Chapter 5

Essay IV

Abstract

This paper presents a bioeconomic model for the Barents Sea capelin and cod fisheries including juvenile herring in the biological model, as the young herring influence the cod-capelin system. The objective of the study is to reveal new perspectives on management of capelin and cod given by a multi-species angle of attack. Balancing the model-complexity of biology and economics is stressed in the investigation of deterministic optimal catch strategies. Optimality is measured as the maximum economic rent in the fishery.

The three species, capelin, cod and juvenile herring, constitute a very dynamic system, also because prey-predation relations are functions of ages within each stock. The biological model relates total biomass of each species through growth functions that include predation effects. The growth functions are based on assessment data from the Institute of Marine Research in Bergen, Norway, and the estimation is done by multivariate regression analysis assuming logistic like growth. Optimal fishing strategies are defined by employing a numerical feedback rule for optimal fishing through dynamic programming. The feedback rule suggests that previous TAC (total allowable catch) levels on average have been too large for both capelin and cod over the past 30 years, according to the management objectives assumed in the study. Moreover, presence of some herring in the system is important for the economic yield although the herring fishery is closed. This indicates that a focus only on the capelin-predator role of herring is too narrow, as herring are also an important prey for cod. Moreover, very interesting multispecies effects on the optimal catch strategy are discovered.

5.1   Introduction

The Barents Sea is one of the most productive ocean areas in the world (O'Brien et. al. 2004). It represents a highly diverse arctic ecosystem (Larsen et.al. 2001), whereof three fish species are of major commercial importance. The key species are the plankton feeders capelin (*Mallotus villosus*) and herring (*Clupea harengus*), as well as their main fish predator, North-East Arctic Cod (*Gadus morhua*, from now on *cod*). The relationships between the three species are highly dynamic, and the cod-capelin relationship is particularly important in the Barents Sea ecosystem (Bogstad, Hauge and Ulltang 1997).

Therefore we choose a growth model consisting of capelin, cod and herring as foundation for the decision-making on total allowable catches (TACs) for capelin and cod.

The herring stock has shown substantial natural fluctuations during the last century (Dragesund, Hamre and Ulltang 1980, Toresen and Østvedt 2000), and a high harvest pressure and unfavorable climatic conditions led to stock depletion around the 1970s (Engelhard and Heino 2004, Røttingen 2003). Although herring is mainly a Norwegian Sea species, the juvenile part of the stock occasionally enters the Barents Sea area. This inflow of young herring into the Barents Sea basin has a substantial impact on the system; the cod-capelin relation is particularly affected. Strong year classes of herring move into the Barents Sea by larval drift and feed in that area until they move back into the Norwegian Sea at 3 to 4 years of age (Huse, Railsback and Fernö 2002), but the stock is not harvested during its stay in the Barents Sea.

Predation from juvenile herring in the Barents Sea causes mass death of capelin fry, which has major impacts on other capelin dependent species, as at the end of the 1980s when there was a rather large cod stock (Mehl 1991) and herring had recovered from the collapse in the late 1960's. A strong year class of herring occurred in the south of the Barents Sea in 1983 to 1986, causing a capelin stock collapse in 1985 to 1986. According to Hamre (2003) this collapse led to cod starvation and had a negative impact also on other fish predators like sea birds and marine mammals. The capelin stock is the largest pelagic fish stock in the Barents Sea and also potentially the largest capelin stock in the world (Gjøsæther 1998, Gjøsæther and Bogstad 1998), and is of crucial importance for the growth of juvenile cod (Dalpadado and Bogstad 2004).

The strong dependence between the biomasses of the three species in our model calls for a management regime that is aware of it. Single-species models fail to explain the large fluctuations in the stocks, and therefore feedback strategies for optimal catch based on single-species growth functions misses the target of optimality.

This work presents a 3D (three-species) biological model where the catch is optimized on the basis of a multi-species feedback strategy. The unique element is that the feedback strategy, which maximizes economic rent from the fishery, is based on the size of all three stocks. With this model it is possible to identify multi-species effects which give greater insights than a single-species or two-species model can do. Identifying such insights is the main goal of this work.

It is not an easy task to include capelin, cod and Barents Sea herring in a limited multi-species growth model and get good statistical results. The picture is complicated by other factors such as primary production and predators not included in our model. Especially when it comes to herring, stock size and catch of the mature population further south are of great importance to the extent of new recruits in the Barents Sea. Year class dynamics within each species is also very important for the fluctuations in stock biomass. Nevertheless, limitation is essential for solvability of the bioeconomic model

within the frames of numerical dynamic programming, which is the instrument of use in this article. It is very computer-intensive, and we have therefore limited ourselves to work with a three-dimensional state space (capelin, cod and herring). Although higher dimensions are possible in theory (Grüne and Semmler 2004), such a rise would result in a dramatic growth of throughput, and is therefore beyond the scope selected for this article. It would, however, be interesting as a future extension to include time-dependence as a cyclic driving force. The time-dependence could be represented by a fourth dimension of the growth model (Capelin, cod, herring and time), some of which could be put into lump stoachastic terms and mainly influence herring growth. The model would then constitute a stochastic optimal dynamic programming problem.

Very interesting multi-species perspectives concerning the optimal catch of capelin are revealed in this work. Also, applied to historical stock data, the model suggests that the catches of both capelin and cod have been much too high. Finally, the interpretation of herring as a tragedy affecting the capelin-cod system is partly challenged by the optimal feedback strategy.

5.2   Model description

This section presents the bioeconomic control model and all of its assumptions. A solution procedure is also sketched.

5.2.1   *Management model*

Our bioeconomic control model is based on a biological multi-species growth model created for this article and on an economic profit model used in a previous article (Agnarsson et al. (2008)).The system of the two models constitutes an optimal control problem, where the objective is to find optimal total Norwegian and Russian harvest levels from a Norwegian point of view when the Norwegian and Russian fraction of total harvest is pre-decided. If we assume $\underline{x}$ is stock biomasses and $\underline{u}$ is corresponding harvest, the continuous time Hamilton-Jacobi-Bellman equation with discount rate δ can be written as (see Kushner and Dupuis 2001)

$$\delta V(\underline{x}) = \max_{\underline{u}} \left\{ \Pi(\underline{x}, \underline{u}) + \nabla V(\underline{x})(\underline{F}(\underline{x}) - \underline{u}) \right\}, \tag{5.1}$$

where the optimal value function $V(\underline{x})$ represents total future discounted profit from optimal harvest and ∇ is the gradient operator. This equation describes the optimal balance between immediate harvest and saving of stocks for future exploitation, and gives necessary conditions for optimum (see Kamien and Schwarz(1991) or Bertsekas (2001)).$\Pi(\underline{x}, \underline{u})$ represents immediate profit (economic model) and $(\underline{F}(\underline{x}) - \underline{u})$ represents

biological growth of the stocks. From equation (5.1) it is obvious that as far as optimality is concerned, both the biological and economic submodels play an important role in the solution. That is, we are primarily interested in determining a reasonable economical and biological balanced harvest strategy through harvest feedback policies taking the stock estimates at any particular time into consideration.

5.2.2 *Biological growth model*

The complexity of the nature can not be fully covered in a model used for management purposes, but a good model should be consistent with some of the main features of real marine life. In this article, predation and other multi-species interaction are given the greatest priority.

The three stocks in our growth model, namely capelin, cod and juvenile herring aged one to three, have historically shown a cyclic behavior, which includes several collapses in the capelin stock owing to high inflow of herring fry to the Barents Sea and good growth conditions for both cod and herring (Hamre 2003). It is reasonable to assume that there exists for all the stocks a saturation level affected by food supply, predation pressure and other external effects like marine temperatures and ocean currents. We focus on saturation and predation, and choose logistic like growth modified by predation as functional form for both capelin and cod. Furthermore, the predation reflects the well-documented fact that cod seems to prefer capelin as prey, but turns over to herring and cannibalism in years of capelin scarcity (Hamre 2004).

The growth function assumed for herring is more complicated than the capelin and cod growth. Since only juvenile herring are found in the Barents Sea, it is somewhat difficult to implement herring growth in a closed model for this area. Historical stock sizes of the Barents Sea fraction of the population are not available. Assessment estimates are available, however, for each year class of herring, and it is known that only the three- to four-year-old first year classes are found in the Barents Sea, and that those aged one to three heavily predate on capelin when overlapping between the two species takes place (Hjermann, Ottersen and Stenseth 2004). Therefore, in the growth of capelin and cod, these age classes represent herring influence. The growth of juvenile herring itself is fully explained by three factors: First, its own stock size, second, multi-species interactions with capelin and cod and, finally, external inflow of herring fry from the North Sea.

A major argument against including older herring in the model is that the emphasis should be on the joint Norwegian and Russian resources of the Barents Sea. From that perspective, to endogenize older year classes of the herring stock, which is located in the North Sea, destroys the possibility of a closed model since those stocks are shared with other partners than Russia. Moreover, it would have been difficult to find the

connection between the size of the mature fraction in the south and the juvenile fraction in the north, and it should also be mentioned that the curse of dimensionality is a very convincing argument against including a fourth stock in the growth model.

Assume stock biomasses are given by the vector $\underline{x} = [x_1, x_2, x_3]$ and harvest by $\underline{u} = [u_1, u_2, u_3]$, where the indexes 1, 2 and 3 represent capelin, cod and herring respectively. The state equations with deterministic evolution are given by

$$\dot{x}_i = F_i(\underline{x}) - u_i, \quad i = 1, 2, 3 \tag{5.2}$$

where F_i is surplus growth of species i. Since herring is not exploited in the Barents Sea, $u_3 = 0$. This means that herring in principle could be removed from the model and be replaced by $\dot{x}_3 = 1$. Then, instead of being herring, x_3 is a direct measure of time. Another possibility is to let $\dot{x}_3 = F_3(x_3)$ be cyclic and measure time-dependent inflow of herring. These are good alternative forms if cyclic time-dependence is more important to the model results than predation-prey relationships with herring, and a lot of literature claims that long-term biomass cycles play an important role in the Barents Sea ecosystem (e.g. Yndestad and Stene (2002), Yndestad (2003)). Unfortunately, we did not succeed in finding such forms that fitted well with our data.

The forms of the growth functions were based on assumptions about the overall structure of the growth dynamics and on numerous statistical fitness-testing with relative growth for stock data. The growth functions are

$$F_1(\underline{x}) = r_1 x_1 \left(1 - \frac{x_1}{k_1} - b_1(x_2 + x_3)\right)$$

$$F_2(\underline{x}) = r_2 x_2 \left(1 - \frac{x_2}{k_2} + b_2(1 + x_1)\sqrt{x_3}\right) \tag{5.3}$$

$$F_3(\underline{x}) = \tilde{r}_3(x_1, x_2) x_3 \left(1 - \frac{x_3}{\tilde{k}_3(x_1, x_2)} + b_3 \frac{x_1(100 + x_1 + x_2)}{1 + x_3}\right) + C, \text{ where}$$

$$\tilde{r}_3(x_1, x_2) = \frac{r_3}{100 + x_1 + x_2}, \tag{5.4}$$

$$\tilde{k}_3(x_1, x_2) = k_3 \tilde{r}_3(x_1, x_2) \tag{5.5}$$

$$r_i, b_i, k_i, C > 0 \text{ for all } i.$$

Generally the growth function for each species is concave with respect to its own species. The growth functions for capelin and cod have straight forward logarithmic forms with r_1 and r_2 representing the intrinsic growth rate. The parameters k_1 and k_2 are sustainable biomass capacities when multi-species interactions with endogenized species are disregarded. Multi-species interactions adjust the sustainable biomass capacities. For capelin the capacity is reduced below k_1 according to the last term of the capelin growth ($b_1 > 0$). Also, this term implies that biomass of cod affects capelin in the same way as

biomass of herring. Cod growth is improved by multi-species interactions ($b_2 > 0$). The term $(1 + x_1)\sqrt{x_3}$ implies for the first that biomass of capelin is more important to cod growth than biomass of herring. Second, it implies that with capelin extinct from the Barents sea ($x_1 = 0$), herring biomass is still beneficial for cod growth.

The growth of herring is somewhat different from that of capelin and cod. It is presented in a quasi-logarithmic form to fit with the forms used for capelin and cod in equation (5.3). An alternative form, which will be referred to later in the article in connection with the parameter estimation, is

$$F_3(\underline{x}) = x_3\big(C(7)\,\frac{1}{100 + x_1 + x_2} + C(8)\,x_3 + C(9)\,\frac{x_1}{1 + x_3}\big) + C. \tag{5.6}$$

The interpretation of $\tilde{r}_3(\cdot)$ as intrinsic growth rate and $\tilde{k}_3(\cdot)$ as carrying capacity for herring is a little debatable owing to the dependences on the biomass of capelin and cod. Let us therefore turn to (5.6). The parameter C reflects the positive inflow of herring (year class 0) from the Norwegian Sea to the Barents Sea. The fact that $F_3\big([x_1, x_2, 0]\big) = C$ implies that herring cannot become extinct. This characteristic is reasonable since mature herring is not a part of the model.

The first term of (5.6) is a growth term ($C(7) > 0$ is assumed) depending on the total biomass of the other endogen species. This growth term is reduced by large biomasses of capelin and cod. The number 100 in the denominator prevents excessive growth when capelin and cod biomasses are both low. The second term secures concavity with respect to herring if $C(8) < 0$, and the third term is a growth term representing multi-species interactions between herring and capelin (herring predation).

5.2.3 *Economic model*

The optimal value function represents the maximum discounted future profit from the fishery. If $\mathcal{U}$ is the set of admissible controls, the optimal value function is given by

$$V(\underline{x}) \equiv \max_{\underline{u} \in \mathcal{U}} \int_0^\infty \Pi\big(\underline{x}, \underline{u}, \underline{\alpha} \cdot \underline{u}\big)e^{-\delta t}\, dt, \tag{5.7}$$

under the conditions in (5.2) with $u_3 = 0$.

Here δ is the social time discount rate, $\underline{u}$ is total landings, $\underline{\alpha}$ is the Norwegian fraction of total landings and Π is the current profit of the fishery.

The formulation implies that the foreign (Russian) catch could be controlled, which is partly compatible with the joint Norwegian and Russian management of the Barents Sea fisheries. Moreover, it is assumed that there are no interactions in the market for fish, meaning that profit from cod and capelin can be added together. The same is true for the cost functions since capelin and cod are not fished jointly. Further, it is assumed

that there is no economic protection on capelin; that is, landing costs are considered independent of the stock size (finding costs are neglected). Beyond that, capelin is not price-elastic, but Russian and Norwegian landings of cod are large enough to influence the price level, $P_2(u_2)$. The Norwegian profit function can be written

$$\Pi(\underline{x}, \underline{u}, \underline{\alpha u}) = CaP(\alpha_1 u_1) + CoP(x_2, u_2, \alpha_2 u_2), \text{ where} \tag{5.8}$$

$$CaP(\alpha_1 u_1) = p_1 \cdot \alpha_1 u_1 - c_1(\alpha_1 u_1)^{c_2}, \tag{5.9}$$

$$CoP(x_2, u_2, \alpha_2 u_2) = (p_2 - p_3 u_2)\alpha_2 u_2 - \frac{c_3}{x_2}(\alpha_2 u_2)^{c_4}, \tag{5.10}$$

and all the coeffisients $\underline{\alpha}, p_1, p_2, p_3, c_1, c_2, c_3$ and $c_4 > 0$.

5.2.4 *Parameter Estimation*

The parameters of the growth, functions (5.3), (5.4) and (5.5), are fitted to the discrete relative growth

$$\underline{Y}(t) \equiv \big(\underline{x}(t+1) - \underline{x}(t) - \underline{u}(t)\big)/\underline{x}(t), \tag{5.11}$$

which is given by historical data from 1973 to 2005 (see Table 5.4 in Appendix 5.6). These data are assumed stationary, but this cannot be confirmed or disconfirmed with the amount of data available. If, however, the stationarity assumption is true co-integration and unit root are not a problem.

The residuals or error terms from the data fitting are given by $\underline{R}_t = \big|\underline{Y}(t) - \underline{F}(\underline{x}(t))/\underline{x}\big|$. Parameters for the three growth functions are found simultaneously with the *Full information maximum likelihood method* to minimize the sum of residuals $\sum_{t=1973}^{2005} R_t$. The residuals are plotted in Figure 5.6.

The statistical results for each of the species are listed in Table 5.2 along with the coefficients $C(1) - C(9)$. The growth functions in equation (5.3) are the same functions that are used in the regression analysis, but in equation (5.3) they are factorized in a manner that stresses their logarithmic behavior. The value of the growth function coefficients and their connections with the regression analysis coefficients are listed in Table 5.1, where $C(i), \quad i \in [1, 9]$, refers to the coefficients in Table 5.2. The parameter C in the herring growth (see equation (5.3)) representing exogenous inflow of juvenile herring is a priory given the value 50 (50 thousand tons). This value should be reasonable although C is a number that depends on the time of year that is the basis for the data used.

All coefficients have signs corresponding with the fact that cod predate both capelin and herring whereas herring predate capelin. This implies that b_1, b_2, b_3, k_1, k_2 and k_3 are greater than zero, which is in accordance with the model formulation (see equation (5.3)).

All the economic parameters apart from the discount rate $\delta = 0.05$ are collected from (Agnarsson et al. (2008)). They are summarized in Table 5.3, but for further information about the economic model refer to the original source in (Agnarsson et al (2008)).

5.2.5 *Solution procedure*

The optimal control problem defined in eq. (5.2) and (5.7) is solved by numerical dynamic programming. The solution is found on feedback form, which implies that optimal catches of capelin and cod are functions of stock sizes only, and we can write the optimal harvest policy as $\underline{u}^\star \equiv \tilde{u}^\star(\underline{x})$. The problem, however, is not algebraically solvable, and the solution must be obtained by numerical techniques.

The main procedure used to find the optimal policy starts out as described in Grüne and Semmler (2004), but the approach differs somewhat. A discrete representation of the continuous-time problem in (5.2) and (5.7) could be expressed by a discrete first-order approximation given by

$$V_h(\underline{x}) \approx \max_u h \sum_{i=0}^{\infty} \beta^i \Pi(\underline{x}_h(i), u(i)) \tag{5.12}$$

$$\underline{x}_h(0) = \underline{x}, \quad \underline{x}_h(i+1) = \underline{\varphi}(\underline{x}, \underline{u}) \equiv \underline{x}_h(i) + \big(\underline{F}\big(\underline{x}_h(i)\big) - \underline{u}(i)\big)h, \tag{5.13}$$

where $0 < h << 1$ is the discrete time step and $\beta = 1 - \delta h$ is the corresponding approximation to the discrete discount rate as shown by Grüne and Semmler (2004).

By inserting (5.13) into (5.12) and extracting profit of the first period from the summation sign, we get the discrete version of the Hamilton-Jacobi-Bellman equation

$$V_h(\underline{x}) = \max_{\underline{u}} \left\{ h\Pi(\underline{x}, \underline{u}) + \beta V_h\big(\underline{\varphi}(\underline{x}, \underline{u})\big) \right\}. \tag{5.14}$$

Optimal value is obtained by solving this equation for every possible stock combination on a selected stock-grid.

Now we define the linear operator

$$L(u)(V_h) \equiv h\Pi(\underline{x}, \underline{u}) + \beta V_h\big(\underline{\varphi}(\underline{x}, \underline{u})\big), \tag{5.15}$$

and the dynamic programming operator

$$T_h(V_h)(\underline{x}) \equiv \max_{u \in \mathcal{U}} \big\{ L(u)(V_h) \big\}. \tag{5.16}$$

V_h being the solution to the fixed-point equation

$$V_h(\underline{x}) = T_h(V_h)(\underline{x}). \tag{5.17}$$

We use the dynamic programming operator (5.16) to solve the Hamilton-Jacobi-Bellman equation (5.14) with fixed point iteration starting with the initial value $V(\underline{x}_0) = 0$, where $\underline{x}_0$ is the $\underline{x}-$grid chosen. To speed up convergence we switch from this policy iteration to less numerically expensive value iterations. In the value-iterations we fix the policy, $\underline{u}^f(\underline{x})$, and therefore also the profit $\Pi(\underline{x}, \underline{u}^f)$ when using the linear value-iteration operator (5.15). When the value-iterations

$$V_h^{n+1} = L(u^f)V_h^n \tag{5.18}$$

stabilize, we shift to policy-iterations in accordance with equation (5.16), before returning again to value-iteration. The alternation between value and policy iteration continues until convergence.

Utilization of first-order conditions

The Hamilton-Jacobi-Belmann equation (5.14) can be solved in various ways, but in three-dimensional space through-put and computer-time become critical points. In the iterative process there are only two limitations in policy space. The feasible policy, that is the harvest on each species, is non-negative and no higher than the size available. Instead of working with a discrete control space, $\tilde{u} = (\tilde{\underline{u}}_0, \tilde{\underline{u}}_2, \cdots, \tilde{\underline{u}}_n)$, and for each fixed-point iteration doing a crude search for the "argmax" to (5.16) among all of the $\tilde{\underline{u}}_i$'s, we check only boundary values and the inner point solution to equation (5.16) when $V_h\varphi(\underline{x}, \underline{u})$ is replaced by its first-order Taylor approximation to reduce the number of terms.

$$V_h\big(\varphi(\underline{x}, \underline{u})\big) \approx V_h(\underline{x}) + (\nabla V_h)^T(\underline{x})(\underline{F}(\underline{x}) - \underline{u})h. \tag{5.19}$$

That is, inserting (5.19) into (5.16) reduces the problem to

$$V_h(\underline{x}) = \frac{h}{1-\beta} \max_{\underline{u}} \left\{ \Pi(\underline{x}, \underline{u}) + \beta(\nabla V_h)^T(\underline{x})(\underline{F}(\underline{x}) - \underline{u}) \right\}, \tag{5.20}$$

which may be tested for three control values only, namely the lower bound $\underline{u} = \underline{0}$, the upper bound and the interior solution solving

$$\frac{\partial \Pi(\underline{x}, \underline{u})}{\partial \underline{u}} - \beta \nabla V = \underline{0} \tag{5.21}$$

with respect to $\underline{u}$. Optimal $\underline{u}$-value in each fixed point iteration step will be the one that gives the highest value to (5.16).

5.3   Results

In this section a brief description of the results will be given.

5.3.1   *The optimal value function*

The optimal value function, $V(\underline{x})$, is a theoretical function relating the present situation of the biological state space (biomass of capelin, cod and herring) to current value of all future profit from the fishery. This function is used to deduce optimal TAC levels in the management model. The estimated size of the function, however, being a measure of the total future value of the fishery as a function of stock sizes, also gives some interesting interpretations. Since a simultaneous graphical presentation of this function on the whole capelin, cod and herring stock range would imply a four-dimensional figure, which is hard to depict, it is presented with one of the stocks fixed in Figure (5.2(a)-5.2(d)). This means that only three dimensions of the four-dimensional value functions are presented in the figures.

   The shapes of the figures verify this rather intuitive result. Absence of stocks of both capelin and cod means zero profit, and a rise in one or both of them means a rise in future profit and therefore also rises in the value function. The value function grows less, however, for extreme values of the stocks.

   Since extinction of either capelin or cod means that the extinct stock will never return, it also implies a low optimal value function. If either capelin or cod is depleted, there is a dramatic fall in the value function. On the surfaces in Figure 5.2(a) - 5.2(d) this can be clearly observed, as the edges are very steep from zero stock value to the first non-zero value along both the capelin axis and the cod axis.

   According to the growth model, herring predate capelin. The economic trace of this effect is interesting to observe in the cod/herring grid of Figure 5.2(d). Without cod (along the herring axis) the optimal value function approaches zero for high levels of herring. With some cod in the model the influence from herring is also clearly negative for the optimal value function, but this trend declines when the cod biomass is high. Figure 5.2(c) shows minor influence from herring when the cod stock is 3.5 million tons. Owing to the large scale of the Z-axis in this figure it is impossible to see the exact influence from herring. A smaller scale, e.g. $2.7 \cdot 10^6$ - $2.9 \cdot 10^6$, would have shown positive influence from herring when the herring stock is not too high and the capelin stock is high.

5.3.2   *Optimal Catches*

In this section we focus on total Norwegian and Russian landings of capelin and cod. The optimality condition maximizes discounted Norwegian profit from the fishery when the Norwegian fraction of the total quota is $\alpha_1 = 0.6$ and $\alpha_2 = 0.5$ for capelin and cod respectively. New TACs are decided once a year.

   Optimal TAC levels for capelin are presented for several different initial conditions

in Figure 5.3(a)-5.3(f). The main trend is that optimal TAC of capelin grows with the capelin stock, but the surfaces also show clear dependence on the levels of cod and juvenile herring. In Figure 5.3(a) we observe that TACs of capelin fall slightly with growing cod stock for low levels of cod. There is even a small discontinuity between zero-level of cod and the first non-zero level. This is a discontinuity between "bliss", which is the TAC level that maximizes current profit, and lower TAC levels in the presence of cod. The discontinuity becomes more explicit with more herring present, as in Figure 5.3(b). The trend that more cod means lower capelin TACs is only confirmed for low levels of cod in that figure. For very high levels of cod it might be more valuable to catch more of the capelin instead of leaving them for cod and herring predation.

The optimal TAC levels for capelin when cod are extinct show a very interesting pattern. When there is much capelin the herring stock alone is no threat to the capelin stock and capelin should be harvested at bliss level. When the capelin stock is low, however, and the herring stock is high, the TAC should be lower or set to zero (see Figure 5.3(c)). The exception is when the capelin stock is below one million tons at the same time as the herring stock is above two million tons. Then capelin should be harvested even though the stock is low. (See the ridge in the TAC for capelin when the stock is low.) This pattern is very interesting, specially since it disappears when the cod stock is high (see Figure 5.3(e) and 5.3(f)). When the cod stock is very low, e.g 150 000 tons as in Figure 5.3(d), the ridge is still visible, but it has been displaced towards higher herring biomasses.

There is an intuitive correlation between the cod biomass and the optimal catch of cod, since cod is the top predator in the model. This intuitive correlation is confirmed in Figure 5.4(a)-5.4(d). The sizes of the capelin and herring stocks do not seem to be very important for optimal TAC of cod. Only in the absence of capelin are there interesting changes. The optimal TAC of cod then gets lower. This is visible in both in Figure 5.4(a) and 5.4(b)). An even more interesting observation is that a very low level of capelin combined with high levels of both herring and cod gives a small peak in the optimal TAC of cod. This peak is visible as a slim ridge in the upper left of Figure 5.4(b).

Optimal catch versus historic catch

In Figure 5.5(a) and 5.5(b) optimal catch (given historical stock data) of capelin and cod respectively is plotted together with actual catch for the period 1973-2005. The capelin curves reveal a dramatic overfishing from the beginning of the period until the closure of the capelin fishery in 1987. In later years the fishery of capelin has been more in accordance with optimal TACs.

The results for cod are similar. The actual harvest has been dramatically higher than what is optimal according to these simulations.

For both the capelin and cod catches it is obvious that variations in the optimal TACs are much lower than the variation in the historic catches. Moreover, the optimal TACs show a tendency to forestall the historical catch within a year or two.

Given the observations in Figure 5.5(a) and 5.5(b) it is obvious that the historical biomasses of cod from 1973 onwards would have been much higher with optimal management. Figure 5.6(a)-5.6(d) show how past stock biomass development and catch could have been given use of the optimal control rule and trust on the biological growth model (see equation 5.3). As expected, Figure 5.6(c) paints a picture of a much higher cod stock than the last 35 years have shown. As far as the capelin stock is concerned, the optimal management would have given a biomass between three and five million tons, which is rather high given the high cod stocks in the optimal scenario. (See Figure 5.6(a).) The falling trend of capelin must be seen in the light of the very high stocks in 1973, which was probably a result of the herring collapse a few years earlier.

The optimal catch of capelin is rather low compared with the actual catch between 1973 and the mid-eighties, but after that the optimal average is bigger. (See Figure 5.6(b).) For cod, the optimal catch becomes higher than the actual catch in 1978. There are also, however, two periods after 1978 when the actual catch is bigger but owing to much lower biomasses in the sea the cost corresponding to actual catch is much higher than it would have been with optimal catches. For the period as a whole (1973-2005), assuming biological growth in accordance with equation (5.3) and use of the optimal feedback rule, average actual catch of cod would have been almost 13 % lower than historic numbers, but the average biomass would have been more than double the historical average.

5.3.3   *Paths towards equilibrium*

Our deterministic growth model with optimal management implies that the stocks will move towards equilibrium levels. These theoretical levels have not been observed in the real marine ecosystem, and there are several reasons for this discrepancy. A lot of varying factors and stochasticity cause huge fluctuations in the stocks. Moreover, the management system in place has not managed the stocks according to our model and hence it has created a different path. Yet, paths towards equilibrium may be interesting from a theoretical point of view, and it is possible that the real biomasses would have fluctuated around these paths with management governed by our prescribed harvest policy.

The paths towards equilibrium resulting from a total closure of the capelin and cod fishery may tell us much about the biological growth model used. In the 2D curves of Figure 5.7(a) - 5.7(c), where in each figure one of the stocks is suppressed, we can see the three equilibriums of the biological model. Note that the external inflow of

young herring from the Norwegian Sea means that there is no equilibrium with the absence of herring. Therefore, with initial absence of herring as in figure 5.7(a), the phase-plot shows evolution towards an interior equilibrium (all model stocks present) of $(2806, 4021, 2040)$. In addition $(0, 3192, 2666)$ is an equilibrium without capelin. Note that the equilibrium without capelin leaves cod in a poorer situation than the interior equilibrium with all stocks present.

The two equilibriums shown in figure 5.7(a) are the same two that are shown in figure 5.7(b). The only difference is that the axes have changed.

In the absence of cod there are two biological equilibriums as well, but only one of them is visible in 5.7(c). The visible equilibrium shows a very high capelin stock (more than six million tons) and a medium high herring stock. The equilibrium not shown in this figure has a very low capelin stock and a very high herring stock.

The management that optimizes the model of course gives other equilibriums and other paths towards them. Figure 5.8 is a 3D phase-plot showing paths towards equilibrium (marked with a red ring) from different initial conditions when new TACs are set each year according to the optimal management strategy. Although all the initial stock size combinations in figure 5.8 are at the edge, and also outside the edge, of a stock space that is reasonable based on historical biomass estimates, most of the combinations end up in the same interior equilibrium with a capelin stock of about 2.7 million tons, a cod stock of 3.2 million tons and a herring stock of almost two million tons. In addition to the interior equilibrium there is also an equilibrium without capelin. The cod stock is about the same for this equilibrium, but the herring stock is a little higher. The equilibrium without cod (when initial cod biomass is zero) is not shown in the figure.

In figure 5.8 we also observe over- and undershooting for all three stocks before settlement in equilibrium. The overshooting for herring is extreme for the initial condition with low level of capelin and cod and high level of herring. The curve with these initial conditions disappears out of the figure because the herring stock grows so much. It comes down again, however, and settles in the equilibrium without capelin.

Figures 5.9(a)-5.9(c), 5.10(a)-5.10(b), and 5.11(a)-5.11(c) illustrate more detailed phase plots in two dimensions when the harvest is according to the optimal control rule. In each of these figures one of the stocks is suppressed from the figure because of the dimension reduction. The suppressed stock, however, works indirectly in the development of the other two stocks.

Initial absence of capelin (see figure 5.10(a)) may give another equilibrium than the equilibrium without capelin shown in figure 5.8, where capelin is extinct owing to predation pressure and harvest. There are at least two equilibriums without capelin and with both cod and herring. The interior equilibrium in 5.10(b) is the same as the interior equilibrium in figure 5.8, namely $(2.73, 3.18, 1.98)$ million tons.

The most interesting observation from figure 5.11(a) - 5.11(c) is all the over and un-

dershooting in the capelin-herring plane. The high capelin equilibrium of about 5.5 million tons should also be noted in figure (5.11(a). In addition, there is also another equilibrium not shown in this figure with a very high herring stock and a very low capelin stock.

The spiraling behavior of the paths towards the interior equilibrium (see Figure 5.8), means that the stocks are fluctuating before settlement in equilibrium. This fluctuation is confirmed in Figure 5.12(a)-5.12(c). These figures show evolution towards equilibrium on the time-stock plane.

5.4   Discussions

5.4.1   *Biological model*

A limitation in the model is the fact that we have to choose between an autonomous (time-independent) growth model with three species (capelin, cod and herring) or a time-dependent growth model with only two species (capelin and cod). A series of tests with regression analysis for a number of different growth functions, indicated that the data were best represented when herring were included instead of cyclic time-dependent fluctuations. It will be interesting to further investigate time-dependent fluctuations in future works, but it ought to be done in a more general stochastic setting.

The choice of the specific growth has some model-technical basis as a closed system is essential for optimizing when sizes of external factors beyond control are unknown. (Catch quotas of mature capelin, ocean currents determining inflow of herring fry to the Barents area etc...). Such factors are very difficult to quantify and recognize, and are for the time being not ready for use in this model. The growth in this article, however, is consistent with expected predation relationships and gives a better representation than a purely stochastic inflow, which cannot explain the forces of importance. At least this model aims to explain some of the variability in the herring stock, and independently of the relevance of the predation terms, the growth functions should cover for some of the stock dynamics. As far as the herring growth is concerned, the regression analysis performed indicates that the stock of capelin and cod to some extent is a proxy. On the other hand, herring can be interpreted as playing the role of a time-dependency in the capelin-cod system representing effects of both herring and other non-autonomies.

One might argue that the growth function for herring is not satisfactory from a biological point of view, as the growth is only passed for and based on the immature part of the stock in addition to predation relationships. It may be a little too much to claim that these factors are the most important for growth of juvenile herring in the marine ecosystem, but it is very difficult to quantify others.

Although we include inflow of juvenile herring in the herring growth, we disregard dependency between the size of this inflow and the size of the mature fraction of herring in the North Sea and the Norwegian Sea. That dependency must be expected to be important to herring recruitment (Hamre and Hatlebakk 1998) in spite the fact that biomass of spawners has turned out to be only a weak indicator of egg production in some populations. (Marshall et. al 1999, Fiksen and Slotte 2002.) We use a growth model, however, not a recruitment model. That is, the aim is not to describe bottom-up factors in recruitment and mortality, but rather to explain some of the top-down stock dynamics which is relevant for more realistic economical and informational optimization. Given the very high uncertainty about stock data, the error made when using a constant factor $C = 50$ (50 thousand tons) in equation (5.3) to represent external inflow of herring should not detract from the validity of the biological model.

The best argument for the biological growth model is that all the parameters from the regression analysis are reasonable. The signs of b_1, b_2 and b_3 are positive, and this is in accordance with the assumed predation pattern for all the species. (Cod predate capelin and herring whereas herring predate capelin.) Moreover, the biological carrying capacity for capelin and cod of respectively 7.89 and 3.19 million tons (see Table 5.1) makes sense. Given the extremely fluctuating nature of the system described and the high inaccuracy in the available assessment data, the general agreement between commonsense and the size of the parameters found in the regression analysis is a very strong argument for the growth model used.

Moreover, the statistical results are acceptable as herring are fitted with only three parameters. The significance of some of the parameters could have been more convincing and the residuals could have been lower, but we should remember that herring have shown very large fluctuation and faced a collapse at the beginning of the period our data are built upon. Therefore we cannot expect them to be lower.

As far as the biological equilibriums are concerned, there are several equilibriums depending on the initial stock sizes. The formulation implies that herring will never go extinct and that cod may only do that as a result of over-fishing. Too much herring and cod might be a catastrophe for capelin owing to predation. That is obvious from the equilibrium $(0, 3192, 3316)$, where capelin have died out.

An interior biological equilibrium is $(2261, 4042, 1769)$, where all three stocks are in good condition. One might argue that the cod level in this equilibrium is too low compared to historically gigantic cod levels reported, but extreme cod levels do not agree very well with the data this analysis is built upon. Also, since the cod stock has a fluctuating nature, the highest sizes reported should probably not be regarded as close to a natural no-fishing equilibrium level.

5.4.2 *The optimal value function*

If we keep in mind that the value function represents the total future discounted profit from harvest of capelin and cod, it is clear that the surfaces must go through the origin between the capelin and cod axes. Absence of both capelin and cod means zero profit. Subsequently, absence of capelin or cod in one moment of time implies that the stock is exterminated for all future, whereas small positive initial level leads to higher future levels under optimal policy. The difference in economic rent between these two situations explains the discontinuity between zero-levels of capelin and cod and the first non-zero levels.

Generally, a rise in one or both stocks means a rise in future profit and therefore rises in the value function as well. Extreme values are not sustainable, however, and if the stocks get very high, they will fall soon independently of the fishing policy and leave little extra profit. Remember also that the profit function (see eq. (5.8)) is concavely related to harvest. Consequently, extreme harvest in a limited moment of time will not result in extreme profit.

It is interesting that although herring is not harvested in the model a modest presence has a positive influence on the economics. This might not be very surprising since herring meat is on the cod-menu, but it contradicts the quite common one-track interpretation of herring as juvenile capelin predator and a tragedy to the capelin-cod system. Nevertheless, for certain initial conditions herring affects the capelin-cod system in a negative way in this model as well. With a low initial stock size of capelin, high herring biomasses may lead to capelin extinction. With low initial levels of cod the biological model gives a very high growth of herring, and, as seen from Figure 5.2(d), this gives a very poor fishery for both capelin and cod. Yet the main impression when we study the optimal value function is that herring, in spite of its predation of capelin, contributes to cod growth. Therefore, when the stock is not too big, herring has a positive influence on the profit in the model. This finding finds support in Helstad (2000), who also concludes that the rebuilding of the herring stock after its collapse in the late 1960s has been optimal from an economic point of view, when considering the three species of relevance in this article.

5.4.3 *Optimal catch policy*

From a multi-species perspective there are many interesting questions concerning optimal catch. On the fishmarket the value of cod is a lot higher than the value of capelin. Since capelin is the most important food for juvenile cod it is not obviously that capelin should be harvested at all. The important role of herring in the capelin-cod system distorts the situation even more, and according to the biological model capelin will become

extinct given certain initial conditions. That is obviously not a desirable scenario. Given the worst possible initial conditions, however, it is probably not possible to prevent it in a real situation. Within the frames of this model it would have been possible, allowing for harvest of juvenile herring, but that strategy is not investigated in this work.

If the initial conditions given are of a kind that makes the extinction of capelin un-preventable, what is the best response from an economic point of view? Is it to let cod and herring alone finish their job of hunting down the capelin stock or is it to contribute to the extinction by fishing as much capelin as possible to earn at least something on the remaining stock? Given that it is assumed no finding costs (costs are independent of stock size) in the capelin fishery (see equation 5.9) the latter alternative is quite likely. Therefore it is not surprising that this alternative is confirmed in Figure 5.3(b), 5.3(c) and 5.3(d).

What is seen in these figures is that capelin is heavily harvested for certain initial conditions where the stock itself is so low that extinction is the only outcome accord-ing to the biological model. Figure 5.3(c) illustrates this very clearly. If the survival of the stock is not vulnerable (enough capelin and not too much herring) the optimal catch is rather high, but for slightly different biomass combinations where the preda-tion pressure on capelin becomes critical, the fishery is closed. Yet, with even worse initial conditions for capelin, the fishery is open. The conclusion is that it is better to profit from the remaining capelin stock by heavy fishing than let herring and cod finish the extinction job alone. This is, however, a rather theoretical conclusion. In reality it is probably unlikely that capelin, from almost any initial conditions, could be driven to extinction by cod and herring without continuous contribution from man all the way. In this model, though, it is possible. That is mainly a result of high herring growth when both the capelin and cod stock is low. Then the term with $C(7)$ as coefficient in equation (5.6) dominates the other terms - also for rather high stocks of herring. The reason is that $C(7)$ is quite big according to the parameter estimation (see Table 5.1). This explains the high catch when there is little capelin and the stock of cod is extinct (along the capelin axis for cod$= 0$) in figure 5.3(b) and it explains the high catch of capelin for very high levels of herring and very low levels of capelin in figure 5.3(d) (lower right corner).

Except for the very special features just mentioned, the findings illustrated in figure 5.3(a)-5.3(f) are pretty straight-forward. In the absence of cod there is no pressure on the capelin stock. This explains the discontinuities visible in figure 5.3(a) and 5.3(b) between the capelin catch given by zero cod stock and the catch given by the first non-zero cod stock level. Actually, absence of cod leads to capelin TACs on "bliss-level" for most levels of herring when the capelin stock is above a certain lower limit. This is shown in figure 5.3(b), where the herring stock is four million tons, which is a very high biomass level for juvenile herring.

The main trend in figure 5.3(a)-5.3(f), that optimal capelin-TACs increase with the

capelin stock, is very intuitive and could be assumed without a multi-species model. The trend that optimal TAC of capelin is negatively correlated with the herring stock is also intuitive (see Figure 5.3(e) and 5.3(f)). A high herring stock leads to a high predation pressure on capelin and therefore the TAC of capelin must be kept low to maintain the stock.

Although cod biomass affects capelin growth just as much as herring according to equation (5.3), the TAC of capelin and the cod biomass is not as negatively correlated as TAC of capelin and herring biomass. To understand this we must remember that cod predates herring. A high cod stock therefore prevents herring growth, and is consequently not as critical to capelin as herring.

Cod is economically the most important species and the top predator of this model. Moreover, profit from cod is concave as a function of harvest. One should thus expect stable and high, but sustainable, optimal TACs of cod. Furthermore, a rather weak dependence of the other two species should be expected. Both expectations are confirmed in figure 5.4(a)-5.4(d). There are only a few discrepancies with these main findings. The first is that TACs of cod fall slightly in the absence of capelin. This can be observed along the cod-axis in figure 5.4(b). The reason is that the future growth conditions for cod are poorer in absence of capelin and the situation demands a lower cod harvest. The most interesting multi-species effect in the TAC of cod is the ridge visible on the left of figure 5.4(b). For large cod stocks there is a peak in the TAC on the first non-zero level of capelin. For smaller cod stocks the ridge shifts to a little "dale". The key to understanding this phenomenon is to be aware of the fact that the capelin stock is under very strong pressure when there is little capelin and very much cod and herring. What is observed is a rescue operation of the capelin stock. As can be seen from figure 5.9(c), the capelin stock might be saved for most stock combinations of capelin and cod even when the initial herring stock is above five million tons (six million tons in that figure).

Cod affects capelin growth both directly and indirectly through the growth functions in equation (5.3). For high cod stocks the direct negative effect on capelin is beyond doubt the most important. A high cod TAC that reduces the stock may therefore ease the pressure on the capelin stock. This explains the ridge in the capelin TAC when high levels of cod coincide with the first non-zero level of capelin. For low levels of cod, however, a rise in the cod stock weakens herring growth very much, according to equation (5.3). This direct effect on herring has an indirect positive effect on capelin growth, which is more important than the direct negative effect from cod. This explains the small (hardly visible) "dale" in the TAC of cod for low levels of cod and low levels of capelin.

When comparing our results with actual historical catches we find that optimal catches of cod are much lower. Similar results have been documented before (Arnason, Sandal, Steinshamn and Vestergaard 2004), (Kugarajh, Sandal and Berge 2006), and

the magnitude of the discrepancy should be a wake-up call. Optimal catches of capelin are also much lower than actual historical catches, but the management of capelin has changed. Before the 1990s the fishery was open all year, but lately it has only been open in a short post-spawning period. Catches of mature capelin after spawning may not harm the capelin stock too much since a high fraction of the capelin population spawns only once (Yndestad and Stene (2002)). The capelin management seems to have improved after the collapse in the late eighties.

Although catches of both capelin and cod have been much too high, there is a clear correlation between actual catch and optimal catch. The positive correlation of 0.75 for capelin and 076 for cod is rather large. We typically observe, however, that changes in actual catch are a little delayed compared with changes in optimal catch. (See Figure 5.5(a) and 5.5(b)). This indicates that multi-species feedback rules can improve the stability of the stocks and reduce the fluctuation.

The sizes of the optimal equilibrium catch curves for capelin and cod in Figure 5.5(a) and 5.5(b) are respectively about 36 % and 34 % of actual average catches in the last thirty years. At the same time, equilibrium level yields stock sizes considerably larger than average stock sizes, especially for capelin and herring. Average stock size of capelin, cod and juvenile herring year 0-2 in the period from 1973 to 2005 is respectively 3.017, 1.522 and 1.084 (million tons), which is considerably lower than the interior equilibrium with the capelin stock of about 2.7 million tons, a cod stock of 3.2 million tons and a herring stock of almost two million tons shown in Figure 5.8. The conclusion is that lower harvest pressure on both capelin and cod would give a much higher cod stock and improve the efficiency of the cod fishery.

Given the growth functions in equation (5.3) and management by the feedback rule suggested, historical stock and catch curves would have been very different. Whereas Figure 5.5(a) and 5.5(b) shows historical optimal catch given the stock biomass assumed from Table 5.4 each year, figure 5.6(a) - 5.6(d) shows historical optimal stocks and catches given by the growth functions in (5.3) and the feedback rule. The discrepancies between historical data and how they could have looked with management from the feedback rule are enormous. The effect of the lower catch rates on both capelin and cod is that the cod stock gets much bigger whereas capelin biomass is kept rather low owing to higher predation pressure from cod. Nevertheless, the optimal biomass of capelin in Figure 5.6(a) is on average more than 10 % higher than the historical biomass. The very low optimal catch of capelin shown in both Figure 5.5(a) and 5.6(b) is a clear sign of multi-species effects. The direct costs in the capelin fishery are not very high, and therefore a single-species cost/utility analysis of the capelin fishery results in rather a high harvest and a low biomass, but when cod and herring are included in the analysis, the benefit of a more conservative capelin management is illumined.

As far as cod is concerned, low optimal catch is not a result of a multi-species effect.

It is rather a question of cost-effectiveness. Large catches give lower cod stocks and lower cod stocks increase the cost of each unit fished. The actual biomass average for the cod stock from 1973 to 2005 is only 49 % of what it could have been according to the growth equations (5.3) and the feedback rule.

5.5   Conclusions

This article is a demonstration of optimal dynamic programming as a useful technique in multi-species management modeling. So far multi-species management has been mostly concerned with stock goals, and less with economics and other important objectives utilizing information outside the biological sphere. We have tried to balance model-complexity of biology and economics in the search for optimal feedback catch-curves for capelin and cod. The aim has been to identify multi-species effects in the optimal feedback. The importance of a multi-species basis in the management of the Barents Sea capelin has certainly been demonstrated. Stocks like capelin, which are more important as prey for other species than as harvest for man, should always be managed in a multi-species context.

As far as the concrete results are concerned, the principles of the findings are intuitive. They show that optimal catch of capelin is very dependent on the size of both the cod and herring stock as well as of the capelin stock itself. Furthermore, optimal catch of cod is less dependent on capelin and herring, but a fully depleted capelin stock will result in less cod growth and should therefore also lead to lower cod landings.

Comparisons between historical catches and long-term optimal catches show that the former have been drawn much too heavily from both capelin and cod, but the management of capelin appears improved after the collapse in the late 1980s. The extent of overharvesting is almost incredible. According to the analysis, the feedback solution would have given a cod stock average that is more than twice the size of the actual average between the years 1973 and 2005.

Perhaps the most interesting observation from a comparison between historical catch curves and the feedback solution is that the feedback solutions appear to forestall future actual catches. Typically the feedback catch curves drop a couple of years before the actual catch curves are forced to drop owing to low biomasses. This is a very strong argument for management that is aware of multi-species effects.

Another important finding is that the presence of some herring in the Barents Sea system apparently gives higher economical yield from capelin and cod than the absence of herring would have done.

Our work should not be considered to be the definitive answer to the best management of the Norwegian and Russian resources of capelin and cod. Rather, it is a

first attempt at combining complicated biology with economics and giving a clear-cut answer to the optimal response to the biological model and the economic models relied upon. In practical management of the fisheries, however, it is rather obvious that the kind of feedback-policy suggested in this work is better for the economy than the pure biological based existing management strategies for capelin and cod. Along with more biological ecosystem based models to test consequences of the feedback solution, a management model of this kind could be interesting to practical politics.

Nomenclature

Variables

$\underline{x}$	Stock levels
$\underline{u}$	Catch levels
δ	Discount rate
$[\alpha_1, \alpha_2]$	Norwegian share of TAC for capelin and cod

Functions

$V(\underline{x})$	The optimal value function
$\Pi(\underline{x}, \underline{u})$	Current profit
$F_i(\underline{x})$	Biological growth function for species i
$CaP(u_1)$	Capelin profit
$CoP(x_2, u_2)$	Cod profit
$P_2(u_2)$	Price for a unit of cod
$C_1(u_1)$	Cost function for catch of capelin
$C_2(x_2, u_2)$	Cost function for catch of cod

Parameters

p_1, p_2 and p_3	Income parameters
c_1, c_2, c_3 and c_4	Cost parameters

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Table 5.1: Coefficients for the growth functions, $C(\cdot)$, are found in the regression analysis results summarized in table 5.2.

	r_1	r_2	r_3	k_1	k_2	k_3
Formula	$r_1 = C(1)$	$r_2 = C(4)$	$r_3 = C(7)$	$k_1 = -\frac{C(1)}{C(2)}$	$k_2 = -\frac{C(4)}{C(5)}$	$k_3 = -\frac{C(7)}{C(8)}$
Value	1.8515	0.5490	1380.7	7890.1	3191.3	$1.0572 \cdot 10^7$

	b_1	b_2	b_3
Formula	$b_1 = -\frac{C(3)}{r_1}$	$b_2 = \frac{C(6)}{C(4)}$	$b_3 = \frac{C(9)}{C(7)}$
Value	$1.1142 \cdot 10^{-4}$	$2.30 \cdot 10^{-6}$	$1.87 \cdot 10^{-6}$

Table 5.2: Regression analysis of relative growth for capelin, cod and herring. The analysis is based on official stock and catch data from ICES.

Estimation Method: Full Information Maximum Likelihood (Marquardt)

Sample: 1973 2004

Included observations: 32

Total system (balanced) observations 96

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.851524	1.080013	1.714354	0.0865
C(2)	-0.000233	0.000210	-1.115061	0.2648
C(3)	-0.000211	0.000208	-1.015541	0.3098
C(4)	0.548985	0.131277	4.181881	0.0000
C(5)	-0.000172	7.51 E-05	-2.289525	0.0220
C(6)	1.26E-06	4.52E-07	2.794831	0.0052
C(7)	1380.662	1186.601	1.163544	0.2446
C(8)	-0.000131	0.000438	-0.298199	0.7656
C(9)	0.002591	0.005169	0.501171	0.6163

Log Likelihood		-74.99661	
Determinant Residual Covariance		0.021789	

Eq.:	$Y_1(t) = C(1) + C(2)\, x_1(t) + C(3)\, (x_2(t) + x_3(t))$		
Obs.: 32			

R-squared	0.244569	Mean dependent var	0.578742
Adjusted R-squared	0.192470	S.D. dependent var	1.192886
S.E. of regression	1.071959	Sum squared resid	33.32379

Eq.:	$Y_2(t) = C(4) + C(5)\, x_2(t) + C(6)\, x_1(t)(1 + \sqrt{x_3(t)})$		
Obs.: 32			

R-squared	0.462158	Mean dependent var	0.36970
Adjusted R-squared	0.425066	S.D. dependent var	0.155888
S.E. of regression	0.118201	Sum squared resid	0.405173

Eq.:	$Y_3(t) = C(7)\, \frac{1}{100 + x_1(t) + x_2(t)} + C(8)\, x_3(t) + C(9)\, \frac{x_1(t)}{1 + x_3(t)} + \frac{50}{x_3(t)}$		
Obs.: 32			

R-squared	0.849223	Mean dependent var	1.004244
Adjusted R-squared	0.838825	S.D. dependent var	3.410217
S.E. of regression	1.369087	Sum squared resid	54.35754

Table 5.3: Economic parameters.

	demand parameters		cost parameters	
capelin	$p_1 = 1$		$c_1 = 0.07,$	$c_2 = 1.4$
cod	$p_2 = 12.65,$	$p_3 = 0.00839$	$c_3 = 5848.1,$	$c_4 = 1.1$

Table 5.4: Offical stock and catch data from ICES. All numbers are in million kg.

Year	Capelin	Cod	herring	Catch of capelin	Catch of cod
1973	5144	2421	166	1336	793
1974	5733	2250	251	1149	1102
1975	7806	2055	196	1439	829
1976	6417	1962	215	2587	867
1977	4796	1971	196	2987	905
1978	4247	1578	185	1916	699
1979	4162	1131	287	1783	441
1980	6715	877	219	1648	380
1981	3895	993	58	1986	399
1982	3779	753	3	1760	364
1983	4230	854	65	2358	290
1984	2964	1031	94	1477	278
1985	860	1179	34	868	308
1986	120	1348	43	123	430
1987	101	1135	77	0	523
1988	428	922	553	0	435
1989	864	933	1420	0	332
1990	5831	1081	1479	0	212
1991	7287	1776	774	929	319
1992	5150	2110	1145	1123	513
1993	796	2462	3159	586	582
1994	200	2240	3179	0	771
1995	193	1909	1331	0	740
1996	503	1795	95	0	732
1997	911	1637	172	1	762
1998	2056	1300	196	1	593
1999	2776	1200	468	105	485
2000	4273	1183	789	410	415
2001	3630	1448	1375	575	426
2002	2210	1649	5803	659	535
2003	533	1695	6370	282	552
2004	628	1700	3404	0	606
2005	324	1634	1970	1	641
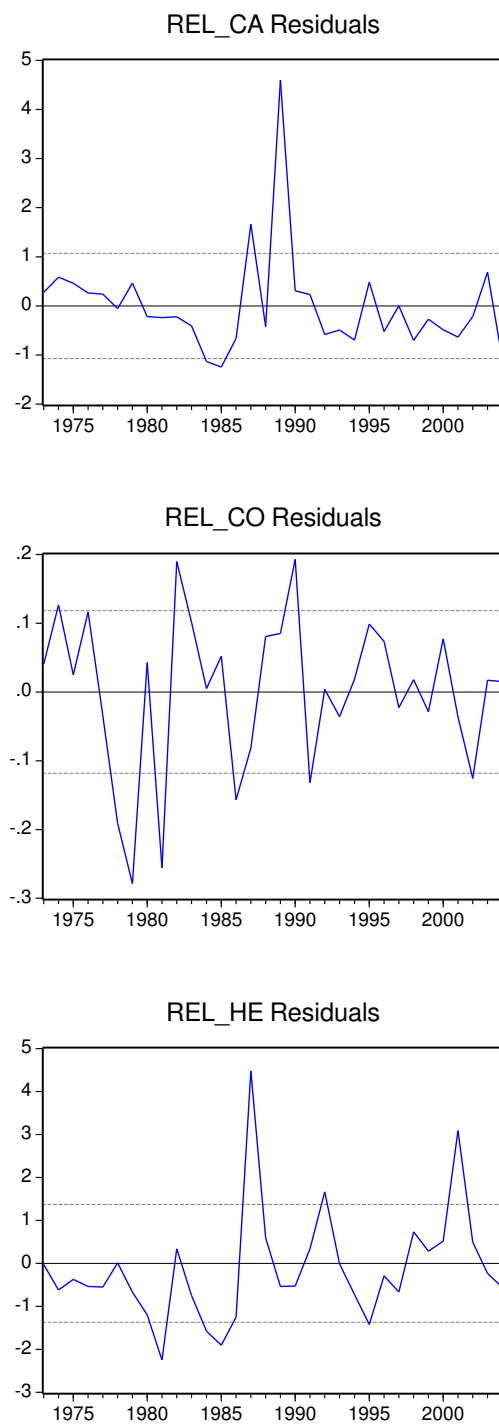
Figure 5.1: The residuals (error terms) of the biological fitting.
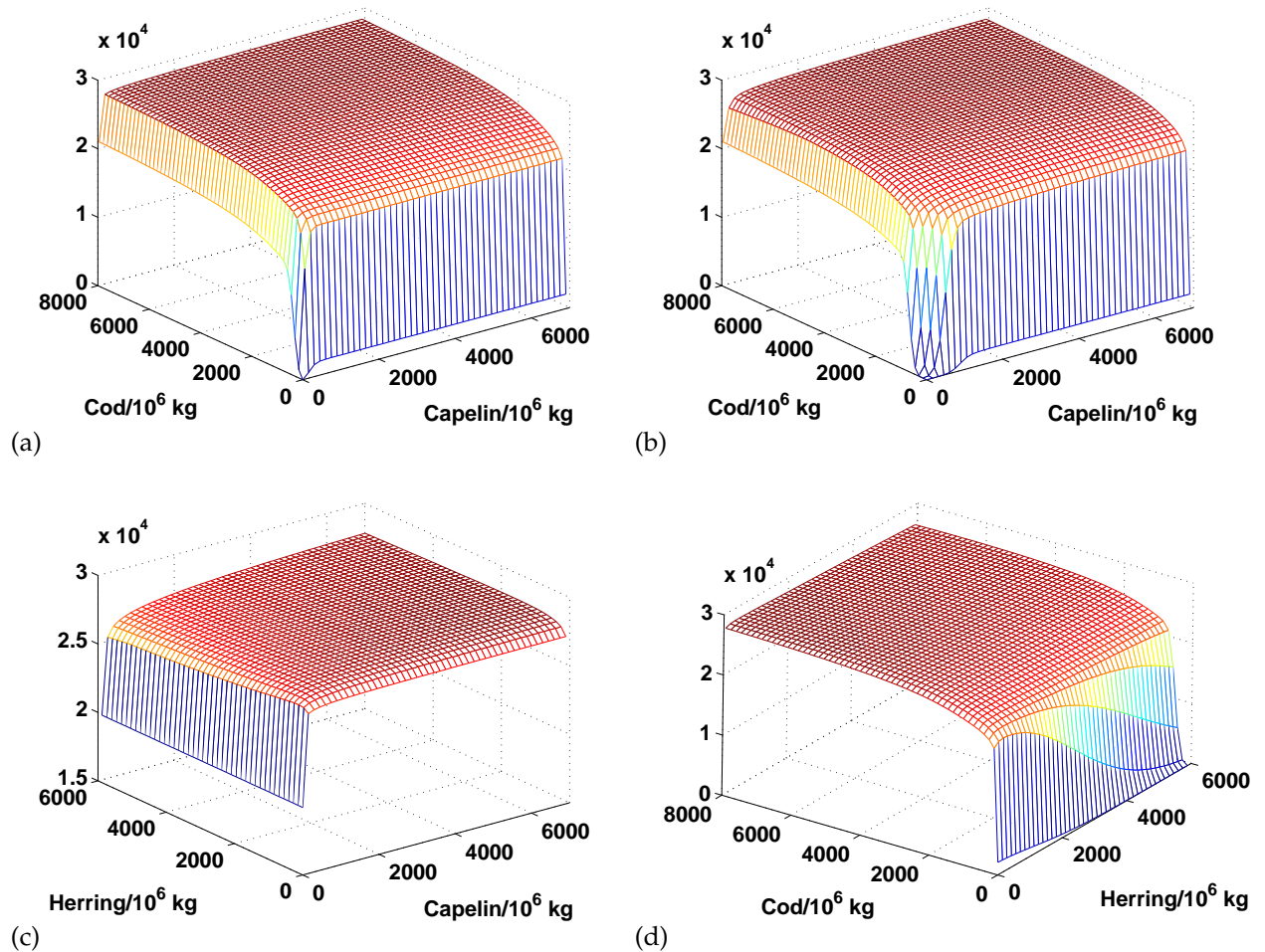
Figure 5.2: The optimal value function (a) with low herring level (280 000 tons), (b) with 4 nillion tons of juvenile herring, (c) with 3.5 million tons of cod and (d) with 1/2 million tons of capelin present.
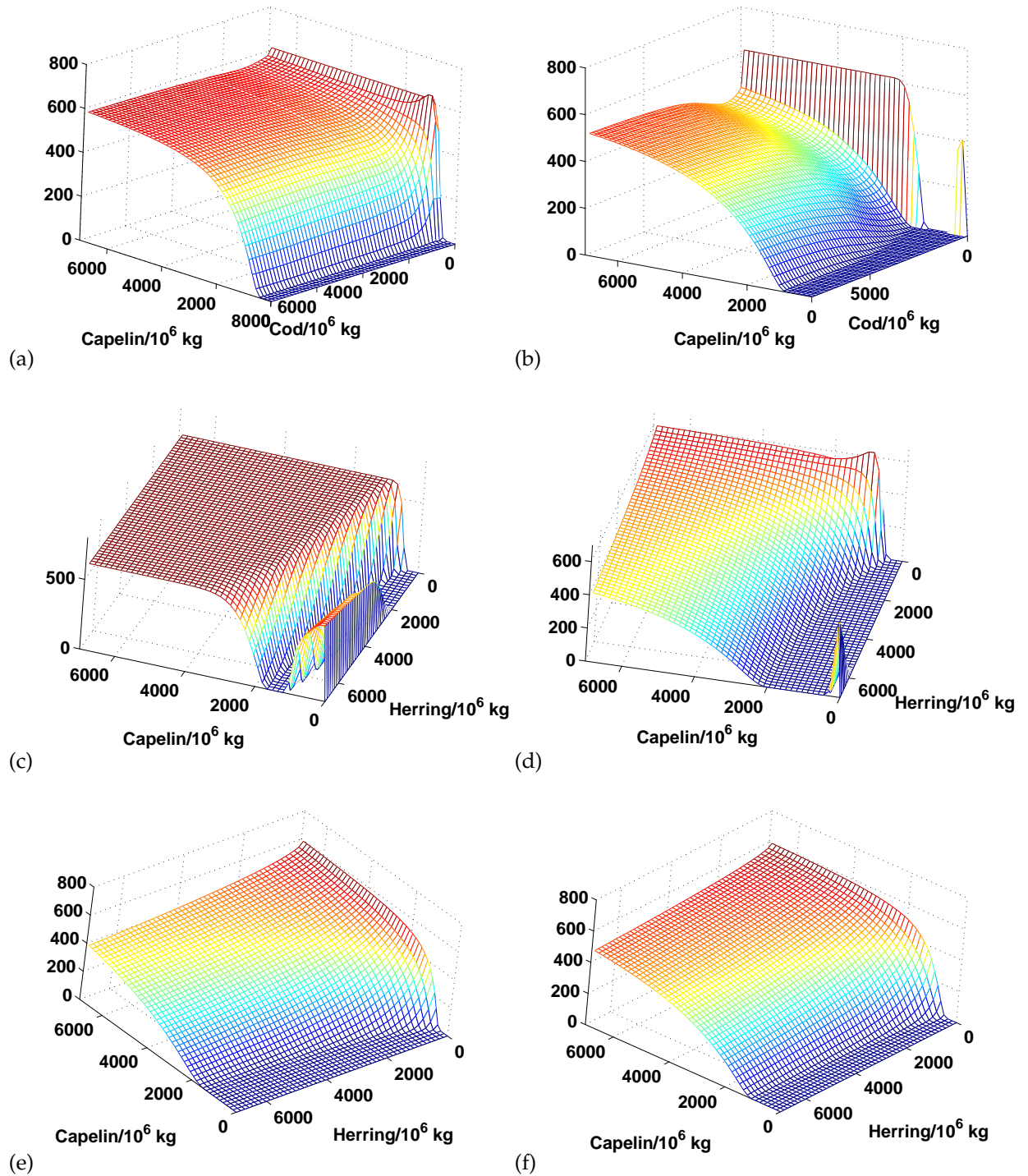
Figure 5.3: Optimal TACs (thousand tons) of capelin. Juvenile herring is fixed in figure (a)-(b) and in figure (c)-(f) cod is fixed. (a) 280 000 tons of herring, (b) 4 million tons of herring, (c) absence of cod, (d) 150 000 tons of cod, (e) 1.5 million tons of cod and (f) 4.5 million tons of cod.
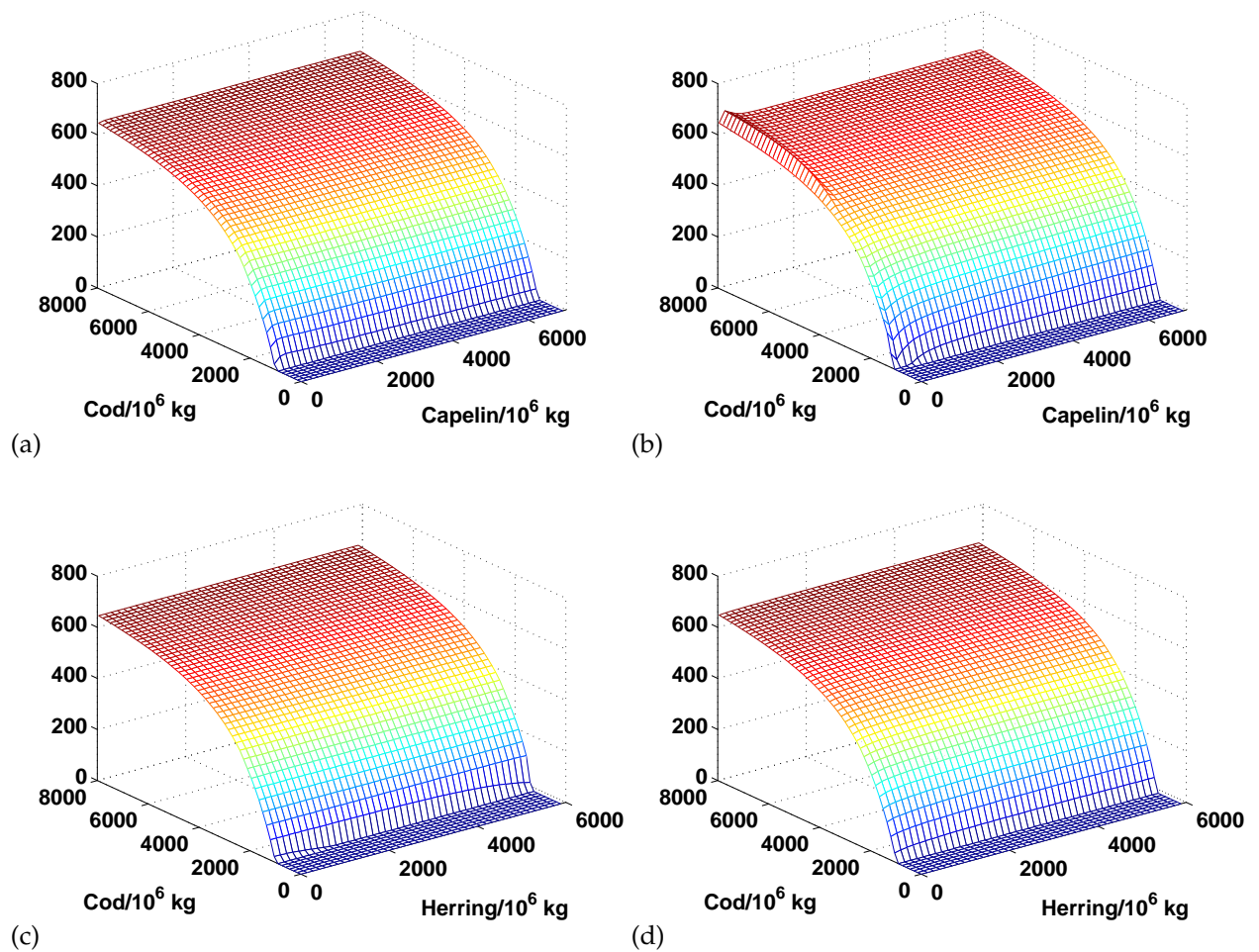
Figure 5.4: Optimal TACs (thousand tons) of cod as a function of capelin and cod with (a) 280 000 tons herring present, with (b) 5 million tons of juvenile herring present, and optimal catch of cod as a function of cod and herring under (c) absence of capelin and with (d) 3 million tons of capelin present.
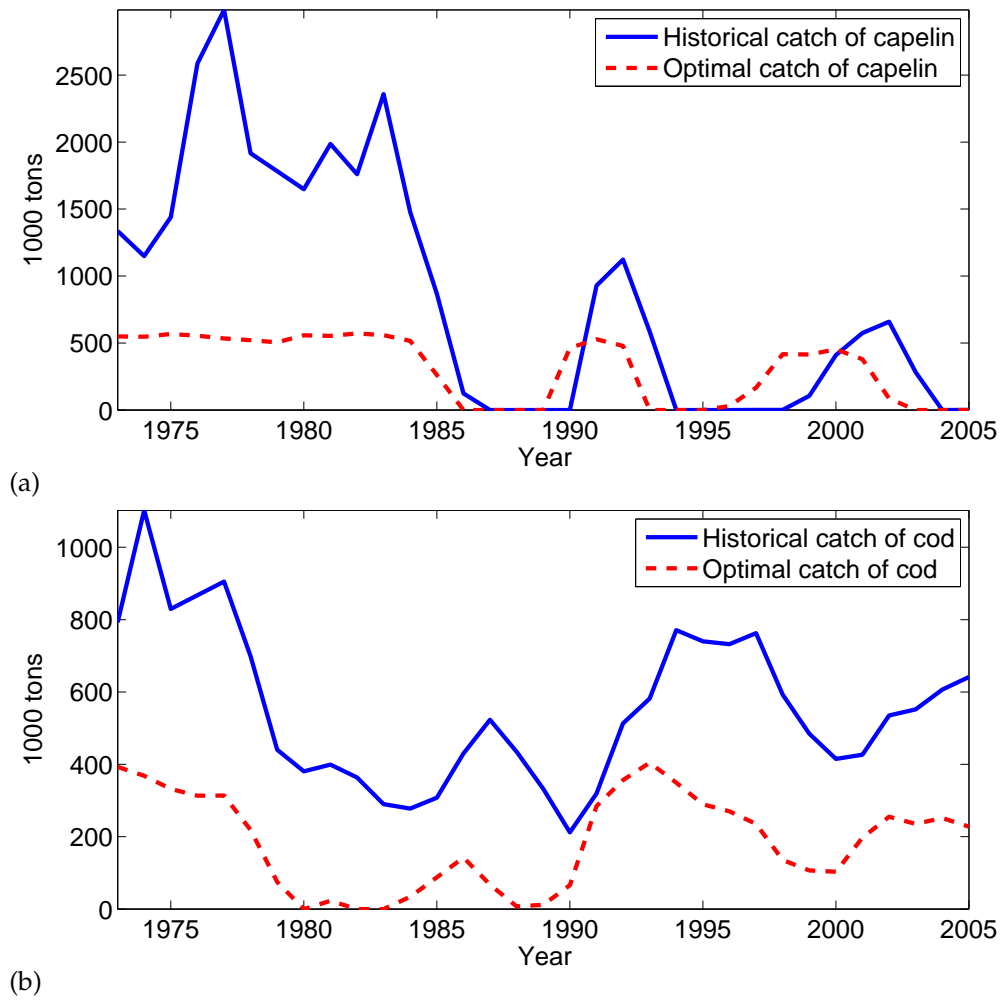
Figure 5.5: Historical catches vs. optimal catches given historical biomasses for (a) Capelin and (b) Cod.
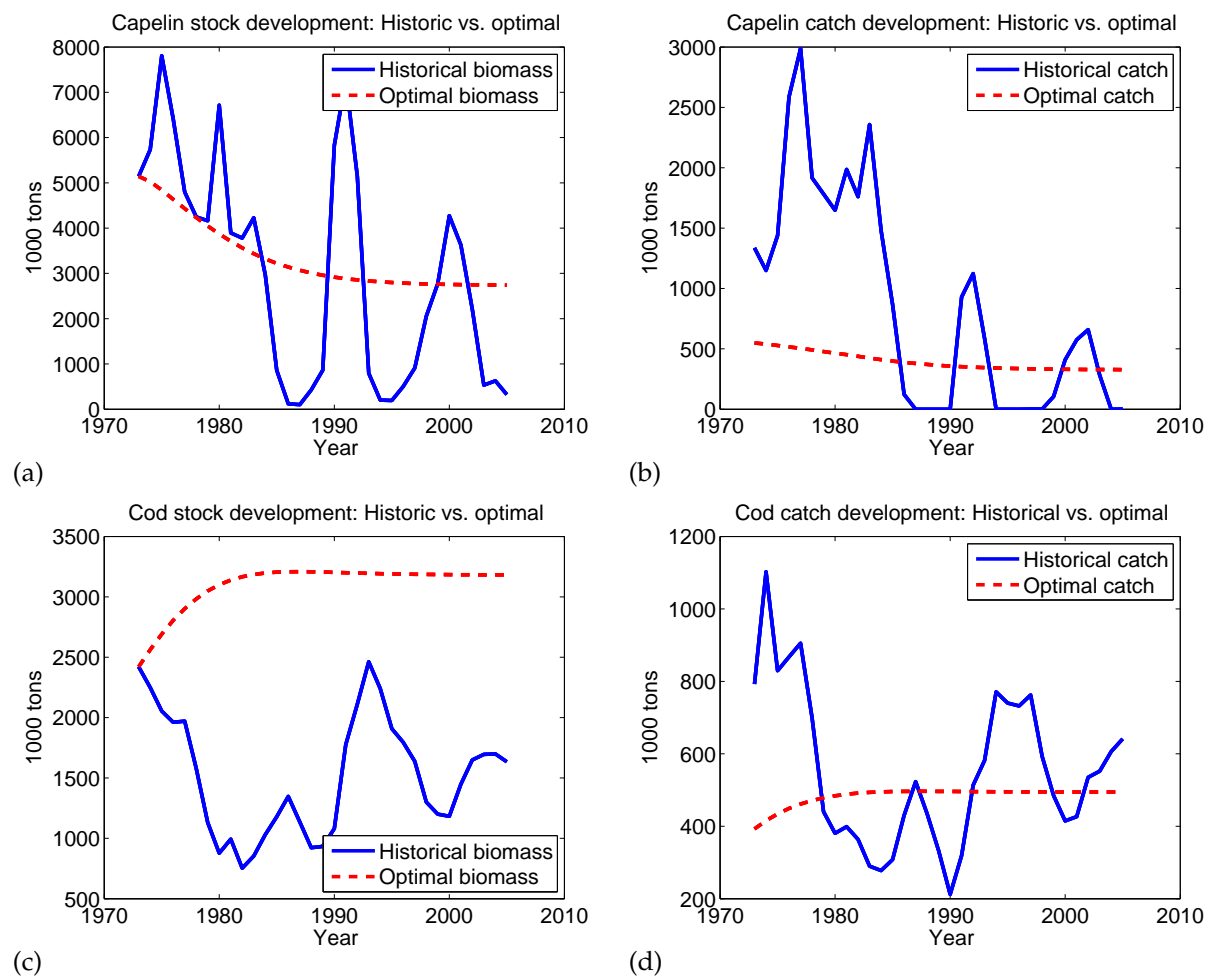
Figure 5.6: Historical biomasses and catches vs. how they could have been with optimal management. (a) Capelin stock, (b) Capelin catch, (c) Cod stock and (d) Cod catch.
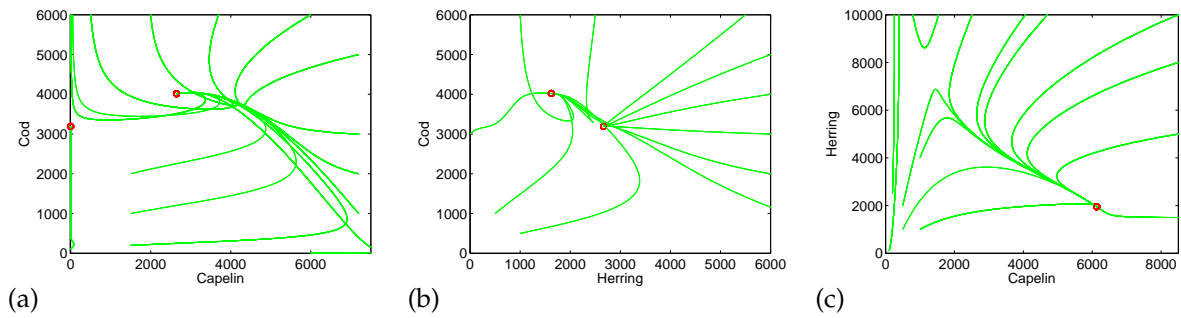
Figure 5.7: Paths towards the biological equilibriums (red rings) resulting from a total closure of the capelin and cod fishery. (a) Initial absence of herring, (b) absence of capelin and (c) absence of cod.
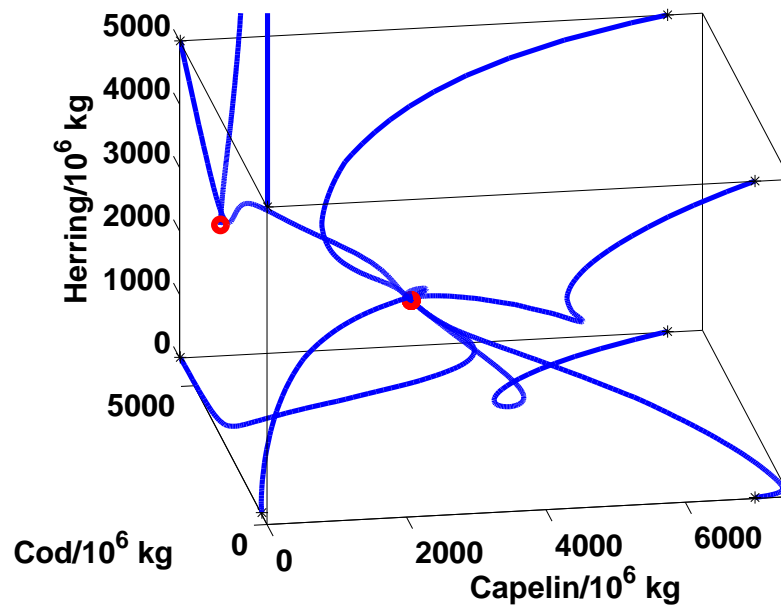
Figure 5.8: Paths towards equilibrium in the 3D capelin-cod-herring plane for different initial stock sizes of capelin, cod and herring. Equilibrium marked with a red ring, and initial positions marked with "∗".
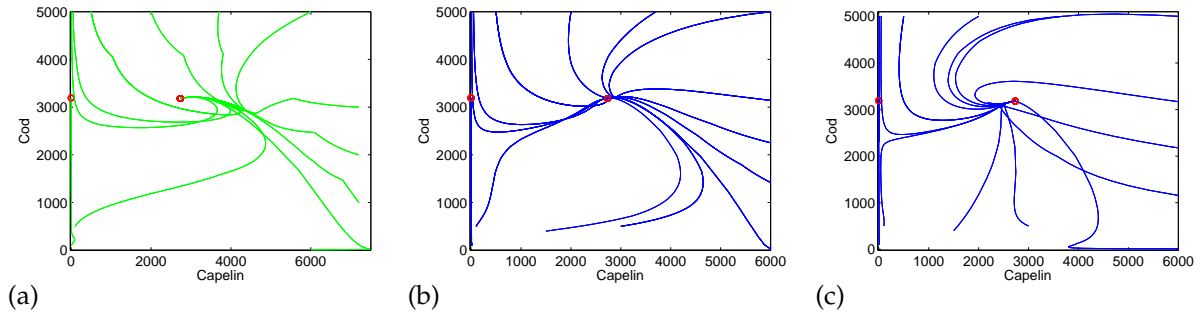
Figure 5.9: Paths towards equilibrium stock sizes on the capelin-cod plane for different initial conditions of capelin, cod and herring. (a) Absence of herring and initial herring stock of (b) 1.5 million tons and (c) 6 million tons.
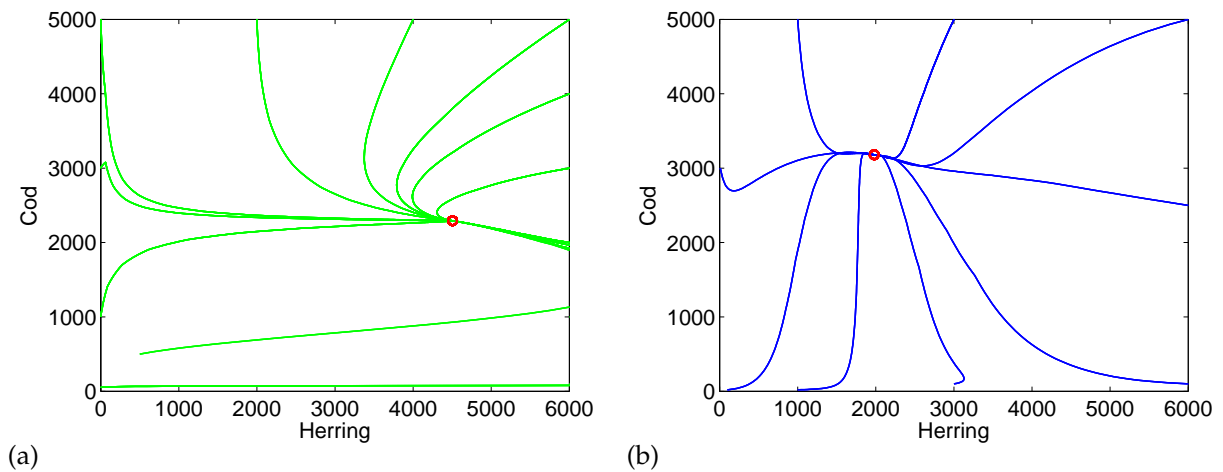

Figure 5.10: Paths towards equilibrium on the herring/cod plane for different initial biomasses of capelin, cod and herring. (a) Absence of capelin and (b) initial capelin stock of 2.5 million tons
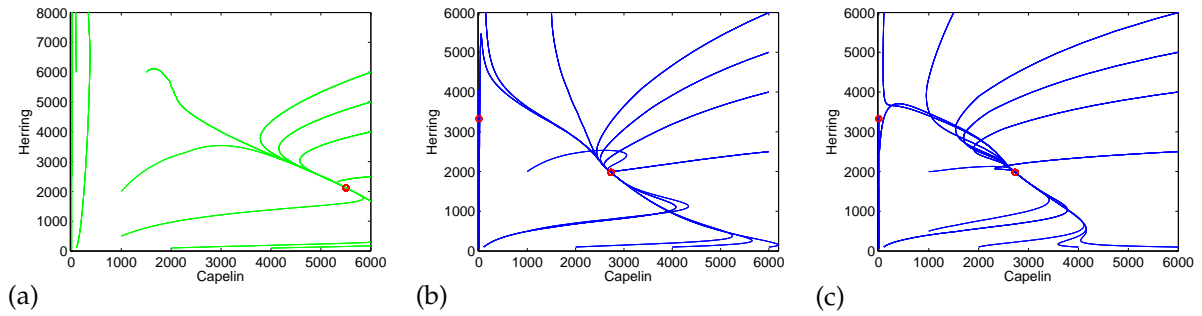
Figure 5.11: Paths towards equilibrium on the capelin/herring plane for different initial biomasses of capelin, cod and herring. (a) Absence of cod, initial cod stock of (b) 1 million tons and (c) 5 million tons.
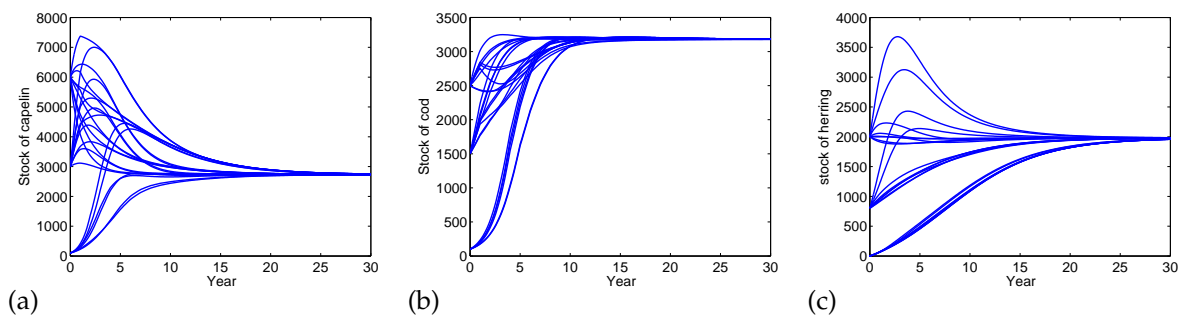

Figure 5.12: Stocks of capelin, herring and cod approaching equilibrium from several different initial conditions.