulated as constrained problems. This is because in most instances a complex problem such as, for example, the detailed production policy of a giant corporation, the planning of a large government agency, or even the design of a complex device cannot be directly treated in its entirety accounting for all possible choices, but instead must be decomposed into separate subproblem with each subproblem having constraints that are imposed to restrict its scope. Thus, in a planning problem, budget constraints are commonly imposed to decouple that one problem from a more global one. Therefore, one frequently encounters general nonlinear constrained mathematical programming problems [30] [31]. The general mathematical programming problem can be stated in the form:

$$\begin{aligned}
&\text{minimize } f(\mathrm{x}) \\
&\text{subject to } h_j(\mathrm{x}) = 0, i = 1, 2, \ldots, m \\
&\qquad\qquad\quad g_j(\mathrm{x}) \leq 0, j = 1, 2, p \\
&\qquad\qquad\quad \mathrm{x} \in S
\end{aligned}$$

Dynamic programming

This method was introduced by Bellman in the 1950s also called recursive optimization, it decomposes a given n-dimensional problem into n-unidimensional sub problems. Thus, the system consists of n steps (or n decision levels) to be resolved sequentially. According to Chevalier, the transition from one step to another occurs according to evolution laws of the system and a decision. This principle ensures that for every consecutive decision process, every sub-policy of an optimal policy is also optimal. The use of dynamic programming to describe the optimal value of the criterion at a given step as a function of its value at the previous step depends on the existence of a recursive equation. From the initial combinatorial problem, the method consists of generating its subproblems, solving them and determining the optimal trajectory. This is carried out by calculating a criterion for a subset k based on knowledge of this criterion for each subset $k-1$, thus bringing the number of considered subsets to $2n$ (n being the number of elements considered in the problem). Owing to its exponential nature, which arises from the exponential generation of sub-problems, the method becomes very memory intensive [30].

Branch and bound algorithm

It is not possible to produce a complete enumeration of every feasible solution for a given combinatorial optimization problem. There are, however, some techniques for the realization of an intelligent enumeration by exploring only certain subsets of solutions in which the optimal solution may be found. The basic idea is, therefore, to separate the set of solutions into subsets, and then evaluate them to see if the optimal solution may be contained therein. This approach, called a branch and bound (BAB) algorithm, requires the study of some properties (dominance properties and lower and upper bounds) of the problem to make use of tools to eliminate the bad solution subsets, which are said to be dominated [30].

Approximate methods

The optimal resolution of (NP-hard) optimization problems encountered in logistics and industrial management requires significant computation times, which are incompatible with the industrial requirements of operational management problems. Approximate methods are used to obtain acceptable solutions that are as close as possible to the optimal solution, but with an acceptable computation time. Unlike exact methods, which are intensive in both computation time and memory, and are limited as the size of the problem increases, approximate methods give importance to solution speed, especially in industrial environments where the time factor is important. The methods to be presented are essentially based on metaheuristics known for their ability to solve NP-hard problems. These methods are simulated annealing, genetic algorithms, tabu search, ant colonies and the particle swarm method. They constitute a family of optimization algorithms that are generally stochastic, and whose aim is to solve complex problems arising in operational research. They may be applied in different domains to solve various types of problems such as layout, transport, and vehicle routing [29] [32].

2.3 Decision Support Tools

2.3.1 Agent Based Modeling

Agent-based Modeling (ABM) or sometimes Agent-based Modeling Simulation (ABMS) is a method of modeling systems which are made up of related agents acting independently. Agent-based modeling provides a unique opportunity for firms to conduct research electronically as well as aiding in decision making. It has revolutionized the way scientific research is conducted spanning all the major fields. Some of the applications include supply chain analysis, modeling of disease outbreaks and disaster simulation thus fields of application ranges across socio-economic, biological, archaeological and military boundaries just to mention a few [33] [34].

A key part of agent-based modeling is the classification of what is considered an agent. This classification is done based on the following:

- An agent is a discrete entity with unique attributes and a set of rules controlling how decisions are made as well as how the agent behaves.
- Agents exist in environments which serve as a medium of interaction with other agents
- An agent is sometimes goal-oriented which enables it to contrast actual behavior to a set of predetermined goals.
- Agents are self-governing and as such function independently.

- Agents are self-adaptive [35].

Agent-based modeling has become very useful to the research community with its adoption being necessitated by a number of reasons key among them as follows:

1. System analysis over time have reached more complex levels with regards to their inter-dependencies with the more traditional methods no more adequate.

2. Certain systems have originally been too complicated to realistically model without making some idealized assumptions.

3. Advancements in database systems have enhanced the support for micro-simulations.

4. The ever-improving computational power of computer systems have made it possible to develop more complex models [36].

An agent-based model is developed in a similar fashion as other types of models as shown in figure 2.5

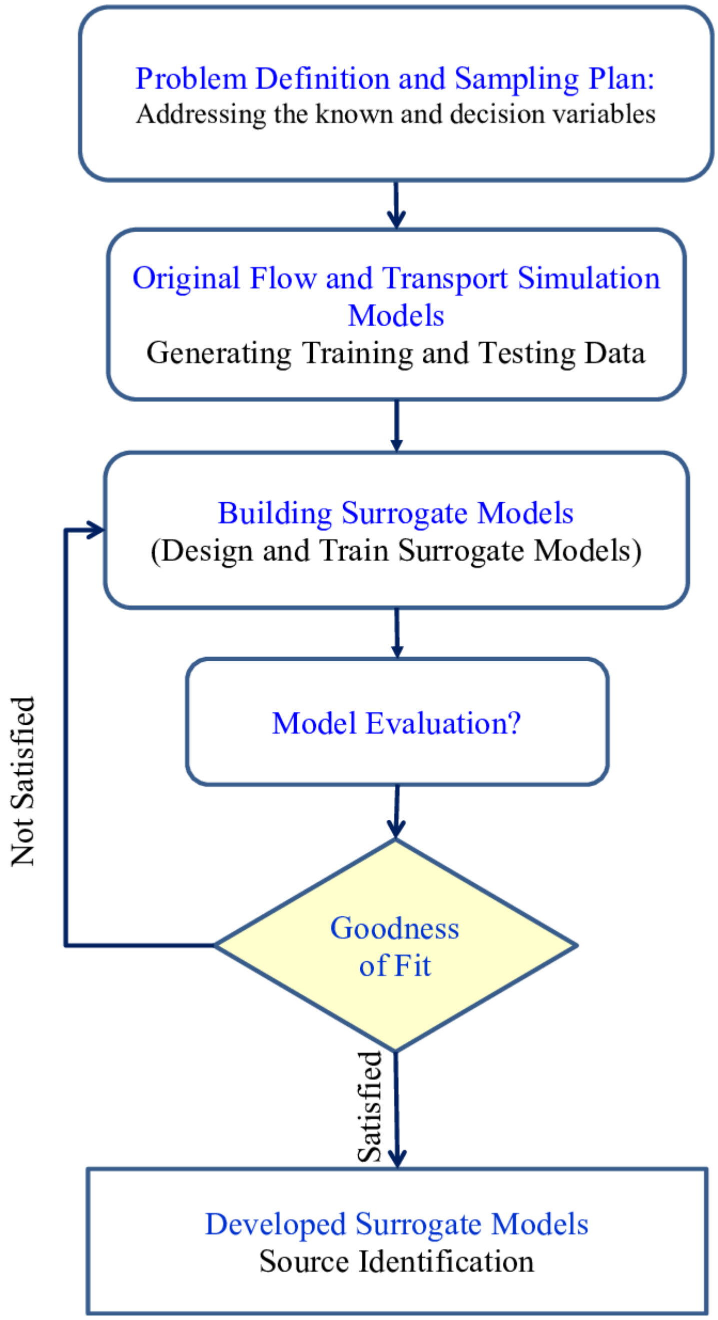


Figure 2.5: Typical steps involved in creating a model

However, because of the agent-focused mindset in developing an agent-based model, extra steps are considered which distinguishes it from other models. The following steps are considered when building such a model:

1. Identification of the agents and determination of the behavior of agents.

2. Identification of the agent interactions and a basis of agent interaction

3. Agent related data acquisition

4. Agent behavior model validation

5. Model execution and analysis of results

There are several software on the market which is used in agent-based modeling by both researchers and firms alike. Some of the popular options are:

- Anylogic
- NetLogo
- MASON
- Swarm
- Repast (Recursive Porous Agent Simulation Toolkit)
- JADE (Java Agent Development Framework) [37]
- LeCas [2]

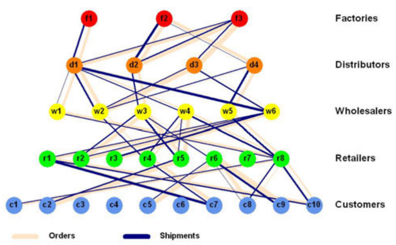


Figure 2.6: Supply chain agents

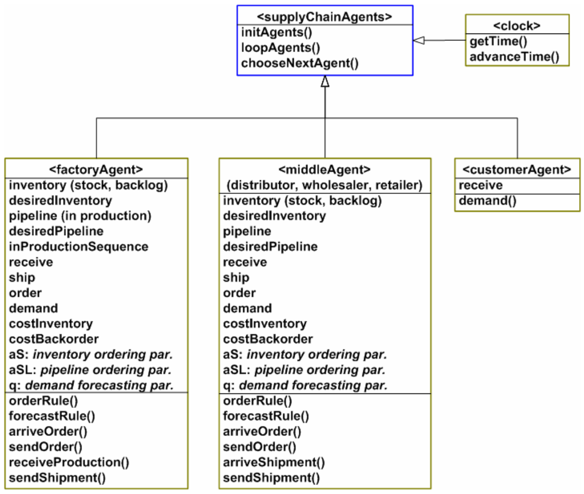


Figure 2.8: Supply Chain class diagram

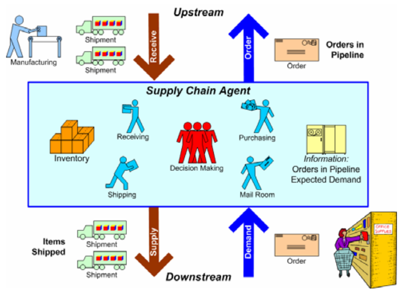


Figure 2.7: TSupply chain wholesaler agent

Agent-based modeling can ideally applied in the when [38] [36] :

- there is a natural representation as agents
- there are decisions and behaviors that can be defined discretely (with boundaries)
- it is important that agents adapt and change their behaviors
- it is important that agents learn and engage in dynamic strategic behaviors
- it is important that agents have a dynamic relationship with other agents, and agent relationships form and dissolve
- it is important that agents form organizations, and adaptation and learn-

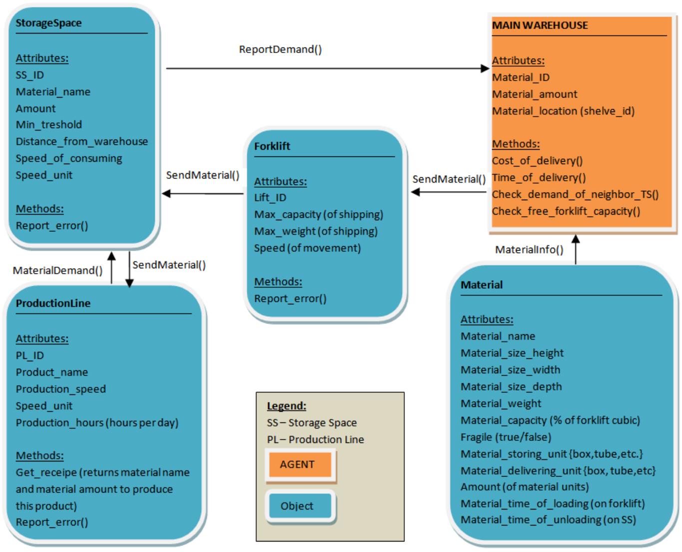


Figure 2.9: Agent relations in a warehouse

ing are important at the organization level

- it is important that agents have a spatial component to their behaviors and interactions
- the past is no predictor of the future
- scaling-up to arbitrary levels is important
- process structural change needs to be a result of the model, rather than a model input

Maka et al. [37] applied the principle of agent-based modeling to a warehouse logistics system and developed the framework in figure 2.9

2.3.2 Anylogic

Anylogic is a very useful software for the simulation of complex systems because it portrays real events with a high degree of accuracy. It has a wide range of applications with the incorporation of GIS (Geographic Information System) maps [39]. The Anylogic simulation software presents a great method for modelling networks through its extensive library. It supports simulaton methods such as system dynamics, agent based modeling as well as discrete event simulation (DES). AnyLogic provides the ability to combine system dynamics model components with components developed using agent based or discrete event methods. This can be achieved in numerous ways such as modeling the consumer market using system dynamics and the supply chain using the agent-based approach. Also, the population of a city can be modeled as agents and the underlying economic or infrastructural background in an SD style. System dynamics diagrams can be incorporated in agents like modeling the production processes inside a company, while the company may be an agent at a higher level. Technically, interfaces and feedback between system dynamic and agent-based or discrete event are very easy. Some system dynamics variables can be used in the decision logic of agents or be parameters of process flowcharts, and the latter in turn may modify other associated variables [40].

The principle of queuing theory coupled with the application of the Anylogic simulation software was used to access the service process at the terminal of containers. Based on the derived key indicators, an analysis of the pros and cons of simulation-based modeling with regards to logistics at container terminals was made [41]. The efficiency of the transportation of containers at harbors is heavily dependent on how efficient the internal processes at the harbor itself are. Thus, increasing the efficiency and utilization at these harbors have been of the topmost priority of both the main industry players and their counterparts in the academia. Various models have been developed by different researchers to address these concerns. Some of these models include a model for the loading and unloading of barges. The single-stage queuing model has also been used to analyze the transfer capacity of ports and the available distance for container ships to maneuver. Other operations research techniques such as mixed linear integer programming, the analytic model as well as the model of nonlinear mathematical programming has been utilized to assess several other processes at container ports. A comparison was made between the use of discrete event simulation and modeling of the port's key operation based on the queuing theory and it was concluded that the discrete event simulation was more efficient for this since it was better at taking real activities into consideration thus portraying a more realistic picture.

The supply chain environment is characterized by the pull and push production systems. In [42], the Anylogic software was used to study the differences between pull and push operations within an enterprise. Based on customer requirements as well as the capacity and objectives an enterprise, the order completion time and inventory levels well contrasted. Based on models developed using Anylogic, it was discovered that an enterprise is better served by adopting either of these two production systems based on their position in the supply chain and its objectives and goals as an enterprise. In the scenario of a universal product which has a lower uncertainty with regards to demand, the push production approach should be utilized when it is desirable to reduce the cost of production. Whereas when the uncertainty of demand is high and

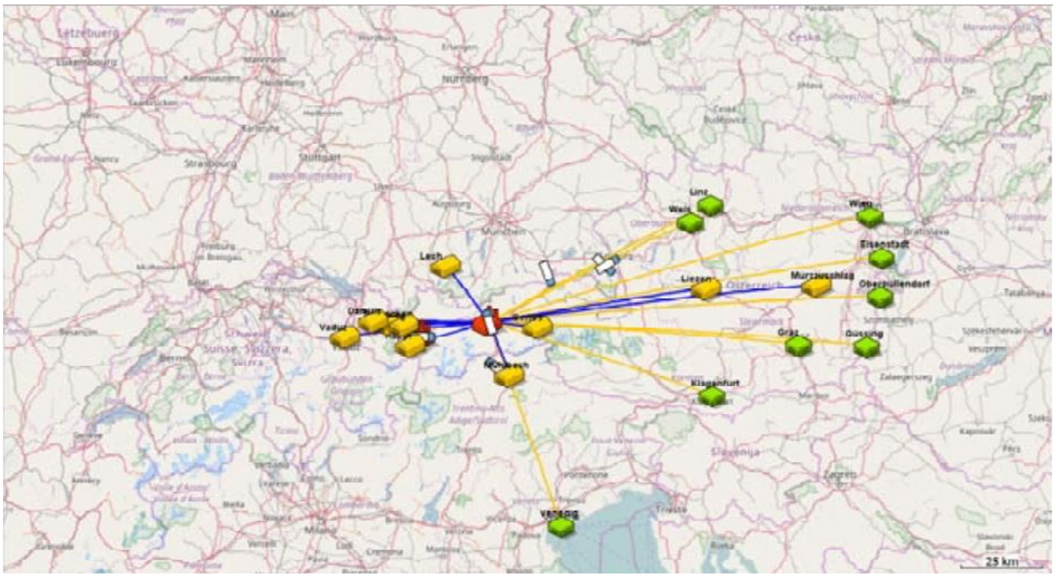


Figure 2.10: A typical supply chain model in Anylogic [43]

key performance indicators such as leas time and service levels are expected to be low and high respectively, a pull-based production system becomes more relevant. Thus, both the pull and push production systems have their pros and cons and as such an enterprise has to determine which system provides a better strategic fit so as to derived the best results.

2.3.3 AIMMS

Advanced Integrated Multidimensional Modeling Software (AIMMS) provides an all-rounded and easy-to-use development environment for creating fully functional Analytic Decision Support (ADS) applications ready for use by end-users. The software runs in different modes to support two primary user groups:

1. Modelers (those developing applications)

2. End users (for decision making)

AIMMS provides decision makers with the power of the most advanced mathematical modeling techniques to enhance their decision-making process.

Analytic decision support applications provide interactive decision support systems with a strong internal emphasis on advanced computational techniques which is associative with extensive problem analysis on the outside. These applications usually:

- represent a complex and large-scale reality,
- organize and use large amounts of interrelated multidimensional data based on corporate and market information,
- represent a complex and large-scale reality,
- use advanced arithmetic manipulations and/or optimization tools to find a solution,
- apply analytic techniques or perform "what-if" experiments to assess the consequences of deciding under different scenarios,
- employ advanced visualization techniques to provide an insight into the solution and/or the problem complexity.

The world has become increasingly more complex and decision makers across the world are constantly in search of advanced decision support tools to provide insight into their decision problems, monitor the consequences of previous decisions, and assist in taking decisions on a regular basis. There is substantial evidence that analytic decision support applications are becoming increasingly popular throughout industry and government, as the improved decisions generated by ADS applications imply increased profit and efficiency.

Major developments over the last decade have increased the suitability of analytic decision support to tackle such problems:

- corporate databases are becoming increasingly mature and allow a quick follow-up to market changes,
- the increasing speed of PCs allows interactive use, even with complex applications,
- the visually attractive and convenient presentation using the standardized and user-friendly Windows environment makes complex processes more accessible to decision makers, and
- the availability of standardized and improved optimization tools allows ADS application developers to specify the problem without having to specify a complicated algorithm to solve it.

Analytic decision support can be applied in a wide variety of decision support problems including the following:

- strategic and tactical planning of resources in industry and government,
- operational scheduling of machines, vehicles, product flow and personnel,
- strategic evaluation studies in the areas of energy, environment, forestry and social policies,
- financial decision-making to support asset-liability management,
- economic decision-making to control market clearing and economic development, and
- technical decision-making to support the design and calibration of systems and objectives.

The three main classes of constrained optimization models are known as linear, integer, and nonlinear programming models. These types have a lot in common. They share the same general structure of optimization with restrictions. Linear programming is the simplest of the three. As the name indicates, a linear programming model only consists of linear expressions.

Mixed Integer Linear Programming and the Ant Colony Optimization were adopted in finding optimal solutions to the Capacitated Vehicle Routing Problem (CVRP) by Dassisti et al. [44]. Brekelmans et al. [45] used the AIMMS Outer Approximation (AOA) method that is implemented in AIMMS to solve the MINLP problems for the optimization model developed to optimize dike heights in the Netherlands. Wang et al. [46] also used the AIMMS modeling language to develop a look-ahead model for grid and market operators to be used as a management tool for operational uncertainties. Caccetta et al. [47] implemented a mixed linear integer programming model for developing a university timetable by using the AIMMS modeling language and the CPLEX solver.

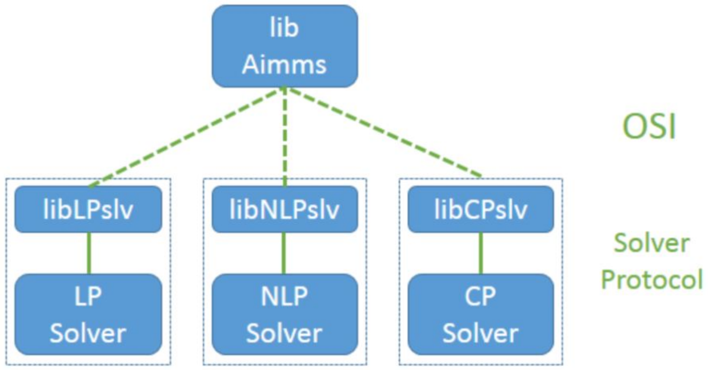


Figure 2.11: AIMMS OSI Model [48]

The AIMMS package integrates externally supplied solution algorithms for mathematical programming and constraint programming with model development, application integration, and application deployment. The AIMMS Open Solver Interface to integrate solution algorithms is extended to communicate instruction arrays in order to communicate non-linear expressions to the CP (Constraint Programming) solvers.

An architecture of the AIMMS OSI (Open Solver Interface) model is represented in Figure 2.12.

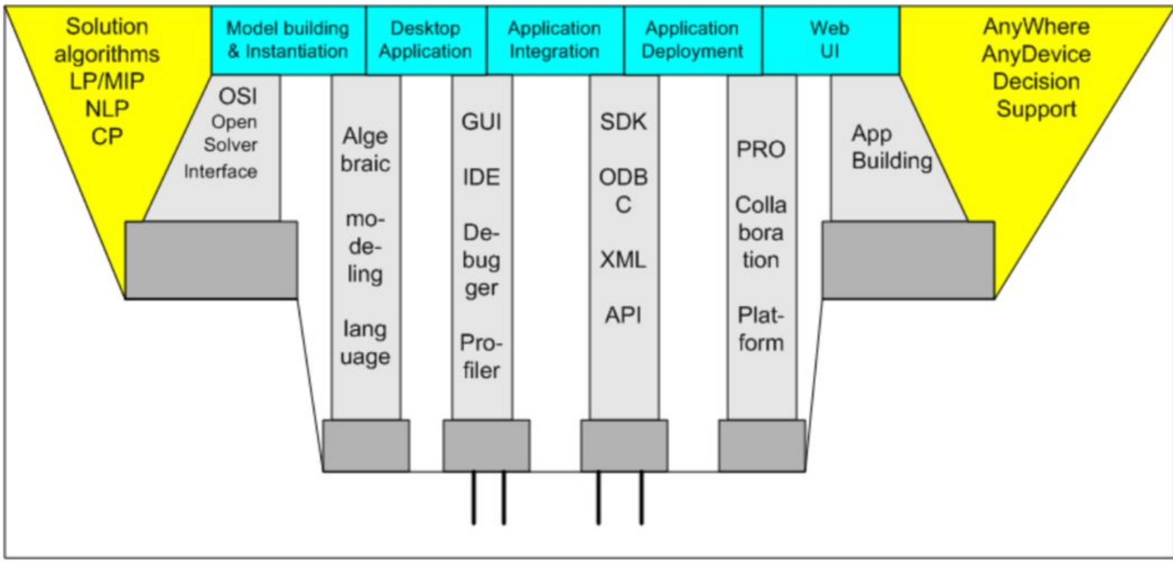


Figure 2.12: AIMMS OSI Model [48]

The AIMMS Open Solver Interface (OSI) is responsible for communicating instances of mathematical programming and constraint programming problems to solution algorithms, setting their control parameters, managing their solution processes, and retrieving solution information. The AIMMS language permits the algebraic specification of the constraints that make up a mathematical program or a constraint program. In addition, this language harbors the algorithmic capabilities Stochastic Programming, Robust Optimization, Outer Approximation, and Automatic Benders Decomposition.

The AIMMS Graphical User Interface (GUI) is used to develop pages on a desktop presenting the input data and the solution. The AIMMS Integrated Development Environment (IDE) lets the model builder act on the structure of the model in a graphical manner and supports debugging, profiling, and analysis of mathematical program instantiations. To integrate AIMMS applications with other applications, a Source Development Kit (SDK) is made available allowing access to the data structure of AIMMS from within languages such as C++, Java, and .net. In addition, the ODBC standard is used to read data from, and write data to relational databases. Moreover, AIMMS also supports the XML standard for data communication with other packages.

The next pillar is to separate the inspecting and interacting with model data from the actual computation of results as is facilitated by the AIMMS PRO collaboration platform. Simply put, data is viewed and changed in lightweight

AIMMS sessions running on Windows laptops and data is computed and problems solved on capable Linux or Windows servers. Finally, the abutment on the right is the app building for supporting Web UI. The objective is to present the model data, both input and solution, in a browser running on any device. This development is currently ongoing.

AIMMS is not the only solution algorithm independent modeling system addressing this gap, nor the first. In mathematical programming, GAMS is generally considered to be the first. However, since the inception of its modeling language, AIMMS put focus on the development of a GUI, IDE, and deployment features and is now a leading vendor in tools for decision support systems [48].

The AIMMS modeling language is linked to many first-class solvers for solving optimization problems. All linear solvers in AIMMS (CPLEX, XA, XPRESS, MOSEK) use a presolve algorithm whereby the problem input is examined for logical reduction opportunities. The goal is to reduce the size of the problem. A reduction in problem size in general leads to a reduction in total run time (including the time spent in the presolve algorithm itself). Of all nonlinear solvers in AIMMS (CONOPT, KNITRO, MINOS, SNOPT, BARON, LGO, AOA) only CONOPT and BARON use pre-processing techniques. When CONOPT solves a model, it tries to detect recursive or triangular equations that can be solved before the optimization is started. The equations identified in this way can be solved independent of the optimization, and they can therefore be removed from the optimization process [49] [50].

2.3.4 Gurobi

The Gurobi Optimizer considered one of the best if not the best commercial optimization solver for linear programming (LP), quadratic programming (QP), quadratically constrained programming (QCP), mixed integer linear programming (MILP), mixed-integer quadratic programming (MIQP), and mixed-integer quadratically constrained programming (MIQCP). It has a flexible environment which enables integration with popular programming languages such as Python, C++, MATLAB, Java, R, etc [51].

Gurobi provides the following advantages:

- High-level optimization modeling constructs embedded in Python programming language.
- Combines expressiveness of a modeling language with the power and flexibility of a programming language.

- Bring feel of a modeling language to the Python interface
- Allows for code that is easy to write and maintain.
- Requires minimal programming skills to get started [52].

2.4 Multi-Criteria Decision Making

In most practical problems, it is the desire of the decision maker to target multiple objectives or take into consideration at least two measures or factors. This desire transforms the decision making problem to a multi-objective decision-making (MODM) problem or a multi-attribute decision making (MADM) problem. These classes of problems can all be categorized under multi-criteria decision making (MCDM) problems [10].

Usually, the decision-making problems which are characterized by spatial or geographical information are referred to as location decisions. These kinds of decisions have become a major part of operations research and management science and is sometimes referred to as location science. Facility location, location science and location models are terminologies which are used interchangeably. Facility location is a section of operations research associated with locating or positioning at least a new facility among several existing facilities to optimize (minimize or maximize) at least one objective function such as cost, profit, revenue, travel distance, service, waiting time, coverage and market shares.

2.4.1 Multi-Attribute Decision Making

Usually in the Multi-Attribute Decision Making, there exists a limitation of the number of predetermined alternatives. These alternatives must satisfy an objective in a stipulated level and the decision maker (DM) selects the best solution or solutions (in certain instances) among all possibilities, based on the priority of each objective and the relationship between them. Various techniques exist which are utilized in tackling the problem of multi-attribute decision making. Popular among these techniques are:

- Dominant
- maximin
- maximax

Table 2.3: Classification of logistics network design problems [53]

Objectives	
Max robustness	Ro
Max responsiveness	Res
Min cost/max profit	C
Outputs	
Inventory	I
Number of vehicles	NV
Demand satisfaction quantity	DS
Price of products	P
Transportation amount	TA
Service region	SR
Facility capacity	FC
Location/alocation	L
Modeling	
Continuous	
Continuous aproximation	CA
Discrete	
Stochastic mixed integer programming	SMIP
Mixed integer non-linear programming	MINLP
Mixed integer linear programming	MILP
Problem definition	
Periods	
Multi-period	MPr
Single period	SPr
Number of facilities to be opened	
Endogenous (undetermined)	En
Exogenous (determined)	Ex
Product	
Single-product	SP
Multi-product	MP
Flow capacity	
Uncapacitated flow	UCF
Capacitated flow	CF
Demand	
Stochastic	S
Deterministic	D
Facility capacity	
Uncapacitated	UC
Capacitated	Ca
Logistics network stages	
Forward logistics stales	
Distribution centers	DisC
Production centers	PC
Supply centers	SC
Reverse logistics stages	
Redistribution centers	RDisC
Disposal centers	DC
Recycling centers	RYC
Recovery centers	RCC
Collection/inspection centers	CIC

- Simple additive weighting
- Hierarchical trade-off
- Conjunctive method
- Disjunctive method
- Lexicographic method
- Elimination by aspects

- Permutation method
- Linear assignment method
- Linear programming techniques or multidimensional analysis of preference.

2.4.2 The Multi-Objective Decision Making

The Multi-Objective Decision Making techniques attempt to design the best possible alternative by taking into consideration the various interactions within the design constraints which best satisfy the decision maker by attaining some acceptable levels of a set of objectives. In multi-objective decision planning and learning, much attention is paid to producing optimal solution sets that contain an optimal policy for every possible user preference profile [54]. The multi-objective decision making problems have various components, the more popular characteristics being:

- A set of quantifiable objectives
- A set of well defined constraints
- A process of obtaining some trade-off information

The solution procedure of the multiple criteria decision problem can be categorized into the following major stages [55]:

1. Identification and verbal description of the decision problem; recognition of its category and definition of major participants of the decision process.
2. Mathematical formulation of the decision problem, including:
    - definition of the set of variants
    - construction of the consistent family of criteria
3. Modeling of the decision maker's preferences
4. Selection of the most suitable multi-criteria decision making(or analysis)method to solve the considered decision problem.
5. Running computational experiments with an application of the selected MCDM/A method. Aggregating the decision maker's preferences using a

particular model of synthesis.

Among methods used in tackling multi-objective decision making problems are:

- global criterion method
- utility function
- metric L-P methods
- the Analytic Hierarchy Process
- bounded objective method
- lexicographic method
- goal programming
- parametric method
- C-constraint method adaptive search method

All these techniques can be categorized into three major groups as follows:

1. Classical approaches try to convert the multi-objective problem into a single objective problem and optimize new single objective problem.
2. Pareto optimal approach.
3. Evolutionary algorithms are used when the problem is too complex to be solved by neither the classical nor the Pareto optimal approach.

Multi-objective location problems are further sub-divided into bi-objective problems and k-objective problems as shown in Figure 2.13

Figure 2.14 illustrates the general goal, criteria (C), and location alternatives (L), which depicts the hierarchy for the location selection problem. The hierarchy shows that the general goal which is selected as the best location in the first level. The second level shows the five criteria subjective factors and the third level has five location alternatives for selection. All of these levels contribute to the realization of the general goal. The size of the hierarchy process is dependent on the number of factors and alternatives present.

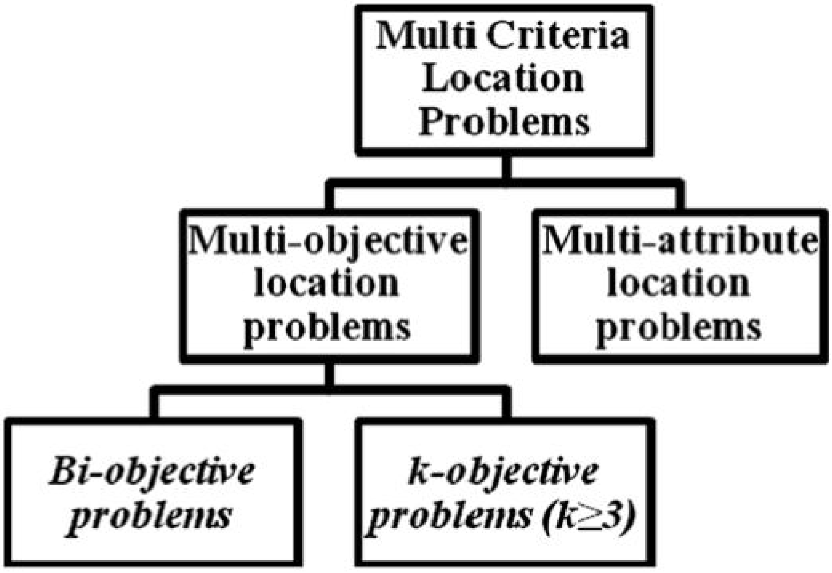


Figure 2.13: The classification of multi-criteria location problems [10]

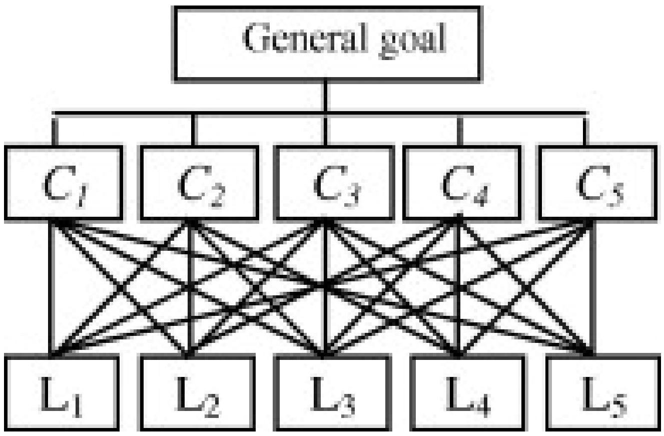


Figure 2.14: The hierarchy process for the location selection [56]

2.4.3 Multi-Criteria Decision Analysis Techniques

Analytical Hierarchy Process (AHP)

The AHP proposed by Saaty [57] is a flexible, quantitative method for selecting among alternatives based on their relative performance with respect to one or more criteria of interest. AHP resolves complex decisions by structuring the alternatives into a hierarchical framework. The hierarchy is constructed through pairwise comparisons of individual judgments, rather than attempting to prioritize the entire list of decisions and criteria simultaneously.

The AHP procedure usually involves six steps :

1. Define the unstructured problem, stating clearly its objectives and outcomes.
2. Decompose the complex problem into decision elements (detailed criteria and alternatives).
3. Employ pairwise comparisons among decision elements to form comparison matrices.
4. Use the eigenvalue method (or some other method) to estimate the relative weights of the decision elements.
5. Calculate the consistency properties of the matrices to ensure that the judgments of decision-makers are consistent.
6. Aggregate the weighted decision elements to obtain an overall rating for the alternatives.

The measurement scale for the pairwise comparisons, where verbal judgments are expressed by a degree of preference: equally preferred = 1, moderately preferred = 3, strongly preferred = 5, very strongly preferred = 7 and extremely preferred = 9. The numbers 2, 4, 6 and 8 are used to distinguish similar alternatives. Reciprocals of these numbers are used to express the inverse relationship. The consistency index (CI) is calculated as

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n - 1} \tag{2.1}$$

where $\lambda_{\max}$ is the biggest eigenvalue of the pairwise comparison matrix. The consistency index of a randomly generated reciprocal matrix is called the random index (RI). The next ratio to be calculated is the CR (Consistency Ratio). If the CR is less than 0.1, the judgments are consistent and the derived weights can be used. The formula for calculating CR is simply

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} \tag{2.2}$$

Fuzzy Analytical Hierarchy Process (FAHP)

Despite the popularity of the AHP method, it is often criticized for its inability to incorporate the inherent uncertainty and imprecision associated with mapping the decision-maker's perceptions to exact numbers. Since fuzziness is a common characteristic of decision-making problems, the FAHP method was developed to address this problem. It allows decision-makers to express approximate or flexible preferences using fuzzy numbers where adding fuzziness to the input, implies adding fuzziness to the judgment.

Fuzzy set theory is a mathematical theory designed to model the fuzziness of human cognitive processes. It is essentially a generalization of set theory where the classes lack sharp boundaries. The membership function $\mu_A(x)$ of a fuzzy set operates over the range of real numbers, generally scaled to the interval [0, 1].

Hence, FAHP uses a range of values to express the decision-maker's uncertainty. The decision-maker is free to select a range of values that reflects his confidence. Alternatively, he can specify his attitude in general terms as optimistic, pessimistic or moderate, representing high, low, and middle ranges of values respectively.

An expert's uncertain judgment can be represented by a fuzzy number. A triangular fuzzy number is a special kind of fuzzy number whose membership function is defined by three real numbers (l, m, u). This membership function is illustrated in Figure 2.15 and represented mathematically as:

$$\mu_A(x) = \begin{cases} (x-l)/(m-l), & l \leq x \leq m \\ (u-x)/(u-m), & m \leq x \leq u \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$

Thus, l, m, and u are the lower, mean and upper bounds of the triangular fuzzy number. The membership function μ represents the degree to which any given element x in the domain X belongs to the fuzzy number A.

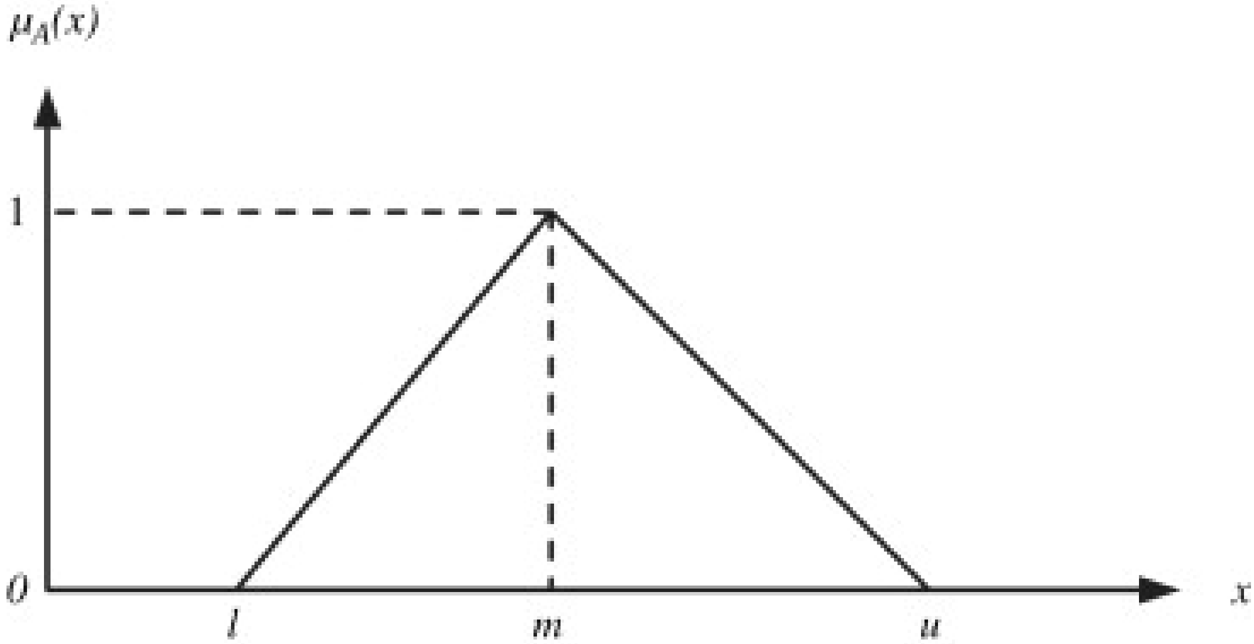


Figure 2.15: Fuzzy triangular number A = (l, m, u) [58]

Fuzzy extent analysis

If the expert judgments are expressed as triangular fuzzy numbers, the triangular fuzzy comparison matrix is

$$\widetilde{A} = \left(\tilde{a}_{ij}\right)_{n\times n} = \begin{bmatrix} (1,1,1) & (l_{12}, m_{12}, u_{12}) & \cdots & (l_{1n}, m_{1n}, u_{1n}) \\ (l_{21}, m_{21}, u_{21}) & (1,1,1) & \ldots & (l_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) & \cdots & (1,1,1) \end{bmatrix}$$

where $\tilde{a}_{ij} = \left(l_{ij}, m_{ij}, u_{ij}\right)$ and $\tilde{a}_{ij}^{-1} = \left(1/u_{ji}, 1/m_{ji}, 1/l_{ji}\right)$

for $i, j, = 1, \ldots, n$ and $i \neq j$ The steps of Chang's fuzzy extent analysis can be summarized as follows:

First, sum each row of the fuzzy comparison matrix $\widetilde{A}$. Then normalize the row sums (obtaining their fuzzy synthetic extent) by the fuzzy arithmetic operation

$$\widetilde{S}_i = \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \left[\sum_{k=1}^{n}\sum_{j=1}^{n} \tilde{a}_{kj}\right]^{-1} = \left(\frac{\sum_{j=1}^{n} l_{ij}}{\sum_{k=1}^{n}\sum_{j=1}^{n} u_{kj}}, \frac{\sum_{j=1}^{n} m_{ij}}{\sum_{k=1}^{n}\sum_{j=1}^{n} m_{kj}}, \frac{\sum_{j=1}^{n} u_{ij}}{\sum_{j=1}^{n}\sum_{j=1}^{n} l_{kj}}\right)$$
$$i = 1, \ldots, n \tag{2.4}$$

where $\otimes$ denotes the extended multiplication of two fuzzy numbers. These fuzzy triangular numbers are known as the relative weights for each alternative under a given criterion, and are also used to represent the weight of each criterion with respect to the total objective. A weighted summation is then used to obtain the overall performance of each alternative.

The degree of possibility is then for $\widetilde{S}_i \geq \widetilde{S}_j$ by the following equation:

$$V\left(\widetilde{S}_i \geq \widetilde{S}_j\right) = \sup_{y \geq x}\left[\min\left(\widetilde{S}_j(x), \widetilde{S}_i(y)\right)\right] \tag{2.5}$$

Which can also as be represented as :

$$V\left(\widetilde{S}_i \geq \widetilde{S}_j\right) = \begin{cases} 1 & m_i \geq m_j \\ \frac{u_i - l_j}{(u_i - m_i) + (m_j - l_j)} & l_j \leq u_i \quad i, j = 1, \ldots, n; j \neq i \\ 0 & \text{otherwise} \end{cases} \tag{2.6}$$

where $\widetilde{S}_i = (l_i, m_i, u_i)$ and $\widetilde{S}_j = \left(l_j, m_j, u_j\right)$

Weighted MOORA Method

The major advantage of using the MOORA methodology is the relative ease by which the results can be obtained. However this is translated into a less accurate result as the weight of individual. The procedure of the Weighted MOORA method are as follows:

- Construct the decision matrix which shows the performance of different alternative with respect to various criteria as shown in 2.7

$$\mathrm{X} = \begin{bmatrix} X_{11} & \cdots & X_{1n} \\ \vdots & \ddots & \vdots \\ X_{m1} & \cdots & X_{mn} \end{bmatrix} \tag{2.7}$$

Where X_{ij} is the performance measure of ith alternative on jth criterion, m is the number of alternatives and n is the number of criteria.

- Normalize the decision matrix using 2.8

$$\mathrm{r_{ij}} = \frac{\mathrm{x_{ij}}}{\sum_{\mathrm{n=1}}^{\mathrm{m}} \mathrm{x_{ij}}} \tag{2.8}$$

Where $\mathrm{i} = 1, 2 \ldots \mathrm{m}; \mathrm{j} = 1, 2, 3 \ldots\ldots \mathrm{n}$

- Assign personal preference to each of the parameter and multiply the parameter weights to the normalized decision matrix.

- Calculate the assessment values by finding the difference between the sum of beneficial and non-beneficial criteria as given in 2.4.3

Assessment Value =

$$\sum_{j=1}^{g} x_{\mathrm{ij}}^{*} - \sum_{j=g+1}^{n} x_{ij}^{*}$$

Where g is the number of criteria to be maximized, $(n-g)$ is the number of criteria to be minimized.

- Finally, rank the assessment values in decreasing order to get the global rank of the alternatives [59] [60].

Fuzzy Set Theory

Set $\widetilde{A}$ as a fuzzy set on discourse domain X, and setting A_α as the fuzzy cut of $\widetilde{A}$, $(\alpha \in [0,1])$, the result is $\widetilde{A} = \cup_{\alpha\in[0,1]} \alpha A_\alpha$,where αA_α is the dot product of a constant and common set, and can be regarded as a special fuzzy set on domain X. The membership function of it can be defined as below [61]:

$$\mu_\alpha A_\alpha(\mathrm{x}) = \begin{cases} \alpha & \mathrm{x} \in \mathrm{A}_\alpha \\ 0 & \mathrm{x}! \in \mathrm{A}_\alpha \end{cases}$$

If $\widetilde{A}$ is a fuzzy set on discourse domain of all the real number R, a relationship exists as below:

$$\mu_{\mathrm{A}}(\lambda \mathrm{x} + (1-\lambda)\mathrm{y}) \geq (\mu_{\mathrm{A}}(\mathrm{x}) \wedge \mu_{\mathrm{A}}(\mathrm{y}))$$
$$\text{Subject to} \quad \forall \mathrm{x}, \mathrm{y} \in \mathrm{R}, \forall \lambda \in [0,1]$$

When $\widetilde{A}$ is a triangular fuzzy number, it should have the membership functions as below.

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} (\mathrm{x}-\mathrm{n}_1)/(\mathrm{n}_2-\mathrm{n}_1), \mathrm{n}_1 \leq \mathrm{x} \leq \mathrm{n}_2 \\ (\mathrm{n}_3-\mathrm{x})/(\mathrm{n}_3-\mathrm{n}_2), \mathrm{n}_2 \leq \mathrm{x} \leq \mathrm{n}_3 \\ 0 & \text{, otherwise} \end{cases}$$

If $\widetilde{A}$ is a L-R fuzzy number, $L(x)$ and $R(x)$ are respectively the left benchmark function and right benchmark function of $\widetilde{A}$, then the general expression of its membership function can be presented as :

$$\mu_{\tilde{\mathrm{A}}}(\mathrm{x}) = \begin{cases} \mathrm{L}\left(\frac{\mathrm{m}-\mathrm{x}}{\alpha}\right) & \mathrm{x} \leq \mathrm{m}, \alpha > 0 \\ \mathrm{R}\left(\frac{\mathrm{x}-\mathrm{m}}{\beta}\right) & \mathrm{x} > m, \beta > 0 \end{cases}$$

m is the mean value of $\widetilde{A}$; α and β are the left and right extension of $\widetilde{A}$

Basic Principle of Entropy Weight Method

Information entropy is a measure of uncertainty, which can measure the useful information provided by the data. In the evaluation system of m factors and n objects, entropy of factor i is defined as:

$$H_i = -k \sum_{j=1}^{n} f_{ij} \ln f_{ij} \quad i = 1, 2, \cdots, m$$

Where: $f_{ij} = \frac{y_{ij}}{\sum_{j=1}^{n} y_{ij}}, k = \frac{1}{\ln n}$

Define f_{ij} In $f_{ij} = 0$ while $f_{\mathrm{ij}} = 0$. Where y_{ij} is defined as eigenvalue of evaluation object j corresponding to factor i (eigenvalue here has been under standardized process). In the evaluation system of m factors and n objects, entropy weight of factor i was defined as:

$$\omega_1 = \frac{1 - H_i}{m - \sum_{i=1}^{m} H_i} \tag{2.9}$$

The decision-making of applying entropy weight method is based on entropy weight, and that is obtained from the basic information provided by the basic data. The more numerical difference of the factor, the more important it will be in the comprehensive evaluation, or vice versa. As the weight deduced by entropy weight method is entirely dependent on the basic data, which will make sure rationality and effectiveness of the result [62]

2.4.4 Facility Location Problem

Usually in a supply chain, a frequently occurring problem in distribution system design is the optimal location of intermediate distribution points between manufacturing plants and customer zones. The intermediate locations serve as storage for the various products pending delivery to their designated customer zones. Within a supply chain network, a facility refers to a production plant, a distribution point or even a warehouse. The facility location problem has received an extensive amount of research focus from the areas of operation research and management science. Because every supply chain network is unique in it's own way, it is quite impractical to design a model which fits every scenario of a facility location problem [63].

Zheng and Liang [63] studied the challenge of dynamic facility location within a supply chain network. As a result of the complexity coupled with the need to obtain a near optimal a two-phase modeling framework is proposed. The model focuses on certain key areas of designing a supply chain such as:

- Partner selection
- Relocation of existing facilities
- Inventory management
- Transportation
- Supply decisions
- Availability of budget

The two phases of the model involve selecting a right partner from a list of prospective partners while the second phase deals with decision making based on the optimal solution. The modeling language ILOG OPL Studio 3.6 was used to implement and solve the model.

2.4.5 Types Of Facility Location Models

Facility location models can be broadly classified as follows [64]:

- The shape or topography of the set of potential plants yields models in the plane, network location models, and discrete location or mixed-integer programming models, respectively. For each of the sub-classes distances are calculated using some metric.
- Objectives may be either of the minsum or the minmax type. Minsum models are designed to minimize average distances while minmax models have to minimize maximum distances. Predominantly, minsum models embrace location problems of private companies while minmax models focus on location problems arising in the public sector.
- Models without capacity constraints do not restrict demand allocation. If capacity constraints for the potential sites have to be obeyed demand has to be allocated carefully. In the latter case we have to examine whether single-sourcing or multiple-sourcing is essential.
- Single-stage models focus on distribution systems covering only one stage explicitly. In multi-stage models the flow of goods comprising several hierarchical stages has to be examined.

- Single-product models are characterized by the fact that demand, cost and capacity for several products can be aggregated to a single homogeneous product. If products are in homogeneous their effect on the design of the distribution system has to be analyzed, viz. multi-product models have to be studied.

- Frequently, location models base on the assumption that demand is inelastic, that is, demand is independent of spatial decisions. If demand is elastic the relationship between, e.g., distance and demand has to be taken into account explicitly. In the latter case cost minimization has to be replaced through, for example, revenue maximization.

- Static models try to optimize system performance for one representative period. By contrast dynamic models reflect data (cost, demand, capacities, etc.) varying over time within a given planning horizon.

- In practice model input is usually not known with certainty. Data are based on forecasts and, hence, are likely to be uncertain. As a consequence, we have either deterministic models if input is (assumed to be) known with certainty or probabilistic models if input is subject to uncertainty.

- In classical models the quality of demand allocation is measured on isolation for each pair of supply and demand points. Unfortunately, if demand is satisfied through delivery tours then, for instance, delivery cost cannot be calculated for each pair of supply and demand points separately. Combined location/routing models elaborate on this interrelationship.

Typically, location problems are classified either as being single criterion or multi-criteria with the objective taking various forms such as :

- Minimizing the number of located facilities

- Minimizing the total setup cost

- Minimizing fixed cost

- Maximizing responsiveness

- Minimizing maximum time and distance traveled

A decision problem may have a single criterion or a single aggregate measure such as cost. In such a scenario, decisions can be taken implicitly by determining the solution with the best value of the single criterion or aggregate measure. This is a classic case of an optimization problem: the objective function is the single criterion and the constraints are the requirements on the alternatives. Different optimization techniques can be used to solve the optimization problem based on the nature and functional description of the problem.

In a scenario where a finite number of criteria exists but the number of the feasible alternatives (the ones meeting the requirements) is infinite, the problem belongs to the field of multiple criteria optimization. The techniques of multiple criteria optimization can also be used when the number of feasible solutions is finite but they are given only in implicit form [65] [66].

2.4.6 Multi-Period (Discrete Time) Facility Location Problem

It is important for decision makers to think about developing robust facility location models that are operable for a considerable time horizon which are capable of being defined in terms of multi-period time horizons. By taking this into consideration, a firm can derive the following benefits [10] :

- the appropriate timing of location decision
- clarifying the best location
- allowing a firm to better anticipate any favorable/unfavorable fluctuations in market demand in the corresponding time horizon, whereas single-period models (continuous time horizon) do not show such characteristics.

Multi-period models provide the added advantage with their correspondence with dynamic models because in each subordinate planning horizon, a decision maker can deal with changing parameters much more effectively as compared to single-period models in which the decision maker is hardly able to cope with the uncertain essence of changing parameters.

Based on the nature of multi-period models, new decision variables such as transportation plan and time-staged establishment of facilities are added to the related problem. Since multi-period models and location have many common elements, they are usually considered as a single model. An example of such a model which incorporates both models in shown:

Minimize Z

$$\sum_{k=1}^{K}\sum_{j=1}^{n}\sum_{i=1}^{m} A_{ijk}x_{ijk} + \sum_{k=2}^{K}\sum_{i=1}^{m}\left(c'_{ik}y'_{ik} + c''_{ik}y''_{ik}\right) \tag{2.10}$$

Subject to:

$$\sum_{i=1}^{m} x_{ijk} = 1 \quad \forall j, k \tag{2.11}$$

$$\sum_{j=1}^{n} x_{ijk} \leqslant n x_{ik} \quad \forall i, k \tag{2.12}$$

$$\sum_{i=1}^{m} x_{iik} = G \quad \forall k \tag{2.13}$$

$$\sum_{i=1}^{m} y'_{ik} \leqslant m_k, \qquad \text{for } k = 2, \ldots, K \tag{2.14}$$

$$x_{iik} - x_{ii,k-1} + y'_{ik} - y''_{ik} = 0, \qquad \text{for } k = 2, \ldots, K \tag{2.15}$$

$$x_{ijk} = \begin{cases} 1 & \text{if location node } j \text{ gives service to demand node } i \text{ in time span } k, \forall i, j, k \\ 0 & \text{otherwise} \end{cases} \tag{2.16}$$

$$y'_{ik} = \begin{cases} 1 & \text{if a facility is closed at node } i \text{ in time span } k, \forall i, k \\ 0 & \text{otherwise} \end{cases} \tag{2.17}$$

$$y''_{ik} = \begin{cases} 1 & \text{if a facility is erected at node } i \text{ in time span } k, \forall i, k \\ 0 & \text{otherwise} \end{cases} \tag{2.18}$$

3

Methodology

Deciding on facility locations is a very complex and usually risk-prone venture because the international environment is volatile with various forms of uncertainty. The process of global location-allocation decision is made up of both quantitative and qualitative entities. As a result, factors such as global competition, political factors, environmental sustainability, government regulations and economic factors must be taken into consideration for the decision-makers to be able to arrive at a more comprehensive solution [67].

Most often in the process of decision making, emphasis is placed on minimizing the total cost involved in building a new facility or the cost of relocation. However, several other key factors play a critical role in determining the success of the new facility. To this end, in addressing the facility location problem identified in this case study, multiple criteria will be considered in conjunction with a cost minimizing mathematical model. Two different methodologies have been considered (Analytic Hierarchy Process and Mixed Integer Programming) in this work to tackle the problem both qualitatively and quantitatively. The Analytic Hierarchy Process (AHP) was applied as a multi-criteria decision-making methodology to address the problem at hand and subsequently the same problem was formulated as a mixed integer programming model towards the reduction of total cost.

3.1 Analytic Hierarchy Process (AHP) Approach

The goal is to the select the best overall alternative out of the proposed locations. The following locations have been chosen as candidates; France, Norway, Japan, China, Spain, UK and South Africa.

Four main criteria are used for the selection process namely; environmental sustainability, global competition, government regulation and economic related factors.

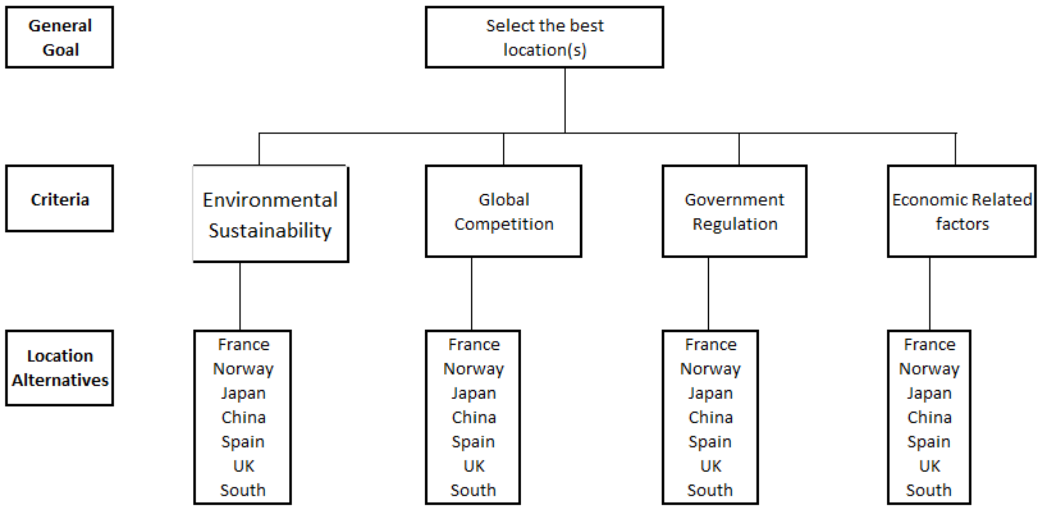


Figure 3.1: The facility location hierarchy

The AHP approach compares only two alternatives at a time. These pairwise comparisons make it easier for the decision maker to isolate and express a level of preference or perceived importance of one of the decision alternatives as compared to the other. This attribute of the AHP methodology is particularly useful in the location selection problem.

Judgment	Verbal equivalent	Comment
1	Equal importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice.
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

Table 3.1: Thomas Saaty's judgment scale [68]

The Thomas Saaty judgment scale in Table 3.1 has been adopted to serve as criteria ranking reference.

3.1.1 Environmental Sustainability

In today's world, thinking of objective functions other than economic functions (like profit, cost, revenue, etc.) is becoming a must. Environmental sustainability has been considered as one of the four main decision criteria because sustainability seeks to protect the natural environment, human and ecological health, at the same time driving innovation and not compromising the quality of life. Thus, it is necessary to consider sustainability as a deciding factor in this facility location problem. Based on data from the Environmental Performance index [69], judgment scale scores have been assigned to the various location alternatives shown in Table 3.2.

	Rank(180)	EPI index	Judgment scale score
France	2	83.95	9
Norway	14	77.49	6
Japan	20	74.69	5
China	120	50.74	3
Spain	12	78.39	7
UK	6	79.89	8
South Africa	142	44.73	2

Table 3.2: Environmental Sustainability ranking

Based on the information gathered, France was identified as the top in rank with an environmental performance index of 83.95 as shown in Table 3.2. Based on this, the matrix for the environmental sustainability was developed as depicted in Table 3.3

	France	Norway	Japan	China	Spain	UK	South Africa
France	1	1.29	1.8	3	1.285	1.125	4.5
Norway	0.778	1	1.2	2	0.857	0.75	3
Japan	0.556	0.833	1	1.667	0.714	0.625	2.5
China	0.333	0.5	0.6	1	0.428	0.375	1.5
Spain	0.778	1.167	1.4	2.333	1	0.875	3.5
UK	0.889	1.333	1.6	2.667	1.142	1	4
South Africa	0.222	0.333	0.4	0.667	0.25	0.25	1

Table 3.3: Criteria Matrix for Environmental Sustainability

Upon computing the normalized matrix, the matrix in Table 3.4 is derived from the criteria matrix.

	France	Norway	Japan	China	Spain	UK	South Africa
France	0.2195	0.1993	0.2250	0.2250	0.2264	0.2250	0.2250
Norway	0.1707	0.1550	0.1500	0.1500	0.1509	0.1500	0.1500
Japan	0.1220	0.1292	0.1250	0.1250	0.1258	0.1250	0.1250
China	0.0732	0.0775	0.0750	0.0750	0.0755	0.0750	0.0750
Spain	0.1707	0.1808	0.1750	0.1750	0.1761	0.1750	0.1750
UK	0.1951	0.2066	0.2000	0.2000	0.2013	0.2000	0.2000
South Africa	0.0488	0.0517	0.0500	0.0500	0.0440	0.0500	0.0500

Table 3.4: Normalized matrix for the environmental sustainability criteria

Finally, the weight of each location alternative with respect to the environmental criteria is obtained as shown in Table 3.5

Location	Weight
France	0.221
Norway	0.154
Japan	0.125
China	0.075
Spain	0.175
UK	0.200
South Africa	0.049
Total	1.000

Table 3.5: Calculated weight for Environmental Sustainability Criteria

The weights were calculated with a consistency ratio of 0.003 which indicates a high degree of consistency. The same methodology is applied to the remaining three criteria.

3.1.2 Global Competition

To successfully compete in the global market, companies must understand that cultural differences play a major role. This criterion provides a unique insight into the drivers of economic growth in the current era of the fourth industrial revolution also known as Industry 4.0.

	Rank(140)	Global Competitiveness index	Judgment scale score
France	17	78	9
Norway	16	78	9
Japan	5	82	8
China	28	73	6
Spain	26	74	7
UK	8	82	8
South Africa	67	61	4

Table 3.6: Global Competitiveness Ranking

Location	Weight
France	0.184
Norway	0.172
Japan	0.158
China	0.118
Spain	0.138
UK	0.158
South Africa	0.072
Total	1.000

Table 3.7: Calculated weight for Global Competition Criteria

3.1.3 Government Regulation

Government regulations such as taxation, the average time to clear customs, contract enforcement, labor regulations as well as regulation on importing and exporting affects the existence of a company within the country. This criterion has the greatest weight among the four. Unfavorable government policy could cause a high level of uncertainty for all companies present.

	Global Regulation score	Judgment scale score
France	100	9
Norway	96.97	8
Japan	86.51	6
China	71.34	4
Spain	100	9
UK	93.76	7
South Africa	59.73	3

Table 3.8: Judgment scale score for Government Regulation

Location	Weight
France	0.182
Norway	0.155
Japan	0.158
China	0.099
Spain	0.138
UK	0.178
South Africa	0.089

Table 3.9: Weighted score of the location alternatives for Government Regulation

3.1.4 Economic Related factors

These refer to the set of fundamental information that impacts a business or an investment's value. Various economic factors need to be considered when determining the current and expected future value of a business or investment portfolio.

	Rank(140)	Global Competitiveness index	Judgment scale score
France	71	63.8	6
Norway	26	73	8
Japan	30	72.1	8
China	100	58.4	5
Spain	57	65.7	7
UK	7	78.9	9
South Africa	102	58.3	5

Table 3.10: Ranking for Economic Related factors

Location	Weight
France	0.129
Norway	0.165
Japan	0.168
China	0.105
Spain	0.147
UK	0.190
South Africa	0.095

Table 3.11: Calculated weight for economic related factors

3.1.5 Priority of Selection Criteria

Criteria	Weight
Environmental Sustainability	0.214
Global Competition	0.267
Government Regulation	0.311
Economic Related factors	0.207
Total	1.000

Table 3.12: Priority of Selection Criteria

3.2 Mixed Integer Programming Approach

3.2.1 Model Formulation

Considering the situation where different products are manufactured at several plants with known production capacities and restrictions. The demands for each product at the various customer point are also known. This demand is satisfied by shipping via intermediate distribution centers, and for reasons of efficiency, each customer point is assigned to a particular distribution center. Associated with each center is a lower and an upper limit on the total throughput of all product. There is also a fixed rental charge and a per unit throughput charge associated with each distribution center. In addition, there is a variable unit cost of transporting a product from a manufacturing plant to a customer zone via a distribution center. This cost also includes the unit production cost.

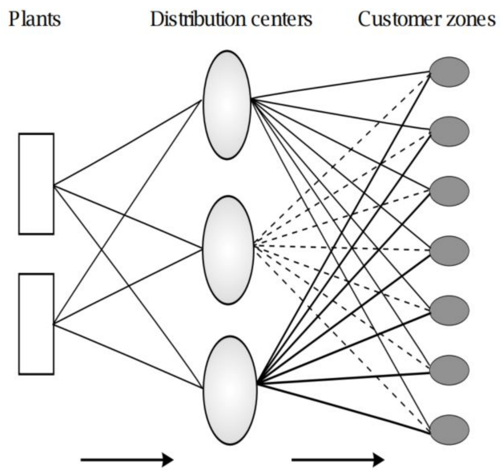


Figure 3.2: A Typical Product Distribution Scheme

The main concern in this kind of scenario is to ascertain which distribution points to be chosen, and the customer zones to be served by the chosen distribution point. The objective is to meet the given demands at minimum total distribution and production cost, subject to plant capacities, distribution center throughput and customer requirement constraints. The problem will be formed as a mixed integer programming model to be used for small and medium-sized data sets. However, for larger data sets, the Benders decomposition method will be developed. Benders decomposition is a technique in mathematical programming that allows the solution of very large linear programming problems that have a special block structure. This block structure often occurs in applications such as stochastic programming as the uncertainty is usually represented with scenarios. This technique is popularly applied to the facility location problem [70].

Sets

First and foremost, the various nodes in the supply chain that can be classified under a unique set will be defined.

C The set of the various customers indexed by c
V The set of the product variants indexed by v
P The set of possible production plants indexed by p
D The set of the possible distribution points indexed by d
B The set for the Bender's cut indexed by β

Parameters

S_{vp} = Production capacity of product variant v at plant p
D_{vc} = The demand of customer c for product variant v
M_{d} = Maximum capacity of distribution point d
m_{d} = Minimum capacity of distribution point d
I_{d} = Inventory cost per unit product at distribution point d
F_{d} = Fixed cost for distribution point d
X_{vpdc} = The variable cost for producing and shipping product variant v from plant p through distribution point d to customer c

Decision Variables

N_{vpdc} = The quantity of product variant v transported from plat p through distribution center d to customer c
a_d = binary to indicate selection of distribution point d
b_{dc} = binary to indicate that customer c is served by distribution point d

Constraints

Supply Constraint

The total quantity of the various products transported to the all customers through the various distribution points cannot exceed the overall capacity of the manufacturing plants thus:

$$\sum_{dc} N_{vpdc} \leq S_{vp} \quad \forall v, p \tag{3.1}$$

Demand Constraint

The demand for each product variant v in each customer section c must be supplied by all plants, but via the selected distribution point.

$$\sum_{p} N_{vpdc} \geq D_{vc} b_{dc} \quad \forall v, d, c \tag{3.2}$$

Throughput Constraint

The total number of product variants v to be delivered to respective customer zones c should be within the throughput limit for each distribution point d.

$$m_d a_d \leq \sum_{vpc} N_{vpdc} = \sum_{vc} D_{vc} b_{dc} \leq M_d a_d \quad \forall d \tag{3.3}$$

Distribution Constraint

Each customer zone c is allocated to one and only one distribution point d

$$\sum_{d} b_{dc} = 1 \quad \forall c \tag{3.4}$$

$$a_d, b_{dc} \in \{0, 1\} \tag{3.5}$$

$$N_{vpdc} \geq 0 \tag{3.6}$$

Objective Function

The aim is to minimize production and transportation costs in addition to the fixed and variable charges for distribution centers and the throughput of products moved through these centers.

$$
\begin{aligned}
\text{minimize} \quad & \sum_{vpdc} X_{vpdc} N_{vpdc} + \sum_{d} \left[F_d a_d + I_d \sum_{vc} D_{vc} b_{dc} \right] \\
\text{subject to} \quad & \sum_{dc} N_{vpdc} \leq S_{vp} \quad \forall v, p \\
& \sum_{p} N_{vpdc} \geq D_{vc} b_{dc} \quad \forall v, d, c \\
& m_d a_d \leq \sum_{vpc} N_{vpdc} = \sum_{vc} D_{vc} b_{dc} \leq M_d a_d \quad \forall d \\
& \sum_{d} b_{dc} = 1 \quad \forall c \\
& a_d, b_{dc} \in \{0, 1\} \\
& N_{vpdc} \geq 0
\end{aligned}
$$

This model is implemented in the AIMMS modeling language as well with Gurobi in the Python programming language.

3.2.2 Implementation in AIMMS

The model is implemented in the AIMMS modeling language. A customized user interface is also design is developed to make what-if analysis and data visualization more convenient.

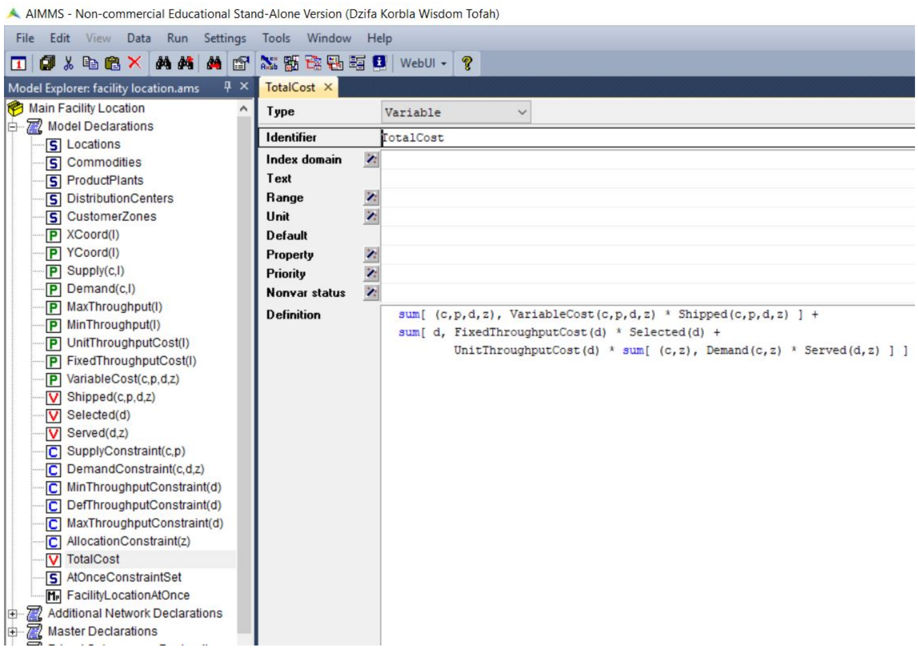


Figure 3.3: Model implementation in AIMMS

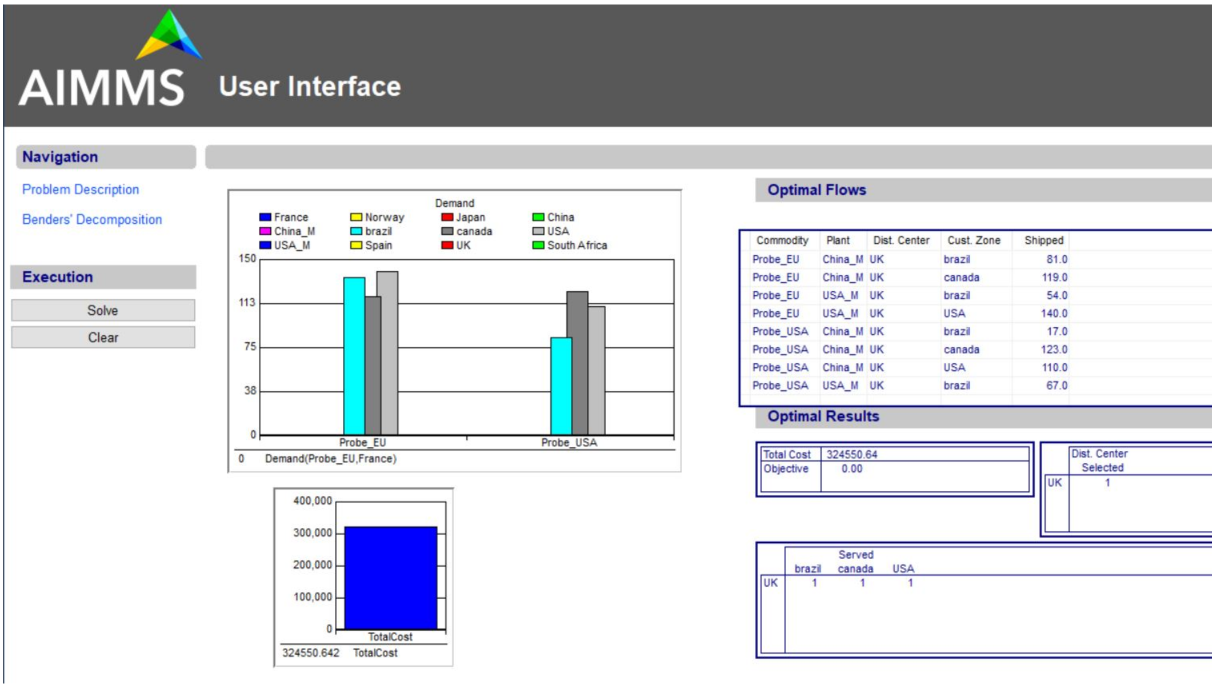


Figure 3.4: Customized user interface in AIMMS

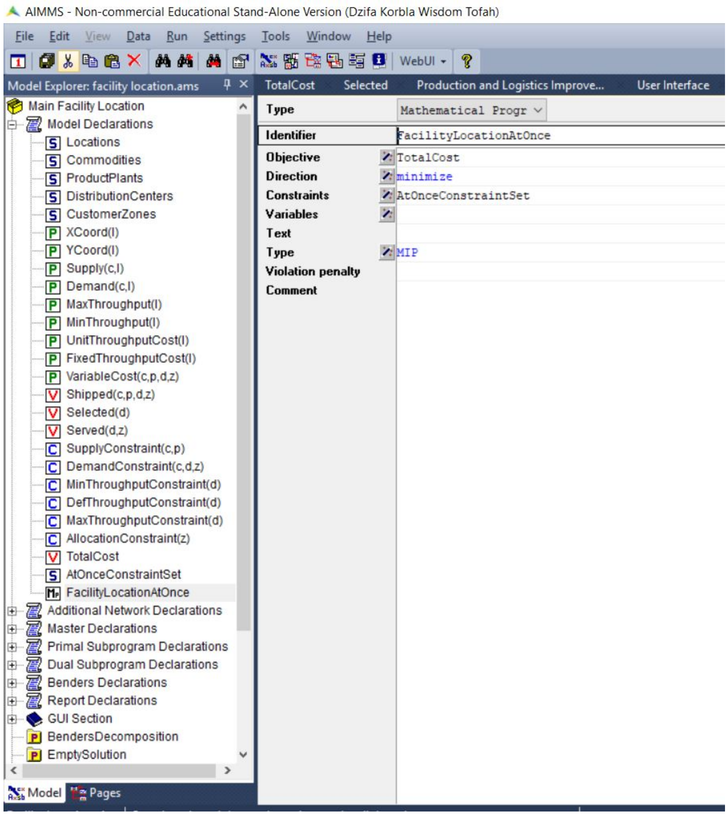


Figure 3.5: Objective function in AIMMS

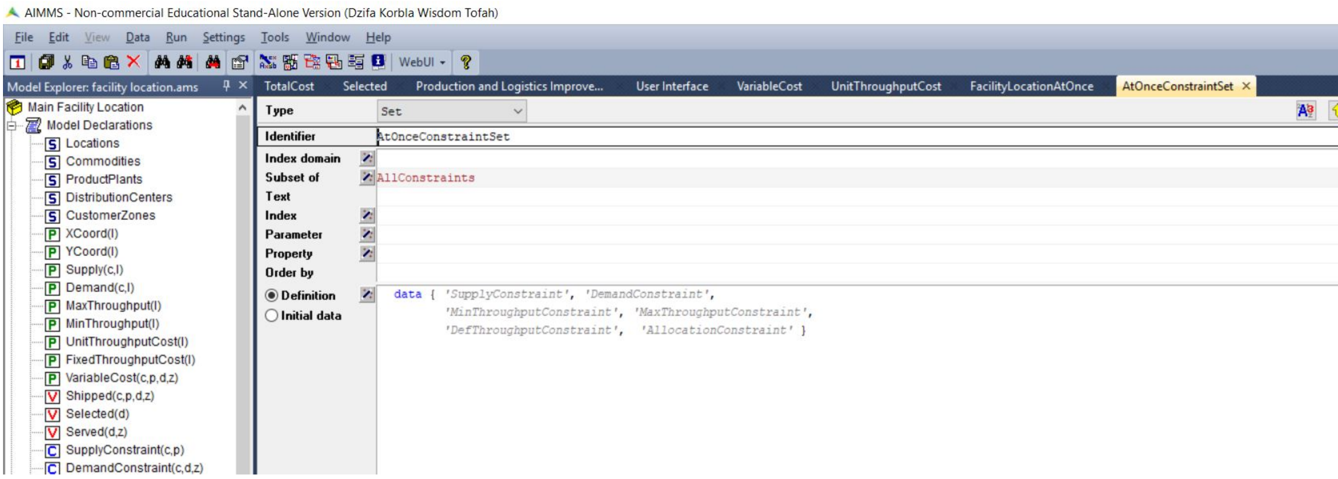


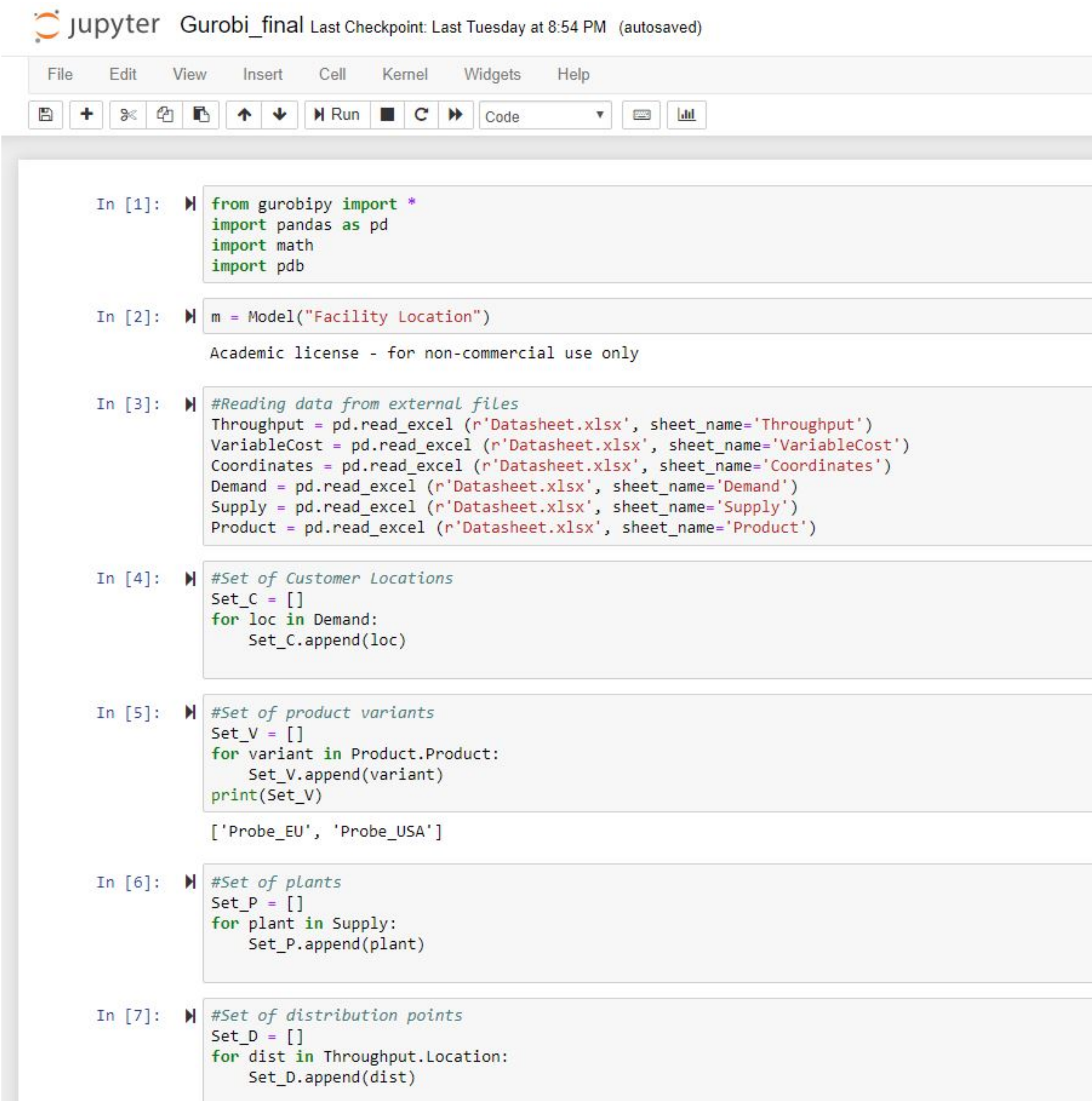
Figure 3.6: Constraints set in AIMMS

The problem is solved as a mixed inter programming problem using the CPLEX

12.0 solver on an Intel(R) Core(TM) i7-6500U CPU @ 2.50GHz (4 CPUs), 2.6GHz with 16GB of RAM.

3.2.3 Implementation in Gurobi

The same model was implemented with Gurobi in Python and a similar result was obtained.




```
In [1]: from gurobipy import *
        import pandas as pd
        import math
        import pdb

In [2]: m = Model("Facility Location")

        Academic license - for non-commercial use only

In [3]: #Reading data from external files
        Throughput = pd.read_excel (r'Datasheet.xlsx', sheet_name='Throughput')
        VariableCost = pd.read_excel (r'Datasheet.xlsx', sheet_name='VariableCost')
        Coordinates = pd.read_excel (r'Datasheet.xlsx', sheet_name='Coordinates')
        Demand = pd.read_excel (r'Datasheet.xlsx', sheet_name='Demand')
        Supply = pd.read_excel (r'Datasheet.xlsx', sheet_name='Supply')
        Product = pd.read_excel (r'Datasheet.xlsx', sheet_name='Product')

In [4]: #Set of Customer Locations
        Set_C = []
        for loc in Demand:
            Set_C.append(loc)

In [5]: #Set of product variants
        Set_V = []
        for variant in Product.Product:
            Set_V.append(variant)
        print(Set_V)

        ['Probe_EU', 'Probe_USA']

In [6]: #Set of plants
        Set_P = []
        for plant in Supply:
            Set_P.append(plant)

In [7]: #Set of distribution points
        Set_D = []
        for dist in Throughput.Location:
            Set_D.append(dist)
```

Figure 3.7: Implementation in Gurobi

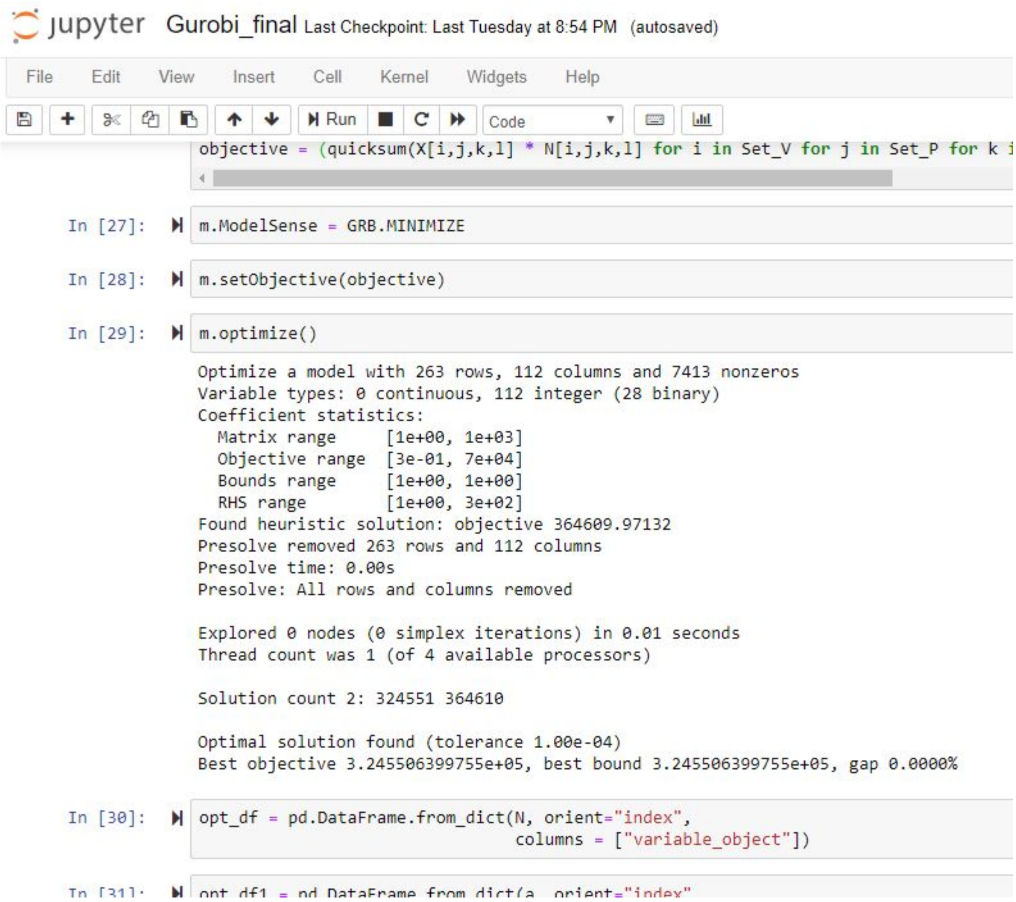


Figure 3.8: Solution in Gurobi

Larger Model Formation Using Bender's Decomposition

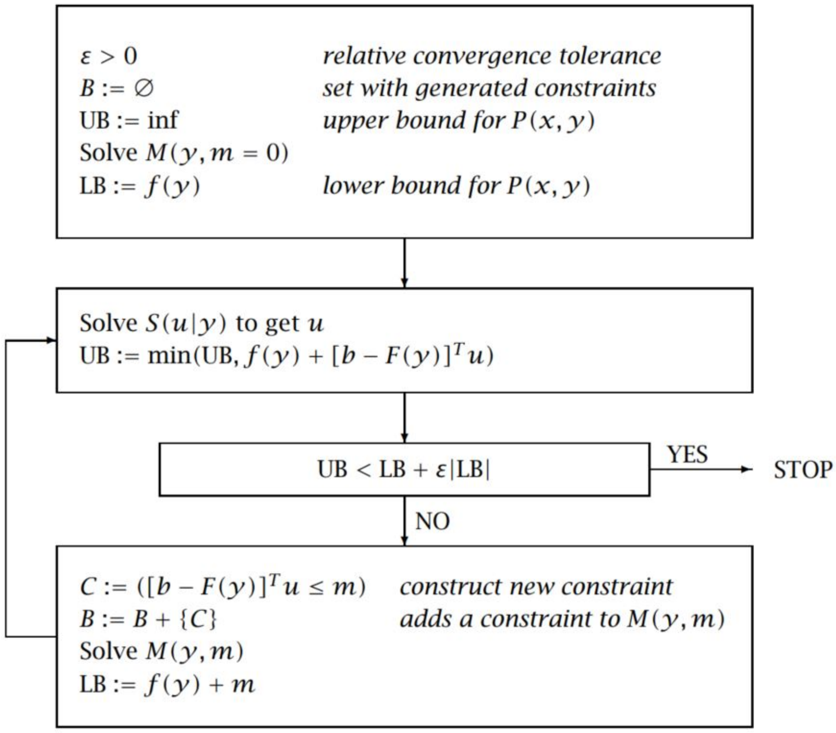


Figure 3.9: A Flow Chart of the Bender's Decomposition Algorithm [71]

The process of developing the Bender's decomposition requires the problem to be divided into two parts namely:

- The master problem
- The dual sub-problem

The variables and constraints will be divided into two groups. Set Y will consist of the binary variables a_d and b_{dc} as well as constraint equations 3.3, 3.4 and 3.5. The linear part will be dualized and will be represented by the continuous variable N_{vpdc} in conjunction with 3.1, 3.2 and 3.6.

The Initial Master Model

A representation of the master model will be given by $M(a, b, m = 0)$ and the objective function will be given by :

$$\sum_{d} \left\{ F_d a_d + R_d \sum_{vc} D_{vc} b_{dc} \right\}$$

The objective function of the master model is subject to:

$$M_d a_d \leq \sum_{vpc} N_{vpdc} = \sum_{vc} D_{vc} b_{dc} \leq m_d a_d \quad \forall d$$

$$\sum_{d} b_{dc} = 1 \quad \forall c$$

$$a_d, b_{dc} \in \{0, 1\}$$

The Dualized Model

The problem to be dualized can be represented by the following objective function:

$$\sum_{vpdc} X_{vpdc} N_{vpdc}$$

This objective function is subject to:

$$\sum_{dc} N_{vpdc} \leq S_{vp} \quad \forall (v, p) | S_{vp} > 0 \tag{3.7}$$

$$\sum_{p} N_{vpdc} = D_{vc} b_{dc} \quad \forall (v, d, c) | b_{dc} = 1 \tag{3.8}$$

$$N_{vpdc} \geq 0 \tag{3.9}$$

The sub-problem will be presented in the form $S(\sigma, \pi | a, b)$ with σ_{vp} corresponding to constraint equation 3.7 and π_{vdc} analogous to constraint equation 3.8

The new formation of the objective function becomes as follows.
Maximize:

$$\sum_{vp} \sigma_{vp} S_{vp} + \sum_{vdc} \pi_{vdc} D_{vc} b_{dc}$$

Subject to:

$$\sigma_{vp} + \pi_{vdc} \leq X_{vpdc} \quad \forall(v, p, d, c)$$
$$\sigma_{vp} \leq 0$$
$$\pi_{vdc} \text{ free}$$
$$\textstyle\sum_p S_{vp} \geq \sum_c D_{vc} \quad \forall v$$
$$\textstyle\sum_c D_{vc} \equiv \sum_{dc} D_{vc} b_{dc} \quad \forall v$$
$$\textstyle\sum_{vp} \sigma_{vp} S_{vp} + \sum_{vdc} \pi_{vdc} D_{vz} b_{dc} \leq m$$

The Final Resulting Master Model

Minimize:

$$\sum_d \left\{ F_d a_d + I_d \sum_{vc} D_{vc} b_{dc} \right\} + m$$

Subject to:

$$\begin{array}{ll} \sum_d y_{dz} = 1 & \forall z \\ \underline{M}_d v_d \leq \sum_{cz} D_{cz} y_{dz} \leq \overline{M} v_d & \forall d \\ \sum_{cp} \sigma_{bcp} S_{cp} + \sum_{cdz} \pi_{bcdz} D_{cz} y_{dz} \leq m & \forall b \\ v_d, y_{dz} \in \{0, 1\} & \end{array}$$

Both the dualized model and the master model are implemented in the AIMMS modeling language.

4

Results and Discussion

4.1 Results From Analytic Hierarchy Process

France was top under the criteria of Global competition, environmental sustainability and government regulation (Figure 4.1). Although UK only had the edge over France in terms of economic related factors, it was consistently ranked in the top three of all four categories

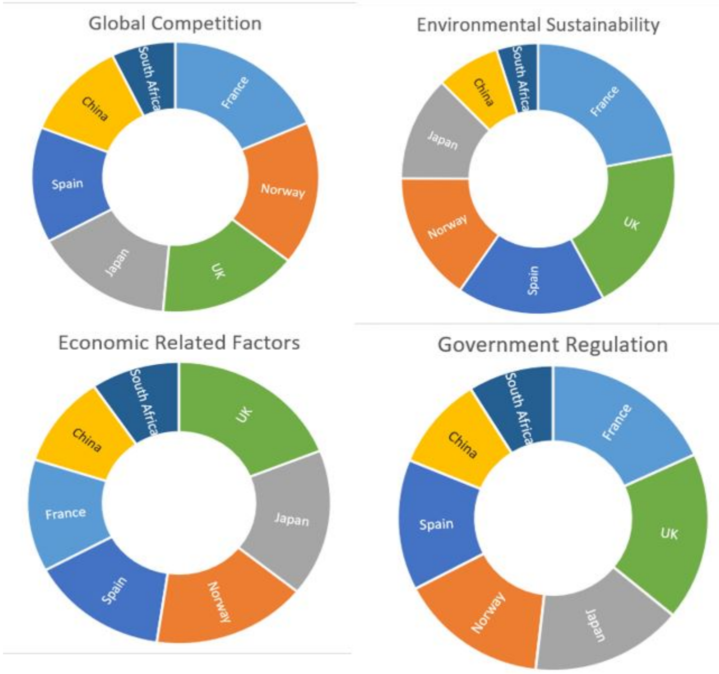


Figure 4.1: Comparison of the location alternatives based on the four criteria

Figure 4.2: Priority Weight of all Criteria

Location	AHP Weighting	Decision Preference
France	0.1798	**First Choice**
Norway	0.1615	Third Choice
Japan	0.1532	Fourth Choice
China	0.1003	Sixth Choice
Spain	0.1481	Fifth Choice
UK	0.1798	**First Choice**
South Africa	0.0773	Seventh Choice

Table 4.1: Overall Ranking of all Alternatives

Based on all the results obtained from the AHP methodology, both France and UK are very strong alternatives for siting the facility with both of them being tied in first place as shown in Table 4.1. It is also worth mentioning that with the exception of South Africa, the other three were not too far off France and UK.

4.1.1 Sensitivity Analysis

Some values of the multi-attribute decision models are often subjective thus the weights of the criteria and the scoring values of the alternatives against the subjective criteria usually contain some uncertainties. It is therefore important to understand how the final ranking, or the ranking values of the alternatives are sensitive to the changes of some input parameters of the decision model. The simplest case is when the value of the weight of a single criterion can vary. For additive multi-attribute models, the ranking values of the alternatives are simple linear functions of this single variable and attractive graphical tools can be applied to present a simple sensitivity analysis to a user [65]. Because there are no clear-cut differences among the location alternatives ranked in the top four, it will be prudent to analyze how changes in the priority weights of the four decision making criteria affects the selection of possible locations.

Case 1

The weight of government regulation is kept constant, global competition and economic related factors are increase with the weight of environmental reduced.

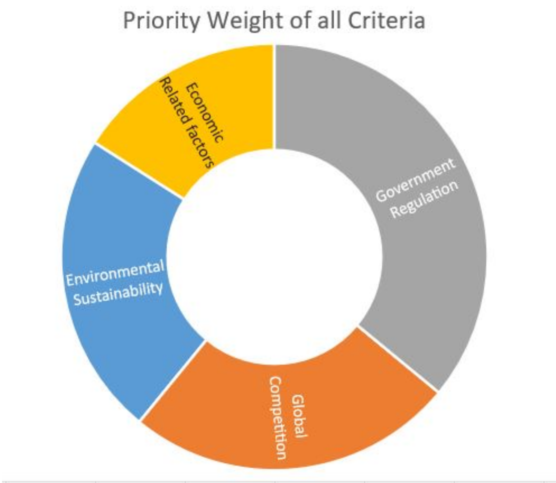


Figure 4.3: Priority Weight of all Criteria for case1

It can be seen from Figure 4.4 that France takes a slight lead over UK when these changes are made.

Figure 4.4: Overall ranking for all alternatives in case1

Case 2

Environmental sustainability was assigned 40 percent of total weight depicted in Figure 4.5 and resulted in the distribution in Figure 4.6. France is slightly preferred with a weight of 0.1899 compared to 0.1847 for UK

Figure 4.5: Priority Weight of all Criteria for case 2

Figure 4.6: Overall ranking for all alternatives in case2

Case 3

Also, Economic related factors was assigned 40 percent of total weight depicted in Figure 4.7 and resulted in the distribution in Figure 4.8. In this scenario, UK has a clear edge over France with a weight of 0.1809 compared to 0.1670 for France.

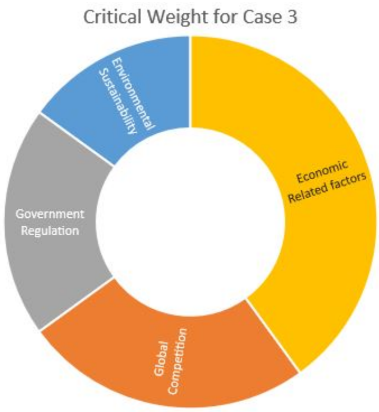


Figure 4.7: Priority Weight of all Criteria for case 3

Figure 4.8: Overall ranking for all alternatives in case 3

Case 4

In the final simulated scenario, all four criteria were assigned an equal weight(Figure 4.9). The result showed that UK was still the preferred location.

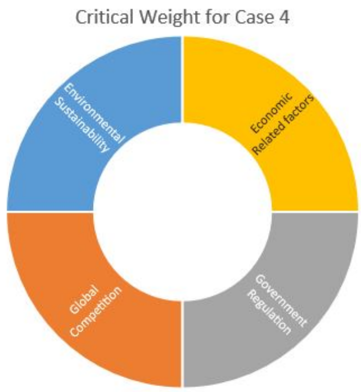


Figure 4.9: Priority Weight of all Criteria for case 4

Figure 4.10: Overall ranking for all alternatives in case 4

4.2 Results From Mixed Integer Programming

Results from the implementation in AIMMS suggest that UK is the best facility location alternative shown in Figure 4.11

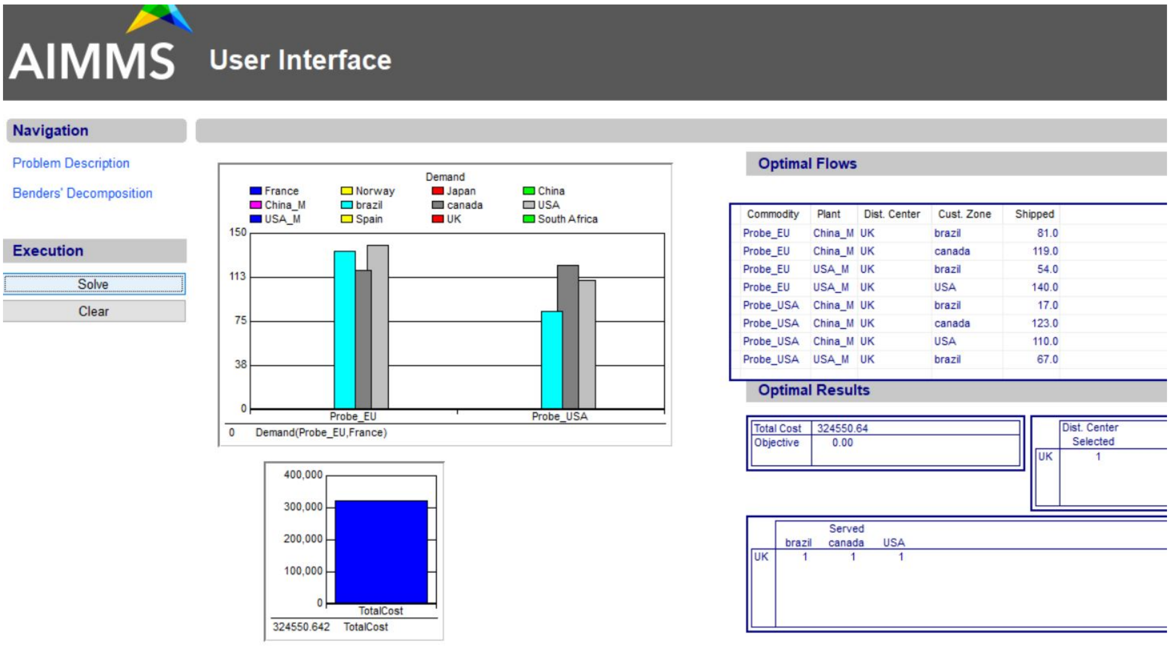


Figure 4.11: Solution from AIMMS optimization modeling software

4.3 Comparison Between AIMMS and Gurobi

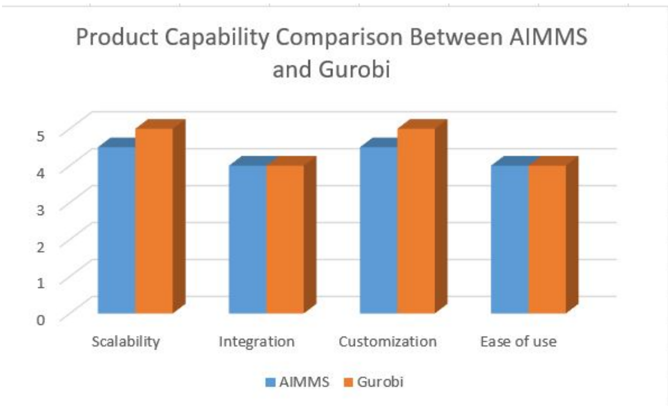


Figure 4.12: Product Capability Comparison Between AIMMS and Gurobi

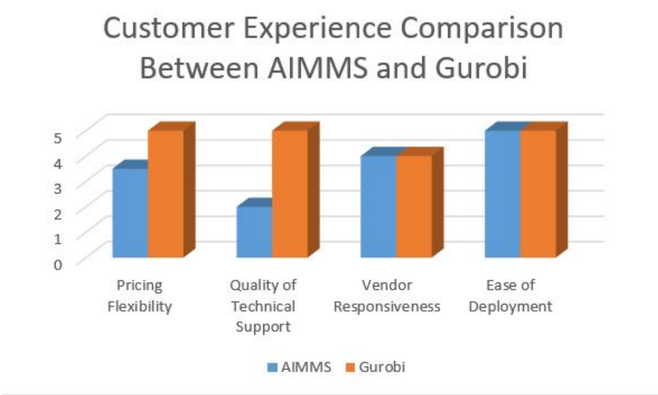


Figure 4.13: User Experience Comparison Between AIMMS and Gurobi

5

Conclusion

In this master's thesis, a facility location decision making model has been developed using Biim Ultrasound as a case study. The framework integrates the Analytic Hierarchy Process (AHP) and a Mixed Integer Programming (MIP) model. During this thesis work, several software tools for optimization have been reviewed and applied in solving the problem presented.

Most of the models used in multi-criteria location problems consider deterministic parameters while solving models including uncertainty, random parameters or with distribution function are more realistic. Therefore, using stochastic optimization and robustness concept in this area can be an important direction [10]. Also, determining robust facility locations which conform to any customization of models' parameters is one of the critical objectives of decision makers because in already existing facilities, the main parameters such as cost, demand, and delivery time are likely to be uncertain during the planning horizon. Therefore, models considering uncertainty would be of interest in such conditions.

Finally, the model developed addresses the issue of optimal facility location however it could be a lot more comprehensive. For instance, instead of using a single period mixed integer programming model, a dynamic multi-objective location model could be formulated to address the effects of time on the decision making process. Furthermore, the AnyLogic simulation software could be used to build an agent-based model utilizing the choices made from the decision models to study their individual behavior as well as the inter-relational behavior within a network.

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Python Code for Analytic Hierarcy Process

```
import numpy as np
from fractions import Fraction

def data_files(file):
    units = []
    with open(file) as fp:
        for line in fp:
            units.append(line.split()[0]) # Used to deal with '\n'
        return units

def comparison(units):
    n = len(units)
    A = np.zeros((n, n))
    for i in range(0, n):
        for j in range(i, n):
            if i == j:
                scale = 1
            else:
                scale = float(Fraction(input(units[i]+' to '+units[j]+':')))
            A[i][j] = scale
            A[j][i] = float(1/scale)
    return A

def weight(A):
    n = A.shape[0]
    e_vals, e_vecs = np.linalg.eig(A)
    lamb = max(e_vals)
    w = e_vecs[:, e_vals.argmax()]
    w = w / np.sum(w) # Normalization
    # Consistency Checking
    ri = {1: 0, 2: 0, 3: 0.58, 4: 0.9, 5: 1.12, 6: 1.24, 7: 1.32,
```

```
    8: 1.41, 9: 1.45, 10: 1.49, 11: 1.51}
    ci = (lamb - n) / (n - 1)
    cr = ci / ri[n]
    print("The normalized eigen vector:")
    print(w)
    print('CR = %f'%cr)
    if cr >= 0.1:
        print("Failed in Consistency check.")
        exit = input("Enter 'q' to quit.")
        raise
    return w, cr

if __name__ == '__main__':
    goal = input("Your Goal: ")
    criteria = data_files('criteria.txt')
    Alternatives = data_files('Alternatives.txt')
    n2 = len(criteria)
    n3 = len(Alternatives)
    A = comparison(criteria)
    print("The matrix A")
    print(A)
    print()
    W2, cr2 = weight(A)
    B = {}
    W3 = np.zeros((n2, n3))
    for i in range(n2):
        print("----------------------------------")
        print("Consider "+criteria[i])
        B[str(i)] = comparison(Alternatives)
        w3, cr3 = weight(B[str(i)])
        W3[i] = w3
    W = np.dot(W2, W3)

    print("------------------------------------")
    print("The final Weight:")
    print(W)
```