



Regional policy analysis in a simple NEG model with vertical linkages

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Working Paper Series in Economics and Management
No. 03/08, March 2008

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Abstract

The paper presents a simple three-region, two-sector general equilibrium model that is used for analysing the effect of regional tax policies. The model includes exogenous asymmetry in terms of transport costs as well as a vertical industry structure that can account for endogenous location development in order to distinguish between the effect of ‘first nature’ and ‘second nature’ on the required subsidy for meeting a population policy target.

JEL classification: D5; R1; J3

Key words: General equilibrium; Regional policy; Labour subsidies

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A first version of this paper was presented at the 47th annual WRSA meeting and a seminar at Western Washington University in February 2008. Thanks to participants for comments.

1. Introduction

One of the great merits of ‘new economic geography’ (NEG) models is the ability to explain core periphery structures endogenously through the economic development (‘second nature’) of location rather than merely assuming core periphery structures based on exogenous attributes (‘first nature’). However, the enthusiasm may have gone too far in disconnecting first and second nature so that the inherent core periphery characteristic of many exogenous attributes is swept under the carpet. Let me give an example: first nature is in many two-region models represented by (iceberg) transport costs. With equal transport costs either way, there are two regions defined by first nature, but there is not a first nature core and periphery. In applied work it would be nice to allow both first nature exogenous core periphery attributes and second nature endogenous core periphery developments to interact. A simple way to allow first nature asymmetry is to add a third region, all three regions located along a line with transport costs depending on distance. Although the analysis may be concentrated to only two of these regions, trade linkages to the third region makes one of them more advantageous than the other in terms of first nature location. In this paper, a simple three region general equilibrium model along these lines is presented. The second nature is introduced through a vertical linkage between an upstream sector producing intermediates under increasing returns for a domestic downstream sector producing consumer goods under constant returns for domestic use and export.

The model is applied to analyze two alternative policies. The first policy is a payroll subsidy in one of the first nature peripheral regions financed by a payroll tax in the first nature core region. We may think of these two regions as part of one country and the remaining region as the world outside so that the policy is a national regional policy. The second policy is laissez-faire, possibly replacing the first because the world outside make complaints about trade distorting state aid.

The paper is organised in 5 sections. The model without policy instruments is presented in Section 2. The ‘policy on’ situation is described in Section 3. The two are compared in Section 4 under the heading ‘Policy performance’ and Section 5 concludes. The microeconomic foundations as well as details on the full model are relegated to Appendix A (microeconomics) and B (macroeconomics).

2. The model

We have three regions: the assisted region, the North n , the assisting region, the South s , and the world outside. Transport costs between n and the world is higher than either between n and s or s and the world because of distance and different accessibility costs. Distance is leading to a geographical disadvantage for n compared to s since higher transport costs imply higher living costs, leading to lower real wage or higher labour costs.

In each of the regions there is an economic base sector, the B sector, and a non-basic sector, the A sector. To simplify, I assume that the B sector produces only final goods, while the A sector only produces intermediates. Hence, we have a vertical industry structure with the B sector downstream and the A sector upstream. Following orthodox trade theory, we abstract from interregional trade in intermediates so only final goods are tradable. Technology is assumed identical in the three regions.

The B sector downstream produces by means of B skilled labour and intermediates from the A sector upstream. The consumers distinguish the products from the B sectors of the three regions by origin only.¹ The firms downstream are price takers in the output market and use a constant to scale technology. In equilibrium no firms earn profits due to free entry. Market prices are determined by equating demand and supply. In order to abstract from currency issues, all regions are treated as if they had a common currency.

The A sector upstream produces differentiated intermediates by means of a single input called A skilled labour. Due to fixed set up costs there are internal increasing returns to scale. The market structure is monopolistic competition. Specialization through the number of intermediate inputs is endogenous and acts as an agglomeration (centripetal) force. The larger the A sector, the more productive is the B sector of the region.²

¹ This is essentially the Armington (1969) assumption.

² An alternative, considered by Skott and Roos (1997), is a fixed number of intermediate inputs.

Both B skilled and A skilled labour is mobile between n and s . Following the new economic geography tradition, spatial equilibrium is simply obtained when the real wage of labour is the same in both regions.³

Under laissez faire the model can essentially be summarized by four simple equations where the variables are relative variables expressing the value in the North relative to the South:

$$wl = 1 \tag{1}$$

$$w = p \tag{2}$$

$$l = m \tag{3}$$

$$p = \tau^{-\alpha/3} w^{1-\alpha} l^{-\beta(1-\alpha)} \tag{4}$$

Here w is the relative downstream producer wage, l and m are the ratios of labour North to South downstream (l) and upstream (m), and p is the relative consumer price level. Greek letters denote parameters: α is characterizing the taste for home made goods ($\alpha / (1 - \alpha)$ is the ratio of home made goods to foreign goods), β is characterizing the strength of the vertical linkage, and τ is the ratio of short distance (the distance between North and South or South and the world) to long distance (the distance between North and the world) iceberg transport costs.

The reduced form model (1) - (4) follows from the full model presented in Appendix B under the following assumptions: there are no taxes and there is full labour mobility between the North and the South in both the upstream and the downstream sectors (recall that labour is sector specific). We will also throughout the paper restrict attention to the situation where there is an exogenous home market effect in the sense that the market share for home made goods is at least as large as the market share for imports from one of the outside regions.

Equation (1) essentially follows from market clearing in all downstream markets, equation (2) is the condition for B-skilled labour mobility equilibrium (downstream) and equation (3) that for A-skilled (upstream), and equation (4) gives the relative consumer price level where we have taken advantage of the restriction imposed by (3) in order to economize on parameters.

³ This need not be interpreted in a literal sense, but may be regarded as a reduced form for more sophisticated behaviour. As shown by Baldwin (2001), replacing myopic with forward looking behaviour in the standard core-periphery model (Fujita et al., 1999) does not imply that the qualitative behaviour of that model changes.

The model is easily solved for w , p , l and m :

$$w = p = \left(\tau^{1/3}\right)^{\frac{\alpha}{\beta(1-\alpha)-\alpha}}$$

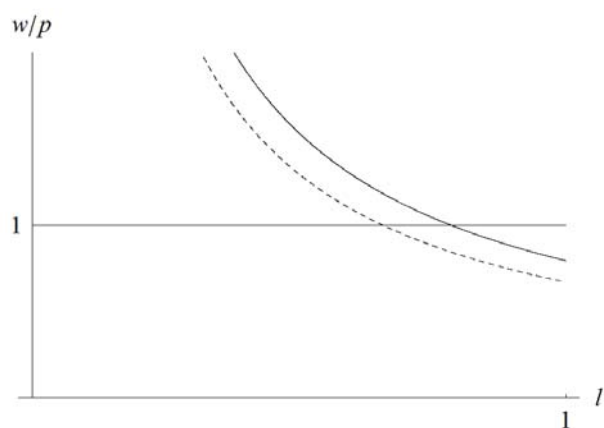
$$l = m = \left(\tau^{1/3}\right)^{\frac{\alpha}{\alpha-\beta(1-\alpha)}}$$

For the model to be well behaved in the sense of generating a higher equilibrium population in the South (advantaged through lower transport costs), we must impose $\alpha / (1 - \alpha) > \beta$. We observe that the restriction is most easily fulfilled when the ratio of home made goods to imported goods is large. For fixed τ we have $\partial l / \partial \alpha > 0$ and $\partial l / \partial \beta < 0$. Hence, an increase in α (weaker preferences for home made goods) acts as a centrifugal force spreading population more equally, whereas an increase in β (higher cost share for intermediates downstream or rising markup over input price upstream) makes the second nature effect through the vertical linkage stronger and acts as a centripetal force concentrating population in the core. Hence, trade linkages and vertical linkages have opposite effects in the model. We may also note that the equilibrium population distribution implies almost total depopulation of the North if β should ever approach the upper limit $\alpha / (1 - \alpha)$.

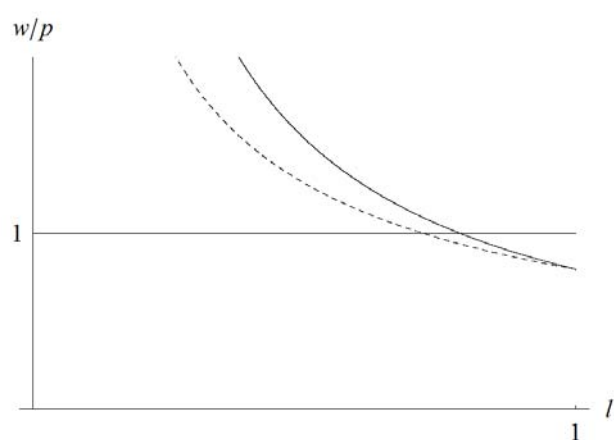
It may be instructive to express relative real wage in terms of the downstream labour ratio (also equal to the upstream labour ratio by equation (3) and therefore also the population ratio). Using equation (1) and (4), we obtain

$$w / p = \tau^{\alpha/3} l^{\beta(1-\alpha)-\alpha}$$

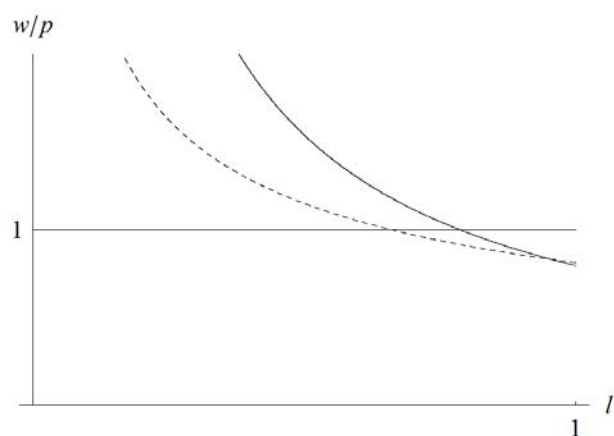
In equilibrium the ratio is equal to one in accordance with equation (2). Figure 1 illustrates the equilibrium for different parameter values. Along the horizontal line the real wage is equal in North and South. The down-sloping curves represent relative real wage as a function of the labour ratio. In Panel a relative distance is increasing, leading to a uniform downward shift in the wage curve as the intercept $\tau^{\alpha/3}$ becomes smaller when τ falls (the dashed curve). In Panel b the vertical linkage becomes stronger, again leading to a downward shift as the slope becomes flatter when β goes up. In Panel c there is a positive shift in preferences for home made goods and services that lead to the shift in the wage curve as the intercept goes up and the slope becomes flatter at the same time.



Panel a. Increasing relative distance (τ down)



Panel b. Stronger vertical linkage (β up)



Panel c. Increasing preferences for home made (α down)

Figure 1. Equilibrium

Note: The horizontal line represents the equilibrium relative real wage, $w/p = 1$. The solid (dashed) curve represents the wage equation, $w/p = \tau^{\alpha/3} l^{\beta(1-\alpha)-\alpha}$, before (after) the parameter change.

Please, observe that as short distance transport costs fall relative to long distance costs, the crossing that indicates equilibrium is moved to the right, ultimately crossing for l equal to one when there is no transport cost asymmetry left. Hence, when space is neutral, downstream labour is equally split between regions and, by implication, populations too. When space is non-neutral, less people reside in the disadvantaged region.

Unlike the traditional core-periphery model (Krugman, 1991, Fujita et al., 1999), the present model gives a unique equilibrium level for the regional population distribution. However, the actual distribution will shift for a change in parameter values as illustrated in Figure 1. A fall in short distance to long distance transport costs, suggested as a stylized fact by Hummels (1999), will move the equilibrium towards the symmetric distribution. So will an increase in the share of imported goods (implied by a higher α), that may also be viewed as a stylized fact of globalization. A stronger vertical linkage on the other hand, also possibly a stylized fact, will move the distribution in the other direction. Hence, a similar reduction in short distance to long distance transport costs could well have very different effects in different economies depending on industry structure and openness as measured by trade statistics. Or economies characterized by differences in relative distance, could well be very differently affected by changes in the vertical structure or trade. If these changes are deliberately brought about through public policy, effective policy design within the model must obviously take the differences between different areas into account since one size will not fit all, here just as in the real world.

3. Policy on

Assume now that we consider a regional policy with the aim to balance the first nature disadvantage reflected in higher transport costs so that population becomes the same in the North and the South. The model then has to be slightly changed,

$$w[1 - (1 - \alpha)/t_n] = 1 - (1 - \alpha)/t_s \quad (5)$$

$$w[1/t_n - 1] = 1 - 1/t_s \quad (6)$$

$$w = p t_n / t_s \quad (7)$$

$$p = \tau^{-\alpha/3} w^{1-\alpha} \quad (8)$$

The new symbols are t_n and t_s for the payroll tax factors in the North and the South (recall that if t is the tax rate, $1+t$ is defined as the tax factor). Observe that l and n no longer are present because these are kept equal to one through the policy instrument.

The reduced form model (5) - (8) follows from the full model presented in Appendix B under the following assumptions: the tax rate is not differentiated across sectors and the policy instrument is used to exactly balance the first order asymmetry between the North and the South so that population is the same, and there is full labour mobility between the North and the South. Equation (5) is equivalent to equation (1) in the model without policy instruments, equation (2) ensures that subsidies are balanced by taxes, equation (3) is the condition for labour mobility equilibrium, and equation (4) gives the relative consumer price level.

The model given by the four equations can in principle be solved for w , p , t_n and t_s .

With symmetric market shares ($\alpha = 1$), the solution is particularly simple. From (1) it follows that $w = 1$. Substituting in the other equations and solving, we get $p = \tau^{-1/3}$, $t_n = \frac{1 + \tau^{1/3}}{2}$ and

$t_s = \frac{1 + \tau^{-1/3}}{2}$. The solution for t_n and t_s is illustrated in Figure 2.

Although it seems reasonable to treat transport costs as exogenous, it turns out to be more convenient in the general case to let τ be endogenous along with w , p and t_s , and solve conditional on t_n . Doing this for the symmetric case, we obtain $\tau = (2t_n - 1)^3$, $p = 1/(2t_n - 1)$ and $t_s = t_n / (2t_n - 1)$.

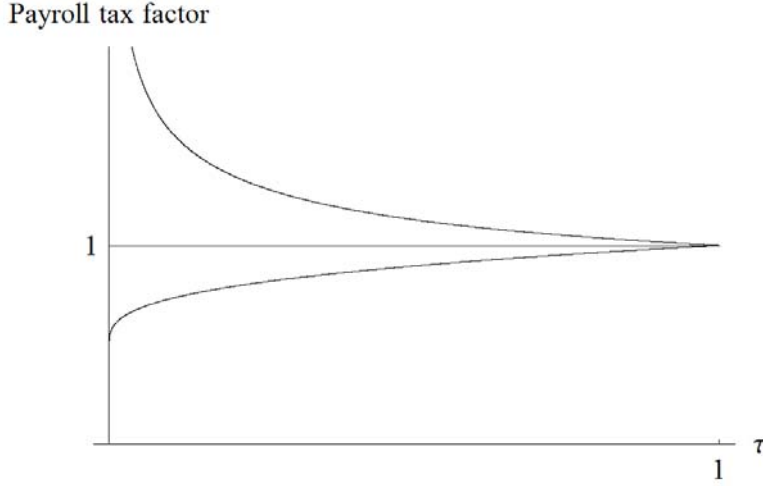


Figure 2. Compensating subsidy

Note: payroll tax factor in the South (above the unit line) and the North (below).

The general solution is just a little more complicated. The solution for w , p , t_s and τ conditional on t_n is given by

$$w = \frac{\alpha t_n}{2(t_n - 1) - \alpha(t_n - 2)} \quad (9)$$

$$p = \frac{\alpha}{2(t_n - 1) + \alpha} \quad (10)$$

$$t_s = \frac{2(t_n - 1) - \alpha(t_n - 2)}{2(t_n - 1) + \alpha} \quad (11)$$

$$\tau = \left(\frac{1}{\alpha}\right)^3 \left[(2(t_n - 1) + \alpha) \left(\frac{t_n}{2(t_n - 1) - \alpha(t_n - 2)} \right)^{1-\alpha} \right]^{3/\alpha} \quad (12)$$

We observe that the value for α places a lower bound on t_n for the model to be well specified. The restriction $t_n > 1 - \alpha/2$ ensures that p is positive. When p is positive, so is w , and t_s is at least unity. But then $t_n \leq 1$ and it follows that $p \geq 1$ and $p \geq w \geq 1$. For $\alpha = 1$ the restriction is fulfilled if t_n exceeds $1/2$. Moving towards autarky ($\alpha \rightarrow 0$) means the restriction is approaching unity so the restriction in fact reflects that t_n is at least t_s in the limit as the first nature disadvantage disappears because there is no trade. More openness through higher α and more equal trade costs through higher τ therefore act as substitutes in the determination of the offsetting subsidy.

We will also need to know how $\tau^{1/3}$ is affected by a marginal change in α for fixed t_n .

Using (12), we have that

$$\partial \ln(\tau^{1/3}) / \partial \alpha = \left(\frac{1}{\alpha}\right)^2 \left[(1-\alpha) \frac{t_n-2}{t_n} w - \frac{1}{\alpha} p + \ln \frac{\alpha}{w} \right] < 0 \quad (13)$$

where we have substituted for equilibrium values for w and p and use the fact that the first and last term in the brackets are non-positive and $-p/\alpha$ is negative. Hence, $\tau^{1/3}$ falls as α is increasing for fixed t_n .

With the solutions at hand for both ‘policy on’ and ‘policy off’, we are in position to assess how the differences in exogenous factors and how the vertical structure condition policy performance.

4. Policy performance

The benefit b of the policy may be measured by the change in the population ratio. Under policy on the ratio is 1 and under policy off it is l , so $b=1-l$. With equal budget shares (the symmetric case), we know that $l = \tau^{1/3}$, hence $b = 1 - \tau^{1/3}$. If we treat τ as exogenous, we observe that if the transport cost asymmetry is increasing (τ goes down) the gap to the target gets wider. This comes as no surprise. Moreover, the gap is entirely due to the transport cost asymmetry since there is no net effect of the vertical industry structure when budget shares are equal. The benefit is therefore entirely a first nature effect and we will take advantage of this when distinguishing between first and second nature in the general case.

We do not pretend to do a full cost benefit analysis of the subsidy, but the fact that more limited cost criteria are often used in practice gives some justification for using the exchequer cost or financial cost (see, e.g., Holden and Swales, 1993), equal to $c = 1 - t_n$. If we express the benefit as a function of t_n under policy on, we have $b = 1 - \tau^{1/3} = 1 - (2t_n - 1) = 2(t_n - 1)$. The benefit cost ratio is therefore equal to $b/c = 2(t_n - 1)/(t_n - 1) = 2$. This means that the gap to the policy target is closed by two percentage points for every percentage point the subsidy is increased. This is the measure of policy performance. We may now turn to the general case with asymmetric budget shares allowing the vertical industry structure to affect the

equilibrium and ask what impact the presence of the second nature effect has on performance. Will a given subsidy close the gap to target more effectively?

In the asymmetric case the benefit is $b = 1 - l = 1 - \left(\tau^{1/3}\right)^{\frac{\alpha}{\alpha - \beta(1 - \alpha)}}$. This can be decomposed into a first nature benefit and a second nature benefit

$$b = \left(1 - \tau^{1/3}\right) + \left(\tau^{1/3} - \left(\tau^{1/3}\right)^{\frac{\alpha}{\alpha - \beta(1 - \alpha)}}\right) \quad (14)$$

The first nature benefit is the gap to be closed whether the vertical linkage has an impact or not. It is caused by the transport cost asymmetry, the first nature difference, between North and South. We know from equation (13) that $\tau^{1/3}$ increases as α falls for fixed t_n , so openness as measured by the share of home made goods and services to imports has an impact on the first nature benefit. A falling α means the ratio of home made to imports becomes larger and this reduce the gap caused by the transport cost asymmetry.

The second nature benefit is the gap that is present only when the vertical linkage has an impact on equilibrium. We know that the second nature effect always is positive for $\alpha < 1$ so second nature makes the benefit or gap between policy on and policy off larger. In order to assess how location of final demand affects the second nature benefit, we need

$$\partial \ln \left(\left(\tau^{1/3} \right)^{\frac{\alpha}{\alpha - \beta(1 - \alpha)}} \right) / \partial \alpha = \frac{\alpha}{\alpha - \beta(1 - \alpha)} \partial \ln \left(\tau^{1/3} \right) / \partial \alpha + \frac{\alpha - \beta(1 - \alpha) - \alpha(1 + \beta)}{(\alpha - \beta(1 - \alpha))^2} \ln \left(\tau^{1/3} \right) \quad (15)$$

The first term is negative, the second positive. Say the first term in the expression for the second nature benefit ($\tau^{1/3}$) falls with x percent when α falls. Then the first term in (15) tells us that the last term in the second nature expression falls with more than x percent since $\alpha / [\alpha - \beta(1 - \alpha)] > 1$, and the last term in (15) just adds to that. Hence, the gap to target caused by second nature gets wider when the home market becomes more important.

Let us now turn to the question of how location of final demand affects the total gap to target or the total benefit. Expanding both terms in (15) using (12) and (13) we get rid of the ambiguity and find that the sign is negative. Hence, the gap to target falls when α falls for fixed t_n . In other words, the gap to the policy target is closed by *less than* two percentage points for every percentage point the subsidy is increased. However, this is due to a smaller first nature gap when the home market becomes more important. As we have seen, the second nature gap gets wider.

Reducing α from unity is making the home market more important at the same time as the vertical linkage comes into play. Hence, two things happen at the same time. Changing β for fixed α is only affecting the second nature effect through the last term, perhaps giving a cleaner design for assessing the effect of second nature. The marginal effect on the last term, keeping α and t_n fixed is given by $\partial \ln \left(\tau^{1/3} \right)^{\frac{\alpha}{\alpha - \beta(1 - \alpha)}} / \partial \beta = \frac{\alpha(1 - \alpha)}{(\alpha - \beta(1 - \alpha))^2} \ln \left(\tau^{1/3} \right) \leq 0$.

Hence, increasing β leads to an increase in the total benefit when $\alpha < 1$, no effect on the first nature benefit and a bigger second nature benefit. We could of course also ask what the effect would be of lowering α and increasing β at the same time compared to the symmetric benchmark case. Since the total effect of the first is to reduce the efficiency below two percentage points, we may wonder if the rise in β would make the total benefit increase sufficiently to boost efficiency beyond the two percentage points. The answer to this obviously depends on the absolute change in the two parameters.

5. Discussion

We have presented a simple ‘new economic geography’ (NEG) model. It is NEG in the sense of being based on a complete microeconomic foundation and allowing a core-periphery structure to emerge endogenously (‘second nature’) in interaction with exogenous geographical asymmetry (‘first nature’). A considerable advantage over most other NEG models is that the model has simple closed form solutions and we do not need numerical simulation. This could make it suitable for rigorously presenting endogenous location in general equilibrium to a wider audience, in particular undergraduate students. Non-neutral space is introduced through a single parameter representing relative distance or transport cost asymmetry, more specifically the ratio of short distance to long distance costs. Neutral space is the limit case as the difference between the two disappears. Economic integration in NEG models is often represented by a uniform reduction in transport costs, but reduction in short distance to long distance transport costs seem in general to be more in accordance with realities (Hummels 1999). In the model this simply means a falling transport cost parameter. Another feature of the model presented, differentiating it from many other NEG models, is

that there are limits to concentration. As argued by Coppella (2007: p. 236), NEG models should be more realistic and allow limits to concentration through the theoretical structure of the model.

Within this model we have done a policy analysis comparing a regional labour subsidy in the first nature periphery region financed through a regional labour tax in the core. The policy target has been to balance the disadvantage caused by transport cost asymmetry in order to obtain equal population in both regions. This policy is contrasted to laissez-faire. Allowing second nature effects, the gap to the policy target, or the policy benefit, for a *fixed* subsidy is increasing when the vertical industry structure in the model becomes stronger. Hence, second nature effects make policy more efficient. We have also seen that an increased share of home made goods makes policy more efficient. Stronger vertical linkages and less dependence on imports therefore seem to be favourable for policy performance. On the other hand, weaker vertical linkages and more openness not only reduce relative performance, but make the gap to the policy target smaller and therefore possibly reduce the incentives for any political interference in the first place.

The model is suitable for analysis of more complex policies in more complex environments. Notably, tax rates may be differentiated between industries within the same region to capture the fact that international agreements are more restrictive to directly subsidizing traded goods than non-traded goods. The policy alternative could then be to abandon the subsidy downstream and maintain it upstream. Another possible extension of interest is to allow for differences in labour mobility between different types of labour and between mobility in the short and the long run. We may even envisage location analysis when labour markets are not clearing, to capture industry wide wage bargaining across different regions.

Appendix A: microeconomic foundations

A.1. Preferences

Consumers have identical Cobb-Douglas preferences regardless of occupation and location of residence.

Everybody supplies one unit of labour, receiving v_j/t_{jA} and w_j/t_{jB} depending on skills. The producer wage rate in sector A upstream is v_j and in the B sector downstream w_j , whereas t_{jA} and t_{jB} are the payroll tax factors.

Individual expenditure systems for A skilled and B skilled are

$$\begin{aligned} p_{ij} y_{ij}^{hA} &= \alpha_{ij} v_j / t_{jA}, \\ p_{ij} y_{ij}^{hB} &= \alpha_{ij} w_j / t_{jB}, \quad i, j = n, s, u. \end{aligned} \tag{A.1}$$

A.2. The downstream economic base sector

The economic base sector in a specific region consists of a large number of firms with identical constant returns to scale technology. Aggregate output is determined by assuming that profits are zero due to free entry and exit.

Skipping indices for region in the rest of the appendix, the unit cost function for a firm is written

$$\ln(c/y) = \beta_w \ln w + \beta_q \ln q, \quad \beta_w + \beta_q = 1. \tag{A.2}$$

Here, w is the wage rate paid by producers, q is a price index of inputs from the non-basic sector, c/y is unit cost, and the Greek letters again parameters. The primal of (A.2) is Cobb-Douglas,

$\ln y = -(\beta_q \ln \beta_q + \beta_w \ln \beta_w) + \beta_w \ln l + \beta_q \ln z$. The price index, q , is defined by

$$q = \left(\sum_k q_k^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad \sigma > 1, \tag{A.3}$$

where q_k is the price paid for input k , and σ is the elasticity of substitution between any pair of inputs. Using

(A.3), I assume a finite number of inputs so large that the integer constraint is not binding. Defining z as a

quantity index of intermediates and z_k as the quantity of input k , the primal of (A.3) is the CES function,

$$z = \left(\sum_k z_k^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}.$$

This technology has several well known attractive properties: a) The cost function is separable in w and q , b) costs decrease when the number of inputs from the non-basic sector increases, and c) no input from the non-

basic sector is essential. Property a) implies that the cost minimising firm may proceed in two steps: First, it may choose how much labour, l , and aggregate input, z , to use conditional on any output level, y . Second, conditional on the optimal level of z , it may choose how much to use of the different inputs from the non-basic sector, z_k . Property b) means that increased specialisation in the non-basic sector rather than subdivision of labour within a single firm, raises productivity.⁴ Property c) implies that the degree of specialisation within any region is endogenous.

Applying Shephard's lemma to the two steps, from (A.2) we obtain the cost shares for l and z ,

$$\begin{aligned} wl / c &= \beta_w \\ qz / c &= \beta_q \end{aligned} \tag{A.4}$$

Hence, the ratio of the wage bill to the cost of intermediates, wl / qz , is constant and equal to β_w / β_q . The larger β_q is, the more important are intermediates for production costs in the basic sector and the stronger is the effect of increased specialisation in the non-basic sector.

From (A.3) we obtain sub cost shares

$$q_k z_k / q = (q_k / q)^{1-\sigma} \quad \forall k. \tag{A.5}$$

We may write (A.5) as $-\ln z_k = \sigma(\ln q_k - \ln q)$ for any k , including $k=s$. Differentiating logarithmically w.r.t. q_s , we obtain the demand elasticity,

$$\varepsilon \equiv \frac{d(-\ln z_s)}{d \ln q_s} = \sigma \left(1 - \frac{q_s z_s}{q} \right). \tag{A.6}$$

When specialisation increases, the sub cost share for input s goes to zero and the demand elasticity is simply equal to the elasticity of substitution.

⁴ This point has perhaps been emphasised most succinctly in the regional context by Nicholas Kaldor (1970, p. 340):

“To explain why certain regions have become highly industrialised, while others have not we must introduce quite different kinds of considerations – what Myrdal (1957) called the principle of ‘circular and cumulative causation’. This is nothing else but the existence of increasing returns to scale – using that term in the broadest sense – in processing activities. These are not just the economies of large-scale production, commonly considered, but the cumulative advantages accruing from the growth of industry itself – the development of skill and know-how; the opportunities for easy communication of ideas and experience; the opportunity of ever-increasing differentiation of processes and of specialisation in human activities. As Allyn Young (1928) pointed out in a famous paper, Adam Smith’s principle of the ‘division of labour’ operates through the constant subdivision of industries, the emergence of new kinds of specialised firms, of steadily increasing differentiation – more than through the expansion in the size of the individual plant or the individual firm.”

A.3. The upstream non-basic sector

The non-basic sector, the A sector, is also assumed to consist of firms with identical technology, but this time increasing returns to scale internal to the firms because of set up costs. The cost function for firm k is written,

$$b_k = (z_k \varsigma_1 + \varsigma_0) \nu \quad (\text{A.7})$$

Here, b_k is total costs and ν is the producer wage rate prevailing in the non-basic sector. The primal to (A.7) is $z_k = (m_k - \varsigma_0) / \varsigma_1$, where m_k is labour input. Marginal cost is $\varsigma_1 \nu$ and the set up cost is $\varsigma_0 \nu$. With internal economies of scale, there must be some kind of imperfect competition to obtain market equilibrium. Following most of the literature in the new economic geography tradition, let us assume that market structure is monopolistic competition. The first order condition for profit maximising is

$$q_k (1 - 1/\varepsilon_k) = \varsigma_1 \nu. \quad (\text{A.8})$$

Assuming specialisation is sufficient to substitute σ for ε_k (cf. eq. (A.6)), the profit maximising price for each differentiated product is equal to a constant mark up over marginal cost,

$$q_k = \frac{\sigma}{\sigma - 1} \varsigma_1 \nu. \quad (\text{A.9})$$

Monopolistic competition implies that profits vanish in equilibrium,

$$q_k z_k - b_k = 0. \quad (\text{A.10})$$

Since there are no profits, only labour input and intermediates are non-tradable, we note that the cost of intermediates for the basic sector is equal to the wage bill for the non-basic sector, $qz = \nu m$. By (A.4),

$$\nu m / \beta_q = \nu l / \beta_w \quad (\text{A.11})$$

as claimed in Appendix B.

Substituting for q_k from (A.9) and b_k from (A.7), we obtain the equilibrium output,

$$z_k = \varsigma_0 (\sigma - 1) / \varsigma_1, \quad (\text{A.12})$$

and labour input,

$$m_k = \varsigma_0 \sigma. \quad (\text{A.13})$$

Full employment means that

$$m = \varsigma_0 \sigma n \quad (\text{A.14})$$

where n is the number of firms. Since B sector productivity rises when the number of intermediate inputs rise and there is internal economies of scale, the number of firms is also equal to the number of products since it is

not profitable for two firms to produce the same product. Using (A.9) and (A.14), we may rewrite (A.3)

logarithmically as

$$\ln q = \frac{1}{1-\sigma} \ln \left(\frac{m}{\zeta_0 \sigma} \right) + \ln \left(\frac{\sigma \zeta_1 v}{\sigma-1} \right) \quad (\text{A.15})$$

In order to simplify, different normalisations are suggested in the literature. We could, e.g., set ζ_0 and ζ_1 in such a way that $\zeta_1 \equiv 1/\zeta_0 - 1 \equiv \sigma - 1$, and write

$$q_k = \sigma v, \quad z_k = 1/\sigma, \quad m_k = 1 \quad (\text{A.16})$$

This means that we can use m for the number of intermediate inputs. Although this kind of normalisations may prove useful for specific purposes, we should be aware that a change in σ implies an automatic change in the cost parameters.⁵

A.4. Mill prices

With free entry, mill prices are just sufficient to cover unit production costs in equilibrium,

$$\ln p = \beta_w \ln w + \beta_q \ln q \quad (\text{A.17})$$

Substituting for $\ln q$ from (A.15), using (A.11), we may express the mill price as a function of the B sector producer wage rate and labour inputs,

$$\ln p = \ln w + \beta_q \ln l + \beta_q \frac{\sigma}{1-\sigma} \ln m + \beta_q \ln z \quad (\text{A.18})$$

where,

$$\ln z = \frac{\sigma}{\sigma-1} \ln \sigma - \ln(\sigma-1) - \frac{1}{1-\sigma} \ln \zeta_0 + \ln \zeta_1 + \ln \left(\frac{\beta_q}{1-\beta_q} \right) \quad (\text{A.19})$$

as claimed in Appendix B. If we impose the normalizations given by (A.16), the expression simplifies to

$$\ln z = \frac{\sigma+1}{\sigma-1} \ln \sigma - \ln(\sigma-1) + \ln \left(\frac{\beta_q}{1-\beta_q} \right) \quad (\text{A.20})$$

⁵ See Neary (2001), p.549, for critical remarks on the use of normalisations in the new economic geography literature.

Appendix B: the macroeconomic model

B.1 The general equilibrium model

The model can be summarised in some simple, intuitive equations. The two equations given by (1) below imply that supply equals domestic and foreign demand for final goods produced in all the regions. Here r_i represents aggregate income in region i , s_{ij} is the aggregate expenditure share for the good produced in region i and consumed in region j , and t_i is the tax factor in region i (if pt is the payroll tax rate, $1+pt$ is defined as the payroll tax factor).

$$r_i(t_i - s_{ii}) = r_j s_{ij} + r_u s_{iu}, \quad i \neq j; i, j = n, s \quad (\text{B.1})$$

The market clearing condition for products made in the outside region u , is not included since adding up implies that only two of the three equations are independent. This is a statement of Walras' law, which basically always is true because of the budget constraints on the market behaviour of each individual. We may also note that imposing market clearing is equivalent to imposing interregional trade equilibrium. There will be balance of payments between the regions, or to paraphrase John Stuart Mill: the produce of a region exchanges for the produce of other regions, at such values as are required in order that the whole of her exports, and net transfers, may exactly pay for the whole of her imports. This is easily seen, by observing that (B.1) can be written as $r_i(1 - s_{ii}) = -r pt_i + r_j s_{ij} + r_u s_{iu}$. The right side is here net transfers plus 'the whole of her exports'. By definition, $r_i = r_i \sum_j s_{ji}$, so $r_i(1 - s_{ii}) = r_i s_{ji} + r_i s_{ui}$ where the right side is 'the whole of her imports', hence equation (B.1) implies that exports net of transfers equal imports.

In order to preserve a constant tax burden when comparing different tax regimes (as we should do according to the widely accepted methodology for comparative studies of tax systems, see, e.g., Hamilton, 1999), without introducing additional taxes in the model, we must have

$$r_n(1 - t_n) = r_s(t_s - 1) \quad (\text{B.2})$$

Subsidies mean that the payroll tax factor is below unity in the assisted region and/or sector, and above unity in the non-assisted area and/or sector. Here, region s is paying for some of the imports to region n through the transfer in form of labour subsidies, so $t_s > 1$ and $t_n < 1$. The tax factor is normalized to unity in region u . The 'policy on' alternative discussed in the main text is obtained by setting

$$t_{nA} = t_{nB} \equiv t_n, \quad t_{sA} = t_{sB} \equiv t_s, \quad t_{uA} = t_{uB} \equiv 1. \quad (\text{B.3})$$

Subscript A and B refer to sector specific factors so the model allows a discussion where the tax rate is differentiated across sectors as well as regions although only regional specificity is discussed in the main text.

An alternative could, e.g., be

$$t_{nB} \neq t_{nA} \equiv t_n, \quad t_{sA} = t_{sB} = t_{nB} \equiv t_s, \quad t_{uA} = t_{uB} \equiv 1. \quad (\text{B.4})$$

This would be a situation where only the upstream sector in the North benefited from subsidies.

Let us now turn to the demand structure of the model. Consider the Cobb-Douglas aggregate demand system,

$$s_{ij} \equiv \frac{p_{ij} y_{ij}}{r_j} = \alpha_{ij}, \quad i, j = n, s, u. \quad (\text{B.5})$$

Greek letters are parameters, p_{ij} is delivered price in region j for the good produced in region i , y_{ij} is quantity of the good. For the expenditure shares to add up, we must impose the restrictions $\sum_i \alpha_{ij} = 1$. To simplify, we assume homogeneous consumers in the sense that everybody is using the same expenditure share on home made, $\alpha_{ii} = \alpha_H, i = n, s, u$, and the same on imports from either source, $\alpha_{ij} = \alpha_F, i \neq j; i, j = n, s, u$. For adding up to hold, this means that $\alpha_H = 1 - 2\alpha_F$. We restrict the discussion to $\alpha_H \geq \alpha_F$. When adding up holds, this means that $\alpha_F \leq 1/3$.

Since labour is the only input and there is zero profit in equilibrium in the markets for intermediates, it is shown in Appendix A that

$$v_i m_i / \beta_q = w_i l_i / \beta_w \quad (\text{B.6})$$

and aggregate nominal income in any region can simply be written,

$$r_i = \frac{w_i l_i}{\beta_w t_i}, \quad i = n, s, u \quad (\text{B.7})$$

The producer wage rate and employment in sector A is v_i and m_i - in the B sector w_i and l_i . The cost share for B skilled labour is β_w , and for intermediates in the production of final goods, β_q . The inverse of the tax factor in any region is the weighted average for the two sectors with β_w and β_q as weights, $1/t_i = \beta_q / t_{iA} + \beta_w / t_{iB}$.

Hence, the right side of (B.7) gives us the income distribution since income earned by A -skilled is equal to $(w_i l_i / \beta_w) \beta_q / t_{iA}$, and by B -skilled, $(w_i l_i / \beta_w) \beta_w / t_{iB}$.

What about the labour markets and mobility? I have assumed that everybody supplies one unit of labour, that they are either A skilled or B skilled and that they cannot be retrained. The number of B skilled people in region u is fixed and equal to \bar{l}_u . So is the number of A skilled in region u , equal to \bar{m}_u . Region n and s share a common pool of potential mobile A skilled and B skilled workers, \bar{m} and \bar{l} . We must have:

$$m_u \leq \bar{m}_u, l_u \leq \bar{l}_u, m_n + m_s \leq \bar{m}, l_n + l_s \leq \bar{l} \quad (\text{B.8})$$

We assume that all labour markets clear so that we have full employment, i.e., all the restrictions given by (B.8) are effective.

When we allow mobility, mobile workers locate wherever the real wages are highest. In full spatial equilibrium, real wages in both sectors in region n and s are equal. There is mobility equilibrium for A skilled labour if

$$\ln(w_n / t_{nA}) + \ln(l_n / m_n) = \ln(w_s / t_s) + \ln(l_s / m_s) + \ln(cpi_n / cpi_s) \quad (\text{B.9})$$

where cpi_j , $j = n, s$, denote the regional consumer price indices.

There is mobility equilibrium for B skilled labour if

$$\ln(w_n / t_{nB}) = \ln(w_s / t_s) + \ln(cpi_n / cpi_s) \quad (\text{B.10})$$

Under the ‘policy on’ regime, (B.10) is the same restriction as (B.9). Hence, one instrument (the tax factor in n) is sufficient to obtain desired levels for the target variables, m_n and l_n .

Three alternative model specifications concerning labour mobility could be considered:

- i Full mobility. Restriction (B.9) and (B.10) are imposed.
- ii Partial mobility (mobility in the A sector only). Restriction (B.9) is still valid, but now (B.8) should be extended, imposing the restrictions $l_n = \bar{l}_n$ and $l_s = \bar{l}_s$.
- iii No mobility. Now neither (B.9) nor (B.10) is valid, and to the list of restrictions under case ii, we must add $m_n = \bar{m}_n$ and $m_s = \bar{m}_s$.

Only the first alternative is discussed in the main text.

The consumer price index of region j is based on the perfect price index consistent with Cobb-Douglas preferences,

$$cpi_j = \prod_i p_{ij}^{s_{ij}} \quad (\text{B.11})$$

Delivered prices are in general different from mill prices because of transport costs. I assume only y_{ij} units arrive when $\tau_{ij} y_{ij}$ units are shipped. Hence, τ_{ij} represent transport costs.⁶ The relationship between delivered prices and mill prices are

$$p_{ij} = \tau_{ij} p_i, \quad i, j = n, s, u \quad (\text{B.12})$$

In order to simplify, we will throughout assume symmetrical transport costs, $\tau_{ij} = \tau_{ji}$, that transport costs between region s and the two other regions are equal and smaller than between n and u , $\tau_L \equiv \tau_{nu} > \tau_{su} = \tau_{ns} \equiv \tau_s$, and ignore domestic distribution costs, $\tau_{ii} = 1$. It is convenient to denote τ_s / τ_L by τ . In order to close the model, we must determine mill prices. The mill prices can be obtained using

$$p_i = w_i \left(l_i m_i^{\sigma/(1-\sigma)} z \right)^{\beta_q}, \quad i = n, s, u \quad (\text{B.13})$$

where the factor z depends on a vector of parameters shared by all regions (on the assumption of identical technology). The vector of parameters consist of the cost share for intermediates (β_q), the elasticity of substitution between any variants of intermediates (σ), and the marginal (ζ_1) and fixed (ζ_0) cost per money

⁶ This is the approach favoured in the new economic geography literature in order to avoid introducing a separate transport sector in the models. Many transport economists would probably not be happy to learn that their subject has been swept under the carpet. At this point there is a large potential to enrich the model. Transport costs depend on more than distance: transport infrastructure quality, substitution possibilities between different transport modes, input prices, logistics and thickness of transport markets. Unfortunately, the results of the simpler model may not survive alternative specifications of transport technology. Neary (2001) refers to older trade theory literature attempting to model an explicit transportation sector: “That approach never led to simple or easily summarizable results, and is now largely forgotten. Perhaps it is due a revival.” (p.550)

wage unit in the production of intermediates (the notation refers to the notation used in the Appendix A where the details leading to (B.13) are found). Alternatively, accepting the normalization given by (A.16), the factor z depends on only two parameters: β_q and σ .

B.2 A six-equation summary

Expressed in terms of the basic sector producer wage rate, the model presented in the previous Section may be summarized by the following equation system,

$$\frac{w_n l_n}{\beta_w} \left[1 - \alpha_H \left(\frac{\beta_q}{t_{nA}} + \frac{\beta_w}{t_{nB}} \right) \right] = \frac{w_s l_s \alpha_F}{\beta_w t_s} + r_u \alpha_F \quad (\text{B.14})$$

$$\frac{w_s l_s}{\beta_w t_s} (t_s - \alpha_H) = \frac{w_n l_n \alpha_F}{\beta_w} \left(\frac{\beta_q}{t_{nA}} + \frac{\beta_w}{t_{nB}} \right) + r_u \alpha_F \quad (\text{B.15})$$

$$w_n l_n \left[\left(\frac{\beta_q}{t_{nA}} + \frac{\beta_w}{t_{nB}} \right) - 1 \right] = \frac{w_s l_s}{t_s} (t_s - 1) \quad (\text{B.16})$$

$$\frac{w_n l_n}{m_n t_{nA}} = \frac{w_s l_s}{m_s t_s} \frac{cpi_n}{cpi_s} \quad (\text{B.17})$$

$$\frac{w_n}{t_{nB}} = \frac{w_s}{t_s} \frac{cpi_n}{cpi_s} \quad (\text{B.18})$$

$$\frac{cpi_n}{cpi_s} = \left(\frac{1}{\tau} \right)^{\alpha_F} \left(\frac{w_n}{w_s} \left(\frac{l_n}{l_s} \right)^{\beta_q} \left(\frac{m_n}{m_s} \right)^{\sigma \beta_q / (1-\sigma)} \right)^{\alpha_H - \alpha_F} \quad (\text{B.19})$$

Equation (B.14) and (B.15) are the market clearing conditions for products made in region n and s, corresponding to (B.1). (B.16) corresponds to (B.2), the restriction that tax revenues should be kept constant. (B.17) and (B.18) are the mobility equilibrium conditions, corresponding to (B.9) and (B.10) (to go from (B.9) to (B.17), use (A.11) in Appendix A). Equation (B.19) is obtained by plain substitution.

Under the ‘policy on’ regime discussed in the main text we have that $t_{nA} = t_{nB} = t_n$ and the policy instrument, t_n , is used to obtain a desired level for employment in the basic sector in the periphery, say the symmetric distribution $l_n = l_s = \bar{l} / 2$. Then, for (B.17) and (B.18) to hold, employment in the non basic sector must be given by $m_n = m_s = \bar{m} / 2$, and total population equally distributed, so we do indeed have a symmetrical outcome. With symmetry imposed, (B.17) is identical to (B.18), so we may ignore (B.17). If we concentrate on the relative producer wage rate between n and s, we may use (B.14) and (B.15) to cancel out the term related to

region u . We are then left with the four equations presented as the model (5)-(8) in the main text (with some simplified notation).

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