Resource Allocation by Contest or Bargaining

Eirik Mikkelsen[1]

Department of Economics, The Norwegian College of Fisheries Science, University of Tromsø, 9037 Tromsø, Norway.

Abstract

We consider resource allocation between two players through contest or Nash bargaining, assuming that player 2's contest effort or resource use exerts an externality on player 1. A sequential move contest where the effectively lowest valuation player (the underdog) moves first maximises individual and thus also sum of payoffs. The interests of regulator and players are thus aligned. In Nash bargaining between the players, a threat point of no allocation produces the highest sum of payoffs. Then the externality source fully gets the cost (benefit) of a negative (positive) externality. If contest outcomes are used as bargaining threat points, the highest sum of payoffs is for the favourite-moves-first contest. That contest, however, gives lower sum of payoffs as a contest, and is thus not immediately credible as threat point. If the regulator can commit to play the contest the players jointly recommend, or have as threat point to bargaining the contest the players jointly promote, sum of payoffs from contests and from Nash bargaining can be maximised, even if the regulator does not have full information about valuation and externality. It requires that the players have sufficient information to know who is favourite and who is underdog.

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[1] E-mail: eirik@samf.norut.no; Tel: +47 9593 5362. I thank Derek Clark for valuable comments.

Introduction

Competition over access to natural resources takes many forms. Sometimes it is a matter of writing applications and having a dialogue with a regulator, alone or as part of a planning process. Lobbying for resource access is also common, inside or outside of a structured planning process, as is bribing officials. Sometimes the regulator is looking for spin-offs of resource use, like job creation and rural development, and will grant resource access to those that best render this probable (sometimes coined "a beauty contest"). In economics all the competition types above can be classified as *contests*.

Epstein and Nitzan (2006) argue that contest-models can be used to study lobbying in a large variety of democratic political environments; contest models can capture the basic relationship between government objectives, public policy, and the characteristics of the interest groups that try to influence that policy. A contest is when actors invest resources/effort in order to influence their chances of winning a prize, or a share of a prize, and the invested resources are sunk (Konrad 2006). The prize would here be access to a natural resource. Contests have been studied extensively, under many different assumptions/settings.[2] Externalities of effort have, however, not been explicitly included in many contest models (see more in section 2).

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[2] Contests have at least been used to study war, sports, R&D contests/patent races, firm-internal labour markets, litigation, education filters, marketing, "beauty contests"/lobbying/rent-seeking, political campaigning and committee bribing. Surveys and collections of seminal papers for contests: (Buchanan *et al.* 1980; Nitzan 1994; Lockard & Tullock 2001; Konrad 2006; Congleton *et al.* 2007; Garfinkel & Skaperdas 2007). Several other papers have dealt with rent-seeking in natural resource allocations (i.a. Boyce 1998; Edwards 2001; i.a. Bergland *et al.* 2002)

In this paper we first consider natural resource contests when an asymmetric external effect may be present. Our study is motivated by coastal zone planning processes. Agents may compete for exclusive access to a location or some other coastal resource, but in many cases it is for a share of a resource. Own access to and use of coastal resources is always valuable for a user, while others' use may entail external effects (negative or positive). The stock-externality in fisheries affecting harvesting costs is an example (Clark 1992). It is a symmetric externality, at least when fishing vessels are identical. In other cases there will be asymmetric externalities, e.g. may owners of holiday homes or tourism resorts experience negative effects if industrial activities take place nearby (fish farming, processing, manufacturing, shipping (Anonymous 2002)), fisheries may be affected by aquaculture (Mikkelsen 2006), and there may be conflicts between wind power farms and fishing (Kannen 2005).

External effects on players' payoffs could emanate directly from the effort other players spend in the contest, by affecting the costs or effect of contest effort. Alternatively, an actor's resource access may lead to a stream of profits, the size of which depends on other actors' access. This can either be due to changed physical productivity or production costs, or a change in the price received for the product.

Our basic model is a one-shot game with simultaneous or sequential moves, and only 2 players. Order of play can be thought of as an instrument a regulator can use to maximise benefits to society. We investigate how the players' optimal effort depends on the sign and size of an asymmetric externality, as well as the set-up of the contest. This in turn affects payoffs to individual players and society at large. We believe that despite the model's simplicity it can be relevant for policymakers in many different settings, including coastal zone management, where stakeholders compete for resource shares and there may be external effects of resource use.

Although the contest model will be relevant for many battles over coastal resources, in coastal zone planning stakeholders often are in a dialogue and try to make arrangements for coexistence. We therefore find it reasonable to compare the contest outcomes with Nash bargaining over resource shares. Also, regulators may encourage bargaining to avoid wasteful rent-seeking, and since stakeholders are often better informed about costs, benefits and externalities than the regulator. We consider the contest outcomes or no allocation at all as threat points, the possible outcomes if bargaining does not lead to agreement.

The paper is organised as follows. In section two, we survey the literature on contests and relate our work to this, before we present our three model-variants in the third section. The models' results are compared to each other and discussed in the fourth section, Nash bargaining is in section five, and in section six we discuss and sum up.

Contests

Let x_i be some amount of resources (time, money, effort) irretrievably expended by agent i (i=1,2..n) to influence the probability of winning a prize A. More formally, the agents have payoff functions:

$$\pi_i = A\,p_i(x_1, x_2, \dots x_n) - c(x_i) \quad \textit{for } i=1\dots n \qquad (1)$$

Here $c(x_i)$ is the agent's cost of expending effort x_i, in the same terms as the prize A. The probability function $p_i(x_1, x_2, \dots, x_n)$, often denoted the "contest success function" (CSF), is for agent i usually increasing in own effort x_i, but falling in other agents' effort. The most popular form of CSF is usually attributed to Tullock (1980):

$$p_i(x_1, x_2, \ldots x_n) = \frac{x_i^r}{\sum x_j^r} \qquad (2)$$

The parameter r is the elasticity of the odds of winning ($p_i(\cdot)/(1-p_i(\cdot))$). The CSF can be interpreted as the probability with which a contestant wins the whole prize (in an "imperfectly discriminating contest"), or as the share of the prize a contestant receives.

The Tullock CSF is widely used in contests, and there seems to be two main ways to justify this. Based on reasonable axioms on how conflict resolution should depend on contestants' efforts, the Tullock CSF appear as the only possible CSF (Skaperdas 1996; Kooreman & Schoonbeek 1997; Clark & Riis 1998). This is coined "axiomatic reasoning" (Konrad 2006). It is also possible to justify the Tullock CSF from a microeconomics basis. From assumptions on the utility function of the contest administrator, and the effort the contestants spend in a probabilistic search for a good proposal that will win them the prize, a Tullock contest structure emerges (Hirshleifer & Riley 1992; Fullerton & McAfee 1999; Baye & Hoppe 2003; Epstein & Nitzan 2006). These papers even make a strong case for the Tullock CSF with r=1. This is despite that the Tullock-type CSF, particularly with r=1, strongly resembles a lottery, where x_i is the number of lottery tickets acquired by agent i, and $\sum x_j$ is the total number of tickets in the lottery.[3,4]

Major issues in the contest literature are optimal choices of effort for the agents, and the amount of rent dissipation in equilibrium (how large part of the prize A is spent as contest

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[3] Of course, some agents in real life sometimes feel that resource allocation outcomes from planning processes *are* like a lottery, but this is beside the point.

[4] Another way of looking at lobbying is that contestants spend effort in a probabilistic search for information that increase their chances of winning the prize (e.g. Lagerlöf 1997; 2006).

effort by the agents). If the size of the prize depends on effort in the contest, sum net payoff is the reasonable measure of the efficiency of different contests setups and outcomes, rather than rent dissipation.

When an agent unilaterally increases his effort in a contest, everybody else's probability of winning goes down, as does the expected return on their effort. There are thus mutual negative externalities between the contestants. These externalities can even be seen as the constituting element of contests (Konrad 2006). However, some authors have also considered other type of externalities in contests. Linster (1993) considers externalities in Tullock contests generally. Long and Vousden (1987) considered rent-seeking contests where the sum total of effort affects the prize in a probabilistic way (the probability that the prize is larger than a given size, but yet below a maximum size, increases with sum effort). Chung (1996) models a game where the size of the prize itself increases with aggregate effort, based on the Tullock CSF. Baye and Hoppe (2003) show that patent race models under certain conditions can be viewed as Tullock contest with a positive externality of effort on the size of the prize.

Shaffer (2006) presents two two-player contest models, aiming to include symmetric externalities of effort on prize, also using the Tullock CSF. In Shaffer's linear externality model, what she sees as an externality coefficient of contest effort on the prize may as well be interpreted as affecting the unit cost of rent-seeking effort directly, and being independent of the players' effort.

Shaffer states in the "linear externalities" model that the players' efforts generate net payoffs of the form:

$$\pi_1 = \frac{Ax(1-\gamma x-\gamma y)}{x+y} - x; \quad \pi_2 = \frac{Ay(1-\gamma x-\gamma y)}{x+y} - y \tag{3}$$

for players 1 and 2, respectively. Player 1 invests an amount x in rent-seeking, and player 2 invests y, the basic prize is A, and γ is the coefficient of destruction (if $\gamma>0$) or enhancement (if $\gamma<0$) of the prize by rent-seeking effort x and y (to follow Shaffer's description).

Shaffer finds that in the non-cooperative Nash-equilibrium, efforts are

$$x^* = y^* = \frac{A}{4(1+\gamma A)} \tag{4}$$

We can transform the payoff-functions in (3):

$$\pi_1 = \frac{Ax}{x+y} - (1+\gamma A)x; \quad \pi_2 = \frac{Ay}{x+y} - (1+\gamma A)y \tag{5}$$

This means Shaffer's model can be interpreted as the standard model of Tullock (1980), with a unit cost of rent-seeking effort of $c=(1+\gamma A)$.[5]

From the Nash equilibrium rent-seeking efforts in (4), Shaffer deduces that "If the prize is large and the contest is strongly productive, we might observe $\gamma < -1/A$, in which case $\partial x/\partial\gamma > 0$ and $\partial y/\partial\gamma > 0$". Clearly, if $\gamma < -1/A$, the Nash equilibrium efforts would be negative. Although Shaffer has not explicitly assumed that efforts are non-negative, it is a common assumption in contest models. The problem with assuming $\gamma < -1/A$, however, goes deeper. When $\gamma < -1/A$, it is clear from (3) that payoffs increase indefinitely with each

[5] We also see that the equilibrium efforts in (4) follows readily from the standard model's solution (x*=A/(4c)) (see e.g. (Hillman & Riley 1989)).

player's own rent-seeking effort. Rent-seeking effort should thus approach infinity to maximise payoffs, and a Nash equilibrium would be impossible.

Trying to model externalities of effort in 2-player Tullock models with linear costs one ends up with a variant of the standard model with either altered costs of effort (like in Shaffer 2006), or with altered (valuation of the) prize. Note that for Tullock models with linear cost functions a change in unit cost is structurally equivalent[6] to a change in the prize, and vice versa. The model we will present below, with asymmetric externalities, gives asymmetric change in valuation or cost of effort.[7]

Contests with asymmetric externalities

In our two-player model player 1 has effort x (≥ 0), player 2 effort y (≥ 0), and $x+y>0$. They compete for shares of the prize A (A>0). There is a direct linear externality from player 2's share of the prize to player 1's net payoffs, with marginal effect of γ ($\gamma < 0$ ($\gamma > 0$) is a negative (positive) externality).[8] Player 2 is not a victim of any external effects. Net payoffs for player 1 and 2 are:[9]

[6] Meaning that the optimisation problem is the same, but that the scaling or denomination of payoffs may differ.

[7] Several papers on contests have considered asymmetric valuation or costs of effort (i.a. Hillman & Riley 1989; i.a. Baik 1994; Nti 1999; Baik 2004; Ryvkin 2007), but not with an externality of effort as the origin of the asymmetry.

[8] Note that in our model the interpretation of the sign of γ is opposite to that of Shaffer (2006).

[9] Define for completeness that if $x = y = 0$, $\pi_1 = (A+\gamma)/2$ and $\pi_2 = A/2$, i.e. that both receive one half of the resource in this case.

$$\pi_1 = A\frac{x}{x+y} + \gamma\frac{y}{x+y} - x\,, \quad \pi_2 = A\frac{y}{x+y} - y \tag{6}$$

For our externality-interpretation of the model to make sense, we assume throughout that the prize is divisible. The only way that player 1 can reduce a negative external effect is to increase own effort relative to player 2's effort, reducing player 2's share of the prize. We look at outcomes in three different games where the players have payoff functions as in (6). The first is a simultaneous move game (S0), the second a sequential game where player 1, the victim of the externality, moves first (S1), and the third a sequential game where player 2 moves first (S2).

Note that in the two-player symmetric simultaneous move contest without externalities, the players' equilibrium efforts and payoffs are all A/4 (Hillman & Riley 1989). This is a benchmark case for our analysis.

Simultaneous move contest (S0)

The first-order conditions for profit-maximisation with respect to own effort are, for player 1 and 2 respectively:

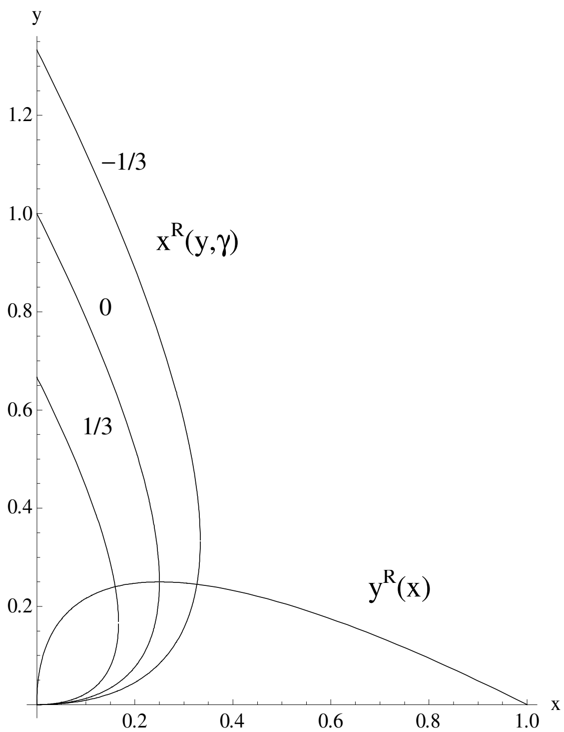
$$\frac{d\pi_1}{dx} = \frac{(A-\gamma)y}{(x+y)^2} - 1 = 0\,, \quad \frac{d\pi_2}{dy} = \frac{Ax}{(x+y)^2} - 1 = 0 \tag{7}$$

The second order conditions for payoff maximisation in an interior solution are both fulfilled for $A - \gamma \geq 0$. Reaction functions for player 1 and 2 are:

$$x^R = \sqrt{(A-\gamma)y} - y\,, \quad y^R = \sqrt{Ax} - x \tag{8}$$

They are valid for $0<y\leq A-\gamma$ and $0<x\leq A$. Note that only player 1's reaction function depends on γ, the externality coefficient. The figure below shows plots of the reaction functions for different values of γ.

Figure 2- Reaction functions. Numbers labelling curves give value of γ for player 1's different reaction curves.[10]



Player 1 responds to increases in player 2's effort by increasing own effort, as long as $y<(A-\gamma)/4$, else increased y is responded to by reducing x. We see that the lower γ is, the more effort should player 1 invest for a given y, getting a larger share of the prize, and thus either reducing a negative external effect or compensating for a reduced positive externality.

The intersections of the reaction functions give efforts in the Nash-equilibrium of the simultaneous move game, depending on γ, with efforts for player 1 and 2 equal to:

[10] All figures are examples using A=1.

$$x^{*S0} = \frac{A(A-\gamma)^2}{4(A-\frac{\gamma}{2})^2},\ y^{*S0} = \frac{A^2(A-\gamma)}{4(A-\frac{\gamma}{2})^2}. \qquad (9)$$

When the second order conditions are fulfilled x^* and y^* are always positive. For negative externalities (γ<0), player 1's equilibrium effort will be larger than A/4, the benchmark from the symmetric case without externalities. For a positive externality, the opposite is true. For player 2 equilibrium effort is lower than A/4 when an externality present (γ≠0). With a strong positive externality (γ goes towards A), both players' effort goes towards zero.

Due to the asymmetric externality, around the Nash equilibrium, the players' optimal responses to effort increases by the other player are also asymmetric. For a positive externality, if player 1 should get more aggressive (increase his effort x), player 2 would respond by increasing his effort too, but if player 2 gets more aggressive, player 1 responds by reducing his effort. The opposite is the case if γ<0. The response is given by the slope of the reaction functions, which again is given by the cross partial derivatives of the profit functions for each player:

$$\frac{\partial^2 \pi_1}{\partial x \partial y} = \frac{(x-y)(A-\gamma)}{(x+y)^3}; \qquad \frac{\partial^2 \pi_2}{\partial y \partial x} = \frac{(y-x)A}{(x+y)^3}. \qquad (10)$$

These change sign depending on whether x is larger or smaller than y, and they will always have opposite sign, unless x=y. When an increase in player 1's effort gives player 2 a larger marginal profit of own effort, an increase in player 2's effort gives player 1 a smaller marginal profit of his own profit, and vice versa.

The effects that the externality has on efforts are:

$$\frac{\partial x^{*S0}}{\partial \gamma} = \frac{-2A^2(A-\gamma)}{(2A-\gamma)^3} < 0\ ;\ \frac{\partial y^{*S0}}{\partial \gamma} = \frac{-\gamma A^2}{(2A-\gamma)^3} \gtreqless 0 \text{ for } \gamma \lesseqgtr 0. \qquad (11)$$

In equilibrium, player 1's effort always decreases with γ. If a positive externality gets stronger, player 1 lets player 2 get a larger share of the prize, since the reduced share of the prize is compensated by the increase in the positive external effect and the reduction in own effort. Similar, but opposite, reasoning applies if a negative externality is reduced.

For player 2, if a negative externality is reduced, he increases his effort in equilibrium, but if a positive externality is increased, he decreases effort. Player 2's response is of course a response to the change in player 1's effort due to the change in γ, rather than an own response to the change in γ. As player 1's reaction curve in Figure 2 shifts due to changes in γ, we see how the optimal effort for player 2 changes too. When there is a reduction in a negative externality, and a corresponding reduction in player 1's effort, the marginal cost of effort for player 2 is lower than the marginal value of the extra share of the prize that that extra effort gives. For a positive externality the situation is opposite.

Larger γ always leads to a smaller share of the prize for player 1 and increased share for player 2 in equilibrium. When both players reduce their efforts in response to increased γ, player 2's reduction is always smaller than player 1's reduction.

Sum effort in equilibrium is

$$x^{*S0} + y^{*S0} = \frac{A(A-\gamma)}{2(A-\frac{\gamma}{2})}. \quad (12)$$

which decreases monotonically in γ, and is zero when we have a positive externality of same magnitude as the prize. Clearly, for a negative externality sum efforts are higher than the A/2 benchmark for no externality, and vice versa for a positive externality.

Profits in equilibrium are, for player 1 and 2 respectively

$$\pi_1^{*S0} = \frac{A}{4}\frac{(A^2 + A\gamma - \gamma^2)}{(A-\frac{\gamma}{2})^2},\ \pi_2^{*S0} = \frac{A}{4}\frac{A^2}{(A-\frac{\gamma}{2})^2}. \quad (13)$$

While player 2's profits are always positive in equilibrium, player 1's profits are negative when $\gamma < \frac{A}{2}(1-\sqrt{5})$ (that is $\gamma <$ ca. –0.62 A). Note that choosing the effort level in (9) is not in conflict with a Nash equilibrium, since if player 1 tried to avoid the negative payoff by choosing x=0, player 2 would choose y=ε, where ε is just above zero, leaving player 1 with the payoff γ. That payoff is even less than in the interior equilibrium represented by (13).

Sum profits in equilibrium are

$$\sum \pi_i^{*S0} = \frac{A(A+\gamma)}{2A-\gamma}, \quad (14)$$

which is negative when γ<–A. It has a maximum value of 2A, when γ=A.

Sequential move contest with externality victim first (S1)

By analyzing how player 2 will react to the effort level set by player 1, player 1 can get an advantage by moving first.[11] Putting player 2's reaction function into player 1's original payoff-function (6) we get a new payoff function for player 1:

$$\pi_1^{S1} = \sqrt{Ax} + \gamma\left(1 - \frac{\sqrt{x}}{\sqrt{A}}\right) - x \qquad (15)$$

The first order condition for player 1 then becomes:

$$\frac{A-\gamma}{\sqrt{Ax}} - 2 = 0 \qquad (16)$$

The second order condition for profit maximisation is still fulfilled for $\gamma \leq A$. Effort in equilibrium for player 1 and 2 are:

$$x^{*S1} = \frac{A}{4} - \frac{\gamma}{2}\left(1 - \frac{\gamma}{2A}\right);\; y^{*S1} = \frac{A}{4} - \frac{\gamma^2}{4A} \qquad (17)$$

Together with the second order condition, the limitation $x \leq A$ from player 2's reaction function means $-A \leqslant \gamma \leqslant A$.

As can be seen from Figure 3, or by differentiating the optimal efforts in (17) wrt γ, the players' response to increased γ is qualitatively the same as in the simultaneous game S0.

Sum effort is $(A-\gamma)/2$, approaching zero as γ goes towards A, and approaching A as γ approaches $-A$ (from the positive side). If player 1 get as much from player 2's share as from

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[11] Contests that may have sequential moves are studied by several others previously (i.a. Dixit 1987; Baik & Shogren 1992; Leininger 1993; i.a. Baik 1994).

his own (when γ=A), he does not care about the size of the share he gets. On the other hand, with a strong negative externality (γ→ –A), player 1 really wants to avoid that player 2 gets anything.

Payoffs for player 1 and 2 are equal in equilibrium:

$$\pi_i^{*S1} = \frac{(A+\gamma)^2}{4A},\ i=1,2. \qquad (18)$$

As γ approaches –A from the positive side, the individual payoff falls towards zero. The sum of payoffs is just twice the individual payoff.

Sequential move contest with externality source first (S2)

The reaction function for player 1 is $x^R = \sqrt{(A-\gamma)y} - y$, with y≤(A–γ) as a requirement for player 1's effort to be non-negative. Inserting this into player 2's payoff function gives

$$\pi_2^{S2} = \frac{A\sqrt{y}}{\sqrt{A-\gamma}} - y \qquad (19)$$

given that γ≤A. The first order condition for player 2 then becomes

$$\frac{A}{\sqrt{y(A-\gamma)}} - 2 = 0 \qquad (20)$$

This gives the following effort in equilibrium for player 1 and 2:

$$x^{*S2} = \frac{A}{4}\left(1 - \frac{\gamma}{A-\gamma}\right),\ y^{*S2} = \frac{A}{4}\left(\frac{A}{A-\gamma}\right) \qquad (21)$$

This equilibrium only exists for γ≤A/2 (else player 1 gets negative effort). Optimal effort for player 1 decreases monotonically with γ, while for player 2 optimal effort increases monotonically with γ. The latter is in contrast to in the other two games, where player 2's effort in equilibrium has a maximum when γ=0. Sum effort in equilibrium is A/2.

Payoffs in equilibrium are the same for player 1 and 2

$$\pi_i^{*S2} = \frac{A}{4}\left(\frac{A}{A-\gamma}\right),\ i=1,2. \qquad (22)$$

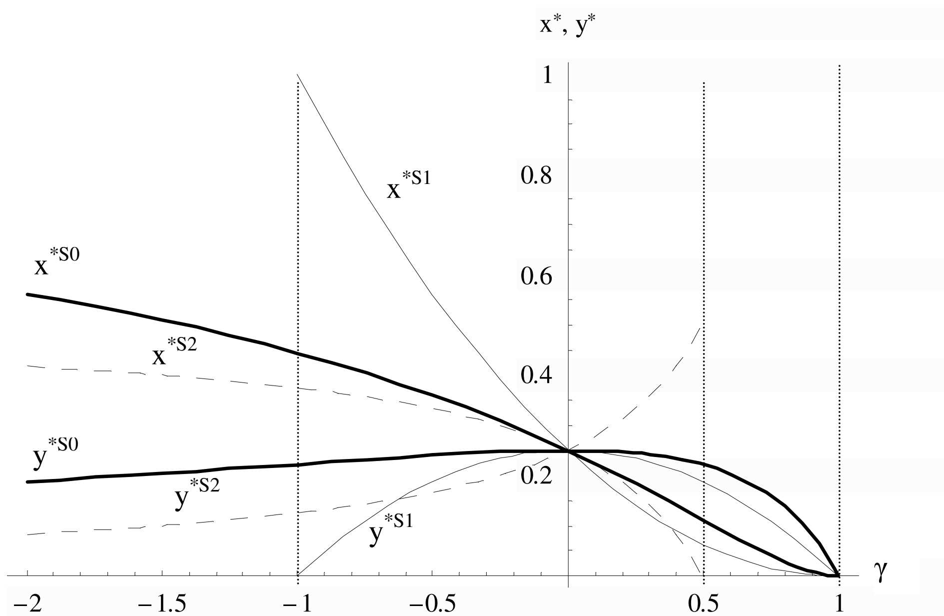
Individual payoffs increase with increasing γ, and then naturally so does the sum, which is just twice the individual payoff.

The effect of order of play in contests

Comparing efforts

When player 1 is subject to a negative externality, his effort will be higher in equilibrium than with no externality, or a positive externality, as Figure 3 shows. For player 2, equilibrium effort is lower when he inflicts an externality (positive or negative) on player 1, compared to when no externality is present. The exception is when player 2 moves first (in a sequential game) and there is a positive externality.

Figure 3: Optimal effort in equilibrium in the three contests, depending on γ.



Our findings on how equilibrium effort depends on the externality in the simultaneous move game is similar to results in Baik (1994). He has a contest model with asymmetric valuation. Player 1's valuation is αv ($\alpha>0$, $v>0$), while player 2's valuation is just v. As α increases from zero, both players' optimal effort increases until $\alpha=1$ and we have symmetric valuation, thereafter player 1's optimal effort level keeps on increasing, while for player 2 it is reduced.

To see that these two findings are comparable, consider the following. Since the CSF for player 2 in my model, $(y/(x+y))$, is equal to $(1-x/(x+y))$, we can rewrite player 1's payoff function:

$$\pi_1 = \frac{(A-\gamma)x}{x+y} + \gamma - x \qquad (23)$$

In this form it is clear that player 1 has a larger marginal payoff of effort than player 2, given that $\gamma<0$, that is, the externality from player 2 is negative. Then it is also natural that player 1 ends up with larger effort in equilibrium than player 2 under this condition.

If γ =A, player 1's valuation is zero, and as γ decreases from A, the valuation increases. This is the same as α increasing from zero in Baik (1994). We both find that with the simplest Tullock-type CSF, like in my model, total effort always increases as player 1's valuation increases.

The results also compares well with Nti (1999). He finds for a two-player simultaneous move Tullock contest that the effort of the "favoured" player (the one with the highest valuation) increases if his own or the other contestant's (the underdog's) valuation increases.[12] The effort of the underdog increases in own valuation but falls in the valuation of the favoured player. In the model we present, as γ changes sign, say from minus to plus, players 1 and 2 also switch roles, from favourite-underdog to underdog-favourite.

I find that if the underdog is allowed to move first, both players reduce effort levels compared to the simultaneous move game, just like Baik (1994) and Dixit (1987). On the other hand, if the stronger player is allowed to move first, he overcommits and chooses a higher effort level than in the simultaneous game.

[12] The terms favourite and underdog are natural to use since asymmetries in valuation is analytically equivalent to asymmetries in the cost of effort. The low cost/high valuation player is the favourite, and the other is the underdog.

Sum effort falls monotonically with γ in games S0 and S1.[13] For a strong negative externality ($\gamma<-A$) game S0 has higher sum effort than S2 (S1 is not defined). With a weaker negative externality S1 gives the highest sum effort, followed by S0 and then S2. For a moderate positive externality ($0<\gamma<A/2$) that order is reversed, with S2 having the highest sum effort followed by S0 and S1. For a stronger positive externality ($A/2<\gamma<A$) contest S0 has higher sum effort than S1 (S2 is not defined).

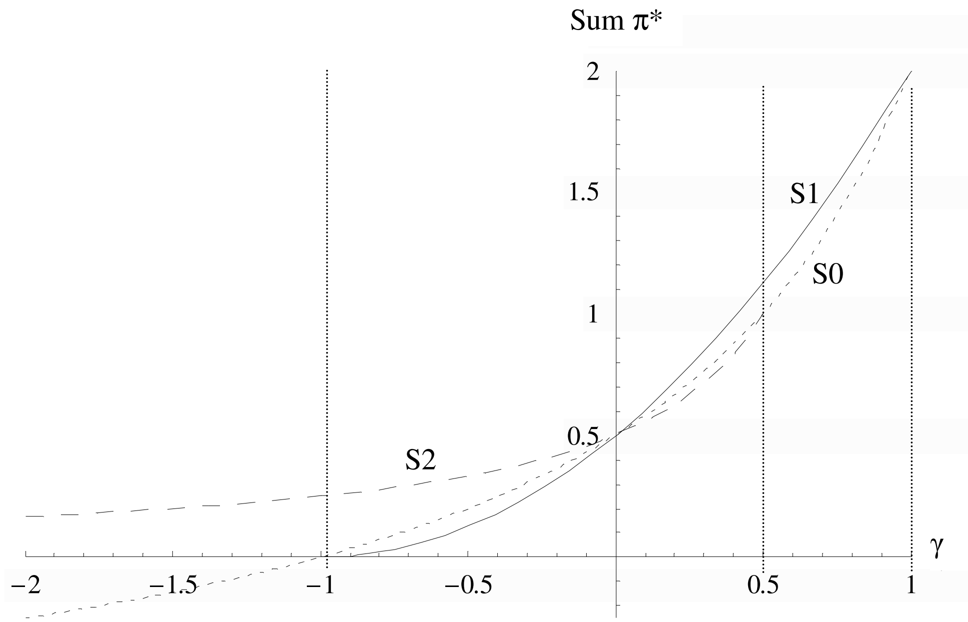
Comparing sum of payoffs

The sum of payoffs in the equilibria of the different games (S0, S1 and S2) varies with size and magnitude of the external effect. One of the sequential games has always larger than sum of payoffs than the simultaneous move game (except for $\gamma=0$), as can be seen from Figure 4. If there is a negative externality ($\gamma<0$) game S2 gives the largest sum of payoffs, and if there is a positive externality ($\gamma>0$) S1 gives the largest sum of payoffs. This means that when the player with the lowest valuation is allowed to move first, the sum of payoffs will be the largest in equilibrium. This is of course as undercommitment by the underdog allows also the favourite to have less effort than in the simultaneous game.

There is a clear policy recommendation from this. If a regulator can choose the order of play in a contest over natural resources, to maximise benefits to society one of the sequential games should be chosen. More specifically, the player with the lowest valuation should be allowed to choose his effort level in the contest first.

[13] Remember that in S2 it is constant at A/2.

Figure 4: Sum of payoffs in equilibrium in the three contests, depending on γ.



It is also interesting to see what sequence of play individual players would prefer depending on the value of γ; will players oppose or support a regulator's choice of game? We analyse this by comparing individual payoffs in the different games, in the next section.

Comparing individual payoffs

The payoff for individual players in the three equilibria ranks as follows, depending on the sign and magnitude of the external effect, given by γ:

For player 1:

$\pi_1^{S1} > \pi_1^{S0}$ always.

$\pi_1^{S0} > \pi_1^{S2}$ for $0 < \gamma \leq A/2$, else $\pi_1^{S0} < \pi_1^{S2}$.

For player 2:

$\pi_2^{S2} > \pi_2^{S0}$ always.

$\pi_2^{S0} > \pi_2^{S1}$ for $A(1-\sqrt{17})/2$ (~ −1.56 A) $< \gamma < 0$, else $\pi_2^{S0} < \pi_2^{S1}$.

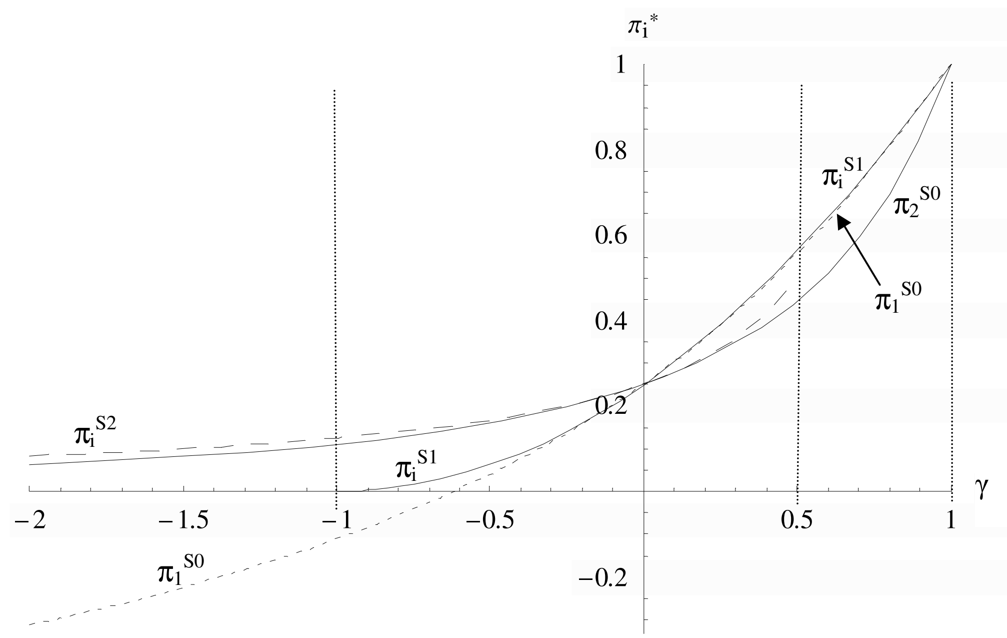
For both players:

$\pi_i^{S1} > \pi_i^{S2}$ for $\gamma < -A(1+\sqrt{5})/2(\sim -1.62\ A)$ or $0 < \gamma \leq A/2$, else $\pi_i^{S1} < \pi_i^{S2}$, for 1=1,2.

When a player moves first, it is natural that he gets at least as high payoff as in the simultaneous move game, since the effort choice of the simultaneous game is also possible to play when he moves first. We find, however, that the individual payoffs are for both players always larger in a sequential move game than in the simultaneous move game (except for γ=0). Which sequential game the players prefer depends on γ-value. Since equilibrium payoffs in each of the sequential games are the same for both players, they actually prefer the same sequential game given the value of γ. For some γ-values, player 1 will prefer the game where player 2 moves first, and for other values of γ, player 2 will prefer the game where player 1 moves first. This can also be seen by inspecting Figure 5. The results conform with two-player models with asymmetric valuation, where the players first choose whether to commit "early" or "late", and then set their effort levels (Baik & Shogren 1992; Leininger 1993). They also find that the underdog wants to move early, and the favourite late. The result does not hold for all contest types, but holds for the basic Tullock contest (Konrad 2006; 56).

What is perhaps most surprising is that both players' individual interests and the collective interests are aligned in choice of game, given the type of externality. Here it is natural, as the players have the same payoffs in each of the sequential games. Since the societal objective of sum of payoffs is just the sum of the players' payoffs, it is clear that individual and collective interests are aligned. In the discussion-section we consider the usefulness of this for a regulator that has limited knowledge over the players' valuation of resource access and sign and magnitude of external effects.

Figure 5: Payoffs for player 1 and 2 in the three contests, depending on γ



Nash bargaining

It may be reasonable to model planning processes and lobbying, where stakeholders try to influence a regulator's decision over access to coastal resources, as contests. However, for a coastal zone regulator, an alternative to deciding resource access himself is to let stakeholders bargain between themselves on how to share resources. This may be an attractive alternative if the regulator is unsure about the players' valuation of resource access and size and direction of externalities, and the players know more than him. Bargaining may also reduce wasteful rent-seeking. Of course, real life bargaining is not costless, even though it is often assumed so in economic models.

We will consider how different threat points affect payoffs in the bargaining outcomes, for individual stakeholders and in sum. "No resource-allocation" and the different contests in the last section are used as threat points. Whether a player would opt for the threat point outcome rather than the outcome of the Nash bargaining solution[14] is also briefly considered. In the discussion section afterwards we also consider how less than perfect information on behalf of the regulator can affect the possibility of realising the outcome with the highest sum of payoffs.

Denote β as the share of the resource that player 1 gets, and (1–β) similarly for player 2. Then the payoffs for players 1 and 2 in a bargaining solution are:

$$\pi_{1B} = A\beta + \gamma(1-\beta);\ \pi_{2B} = A(1-\beta) \tag{24}$$

The Nash product to be maximised by choice of β is then:[15]

$$(\pi_{1B} - \pi_{10})(\pi_{2B} - \pi_{20}) \tag{25}$$

where (π_{10}, π_{20}) is the threat point of the bargaining; the payoff players 1 and 2 will get if they do not achieve agreement. The solution to this bargaining is

$$\beta^*(\pi_{10}, \pi_{20}) = \frac{A(A - 2\gamma + (\pi_{10} - \pi_{20})) + \gamma\pi_{20}}{2A(A-\gamma)} \tag{26}$$

Assuming that π_{10} and π_{20} are independent of γ, larger γ leads to a lower share for player 1 in equilibrium (lower β^*). With a stronger positive externality, or a reduced negative externality, player 1 is less eager to fight over shares of the natural resource. We also see that increased

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[14] The Nash bargaining solution is the only bargaining solution simultaneously satisfying four axioms often considered reasonable (Clark 1995). It maximises the product of the gains from bargaining above the disagreement point. Other bargaining solutions are i.a. the Kalai-Smorodinsky solution and the utilitarian (Clark 1995). They implicitly emphasize other principles for sharing of gains. We have chosen to use the Nash bargaining solution, simply as it is the most commonly used bargaining solution. Which bargaining solution that will be used in real-life bargaining situation depends on the stakeholders' cultural background, ethics and more.

[15] See e.g. (Muthoo 1999) chapter 2.

payoff for player 1 in the threat point increases β^*, while increased payoff for player 2 in the threat point reduces β^*. If your outside options are bettered, then so will the bargaining outcome be too, but if the outside options are weakened, the opposite is true. Consequently player 1 wants π_{10} to be as large as possible, and π_{20} as small as possible. The opposite applies for player 2.

If the players get nothing without agreement ((0,0) is threat point), the outcome of the bargaining is

$$\beta^*(0,0)=\frac{1}{2}\left(1-\frac{\gamma}{(A-\gamma)}\right), \qquad (27)$$

which is positive only for $\gamma \leq A/2$. For a positive externality (γ>0) player 1 gets less than half of the resource, and vice versa for a negative externality. With this sharing rule the players get equilibrium payoffs of:

$$\pi^*_{1B}=\frac{A}{2};\ \pi^*_{2B}=\frac{A}{2}\frac{A}{A-\gamma}. \qquad (28)$$

Note that player 1 gets the same payoff independent of the sign and magnitude of the externality. If there is a negative externality (γ<0), player 2 gets a smaller equilibrium payoff than player 1, and vice versa (provided A> γ). This means, for example if player 2's use of the resource will create pollution that harms player 1, bargaining with no resource allocation as the threat point, makes the polluter pay.

That the stakeholders should get no resources at all if they don't agree on a sharing rule will be unlikely in many settings. Rather, a regulator may offer the agents to negotiate over the sharing of the resource, and announce that if they do not reach agreement the sharing will be decided by a contest, like those described in previous sections.[16] One of the contest outcomes thus constitutes the threat point. Table 2 gives payoffs from bargaining for all the different threat points.

Table 2: Payoffs from bargaining, with no allocation (0,0) or contests S0, S1, S2 as threat points

Threat point	π_1	π_2	**Sum of payoffs**
(0,0)	$\frac{A}{2}$	$\frac{A}{2}\frac{A}{(A-\gamma)}$	$A+\frac{A\gamma}{2(A-\gamma)}$
S0	$\frac{A}{2-\frac{\gamma}{A}}$	$\frac{A}{2-\frac{\gamma}{A}}$	$\frac{A}{1-\frac{\gamma}{2A}}$
S1	$\frac{A}{2}+\frac{1}{8}+\frac{\gamma^2}{4A}\left(1+\frac{\gamma}{2A}\right)$	$\frac{A}{2}\left(\frac{A-\frac{3}{4}\gamma}{A-\gamma}\right)$	$A+\gamma\left(\frac{1}{2}+\frac{\gamma^2}{8A}\left(3+\frac{\gamma}{A}\right)\right)$
S2	$\frac{A}{2}\left(\frac{A-\frac{3}{4}\gamma}{A-\gamma}\right)$	$\frac{A}{2}\left(\frac{A}{A-\gamma}\right)\left(\frac{A-\frac{5}{4}\gamma}{A-\gamma}\right)$	$A\left(\frac{A^2-\frac{3}{4}A\gamma+\frac{3}{8}\gamma^2}{(A-\gamma)^2}\right)$

In Figure 6, sum of payoffs are plotted as a function of γ for the bargaining outcomes of different threat points. We see that the bargaining outcome with "no allocation" as threat point gives the largest sum benefit (only defined for $\gamma<A/2$).[17] For the possibly more credible threat points of contest outcomes, which threat point that gives the largest sum of payoffs varies with size and direction of the externality. However, the bargaining outcome with one of

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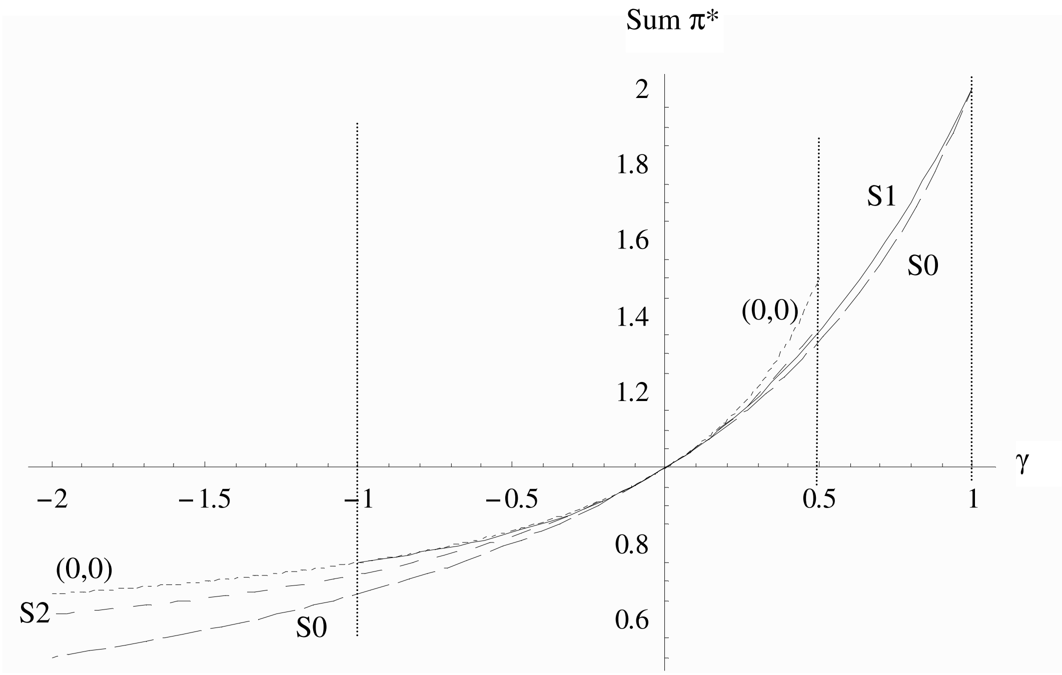
[16] Grepperud and Pedersen (2003) is an example of a bargaining game where a non-cooperative game is the threat point. Unlike our setting, their game is between a regulator and a single resource user/polluter.

[17] Although difficult to see from Figure 6, this can be verified by algebra or more detailed plots.

the sequential move contests as threat points always dominates the simultaneous move contest.

Figure 6: Sum of payoffs from bargaining, given no allocation (0,0) or the contest outcomes (S0, S1, S2) as threat points

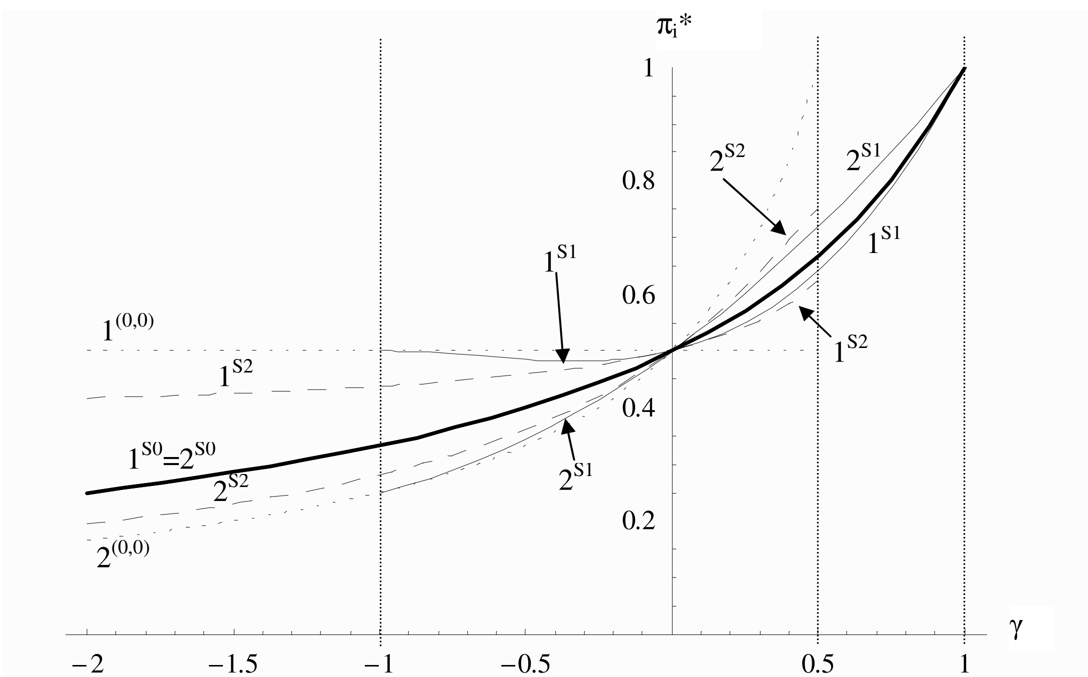


If there is a strong negative externality (γ<–A), contest S1 is not defined, and S2 as threat point gives the largest sum of payoffs in the bargaining outcome. For a less strong negative externality (–A<γ<0), S1 as threat point gives the highest sum of payoffs. For a moderate positive externality (0<γ<A/2) contest S2 gives the largest sum of payoffs from bargaining. For a strong positive externality (A/2<γ<A) S2 is not defined and S1 gives the largest sum of payoffs.

In the contests, letting the player with the lowest valuation choose effort level first gives the highest sum of payoffs. In bargaining, using the contest where the player with the highest valuation chooses effort level first as threat point gives the highest sum of payoffs (when both sequential contests are defined). If the regulator does not bind himself to using the right contest in case of bargaining breakdown, players may not see it as a credible threat, and bargain as if the other sequential contest is the real threat point, thus not realising the higher sum of payoffs.

Figure 7 shows the individual payoffs from Nash bargaining given different threat points. If there is a negative externality, player 1 prefers no allocation (0,0) as the threat point. Naturally this gives the lowest possible payoff to player 2. Conversely, with a positive externality, player 2 gets the highest payoff when (0,0) is threat point, leaving player 1 with the smallest possible payoff.[18]

Figure 7: Individual payoffs from bargaining as a function of γ, given threat points.



Note: $1^{(0,0)}$ is player 1's payoff when no allocation is threat point, 2^{S1} is player 2's payoff when contest S1 is threat point, and so on.

[18] Remember that the bargaining outcome is not defined with (0,0) as threat point for γ>A/2.

If we concentrate on the contest outcomes as possible threat points we find the following. With a strong negative externality ($\gamma<-A$) player 1 prefers contest S2 as the threat point (S1 is not defined). If there is a moderately strong negative externality ($-A<\gamma<0$) he prefers contest S1 as threat point. For a positive externality player 1 prefers contest S0 as threat point. With a strong positive externality ($A/2<\gamma<A$) player 2 prefers contest S1 as the threat point (S2 is not defined). If there is a less strong positive externality ($0<\gamma<A/2$) he prefers contest S2 as the threat point. For a negative externality player 2 prefers contest S0 as threat point. All these findings also follow directly from (26) and the observations on how (π_{10}, π_{20}) influence β^*.

Overall, of all bargaining outcomes, the regulator and the player with the effectively higher valuation prefer the one with no allocation as threat point (when all contest outcomes are defined). Among the bargaining games with contest outcomes as threat points, the regulator and the player with the higher valuation prefers the contest where the favourite moves first. In fact, the regulator and the player with the higher valuation, player 1 if there is a negative externality and player 2 if there is a positive externality, has the same ranking of the bargaining games according to threat point. The lower valuation player prefers the bargaining game with simultaneous move contest as threat point. Among the sequential move contests he prefers the game where he moves first. The ranking of bargaining games by threat point is summed up in Table 3.

Whether the regulator can realise the largest sum of payoffs through bargaining with one of the sequential contests as threat points depends not only on him knowing which player is favourite and which is underdog. If the underdog's payoff in the bargaining outcome will be lower than in the contest making up the threat point, he will cause a breakdown of the bargaining to realise the higher payoff of the contest. However, this will not happen. With a positive externality, player 2 is the favourite, and the bargaining game with the largest sum of payoffs is the one where contest S2 is threat point. For the values of γ where contest S2 is defined ($\gamma < A/2$), player 1 prefers the bargaining outcome. If the positive externality is stronger than that, player 1 will choose zero effort in contest S2.[19] With a negative externality, player 2 is the underdog, and the bargaining game with contest S1 as threat point has the largest sum of payoffs. Player 2 prefers the bargaining outcome over the outcome of contest S1 itself as long as S1 is defined ($-A \leq \gamma < 0$). If the negative externality is stronger than that player 2 will have zero effort in contest S1.

Discussion and conclusion

In this paper we have considered two ways of allocating a resource between two players. The first is a contest, the second bargaining. In both we assume that player 1 receives an external effect from player 2, depending on player 2's contest effort or use of the resource. We have considered both positive and negative externalities.

We investigate contest that are either a simultaneous move game (S0) or sequential move games where player 1 moves first (S1) or player 2 moves first (S2). In all the contests it is optimal for player 1 to increase contest effort in equilibrium if a negative externality gets stronger, or a positive externality is reduced. Then he gets a larger share of the prize, reduces player 2's share, and also reduces the negative externality or compensates for a reduction in the positive externality.

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[19] The non-negativity restriction on effort is the reason why contest S2 is not defined for $\gamma \geq A/2$.

If the players value shares of the resource equally, independent of the externality, player 1 has the largest valuation if there is a negative externality, and player 2 if there is a positive externality. If a regulator can set up the resource contest, he should let the underdog, the player which has the lowest valuation of the natural resource when also the externality is accounted for, move first. Sum of payoffs are then maximised. In such a contest, the players' individual interests and the regulator's interest are aligned; the regulator and the underdog want the underdog to move first, and the favourite wants to move last, as can be seen from Table 3. In each of the sequential move contests the players get the same payoff in equilibrium. A regulator may find the equitable sharing a positive feature of these contests.

Table 3: Ranking of games by payoffs, given value of γ

	$\sum\pi$	π_1	π_2	$\sum\pi$	π_1	π_2	$\sum\pi$	π_1	π_2	$\sum\pi$	π_1	π_2
C	S2 S0	S2 S0	S2 S0	S2 S0 S1	S2 S1 S0	S2 S1 S0	S1 S0 S2	S1 S0 S2	S1 S2 S0	S1 S0	S1 S0	S1 S0
NB	S2 S0	S2 S0	S0 S2	S1 S2 S0	S1 S2 S0	S0 S2 S1	S2 S1 S0	S0 S1 S2	S2 S1 S0	S1 S0	S0 S1	S1 S0

γ axis: –A | 0 | A/2 | A

Note: NB indicates Nash Bargaining and C contests. We only consider contests as threat points for the bargaining. $\sum\pi$ means sum of payoffs, and π_1 and π_1 payoffs for player 1 and 2. The ranking in each column gives the game that gives highest payoff (top) to lowest payoff (bottom) in sum or for that player. Note the axis on the bottom for the γ-values. Remember that for γ<–A contest S1 is not defined, and for γ>A/2 contest S2 is not defined.

Information asymmetries between the regulator and the players can pose big challenges for setting up such a contest. Imagine a fisher and a fish-farmer in a contest as described above, with a negative externality of farming on fishing. The regulator is, however, not certain about sign and magnitude of the externality, but he expects the players want to maximise their own profits, and he wants to maximise sum profits. The regulator can then simply ask the players who should be allowed to move first in the contest, and they should provide the true answer, as everybody's interests are aligned. The correctness of that assumption depends crucially on the player with the lower valuation, in this case the fisher, being certain resource allocation will be through a contest. If the fisher assumes the regulator will use the players' information on who should move first to directly allocate resources, he has incentive to lie. This is because the regulator should give *all* of the resource to the player with the highest valuation, in order to maximise overall profits. Naturally then, the lower-valuation player will not reveal himself to the regulator. This is regardless of whether there is a negative externality (externality victim has the highest valuation), or a positive externality (externality source has the highest valuation).

Contests imply waste of effort to affect allocation. An alternative method, with possibly less waste of effort, is to let the players bargain over resource sharing. A bargaining outcome is sensitive to the threat point, the payoffs the players get if they do not reach agreement. The regulator could state that there will be no allocation of the natural resource if the players do not reach agreement. Then, in the bargaining outcome, the externality victim ends up with the same payoff regardless of sign and magnitude of the externality. If there is a negative externality the player which is source of the externality ends up with a lower payoff than the victim, and if there is a positive externality he ends up with a higher payoff than the victim. The players actually agree to share the resource so that "the polluter pays" and the victim is as well off as if there were no externality. A regulator may find it attractive to use no allocation as threat point for the bargaining due to this, and since it also gives the highest sum of payoffs of all bargaining games considered here. However, it may not be very credible as threat point. Which real-world regulator would leave resources unused when their use could bring benefits to society? Rather, the regulator may decide that if the players can not reach agreement resource sharing will be decided by him. The players will naturally try to influence the regulator's decision. The process may then again be described or illustrated as a contest.

Using the sequential contest where the favourite moves first as threat point gives the highest sum of payoffs in the bargaining outcome. However, the stakeholders may nevertheless expect that the other sequential contest, where the underdog moves first, will be used in case of breakdown. This is because it gives the largest sum of payoffs, *as a contest*. If the regulator can not credibly establish the favourite-moves-first contest as the threat point to bargaining breakdown, the highest sum of payoffs will not be realised from bargaining.

Unlike for the contests, the regulator's interests are not aligned with both players' in the bargaining, as can be seen from Table 3. While the favourite and the regulator share interest over which contest should be used as threat point, the underdog gets the highest payoff if a different contest is used as threat point.

Would it be possible for the regulator to use bargaining to maximise sum of payoffs if he has limited knowledge about the players' valuation and the externality? Consider the same example as above in this section, with a fish-farmer exerting a negative externality on a fisher, the regulator unsure about players' valuation and the externality, but players have full knowledge over them. Let us start by assuming that the regulator credibly has committed to use as threat point any sequential contest the players jointly promote. An obvious challenge is that the farmer and the fisher prefer different threat points, as we have seen. But would it be possible for the fisher to promise the fish-farmer compensation if he would agree to promote as threat point the contest maximising the fisher's payoff, the one where the fisher moves first? Clearly this is the case, as sum of payoffs are always higher from bargaining when the favourite moves first in the threat point contest. The fisher can give the farmer a side payment that will make both their payoffs at least as good as from the bargaining game with the farmer-moving-first contest as threat point.

Summing up, we have found that it is possible to maximise sum of payoffs from contests and from Nash bargaining when an asymmetric externality is present, even if the regulator does not have full information about players' valuation and the externality. It requires that the players have sufficient information to know who is underdog and who is favourite, and that the regulator can commit to either playing the contest the players jointly recommend, or have as threat point to bargaining the contest the players jointly promote. The latter will require a side payment from the favourite to the underdog.

These results have interesting policy implications for managing contests over natural resource, or setting up bargaining over resource allocation, both when the resources are in the coastal zone and elsewhere. The results should be investigated for games where the players have different valuation at the outset independent of an externality, and also for games with more than two players.

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