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Assessment of the Remaining Carbon Budget: A Comparison of a Simple Response Model and the MAGICC Model.

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“Difficult to see. Always in motion is the future.”
–Yoda

Abstract

We are changing the global climate by altering the Earths energy balance through the emission of greenhouse gasses. The international community aims to prevent dangerous warming with mitigative efforts. A remaining carbon budget (RCB) can roughly quantify an allowable amount of emissions for keeping the temperature below a set target. We have built a simple climate model using impulse response functions and parameterizations of the forcing of atmospheric radiatively active agents. The Simple Response Model (SRM) emulates CMIP5 ensemble models over several SSP emission scenarios. We deliver compelling visualization of the risk of warming associated with carbon budgets using probability density functions around TCREs. The risk of warming and uncertainty in the carbon budget increase with less ambitious targets. By comparing the SRM with the Model for the Assessment of Greenhouse Gas Induced Climate Change (MAGICC) model, we verify the SRMs results.

Results from incorporating regional Arctic amplification show more substantial uncertainties and more damaging temperature responses. We added possible nonlinear effects into the model framework, proving it possible under reasonable levels of additional forcing. The linearity of the TCRE falls apart for strongly nonlinear Earth system feedbacks.

Acknowledgements

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1

Introduction

This paper is based on the research project I have been involved in the last year of my master's programme. Me and two fellow students are involved in the research project lead by our supervisor, Martin Rypdal. My two fellow students, Andreas Johansen and Endre Falck Mentzoni, are also writing their theses based on our collective effort in the research project. In the project, we have built a climate model in the form of a simple response model (SRM), based on impulse response functions. The SRM can estimate carbon budgets and assess the risk of climate change, represented by the temperature response associated with the budget size. Endre Falck Mentzoni worked on implementing an additional, temperature-dependent, forcing term to the model framework, making it possible to research the impact of nonlinear effects on the model output. Andreas Johansen worked on translating the global temperature response from the model into a regional temperature response in the Arctic region, giving more insight into possible local challenges related to different sizes of carbon budgets.

Human activity is changing the energy balance and climate of the Earth through altering the concentrations of radiative gasses such as carbon dioxide (CO_2) and methane (CH_4). Advances in measurement systems, such as satellites, have increased observations of the climate systems tremendously. More sophisticated models describe the climate system better, giving better estimates of uncertainties in climate projections (Cubasch et al., 2013)

The international community has agreed through the Paris agreement to pursue efforts to limit global temperature increase to 1.5°C above pre-industrial levels and to hold warming levels well below 2°C. How do we keep track of our progress towards such goals? An emission budget quantifies the amount of greenhouse gasses that can be emitted and still stay within a specified temperature goal. After the Paris agreement, the The Intergovernmental Panel on Climate Change (IPCC) made a special report on the goal of limiting global temperature increase to 1.5°C (SR15). The report states an estimated remaining carbon budget (RCB), using global mean surface temperature (GMST), of 770 gigatonnes (Gt) of CO_2 for a 50% probability of limiting warming to 1.5°C (Hoegh-Guldberg et al., 2018). Our SRM-approach uses emulations of climate models in the CMIP5 ensemble and emission scenarios called Shared Socioeconomic Pathways (SSPs). We compute the probability of different maxim temperatures for given sizes of the RCB.

In this thesis I describe the Model for the Assessment of Greenhouse Gas Induced Climate Change (MAGICC) in detail, and the temperature responses from the SRM and MAGICC models are compared to validate the responses of the SRM. The results for the RCBs and associated risks of maximum temperature increase estimated from the SRM. The comparison is also carried out with additional temperature dependent forcing terms in the SRM (see Mentzoni (2020)) and for the regional temperature response in the Arctic when including Arctic temperature Amplification (see Johansen (2020)).

2

Background Theory

2.1 Climate forcing and feedback

Climate forcings are external factors driving the climate system. There are natural forcings, such as changes in the orbit of the Earth around the sun or volcanic eruptions, and anthropogenic, i.e., human-induced, forcings. The most notable of the anthropogenic forcings are the changes in atmosphere's composition produced by the emission of greenhouse gasses such as carbon dioxide (CO_2) or aerosols (Kaper and Engler, 2013).

More specifically, forcing is an energy imbalance in the climate system, imposed either naturally or anthropogenically. Nonradiative forcing creates an energy imbalance that does not involve radiation. Such forcing results in a redistribution of energy and does not directly affect incoming and outgoing radiation. Even though the radiative forcing from well-mixed greenhouse gasses is well known, the effect and implementation of nonradiative forcings are one of the big challenges/ uncertainties in climate models today.

One of the strengths of the forcing framework is the nearly linear relationship between the top of atmosphere radiative forcing and the equilibrium response of the GMST, for a wide range of forcings. A considerable uncertainty associated with forcing is the interplay between regional and global effects; how regional forcing agents may alter the global climate response and vice versa.

A climate feedback is a process internally in the climate system that strengthens or weakens the climate response to forcing. Positive feedbacks will strengthen the climate response to the forcing, and negative feedbacks will weaken it. An example is the ice-albedo feedback loop caused by warming: warming leads to an increase in the melting of sea ice. Less sea ice gives a darker planet surface which absorbs more sunlight than a brighter surface, leading to warming, and melting more sea ice. Since the melting of sea ice is intensified through the loop, it is a positive feedback process (Council et al., 2005).

2.2 Tipping points and bifurcation in the climate system

Potentially, there are points or thresholds in the climate system, that if exceeded, may lead to significant, and often irreversible, changes in the system. Tipping points represent an essential risk from global warming (Hoegh-Guldberg et al., 2018). These thresholds are often closely associated with bifurcation points, where a small, smooth variation of a parameter, leads to a qualitative change in the system. These qualitative changes can be irreversible, or at least, it can be challenging to return to the initial state before (Kaper and Engler, 2013). Such a change can be a sudden transition into an ice-free arctic or sudden changes in ocean currents due to changes in ocean salinity.

2.3 Climate models

In Kaper and Engler (2013) it is stated that as mathematicians we take the differential equations and apply the tools of the trade to extract information about the behaviour of the physical pendulum or, in our case, physical processes of climate change. Climate scientists are assessing assumptions and mathematicians extracting information: selecting variables that describe the state of the climate system, describing their evolution, and translating science into mathematical language. When the mathematical description gets too complex, one can use numerical simulations with supercomputers, a common approach in the climate science community. An alternative is the (complex) system-level approach. If this system-level approach is successful, it is possible to derive simpler conceptual models to describe the behaviour of the system, whereas the numerical-simulation approach is more concenred with the details of the processes (Kaper and Engler, 2013).

2.3.1 Conceptual models

A conceptual approach to Earth's climate system can be to view it as a heat engine driven by the sun. The system stays in equilibrium by radiating out the same amount of energy as it receives from the sun. Imbalances in energy in will lead to cooling or heating of the climate system. A simple energy balance model is presented in the following equation:

$$C\frac{dT}{dt} = E_{\text{in}} - E_{\text{out}}\,, \tag{2.1}$$

where C is the heat capacity (W yr m^{-2}) of the system, T is temperature, and E is energy (W m^{-2}).

An energy balance model is one of the simplest examples of a conceptual climate model. It can be as simple as viewing Earth as a homogeneous sphere that can absorb and emit energy. More elements such as varying heat capacity and albedo can be implemented to describe the processes better. However, it still relies on the premises of energy in and out, and the relation to the temperature of the system. A model like this is not be able to tell us anything about how deforestation may alter the local temperature in China. However, it can give some information about the qualitative behaviour of the climate system.(Kaper and Engler, 2013)

2.3.2 Simple models

Intermediate or low complexity models are useful complementary tools to the more sophisticated models. These models can describe individual processes and we can evaluate multiple scenarios. Simple models are great for emulating sophisticated models and reproducing coarse-grained (in space or time) results of such models. The computational cost of sophisticated models is substantial, and we cannot run large ensembles. This limits their ability to produce probabilistic studies where one evaluates large sets of emission scenarios (Meinshausen et al., 2011).

2.3.3 Integrated Assessment Models (IAMs)

Integrated assessment models (IAMs) combine models from different disciplines into a combined assessment. For instance climate models and energy-economics models. They provide a useful framework for constructing mitigation scenarios. In principle, IAMs can be used for finding the most cost-efficient way to provide the needed energy to keep economic growth while still limiting global warming to a specific target. IAMs play an essential role in the IPCC assessment reports, pointing to feasible goals for climate mitigation in the context of energy economics. They are used to build the RCP scenarios (Hare et al., 2018).

Shared Socioeconomic Pathways (SSPs)

SSPs are scenarios based on a combination of climate model projections, socioeconomic conditions, and assumptions about climate policies. They can illustrate the mitigation efforts needed to reach specific climate targets and characterize the adaptive measures needed in response to climate change.

Climate targets are crucial factors in SSPs. In the SSPs, climate outcomes are based on Representative Concentration Pathways (RCPs) and by the climate model projections based on them. These model projections are part of the Coupled Model Intercomparison Project 5 (CMIP5) (O'Neill et al., 2014).

2.3.4 Box models

Box models describe the temperature and heat exchange between coarse climate components in a simplified manner. A simple box model can contain an ocean box, a land box and an atmosphere box. The internal climate dynamics is often modelled as a stochastic process, and important physical processes remain unresolved. For example, if we model the north Atlantic ocean as one box, the model cannot describe ocean currents within the box, only movements between boxes (Goosse et al., 2010).

The response of climate variables or greenhouse gasses, such as CO_2, to an emission pulse, are often used to build reduced-form climate models or box models as they can capture the behaviour of more complex models.

2.3.5 General Circulation Models (GCM)

General Circulation Models (GSMs) are complex climate models, but do not describe biochemical feedbacks. They model the atmosphere, land, ocean and sea ice to understand the dynamics (Flato et al., 2014).

The Lorenz equations are an excellent example of a GCM in a very elementary form. Edward N. Lorenz modelled Earth's atmosphere to investigate long-term weather forecasting. He based the model on heat conduction in an incompressible fluid situated between to infinitely wide horizontal planes, heated from below. From the hydrodynamic equations, Lorenz found a set of nonlinear autonomous differential equations, shown in Equation2.2

$$
\begin{aligned}
\dot{x} &= -\sigma x + \sigma y \\
\dot{y} &= \rho x - y - xz \\
\dot{z} &= -\beta z + xy
\end{aligned}
\tag{2.2}
$$

Here x, y and z represent variables of state, representing, respectively, the spatial average of the hydrodynamic velocity, temperature and temperature gradient (Kaper and Engler, 2013).

This is a GCM on a very conceptual level which is more useful for gaining insight into the dynamics of the system rather than using it to predict actual changes in weather regimes.

2.3.6 Earth System Models (ESM)

Earth System Models (ESMs) have become extremely complex, and they demand state of the art high-performance computing resources. ESMs are generally expansions of AOGCMs including biogeochemical cycles, e.g. the carbon cycle. ESMs are the most comprehensive tools for simulating historical and future climate responses to external forcing.

An ESM of Intermediate Complexity (EMIC) is a type of ESM which often include relevant components of the Earth system but in a more idealized manner. By idealizing components, they can include some components not yet included in ESMs. For instance, ice sheets can be implemented in EMICs. This idealized setup is suitable for running experiments on long timescales, and can be used to improve understanding of climate feedbacks on millennial timescales (Flato et al., 2014).

2.4 Solving box models

This section goes through the solution of the Simple Response Model, using the standard approach with Greens functions. Starting from a one-box model on the form:

$$C\frac{dT}{dt} = -\lambda T + f(t) \tag{2.3}$$

Defining a differential operator $\mathcal{L}$, Equation 2.3 can be rewritten as

$$\mathcal{L}T = f(t)\,, \tag{2.4}$$

where

$$\mathcal{L} = C\frac{d}{dt} + \lambda\,.$$

The definition means that

$$(\mathcal{L}T) = C\frac{dT}{dt} + \lambda T\,.$$

The differential operator can easily be generalized to all N-box models. The important part is its linearity. The climate models used in our SRM is on the form of Equation 2.4.

Equation 2.4 can be solved, using a Greens function. We find a function $\tilde{G}(t)$, so that:

$$\left(\mathcal{L}\tilde{G}\right)(t) = \delta(t)\,,$$

because then

$$\int_0^\infty \tilde{G}(t-s)f(s)ds$$

is a solution of Equation 2.4. The proof is as follows:

$$\begin{aligned}
(\mathcal{L}T)(t) &= \mathcal{L}\int_0^\infty \tilde{G}(t-s)f(s)ds \\
&= \int_0^\infty \left(\mathcal{L}\tilde{G}\right)(t-s)f(s)ds \\
&= \int_0^\infty \delta(t-s)f(s)ds \\
&= f(t)
\end{aligned}$$

Explicit formulas for the Green's function can be found by taking Fourier transforms.

2.4.1 Addition of temperature dependent forcing

This section explains the solution of the N-box model with an added temperature dependent forcing term to incorporate nonlinear effects:

$$\mathcal{L}T = f(t) + F(T) \tag{2.5}$$

Equation 2.5 is solved by:

$$T(t) = \int_0^\infty \tilde{G}(t-s)\left[f(s) + F(T(s))\right]ds$$

Proof:

$$\begin{aligned}(\mathcal{L}T)\,(t) &= \int_0^\infty \left[\mathcal{L}\tilde{G}(t-s)\right] [f(s)+F(T(s))]\,ds \\ &= \int_0^\infty \delta(t-s)\,[f(s)+F(T(s))]\,ds \\ &= f(t)+F(T(t))\end{aligned}$$

Define an integral operator:

$$(\mathcal{R}T)(t) = \int_0^\infty \tilde{G}(t-s)\,[f(s)+F(T(s))]\,ds.$$

Equation 2.5 is now equivalent to:

$$\mathcal{R}T = T. \tag{2.6}$$

Equation 2.6 is solved by iteration

$$T_{n+1} = \mathcal{R}(T_n).$$

The convergence of the iteration can be proven analytically through the "contracting mapping principle", this is a sufficient condition for convergence but not a requirement. The convergence is verified numerically in this work. Note that the temperature-dependent forcing acts as a nonlinear forcing in the case where $F(T)$ is a nonlinear function. If $F(T)$ is a linear function, on the form aX+b, the forcing effect is still linear.

2.5 Remaining Carbon Budget (RCB) and the Trancient Climate Response to Cumulative Emissions (TCRE)

Rogelj et al. (2019) define the carbon budget as the finite total amount of CO2 that can be emitted into the atmosphere by human activities while still holding global warming to a desired temperature limit.

One of the building blocks, and arguably the most important, in this simple framework is the Transient Climate Response to Cumulative Emissions of CO2 (TCRE). The TCRE is the approximate linear relationship between cumulative anthropogenic emissions and an increase in temperature, shown in Figure 2.1. It is this trend that allows us to estimate how much we can emit concerning set goals for human-induced climate change Rogelj et al. (2019).

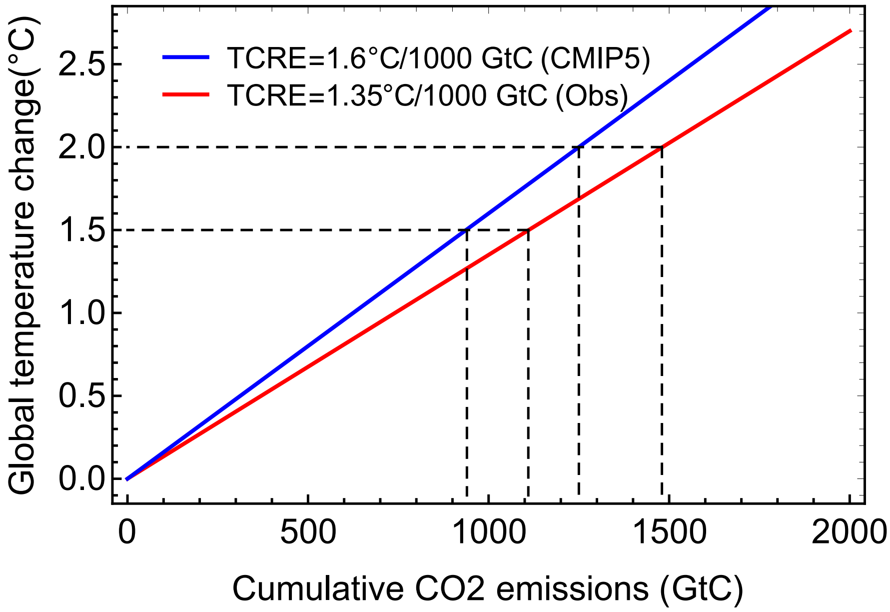

Figure 2.1: Transient Climate Response to Cumulative Emissions of CO_2 (TCRE)(°C / GtC) reproduced from Matthews et al. (2017). The figure illustrates the relationship between Cumulative CO_2 Emissions (GtC) and Global temperature change (°C). TCRE from the CMIP5 ensemble (blue) and TCRE estimated from observations (red).

The slope of the TCRE is essential as it directly affects the carbon budget. Different approaches to estimating TCRE can lead to different values. In Figure 2.1, two different slopes are shown. One derived from models in the CMIP5 ensemble, used in the IPCC's Fifth Assessment Report (AR5) (Myhre et al., 2013). The red line is from observational data.

It is worth mentioning that the TCRE is not a law of nature. It is more of an emergent relationship. The TCRE, with a probability of 66%, lies between 0.8 and 2.5 °C pr. 1000 PgC, valid for cumulative emissions lower than 2000 PgC. The range of validity for the linearity is below 2000 PgC. A reasonable level of path independence and a monotonously increasing carbon emissions scenario are also important conditions for using the TCRE in a carbon-budget setting. By assuming some probability distribution around the TCRE, it is possible to derive probabilities of warming for carbon budgets. (Rogelj et al., 2016)

Unfortunately, the more robust carbon-budget definitions are often the least useful for policymaking. An example is a budget for CO_2-induced warming only. It is the most robust translation of TCRE into a carbon budget; it does not account for non-CO_2 forcing, which also cause warming. Using multi-gas emission scenarios combined with the TCRE-based approach can better account for non-CO_2 forcing (Rogelj et al., 2016).

2.5.1 Non-CO_2 emissions

The strictly anthropogenic emissions of CO_2 are, of course, not the only factor influencing the climate system and the temperature increase. The increase in temperature due to human activities may cause other parts of the climate system to respond, adding more greenhouse gasses and altering the initial temperature response. Thawing of permafrost due to an increase in temperature is such a process. Including processes like thawing of permafrost will also alter the carbon budget.

There is also the warming due to other greenhouse gasses than CO_2 and aerosols. These factors also have the potential for warming and should, in some way, be considered in the carbon budget (Rogelj et al., 2019). As more scenarios are considered, and greater differences in carbon budgets emerge, it is vital to have a robust methodological framework. Such a framework is proposed by Rogelj et al. (2019), Shown in Equation 2.7:

$$B_{\text{lim}} = \frac{T_{\text{lim}} - T_{\text{hist}} - T_{\text{nonCO2}} - T_{\text{ZEC}}}{TCRE - E_{\text{esfb}}}\,. \tag{2.7}$$

Here B_{lim} is the RCB, T_{lim} is the temperature limit, T_{hist} is the historical human induced warming to date, T_{nonCO2} is the expected future warming contributed by non-CO_2, $TZEC$ is the zero-emissions commitment and E_{esfb} is the adjustment due to sources of unrepresented Earth System Feedback processes (Rogelj et al., 2019).

The term E_{esfb} is of extra interests as CO_2 and methane released from the thawing of permafrost as well as methane from wetlands are possible tipping points and sources of nonlinearities in the climate system. The thawing of permafrost will, in this framework, fall under the category of Earth system feedbacks.

2.5.2 Emissions pathways

In an experiment by Gasser et al. (2018) a carbon model (Oscar) of intermediate complexity is used to run different emission scenarios and estimate carbon budgets with and without including the permafrost feedback. The type of scenarios explored are: Exceedance-, avoidance- and capture budgets. Figure2.2

shows the different scenario types.

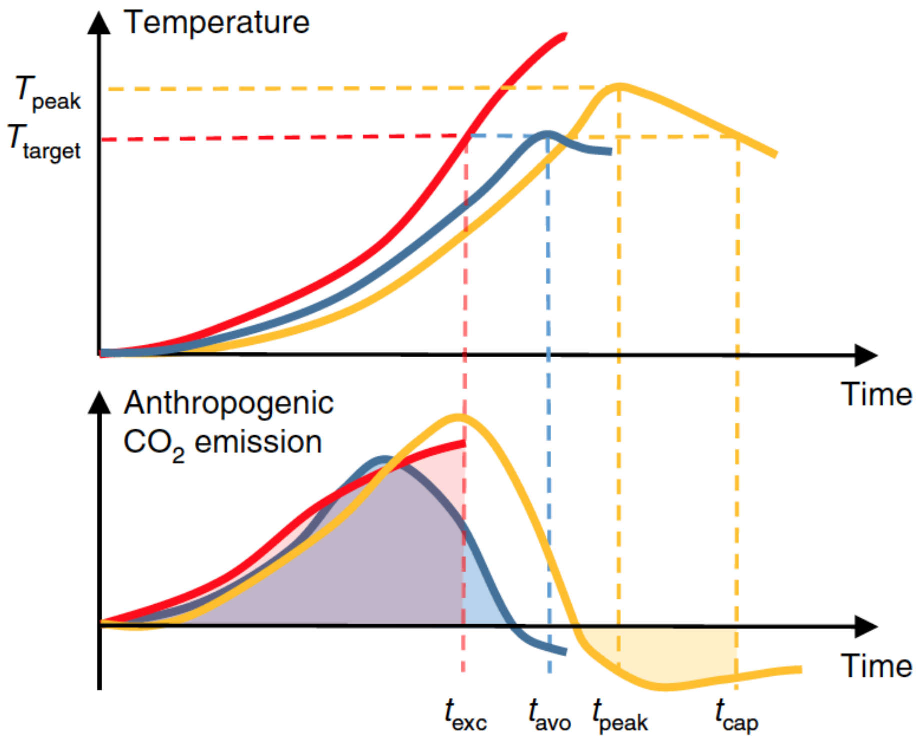

Figure 2.2: Illustration of the three main types of emission scenarios. Exceedance budgets are in red, avoidance budgets in blue and capture budgets in yellow. The upper figure shows the temperature and the lower shows anthropogenic emissions of CO2 (Gasser et al., 2018).

The exceedance budget quantifies the amount of CO2 that we can emit before exceeding a temperature limit. These budgets are, however, a bit imprecise, as they do not account for the system's dynamics after the system reaches the temperature target. The inability to account for the lag in the climate system when assessing the carbon budget is a weakness. It is also worth mentioning that in the AR5, exceedance budgets are the only type assessed using complex models (Gasser et al., 2018).

For the Paris agreement, avoidance carbon budgets are the most relevant. Avoidance budgets quantify the amount of CO2 that can be emitted and still stay below a given temperature limit. The capture budgets quantifies the removal of CO2 needed in case of an overshoot in temperature. Combining avoidance and capture budgets gives the net overshoot budget. There are two subgroups of avoidance budgets: No net negative emissions (NetNegEmo), and scenarios where net negative emissions are extensively implemented (NetNegEm+). A substantial drawback of the avoidance budgets (and overshoot) is its computation cost. This computational cost is especially problematic when using high complexity models (Gasser et al., 2018). Low computational cost is also one of the strengths of simpler models, where, if well-implemented, these types of scenarios, pose no problems regarding requirements to computational power or computation time.

2.5.3 Mitigation and adaptation

How extensively and fast we carry out climate mitigation will affect prospects for climate-resilient pathways in the future. Our ability to take advantage of synergies between mitigation and adaptation may also decrease over time. Extensive mitigation may come with some risk, but these risks are not as severe or irreversible as possible risks from climate change. Risks from mitigation could be adverse side effects of large-scale deployment of low-carbon technology options. There is also the economic risk of both adaptation and mitigation. The timescale of risks is also different, as the timescale of climate change-related risks could be millennia.

Adaptation is more limited in its use than mitigation. It can reduce the risk of severe climate impacts, but severe climate change limit the potential of adaptive measures (IPCC and Team, 2014).

3

MAGICC

This chapter is in its entirety based on the article from Meinshausen et al. (2011) that describes the MAGICC climate model. The intention is to compare and illustrate the differences between the SRM and MAGICC, giving more insight into the construction of a climate model.

MAGICC or "Model for the Assessment of Greenhouse Induced Climate Change" is a low complexity climate model. A low complexity model gives excellent flexibility in emulating the behaviour of more complex models. MAGICC will be compared with the SRM to validate the SRM's results. The motivation for comparing our SRM with MAGICC is that MAGICC is extensively used in carbon-budget assessments.

The modelling flow in MAGICC is going from emissions to concentrations, from concentrations to forcings, and from forcing to temperature. The emissions, lifetimes and interaction between species governs the concentration of GHGs, tropospheric ozone and aerosols. Radiative forcing is estimated from different parameterizations, accounting for processes like saturation and interplay between species. The upwelling-diffusion climate model estimates the global or hemispherically averaged temperature.

3.1 Terrestrial carbon cycle

MAGICC uses a three-box system to model the terrestrial carbon cycle. A living-plants box, a detritus box and a soil box. These boxes are all exchanging carbon between each other and the atmosphere. The net primary production (NPP) of the terrestrial carbon cycle is simulated.

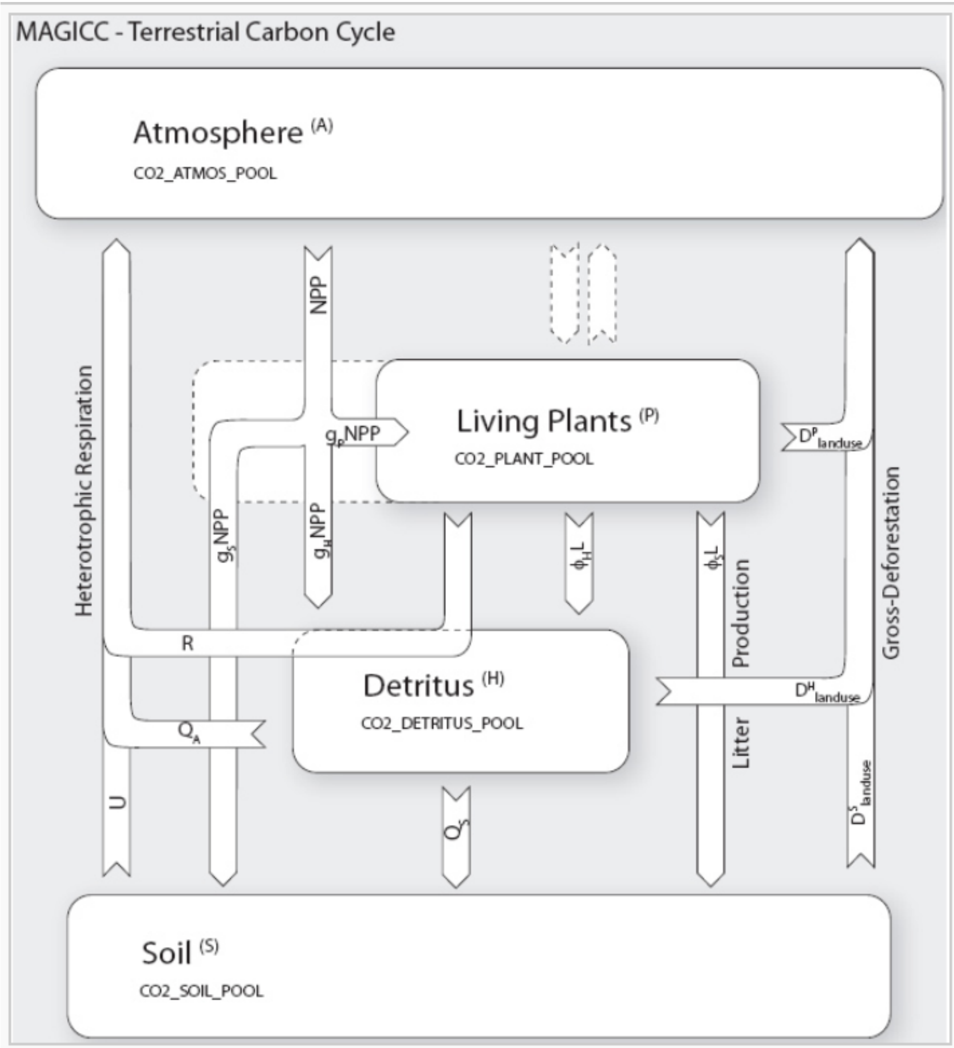

Figure 3.1: Schematic representation of the terrestrial carbon cycle in MAGICC, showing carbon pools (boxes) and fluxes (Meinshausen et al., 2011).

3.1.1 Living plants-box, mass balance

The mass balance of the Living plants-box is expressed in 3.1:

$$\Delta P/\Delta t = g_P \mathrm{NPP} - R - L - D^P_{\mathrm{gross}}, \tag{3.1}$$

where L is litter production, D is part of gross deforestation and R is heterotrophic respiration.

3.1.2 Detritus box, mass balance

The mass balance of the detritus box is expressed in 3.2:

$$\Delta H/\Delta t = g_H \mathrm{NPP} + \phi_H L - Q_{\mathrm{A}} - Q_{\mathrm{S}} - D^H_{\mathrm{lu}}. \tag{3.2}$$

Here $\phi_H L$ is litter production, Q_{A} is non-land use related oxidation, Q_{S} is the sink to the Soil box and D^H_{lu} is a sink to the atmosphere due to land use.

3.1.3 Soil box, mass balance

The mass balance of the Soil box is expressed in 3.3:

$$\Delta S/\Delta t = g_S \mathrm{NPP} + \phi_S L + Q_{\mathrm{S}} - U - D^S_{\mathrm{lu}}, \tag{3.3}$$

where $\phi_S L$ is a source from litter production, Q_{S} is gain from the detritus box, U is non-land use related oxidation, and D^S_{lu} is the loss to the atmosphere due to land use.

The decay rates (L, Q and U) are assumed proportional to the respective pool's box masses (P, H and S). Turnover times vary depending on initial steady-state conditions for box sizes and fluxes.

3.1.4 Constant relaxation times, regrowth terms

If perturbed by carbon release or uptake due to a one-off change in land use, the boxes will relax back to their original state, as in a regrowth term. No changes in fertilization nor any temperature feedback terms are assumed.

Deforestation ΣD_{gross}=D^P_{gross} + D^H_{gross} + D^S_{gross} represents the gross land use emissions, related to net land use emissions E_{lu} by regrowth ΣG=G^P + G^H + G^S

$$\begin{aligned} \Sigma D_{\text{gross}} - \Sigma \mathrm{G} &= E_{\text{lu}} \\ D^P_{\text{gross}} - \mathrm{G}^P &= d_P E_{\text{lu}} \\ D^H_{\text{gross}} - \mathrm{G}^H &= d_H E_{\text{lu}} \\ D^S_{\text{gross}} - \mathrm{G}^S &= d_S E_{\text{lu}} \,. \end{aligned} \tag{3.4}$$

Human activity may lead to persistent changes in the cycle due to land-use activities, and hence affecting gross land-use related emissions, leading to the system not relaxing back to its initial states P_0, H_0 or S_0. ψ. Note that (0≤ψ≤1) denotes the part of gross deforestation that does not regrow, giving the time-dependent relaxation times in Equation3.5:

$$\begin{aligned} \tau^P(t) &= \left(P_0 - \psi \int_0^t d_P E_{\text{lu}}(t')dt'\right)/L_0 \\ \tau^H(t) &= \left(H_0 - \psi \int_0^t d_H E_{\text{lu}}(t')dt'\right)/Q_0 \,. \\ \tau^S(t) &= \left(S_0 - \psi \int_0^t d_S E_{\text{lu}}(t')dt'\right)/U_0 \end{aligned} \tag{3.5}$$

3.2 Formulation for CO2 fertilization

The enhancement in NPP due to elevation in CO2 is referred to as CO2 fertilization. MAGICC models the CO2 fertilization in two ways, as well as using a combination of the two.

3.2.1 Logarithmic form and rectangular hyperbolic or sigmoidal growth function

The logarithmic form indicates a fertilization parameter, $\beta_{\text{m}} = 1$ and $\beta_{\text{m}} = 2$ for the rectangular hyperbolic or sigmoidal growth function (From (Gates, 1985)). The latest version of MAGICC allows for a combination of the two variations to be used, where ($1 \leq \beta_{\text{m}} \leq 2$).

The logarithmic form calculates enhancement in NPP as proportional to the logarithmic change in CO2 concentration, C, above preindustrial level C_0:

$$\beta_{\log} = 1 + \beta_s \ln\left(\mathrm{C}/\mathrm{C}_0\right)$$

The rectangular hyperbolic parameterization is given by:

$$N = \frac{C - C_\mathrm{b}}{1 + b(C - C_\mathrm{b})} = \frac{N_0(1 + b(C_0 - C_\mathrm{b}))(C - C_\mathrm{b})}{(C_0 - C_\mathrm{b})(1 + b(C - C_\mathrm{b}))}, \qquad (3.6)$$

where N_0 is the net primary production and C_0 the CO2 concentrations at pre-industrial conditions, C_b the concentration value at which NPP is zero (default setting: $C_\mathrm{b} = 31$ ppm (Method from Gifford,1993)

The NPP enhancement due to a CO2 increase from 340ppm to 680ppm is expressed by the CO2 fertilization factor β_s. It is valid under both formulations. Thus, MAGICC first determines the NPP ratio r for a given β_s fertilization factor according to:

$$r = \frac{N(680)}{N(340)} = \frac{N_0(1 + \beta_s \ln(680/C_0))}{N_0(1 + \beta_s \ln(340/C_0))}$$

Following from here, b is determined by

$$b = \frac{(680 - C_\mathrm{b}) - r(340 - C_\mathrm{b})}{(r - 1)(680 - C_\mathrm{b})(340 - C_\mathrm{b})}$$

which can in turn be used in Equation3.6 to calculate the effective CO2 fertilization factor β_sig at time t as

$$\beta_\mathrm{sig}(t) = \frac{1/(C_0 - C_\mathrm{b}) + b}{1/(C(t) - C_\mathrm{b}) + b}$$

In MAGICC6 any linear combination of the two fertilization parameterizations can be chosen ($1 \leq \beta_m \leq 2$). This gives added flexibility. The effective parameterization factor β_eff is given by:

$$\beta_\mathrm{eff}(t) = (2 - \beta_m)\beta_\mathrm{log} + (\beta_m - 1)\beta_\mathrm{sig}$$

NPP is affected by the CO2 fertilization effect so that $\beta_\mathrm{eff} = NPP - NPP0$. The fertilization factor is applied by the terrestrial carbon cycle to one of the heterotrophic respiration fluxes R that cycles through the detritus box, which makes up 18.5 % of the total heterotrophic respiration ($\sum$ R$=R + U_a + Q$) at the initial steady-state.

Note: The methodology behind making these natural processes follow predetermined functions is very similar to the implementation of a nonlinear forcing factor in the framework of the SRM. However, the implications and results are very different.

3.3 Temperature effect on respiration and decomposition

The modelled carbon cycle feedbacks use the GMST as a proxy for carbon cycle relevant temperatures.

The terrestrial carbon fluxes NPP, and the heterotrophic respiration/decomposition fluxes R, Q and U are scaled assuming an exponential relationship,

$$F_i(t) = F_i'(t) \cdot \exp(\sigma_i \Delta T(t))$$

where $\Delta T(t)$ is the temperature relative to a reference year, e.g. for 1990 or 1900, and F_i' (F_i) denotes the (feedback-adjusted) fluxes NPP, R, Q and U. The parameters σ_i (K^{-1}) are their respective sensitivities to temperature changes.

3.4 Ocean carbon cycle

An efficient impulse response function describes the perturbation in ocean surface dissolved organic carbon.

The sea-to-air flux, F_{ocn} is given by:

$$\mathrm{F}_{\text{ocn}} = k(C - \rho\mathrm{CO}_2) \tag{3.7}$$

Here C is the partial pressure for CO2 in the atmosphere and ρCO2 is the pressure at the surface layer of the ocean. k is the global average gas exchange coefficient (Joos et al., 2001)

This framework of using impulse response functions for modelling CO2 concentration is also the basis for the carbon model in the SRM, where impulse response functions describe how concentration are related to CO2 emissions.

3.5 Non-CO2 concentrations

From emissions to concentrations. The formulas used in MAGICC to convert emissions to concentrations.

3.5.1 Methane

Methane concentrations are deduced based on natural, fossil and land-use related emissions. The atmospheric lifetime of methane is given by its chemical lifetime in the atmosphere, as well as by sinks to soil and other sinks.

MAGICC models the interactions between methane and other atmospheric components, like methane feedbacks on tropospheric OH. The results of the OxComp work (Ehhalt et al., 2001) provides the parameters needed to model changes in tropospheric OH abundances:

$$\Delta \ln(\text{tropOH}) = S^{\text{OH}}_{\text{CH}_4} \Delta \ln(\text{CH}_4) + S^{\text{OH}}_{\text{NOx}} E_{\text{NOx}} + S^{\text{OH}}_{\text{CO}} E_{\text{CO}} + \text{S}^{\text{OH}}_{\text{VOC}} \text{E}_{\text{VOC}},$$

where S^{OH}_{x} is the sensitivity of tropospheric OH with respect to the other constituents. It is Important to note that increasing abundance of tropospheric OH will decrease the lifetime τ' of Methane in the troposphere, approximated by this exponential relationship:

$$\tau'_{\text{CH}_4,\text{tropos}} = \tau^0_{\text{CH}_4,\text{tropos}} \exp^{\Delta \ln(\text{tropOH})}$$

Accounting for the change in chemical reaction speed due to changes in temperature gives an adjusted tropospheric lifetime of CH_4:

$$\tau_{\text{CH}_4,\text{tropos}} = \frac{\tau^0_{\text{CH}_4,\text{tropos}}}{\frac{\tau^0_{\text{CH}_4,\text{tropos}}}{\tau'_{\text{CH}_4,\text{tropos}}} + S_{\tau_{\text{CH}_4}} \Delta T}$$

ΔT is the temperature change and $S_{\tau_{\text{CH}_4}}$ is the temperature sensitivity coefficient.

3.5.2 Nitrous oxide

Estimating nitrous oxide is done in the same way as for methane. The tropospheric N_2O takes some time to transfer to the main stratospheric sink. Thus the average concentration period is shifted by 3 yrs. The atmospheric lifetime of nitrous oxide is also affected by its concentration in the atmosphere. It has a feedback effect on itself, approximated by:

$$\tau_{N_2O} = \tau^0_{N_2O}\left(\frac{C_{N_2O}}{C^0_{N_2O}}\right)^{S_{\tau_{N_2O}}} \tag{3.8}$$

Here, "0" indicates a pre-industrial reference level and $S_{\tau_{N_2O}}$ is a sensitivity coefficient.

3.5.3 Tropospheric aerosols

MAGICC approximates the atmospheric abundance of tropospheric aerosols from emissions. Historical emission data or proxes are the basis for constructing emission scenarios. The MESSAGE emissions scenario modelling group (Rao et al., 2005) included BC and OC emissions in scenarios. From these scenarios, a scaling factor for aerosols, from carbon monoxide, varying linearly in time until 2100, was found by analyzing MESSAGE scenarios.

3.6 Radiative forcing

Generally, the radiative forcing applied in MAGICC is at the tropopause level after a stratospheric temperature adjustment.

3.6.1 Carbon dioxide

MAGICC includes the saturation effect of increased CO2 concentrations. The forcing efficiency of CO2 decreases as CO2 concentration increases. In MAGICC, the adjusted radiative forcing ΔQ_{CO_2}, by CO2, is given by:

$$\Delta Q_{CO_2} = \alpha_{CO_2} \ln(C/C_0)$$

Where C is the concentration of CO2 (ppm) above a pre-industrial level, C_0 and α_{CO_2} is a scaling parameter (Myhre et al., 1998). For AOGCM-specific CO2 forcing, MAGICC sets the adjusted radiative forcing to: $\alpha_{CO_2} = \frac{\Delta Q_{2\times}}{\ln(2)}$.

3.6.2 Methane and nitrous oxide

Nitrous oxide and methane have overlapping absorption bands, affecting radiative forcing. The expression for their individual forcing effects accounts for the overlapping absorption bands. Equation 3.9 shows the expressions for methane forcing, ΔQ_{CH_4}, and for nitrous oxide forcing, ΔQ_{N_2O}:

$$\begin{aligned} \Delta Q_{CH_4} &= \alpha_{CH_4}(\sqrt{C_{CH_4}} - \sqrt{C^0_{CH_4}} - f(C_{CH_4}, C^0_{N_2O}) - f(C^0_{CH_4}, C^0_{N_2O}) \\ \Delta Q_{N_2O} &= \alpha_{N_2O}(\sqrt{C_{N_2O}} - \sqrt{C^0_{N_2O}}) - f(C^0_{CH_4}, C_{N_2O}) - f(C^0_{CH_4}, C^0_{N_2O}) \end{aligned} \tag{3.9}$$

The overlap is expressed by:

$$f(M, N) = 0.47 \ln \left(1 + 0.6356 \left(\frac{MN}{10^6}\right)^{0.75} + 0.007 \frac{M}{10^3} \left(\frac{MN}{10^6}\right)^{1.52}\right).$$

Here, C denotes concentration, M and N are CH4 and N2O concentrations in ppb (parts pr. billion), and the subscript "o" denotes the unperturbed concentration (pre-industrial concentration).

MAGICC also adds a forcing factor for methane due to the enhancement in stratospheric water vapour induced by methane, given by:

$$\Delta Q^{\mathrm{stratoH2O}}_{CH_4} = \beta \alpha_{CH_4} \left(\sqrt{C_{CH_4}} - \sqrt{C^0_{CH_4}}\right).$$

3.6.3 Tropospheric ozone

MAGICC uses the change in hemispheric tropospheric ozone concentrations, from Ehhalt et al. (2001), parameterized as:

$$\Delta(\mathrm{tropO_3}) = S^{O_3}_{CH_4} \Delta \ln(CH_4) + S^{O_3}_{NOx} E_{NOx} + S^{O_3}_{CO} E_{CO} + S^{O_3}_{VOC} E_{VOC},$$

where $S^{O_3}_x$ are the respective sensitivity coefficients of tropospheric ozone to methane concentrations and precursor emissions. From the abundance of tropospheric ozone, the radiative forcing is approximated as $\Delta Q_{\mathrm{tropO_3}} = \alpha_{\mathrm{tropO3}} \Delta(\mathrm{tropO_3})$ with α_{tropO3} being the radiative efficiency factor.

3.6.4 Halogenated gasses

MAGICC derives the radiative forcing of halogenated gasses from their atmospheric concentration's radiative efficiencies (following (Ehhalt et al. (2001), Table 4.11). MAGICC uses a land-ocean forcing contrast for CFC-11 (Chlorofluorocarbon) from Hansen et al. (2005). The forcing contrast of gases with a lifetime less than one year is assumed to be equal to the emission ratio. MAGICC assumes forcing contrasts equal to CFC-11 for gasses with lifetimes longer than eight years. In the case of medium lifetimes, MAGICC applies a linear scaling factor between the two mentioned lifetimes.

3.6.5 Stratospheric ozone

A reduction in stratospheric ozone will result in a negative global-mean radiative forcing. MAGICC assumes the depletion of stratospheric ozone to be dependent on the effective stratospheric chlorine (EESC) concentrations. EESC concentrations are from the work of Daniel et al. (1999).

3.6.6 Tropospheric aerosols

Due to short lifetimes, hemispheric emissions approximate the concentration of tropospheric aerosols. A linear relationship between abundance and forcing approximates the direct effect from sulfate, nitrate, black carbon and organic carbon.

3.7 The upwelling-diffusion climate model

Figure3.2 shows a representation of the upwelling-diffusion energy balance module used in MAGICC. The module consists of a northern hemisphere and a southern hemisphere. Each hemisphere consists of two atmospheric boxes—one box over land and one box coupled to a mixed-layer ocean. Each of the atmospheric boxes has zero heat capacity.

Vertical diffusion and advection drive the heat exchange between ocean layers. This version (MAGICC6) uses an upwelling-diffusion-entrainment (UDE) ocean model with a depth-dependent ocean area.

3.7.1 Partitioning of feedbacks

MAGICC uses different feedback parameters over land and oceans to improve the comparability between MAGICC and AOGCMs. AOGCM results give an adjustable land to ocean warming ratio in equilibrium, needed for using different feedback parameters. Since the oceanic heat uptake is zero at equilibrium, the

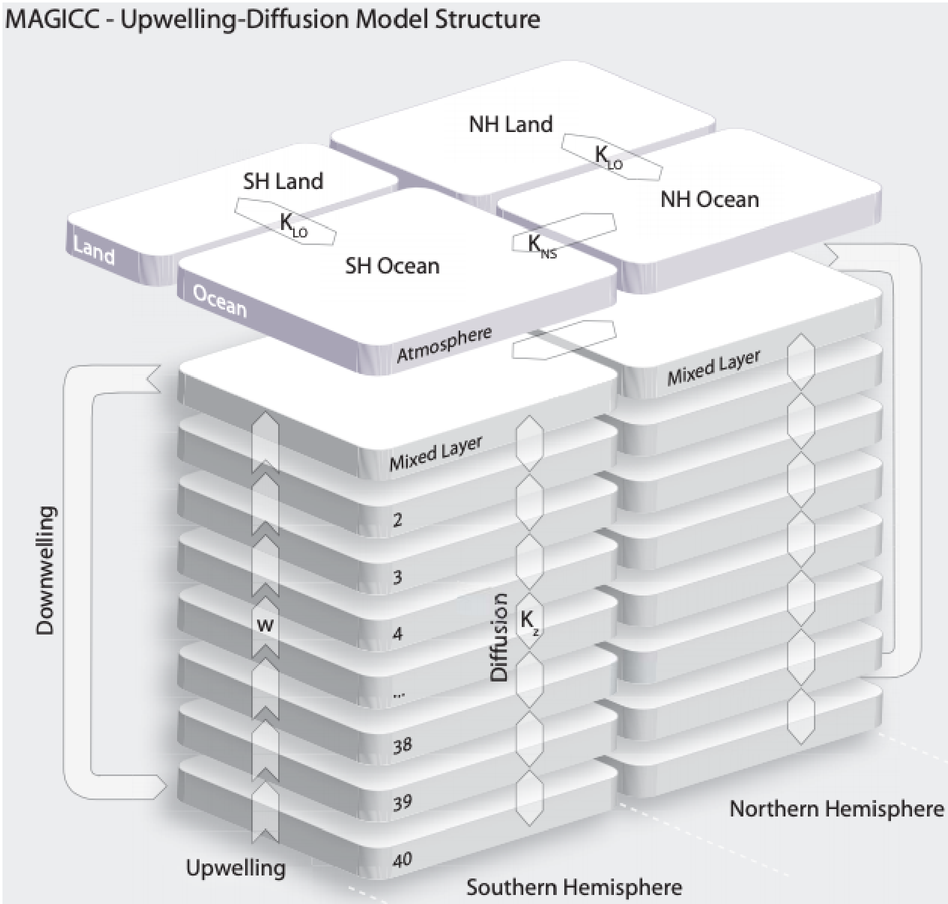

Figure 3.2: Schematic representation of MAGICCs hemispherically averaged upwelling-diffusion energy balance module. deepwater formation, upwelling, diffusion, and heat exchange between the hemispheres are the included processes for heat transport in the ocean (Meinshausen et al., 2011).

global energy balance equation is:

$$\Delta Q_G = \lambda_G \Delta T_G = f_L \lambda_L \Delta T_L + f_O \lambda_O \Delta T_O \,,$$

where ΔQ_G is the global-mean forcing, λ_G is the feedback and ΔT_G is the temperature change. The right-hand side uses the area fractions f, feedbacks λ, and mean temperature changes, ΔT for the ocean (O) and land (L).

3.7.2 Revised land-ocean heat exchange formulation

The partitioning of feedbacks over land and ocean may lead to change in effective climate sensitivities over time. MAGICC includes a heat transport enhancement factor μ to control the relative temperature changes over ocean and land. This factor allows MAGICC to simulate some AOGCM responses better.

3.7.3 Accounting for climate-state dependent feedbacks

Climate feedbacks being climate-state dependent explains higher effective climate sensitivities for higher forcing or higher temperatures. This problem of climate feedbacks changing along with changes in climate forcing is dealt with by scaling MAGICC's land and ocean feedback parameters.

4

The Simple Response Model (SRM)

In essence, the SRM transforms an emission time-series into a temperature time-series or a climate scenario, in this order: Emissions ↔ Concentration ↔ Forcing ↔ Temperature. The two main components of the model is a carbon module, transforming carbon emissions into carbon concentrations, and a climate module, transforming forcing into temperature. The model framework is based around impulse response functions for N-box models and simple parameterizations for atmospheric forcing.

4.1 Emissions

4.1.1 Carbon emissions

The SRM runs using carbon or carbon dioxide emission scenarios. All scenarios follow the same historic emission scenario up to present. Then, for the emission scenario applied, the model will calculate a resulting global mean surface temperature (GMST). This scenario versatility is one of the strengths of this simple modelling framework.

4.1.2 Methane emissions

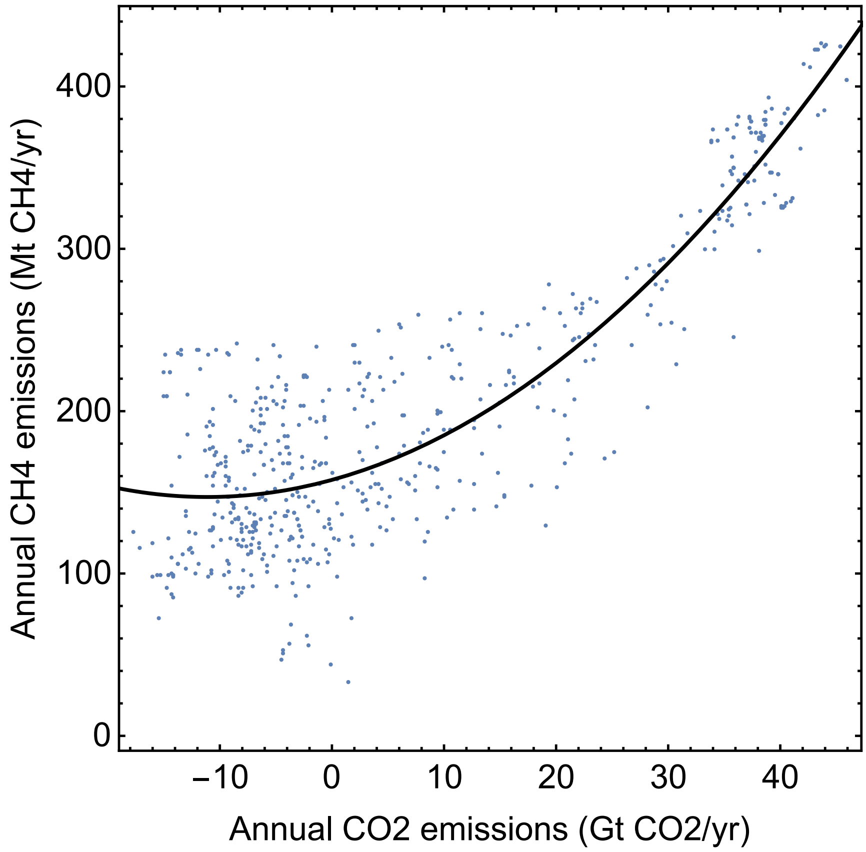

Figure 4.1: Relationship between annual emissions of carbon dioxide and methane. Annual CO_2 emissions (Gt CO_2/yr) plotted against annual CH_4 emissions (Mt CH_4/yr), plotted from the data-set from Huppmann et al. (2018).

Figure 4.1 shows the relationship between the emission of CO_2 and CH_4, in the IAMC 1.5°C Scenario Explorer and Data hosted by IIASA, release 1.1 (Huppmann et al., 2018). This scenario explorer includes an ensemble of mitigation pathways underpinning the SR15. A relatively strong linear trend is observable in the figure for emissions larger than 15 Gt CO_2. The change in methane emissions stagnates at annual emissions of around 150 megatonnes (Mt) as carbon dioxide emissions move towards zero and negative emissions. This relationship between annual emissions is the basis for constructing our methane emission scenario:

$$E_{\mathrm{m}}(t) = U(E_{\mathrm{CO_2}}(t)), \tag{4.1}$$

where $E_{\mathrm{m}}(t)$ is the methane emissions, $E_{\mathrm{CO2}}(t)$ is the carbon dioxide emissions and U is the quadratic function in Figure 4.1. The methane emission scenario will not go below a certain threshold as it is unlikely that methane emissions will be negative in the case of carbon dioxide emissions moving towards zero or negative, through, e.g. Carbon Capture and Storage (CCS).

4.1.3 Aerosol emissions and concentration

Aerosols have a negligible lifetime relative to the timescales of interest in this study. The short lifetime means that the concentration will solely depend on annual emissions. The emission scenario is constructed as a linear function of the carbon dioxide emissions:

$$E_{\mathrm{a}}(t) = bE_{\mathrm{CO2}}(t)$$

4.2 Concentrations

4.2.1 The carbon module

The SRM uses a carbon module to model the carbon concentration in the atmosphere from the carbon dioxide emission scenario. The carbon model emulates the results or trends from more complex models. The carbon module is based on the work by Joos et al. (2013) on carbon dioxide and impulse response functions. Figure4.2 shows the impulse response function emulated in the carbon module.

The climate module uses four exponential functions in its response function. All four are needed to describe the decay in concentration (after an impulse emission) on different time-scales. The module use fits for the upper and the lower ranges in the ensemble. Our model for carbon dioxide concentration is the following convolution integral:

$$\int_{t_0}^{t} G_C(t-s)E(s)ds\,, \tag{4.2}$$

where,

$$\begin{aligned} G_C(t) = CF(C_0 + C_1 \exp\left(\frac{-\tau_1}{t}\right) + C_2 \exp\left(\frac{-\tau_2}{t}\right) \\ + C_3 \exp\left(\frac{-\tau_3}{t}\right) + C_4 \exp\left(\frac{-\tau_4}{t}\right)\,. \end{aligned} \tag{4.3}$$

Here $E(s)$ is the emission scenario for carbon dioxide emissions as a function

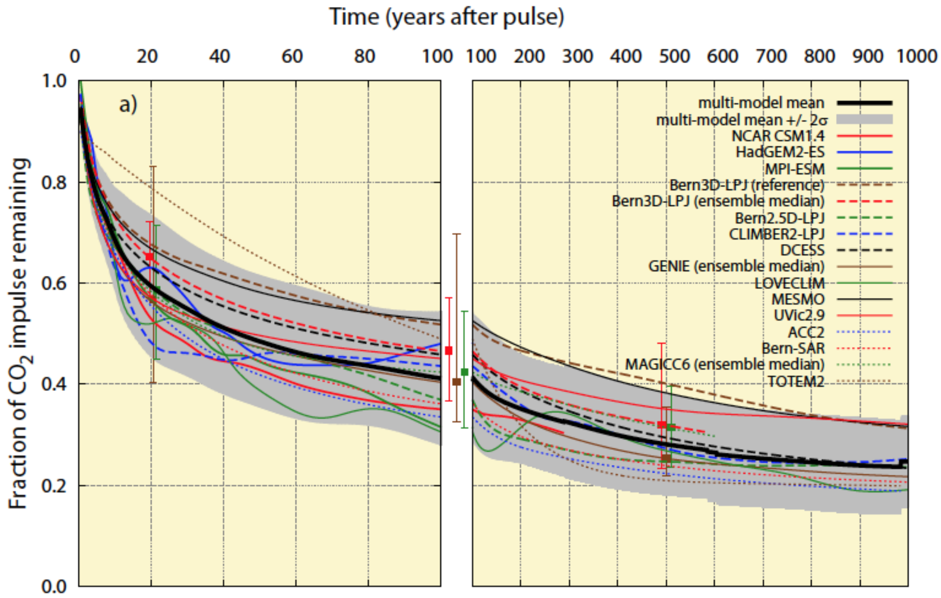

Figure 4.2: The evolution of the impulse response function for CO2 for an emission pulse of 100 Gt carbon. An ensemble of models of varying complexity is used (Joos et al., 2013).

of time. $C0$, $C1$, $C2$, $C3$ and $C4$ are the fitted parameters, where the sum of the parameters add up to one $\left(\sum_{n=0}^{4} C_i = 1\right)$. τ_1, τ_2, τ_3 and τ_4 represent the different timescales for the exponential functions. They are set to be 1, 10, 100 and 1000 yrs.

4.2.2 Methane concentration

The SRM computes the methane concentration using the same framework. Methane has a relatively short lifetime in the atmosphere, so only one exponential function is needed to describe its evolution. Equation 4.4 shows our methane model and Equation 4.5 displays the exponential function.

$$\int_{t_0}^{t} G_{\mathrm{M}}(t-s)E_{\mathrm{M}}(s)ds \tag{4.4}$$

$$G_{\mathrm{M}}(t) = C \exp\left(\frac{-\tau_{\mathrm{M}}}{t}\right) . \tag{4.5}$$

Here E_{M} is the methane emission scenario, and τ_{M} is the lifetime of methane in the atmosphere. C is a constant that fixes the model's concentration to be consistent with today's concentration.

4.3 Radiative forcing

The SRM uses parameterizations, not response functions, to estimate the radiative forcing. The parameterizations are based on the work by Myhre et al. (1998).

4.3.1 Carbon dioxide forcing

The SRM translates the concentration of CO2 to forcing through the relation in Equation 4.6:

$$F_{\mathrm{CO_2}} = 5.35 \ln\left(1 + \frac{[CO2] - 280\mathrm{ppm}}{280\mathrm{ppm}}\right). \tag{4.6}$$

Here, $F_{\mathrm{CO_2}}$ ($\mathrm{Wm^{-2}}$) is the radiative forcing from the atmospheric concentration of CO2, [CO2] is the carbon dioxide concentration, 280ppm is the preindustrial/ unperturbed concentration of CO2, and 5.35 is a conversion factor (Myhre et al., 1998).

4.3.2 Methane forcing

The concentration of atmospheric methane is converted to forcing through the relation in Equation 4.7.

$$F_M = 0.036\left(\sqrt{[\mathrm{methane}]} - \sqrt{700}\mathrm{ppb}\right), \tag{4.7}$$

where F_M represents the methane induced radiative forcing ($\mathrm{Wm^{-2}}$), [methane] is the atmospheric concentration of methane in ppb, 700 ppb is the preindustrial methane concentration, and 0.036 is a conversion factor (Myhre et al., 1998).

4.3.3 Aerosol forcing

In contrast to both carbon dioxide and methane, aerosols do not contribute to an overall increase in radiative forcing. There are some aerosols which may contribute positively to the net radiative forcing, but summing the forcing effect from aerosols as a group contributes to a cooling effect (Myhre et al., 2013). The way we have incorporated it in our model is, however, very similar to Methane. In the same way as Methane, aerosols are assumed to follow the CO2 emission scenarios. The radiative forcing from atmospheric aerosol concentrations is negative. So the emission scenario, multiplied by a factor of -0.2 gives the atmospheric aerosol forcing. We suspect that aerosols will not reach zero or negative emissions in the same way that CO2 might. Therefore we set a minimum forcing effect by aerosols at -0.4 Wm^{-2}. Due to the short atmospheric lifetime of aerosols and the resolution of the SRM being one year, the concentration of aerosols is the same as the emissions.

4.4 The climate module

The SRMs climate module translates the total effective forcing into GMST. It uses a set of exponential response functions to emulate behaviour of more complex Earth-system models or AOGCMs, such as the models in the CMIP5 ensemble. The SRM computes the temperature response from the impulse function on matrix form.

4.5 Current configuration

Figure 4.3 shows data for Global CO2 Emissions from Fossil-Fuel Burning, Cement Manufacture, and Gas Flaring from 1751 to 2011 (Boden et al., 2015).

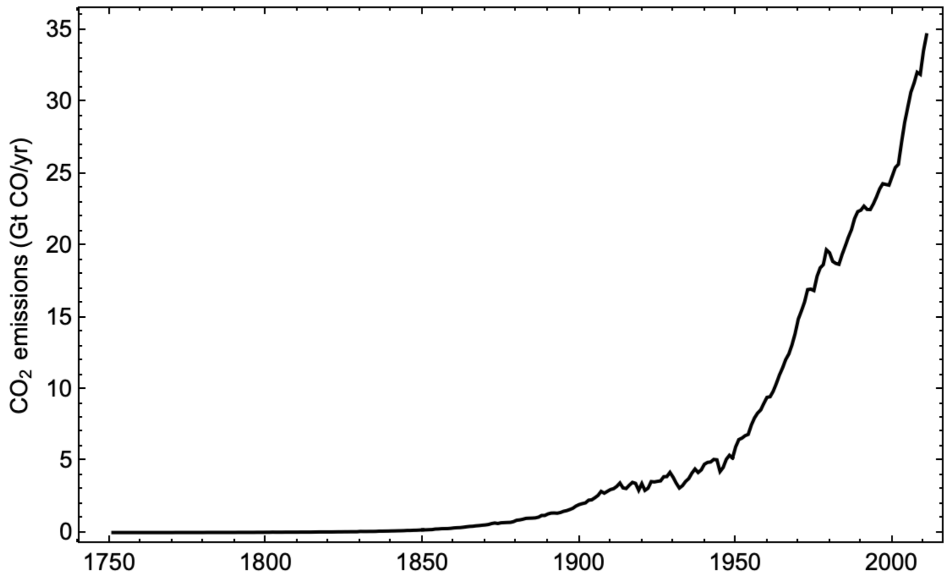

Figure 4.3: Accumulated historic global emissions of carbon dioxide from Fossil-Fuel Burning, Cement Manufacture, and Gas Flaring (Gt CO2 pr. year) from 1751 to 2011.

This data is the historic anthropogenic emission scenario in the SRM, combining the historic emissions with the scenarios for future emissions gives the full emission scenario.

The current version of the SRM is running on CO_2 emission data extracted from the SSP IAM V2 data (Riahi et al., 2017; Rogelj et al., 2018). The climate module uses parameter data from the CMIP5 model ensemble (Cummins et al., 2020) to emulate the behaviour of these complex climate models. The parameters for the exponential functions for the climate module depend on the model. The climate module uses two to four exponential functions, depending on the number of parameters given for each model in the data. So for every model and every emission scenario in the ensemble, the SRM model setup can compute a temperature response.

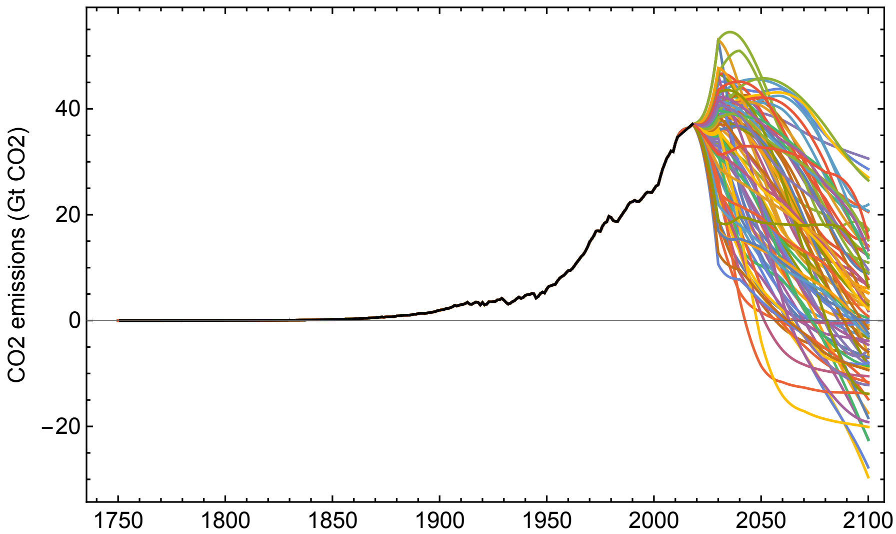

Figure 4.4: Carbon dioxide emission scenarios (Gt CO_2) from the IAM SSP V2 data added to the historic emission scenario, running until year 2100.

There are a total of 86 scenarios, shown in Figure 4.4, evaluated across 14 climate models. We have decided to omit some scenarios due to the evolution in CO_2 emissions they represent (See Appendix A for the figure including all 127 scenarios). These scenarios are so unambitious that they are uninteresting with respect to carbon budgets.

The already incorporated framework for methane and aerosols give concentrations and forcing from the carbon dioxide emission scenarios and add to the total forcing, shown in Figure 4.5 (see appendix A for the figure showing all forcing agents added up). The total forcing is then run through the climate module to calculate the temperature response.

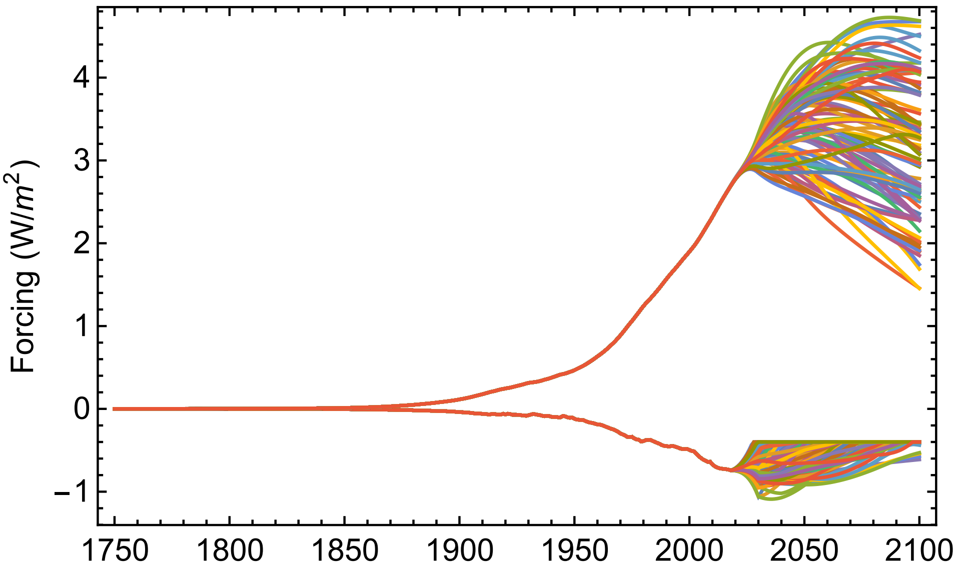

Figure 4.5: Radiative forcing (W/m^2) estimated from the emission scenarios, including methane and aerosol emissions.

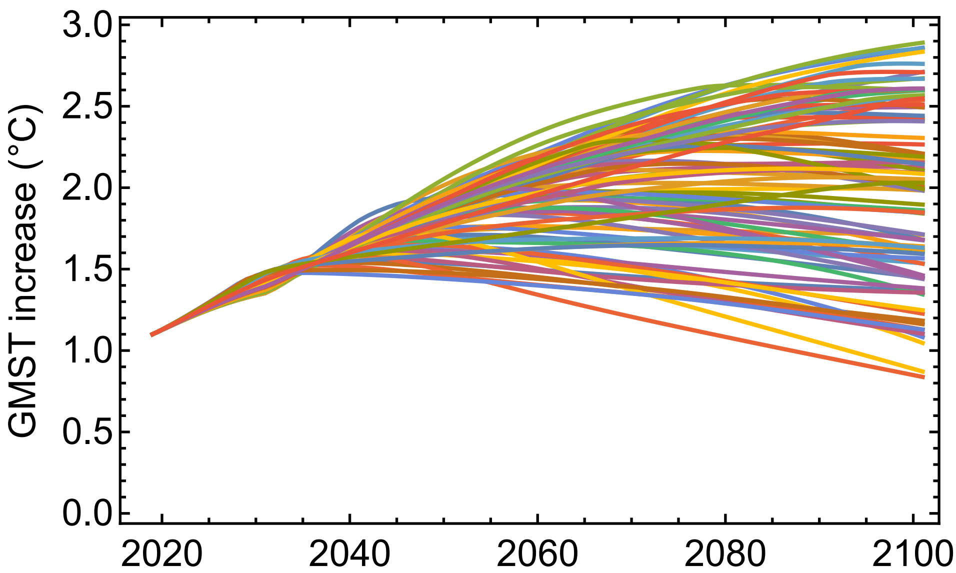

Figure 4.6: Global mean surface temperature response estimates from the scenarios in the SSP IAM data.

4.5.1 TCRE

The SRM runs every scenario (SSP) in the data through the SRM with two carbon models and 14 climate models, including internal variability (See appendix A for internal variability plot). Every combination of climate model and carbon model will yield a specific TCRE, as shown in Figure 4.7. Here every dot is an SSP scenario in different combinations of carbon and climate models.

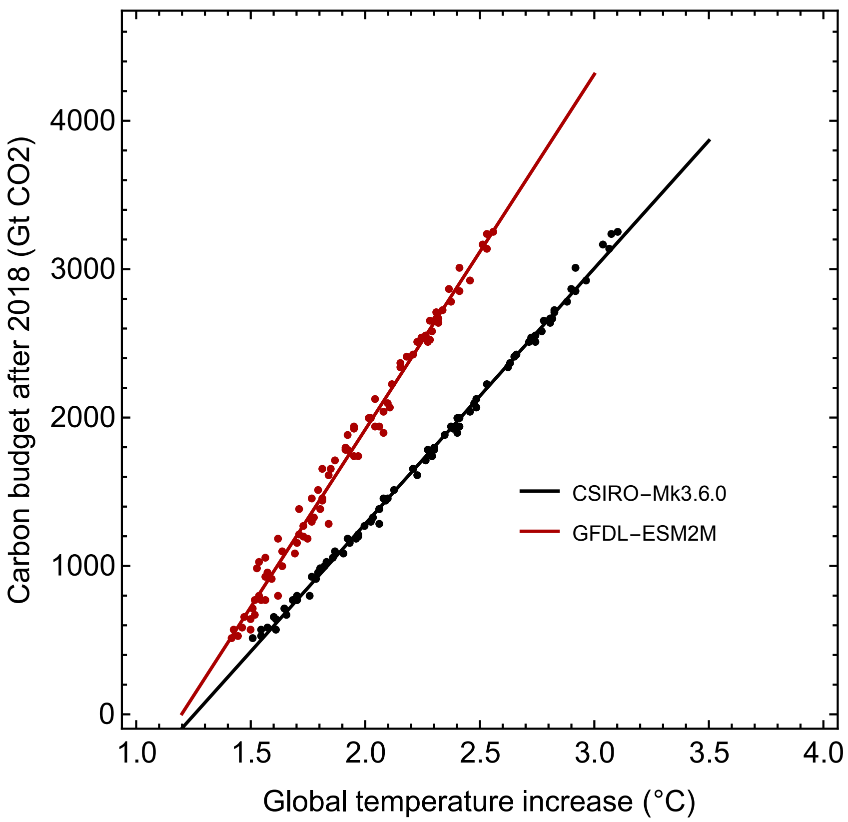

Figure 4.7: Transient Climate Response to the Cumulative Emissions of CO2 (Gt CO2) for two different climate models in the ensemble (CSIRO-Mk3.6.0 and GFDL-ESM2M) calculated with the mean of the two carbon models.

We can see from Figure4.7 that varying climate models can lead to substantial variations in slope and temperature responses.

4.5.2 Internal variability

Internal variability is included and modelled as a white noise function with zero mean and a standard deviation of one (see appendix A for the internal variability for every model used in the study). Modelling it as white noise should coincide with the cyclic or oscillatory nature of internal climate variability. The strength of the white noise process, modelling the internal variability, varies with climate models.

4.5.3 Pdf generating method

The SRM implements a method from Cox et al. (2018) for constructing conditional probability density functions (pdfs). In the SRM, the method is adapted for generating pdfs for RCBs from the TCREs, as shown in Figure 4.8:

This method is applied to all the model combinations, generating multiple pdfs. Combining all pdfs, a probabilistic representation, shown in Figrue4.9 **(d)**, is generated.

This probabilistic representation is a carbon budget plot, including the probability of different temperature responses as a function of carbon budget size. By varying carbon models, climate models and the inclusion of internal variability, we can assess how much each component contribute to the carbon budget uncertainty. Figure4.9 visualizes the uncertainty of the temperature response for each component and the combined uncertainty of the temperature response.

4.6 Results

To determine the impact of varying climate models, carbon models and internal variability, Figure 4.9 is evaluated. Plot **(a)** shows that the 14 climate models add most uncertainty to the maximum temperature increase, as they contribute to most of the spreading. Plot **(b)** shows that uncertainty in the carbon model shows a small spread as the budgets gets larger. Internal variability, shown in plot **c)**, does not show any indications of spreading as budgets grow. We conclude that the climate models is the most dominant source of uncertainty.

From **(d)** the total risk of maximum temperature increase associated with carbon budget size, can be assessed. Looking at a goal of a 2 °C maximum temperature increase and a carbon budget of around 1500 Gt of CO2. There will,

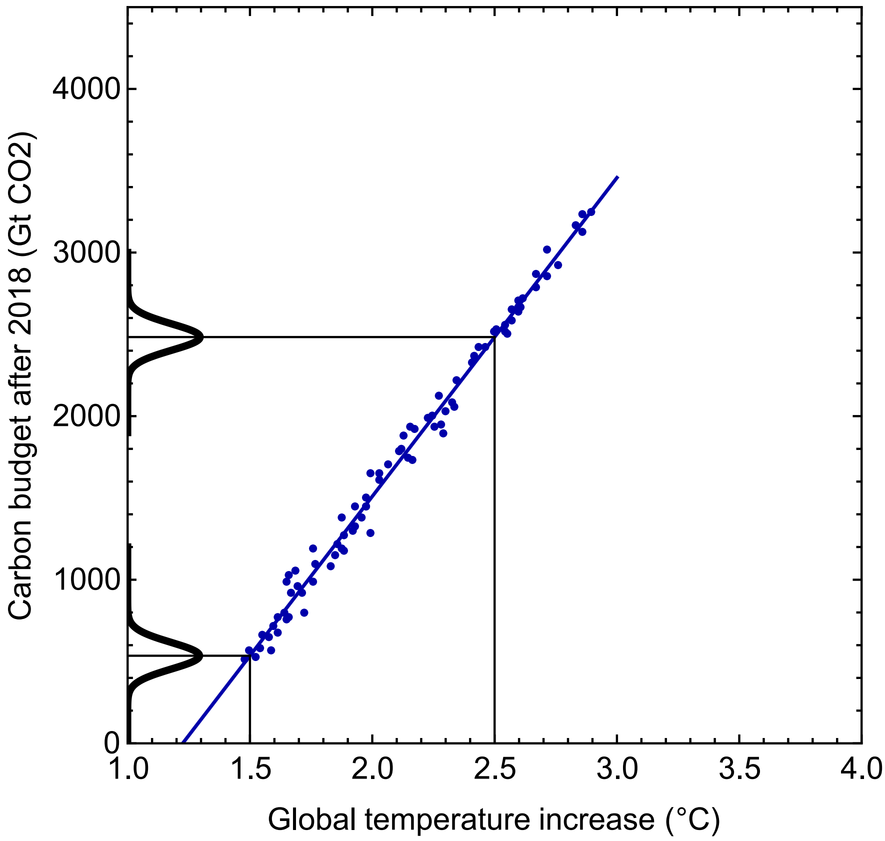

Figure 4.8: TCRE with probability density functions for the RCBs associated with the 1.5 and 2.5 °C temperature targets for one climate model in the ensemble.

associated with this carbon budget, be an even chance (50%) of a maximum temperature increase of 2 °C. This carbon budget also comes with a 10% probability or risk of a 2.5 °C maximum temperature increase.

It is also notable how the uncertainty increases as the carbon budget grows larger. This spreading increases uncertainties in maximum temperature response for larger carbon budgets. A carbon budget of 3000 Gt of CO_2 will have an even chance of reaching a maximum temperature increase of 2.75 °C, with a 10% risk of a maxim temperature increase of almost 3.5 °C. This represents the chance of a 0.75 °C increase to what was initially aimed for. For a 1500 Gt budget this miss would be of around 0.5 °C.

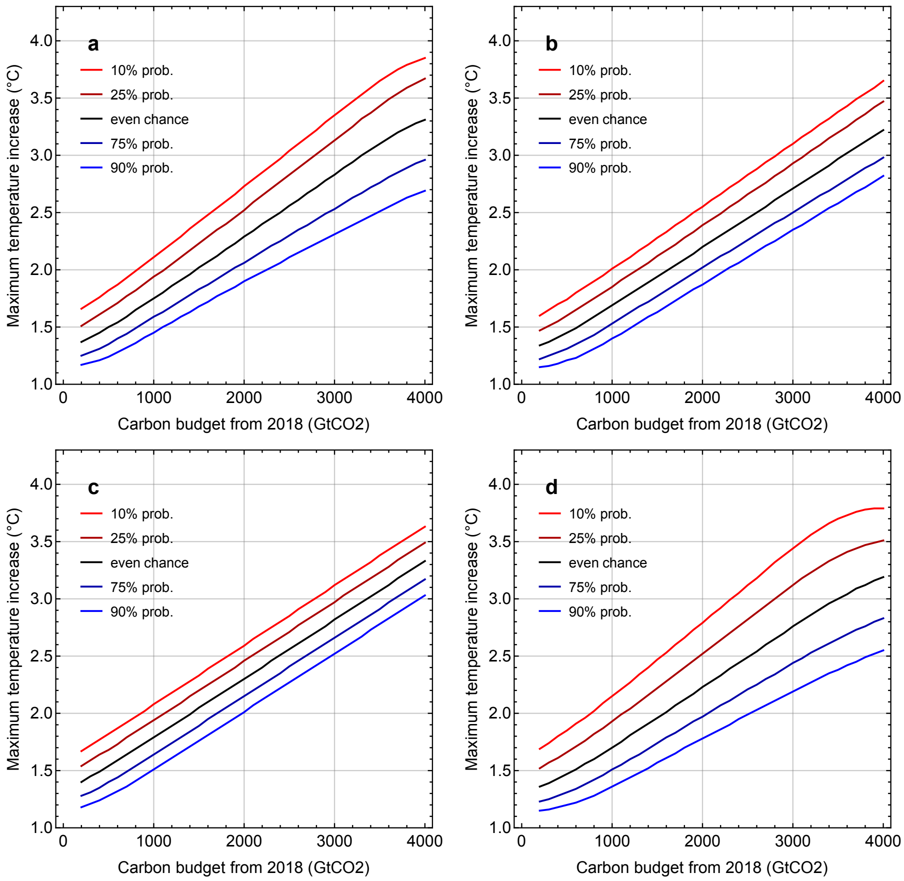

Figure 4.9: Probability of maximum temperature increase (°C) for various carbon budgets (GtCO2). All figures represent the probability for a maximum temperature increase for cumulative emissions of CO2. **a)** Using the 14 climate models from CMIP5. **b)** Varying the carbon models, using the mean climate model. **c)** Adding internal variability, using the mean climate model and mean carbon model mean. **d)** Taking all sources of uncertainty into account.

5

Comparing the SRM and MAGICC

The SSP data used for the emission scenarios for the SRM included runs with the low complexity climate model, MAGICC. The SSP data-set includes decadal temperature responses estimated with MAGICC. The two models are compared by extracting MAGICCs temperature response to the emission scenarios. The temperature responses of each model are compared using a scatter plot. The comparison includes the implementation of nonlinear forcing to the SRM by Mentzoni (2020).

The SRM has not, in any way, been calibrated to fit MAGICC's results before this comparison.

5.1 Model differences between the SRM and MAGICC

Both are in the category of low complexity models. The SRM is very result-oriented in computing only the response of the system to changes in emissions. MAGICC is based on an energy balance model and is more descriptive of the climate system and its components in a more process-based approach. The SRM is, in all its simplicity, a response model with some added parameterizations for radiative forcing. MAGICC has a more box model-like setup, describing movements between ocean, land and hemispheres.

The SRM includes parameterizations for the radiative forcing of CO2, CH4 and aerosols. MAGICC includes all these species, including nitrous oxide, tropospheric/ stratospheric ozone and halogenated gasses. MAGICC also includes the interplay between different species, such as overlapping absorption bands and how effective the different species are as climate forcers depending on concentration.

The SRM's and MAGICC's modelling of the carbon cycle indicates the differences in their complexity and their approach to describing the climate system. The SRM takes a system-level approach, describing the general trends in the carbon cycle without describing internal processes. MAGICCs carbon cycle includes a globally averaged box model of the terrestrial carbon cycle, and describes the processes involved in the carbon cycle and calculates a response, while the SRM calculates the response based on statistics.

5.2 Comparing temperature responses between the original SRM and MAGICC

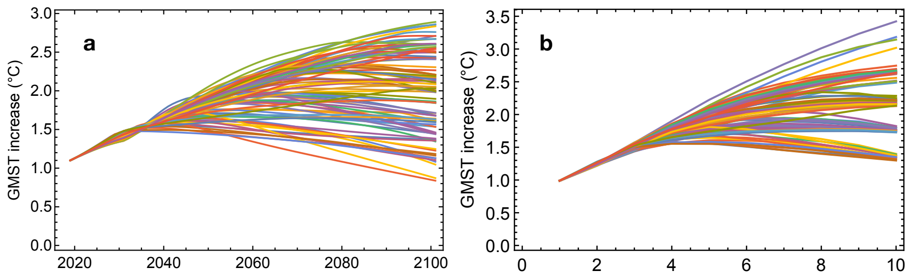

Figure 5.1: Global Mean Surface Temperature (GMST) increase (°C) comparison from 2020 to 2100 between a) the normal run of the SRM and **b)** the MAGICC model.

Figure5.1 shows the GMST increase of the SRM and MAGICC when evaluating scenarios from the SSP database. A visual representation like this already shows substantial similarities between the results from the two models. Both models appear similar in shape. MAGICC appears to be computing a higher maximum temperature and a higher minimum temperature.

A more systematic approach is using scatter plots, as shown in Figure5.2. The scatterplots better illustrate the similarities between the temperature responses of the two models.

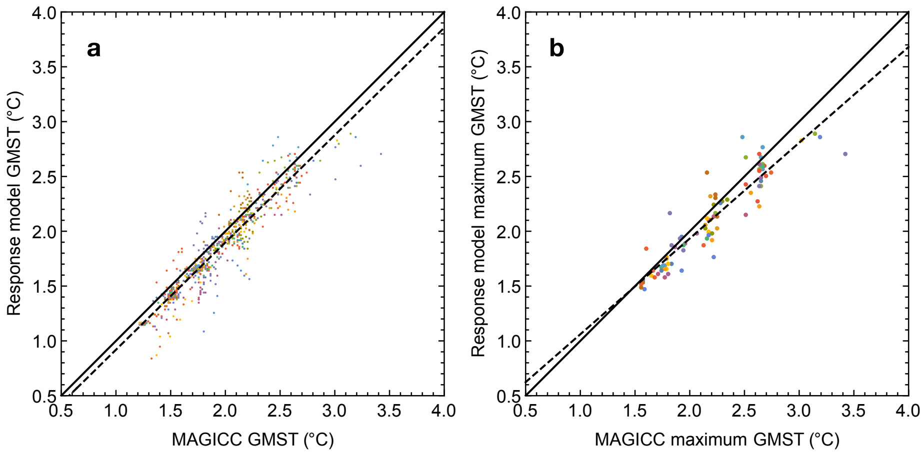

Figure 5.2: Scatter plot comparing the temperature responses of The SRM and MAGICC models a) Comparison between all MAGICC temperatures and corresponding SRM temperatures. **b)** Comparison of the maximum temperature for each emission scenario.

The scatter plot shows, for all MAGICC temperatures in the data-set, that the two models perform very similarly in modelling temperature response based on the SSP dataset. The dashed line is slightly below the full line, which means that the SRM computes a slightly lower temperature for the whole data set. Plot **b)** in Figure5.2 visualizes the maximum temperature for each scenario. The SRM calculates the RCBs based on maximum temperature. Therefore it is of interest to compare how the models perform in computing maximum temperatures as well. Here we can see the dashed line starting above the thick line, crossing it around 1.5°C, as temperature increases. This implies higher maximum temperatures for the SRM at low temperatures or early in the scenario, but higher maximum temperatures for MAGICC later in the scenario. This trend appears reasonable from the difference in the general mean temperature increase shown in Figure5.1. Maximum temperature in a scenario is an important factor when addressing concerns about peak warming. The concerns are especially valid in the context of possible tipping points triggered at some temperature.

5.3 Comparing temperature responses between the nonlinear SRM and MAGICC

In this section, the results from Endre Falck Mentzoni's work on nonlinear forcing is added to the comparison between the SRM and MAGICC. One parameterization of each of the state-dependent forcing terms (linear and hyperbolic) is selected and compared, as in section 5.2, to the temperature responses from MAGICC. Endre Falck Mentzoni's work includes more parameterizations of the added temperature dependent forcing functions that are not included in this comparison (See appendix A).

5.3.1 Linear addition to forcing

This section highlights the comparison of the SRM and MAGICC after adding a state-dependent forcing factor modelled as a linear function on the form:

$$F(t) = 0.2T,$$

where F is forcing (wm^{-2}), 0.2 is some factor (Wm^{-2}°C^{-1}) and T is temperature (°C). Figure 5.3 shows the similarities between the SRM and MAGICC after the implementation.

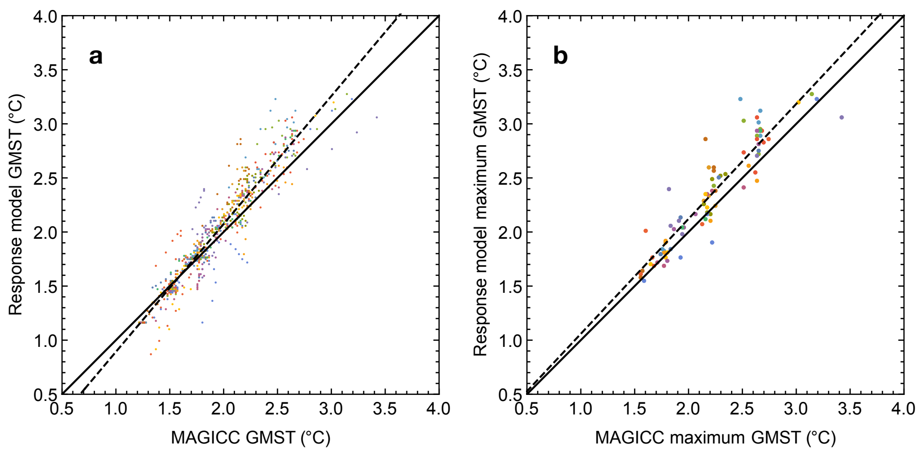

Figure 5.3: Scatter plot comparing temperature responses between MAGICC and the SRM with an added linear temperature dependent forcing function (0.2T) component included in the SRM. a) Comparison between all MAGICC temperatures and corresponding temperatures in the SRM. **b)** Comparison between maximum temperatures for every scenario.

We can observe in **(b)** that the dashed line is above the thick line, indicating that the SRM estimates a higher maximum temperature than MAGICC when including the nonlinearity. The difference also seems to be increasing along with the increasing temperature. This result is very intuitive since the added forcing is a function that increases forcing with increasing temperatures. In **(a)** the temperature appears to behave differently from the linear case, where the dashed line followed the thick line for all temperatures. In this case, MAGICC predicts the highest maximum temperature at lower temperatures. As the temperature increases, the SRM predicts higher temperatures. The linear function has a smaller addition to the total forcing at lower temperatures and a higher impact at high temperatures, due to its dependency on temperature. A result of this is that the linear trend between the two models is biased toward higher temperatures. The relation between MAGICC temperatures and temperatures from the SRM is not conclusive at the lower temperatures in the plot.

5.3.2 Hyperbolic tangent forcing addition

Figure6.1 illustrates the comparison of the SRM and MAGICC where the SRM has an added nonlinear forcing term that follows the hyperbolic tangent function

$$F(t) = 1 * 0.5 * (1 + \tanh(T - 2)/0.5).$$

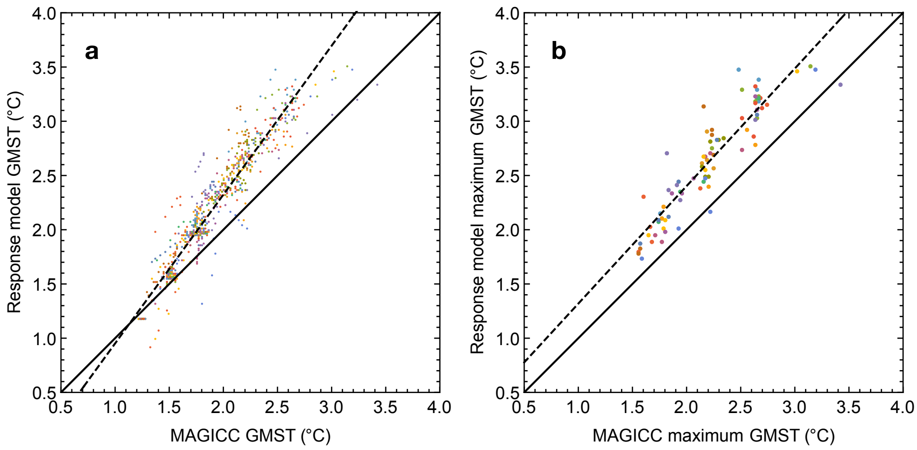

Figure 5.4: Scatter plot comparing global mean surface temperatures (°C) between MAGICC and The SRM with a nonlinear forcing component following the hyperbolic tangent function in the SRM. a) Comparison between all MAGICC temperatures and corresponding SRM temperatures. **b)** Comparison between Maximum temperatures for each corresponding scenario.

The most notable result from adding the nonlinear hyperbolic forcing is the increase in temperature for the SRM compared to MAGICC. The hyperbolic tangent function used in this comparison has a temperature threshold at 2°C.

This is the temperature where it kicks in. Compared to the linear forcing term, the hyperbolic function starts a bit later but is much stronger at the point of entry (See Figure6.1). These differences lead to the changes we see in Figure5.4. The maximum temperatures are much larger now for the SRM compared to MAGICC. Plot **(a)** shows that MAGICC's predictions are warmer at low temperatures, but the SRM gets increasingly warmer at higher temperatures and the difference only seem to get larger. Plot **(b)** shows that even with the implementation of a relatively drastic nonlinear forcing term, the maximum temperatures do not diverge very sharply but continues with a constant difference from the thick line. The cause of the constant difference could be little inclusion of low temperatures in the maximum temperature plot. The maximum temperatures are all affected by the nonlinear hyperbolic forcing resulting in the dotted line "bumping up", compared to the case without extra forcing. Lower maximum temperatures would probably lead to the dotted line diverging more from the thick line.

6

Additional results

6.1 Temperature dependent forcing addition

In the research project Endre Falck Mentzoni worked on implementing nonlinear forcing effects into the SRM framework (Mentzoni, 2020). Adding an extra forcing component depending on the temperature is a way of implementing a possible nonlinearity in our linear model framework.

The implementation of an extra temperature dependent forcing effect changes both the temperature response and the RCB, compared to running the scenarios in the SRM without any added nonlinear effect. The nonlinear forcing effect is modelled either as a hyperbolic function or a simple linear function shown in Figure6.1.

The hyperbolic tangent function is implemented on the form:

$$F(T) = s0.5(1 + \tanh((T - t)/b)$$

Where s, t and b are respectively the strength (Wm^{-2}), threshold (°C) and steepness (°C), and T is temperature. The linear function follows the form of:

$$F(T) = XT$$

where X determines the slope ($\mathrm{Wm}^{-2}{}^{\circ}\mathrm{C}^{-1}$) and T is temperature (°C).

There is uncertainty regarding the implementation of nonlinear effects in climate models. This uncertainty is a reason for using two different functions

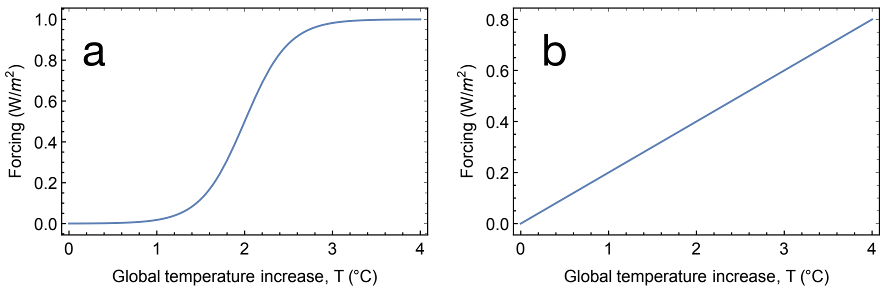

Figure 6.1: Temperature dependent forcing functions. a) Hyperbolic tangent function and **b)** Linear function are the types of functions used to research, respectively, nonlinear and linear added forcing terms.

for researching the effect of the temperature dependent or nonlinear forcing. These two functions are run with different parameter settings to evaluate the span of their impact better. Figures of all the parameter variations are found in Appendix A. The hyperbolic tangent function is varied most with variations in strength (s), threshold (t) and steepness (b), changing respectively the strength of the forcing, the temperature where it kicks in and how steep the increase is. One of the physical interpretations of the hyperbolic forcing function is a tipping point scenario. In a tipping point case, looking at the temperature as a system defining parameter, at some temperature threshold, something will rapidly affect the radiative forcing balance. A temperature threshold can be the temperature where the permafrost starts melting—Emitting a large amount of previously frozen methane into the atmosphere.

6.1.1 TCRE

Does the path independence of the TCRE still hold with the implementation of nonlinearities? In Section 2.5, it was stated that a monotonously increasing carbon emission scenario was an essential factor for using TCRE. Endre Falck Mentzoni compares the TCREs computed by the SRM with and without the inclusion of a temperature dependent forcing function. Figure 6.2 shows the differences is TCREs for two models in the SSP database with and without the extra nonlinear forcing factor.

In **(a)** strong linearity is observable through all data points. In **(b)**, we can observe that all the data points do not align as well as in the case with no additional forcing implemented. In this case, the deviation is not so severe that the nonlinear forcing function is discarded. Implementing too strong nonlinearities yielded cases where finding a linear fit to the TCRE was not

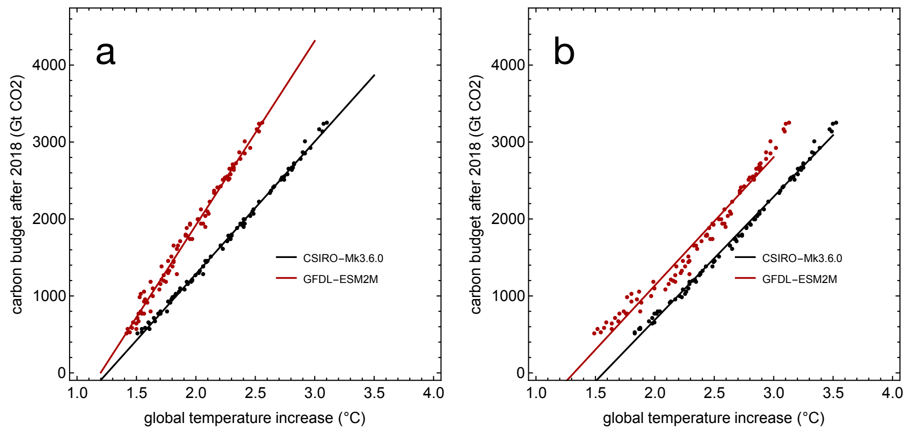

Figure 6.2: Visualizing changes in TCRE from the addition of a nonlinear forcing function. TCREs for two of the models in the SSP database (CSIRO-Mk3.6.0 and GFDL-ESM2M) with a) showing a normal forcing setup without nonlinearities and **b)**including hyperbolic forcing function with strength, $s = 1\text{W}/\text{m}^2$, threshold, $t = 2°\text{C}$ and steepness, $b = 0.5°\text{C}$.

possible. In the cases of implementing too extreme temperature-dependent forcing functions, the carbon budgets do not make any sense. Examples of these functions with corresponding TCREs and carbon budgets are presented in Appendix A. An important note is that the RCB is built on this linearity being valid. This shows that too severe nonlinearities could make the framework collapse and must, therefore, be checked before being implemented.

6.1.2 Carbon budget

As shown in the last section, including a nonlinear forcing effect can alter the resulting TCRE. Altering the TCRE will, in turn, change the probability plot for maximum temperature increase as a function of carbon budget size. Figure 6.3 visualizes how implementing a nonlinear forcing factor can alter the maximum temperature response for different carbon budgets.

Figure 6.3 shows that the uncertainty in smaller carbon budgets increase significantly when implementing the hyperbolic forcing function.

6.2 Arctic amplification

Andreas Johansen worked on translating the global data output from the SRM to regional results for the Arctic, assisted by myself and Endre Falck Mentzoni.

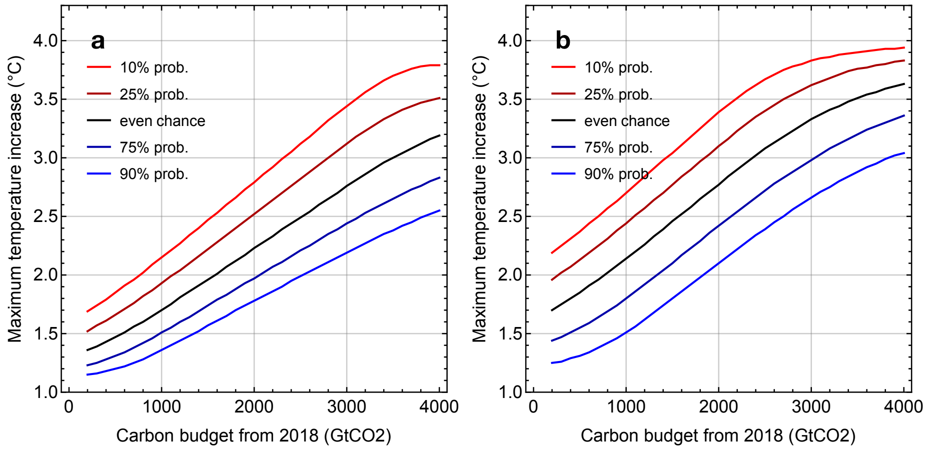

Figure 6.3: Probability of maximum temperature increase (°C) for various carbon budgets. a) From normal SRM run without added temperature dependent forcing forcing. **b)** With added temperature dependent forcing function on hyperbolic tangent form with strength, $s = 1\text{W/m}^2$, threshold, $t = 2°\text{C}$ and steepness, $b = 0.5°\text{C}$.

For further details regarding the implementation of the arctic amplification, see (Johansen, 2020).

Figure 6.4 shows the change in risk of maximum temperature increase related to carbon budget size when including arctic temperature amplification.

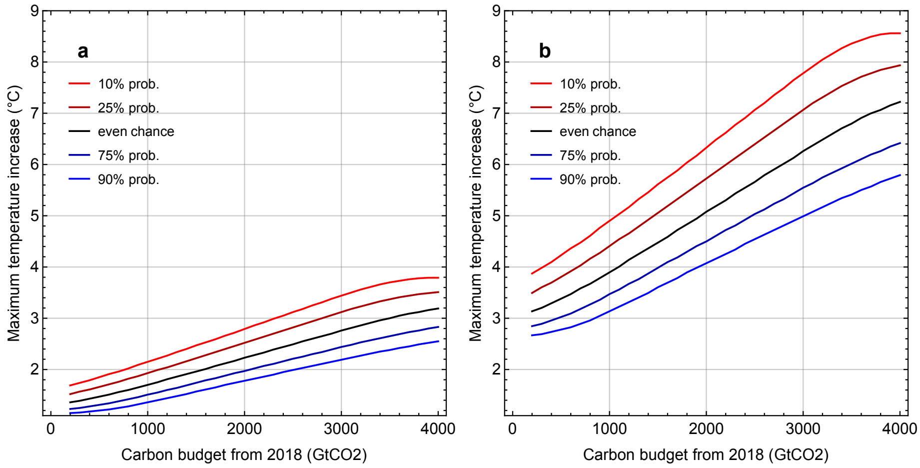

Figure 6.4: Probability of maximum temperature increase (°C) for various carbon budgets. a) As in Figure 4.9 d). **b)** Including arctic temperature amplification.

The inclusion of an Arctic amplification factor leads to a higher maximum temperature increase compared to the average global case. The distance between lines representing the probability of maximum warming also increases, increasing the uncertainty of the actual maximum temperature increase for a given carbon budget. This uncertainty also seems to increase for a larger carbon budget. The magnitude of potential warming when including arctic amplification is also substantial. In the global case, a carbon budget of around 1500 Gt CO_2 gives an even chance of a maximum temperature increase of 2°C. Whereas the same carbon budget, when including arctic amplification, gives an even chance for a maximum temperature increase of around 4.5°C.

The work done on implementing temperature-dependent forcing functions to the SRM framework is also applicable for this case of Arctic amplification. The effects of implementing the added forcing will have the same general effect, looking at the arctic, as when looking at global temperatures—making the range of temperatures more substantial and hence increasing the uncertainty of the maximum temperature increase associated with a given carbon budget. Functions displaying this increased uncertainty are published in Appendix A.

7 Conclusion

The SRM can, using impulse response functions and some atmospheric forcing parameterizations, compute temperature responses very similar to the MAGICC model. Hence, we have constructed a climate model with high scenario flexibility and low computational cost that performs similarly to a frequently used climate model. The SRM can be used as an alternative to MAGICC for reducing underappreciation of model uncertainties. The output showing the risk of warming associated with the size of a carbon budget is a advantageous way of conveying the risk of setting temperature goals related to emission budgets. Doing the "bare minimum" to reach a temperature target with a probability of 50% or 66% probability may still lead to greater warming than anticipated. The SRM clearly shows how the uncertainty in temperature response associated with carbon budget size increase with the size of the carbon budget. This uncertainty makes a solid argument for making more considerable efforts in mitigation rather than for uncertainties in adaptation. The work on Arctic amplification and implementing possible nonlinear effects further accentuates this point.

7.1 Additional work

The model should use scenarios from recent SSPs to ensure that the SRMs output stays relevant. Many factors can lead to the scenarios used now, being outdated in the future. The temperature-dependent forcing framework will be able to implement nonlinear forcing components better once their behaviour is less uncertain.

The addition of more elements to the SRM is very debatable. The SRM has both its strength and weakness in being a very simple model. The aim of the model is not to replicate the climate system as well as possible, but to emulate more complex models. Judging from the comparison with MAGICC, it does a good job emulating. More greenhouse gasses can be implemented relatively quickly to the model framework, such as ozone or nitrous dioxide. However, as already stated, as long as the SRM does a good job emulating the more complex models, the simpler, the better.

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A

Additional figures

Additional figures produced by the SRM throughout the project.

The hyperbolic tangent nonlinear forcing function is on the form:

$$F(T) = \text{strength} * 0.5 * (1 + \text{Tanh}(T - \text{threshold})/\text{steepness}).$$

The parameters strength, threshold and steepness will be denoted as: s, t and b. The linear temperature dependent forcing function is on the form:

$$F(T) = XT,$$

where F is forcing (Wm^{-2}), X is the slope ($\text{Wm}^{-2}{}^{\circ}\text{C}^{-1}$) and T is temperature ($^{\circ}$C)

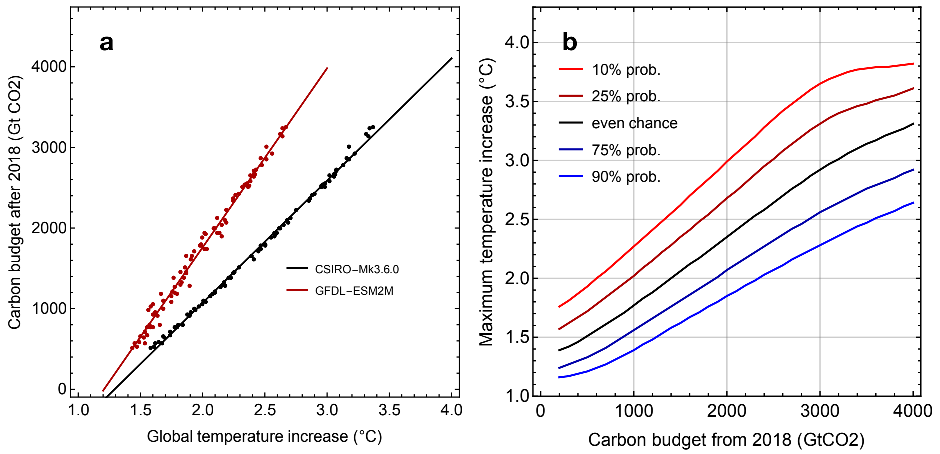

Figure A.1: Temperature dependent forcing addition following a linear function on the form $F = 0.1T$ for **a)** TCREs from CSIRO-Mk3.6.0 and **b)** Probability for maximum temperature increase (°C) for carbon budgets (Gt CO_2)

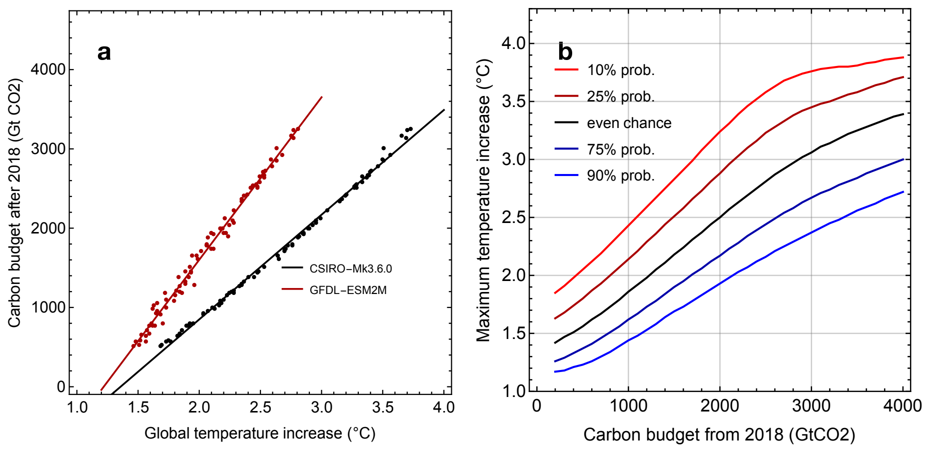

Figure A.2: Same as Figure A.1 with $F = 0.2T$

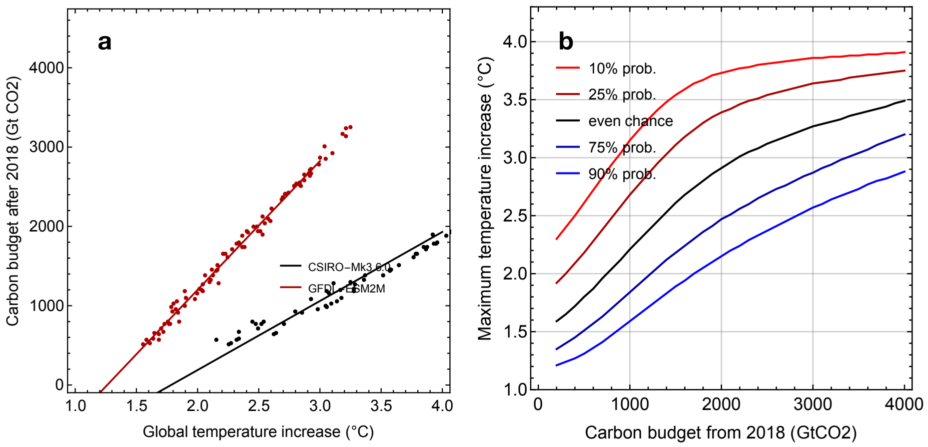

Figure A.3: Same as Figure A.1 with $F = 0.45T$

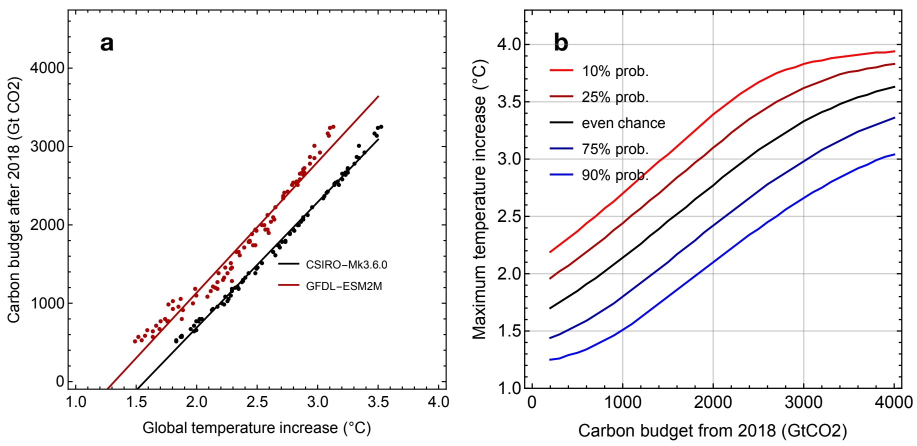

Figure A.4: Temperature dependent forcing addition of a nonlinear function with parameters: s=1, t=2 and b=0.5 for **a)** TCREs from CSIRO-Mk3.6.0 and **b)** Probability for maximum temperature increase (°C) for carbon budgets (Gt CO2)

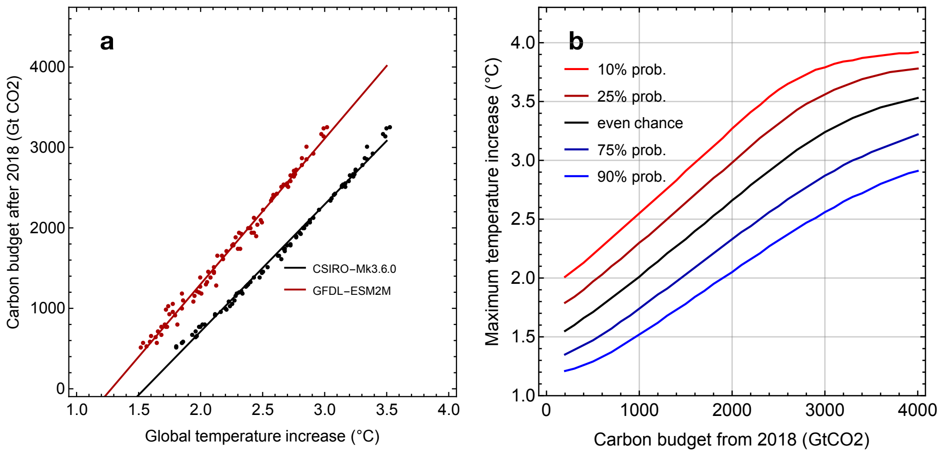

Figure A.5: Same as Figure A.4 with parameters: s=1, t=2 and b=1

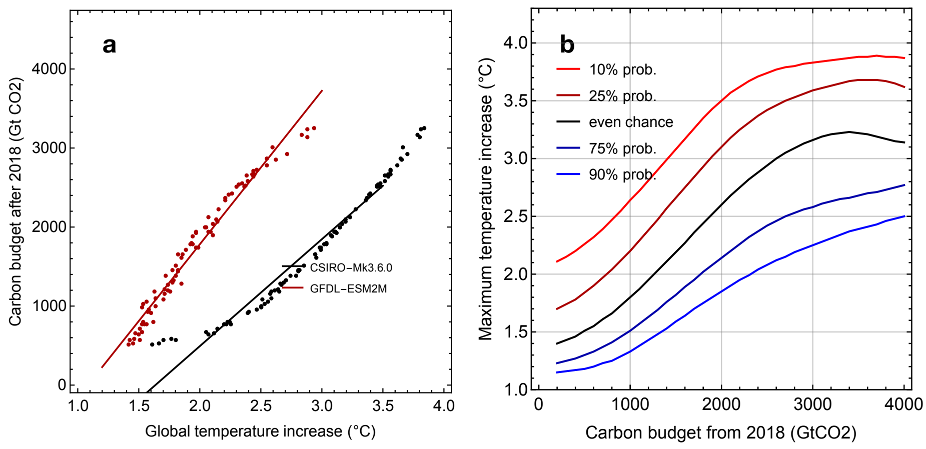

Figure A.6: Same as Figure A.4 with parameters: s=1, t=3 and b=0.5

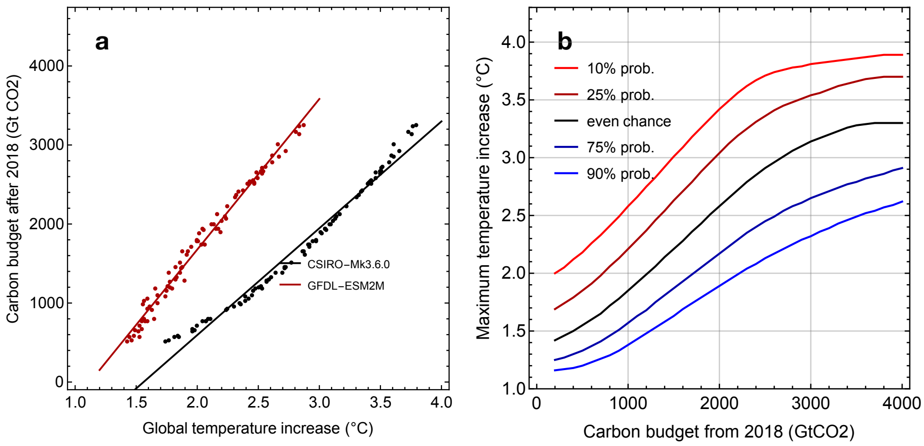

Figure A.7: Same as Figure A.4 with parameters: s=1, t=3 and b=1

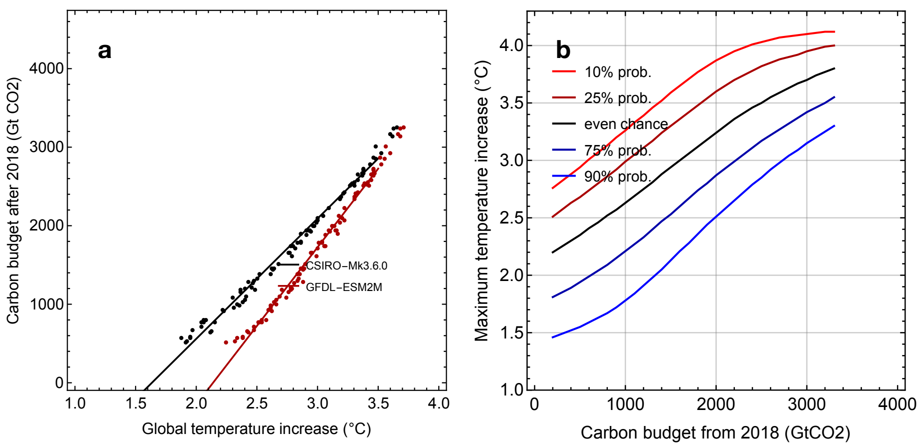

Figure A.8: Same as Figure A.4 with parameters: s=2, t=2 and b=0.5

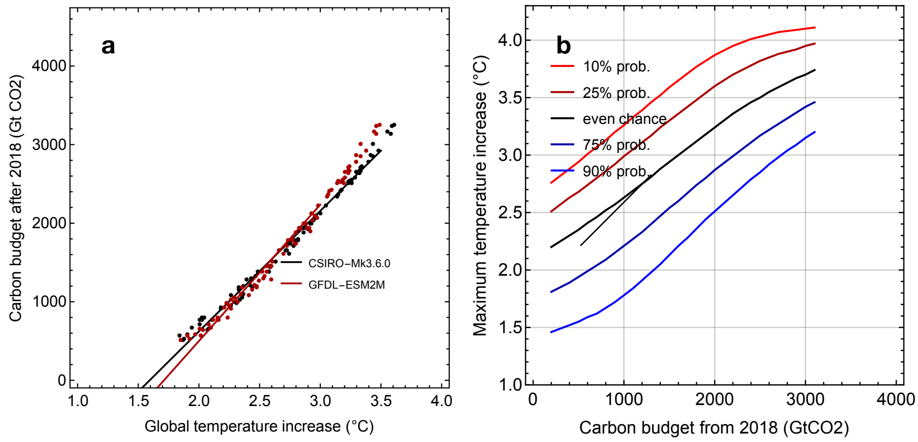

Figure A.9: Same as Figure A.4 with parameters: s=2, t=2 and b=1

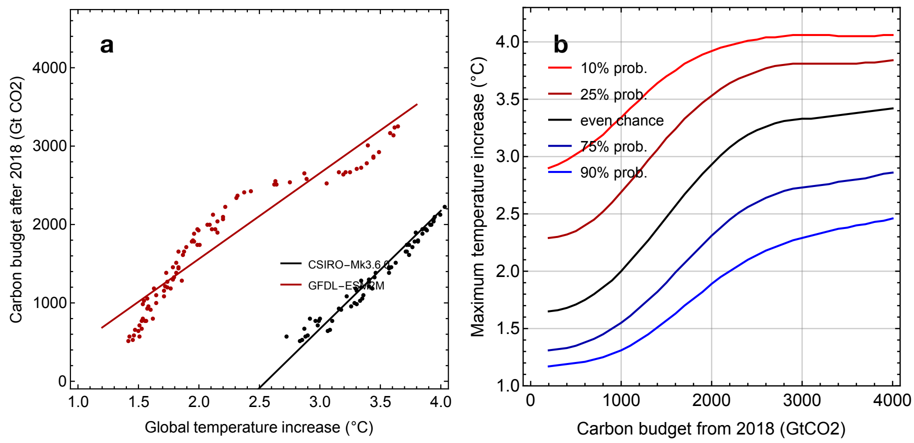

Figure A.10: Same as Figure A.4 with parameters: s=2, t=3 and b=0.5

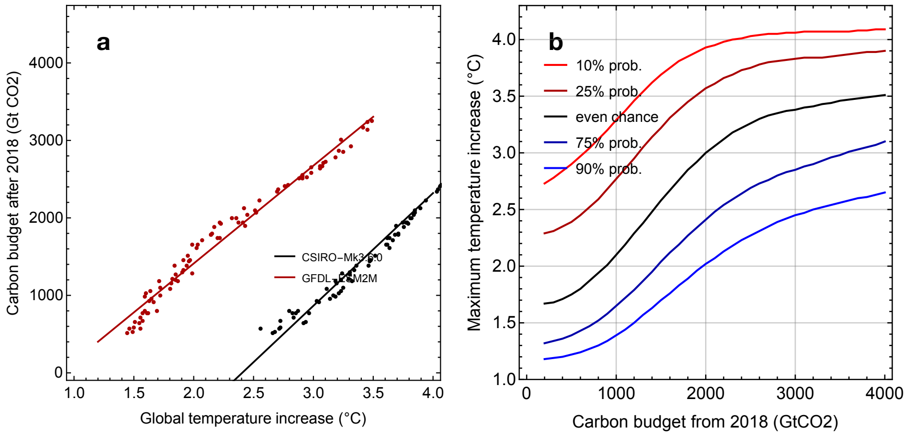

Figure A.11: Same as Figure A.4 with parameters: s=2, t=3 and b=1

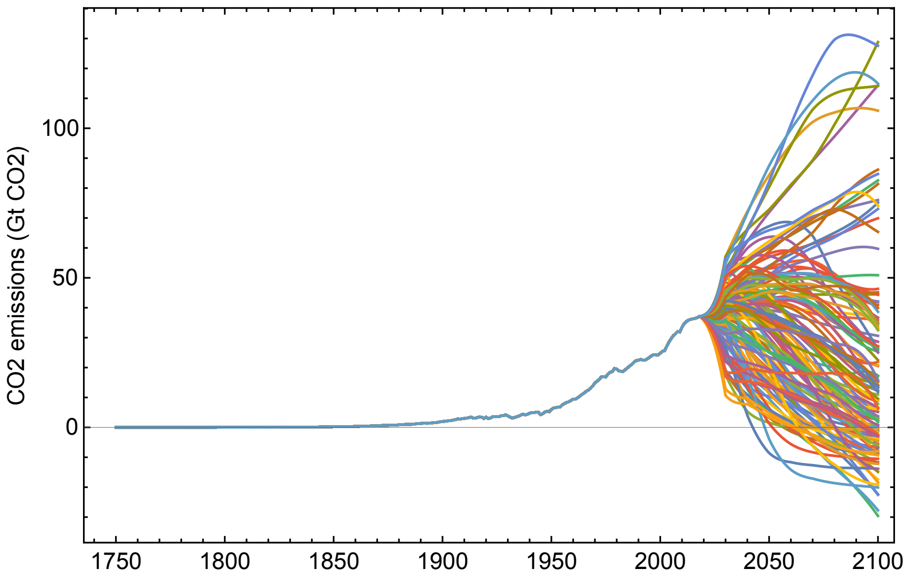

Figure A.12: All 127 emission scenarios from the SSP data-set.

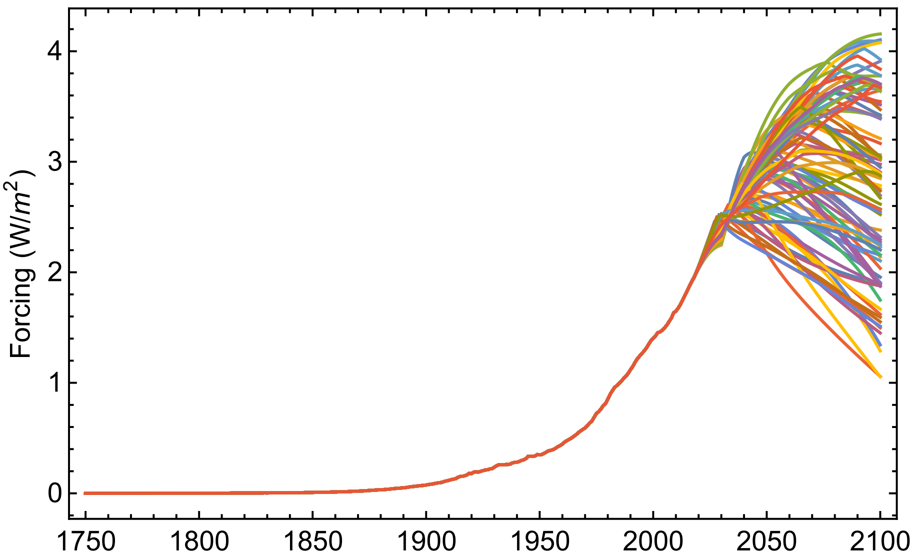

Figure A.13: Combined forcing from all forcing agents in the SRM

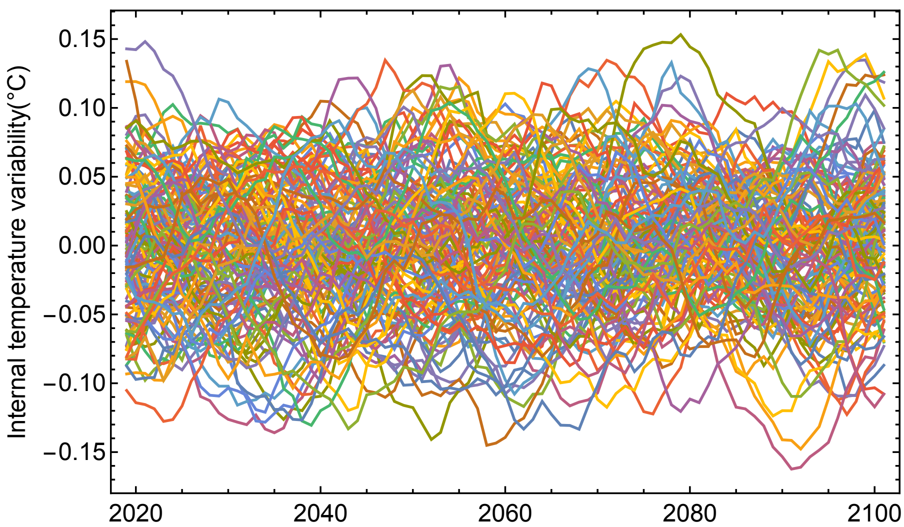

Figure A.14: Internal variability for all 127 scenarios

B

Mathematica code

This appendix provides the full *Mathematica* code fro the SRM used in the research project. The code for the SRM is run with Mathematica version: 12.0.0.0, Platform: Mac OS X x86 (64-bit). The code is produced in collaboration with Andreas Johansen and Endre Falck Mentzoni under the supervision of Martin Rypdal.

```
In[283]:= SetDirectory["OneDrive - UiT Office 365"];
Z = Import["SSP_IAM_V2_201811.csv"];
Z = Map[StringSplit[#, ","] &, Z];
```

In[5]:= **hh=157.65890684920566`+1.8942819330281027`zz+0.08520850267749702`zz^2;**

```
In[6]:= Z[[1]]
Out[6]= {{MODEL, "SCENARIO", "REGION", "VARIABLE", "UNIT",
2005, 2010, 2020, 2030, 2040, 2050, 2060, 2070, 2080, 2090, 2100}}
```

```
In[7]:= RR=Table[Z[[k]][[1]][[4]],{k,1,Length[Z]}];
Union[RR]
```

```
Out[8]= {"Agricultural Demand|Crops", "Agricultural Demand|Crops|Energy",
"Agricultural Demand|Livestock", "Agricultural Production|Crops|Energy",
"Agricultural Production|Crops|Non-Energy","Agricultural Production|Livestock", "Capacity|Electricity",
"Capacity|Electricity|Biomass", "Capacity|Electricity|Coal", "Capacity|Electricity|Gas",
"Capacity|Electricity|Geothermal", "Capacity|Electricity|Hydro", "Capacity|Electricity|Nuclear",
"Capacity|Electricity|Oil", "Capacity|Electricity|Other", "Capacity|Electricity|Solar", "Capacity|Electricity|Solar|CSP",
"Capacity|Electricity|Solar|PV", "Capacity|Electricity|Wind", "Capacity|Electricity|Wind|Offshore",
"Capacity|Electricity|Wind|Onshore", "Consumption", "Diagnostics|MAGICC6|Concentration|CH4",
"Diagnostics|MAGICC6|Concentration|CO2", "Diagnostics|MAGICC6|Concentration|N2O",
"Diagnostics|MAGICC6|Forcing", "Diagnostics|MAGICC6|Forcing|Aerosol", "Diagnostics|MAGICC6|Forcing|CH4",
"Diagnostics|MAGICC6|Forcing|CO2", "Diagnostics|MAGICC6|Forcing|F-Gases",
"Diagnostics|MAGICC6|Forcing|Kyoto Gases", "Diagnostics|MAGICC6|Forcing|N2O",
"Diagnostics|MAGICC6|Temperature|Global Mean", "Emissions|BC", "Emissions|CH4",
"Emissions|CH4|Fossil Fuels and Industry", "Emissions|CH4|Land Use", "Emissions|CO",
"Emissions|CO2", "Emissions|CO2|Carbon Capture and Storage",
"Emissions|CO2|Carbon Capture and Storage|Biomass","Emissions|CO2|Fossil Fuels and Industry",
"Emissions|CO2|Land Use", "Emissions|F-Gases", "Emissions|Kyoto Gases", "Emissions|N2O",
"Emissions|N2O|Land Use", "Emissions|NH3", "Emissions|NOx", "Emissions|OC", "Emissions|Sulfur",
"Emissions|VOC", "Energy Service|Transportation|Freight", "Energy Service|Transportation|Passenger",
"Final Energy","Final Energy|Electricity", "Final Energy|Gases", "Final Energy|Heat",
"Final Energy|Hydrogen", "Final Energy|Industry", "Final Energy|Liquids",
"Final Energy|Residential and Commercial", "Final Energy|Solar","Final Energy|Solids",
"Final Energy|Solids|Biomass","Final Energy|Solids|Biomass|Traditional", "Final Energy|Solids|Coal",
"Final Energy|Transportation", "GDP|PPP", "Harmonized Emissions|BC",
"Harmonized Emissions|CH4|Fossil Fuels and Industry","Harmonized Emissions|CH4|Land Use",
"Harmonized Emissions|CO","Harmonized Emissions|CO2|Fossil Fuels and Industry",
"Harmonized Emissions|CO2|Land Use", "Harmonized Emissions|F-Gases", "Harmonized Emissions|Kyoto Gases",
"Harmonized Emissions|NH3", "Harmonized Emissions|NOx", "Harmonized Emissions|OC",
"Harmonized Emissions|Sulfur", "Harmonized Emissions|VOC","Land Cover|Built-up Area",
"Land Cover|Cropland", "Land Cover|Forest", "Land Cover|Pasture", "Population", "Price|Carbon", "Primary Energy",
"Primary Energy|Biomass", "Primary Energy|Biomass|Traditional", "Primary Energy|Biomass|w/ CCS",
"Primary Energy|Biomass|w/o CCS", "Primary Energy|Coal", "Primary Energy|Coal|w/ CCS",
"Primary Energy|Coal|w/o CCS", "Primary Energy|Fossil", "Primary Energy|Fossil|w/ CCS",
"Primary Energy|Fossil|w/o CCS","Primary Energy|Gas", "Primary Energy|Gas|w/ CCS",
"Primary Energy|Gas|w/o CCS", "Primary Energy|Geothermal","Primary Energy|Hydro",
"Primary Energy|Non-Biomass Renewables","Primary Energy|Nuclear", "Primary Energy|Oil",
"Primary Energy|Oil|w/ CCS", "Primary Energy|Oil|w/o CCS","Primary Energy|Other",
"Primary Energy|Secondary Energy Trade","Primary Energy|Solar", "Primary Energy|Wind",
"Secondary Energy|Electricity", "Secondary Energy|Electricity|Biomass",
"Secondary Energy|Electricity|Biomass|w/ CCS","Secondary Energy|Electricity|Biomass|w/o CCS",
"Secondary Energy|Electricity|Coal","Secondary Energy|Electricity|Coal|w/ CCS",
"Secondary Energy|Electricity|Coal|w/o CCS","Secondary Energy|Electricity|Gas",
"Secondary Energy|Electricity|Gas|w/ CCS", "Secondary Energy|Electricity|Gas|w/o CCS",
"Secondary Energy|Electricity|Geothermal","Secondary Energy|Electricity|Hydro",
"Secondary Energy|Electricity|Non-Biomass Renewables","Secondary Energy|Electricity|Nuclear",
"Secondary Energy|Electricity|Oil", "Secondary Energy|Electricity|Solar", "Secondary Energy|Electricity|Wind",
"Secondary Energy|Gases", "Secondary Energy|Gases|Biomass","Secondary Energy|Gases|Coal",
"Secondary Energy|Gases|Natural Gas", "Secondary Energy|Heat", "Secondary Energy|Heat|Geothermal",
"Secondary Energy|Hydrogen", "Secondary Energy|Hydrogen|Biomass",
"Secondary Energy|Hydrogen|Biomass|w/ CCS", "Secondary Energy|Hydrogen|Biomass|w/o CCS",
"Secondary Energy|Hydrogen|Electricity", "Secondary Energy|Liquids", "Secondary Energy|Liquids|Biomass",
```

```
"Secondary Energy|Liquids|Biomass|w/ CCS", "Secondary Energy|Liquids|Biomass|w/o CCS",
"Secondary Energy|Liquids|Coal", "Secondary Energy|Liquids|Coal|w/ CCS",
"Secondary Energy|Liquids|Coal|w/o CCS", "Secondary Energy|Liquids|Gas",
"Secondary Energy|Liquids|Gas|w/ CCS", "Secondary Energy|Liquids|Gas|w/o CCS",
"Secondary Energy|Liquids|Oil", "Secondary Energy|Solids", "VARIABLE"}

In[9]:= RRR=Table[Z[[k]][[1]][[3]],{k,1,Length[Z]}];
Union[RRR]

Out[10]= {"R5.2ASIA", "R5.2LAM", "R5.2MAF", "R5.2OECD", "R5.2REF", "REGION", "World"}

In[11]:= co2pos1=Position[RR,_?(#=="\"Emissions|CO2|FossilFuelsandIndustry\""&)];
co2pos2 = Position[RR, _ ? (# == "\"Emissions|CO2|Land Use\"" &)];
co2pos3 = Position[RRR, _ ? (# == "\"World\"" &)];

In[14]:= ppos1=Intersection[co2pos3,co2pos1];
ppos2 = Intersection[co2pos3, co2pos2];

In[16]:= Extract[Z,co2pos1][[1]]

Out[16]=  {{AIM/CGE, "SSP1-19", "R5.2ASIA", "Emissions|CO2|Fossil Fuels and Industry", "Mt CO2/yr", 8985.6725,
10008.8152, 11790.747500000001,
6131.6627, 3271.4353000000006, 1678.8029, 638.87, 259.4755, 82.29590000000003, -7.9353000000000105, -
103.9171}}

In[17]:= em1=ToExpression[Map[Drop[Flatten[#],7]&,Extract[Z,ppos1]]];
em2 = ToExpression[Map[Drop[Flatten[#], 7] &, Extract[Z, ppos2]]];

In[19]:= ListPlot[em1,PlotRange→All,Joined→True]

In[20]:= Length[em1]
Out[20]= 127

In[21]:= emissions=Map[#[[1;;2]]&,ToExpression[Map[StringSplit[#] &, Drop[ReadList["emissionsCO2.txt",
String], 31]]]];
emissions = Table[{emissions[[i, 1]], (44 / 12) * emissions[[i, 2]] / 1000.}, {i, 1, Length[emissions]}];
ListPlot[emissions, Joined → True, PlotStyle → {Black, Thick}, PlotRange → All, Axes → False, Frame → True,
FrameStyle → Directive[14, Black],
FrameLabel → {"year", "CO2 emissions (Gt CO/yr)"}]
(*historical emissions*)

EM = Join[emissions, {{2018, 37.1}}];
data2 = Table[Prepend[Table[{t, Interpolation[Join[EM, Transpose[{{2030, 2040, 2050, 2060, 2070, 2080,
2090, 2100}, 0.001 * Drop[em1[[k]], 1]}]]][t]}, {t, 1751, 2100}], {1750, 0}], {k, 1, Length[em1]}]; totliste =
Table[data2[[k]][[All, 2]], {k, 1, Length[data2]}];
PLAll = ListPlot[data2, Joined → True, Frame → True, FrameStyle → Directive[Black, 14], PlotRange → All,
FrameLabel → {None, "CO2 emissions (Gt CO2)"}]
positivepaths = Table[DeleteCases[Map[# * UnitStep[#] &, totliste[[k]][[269 ;; 351]]], _ ? (# == 0 &)], {k, 1,
Length[totliste]}]; ListPlot[positivepaths, Joined → True, PlotRange → All]
(*Before removal of exceedance scenarios*)

In[29]:= RCBliste2=Map[Plus@@#&,positivepaths];

In[175]:= p1=Position[RCBliste2,_?(#<3300&)];

In[31]:= maxtemp=Map[Max[#]&,templiste];
maxtemp2 = Map[Max[#] &, uptempliste];
maxtemp3 = Map[Max[#] &, lowtempliste];

PL1 = ListPlot[Extract[data2, p1], Joined → True,Frame → True, FrameStyle → Directive[Black, 14], PlotRange
→ All];
```

```
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];
FFC = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["", 18],
Scaled[{0.1, 0.9}]]]
(*After removal of exceedance scenarios*)

In[37]:= PL1=ListPlot[data2[[1]],Joined→True,Frame→True, FrameStyle → Directive[Black, 14], PlotStyle →
Darker[Blue]];
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];
FFA = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["a", 18],
Scaled[{0.1, 0.9}]]]

In[40]:= n=Length[data2[[1]]];
futuretime = 2100 - 2020;
τmetan = 12.4;

In[43]:= (* Carbon model *)
τ1=1;
τ2=10;
τ3 = 100;
τ4 = 1000;
c1mean = 0.152;
c2mean = 0.246;
c4mean = 0.134;
c5mean = 0.194;

Gmean = (12/44) *0.47* (c1mean * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]
+ c2mean * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1mean - c2mean - c4mean -
c5mean)* Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]
+ c4mean * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + Table[c5mean * UnitStep[i - j], {i, 1, n}, {j,
1, n}]);

In[52]:= (* Carbon models *)
c1upper = 0.11;
c2upper = 0.212;
c4upper = 0.106;
c5upper = 0.262;
c1lower = 0.18;
c2lower = 0.296;
c4lower = 0.122;
c5lower = 0.148;

Glower = (12 / 44) * 0.47 * (c1lower * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c2lower *
Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1lower - c2lower - c4lower - c5lower) *
Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c4lower * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1,
n}, {j, 1, n}] + Table[c5lower * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

Gupper = (12/44) *0.47* (c1upper * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c2upper *
Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1upper - c2upper - c4upper - c5upper) *
Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c4upper * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i,
1, n}, {j, 1, n}] + Table[c5upper * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

In[62]:=
(*Optimal Estimation of Stochastic Energy Balance Model Parameters *)

In[63]:= (* Climate models *)
models = ReadList["CMIP5parameters.txt", String];
models = Delete[models, {{5}, {12}}];
boxes = StringSplit[models][[All, 2]];
Klimaliste = {};
Γliste = {};
σ2liste = {};
```

```
Monitor[
Do[

Clear[A];
modelnr = p; If[boxes[[p]] == "2",

{C1, C2, κ1, κ2, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1}, {κ2/C2, -κ2/C2}};
g = (MatrixExp[t A].{1 / C1, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If[ boxes[[p]] == "3",

{C1, C2, C3, κ1, κ2, κ3, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1, 0}, {κ2/C2, -(κ2+κ3)/C2, κ3/C2},{0, κ3/C3, - κ3/C3}};

g = (MatrixExp[t A].{1 / C1, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If [ boxes[[p]] == "4",

{C1, C2, C3, C4, κ1, κ2, κ3, κ4, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+κ2)/C1, κ2/C1, 0, 0} , {κ2/C2, -(κ2+κ3)/C2, κ3/C2, 0},{ 0, κ3/C3, -(κ3+κ4)/C3, κ4/C3},
{0, 0, κ4/C4, -κ4/C4}};

g = (MatrixExp[t A].{1 / C1, 0, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

Γliste = Append[Γliste, Γ];
σ2liste = Append[σ2liste, σ2];
, {p, 1, Length[models]}
];
, {p, boxes[[p]]}
];

In[70]:= RCBliste={};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
Δfaeroliste = {};
Δfghgliste = {};
Δfliste = {};
noiseliste = {};

Monitor[
Do[
tot = data2[[u]][[All, 2]]; meanco2 = Gmean.tot + 280;

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 11.9 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *) ;
```

```
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.34 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];

Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfaer= -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ;
Δfaer2 = Drop[Δfaer, Length[EM]] ;
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;

Δfliste = Append[Δfliste, Δf];
Δfaeroliste = Append[Δfaeroliste, Δfaer];
Δfghgliste = Append[Δfghgliste, Δfco2 + Δfmetan];

Tliste = {};
Do[
T2 = Klimaliste[[p]].Δf;
noise = σ2liste[[p]] * (Klimaliste[[p]].RandomReal[NormalDistribution[0, 1], Length[Δf]]);
noise = Drop[noise, 268 - 20];
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.1+T2-T2[[1]];
noiseliste = Append[noiseliste, noise]; Tliste = Append[Tliste, T2];
, {p, 1, Length[models]}];

middel =Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}]
, u];
```

In[183]:= **Length[noise]**
Out[183]= 103

In[82]:= **Length[T2]**
Out[82]= 83

In[83]:=
```
window=10;
noiseliste2 = Table[MovingAverage[noiseliste[[i]], window][[1 ;; Length[T2]]], {i, 1, Length[noiseliste]}];
noiseliste2 = Transpose[Partition[noiseliste2, 14]];
```

In[86]:= **Length[noiseliste2]**
Out[86]= 14

In[87]:= **Dimensions[noiseliste2]**
Out[87]= {14, 127, 83}

```
In[88]:= Length[noiseliste2[[1]]]
Out[88]= 127

PLNoise = ListPlot[Map[Transpose[{2018 + Range[Length[templiste[[1]]]], #}] &, noiseliste2[[3]]], Joined →
True];
FFNoise = Show[PLNoise, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle →
Directive[Black, 14], Joined → True, FrameLabel → {None, "Internal temperature variability(°C)"}, Epilog →
Inset[Style["", 18], Scaled[{0.1, 0.9}]]]
(*Plot of internal variability*)

In[90]:= FFE=ListPlot[Map[Transpose[{2018+Range[Length[templiste[[1]]]],#}]&, Extract[templiste, p1]],
PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel
→ {None, "GMST increase (°C)"},Epilog → Inset[Style["e", 18], Scaled[{0.1, 0.9}]]]
(*Temperature response*)

In[121]:= PL3=ListPlot[Map[Transpose[{1749+Range[Length[Δfaeroliste[[1]]]],#}]&, Extract[Δfliste, p1]], Joined
→ True];
FF3 = Show[PL3, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black,
14], FrameLabel → {None, "Forcing (W/m^2)"}, Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]]]
(*Combined forcing for both ghg's and aerosols*)

In[150]:= PL1=ListPlot[Map[Transpose[{1749+Range[Length[Δfaeroliste[[1]]]],#}]&, Extract[Δfaeroliste, p1]],
Joined → True];
PL2 = ListPlot[Map[Transpose[{1749 + Range[Length[Δfaeroliste[[1]]]], #}] &, Extract[Δfghgliste, p1]], Joined
→ True];
FFD = Show[{PL1, PL2}, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle →
Directive[Black, 14], FrameLabel → {None, "Forcing (W/m^2)"}, Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]]]
(*Split forcing for ghg's and aerosols*)

In[94]:=modellfarger={,,,,,,,,,,,,,,, };
In[95]:= pan=LineLegend[modellfarger,Map[StringSplit[#]&,models][[All,1]]]
Out[95]= BCC-CSM1-1 BNU-ESM CanESM2 CCSM4 CSIRO-Mk3.6.0 FGOALS-s2 GFDL-ESM2M GISS-E2-R
HadGEM2-ES INM-CM4 MIROC5 MPI-ESM-LR MRI-CGCM3 NorESM1-M

FFB = ListPlot[Map[Transpose[{2018 + Range[Length[alltliste[[1]]]], #}] &, alltliste[[1 ;; 14]]], PlotRange →
All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None,
"GMST increase (°C)"},Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]],PlotStyle → Map[{#} &, modellfarger]]
(*Plot for temperature response, 1 scenario and 14 ESMs*)

In[*]:= Grid[{{Show[FFA,ImageSize→400],Show[FFB,ImageSize→400],pan}}]
(*Grid plot for one scenario, 14 ESMs*)

Grid[{{Show[FFC, ImageSize → 400,Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]]], Show[FFD, ImageSize →
400, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]]] , Show[FFE, ImageSize → 400, Epilog → Inset[Style["c",
18], Scaled[{0.1, 0.9}]]]}}]
(*Grid plot for emissions, forcing and GMST for 86 scenarios and 1 esm*)

maxtemp = Map[Max[#] &, templiste]; (*mean*)
maxtemp2 = Map[Max[#] &, uptempliste]; (*+1sd*)
maxtemp3 = Map[Max[#] &, lowtempliste] (*-1sd*)

In[*]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1], AspectRatio → 1, PlotRange → All];
PL3 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Red];
PL4 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Red];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz]; PL2 = Plot[gg, {zz, 1.2, 3}];
gg2 = Fit[Extract[Transpose[{maxtemp2, RCBliste2}], p1], {zz, 1}, zz]; PL4 = Plot[gg2, {zz, 1.2, 3}];
Show[{PL1, PL2}, PlotRange → All]
(*TCRE plot w/o pdf*)
```

In[*]:= pairs=Extract[Transpose[{maxtemp,RCBliste2}],p1];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];

σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];

In[*]:= pdf=(PDF[NormalDistribution[gg,σf[zz]]][p])/.zz→1.5;
pdf2 = (PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → 2.5;
Plot[{pdf, pdf2}, {p, 0, 5200}, PlotRange → All]

In[*]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1],AspectRatio→1, PlotRange → {{1, 4}, {0, 4000}}, PlotStyle → Darker[Blue]];
PL2 = Plot[gg, {zz, 1, 3}, PlotStyle → Darker[Blue]];
l1 = Graphics[{Black, Line[{{1.5, 0}, {1.5, gg /. zz → 1.5}}]}];
l2 = Graphics[{Black, Line[{{1.5, gg /. zz → 1.5}, {1, gg /. zz → 1.5}}]}]; inset = ParametricPlot[{1 + 60 * pdf, p}, {p, 0, 1200}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];

l11 = Graphics[{Black, Line[{{2.5, 0}, {2.5, gg /. zz → 2.5}}]}];
l22 = Graphics[{Black, Line[{{2.5, gg /. zz → 2.5}, {1, gg /. zz → 2.5}}]}]; inset2 = ParametricPlot[{1 + 60 * pdf2, p}, {p, 1900, 3000}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];

FA = Show[{PL1, PL2, l1, l2, l11, l22, inset, inset2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {"global temperature increase (°C)", "carbon budget after 2018 (Gt CO2)"}, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]], ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]
(*TCRE with pdf, 1 ESM*)

ALL CLIMATE MODELS

In[*]:= maxtemp=Partition[Map[Max[#]&,alltliste],14][[All,5]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Black];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 3.5}, PlotStyle → Black];
QL1 = Show[{PL1, PL2}, PlotRange → All];
maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, 7]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Darker[Red]];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz]; PL2 = Plot[gg, {zz, 1.2, 3}, PlotStyle → Darker[Red]];
QL2 = Show[{PL1, PL2}, PlotRange → All];
FB = Show[{QL1, QL2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {"global temperature increase (°C)", "carbon budget after 2018 (Gt CO2)"}, Epilog → {Inset[Style["b", 18], Scaled[{0.1, 0.9}]], Inset[LineLegend[{Black, Darker[Red]}, {"CSIRO-Mk3.6.0", "GFDL-ESM2M"}], Scaled[{0.7, 0.3}]]}, ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]

In[*]:= Grid[{{FA,FB}}]
(*TCREs for two ESM's*)

(*MEAN TCRE CALCULATION*)

In[*]:= maxtemp1=Partition[Map[Max[#]&,alltliste],14][[All,1]];
maxtemp2 = Partition[Map[Max[#] &, alltliste], 14][[All, 2]];
maxtemp3 = Partition[Map[Max[#] &, alltliste], 14][[All, 3]];
maxtemp4 = Partition[Map[Max[#] &, alltliste], 14][[All, 4]];
maxtemp5 = Partition[Map[Max[#] &, alltliste], 14][[All, 5]];
maxtemp6 = Partition[Map[Max[#] &, alltliste], 14][[All, 6]];
maxtemp7 = Partition[Map[Max[#] &, alltliste], 14][[All, 7]];
maxtemp8 = Partition[Map[Max[#] &, alltliste], 14][[All, 8]];
maxtemp9 = Partition[Map[Max[#] &, alltliste], 14][[All, 9]];

```
maxtemp10 = Partition[Map[Max[#] &, alltliste], 14][[All, 10]];
maxtemp11 = Partition[Map[Max[#] &, alltliste], 14][[All, 11]];
maxtemp12 = Partition[Map[Max[#] &, alltliste], 14][[All, 12]];
maxtemp13 = Partition[Map[Max[#] &, alltliste], 14][[All, 13]];
maxtemp14 = Partition[Map[Max[#] &, alltliste], 14][[All, 14]];

ggm1 = Fit[Extract[Transpose[{maxtemp1, RCBliste2}], p1], {zz, 1}, zz];
ggm2 = Fit[Extract[Transpose[{maxtemp2, RCBliste2}], p1], {zz, 1}, zz];
ggm3 = Fit[Extract[Transpose[{maxtemp3, RCBliste2}], p1], {zz, 1}, zz];
ggm4 = Fit[Extract[Transpose[{maxtemp4, RCBliste2}], p1], {zz, 1}, zz];
ggm5 = Fit[Extract[Transpose[{maxtemp5, RCBliste2}], p1], {zz, 1}, zz];
ggm6 = Fit[Extract[Transpose[{maxtemp6, RCBliste2}], p1], {zz, 1}, zz];
ggm7 = Fit[Extract[Transpose[{maxtemp7, RCBliste2}], p1], {zz, 1}, zz];
ggm8 = Fit[Extract[Transpose[{maxtemp8, RCBliste2}], p1], {zz, 1}, zz];
ggm9 = Fit[Extract[Transpose[{maxtemp9, RCBliste2}], p1], {zz, 1}, zz];
ggm10 = Fit[Extract[Transpose[{maxtemp10, RCBliste2}], p1], {zz, 1}, zz];
ggm11 = Fit[Extract[Transpose[{maxtemp11, RCBliste2}], p1], {zz, 1}, zz];
ggm12 = Fit[Extract[Transpose[{maxtemp12, RCBliste2}], p1], {zz, 1}, zz];
ggm13 = Fit[Extract[Transpose[{maxtemp13, RCBliste2}], p1], {zz, 1}, zz];
ggm14 = Fit[Extract[Transpose[{maxtemp14, RCBliste2}], p1], {zz, 1}, zz];

PLm5 = ListPlot[Extract[Transpose[{maxtemp5, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle
→ Black];
PLm7 = ListPlot[Extract[Transpose[{maxtemp7, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle
→ Darker[Red]];
PLm7dot = Plot[ggm7, {zz, 1.2, 3}, PlotStyle → Darker[Red]]; PLm5dot = Plot[ggm5, {zz, 1.2, 3}, PlotStyle →
Black];

meanTCRE=
(ggm1+ggm2+ggm3+ggm4+ggm5+ggm6+ggm7+ggm8+ggm9+ggm10+ggm11+ggm12+ggm13+ggm14)/14;

PLmeanTCRE = Plot[meanTCRE, {zz, 1.2, 3}, PlotStyle → {Black, Dashed}];
QLmeanTCRE = Show[{PLm5dot, PLm5, PLm7dot, PLm7, PLmeanTCRE}, Axes → False, Frame → True,
FrameStyle → Directive[Black, 14], AspectRatio → 1, FrameLabel → {"global temperature increase (°C)",
"carbon budget after 2018 (Gt CO2)"}, ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}},Epilog →
Inset[LineLegend[{Black, {Black, Dashed}, Darker[Red]}, {"CSIRO-Mk3.6.0", "Mean TCRE", "GFDL-ESM2M"}],
Scaled[{0.7, 0.3}]]]
(*mean TCRE plot*)
```

PDF estimation

```
In[*]:= smliste={};
tliste = {};
Monitor[ Do[

pdfliste = {}; Do[
maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, kk]];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
pairs = Extract[Transpose[{maxtemp, RCBliste2}], p1];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
, {kk, 1, 14}];

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x]; sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];
```

```
In[*]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]
In[*]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[*]:= bliste={};
Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]]; y = y/ ((Plus@@y) * Δt);
t1= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.90 &)]][[1]] - 1]];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.75 &)]][[1]] - 1]];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.5 &)]][[1]] - 1]];
t4= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.25 &)]][[1]] - 1]];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.10 &)]][[1]] - 1]];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}];
, {budget, 200, 4000, 100}]

In[*]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};

In[*]:= GGA=ListPlot[{Transpose[{bliste[[All,1]],bliste[[All,2]]}], Transpose[{bliste[[All, 1]], bliste[[All, 3]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], bliste[[All, 5]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 6]]}]}, Joined → True, AspectRatio → 1, PlotRange → {1, 4}, Axes → False,
Frame → True, FrameStyle → Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines →
Automatic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)"},
PlotLegends → Placed[{"10% prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05,
0.7}], {0, 0.5}}]]

(*RCB plot one carbon, 14 ESMs*)
```

TWO CARBON MODELS, MEAN CLIMATE MODEL

```
In[*]:= RCBliste={};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
Monitor[
Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
```

```
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
Δfaer= -0.02tot; (* aerosols *)
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ;
Δfaer2 = Drop[Δfaer, Length[EM]] ;
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];  (*-0.4 asymptote*)
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;

Tliste = {};
Do[
(*T2=Klimaliste[[p]].Δf;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268]; T2=1.0+T2-T2[[1]];
Tliste=Append[Tliste,T2];*)

T2 = Klimaliste[[p]].Δfupper;
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268]; T2=1.1+T2-T2[[1]];
Tliste = Append[Tliste, T2];

T2 = Klimaliste[[p]].Δflower;
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268]; T2=1.0+T2-T2[[1]];
Tliste = Append[Tliste, T2];
, {p, 1, Length[models]}];

Tlisteupper = Partition[Tliste, 2][[All, 1]];
Tlistelower = Partition[Tliste, 2][[All, 2]];
meanupper = Map[Mean[#] &, Transpose[Tlisteupper]];
meanlower = Map[Mean[#] &, Transpose[Tlistelower]];
Tliste = {meanupper, meanlower};

middel =Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];

RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}]
,u]
```

```
In[*]:= Dimensions[alltliste]
Out[*]= {254, 83}
```

```
In[*]:= 3556/14
Out[*]= 254
```

```
In[*]:= CM = {};
cm1 =
Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2], p1][[kk]], 2][[ All, 1]], {kk, 1, Length[p1]}];
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2], p1][[kk]], 2][[All, 2]], {kk, 1,
Length[p1]}];

Do[
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]];
```

```
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]];
, {j, 1, Length[Transpose[cm2]]}];

In[*]:= ListPlot[{CM[[1]],CM[[2]]}]

In[*]:= smliste={};
tliste = {};

Monitor[ Do[
pdfliste = {}; Do[
gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
, {kk, 1, 2}];

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];

In[*]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[*]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[*]:= bliste={};
Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]]; y = y/ ((Plus@@y) * Δt);
t1= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.90 &)]][[1]] - 1]];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.75 &)]][[1]] - 1]];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.5 &)]][[1]] - 1]];
t4= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.25 &)]][[1]] - 1]];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.10 &)]][[1]] - 1]];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}]; , {budget, 200, 4000, 100}]

In[*]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};

In[*]:= a=ListPlot[{Transpose[{bliste[[All,1]],bliste[[All,2]]}], Transpose[{bliste[[All, 1]], bliste[[All, 3]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], bliste[[All, 5]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 6]]}]}, Joined → True, AspectRatio → 1, PlotRange → {1, 4}, Axes → False,
Frame → True, FrameStyle → Directive[Black, 14],PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines →
Automatic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)"},
PlotLegends → Placed[{"10% prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05,
0.7}], {0, 0.5}}]]; l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]}];
GGB = Show[{a}]

(*RCB estimate two carbons, mean ESM*)
```

INTERNAL VARIABILITY

```
RCBliste = {};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};

Monitor[ Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
(* aerosols *)
Δfaer= -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ; Δfaer2 = Drop[Δfaer, Length[EM]] ; Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;

Tliste = {};
Do[
T2 = Klimaliste[[p]].Δf;
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.1+T2-T2[[1]];
Tliste = Append[Tliste, T2 + StandardDeviation[Flatten[noiseliste2[[p]]]]];
, {p, 1, Length[models]}];

Tlisteupper = Partition[Tliste, 2][[All, 1]];
Tlistelower = Partition[Tliste, 2][[All, 2]];
mupper = {};
mlower = {};

Do[
meanupper = Map[Mean[#] &, Transpose[Tlisteupper]] + StandardDeviation[Flatten[noiseliste2[[p]]]];
meanlower = Map[Mean[#] &, Transpose[Tlistelower]] - StandardDeviation[Flatten[noiseliste2[[p]]]];
mupper = Append[mupper, meanupper]; mlower = Append[mlower, meanlower]; , {p, 1, 14}];
Tliste = Join[mupper, mlower];

middel =Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
```

```
RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}]
,u]
```

In[*]:= Dimensions[alltliste]
Out[*]= {3556, 83}

```
In[*]:= CM = {};
cm1 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[ All, 1]], {kk, 1, Length[p1]}];
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[ kk]], 2][[All, 2]], {kk, 1, Length[p1]}];

Do[ CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]];
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]]; , {j, 1, Length[Transpose[cm2]]}];
```

```
In[*]:= ListPlot[{CM[[1]],CM[[2]]}]
```

```
In[*]:= smliste={};
tliste = {};
Monitor[ Do[

pdfliste = {}; Do[
gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
,{kk,1,2*14}];

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];
```

```
In[*]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]
```

```
In[*]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]
```

```
In[*]:= bliste={};
Do[ Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt);
t1= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.90 &)]][[1]] - 1]];
```

```
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.75 &)]][[1]] - 1]];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.5 &)]][[1]] - 1]];
t4= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.25 &)]][[1]] - 1]];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.10 &)]][[1]] - 1]];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}];
, {budget, 200, 4000, 100}]

In[*]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};

In[*]:= a=ListPlot[{Transpose[{bliste[[All,1]],bliste[[All,2]]}], Transpose[{bliste[[All, 1]], bliste[[All, 3]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], bliste[[All, 5]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 6]]}]}, Joined → True, AspectRatio → 1, PlotRange → {1, 4.0}, Axes →
False, Frame → True, FrameStyle → Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines →
Automatic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)"},
PlotLegends → Placed[{"10% prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05,
0.7}], {0, 0.5}}]];
l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]}];
GGC = Show[{a}]
(*RCB estimate internal variability*)
```

COMBINATION BETWEEN TWO CARBON MODELS, 14 ESMs AND INTERNAL VARIABILITY

```
RCBliste = {};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};

Monitor[ Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];

Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
(* aerosols *)
Δfaer= -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ; Δfaer2 = Drop[Δfaer, Length[EM]] ; Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;

Tliste = {};
Do[
```

```
T2 = Klimaliste[[p]].Δfupper;
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.1+T2-T2[[1]];
Tliste = Append[Tliste, T2 + StandardDeviation[Flatten[noiseliste2[[p]]]]];

(*T2=Klimaliste[[p]].Δfupper;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.1+T2-T2[[1]]; Tliste=Append[Tliste,T2-StandardDeviation[Flatten[noiseliste2[[p]]]]];*)

(*T2=Klimaliste[[p]].Δflower;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.0+T2-T2[[1]]; Tliste=Append[Tliste,T2+StandardDeviation[Flatten[noiseliste2[[p]]]]];*)

T2 = Klimaliste[[p]].Δflower;
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.0+T2-T2[[1]];
Tliste = Append[Tliste, T2 - StandardDeviation[Flatten[noiseliste2[[p]]]]];

, {p, 1, Length[models]}];

middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];

alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}]
,u]
```

```
In[*]:= Dimensions[alltliste]
Out[*]= {3556, 83}
```

```
In[*]:= CM = {};
cm1 = Table[ Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[ All, 1]], {kk, 1,
Length[p1]}];
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[ kk]], 2][[All, 2]], {kk, 1,
Length[p1]}];

Do[
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]];
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]];
, {j, 1, Length[Transpose[cm2]]}];
```

```
In[*]:= ListPlot[{CM[[1]],CM[[2]]}]
```

```
In[*]:= smliste={};
tliste = {};
Monitor[ Do[
pdfliste = {}; Do[
```

```
gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
,{kk,1,2*14}];

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];

In[*]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[*]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[*]:= bliste={};
Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt);
t1= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.90 &)]][[1]] - 1]];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.75 &)]][[1]] - 1]];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.5 &)]][[1]] - 1]];
t4= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.25 &)]][[1]] - 1]];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.10 &)]][[1]] - 1]];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}];
, {budget, 200, 4000, 100}]

In[*]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};

In[*]:= a=ListPlot[{Transpose[{bliste[[All,1]],bliste[[All,2]]}], Transpose[{bliste[[All, 1]], bliste[[All, 3]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], bliste[[All, 5]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 6]]}]}, Joined → True, AspectRatio → 1, PlotRange → {1, 4.0}, Axes →
False, Frame → True, FrameStyle → Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines →
Automatic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)"},
PlotLegends → Placed[{"10% prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05,
0.7}], {0, 0.5}}]];
l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]}];
GGD = Show[{a}]
(*RCB estimate for 2 carbon models, 14 ESMs and internal variability*)

(*Comparison plot*)
```

(*GGA = all climate - GGB = mean climate + 2 carbon - GGC = mean climate + mean carbon + internal - GGD = All climate + 2 carbon + internal*)

```
Grid[{{
Show[GGA, PlotRange → {{0, 4000}, {1.1, 4.3}}, ImageSize → 380, Epilog → Inset[Style["a", 18], Scaled[{0.1,
0.9}]]],
```

```
Show[GGB, PlotRange → {{0, 4000}, {1.1, 4.3}}, ImageSize → 380, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]]]},

{Show[GGC, PlotRange → {{0, 4000}, {1.1, 4.3}}, ImageSize → 380, Epilog → Inset[Style["c", 18], Scaled[{0.1, 0.9}]]],
Show[GGD, PlotRange → {{0, 4000}, {1.1, 4.3}}, ImageSize → 380, Epilog → Inset[Style["d", 18], Scaled[{0.1, 0.9}]]]}}]

(*Grid plot *)
```

CODE FOR IMPLEMENTATION OF NON-LINEAR FRAMEWORK.

```
In[283]:= SetDirectory["OneDrive - UiT Office 365"];
Z = Import["SSP_IAM_V2_201811.csv"];
Z = Map[StringSplit[#, ","] &, Z];

In[5]:=  hh=157.65890684920566`+1.8942819330281027`zz+0.08520850267749702`zz^2;

In[6]:= Z[[1]]
Out[6]= {{MODEL, "SCENARIO", "REGION", "VARIABLE", "UNIT",
2005, 2010, 2020, 2030, 2040, 2050, 2060, 2070, 2080, 2090, 2100}}

In[7]:= RR=Table[Z[[k]][[1]][[4]],{k,1,Length[Z]}];
Union[RR]
```

```
Out[8]= {"Agricultural Demand|Crops", "Agricultural Demand|Crops|Energy",
"Agricultural Demand|Livestock", "Agricultural Production|Crops|Energy",
"Agricultural Production|Crops|Non-Energy","Agricultural Production|Livestock", "Capacity|Electricity",
"Capacity|Electricity|Biomass", "Capacity|Electricity|Coal", "Capacity|Electricity|Gas",
"Capacity|Electricity|Geothermal", "Capacity|Electricity|Hydro", "Capacity|Electricity|Nuclear",
"Capacity|Electricity|Oil", "Capacity|Electricity|Other", "Capacity|Electricity|Solar", "Capacity|Electricity|Solar|CSP",
"Capacity|Electricity|Solar|PV", "Capacity|Electricity|Wind", "Capacity|Electricity|Wind|Offshore",
"Capacity|Electricity|Wind|Onshore", "Consumption", "Diagnostics|MAGICC6|Concentration|CH4",
"Diagnostics|MAGICC6|Concentration|CO2", "Diagnostics|MAGICC6|Concentration|N2O",
"Diagnostics|MAGICC6|Forcing", "Diagnostics|MAGICC6|Forcing|Aerosol", "Diagnostics|MAGICC6|Forcing|CH4",
"Diagnostics|MAGICC6|Forcing|CO2", "Diagnostics|MAGICC6|Forcing|F-Gases",
"Diagnostics|MAGICC6|Forcing|Kyoto Gases", "Diagnostics|MAGICC6|Forcing|N2O",
"Diagnostics|MAGICC6|Temperature|Global Mean", "Emissions|BC", "Emissions|CH4",
"Emissions|CH4|Fossil Fuels and Industry", "Emissions|CH4|Land Use", "Emissions|CO",
"Emissions|CO2", "Emissions|CO2|Carbon Capture and Storage",
"Emissions|CO2|Carbon Capture and Storage|Biomass","Emissions|CO2|Fossil Fuels and Industry",
"Emissions|CO2|Land Use", "Emissions|F-Gases", "Emissions|Kyoto Gases", "Emissions|N2O",
"Emissions|N2O|Land Use", "Emissions|NH3", "Emissions|NOx", "Emissions|OC", "Emissions|Sulfur",
"Emissions|VOC", "Energy Service|Transportation|Freight", "Energy Service|Transportation|Passenger",
"Final Energy","Final Energy|Electricity", "Final Energy|Gases", "Final Energy|Heat",
"Final Energy|Hydrogen", "Final Energy|Industry", "Final Energy|Liquids",
"Final Energy|Residential and Commercial", "Final Energy|Solar","Final Energy|Solids",
"Final Energy|Solids|Biomass","Final Energy|Solids|Biomass|Traditional", "Final Energy|Solids|Coal",
"Final Energy|Transportation", "GDP|PPP", "Harmonized Emissions|BC",
"Harmonized Emissions|CH4|Fossil Fuels and Industry","Harmonized Emissions|CH4|Land Use",
"Harmonized Emissions|CO","Harmonized Emissions|CO2|Fossil Fuels and Industry",
"Harmonized Emissions|CO2|Land Use", "Harmonized Emissions|F-Gases", "Harmonized Emissions|Kyoto Gases",
"Harmonized Emissions|NH3", "Harmonized Emissions|NOx", "Harmonized Emissions|OC",
"Harmonized Emissions|Sulfur", "Harmonized Emissions|VOC","Land Cover|Built-up Area",
"Land Cover|Cropland", "Land Cover|Forest", "Land Cover|Pasture", "Population", "Price|Carbon", "Primary Energy",
"Primary Energy|Biomass", "Primary Energy|Biomass|Traditional", "Primary Energy|Biomass|w/ CCS",
"Primary Energy|Biomass|w/o CCS", "Primary Energy|Coal", "Primary Energy|Coal|w/ CCS",
"Primary Energy|Coal|w/o CCS", "Primary Energy|Fossil", "Primary Energy|Fossil|w/ CCS",
"Primary Energy|Fossil|w/o CCS","Primary Energy|Gas", "Primary Energy|Gas|w/ CCS",
"Primary Energy|Gas|w/o CCS", "Primary Energy|Geothermal","Primary Energy|Hydro",
"Primary Energy|Non-Biomass Renewables","Primary Energy|Nuclear", "Primary Energy|Oil",
"Primary Energy|Oil|w/ CCS", "Primary Energy|Oil|w/o CCS","Primary Energy|Other",
"Primary Energy|Secondary Energy Trade","Primary Energy|Solar", "Primary Energy|Wind",
"Secondary Energy|Electricity", "Secondary Energy|Electricity|Biomass",
"Secondary Energy|Electricity|Biomass|w/ CCS","Secondary Energy|Electricity|Biomass|w/o CCS",
"Secondary Energy|Electricity|Coal","Secondary Energy|Electricity|Coal|w/ CCS",
"Secondary Energy|Electricity|Coal|w/o CCS","Secondary Energy|Electricity|Gas",
"Secondary Energy|Electricity|Gas|w/ CCS", "Secondary Energy|Electricity|Gas|w/o CCS",
"Secondary Energy|Electricity|Geothermal","Secondary Energy|Electricity|Hydro",
"Secondary Energy|Electricity|Non-Biomass Renewables","Secondary Energy|Electricity|Nuclear",
"Secondary Energy|Electricity|Oil", "Secondary Energy|Electricity|Solar", "Secondary Energy|Electricity|Wind",
"Secondary Energy|Gases", "Secondary Energy|Gases|Biomass","Secondary Energy|Gases|Coal",
"Secondary Energy|Gases|Natural Gas", "Secondary Energy|Heat", "Secondary Energy|Heat|Geothermal",
"Secondary Energy|Hydrogen", "Secondary Energy|Hydrogen|Biomass",
"Secondary Energy|Hydrogen|Biomass|w/ CCS", "Secondary Energy|Hydrogen|Biomass|w/o CCS",
"Secondary Energy|Hydrogen|Electricity", "Secondary Energy|Liquids", "Secondary Energy|Liquids|Biomass",
"Secondary Energy|Liquids|Biomass|w/ CCS", "Secondary Energy|Liquids|Biomass|w/o CCS",
"Secondary Energy|Liquids|Coal", "Secondary Energy|Liquids|Coal|w/ CCS",
"Secondary Energy|Liquids|Coal|w/o CCS", "Secondary Energy|Liquids|Gas",
"Secondary Energy|Liquids|Gas|w/ CCS", "Secondary Energy|Liquids|Gas|w/o CCS",
"Secondary Energy|Liquids|Oil", "Secondary Energy|Solids", "VARIABLE"}
```

```
In[9]:= RRR=Table[Z[[k]][[1]][[3]],{k,1,Length[Z]}];
Union[RRR]
```

```
Out[10]= {"R5.2ASIA", "R5.2LAM", "R5.2MAF", "R5.2OECD", "R5.2REF", "REGION", "World"}
```

```
In[11]:= co2pos1=Position[RR,_?(#=="\"Emissions|CO2|FossilFuelsandIndustry\""&)];
co2pos2 = Position[RR, _ ? (# == "\"Emissions|CO2|Land Use\"" &)];
co2pos3 = Position[RRR, _ ? (# == "\"World\"" &)];
```

```
In[14]:= ppos1=Intersection[co2pos3,co2pos1];
ppos2 = Intersection[co2pos3, co2pos2];

In[16]:= Extract[Z,co2pos1][[1]]

Out[16]= {{AIM/CGE, "SSP1-19", "R5.2ASIA", "Emissions|CO2|Fossil Fuels and Industry", "Mt CO2/yr", 8985.6725,
10008.8152, 11790.747500000001,
6131.6627, 3271.4353000000006, 1678.8029, 638.87, 259.4755, 82.29590000000003, -7.9353000000000105, -
103.9171}}

In[17]:= em1=ToExpression[Map[Drop[Flatten[#],7]&,Extract[Z,ppos1]]];
em2 = ToExpression[Map[Drop[Flatten[#], 7] &, Extract[Z, ppos2]]];

In[19]:= ListPlot[em1,PlotRange→All,Joined→True]

In[20]:= Length[em1]
Out[20]= 127

In[21]:= emissions=Map[#[[1;;2]]&,ToExpression[Map[StringSplit[#] &, Drop[ReadList["emissionsCO2.txt",
String], 31]]]];
emissions = Table[{emissions[[i, 1]], (44 / 12) * emissions[[i, 2]] / 1000.}, {i, 1, Length[emissions]}];
ListPlot[emissions, Joined → True, PlotStyle → {Black, Thick}, PlotRange → All, Axes → False, Frame → True,
FrameStyle → Directive[14, Black],
FrameLabel → {"year", "CO2 emissions (Gt CO/yr)"}]
(*historical emissions*)

EM = Join[emissions, {{2018, 37.1}}];
data2 = Table[Prepend[Table[{t, Interpolation[Join[EM, Transpose[{{2030, 2040, 2050, 2060, 2070, 2080,
2090, 2100}, 0.001 * Drop[em1[[k]], 1]}]]][t]}, {t, 1751, 2100}], {1750, 0}], {k, 1, Length[em1]}]; totliste =
Table[data2[[k]][[All, 2]], {k, 1, Length[data2]}];
PLAll = ListPlot[data2, Joined → True, Frame → True, FrameStyle → Directive[Black, 14], PlotRange → All,
FrameLabel → {None, "CO2 emissions (Gt CO2)"}]
positivepaths = Table[DeleteCases[Map[# * UnitStep[#] &, totliste[[k]][[269 ;; 351]]], _ ? (# == 0 &)], {k, 1,
Length[totliste]}]; ListPlot[positivepaths, Joined → True, PlotRange → All]
(*Before removal of exceedance scenarios*)

In[29]:= RCBliste2=Map[Plus@@#&,positivepaths];

In[175]:= p1=Position[RCBliste2,_?(#<3300&)];

In[31]:= maxtemp=Map[Max[#]&,templiste];
maxtemp2 = Map[Max[#] &, uptempliste];
maxtemp3 = Map[Max[#] &, lowtempliste];

PL1 = ListPlot[Extract[data2, p1], Joined → True,Frame → True, FrameStyle → Directive[Black, 14], PlotRange
→ All];
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];
FFC = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["", 18],
Scaled[{0.1, 0.9}]]]
(*After removal of exceedance scenarios*)

In[37]:= PL1=ListPlot[data2[[1]],Joined→True,Frame→True, FrameStyle → Directive[Black, 14], PlotStyle →
Darker[Blue]];
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];
FFA = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["a", 18],
Scaled[{0.1, 0.9}]]]

In[40]:= n=Length[data2[[1]]];
futuretime = 2100 - 2020;
τmetan = 12.4;
```

```
In[43]:= (* Carbon model *)
τ1=1;
τ2=10;
τ3 = 100;
τ4 = 1000;
c1mean = 0.152;
c2mean = 0.246;
c4mean = 0.134;
c5mean = 0.194;

Gmean = (12/44) *0.47* (c1mean * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]
+ c2mean * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1mean - c2mean - c4mean -
c5mean)* Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]
+ c4mean * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + Table[c5mean * UnitStep[i - j], {i, 1, n}, {j,
1, n}]);

In[52]:= (* Carbon models *)
c1upper = 0.11;
c2upper = 0.212;
c4upper = 0.106;
c5upper = 0.262;
c1lower = 0.18;
c2lower = 0.296;
c4lower = 0.122;
c5lower = 0.148;

Glower = (12 / 44) * 0.47 * (c1lower * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c2lower * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1lower - c2lower - c4lower - c5lower) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c4lower * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5lower * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

Gupper = (12/44) *0.47* (c1upper * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c2upper * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1upper - c2upper - c4upper - c5upper) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c4upper * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5upper * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

In[62]:=
(*Optimal Estimation of Stochastic Energy Balance Model Parameters *)

In[63]:= (* Climate models *)
models = ReadList["CMIP5parameters.txt", String];
models = Delete[models, {{5}, {12}}];
boxes = StringSplit[models][[All, 2]];
Klimaliste = {};
Γliste = {};
σ2liste = {};
Monitor[
Do[

Clear[A];
modelnr = p; If[boxes[[p]] == "2",

{C1, C2, κ1, κ2, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1}, {κ2/C2, -κ2/C2}};
g = (MatrixExp[t A].{1 / C1, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If[ boxes[[p]] == "3",
```

```
{C1, C2, C3, κ1, κ2, κ3, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1, 0}, {κ2/C2, -(κ2+κ3)/C2, κ3/C2},{0, κ3/C3, - κ3/C3}};

g = (MatrixExp[t A].{1 / C1, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If [ boxes[[p]] == "4",

{C1, C2, C3, C4, κ1, κ2, κ3, κ4, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+κ2)/C1, κ2/C1, 0, 0} , {κ2/C2, -(κ2+κ3)/C2, κ3/C2, 0},{ 0, κ3/C3, -(κ3+κ4)/C3, κ4/C3},
{0, 0, κ4/C4, -κ4/C4}};

g = (MatrixExp[t A].{1 / C1, 0, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

Γliste = Append[Γliste, Γ];
σ2liste = Append[σ2liste, σ2];
, {p, 1, Length[models]}
];
, {p, boxes[[p]]}
];
```

```
(*Nonlin parameter changes*)

styrke = 1; (*w/m^2*)
terskel = 2; (*grader*)
bratthet = 0.5;
Plot[styrke * 0.5 * (1 + Tanh[(T - terskel) / bratthet]), {T, 0, 4}]
(*Test plot to visualise the non-linear forcing*)

RCBliste={};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
Δfaeroliste = {};
Δfghgliste = {};
Δfliste = {};
noiseliste = {};

Monitor[ Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 11.9 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];
```

```
Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.34 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);

Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfaer= -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ;
Δfaer2 = Drop[Δfaer, Length[EM]] ;
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfliste = Append[Δfliste, Δf];
Δfaeroliste = Append[Δfaeroliste, Δfaer];
Δfghgliste = Append[Δfghgliste, Δfco2 + Δfmetan];

Tliste = {}; Do[
T2 = Klimaliste[[p]].Δf;

(*nonlin loop*)
Do[
T2 = Klimaliste[[p]].(Δf + styrke * 0.5 * (1 + Tanh[(T2 - terskel) / bratthet])); , {10}]; (*number of iterations*)
noise = σ2liste[[p]] * (Klimaliste[[p]].RandomReal[NormalDistribution[0, 1], Length[Δf]]);
noise = Drop[noise, 268 - 20];
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268]; T2=1.1+T2-T2[[1]];
noiseliste = Append[noiseliste, noise];
Tliste = Append[Tliste, T2];
, {p, 1, Length[models]}];

middel =Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];

lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];

RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot]; alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower]; , {u, 1, Length[data2]}]
, u];

In[258]:= Length[noise]
Out[!]= 103

In[259]:= Length[T2]
Out[!]= 83

In[260]:= window=10;
noiseliste2 = Table[MovingAverage[noiseliste[[i]], window][[1 ;; Length[T2]]], {i, 1, Length[noiseliste]}];
noiseliste2 = Transpose[Partition[noiseliste2, 14]];

In[263]:= Length[noiseliste2]
Out[!]= 14

In[264]:= Dimensions[noiseliste2]
Out[!]= {14, 127, 83}
```

```
In[265]:= Length[noiseliste2[[1]]]
Out[!]= 127

PLNoise = ListPlot[Map[Transpose[{2018 + Range[Length[templiste[[1]]]], #}] &, noiseliste2[[3]]], Joined →
True];
FFNoise = Show[PLNoise, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle →
Directive[Black, 14], Joined → True, FrameLabel → {None, "Internal temperature variability(°C)"}, Epilog →
Inset[Style["", 18], Scaled[{0.1, 0.9}]]]

FFE=ListPlot[Map[Transpose[{2018+Range[Length[templiste[[1]]]],#}]&, Extract[templiste, p1]], PlotRange
→ All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None,
"GMST increase (°C)"}, Epilog → Inset[Style["e", 18], Scaled[{0.1, 0.9}]]]
PL1 = ListPlot[Map[Transpose[{1749 + Range[Length[Δfaeroliste[[1]]]], #}] &, Extract[Δfaeroliste, p1]],
Joined → True];
PL2 = ListPlot[Map[Transpose[{1749 + Range[Length[Δfaeroliste[[1]]]], #}] &, Extract[Δfghgliste, p1]], Joined
→ True];
FFD = Show   {PL1, PL2}, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle →
Directive[Black, 14], FrameLabel →    None, "Forcing (W/m^2)"϶, Epilog → Inset[Style["d", 18], Scaled[{0.1,
0.9}]]]

In[267]:=

In[271]:=modellfarger={,,,,,,,,,,,,,,,,, };
In[272]:= pan=LineLegend[modellfarger,Map[StringSplit[#]&,models][[All,1]]]

In[273]:= FFB=ListPlot[Map[Transpose[{2018+Range[Length[alltliste[[1]]]],#}]&, alltliste[[1 ;; 14]]], PlotRange →
All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None,
"GMST increase (°C)"}, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]], PlotStyle → Map[{#} &, modellfarger]]

In[!]:= Grid[{{Show[FFA,ImageSize→400],Show[FFB,ImageSize→400],pan}}]

In[!]:= Grid[{{Show[FFC,ImageSize→400, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]]], Show[FFD, ImageSize
→ 400, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]]] , Show[FFE, ImageSize → 400, Epilog → Inset[Style["c",
18], Scaled[{0.1, 0.9}]]]}}]

In[!]:= maxtemp=Map[Max[#]&,templiste];
maxtemp2 = Map[Max[#] &, uptempliste];
maxtemp3 = Map[Max[#] &, lowtempliste];

In[!]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1], AspectRatio → 1, PlotRange → All];
PL3 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Red];
PL4 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Red];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz]; PL2 = Plot[gg, {zz, 1.2, 3.5}];
gg2 = Fit[Extract[Transpose[{maxtemp2, RCBliste2}], p1], {zz, 1}, zz]; PL4 = Plot[gg2, {zz, 1.2, 3}];
Show[{PL1, PL2}, PlotRange → All]

In[!]:= pairs=Extract[Transpose[{maxtemp,RCBliste2}],p1];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];

In[!]:= pdf=(PDF[NormalDistribution[gg,σf[zz]]][p])/.zz→2.0;
pdf2 = (PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → 3.0;
Plot[{pdf, pdf2}, {p, 0, 5200}, PlotRange → All]

PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → {{1, 4}, {0, 4000}},
PlotStyle → Darker[Blue]];
```

```
PL2 = Plot[gg, {zz, 1, 3.5}, PlotStyle → Darker[Blue]];
l1 = Graphics[{Black, Line[{{2.0, 0}, {2.0, gg /. zz → 2.0}}]}];
l2 = Graphics[{Black, Line[{{2.0, gg /. zz → 2.0}, {1, gg /. zz → 2.0}}]}]; inset = ParametricPlot[{1 + 60 * pdf, p}, {p, 400, 1200}, Axes → False, PlotStyle → {Black, Thickness[0.01]}]; (*{p, budget low, budget high}*)
l11 = Graphics[{Black, Line[{{3.0, 0}, {3.0, gg /. zz → 3.0}}]}];
l22 = Graphics[{Black, Line[{{3.0, gg /. zz → 3.0}, {1, gg /. zz → 3.0}}]}]; inset2 = ParametricPlot[{1 + 60 * pdf2, p}, {p, 1900, 2700}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];(*{p, budget low, budget high}*)

FA = Show[{PL1, PL2, l1, l2, l11, l22, inset, inset2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {"Global temperature increase (°C)", "Carbon budget after 2018 (Gt CO2)"}, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]], ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]
(*TCRE for 1 ESM, 86 scenarios*)
```

ALL CLIMATE MODELS

(*Just to get the TCRE plots*)

```
maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, 5]]; PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Black];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 3.5}, PlotStyle → Black];
QL1 = Show[{PL1, PL2}, PlotRange → All];

maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, 7]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Darker[Red]];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 3.5}, PlotStyle → Darker[Red]];
QL2 = Show[{PL1, PL2}, PlotRange → All];
FB = Show[{QL1, QL2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {"Global temperature increase (°C)", "Carbon budget after 2018 (Gt CO2)"}, Epilog → {Inset[Style["b", 18], Scaled[{0.1, 0.9}]], Inset[LineLegend[{Black, Darker[Red]}, {"CSIRO-Mk3.6.0", "GFDL-ESM2M"}], Scaled[{0.7, 0.3}]]}, ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]

In[!]:= Grid[{{FA,FB}}]
(*Grid plot of TCRE's *)
```

ALL CLIMATE MODELS, 2 CARBON MODELS AND INTERNAL VARIABILITY

```
RCBliste = {};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};

Monitor[ Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];
```

```
Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];

Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280];(* CO2 til forcing*)
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
(* aerosols *)
Δfaer= -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ;
Δfaer2 = Drop[Δfaer, Length[EM]] ;
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;

Tliste = {};
Do[
T2 = Klimaliste[[p]].Δfupper;


(*nonlin loop*)
Do[
T2 = Klimaliste[[p]].
(Δfupper + styrke * 0.5 * (1 + Tanh[(T2 - terskel) / bratthet]));
, {10}]; (*number of iterations*)

T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.1+T2-T2[[1]];
Tliste = Append[Tliste, T2 + StandardDeviation[Flatten[noiseliste2[[p]]]]];

(*T2=Klimaliste[[p]].Δfupper;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.1+T2-T2[[1]]; Tliste=Append[Tliste,T2-StandardDeviation[Flatten[noiseliste2[[p]]]]];*)

(*T2=Klimaliste[[p]].Δflower;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.0+T2-T2[[1]]; Tliste=Append[Tliste,T2+StandardDeviation[Flatten[noiseliste2[[p]]]]];*)

T2 = Klimaliste[[p]].Δflower;
(*nonlin loop*)
Do[
T2 = Klimaliste[[p]].
(Δflower + styrke * 0.5 * (1 + Tanh[(T2 - terskel) / bratthet]));
, {10}]; (*number of iterations*)

T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268]; T2=1.0+T2-T2[[1]];

Tliste = Append[Tliste, T2 - StandardDeviation[Flatten[noiseliste2[[p]]]]];
, {p, 1, Length[models]}];

middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}]; RCB = Plus @@ Drop[tot, 270];
```

```
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}]
,u]

In[!]:= Dimensions[alltliste]
Out[!]= {3556, 83}

In[!]:= CM = {};
cm1 = Table[ Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[ All, 1]], {kk, 1,
Length[p1]}];
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[ kk]], 2][[All, 2]], {kk, 1,
Length[p1]}];

Do[
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]];
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]];
, {j, 1, Length[Transpose[cm2]]}];

In[!]:= ListPlot[{CM[[1]],CM[[2]]}]

In[!]:= smliste={};
tliste = {};

Monitor[ Do[
pdfliste = {}; Do[

gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
,{kk,1,2*14}];

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];

In[!]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[!]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[!]:= bliste={};

Do[
Δt = tliste[[2]] - tliste[[1]];
```

```
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt);

t1= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.90 &)]][[1]] - 1]];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.75 &)]][[1]] - 1]];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.5 &)]][[1]] - 1]];
t4= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.25 &)]][[1]] - 1]];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.10 &)]][[1]] - 1]];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}];
, {budget, 200, 4000, 100}]

In[!]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};
a = ListPlot[{Transpose[{bliste[[All, 1]], bliste[[All, 2]]}],

Transpose[{bliste[[All, 1]], bliste[[All, 3]]}], Transpose[{bliste[[All, 1]], bliste[[All, 4]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 5]]}], Transpose[{bliste[[All, 1]], bliste[[All, 6]]}]}, Joined → True,
AspectRatio → 1, PlotRange → {1, 4.0}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14],
PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines → Automatic, FrameLabel → {"Carbon budget from 2018
(GtCO2)", "Maximum temperature increase (°C)"}, PlotLegends → Placed[{"10% prob.", "25% prob.", "even
chance", "75% prob.", "90% prob."}, {Scaled[{0.05, 0.7}], {0, 0.5}}]]; l = Graphics[{Black, Line[{{1294, 1},
{1294, 2.5}}]}];
GGD = Show[{a}, PlotRange → {{0, 4000}, {1, 4.3}}]

(*RCB estimates for non-linear framework *)
```

CODE FOR IMPLEMENTATION OF THE ARCTIC AMPLIFICATION FACTOR

```
In[283]:= SetDirectory["OneDrive - UiT Office 365"];
Z = Import["SSP_IAM_V2_201811.csv"];
Z = Map[StringSplit[#, ","] &, Z];

In[5]:=  hh=157.65890684920566`+1.8942819330281027`zz+0.08520850267749702`zz^2;

In[6]:= Z[[1]]
Out[6]= {{MODEL, "SCENARIO", "REGION", "VARIABLE", "UNIT",
2005, 2010, 2020, 2030, 2040, 2050, 2060, 2070, 2080, 2090, 2100}}

In[7]:= RR=Table[Z[[k]][[1]][[4]],{k,1,Length[Z]}];
Union[RR]

Out[8]= {"Agricultural Demand|Crops", "Agricultural Demand|Crops|Energy",
"Agricultural Demand|Livestock", "Agricultural Production|Crops|Energy",
"Agricultural Production|Crops|Non-Energy","Agricultural Production|Livestock", "Capacity|Electricity",
"Capacity|Electricity|Biomass", "Capacity|Electricity|Coal", "Capacity|Electricity|Gas",
"Capacity|Electricity|Geothermal", "Capacity|Electricity|Hydro", "Capacity|Electricity|Nuclear",
"Capacity|Electricity|Oil", "Capacity|Electricity|Other", "Capacity|Electricity|Solar", "Capacity|Electricity|Solar|CSP",
"Capacity|Electricity|Solar|PV", "Capacity|Electricity|Wind", "Capacity|Electricity|Wind|Offshore",
"Capacity|Electricity|Wind|Onshore", "Consumption", "Diagnostics|MAGICC6|Concentration|CH4",
"Diagnostics|MAGICC6|Concentration|CO2", "Diagnostics|MAGICC6|Concentration|N2O",
"Diagnostics|MAGICC6|Forcing", "Diagnostics|MAGICC6|Forcing|Aerosol", "Diagnostics|MAGICC6|Forcing|CH4",
```

"Diagnostics|MAGICC6|Forcing|CO2", "Diagnostics|MAGICC6|Forcing|F-Gases",
"Diagnostics|MAGICC6|Forcing|Kyoto Gases", "Diagnostics|MAGICC6|Forcing|N2O",
"Diagnostics|MAGICC6|Temperature|Global Mean", "Emissions|BC", "Emissions|CH4",
"Emissions|CH4|Fossil Fuels and Industry", "Emissions|CH4|Land Use", "Emissions|CO",
"Emissions|CO2", "Emissions|CO2|Carbon Capture and Storage",
"Emissions|CO2|Carbon Capture and Storage|Biomass","Emissions|CO2|Fossil Fuels and Industry",
"Emissions|CO2|Land Use", "Emissions|F-Gases", "Emissions|Kyoto Gases", "Emissions|N2O",
"Emissions|N2O|Land Use", "Emissions|NH3", "Emissions|NOx", "Emissions|OC", "Emissions|Sulfur",
"Emissions|VOC", "Energy Service|Transportation|Freight", "Energy Service|Transportation|Passenger",
"Final Energy","Final Energy|Electricity", "Final Energy|Gases", "Final Energy|Heat",
"Final Energy|Hydrogen", "Final Energy|Industry", "Final Energy|Liquids",
"Final Energy|Residential and Commercial", "Final Energy|Solar","Final Energy|Solids",
"Final Energy|Solids|Biomass","Final Energy|Solids|Biomass|Traditional", "Final Energy|Solids|Coal",
"Final Energy|Transportation", "GDP|PPP", "Harmonized Emissions|BC",
"Harmonized Emissions|CH4|Fossil Fuels and Industry","Harmonized Emissions|CH4|Land Use",
"Harmonized Emissions|CO","Harmonized Emissions|CO2|Fossil Fuels and Industry",
"Harmonized Emissions|CO2|Land Use", "Harmonized Emissions|F-Gases", "Harmonized Emissions|Kyoto Gases",
"Harmonized Emissions|NH3", "Harmonized Emissions|NOx", "Harmonized Emissions|OC",
"Harmonized Emissions|Sulfur", "Harmonized Emissions|VOC","Land Cover|Built-up Area",
"Land Cover|Cropland", "Land Cover|Forest", "Land Cover|Pasture", "Population", "Price|Carbon", "Primary Energy",
"Primary Energy|Biomass", "Primary Energy|Biomass|Traditional", "Primary Energy|Biomass|w/ CCS",
"Primary Energy|Biomass|w/o CCS", "Primary Energy|Coal", "Primary Energy|Coal|w/ CCS",
"Primary Energy|Coal|w/o CCS", "Primary Energy|Fossil", "Primary Energy|Fossil|w/ CCS",
"Primary Energy|Fossil|w/o CCS","Primary Energy|Gas", "Primary Energy|Gas|w/ CCS",
"Primary Energy|Gas|w/o CCS", "Primary Energy|Geothermal","Primary Energy|Hydro",
"Primary Energy|Non-Biomass Renewables","Primary Energy|Nuclear", "Primary Energy|Oil",
"Primary Energy|Oil|w/ CCS", "Primary Energy|Oil|w/o CCS","Primary Energy|Other",
"Primary Energy|Secondary Energy Trade","Primary Energy|Solar", "Primary Energy|Wind",
"Secondary Energy|Electricity", "Secondary Energy|Electricity|Biomass",
"Secondary Energy|Electricity|Biomass|w/ CCS","Secondary Energy|Electricity|Biomass|w/o CCS",
"Secondary Energy|Electricity|Coal","Secondary Energy|Electricity|Coal|w/ CCS",
"Secondary Energy|Electricity|Coal|w/o CCS","Secondary Energy|Electricity|Gas",
"Secondary Energy|Electricity|Gas|w/ CCS", "Secondary Energy|Electricity|Gas|w/o CCS",
"Secondary Energy|Electricity|Geothermal","Secondary Energy|Electricity|Hydro",
"Secondary Energy|Electricity|Non-Biomass Renewables","Secondary Energy|Electricity|Nuclear",
"Secondary Energy|Electricity|Oil", "Secondary Energy|Electricity|Solar", "Secondary Energy|Electricity|Wind",
"Secondary Energy|Gases", "Secondary Energy|Gases|Biomass","Secondary Energy|Gases|Coal",
"Secondary Energy|Gases|Natural Gas", "Secondary Energy|Heat", "Secondary Energy|Heat|Geothermal",
"Secondary Energy|Hydrogen", "Secondary Energy|Hydrogen|Biomass",
"Secondary Energy|Hydrogen|Biomass|w/ CCS", "Secondary Energy|Hydrogen|Biomass|w/o CCS",
"Secondary Energy|Hydrogen|Electricity", "Secondary Energy|Liquids", "Secondary Energy|Liquids|Biomass",
"Secondary Energy|Liquids|Biomass|w/ CCS", "Secondary Energy|Liquids|Biomass|w/o CCS",
"Secondary Energy|Liquids|Coal", "Secondary Energy|Liquids|Coal|w/ CCS",
"Secondary Energy|Liquids|Coal|w/o CCS", "Secondary Energy|Liquids|Gas",
"Secondary Energy|Liquids|Gas|w/ CCS", "Secondary Energy|Liquids|Gas|w/o CCS",
"Secondary Energy|Liquids|Oil", "Secondary Energy|Solids", "VARIABLE"}

In[9]:= **RRR=Table[Z[[k]][[1]][[3]],{k,1,Length[Z]}];**
Union[RRR]

Out[10]= {"R5.2ASIA", "R5.2LAM", "R5.2MAF", "R5.2OECD", "R5.2REF", "REGION", "World"}

In[11]:= **co2pos1=Position[RR,_?(#=="\"Emissions|CO2|FossilFuelsandIndustry\""&)];**
co2pos2 = Position[RR, _ ? (# == "\"Emissions|CO2|Land Use\"" &)];
co2pos3 = Position[RRR, _ ? (# == "\"World\"" &)];

In[14]:= **ppos1=Intersection[co2pos3,co2pos1];**
ppos2 = Intersection[co2pos3, co2pos2];

In[16]:= **Extract[Z,co2pos1][[1]]**

Out[16]= {{AIM/CGE, "SSP1-19", "R5.2ASIA", "Emissions|CO2|Fossil Fuels and Industry", "Mt CO2/yr", 8985.6725,
10008.8152, 11790.747500000001,

6131.6627, 3271.4353000000006, 1678.8029, 638.87, 259.4755, 82.29590000000003, -7.9353000000000105, -103.9171}}

```
In[17]:= em1=ToExpression[Map[Drop[Flatten[#],7]&,Extract[Z,ppos1]]];
em2 = ToExpression[Map[Drop[Flatten[#], 7] &, Extract[Z, ppos2]]];

In[19]:= ListPlot[em1,PlotRange→All,Joined→True]

In[20]:= Length[em1]
Out[20]= 127

In[21]:= emissions=Map[#[[1;;2]]&,ToExpression[Map[StringSplit[#] &, Drop[ReadList["emissionsCO2.txt", String], 31]]]];
emissions = Table[{emissions[[i, 1]], (44 / 12) * emissions[[i, 2]] / 1000.}, {i, 1, Length[emissions]}];
ListPlot[emissions, Joined → True, PlotStyle → {Black, Thick}, PlotRange → All, Axes → False, Frame → True, FrameStyle → Directive[14, Black],
FrameLabel → {"year", "CO₂ emissions (Gt CO/yr)"}]
(*historical emissions*)

EM = Join[emissions, {{2018, 37.1}}];
data2 = Table[Prepend[Table[{t, Interpolation[Join[EM, Transpose[{{2030, 2040, 2050, 2060, 2070, 2080, 2090, 2100}, 0.001 * Drop[em1[[k]], 1]}]]][t]}, {t, 1751, 2100}], {1750, 0}], {k, 1, Length[em1]}]; totliste = Table[data2[[k]][[All, 2]], {k, 1, Length[data2]}];
PLAll = ListPlot[data2, Joined → True, Frame → True, FrameStyle → Directive[Black, 14], PlotRange → All, FrameLabel → {None, "CO2 emissions (Gt CO2)"}]

positivepaths = Table[DeleteCases[Map[# * UnitStep[#] &, totliste[[k]][[269 ;; 351]]], _ ? (# == 0 &)], {k, 1, Length[totliste]}]; ListPlot[positivepaths, Joined → True, PlotRange → All]
(*Before removal of exceedance scenarios*)

In[29]:= RCBliste2=Map[Plus@@#&,positivepaths];

In[175]:= p1=Position[RCBliste2,_?(#<3300&)];

In[31]:= maxtemp=Map[Max[#]&,templiste];
maxtemp2 = Map[Max[#] &, uptempliste];
maxtemp3 = Map[Max[#] &, lowtempliste];

PL1 = ListPlot[Extract[data2, p1], Joined → True,Frame → True, FrameStyle → Directive[Black, 14], PlotRange → All];
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];
FFC = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]]]
(*After removal of exceedance scenarios*)

In[37]:= PL1=ListPlot[data2[[1]],Joined→True,Frame→True, FrameStyle → Directive[Black, 14], PlotStyle → Darker[Blue]];
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];
FFA = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]]]

In[40]:= n=Length[data2[[1]]];
futuretime = 2100 - 2020;
τmetan = 12.4;

In[43]:= (* Carbon model *)
τ1=1;
τ2=10;
τ3 = 100;
τ4 = 1000;
c1mean = 0.152;
c2mean = 0.246;
```

```
c4mean = 0.134;
c5mean = 0.194;

Gmean = (12/44) *0.47* (c1mean * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]
+ c2mean * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1mean - c2mean - c4mean -
c5mean)* Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]
+ c4mean * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + Table[c5mean * UnitStep[i - j], {i, 1, n}, {j,
1, n}]);

In[52]:= (* Carbon models *)
c1upper = 0.11;
c2upper = 0.212;
c4upper = 0.106;
c5upper = 0.262;
c1lower = 0.18;
c2lower = 0.296;
c4lower = 0.122;
c5lower = 0.148;

Glower = (12 / 44) * 0.47 * (c1lower * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c2lower * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1lower - c2lower - c4lower - c5lower) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c4lower * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5lower * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

Gupper = (12/44) *0.47* (c1upper * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c2upper * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1upper - c2upper - c4upper - c5upper) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c4upper * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5upper * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

In[62]:=
(*Optimal Estimation of Stochastic Energy Balance Model Parameters *)

In[63]:= (* Climate models *)
models = ReadList["CMIP5parameters.txt", String];
models = Delete[models, {{5}, {12}}];
boxes = StringSplit[models][[All, 2]];
Klimaliste = {};
Γliste = {};
σ2liste = {};
Monitor[
Do[

Clear[A];
modelnr = p; If[boxes[[p]] == "2",

{C1, C2, κ1, κ2, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1}, {κ2/C2, -κ2/C2}};
g = (MatrixExp[t A].{1 / C1, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If[ boxes[[p]] == "3",

{C1, C2, C3, κ1, κ2, κ3, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1, 0}, {κ2/C2, -(κ2+κ3)/C2, κ3/C2},{0, κ3/C3, - κ3/C3}};

g = (MatrixExp[t A].{1 / C1, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
```

```
Klimaliste = Append[Klimaliste, Gklima];
];

If [ boxes[[p]] == "4",

{C1, C2, C3, C4, κ1, κ2, κ3, κ4, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+κ2)/C1, κ2/C1, 0, 0} , {κ2/C2, -(κ2+κ3)/C2, κ3/C2, 0},{ 0, κ3/C3, -(κ3+κ4)/C3, κ4/C3},
{0, 0, κ4/C4, -κ4/C4}};

g = (MatrixExp[t A].{1 / C1, 0, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

Γliste = Append[Γliste, Γ];
σ2liste = Append[σ2liste, σ2];
, {p, 1, Length[models]}
];
, {p, boxes[[p]]}
];
```

```
(*Nonlin parameter changes*)

styrke = 1; (*w/m^2*)
terskel = 2; (*grader*)
bratthet = 0.5;
Plot[styrke * 0.5 * (1 + Tanh[(T - terskel) / bratthet]), {T, 0, 4}]
(*Test plot to visualise the non-linear forcing*)
```

```
In[!]:= RCBliste={};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
Δfaeroliste = {};
Δfghgliste = {};
Δfliste = {};
noiseliste = {};

Monitor[
Do[
tot = data2[[u]][[All, 2]]; meanco2 = Gmean.tot + 280;
(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 11.9 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.34 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);

Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfaer= -0.02tot;
```

```
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ;
Δfaer2 = Drop[Δfaer, Length[EM]] ;
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfliste = Append[Δfliste, Δf];
Δfaeroliste = Append[Δfaeroliste, Δfaer];
Δfghgliste = Append[Δfghgliste, Δfco2 + Δfmetan];

Tliste = {};
Do[
T2 = Klimaliste[[p]].Δf;

(*nonlin nonlin loop*) (*Do[
T2=Klimaliste[[p]].(Δf+styrke*0.5*(1+Tanh[(T2-terskel)/bratthet])); ,{10}];*)
(*nonlin lin loop*)
Do[
T2 = Klimaliste[[p]].(Δf + 0.2 T2); , {10}];
noise = σ2liste[[p]] * (Klimaliste[[p]].RandomReal[NormalDistribution[0, 1], Length[Δf]]);
noise = Drop[noise, 268 - 20];
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35; T2 = Drop[T2, 268]; T2=1.1+T2-T2[[1]];
noiseliste = Append[noiseliste, noise]; Tliste = Append[Tliste, T2];
, {p, 1, Length[models]}];

middel =Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];

lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];

RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}]
, u];
In[!]:= Length[noise]
Out[!]= 103
In[!]:= Length[T2]
Out[!]= 83

In[!]:= window=10;
noiseliste2 = Table[MovingAverage[noiseliste[[i]], window][[1 ;; Length[T2]]], {i, 1, Length[noiseliste]}];
noiseliste2 = Transpose[Partition[noiseliste2, 14]];

In[!]:= Length[noiseliste2]
Out[!]= 14
In[!]:= Dimensions[noiseliste2]
Out[!]= {14, 127, 83}
In[!]:= Length[noiseliste2[[1]]]
Out[!]= 127

In[!]:= ListPlot[noiseliste2[[3]],Joined→True]
In[!]:= FFE=ListPlot[Map[Transpose[{2018+Range[Length[templiste[[1]]]],#}]&, Extract[templiste, p1]],
PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel
→ {None, "GMST increase (°C)"}, Epilog → Inset[Style["e", 18], Scaled[{0.1, 0.9}]]]
```

```
In[!]:= PL1=ListPlot[Map[Transpose[{1749+Range[Length[Δfaeroliste[[1]]]],#}]&, Extract[Δfaeroliste, p1]], Joined → True];
PL2 = ListPlot[Map[Transpose[{1749 + Range[Length[Δfaeroliste[[1]]]], #}] &, Extract[Δfghgliste, p1]], Joined → True];

FFD = Show[{PL1, PL2}, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "forcing"},Epilog → Inset[Style["d", 18], Scaled[{0.1, 0.9}]]]

modellfarger={,,,,,,,,,,,,,,, };
In[!]:= pan=LineLegend[modellfarger,Map[StringSplit[#]&,models][[All,1]]]
Out[!]= BCC-CSM1-1 BNU-ESM CanESM2 CCSM4 CSIRO-Mk3.6.0 FGOALS-s2 GFDL-ESM2M GISS-E2-R HadGEM2-ES INM-CM4 MIROC5 MPI-ESM-LR MRI-CGCM3 NorESM1-M

In[!]:= FFB=ListPlot[Map[Transpose[{2018+Range[Length[alltliste[[1]]]],#}]&, alltliste[[1 ;; 14]]], PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "GMST increase (°C)"},Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]], PlotStyle → Map[{#} &, modellfarger]]

In[!]:= Grid[{{Show[FFA,ImageSize→400],Show[FFB,ImageSize→400],pan}}]

In[!]:= Grid[{{Show[FFC,ImageSize→400, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]]], Show[FFD, ImageSize → 400, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]]] , Show[FFE, ImageSize → 400, Epilog → Inset[Style["c", 18], Scaled[{0.1, 0.9}]]]}}]

In[!]:= maxtemp=Map[Max[#]&,templiste];
maxtemp2 = Map[Max[#] &, uptempliste];
maxtemp3 = Map[Max[#] &, lowtempliste];

In[!]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1], AspectRatio → 1, PlotRange → All];

PL3 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Red];
PL4 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Red];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz]; PL2 = Plot[gg, {zz, 1.2, 3}];
gg2 = Fit[Extract[Transpose[{maxtemp2, RCBliste2}], p1], {zz, 1}, zz]; PL4 = Plot[gg2, {zz, 1.2, 3}];
Show[{PL1, PL2}, PlotRange → All]

In[!]:= pairs=Extract[Transpose[{maxtemp,RCBliste2}],p1];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];

In[!]:= pdf=(PDF[NormalDistribution[gg,σf[zz]]][p])/.zz→1.5;
pdf2 = (PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → 2.5;
Plot[{pdf, pdf2}, {p, 0, 5200}, PlotRange → All]

In[!]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1],AspectRatio→1, PlotRange → {{1, 4}, {0, 4000}}, PlotStyle → Darker[Blue]];
PL2 = Plot[gg, {zz, 1, 3}, PlotStyle → Darker[Blue]];
l1 = Graphics[{Black, Line[{{1.5, 0}, {1.5, gg /. zz → 1.5}}]}];
l2 = Graphics[{Black, Line[{{1.5, gg /. zz → 1.5}, {1, gg /. zz → 1.5}}]}]; inset = ParametricPlot[{1 + 60 * pdf, p}, {p, 0, 1200}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];

l11 = Graphics[{Black, Line[{{2.5, 0}, {2.5, gg /. zz → 2.5}}]}];
l22 = Graphics[{Black, Line[{{2.5, gg /. zz → 2.5}, {1, gg /. zz → 2.5}}]}]; inset2 = ParametricPlot[{1 + 60 * pdf2, p}, {p, 1900, 3000}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];

FA = Show[{PL1, PL2, l1, l2, l11, l22, inset, inset2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {"global temperature increase (°C)", "carbon budget after 2018 (Gt CO2)"}, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]],ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]
```

```
In[!]:= maxtemp=Partition[Map[Max[#]&,alltliste],14][[All,5]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Black];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 4}, PlotStyle → Black];
QL1 = Show[{PL1, PL2}, PlotRange → All];

maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, 7]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Darker[Red]];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 3.5}, PlotStyle → Darker[Red]];
QL2 = Show[{PL1, PL2}, PlotRange → All];
FB = Show[{QL1, QL2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel →
{"global temperature increase (°C)", "carbon budget after 2018 (Gt CO2)"}, Epilog → {Inset[Style["b", 18],
Scaled[{0.1, 0.9}]], Inset[LineLegend[{Black, Darker[Red]}, {"CSIRO-Mk3.6.0", "GFDL-ESM2M"}],

Scaled[{0.7, 0.3}]]}, ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}] In[!]:= Grid[{{FA,FB}}]
```

14 ESMS FROM CMIP5, 2 CARBONMODELS FROM SRM, INTERNAL VARIABILITY

```
RCBliste = {};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};

Monitor[
Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
(* aerosols *)
Δfaer= -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ;
Δfaer2 = Drop[Δfaer, Length[EM]] ;
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;

Tliste = {};
Do[
T2 = Klimaliste[[p]].Δfupper;
```

```
(*nonlin loop*) (*Do[
T2=Klimaliste[[p]].(Δfupper+styrke*0.5*(1+Tanh[(T2-terskel)/bratthet]));
,{10}];*)

(*nonlin lin loop*) Do[
T2 = Klimaliste[[p]].(Δfupper + 0.2 T2);
, {10}];

T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.1+T2-T2[[1]];
Tliste = Append[Tliste, T2 + StandardDeviation[Flatten[noiseliste2[[p]]]]];

(*T2=Klimaliste[[p]].Δfupper;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.1+T2-T2[[1]]; Tliste=Append[Tliste,T2-StandardDeviation[Flatten[noiseliste2[[p]]]]];*)

(*T2=Klimaliste[[p]].Δflower;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.0+T2-T2[[1]]; Tliste=Append[Tliste,T2+StandardDeviation[Flatten[noiseliste2[[p]]]]];*)

T2 = Klimaliste[[p]].Δflower;

(*nonlin loop*) (*Do[
T2=Klimaliste[[p]].(Δflower+styrke*0.5*(1+Tanh[(T2-terskel)/bratthet]));
,{10}];*)

(*nonlin lin loop*) Do[
T2 = Klimaliste[[p]].(Δflower + 0.2 T2);
, {10}];
(* Comment out everything between: Do[ T2=Klimaliste[[p]].Δfupper; to here to look at linear comparison*)

T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.0+T2-T2[[1]];
Tliste = Append[Tliste, T2 - StandardDeviation[Flatten[noiseliste2[[p]]]]];
, {p, 1, Length[models]}];

middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}]; RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];

alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}]
,u]

In[!]:= Dimensions[alltliste]
Out[!]= {3556, 83}

In[!]:= CM = {};
cm1 = Table[ Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[ All, 1]], {kk, 1,
Length[p1]}];
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[ kk]], 2][[All, 2]], {kk, 1,
Length[p1]}];
```

```
Do[
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]]; CM = Append[CM,
Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]]; , {j, 1, Length[Transpose[cm2]]}];

In[!]:= ListPlot[{CM[[1]],CM[[2]]}]

In[!]:= smliste={};
tliste = {};

Monitor[ Do[
pdfliste = {};
Do[
gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
,{kk,1,2*14}];

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];

In[!]:= (*forsterkningsfaktor a+bT: 0.10034007260683281`+2.2321837475237376` x*)

In[!]:= a=0.10034;
b = 2.23218;
Δtarc = a+b* Δt;

In[!]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[!]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[!]:= bliste={};
Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt);
t1= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.90 &)]][[1]] - 1]];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.75 &)]][[1]] - 1]];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.5 &)]][[1]] - 1]];
t4= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.25 &)]][[1]] - 1]];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _ ? (# > 0.10 &)]][[1]] - 1]];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}]; , {budget, 200, 4000, 100}]

In[!]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};
aa = ListPlot[{Transpose[{bliste[[All, 1]], bliste[[All, 2]]}],
```

```
Transpose[{bliste[[All, 1]], bliste[[All, 3]]}], Transpose[{bliste[[All, 1]], bliste[[All, 4]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 5]]}], Transpose[{bliste[[All, 1]], bliste[[All, 6]]}]}, Joined → True,
AspectRatio → 1, PlotRange → {{1, 9}}, Axes → False,Frame → True, FrameStyle → Directive[Black, 14]
,PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines → Automatic, FrameLabel → {"Carbon budget from 2018
(GtCO2)", "Maximum temperature increase (°C)"}, PlotLegends → Placed[{"10% prob.", "25% prob.", "even
chance", "75% prob.", "90% prob."}, {Scaled[{0.05, 0.7}], {0, 0.5}}]]; l = Graphics[{Black, Line[{{1294, 1},
{1294, 2.5}}]}];
GGD = Show[{aa}]
```

```
In[!]:= Tclow=Graphics[{Black,Line[{{0,8},{4000,8}}]}]; (*Greenland paper critical values*)
Tcupper = Graphics[{Black, Line[{{0, 8.5}, {4000, 8.5}}]}];
```

```
In[!]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};
aaa = ListPlot[{Transpose[{bliste[[All, 1]], a + b * bliste[[All, 2]]}], Transpose[{bliste[[All, 1]], a + b * bliste[[All,
3]]}], Transpose[{bliste[[All, 1]], a + b * bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], a + b * bliste[[All, 5]]}],
Transpose[{bliste[[All, 1]], a + b * bliste[[All, 6]]}]}, Joined → True, AspectRatio → 1, PlotRange → {{1, 9}} , Axes
→ False, Frame → True, FrameStyle → Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines →
Automatic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)"},
PlotLegends → Placed[{"10% prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05,
0.7}], {0, 0.5}}]]; l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]}];
GGD = Show[{aaa, Tclow, Tcupper}, PlotRange → {{0, 4000}, {1.1, 10}}]
```

```
In[!]:= Max[a+b*bliste[[All,2]]]
Out[!]= 8.7612
```

COMPARISON

```
In[!]:= Grid[{{Show[aa,PlotRange→{{0,4000},{1.1,9}}, ImageSize → 400, Epilog → Inset[Style["a", 18], Scaled[{0.1,
0.9}]]], Show[aaa, PlotRange → {{0, 4000}, {1.1, 9}} , ImageSize → 400, Epilog → Inset[Style["b", 18],
Scaled[{0.1, 0.9}]]]
}}]
```

ESTIMATION OF THE ARCTIC AMPLIFICATION FACTOR

```
(* GLOBAL LAND-OCEAN TEMPERATURE*)
SetDirectory["OneDrive - UiT Office 365"];
global = Drop[Drop[Import["GLB.Ts+dSST.csv", {"Data", All, 14}], 2], - 1];
In[!]:= PL1=ListPlot[global,Joined→True,PlotRange → All, DataRange → {1880, 1880 + Length[global]}]
(*base period 1951-1980*)
In[!]:= global;
```

```
(* ANNUAL MEAN LAND-OCEAN TEMPERATURE 64N-90N (ARCTIC IS 66.34N)*)
arctic = Drop[Import["ZonAnn.Ts+dSST.csv", {"Data", All, 8}], 1];
In[!]:= PL2=ListPlot[arctic,Joined→True,PlotRange→All, PlotStyle → Orange, DataRange → {1880, 1880 +
Length[arctic]}]
(*base period 1951-1980*)
In[!]:= Show[{PL1,PL2},PlotRange->All]
(* COMPARISON*)
comp = Transpose[{global, arctic}];
```

```
In[!]:= compPlot=ListPlot[comp,PlotRange→All, AxesLabel → {"global", "arctic"}, AspectRatio → 1]
In[!]:= lm=LinearModelFit[comp,x,x](*forsterkningsfaktor=2.23218*)
Out[!]= FittedModel 0.10034 + 2.23218 x
In[!]:= Fit[comp,{zz,1},zz]
```

```
Out[!]= 0.10034+2.23218zz
In[!]:= fit=Plot[lm[x],{x,-3,5}];
In[!]:= Show[compPlot,fit]
```

CODE FOR COMPARISON OF A SIMPLE RESPONSE MODEL TO MAGICC6.

```
SetDirectory["OneDrive - UiT Office 365"] M = Import["SSP_IAM_V2_201811.csv"];
M[[1]]
Out[!]= {MODEL,"SCENARIO","REGION","VARIABLE","UNIT",2005,2010,2020,2030,2040,2050,2060
,2070,2080,2090,2100}

In[!]:= M=Map[StringSplit[#,","]&,M];
In[!]:= TT=Table[M[[k]][[1]][[4]],{k,1,Length[M]}]; Union[TT];
TTT = Table[M[[k]][[1]][[3]], {k, 1, Length[M]}];

In[!]:= MAG=Position[TT,_?(#=="\"Diagnostics|MAGICC6|Temperature|GlobalMean\""&)];
In[!]:= MAGpos=Position[TTT,_?(#=="\"World\""&)];
In[!]:= mpos1=Intersection[MAGpos,MAG]; Extract[M, MAG][[1]];
In[!]:= temp1=ToExpression[Map[Drop[Flatten[#],6]&,Extract[M,mpos1]]]; ListPlot[temp1, Joined → True]
```

SRM MODEL

```
In[!]:= hh=157.65890684920566`+1.8942819330281027`zz+0.08520850267749702`zz^2;
In[!]:= RR=Table[M[[k]][[1]][[4]],{k,1,Length[M]}];
Union[RR];
In[!]:= RRR=Table[M[[k]][[1]][[3]],{k,1,Length[M]}];

Union[RRR]
Out[!]= {"R5.2ASIA", "R5.2LAM", "R5.2MAF", "R5.2OECD", "R5.2REF", "REGION", "World"}

In[!]:= co2pos1=Position[RR,_?(#=="\"Emissions|CO2|FossilFuelsandIndustry\""&)];
co2pos2 = Position[RR, _ ? (# == "\"Emissions|CO2|Land Use\"" &)];
co2pos3 = Position[RRR, _ ? (# == "\"World\"" &)];
```

```
In[!]:= ppos1=Intersection[co2pos3,co2pos1];
ppos2 = Intersection[co2pos3, co2pos2];

In[!]:= Extract[M,co2pos1][[1]]

Out[!]= {{AIM/CGE, "SSP1-19", "R5.2ASIA", "Emissions|CO2|Fossil Fuels and Industry",
"Mt CO2/yr", 8985.6725, 10008.8152, 11790.747500000001,
6131.6627, 3271.4353000000006, 1678.8029, 638.87, 259.4755, 82.29590000000003, -7.9353000000000105, -
103.9171}}

In[!]:= em1=ToExpression[Map[Drop[Flatten[#],7]&,Extract[M,ppos1]]];
em2 = ToExpression[Map[Drop[Flatten[#], 7] &, Extract[M, ppos2]]];

In[!]:= ListPlot[em1,PlotRange→All,Joined→True]
emissions = Map[#[[1 ;; 2]] &, ToExpression[ Map[StringSplit[#] &, Drop[ReadList["emissionsCO2.txt", String],
31]]]]; In[!]:= emissions=Table[{emissions[[i,1]],(44/12)*emissions[[i,2]]/1000.}
, {i, 1, Length[emissions]}];

In[!]:= EM=Join[emissions,{{2018,37.1}}];
data2 = Table[Prepend[Table[{t, Interpolation[Join[EM, Transpose[{{2030, 2040, 2050, 2060, 2070, 2080,
2090, 2100}, 0.001 * Drop[em1[[k]], 1]}]]][t]}, {t, 1751, 2100}], {1750, 0}], {k, 1, Length[em1]}];
totliste = Table[data2[[k]][[All, 2]], {k, 1, Length[data2]}];

In[!]:= positivepaths= Table[DeleteCases[Map[# * UnitStep[#] &, totliste[[k]][[269 ;; 351]]], _?(#
==0&)],{k,1,Length[totliste]}]; ListPlot[positivepaths, Joined → True]

In[!]:= RCBliste2=Map[Plus@@#&,positivepaths];
In[!]:= p1=Position[RCBliste2,_?(#<3300&)];
In[!]:= ListPlot[data2,Joined→True,PlotRange→All]

In[!]:= n=Length[data2[[1]]];
futuretime = 2100 - 2020;
τmetan = 12.4;

In[43]:= (* Carbon model *)
τ1=1;
τ2=10;
τ3 = 100;
τ4 = 1000;
c1mean = 0.152;
c2mean = 0.246;
c4mean = 0.134;
c5mean = 0.194;

Gmean = (12/44) *0.47* (c1mean * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]+
c2mean * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1mean - c2mean - c4mean - c5mean)* Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]+
c4mean * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5mean * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

In[52]:= (* Carbon models *)
c1upper = 0.11;
c2upper = 0.212;
c4upper = 0.106;
c5upper = 0.262;
c1lower = 0.18;
c2lower = 0.296;
c4lower = 0.122;
c5lower = 0.148;

Glower = (12 / 44) * 0.47 * (c1lower * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c2lower * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1lower - c2lower - c4lower - c5lower) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
```

```
c4lower * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5lower * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

Gupper = (12/44) *0.47* (c1upper * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c2upper * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1upper - c2upper - c4upper - c5upper) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c4upper * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5upper * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

In[62]:=
(*Optimal Estimation of Stochastic Energy Balance Model Parameters *)

In[63]:= (* Climate models *)
models = ReadList["CMIP5parameters.txt", String];
models = Delete[models, {{5}, {12}}];
boxes = StringSplit[models][[All, 2]];
Klimaliste = {};
Γliste = {};
σ2liste = {};
Monitor[
Do[

Clear[A];
modelnr = p; If[boxes[[p]] == "2",

{C1, C2, κ1, κ2, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1}, {κ2/C2, -κ2/C2}};
g = (MatrixExp[t A].{1 / C1, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If[ boxes[[p]] == "3",

{C1, C2, C3, κ1, κ2, κ3, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1, 0}, {κ2/C2, -(κ2+κ3)/C2, κ3/C2},{0, κ3/C3, - κ3/C3}};

g = (MatrixExp[t A].{1 / C1, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If [ boxes[[p]] == "4",

{C1, C2, C3, C4, κ1, κ2, κ3, κ4, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];

A = {{-(κ1+κ2)/C1, κ2/C1, 0, 0} , {κ2/C2, -(κ2+κ3)/C2, κ3/C2, 0},{ 0, κ3/C3, -(κ3+κ4)/C3, κ4/C3},
{0, 0, κ4/C4, -κ4/C4}};

g = (MatrixExp[t A].{1 / C1, 0, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

Γliste = Append[Γliste, Γ];
σ2liste = Append[σ2liste, σ2];
, {p, 1, Length[models]}
];
, {p, boxes[[p]]}
];
```

```
(*Nonlin parameter changes*)

styrke = 1; (*w/m^2*)
terskel = 2; (*grader*)
bratthet = 0.5;
Plot[styrke * 0.5 * (1 + Tanh[(T - terskel) / bratthet]), {T, 0, 4}]
(*Test plot to visualise the non-linear forcing*)

RCBliste = {}; totliste = {}; templiste = {}; uptempliste = {}; lowtempliste = {}; alltliste = {}; Δfaeroliste = {};
Δfghgliste = {}; Δfliste = {}; noiseliste = {}; Monitor[

Do[
tot = data2[[u]][[All, 2]]; meanco2 = Gmean.tot + 280;

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 11.9 tunes 2019 methane emmisions in 2019 to 440 Tg Methane *) del2 = hh /. zz →
tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];


Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.34 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];

Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfaer= -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]] ;
Δfaer2 = Drop[Δfaer, Length[EM]] ;
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfliste = Append[Δfliste, Δf];
Δfaeroliste = Append[Δfaeroliste, Δfaer];
Δfghgliste = Append[Δfghgliste, Δfco2 + Δfmetan];

Tliste = {}; Do[ T2 = Klimaliste[[p]].Δf;

(* Remove this part to run the linear SRM comparison*)

Do[
T2 = Klimaliste[[p]].(Δf + 0.2 T2);
, {10}];

(* End of part *)

noise = σ2liste[[p]] * (Klimaliste[[p]].RandomReal[NormalDistribution[0, 1], Length[Δf]]);
noise = Drop[noise, 268 - 20];
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35; T2 = Drop[T2, 268]; T2=1.1+T2-T2[[1]];
noiseliste = Append[noiseliste, noise]; Tliste = Append[Tliste, T2];
, {p, 1, Length[models]}];

middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];
```

```
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1,
Length[Transpose[Tliste]]}];

RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}]
, u];

In[!]:= Length[noise]
Out[!]= 103

In[!]:= Length[T2]
Out[!]= 83

In[!]:= window=10;
noiseliste2 = Table[MovingAverage[noiseliste[[i]], window][[1 ;; Length[T2]]], {i, 1, Length[noiseliste]}];
noiseliste2 = Transpose[Partition[noiseliste2, 14]];

In[!]:= Length[noiseliste2]
Out[!]= 14

In[!]:= Dimensions[noiseliste2]
Out[!]= {14, 127, 83}

In[!]:= Length[noiseliste2[[1]]]
Out[!]= 127

In[!]:= ListPlot[noiseliste2[[3]],Joined→True]
In[!]:= ListPlot[Extract[templiste,p1],Joined→True]
In[!]:= ListPlot[Extract[temp1,p1],Joined→True,Axes→False,Frame→True, FrameStyle → Directive[Black, 14],
FrameLabel → {None, "GMST increase (°C)"}]

In[!]:= OURMODEL=Extract[templiste,p1];
MAGICC = Extract[temp1, p1];

In[!]:= Length[OURMODEL]
Out[!]= 86
In[!]:= Length[MAGICC]
Out[!]= 86

In[!]:= ListPlot[{OURMODEL[[1]],MAGICC[[1]]}]

In[!]:= parliste={};

Do[
x = Drop[MAGICC[[i]], 1];
y = OURMODEL[[i]][[{3, 13, 23, 33, 43, 53, 63, 73, 83}]]; par = Transpose[{x, y}];
parliste = Append[parliste, par];
, {i, 1, Length[OURMODEL]}]

In[!]:= y=OURMODEL[[1]][[{3,13,23,33,43,53,63,73,83}]]
Out[!]= {1.16787, 1.54466, 1.64649, 1.64045, 1.60513, 1.56572, 1.53297, 1.50672, 1.48457}
PL1 = ListPlot[parliste, AspectRatio → 1, PlotRange → {{0.5, 4}, {0.5, 4}}]; PL2 = Plot[zz, {zz, 0.5, 4}, PlotStyle →
Black];
gg = Fit[Partition[Flatten[parliste], 2], {zz, 1}, zz];
PL3 = Plot[gg, {zz, 0.5, 4}, PlotStyle → {Black, Dashed}];
Show[{PL1, PL2, PL3}, FrameStyle → Directive[Black, 14], Axes → False, Frame → True, FrameLabel →
```

```
{"MAGICC GMST (°C)", "Response model GMST (°C)"}]
(* All 774 datapoints *)
In[!]:= parliste2={};

Do[
x = Drop[MAGICC[[i]], 1];
y = OURMODEL[[i]][[{3, 13, 23, 33, 43, 53, 63, 73, 83}]]; par = {Max[x], Max[y]};
parliste2 = Append[parliste2, par];
, {i, 1, Length[OURMODEL]}];

PL1 = ListPlot[Map[{#} &, parliste2],
AspectRatio → 1, PlotRange → {{0.5, 4}, {0.5, 4}}];

PL2 = Plot[zz, {zz, 0.5, 4}, PlotStyle → Black];
gg = Fit[parliste2, {zz, 1}, zz];
PL3 = Plot[gg, {zz, 0.5, 4}, PlotStyle → {Black, Dashed}]; Show[{PL1, PL2, PL3}, FrameStyle → Directive[Black,
14], Axes → False, Frame → True, FrameLabel → {"MAGICC maximum GMST (°C)", "Response model maximum
GMST (°C)"}]
(* Max temperature comparison for 86 scenarios *)
```