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Meaning making in a sixth-grade mathematics classroom through touch screen technology

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ABSTRACT

This paper presents a case study of two sixth-grade students' use of an iPad as an instructional tool for mathematics. Based on their written and oral responses, we investigated and analyzed their meaning making process with mathematical content in a classroom where the iPad was a central tool for teaching practices. The analyses were based on Steinbring's [(2005). *The construction of new mathematical knowledge in classroom interaction: An epistemological perspective* (Vol. 38). Springer] framework, which we applied to understand how students construct meaning for mathematical concepts. The analysis showed that the students constructed mathematical knowledge by forming associations between contexts represented by the activities and by mathematical symbols and signs. The findings suggest the possibility that touchscreen technology reinforces the link between mathematical content, mathematical representations, and tangible experiences.

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KEYWORDS
Touchscreen technology; meaning making process; construction of mathematical knowledge; context

1. Introduction

In recent years, the use of touchscreen devices and software (iOS-based apps) has been rapidly increasing in teaching and learning (Zhang et al., 2015). Many apps have been designed to aid teaching and learning for school-aged children in a variety of disciplines. Since the widespread use of personal technological devices such as touchscreen tablets is still relatively new and the technology is rapidly changing, more research is needed to better understand its potential for learning and teaching in various disciplines (Kucirkova et al., 2014). This reflects the overall importance of digital technology, which in many countries, is considered an important factor in all areas of society. Research into the educational effects of digital technology has shown that using mobile device apps can have a positive impact on students' learning performances (Dalby & Swan, 2019; Galligan & Hobohm, 2018; Helen 2017; Sedaghatjou & Rodney, 2018; Hilton & Hilton, 2018; Kyriakides et al., 2016; Ronau et al., 2014; Soto & Ambrose, 2016).

This study was a collaboration between The Arctic University of Norway and a school in northern Norway for a project called, 'With Open Doors Toward the World – Using

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Massive Technology in Learning', which took place over a two-year period (2015–2017). The school's rationale for embarking on this project developed from the necessity of improving students' digital competencies in response to digitalization occurring across all levels of society. The pilot project started in 2015 with the participation of only first grade to fourth grade students (Johanson et al., 2018). The research team consisted of representatives with backgrounds in language (Norwegian), pedagogy, social sciences, and mathematics.

In response to this pilot project, the school expanded the use of the iPad to all grade levels (first to seventh grade, $N = 260$) in 2017. Teachers who participated in the pilot became mentors for new teachers after engaging in training seminars to master the use of the iPad and to learn about a variety of professional apps. Touchscreen devices were chosen because they are mobile and are more user-friendly than PCs. In addition, many students already have experience using devices such as mobile telephones and iPads in their everyday lives. The major goal of this study was to investigate iPad use as a methodological tool in the practice of teaching mathematics. The central learning goal was for students to be active participants in the production of their learning through text production and communication, and the iPad was intended to be a central tool in achieving the learning objectives in all teaching practices.

2. Literature review and research problem

Watts et al. (2016) documented the effects of touchscreen technology on mathematics learning by using virtual mathematics apps on a touchscreen device and found that this use of apps had a positive effect on children's learning progress. The participants in their study were pre-school aged children (age 3–4; $N = 35$), kindergarten aged children (age 5–6; $N = 33$), and second grade children (age 7–8; $N = 32$). Specifically, the study found that there was a shift in children's learning progression when they interacted with mathematical apps on touchscreen devices. 'Learning progression', according to the authors, is a construct that describes the hierarchical levels indicating children's understanding of mathematical concepts; these levels comprise a shifting process with incremental steps. Each step indicates a progression from a more limited understanding to a greater understanding of mathematical concepts. Others, including Smith et al. (2006) and Winick et al. (2008), have defined learning progression as a successive sequence of increasingly complex approaches to reasoning about a set of ideas. They suggest that an open-ended number of tasks with a variety of representations and levels of difficulty can support children in refining and shaping their understanding of mathematical ideas, resulting in incremental shifts in learning.

Moyer-Packenham et al. (2016) focused on affordances of mathematics apps for touchscreen devices on children's learning performance and efficiency. The participants in their study were 100 children aged 3–8 in pre-school, kindergarten, and second grade. Each participant used six different mathematics apps in 30–40-minute-long clinical interviews. They described 'affordances' as 'cues of potential uses of an artefact by an agent in a given environment' (as cited in Burlamaqui and Dong (2016)). Moreover, Moyer-Packenham et al. (2016) distinguished affordances as possessing either 'helping' or 'hindering' potential. Apps with 'helping' affordances would, for example, help students maintain their focus on mathematical content and their learning process, while 'hindering' affordances would distract children from their mathematical focus.

The results suggested that access to an affordance, regardless of whether it was helping or hindering, influenced children's learning performance and efficiency. For example, children in kindergarten and second grade showed a significant gain in numeracy after interacting with iPad apps for only one session. They also reported that hindering affordances could, in some cases, make students more conscious of the need to work harder to achieve their goal once they became aware of factors distracting their attention from the tasks.

Similarly, Spenser (2013) studied the use of learning apps for a touchscreen device (iPad) to improve numeracy among early school children. Participants in the research project were 160 five-year-old children from a private school in Dubai, United Arab Emirates. The results showed that children improved their numeracy skills, increased motivation, and gained positive associations with the subject. Additionally, Riconscente (2011) reported on the value of an iPad game in learning fractions. Participants in the project were 122 U.S students in fifth grade. The study employed a repeated measures crossover design to examine the role of an iPad fraction game in improving students' fraction knowledge and attitudes. The results demonstrated that students had significant learning improvement while they played the game. The authors also noted that the learning gains were stable over time, meaning that they were retained even when the students were no longer exposed to the game. Moreover, using the fraction game had positive impact on students' attitudes.

Corbett et al. (2017) conducted a case study that utilized mobile touchscreen technology as an aid in elementary mathematics education. They developed an educational software application for use in elementary mathematics classrooms to study the iPad's potential for improving student achievement in early algebra. The study included a pre-test and post-test for fourth and fifth grade students who had received traditional lessons, compared to students who received instruction using iPads. The results suggested that the iPad instruction did not influence the performance of either the fourth- or fifth-grade students in areas including fluency with whole numbers and correct usage of the equal sign. However, the study identified that utilizing iPads improved students' attention to pre-test tasks. This indicates that the use of iPads in elementary mathematics can improve students' attention to subsequent mathematics activities (Corbett et al., 2017). These studies demonstrate the usefulness of mobile technologies for current and ongoing learning, adding support to their use in elementary classrooms.

3. Research question

The majority of studies on the role and effects of touchscreen technology in mathematics learning have focused on measuring such effects in the context of short-term technology applications during the teaching practice. The current study seeks to investigate the use of these tools over a longer term. The school in this project had been using iPads in all teaching practices. This study does not focus on the impact of mathematics-specific applications on learning mathematics. Instead, the aim of this project is to study how mathematical conceptual learning takes place in a teaching practice in which lessons and educational instructions are disseminated through the iPad. As such, we sought to answer the following research question: How do sixth-grade students make meaning of mathematical concepts when touchscreen technology is used in the teaching practice?

4. Theoretical background

This study builds on socio-cultural learning theories that argue for the individual being an active participant in his or her own education (Cobb & Bowers, 1999). Mathematics is often perceived as an abstract knowledge area, and the learning of mathematical concepts occurs through abstraction processes. In the natural world, learning the properties of a tangible object can often help us develop an understanding of that object. In mathematics, one can consider a 'mathematical object' (Sfard, 2008) as an abstract concept in which its content is, in a sense, hidden (Steinbring, 2005). To represent how mathematical concepts are abstract constructs (Thompson & Sfard, 1994), we can consider the case of prime numbers, which are one set of natural numbers. One can identify whether a natural number is prime by verifying whether the number satisfies the definition of a prime number. The fundamental theorem of arithmetic states that all natural numbers can be written uniquely as products of prime factors (Rosen, 1988). This fact expresses a multiplicative relationship that is initially hidden. Therefore, the presentation of mathematical objects through character signs and symbols is initially not directly related to its contents (Duval, 2017)

There is a broad consensus in mathematics didactics that learning requires the student's active participation in the learning process in order to form supporting content for mathematical concepts (Cobb & Bowers, 1999; Steinbring, 2005). Students' engagement and social action are important prerequisites for learning, and they form the basis for learners' acquisition of their own experiences. Developing mathematical knowledge occurs through abstraction processes in which the learner constructs conceptual content for mathematical signs through a reference context (Figure 1). This process can be illustrated through an epistemological model (Steinbring, 2005, p. 22):

> To understand the abstraction process, we analyzed expressions (signifiers) that students use to symbolize mathematical ideas with respect to context. Indeed, the meaning making process can be understood as a process of relating mathematical signs/symbols to the relevant context. Students' development of mathematical concepts is influenced by the interaction between the sign/symbol and reference model.

According to Sfard (1991), a mathematical object indicates a structural aspect in which several underlying concepts are related to each other – in other words, they form a kind of web of ideas. Thus, perception of an abstract entity as a mathematical object requires structural conception. Sfard (1991, p. 4) describes this relationship:

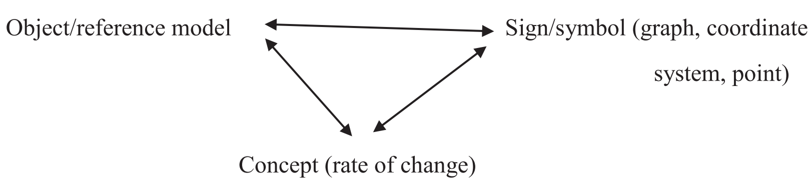

Figure 1. The epistemological triangle visualizing mediation between mathematical signs and reference model.

> Seeing a mathematical entity as an object means being capable of referring to it as if it was a real thing – a static structure, existing somewhere in space and time. It also means being able to recognize the idea 'at a glance' and to manipulate it as a whole, without going into details.

Hershkowitz et al. (2001, p. 202) describe abstraction process as follows:

> A process of abstraction leads from initial unrefined abstract entities to a novel structure. The novel structure comes into existence through reorganization of abstract identities and through establishment of new internal links within the initial entities and external links among them.

They consider the abstraction process as a series of actions (i.e. mental actions) through which an individual constructs more complex structures from an initial conceptual structure. This view is consistent with Steinbring's (2005) epistemological model of the meaning making process as a dialectic interaction between signs/symbols and mathematical objects.

5. Mathematical concepts

The mathematical object considered in this study is the graph, and we targeted the related concepts of the function concept and the rate of change. For the first task, students made a graph of a plant's growth by measuring and plotting their data. Students were familiar with the notion of a coordinate system, but they had not previously used graphs to interpret phenomena. Students are usually introduced to the algebraic expression of functions for the first time in eighth or ninth grade. Therefore, the activities in this project aimed to make a graphical presentation of a real-world situation rather than an algebraic one.

Rate of change is often a key concept for interpreting graphs, and conceptualizing the idea is considered to be challenging (Herbert & Pierce, 2011), even for calculus students (Ubuz, 2007). The rate of change concept builds on students' proportional reasoning, and it is a key aspect in the development of relational understanding for the concept of a function (Dolores-Flores et al., 2019). The slope of a line is the most basic rate of change (Stanton & Moore-Russo, 2012), and it is defined by the ratio $\Delta y/\Delta x = [\mathrm{f}(x_2) - \mathrm{f}(x_1)]/(x_2 - x_1)$ in the interval $[x1, x2]$ (Adams, 2003). The interpretation of change as the ratio $\Delta y/\Delta x$ can be expressed as the slope of a linear graph obtained from a real-world situation. The plant's growth activity represented non-linear functions. However, as the algebraic expression of a biological model often results in a nonlinear function, it is not suitable for sixth grade students. A basic mathematical representation (graph) of plant growth consisting of several line segments was the starting point for students' interpretation of graphs.

Based on consultation with the teacher through conversations at the beginning of the project, students connected segments between two points (data from the plant growth) in order to present a coherent graph. Thompson and Carlson (2017) have argued that, in developing an idea of a constant rate of change, middle school students can apply their developing conception to the non-constant rate by thinking of it as 'having constant rates of change over small (infinitesimal) intervals of its argument, but different constant rates of change over different infinitesimal intervals of its argument' (p. 452). In doing so, students can interpret a nonlinear model by using constant rate of change over small intervals.

The development and conceptualization of the idea of change is dependent on one's perceptions of quantitative reasoning. Thompson (2011) considers 'quantification' to be a process that forms the basis of quantitative reasoning, where quantification as defined as follows: 'Quantification is the process of conceptualizing an object and an attribute of it so that the attribute has a unit of measure, and the attribute's measure entails a proportional relationship (linear, bi-linear or multi-linear) with its unit' (p. 37). Variation and covariation are central ideas that are strongly related to quantitative reasoning. As a concept, variation is formed when one can imagine that an attribute of an object can vary. Covariation reasoning is based on one's awareness of quantification and variation between two quantities in a functional relationship (Thompson, 1994). Research has documented that even young children are able to grasp the basic idea of covariation through observations of change in real world phenomena (Comfrey & Smith, 1994).

6. Research method and materials

The teacher in the targeted class had positive impressions regarding the use of digital technology in her teaching, and she participated in the pilot study that took place from 2016 to 2017. Teachers who participated in the pilot were trained to use apps with an iOS platform (Apple products) designed for education in various school subjects. Since 2017, the school that was the setting for this study has been using iPads across all grades (first to seventh), and the devices have largely replaced books. Participants in this study were students from a sixth grade, mixed gender class ($N = 19$, aged 11–12 years). At the time of the study, the students had used iPad over almost nine months, and they had mastered using different apps. In this paper, two students' responses are accounted for. The duration of the project was four weeks. Apps that the teacher used at the time of the study are outlined below.

6.1. Featured applications

Book Creator (https://apps.apple.com). Book Creator is a text application that students can use to produce text, images, drawings, and sound. A Book Creator file layout is similar to a book in that it allows students to create the layout and forms of their work.

iThoughts (https://apps.apple.com). This is a mind map app that can organize themes and other content into one's own preferred structure. The teacher uses iThoughts to organize the lessons so that the students obtain an overview of the class's theme, activities, content, academic goals, and requirements.

Showbie (https://www.showbie.com/features/). Showbie is an app that students can use to save their work. Teachers can communicate to students through this app and provide feedback.

GeoGebra Graphing Calculator (https://apps.apple.com). GeoGebra is a mathematics app with a variety of features, including construction of geometric shapes, graphing of algebraic expressions, and presentation of descriptive statistics. For the duration of this project, the students had limited use of this app, employing it only to draw graphs based on their data in a coordinate system.

6.2. Task design

The researchers for the project discussed and designed the activities in collaboration with the teacher. The activities referred to real situations that students were familiar with and considered the 'meso-space' (Bessot, 2014), where students could directly experience and apply concrete objects to the development of their mathematical knowledge. According to van Hiele (1986), learning in mathematics assumes that students actively manipulate and investigate objects in relevant contexts. Although his research focused on students' understanding and development in geometry, his perspective supports a constructivist view on learning in mathematics and the methodology used in the present study.

6.2.1. Task 1

For the first activity, the teacher reviewed basic concepts such as the coordinate system and coordinates in a class discussion. Students were to plant sunflower seeds and observe the growth over a three-week period. Further, students were directed to measure the growth of their plants and insert plant growth data into a coordinate system provided as a file on their iPad. Each student cultivated two seeds. This was to ensure that each student had a plant if one of the plants failed to germinate or died out. Finally, students were to make a graph using the GeoGebra app and interpret the growth process. The graph was a continuous curve consisting of segments connecting marked points. Over the three weeks, students gained experience with the coordinate system by plotting their data as points on the coordinate system two times a week.

6.2.2. Task 2

In the second activity, students watched a documentary about salmon fishing in Norway and then interpreted two graphs: one presenting wild Atlantic salmon caught in the sea from 1980 to 2016 and the other showing wild Atlantic salmon caught in rivers in Norway during the same period. Salmon fishing is a well-known and important national topic, and a four-episode documentary was aired on Norwegian national television (NRK, 2017) in 2017 to discuss different perspectives on this theme regarding salmon as a national resource. Norway has more than 400 water sources that are home to Atlantic salmon, and it supports a large proportion of the world's wild Atlantic salmon Therefore, Norway has a special obligation to manage this resource effectively (Forseth et al., 2017). In one of the programmes, researchers caught fish in the Alta River and Fjord for data collection, and this area was very familiar to the students who participated in the study. In the second episode, two graphs of the salmon catch were presented in the same coordinate system. Students were asked to interpret the graphs and predict where the largest salmon catch would be in 2018 (i.e. sea or river) (Figure 2).

Three different recording devices were used to videotape both activities. A desktop camera was used to record whole-class activities, and it was equipped with a wireless microphone in middle of the class to record all interactions. The student's iPads were used to capture small-scale student interactions. In addition, the researcher used a GoPro camera to move about in the classroom and record student communication and utterances. The teacher supervised the activity, and her role was primarily to organize the students and activities. The researcher was an active observer and asked questions that were mainly about students' thoughts and reasoning. The analysis consisted of the following three steps. First, preliminary interpretations resulted from an overall interpretation based on a total of five hours of video recordings. Next, data reduction was conducted by choosing interactions containing some explicit mathematical content. This resulted in choosing data from two students, presented as P1 and P2 in this paper. Finally, the interactions were analyzed

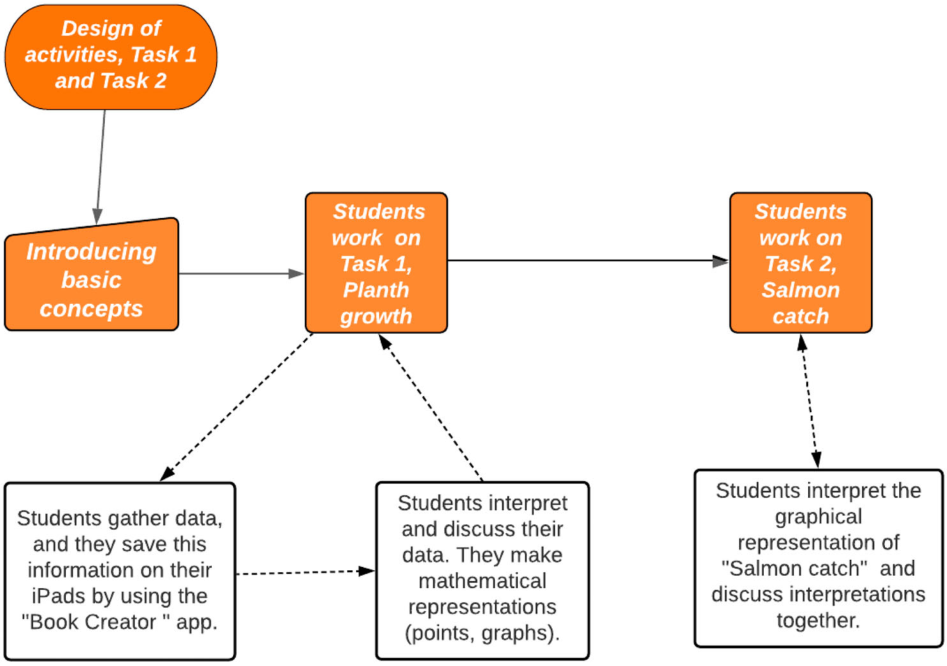

Figure 2. A diagrammatic representation of the methodology.

in depth by means of the analytical framework. The interpretations were discussed among the research team to achieve triangulation. The analysis consisted primarily of interpretations of those mathematical concepts signified by student's utterances (signifiers). Other aspects of touchscreen technology use in the mathematics classroom are identified in the Discussion section.

7. Results

Prior to the plant growth activity, the teacher had already introduced basic concepts including coordinates of a point and the coordinate system. The students had some understanding of these concepts; however, they expressed that the terms 'coordinate' and 'coordinate system' were still unclear concepts for them. In the first activity, the students (designated as P1 and P2) monitored the growth of their plants. Each student had sown two sunflower seeds. They each produced a table of their results and marked the points on a coordinate system. Students documented their measurements by taking pictures of plants where the length and date of the images were stored in the iPad web (Showbie app).

7.1. Case 1: Task 1

For Task 1, P2 measured the growth of his plants over a longer period and plotted the graph. The results suggest that P2 was able to relate the incline of the graph to the idea of 'rate of change'. He interpreted a horizontal part of graph as signifying 'no growth' or $\Delta y/\Delta x = 0$. Furthermore, P2 noted periods when his plants grew fastest by indicating the steepness of the graph and using it to reason out his assertion. Below is an excerpt of his conversation with the researcher, denoted by 'R'.

1: P2: It is fastest on day 9 to day 10 because it has grown 3 cm.
2: R: Ok.
3: P2: It has grown 0 cm first.
4: R: Why do you think it is the fastest?
5: P2: Because it is the steepest hill here [pointing to steepest line segment on the graph].
6: R: If you look at growth between days 10 and 11 and [days] 11 and 12.
7: P2: Umm [confirming].
8: R: What can you say? How much have they grown over the two periods?
9: P2: They have grown [pauses] from day 10–11. They have grown 0.9 cm [reads from the graph].
10: R: What about days 11–12?
11: P2: 0.6 and 0.7 or something like that [looks uncomfortable, as if something is not quite right].
12: R: Can one say that the plant has grown in days 10–11 as fast as days 11–12?
13: P2: Yes.
14: R: Why so?
15: P2: Because it is straight [shows line between two points: $x = 10$ and $x = 12$].
16: R: What happened between 0 and 7?
17: P2: 0 cm.
18: R: What does that mean?
19: P2: It has not grown anything. But from 7 to 8, it has grown slightly, 0.1 cm. That became more and more.

Here, P2 expresses his understanding of the relationship between the slope of the lines and the growth rate. The relationship between the student's argument as 'signifier' and the growth rate as 'signified' is illustrated in Figure 3. P2 uses the mathematical concept of a point, or coordinate, to argue his claim. He reads the values of points J and K (see Figure 3) and calculates the growth between days 9 and 10. The calculation $y_{10} - y_9/x_{10} - x_9$ is executed informally (mental calculation). He considers the concept of rate $\Delta y/\Delta x$ as the slope of the segments and mentions that the rate is greatest (Turn 5) in the interval [9, 10]. When P2 is asked (Turns 6–11) to interpret the graph in the period [10–12], he reads two different values – 0.9 and 0.7 – instead of just the value 0.9. He uses vertical gridlines on the coordinate system to find the intersection of the graph and reads the y-value differences $(y_{11} - y_{10}) = 0.9$ (which is the correct reading). However, $(y_{12} - y_{11})$ is measured incorrectly as 0.7. This is probably because P2 reads the distance from the intersection of the vertical gridline at $x = 12$. His different approach to identifying the slope in these two instances indicates an inconsistency.

P2's argument, based on reading the measurements in the period [10–12], conflicts with his interpretation of slope for the line segment (marked as KL by P2, Figure 1). P2 provided two different values (0.9 and 0.7) for the same rate, which indicates that P2 did not yet comprehend rate of change as a 'quotient' (Byerley & Thompson, 2017). Utterances (Turns 16 and 19) indicate that P2 considers a horizontal line segment as a semiotic representation of zero growth for a given period.

It seems that P2 was still in the process of shaping his concept of 'growth rate'. He interprets a horizontal segment as a representation of constant y-values, where y-values represent the length of the plant. His conception of a horizontal segment is another interpretation of plant growth that is consistent with his generalizations of the relationship

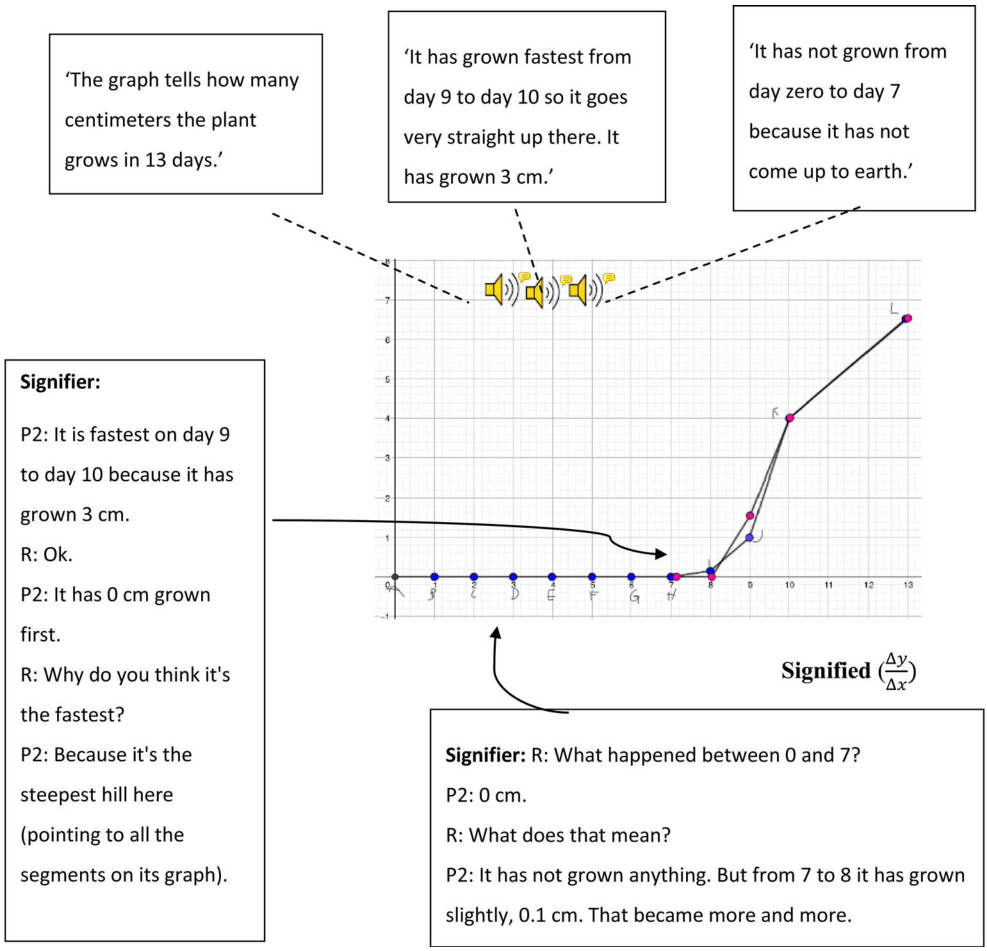

Figure 3. The students' work sheet produced using the Book Creator application.

between the slope of the segment and the rate of change for plant growth. With regard to the relationship between 'signifiers' and 'signified', P2 has created new meaning for the graph's 'change', which is an important aspect of semiotic content for conceptually understanding 'rate of change'.

7.2. Case 2: Task 2

Cases 2 and 3 occurred during the second activity. Students had previously watched parts of a documentary episode about salmon fishing in the sea and rivers in Norway, which was the background for Task 2. They used their iPads to log their answers, and they recorded their explanations and arguments as an audio file. Almost all of the audio files consisted of students reading their written explanations; therefore, their oral utterances did not provide much more information than their written answers. Both P1 and P2 used the steepness of the graphs as to support their interpretations of what had occurred in the given context (salmon fishing). Their arguments (in Turns 20–30, below) indicate covariation reasoning, and they show signs of developing their conceptualization of a 'variable'.

20: R: What can you say about salmon fishing in the sea from 1980–1990?
21: P1: They have been fishing a lot.
22: P2: Yes, very much, fishing went down from 1980 to 1990. It went approximately 300,000 down.
23: R: Ok.
24: P2: In number.
25: R: What about salmon fishing in the river?
26: P1: They do not fish that much.
27: P2: It was very stable.
28: P1: It was almost a straight line there [using hand gestures to express a horizontal line].
29: R: How can you see stability from a graph? [Both are very eager to comment and argue].
30: P1: It's almost like a straightforward stretch. [He zooms out so that the graphs flatten out and become more visible].

P2 and P1's interpretations of the graph indicated that both saw that the rate of change for the Atlantic salmon-graph between 1980 and 1990 was decreasing. Their interpretation of the graph was based on reading the y-coordinates at periods 1980 and 1990, and this reinforced their arguments for the decline of salmon fishing in the sea. Student statements (Turns 21, 22) emphasize the rate of change $\Delta y/\Delta x$ (signified) in the period 1980–1990. Students' covariation reasoning suggests that they consider variation of one quantity (fish) in relation to other quantity (time) as 'declining'. Thus, students can envision the quantity values varying in certain periods in relation to one another; this indicates development of the conceptual meaning of the 'variable' (Thompson & Carlson, 2017). P2 uses the description 'very stable' to express 'no or [slight] change' on the graph for the period of 1980–2016. This is supported by P1, who expresses the graph's development as 'almost a straight line' while using hand gestures to illustrate a horizontal line.

In Task 1 (plant growth), P2 had interpreted a horizontal curve (line segment) as having no growth (i.e. $\Delta y/\Delta x = 0$). By ignoring minor variations in the given interval, both P1 and P2 regarded the graph of river salmon fishing as stable. Zooming out on the graph enhanced the interpretation by both students, making the stability more visible (Turn 30). P1 and P2 were involved in an abstraction process where the rate concept $\Delta y/\Delta x$ was being shaped by establishing a relation between context and graph as a mathematical sign.

7.3. Case 3: Task 2 – predict near future development of salmon fishing

In Activity 2, students were encouraged to make predictions about the future of salmon fishing based on the information from the graphs. P1 and P2 expressed their opinions, which were similar. P2 agreed with P1 and explained more fully:

31: R: What will happen to developments in 2018?
32: P2: I think that it will be similar to 2016. There is only two years difference (referring to the graphs). As you can see from the 2010–2012 … it's very stable further on (makes horizontal movement with hands).

P2 used the word 'stable' as well as a gesture that expressed a horizontal line. When he was asked about the development of salmon in 2018, P2 indicated that there would be no great change in salmon fishing development by expressing the 'signifier' – *'it is similar to 2016'* – in combination with a hand movement that expresses a horizontal line as an indication for no/weak growth (signified $\Delta y/\Delta x \approx 0$, a constant). This interpretation is supported by P2's answer to the question: 'Where was there more salmon fishing – in the sea or in the rivers – in 2018?' The following is his response from his task sheet on the iPad:

Where was the most salmon caught (in the sea or in rivers) in 2018?	P2: *More salmon were caught in rivers than in the sea in 2018. Because the river is stable and the sea is also stable. And the river is higher than the sea; therefore, I think there will be more catch from river than the sea in 2018.*

8. Discussion

The mathematical concepts that were central to the students' activities referenced real world experiences that were known and relevant to the students. In their arguments about experiential phenomena, students used quantitative reasoning. This may indicate that they conceived coordinates as a relation between two measurements – time and length in the first activity, and time and quantity of salmon caught in the second. In their interpretation of the models, students demonstrated the ability to use the coordinate system in both activities. In the first activity, both P1 and P2 interpreted the graph's horizontal position as an indication of no growth of the plants. One possible factor that may have supported each student's abstraction process was the data collection in the first activity over the three-week period, which likely reinforced the relationship between the context (plant growth) and mathematical concepts. The students' first-hand knowledge of their plants' growth supported them in creating meaning and content for mathematical symbols (such as graphs and points). This interpretation suggests the establishment of a connection between the graph's incline and the 'rate of change' as a mathematical concept (Figure 4).

P1 and P2 both consider the attribute (plant length) as a value that did not vary in a certain interval (i.e. the first seven days after cultivating). This implies the construction of the

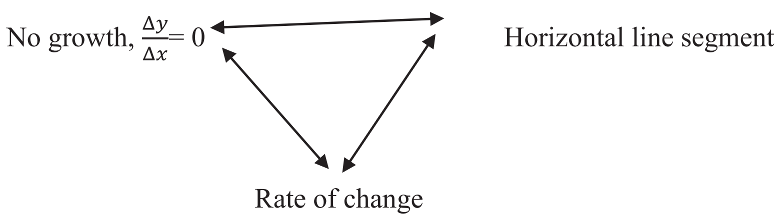

Figure 4. The epistemological triangle visualizing mediation between line segment as mathematical sign and reference model.

Signifier **Signified** $(\frac{\Delta y}{\Delta x})$

R: What about salmon fishing in the river?

P1: They do not fish that much.

P2: It was very stable.

P1: It was almost straight line there [using hand gestures to express a horizontal line].

R: How can you see stability from a graph?

P1: It's almost like going straight ahead.

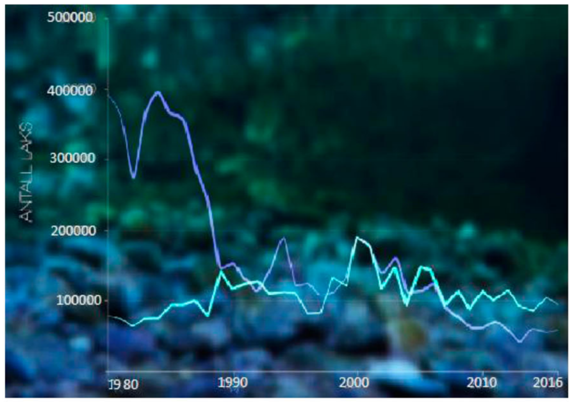

Figure 5. The graph for salmon catch.

new knowledge that a horizontal line segment has the meaning of a constant. Hershkowitz et al. (2001) consider the abstraction process to be a series of actions in which an individual constructs entities (concepts, mathematical objects) with more complex structures based on initial structures. As an illustration of this, P1 and P2 used 'points' on the graph as an initial entity indicating the relation between two quantities (attributes of time and length). Students' interpretation of the graph's steepness is probably an early formation of a new meaning for a more complex structure of 'rate of change'. In their quantitative reasoning about how the development of salmon fishing would play out in the near future, the term 'stability' was used. It is likely that the students' emphasis on the term 'stability' in their reasoning was influenced by their experience in the first activity, in which they constructed horizontal graph positions as representations of zero growth (Figure 5).

In Task 1 on plant growth, students used the iThoughts app to organize their activities by accessing information about content, goals, criteria, descriptions of activity, and reflections of their work. Students were able to retrieve information in the form of a mind map (Figure 6), and they could process their measurements using the GeoGebra app and store their data in a digital folder with the Showbie app. Further, the measurements – presented as tablets, pictures, and written notes – were created by the Book Creator app. This data was then available for each student, the teacher, and parents.

Through the use of the apps, students oriented themselves within the activity to determine what to do at different times. Knowing what to do is important for students' level of involvement because they can orient themselves as they move through the activity. In fact, students could observe that the seeds did not germinate for several days (P1 notes seven days with no sign of growth), and they were able to document this through their measurements (filling in tables) and their photographs taken using the iPads. This could support the formation of new knowledge by establishing a relationship between context and mathematical concepts (e.g. line steepness as an indication of growth rate).

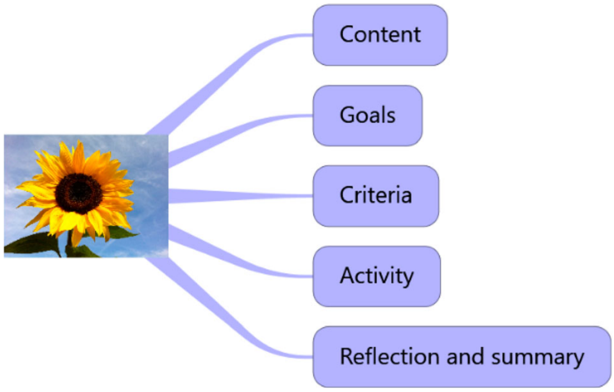

Figure 6. The mind map for the first activity. The content of each component in the mind map could be expanded by touching each icon.

Through formative assessment (Heritage et al., 2009), teachers can apply students' performance, motivation, learning potential (what they would be able to achieve next), and knowledge (Ginsburg, 2009). Students' work in this study includes their data collection, reasoning, and interpretations of tasks, which together consist of multiple representations, such as written text, pictures, and audio. Because mobile technology provides students with multiple modes of representation rather than solely those in written form, teachers are able to access more data to engage in students' learning processes (Soto & Ambrose, 2016). It is likely that student-generated data in this project can provide an important resource for teachers to enhance the formative assessment process.

Teaching activities can indicate innovative approaches to mathematics that differ from the traditional task-paradigm approach (Skovsmose, 2001). For many decades, mathematics teaching has been characterized by a task paradigm in which the teacher or textbook's role in the learning process is authoritative (Mortimer & Scott, 2003). Such an approach to learning contributes to the production of mathematics as a set of rules and formulas (Kaiser & Vollstedt, 2007). In the current study, however, the activities' contexts created the opportunity for students to become active participants in their own learning. The literature suggests that a teacher's approach to mathematics affects students' learning and knowledge. In this study, the teacher's role supported a non-authoritative (Mortimer & Scott, 2003) interaction with students, resulting in learners' voices being expressed during activities. In the class discussion, students' work was presented on a large screen in the classroom. This made their work more open for comments. It is likely that the students' familiarity with the contexts of activities influenced the students' engagement in those activities. This approach of providing relevant context for the content in mathematical learning, therefore, can be understood to have a positive effect on learning.

9. Conclusion and implications

This study shows the early formation of conceptualization of the 'rate of change' for two sixth-grade students. Our results showed that the students successfully constructed their mathematical knowledge employing the touchscreen technology utilized in the school's teaching practice. We traced student's understanding through their arguments about 'signifiers', which indicated the formation of their mathematical knowledge. While this study does not provide a basis for generalizing mobile technology's influence on mathematics learning, it does indicate the possibility that adapting touchscreen technology in mathematics instruction can have a supportive impact on student learning. Students' formations of mathematical concepts were influenced by the interaction between the activities and mathematical representations.

In this case, the technology played primarily a mediating role for students by reinforcing the link between mathematical representations (graphs), mathematical content, and mathematical activities. An important prerequisite for mobile technology to support student learning is to ensure that it is adapted into the teaching in an effective manner. In our case, the choice of real-life contexts was likely a supporting factor for students' meaning making processes. This is because the construction of mathematical knowledge cannot be separated from the social development context (Steinbring, 2005). In general, the connection between digital technology use and mathematics education needs more research focus. Moreover, in future research, study is needed into how the use of touchscreen technology can affect students' communication in learning mathematics.

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References

Adams, R. A. (2003). *Calculus: A complete course* (5th ed., p. 14). Pearson/Addison-Wesley.

Bessot, A. (2014). Designing a simulator in building trades to Transform vocational education. In S. Rezat, M. Hattermann, & A. Peter-Koop (Eds.), *Transformation - A fundamental idea of mathematics education* (pp. 223–239). Springer. https://doi.org/10.1007/978-1-4614-3489-4_12

Burlamaqui, L., & Dong, A. (2016). Affordances: Bringing them out of the woods. *Interactions*, *23*(4), 80–82. https://doi.org/10.1145/2934292

Byerley, C., & Thompson, P. W. (2017). Secondary mathematics teachers' meanings for measure, slope, and rate of change. *The Journal of Mathematical Behavior*, *48*, 168–193. https://doi.org/10.1016/j.jmathb.2017.09.003

Cobb, P., & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher*, *28*(2), 4–15. https://doi.org/10.3102/0013189X028002004

Comfrey, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. *An International Journal*, *26*(2), 135–164. https://doi.org/10.1007/BF01273661

Corbett, M., Ehrlich, J., McQuillan, J., & Zhao, C. (2017). *Utilizing mobile touch screen technology as an aid in elementary mathematics education*. ProQuest dissertations publishing.

Dalby, D., & Swan, M. (2019). Using digital technology to enhance formative assessment in mathematics classrooms. *British Journal of Educational Technology*, *50*(2), 832–845. https://doi.org/10.1111/bjet.12606

Dolores-Flores, C., Rivera-López, M. I., & García-García, J. (2019). Exploring mathematical connections of pre-university students through tasks involving rates of change. *International Journal of Mathematical Education in Science and Technology*, *50*(3), 369–389. https://doi.org/10.1080/0020739X.2018.1507050

Duval, R. (2017). *Understanding the mathematical way of thinking – The registers of semiotic representations*. Springer International Publishing: Imprint: Springer.

Forseth, T., Barlaup, B. T., Finstad, B., Fiske, P., Gjøsæter, H., Falkegård, M., Hindar, A., Mo, T. A., Rikardsen, A. H., Thorstad, E. B., Vøllestad, L. A., & Wennevik, V. (2017). The major

threats to Atlantic salmon in Norway. *ICES Journal of Marine Science*, *74*(6), 1496–1513. https://doi.org/10.1093/icesjms/fsx020
Galligan, L., & Hobohm, C. (2018). Mathematics screencasts for teaching and learning. In N. Calder, K. Larkin, & N. Sinclair (Eds.), *Using mobile technologies in the teaching and learning of mathematics* (pp. 265–282). Springer International Publishing. https://doi.org/10.1007/978-3-319-90179-4_15
Ginsburg, H. P. (2009). The Challenge of Formative Assessment in Mathematics Education: Children's Minds, Teachers' Minds. *Human Development*, *52*(2), 109–128. https://doi.org/10.1159/000202729
Helen, C. (2017). Using mobile learning to support students' understanding in geometry: A design-based research study. *Educational Technology & Society*, *20*(3), 207–219.
Herbert, S., & Pierce, R. (2011). What is rate? Does context or representation matter? *Mathematics Education Research Journal*, *23*(4), 455–477. https://doi.org/10.1007/s13394-011-0026-z
Heritage, M., Kim, J., Vendlinski, T., & Herman, J. (2009). From evidence to action: A seamless process in formative assessment? *Educational Measurement: Issues and Practice*, *28*(3), 24–31. https://doi.org/10.1111/j.1745-3992.2009.00151.x
Hershkowitz, R., Schwarz, B. B., & Dreyfus, T. (2001). Abstraction in context: Epistemic actions. *Journal for Research in Mathematics Education*, *32*(2), 195–222. https://doi.org/10.2307/749673
Hilton, A., & Hilton, A. (2018). Engaging primary school students in mathematics: Can iPads make a difference? *International Journal of Science and Mathematics Education*, *16*(1), 145–165. https://doi.org/10.1007/s10763-016-9771-5
Johanson, L. B., Johanson, L. B., & Karlsen, S. S. (2018). *Restart: å være digital i skole og utdanning* [To be digital in school and education]. Universitetsforl.
Kaiser, G., & Vollstedt, M. (2007). Teachers' views on effective mathematics teaching: Commentaries from a European perspective. *ZDM*, *39*(4), 341–348. https://doi.org/10.1007/s11858-007-0036-1
Kucirkova, N., Messer, D., Sheehy, K., & Fernández Panadero, C. (2014). Children's engagement with educational iPad apps: Insights from a spanish classroom. *Computers & Education*, *71*(C), 175–184. https://doi.org/10.1016/j.compedu.2013.10.003
Kyriakides, A. O., Kyriakides, A. O., Meletiou-Mavrotheris, M., Meletiou-Mavrotheris, M., Prodromou, T., & Prodromou, T. (2016). Mobile technologies in the service of students' learning of mathematics: The example of game application A.L.E.X. In the context of a primary school in Cyprus. *Mathematics Education Research Journal*, *28*(1), 53–78. https://doi.org/10.1007/s13394-015-0163-x
Mortimer, E., & Scott, P. (2003). *Meaning making in secondary science classrooms*. Maidenhead Open University Press.
Moyer-Packenham, P., Bullock, E., Shumway, J., Tucker, S., Watts, C., Westenskow, A., Anderson-Pence, K., Maahs-Fladung, C., Boyer-Thurgood, J., Gulkilik, H., & Jordan, K. (2016). The role of affordances in children's learning performance and efficiency when using virtual manipulative mathematics touch-screen apps. *Mathematics Education Research Journal*, *28*(1), 79–105. https://doi.org/10.1007/s13394-015-0161-z
NRK. (2017). *Den fantastsike villaksen* [The amazing wild salmon]. https://tv.nrk.no/serie/den-fantastisk-villaksen/sesong/1/episode/1/avspiller.
Riconscente, M. M. (2011). *Mobile learning game improves 5th graders' fractions knowledge and attitudes*. GameDesk Institute.
Ronau, R. N., Rakes, C. R., Bush, S. B., Driskell, S. O., Niess, M. L., & Pugalee, D. K. (2014). A survey of mathematics education technology dissertation scope and quality: 1968–2009. *American Educational Research Journal*, *51*(5), 974–1006. https://doi.org/10.3102/0002831214531813
Rosen, K. H. (1988). *Elementary number theory and its applications* (2nd ed.). Addison-Wesley.
Sedaghatjou, M., & Rodney, S. (2018). Collaborative engagement through mobile technology in mathematics learning. In N. Calder, K. Larkin, & N. Sinclair (Eds.), *Using mobile technologies in the teaching and learning of mathematics* (pp. 113–129). Springer International Publishing. https://doi.org/10.1007/978-3-319-90179-4_7

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, *22*(1), 1–36. https://doi.org/10.1007/BF00302715

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Skovsmose, O. (2001). Landscapes of investigation. *ZDM*, *33*(4), 123–132. https://doi.org/10.1007/bf02652747

Smith, C. L., Wiser, M., Anderson, C. W., & Krajcik, J. (2006). FOCUS ARTICLE: Implications of research on children's learning for standards and assessment: A proposed learning progression for matter and the atomic-molecular theory. *Measurement: Interdisciplinary Research & Perspective*, *4*(1-2), 1–98. https://doi.org/10.1080/15366367.2006.9678570

Soto, M., & Ambrose, R. (2016). Screencasts: Formative assessment for mathematical thinking. *Technology, Knowledge and Learning*, *21*(2), 277–283. https://doi.org/10.1007/s10758-015-9272-6

Spenser, P. (2013). *Ipads: Improving numeracy learning in the early years*. Mathematics education: Yesterday, today, and tomorrow: Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia (pp. 610–617). www.scopus.com.

Stanton, M., & Moore-Russo, D. (2012). Conceptualizations of slope: A review of state standards: Conceptualizations of slope. *School Science and Mathematics*, *112*(5), 270–277. https://doi.org/10.1111/j.1949-8594.2012.00135.x

Steinbring, H. (2005). *The construction of new mathematical knowledge in classroom interaction: An epistemological perspective* (Vol. 38). Springer.

Thompson, P., & Carlson, M. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.

Thompson, P. W. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. *Educational Studies in Mathematics*, *26*(2), 229–274. https://doi.org/10.1007/BF01273664

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In L. L. Hatfield, S. Chamberlain, & S. Belbase (Eds.), *WISDOMe monograph: New perspectives and directions for collaborative research in mathematics education* (Vol. 1, pp. 33–57). University of Wyoming. http://patthompson.net/RUME2S19Public/files/36835.pdf.

Thompson, P. W., & Sfard, A. (1994). Problems of reification: Representations and mathematical objects. In D. Kirshner (Ed.), *Proceedings of the Annual meeting of the International Group for the psychology of mathematics education — North America, plenary sessions* (Vol. 1, pp. 1–32). Louisiana State University. https://asu.pure.elsevier.com/en/publications/problems-of-reification-representations-and-mathematical-objects.

Ubuz, B. (2007). Interpreting a graph and constructing its derivative graph: Stability and change in students' conceptions. *International Journal of Mathematical Education in Science and Technology*, *38*(5), 609–637. https://doi.org/10.1080/00207390701359313

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press. http://books.google.no/books?id=0sHaAAAAMAAJ.

Watts, C. M., Moyer-Packenham, P. S., Tucker, S. I., Bullock, E. P., Shumway, J. F., Westenskow, A., Boyer-Thurgood, J., Anderson-Pence, K., Mahamane, S., & Jordan, K. (2016). An examination of children's learning progression shifts while using touch screen virtual manipulative mathematics apps. *Computers in Human Behavior*, *64*(C), 814–828. https://doi.org/10.1016/j.chb.2016.07.029

Winick, D., Avallone, A., Smith, C., & Crovo, M. (2008). *Science framework for the 2009 national assessment of educational progress*. National Assessment Governing Board.

Zhang, M., Trussell, R. P., Gallegos, B., & Asam, R. R. (2015). Using math apps for improving student learning: An exploratory study in an inclusive fourth grade classroom. *TechTrends*, *59*(2), 32–39. https://doi.org/10.1007/s11528-015-0837-y