Mission Performance Trade-offs of Battery-powered sUAS

Anthony Reinier Hovenburg*†, Tor Arne Johansen* and Rune Storvold*‡

*NTNU Centre for Autonomous Marine Operations and System, Dept. of Engineering Cybernetics, Norwegian University of Science and Technology, Trondheim, Norway
†Maritime Robotics A/S, Trondheim, Norway
‡NORUT Northern Research Institute, Tromsoe, Norway

***Abstract*—A sensitivity analysis is presented on the influence of the weight, altitude and speed of battery-powered sUAS on the resulting stall speed, endurance and range. To aid in the determination of the aircraft performance prior to flight, a method is being brought forth that quantifies the impact of these mission parameters. As a case study the P31015 sUAS is used. The P31015 is a concept model of a battery-powered sUAS with a total battery capacity of 977Wh. Since the aerodynamic model of the aircraft was determined through simulations, and the specific propulsion set-up is yet to be determined, the case study remains to be a theoretical approach. The proposed methods and limitations of this study are applicable to other electric sUAS in similar set-up.**

Keywords*—*sUAS, mission performance, sensitivity analysis

I. INTRODUCTION

With the recent technological advancements in small Unmanned Aircraft Systems (sUAS) there has been an increase in the search for suitable applications. Where the commercial development of a manned aircraft is solely reserved to large specialized firms, this is not the case for the development of sUAS. The increasing growth of new sUAS platforms testify to this accessibility to the market. The lower costs and reduced regulatory complexity allow for smaller firms to enter the market and offer tailored solutions to the end-user's specific requirements. With the trend of tailored designs, there is room for a stronger role of the end-user in the design process. In these often multi-disciplinary settings there may be challenges in terms of expectations versus technical possibilities [1]. It is the author's observation that there is often a knowledge gap on the consequences of altering the mission requirements and the resulting consequences on the in-flight performance. This study aims to contribute to the scientific community by offering a clear overview of the trade-offs of the in-flight cruise performance characteristics of a sUAS, and perform a sensitivity analysis on mission-specific flight characteristics. This paper shall demonstrate its proposed theory through analysis of the P31015 sUAS (Fig. 1) as case study. However, the proposed theoretical model (and limitations) are applicable to any electric sUAS in similar configuration. The theoretical framework of this article builds upon the work of Traub [2] and Donateo et al. [3] who studied the effects of the Peukert-constant and battery discharge rate on the in-flight performance of sUAS. Currently the P31015 is a conceptual aircraft, with an aerodynamic model that was approximated through simulations using the AVL software package [4]. The P31015 is an electric-powered sUAS in a conventional pusher configuration. The sUAS was specifically designed to offer strong wind penetrating capabilities and low landing speeds. Propulsion for the intended aircraft shall be delivered by one brushless motor with a maximum shaft power (P_s) of 6kW, while the electric power shall be delivered by two six-cell LiPo battery packs with a total capacity of 977Wh.

II. FLIGHT ENVELOPE

In a level and unaccelerated flight at a given altitude, the net force on the aircraft's body equals zero. This requires that the aircraft produces a lift

Fig. 1. Maritime Robotics P31015 Prototype sUAS

force (L) that equals the aircraft's weight (W), and thrust force (T) that equals the experienced aerodynamic drag force (D). For an electric sUAS the weight is considered constant during the length of the mission. For sUAS flying in subsonic, level and unaccelerated conditions the lift and drag forces are a function of the dynamic pressure (q_∞), wing surface (S) and the specific aircraft's known lift and drag coefficients (C_L, C_D) [5]. This results in:

$$L = W = q_\infty S C_L \tag{1}$$

$$D = T = q_\infty S C_D \tag{2}$$

Where:

$$q_\infty = \frac{1}{2}\rho_\infty v_\infty^2 \tag{3}$$

In level and unaccelerated flight the air density (ρ_∞) is incrementally constant. Demonstrated by Eq. 3 the dynamic pressure is therefore solely a function of the free-stream air velocity (v_∞). As described by [6], rearranging Eq. 1 results in the following expression for v_∞:

$$v_\infty = \sqrt{\frac{2}{\rho_\infty}\left(\frac{W}{S}\right)\frac{1}{C_L}} \tag{4}$$

A. Available power

Eq. 2 expresses that for level and unaccelerated flight the thrust force must equal the drag force that is experienced by the aircraft. As the efficiency of the propeller depends on airspeed, the resulting thrust force is a velocity-dependent variable. The measurement for the propulsion is therefore referred to in power (P) rather than force [7][8][9]. Multiplying the thrust force with airspeed results in the following expression for the available power (P_a):

$$P_a = \eta_p \, T \, v_\infty = \eta_p P_s \tag{5}$$

Today sUAS primarily utilize a fixed-pitch propeller. For the remainder of this study the assumption is made that for each situation an optimal propeller is installed to offer an invariant efficiency. Due to a lack of data the total efficiency of the complete propulsion system (η_p) is assumed to have a constant value of η_p = 0.50. This value lies within the range of the typical propulsion efficiency of a small sUAS, as described in [10].

B. Required power

To be able to compare the required power with the available power, one must also transform the required thrust into the required power (P_r). This is done by multiplying the required thrust with the velocity component, as expressed in Eq. 6. The required power for level and unaccelerated flight is determined by substituting Eq. 4 into Eq. 1 and 2. As proposed by [11] this results in the following expression for P_r:

$$P_r = D \, v_\infty = \sqrt{\frac{2\,W^3}{\rho_\infty \, S}\frac{C_D^2}{C_L^3}} \tag{6}$$

For level and unaccelerated flight a lift force is required that equals the aircraft's weight. Eq. 6 shows that for one specific aircraft design the drag, and consequently the required power, are solely a function of airspeed, as C_L,C_D are speed dependent variables, and the weight, air density and wing surface are constant parameters. When plotting P_r against v_∞, one illustrates what is known as the aircraft's power curve. This curve describes the required power at different airspeeds. With a total aircraft mass of 17.5 kilograms, or weight W of 171.7 Newtons, a wing surface of 0.81m^2 and flying at an altitude of 0m under International Standard Atmospheric (ISA) conditions (ρ_∞ = 1.225kg/m^3), the resulting power curve of the P31015 is shown in Figure 2.

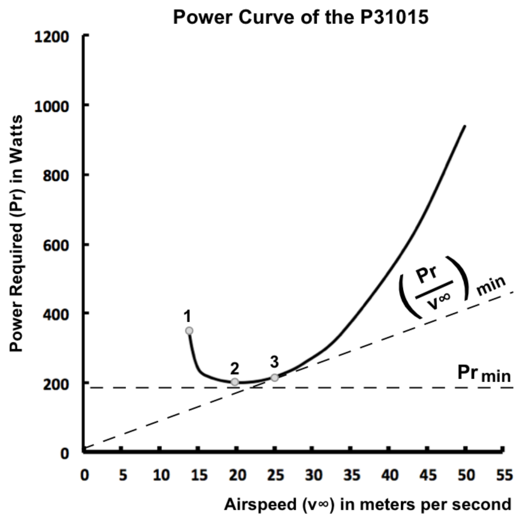

Fig. 2. Power Curve of the P31015 - where $\rho_\infty = 1.225 kg/m^3$

C. Minimum airspeed

As the aircraft slows down, it can only maintain altitude by exchanging the lower airspeed for a higher C_L. However, the C_L of one aircraft is limited to a maximum, $C_{L_{max}}$, after which the aircraft will enter a stall. For most aircraft the practical minimum airspeed is naturally limited to be at the stall speed (v_{stall}) [9]. As described by [6] the expression for v_{stall} in level and unaccelerated flight can be obtained by rearranging Eq. 1 and Eq. 3 into:

$$v_{stall} = \sqrt{\frac{2}{\rho_\infty} \left(\frac{W}{S}\right) \frac{1}{C_{L_{max}}}} \tag{7}$$

Under the conditions described in the previous section the P31015 has a resulting stall steed of 13.8 meters per second. In Fig. 2 the stall speed and corresponding P_r are indicated as point (1).

D. Maximum endurance

Point (2) in Fig. 2 corresponds to the minimum amount of power required ($P_{r_{min}}$) for sustained horizontal flight. When flying at this speed, the electric power consumption per time unit is minimal. Thus the aircraft can stay airborne the longest on one battery load. This point is defined as the aircraft's maximum endurance [8]. This airspeed is relevant for planning long endurance missions. An example of such a mission could be the surveillance of a static object.

Relating to Eq. 6 the required power for flying at the speed that offers the maximum endurance is influenced by the aircraft weight, air density, surface, and lift and drag characteristics. During cruise flight the air density and wing surface are considered constant. In addition battery-powered aircraft have a constant total weight during flight [12]. Therefore the remaining variables are the aircraft's lift and drag characteristics. By flying at the aircraft's minimum ratio between C_D^2 and C_L^3, commonly known as $(C_L^3/C_D^2)_{max}$, the aircraft shall fly at the airspeed where the maximum endurance is achieved [13]. The corresponding airspeed can be found by substituting the value of C_L into Eq. 4. Under the standard conditions, as described in the section for the required power, the P31015 has a maximum endurance of 2.57hr (2hr 34m), at a speed of 20.0 meters per second.

E. Maximum range

Point (3) in Fig. 2 corresponds to the speed and power consumption for achieving maximum range (R_{max}). In contrast to maximum endurance, which aims to minimize the power consumption, the maximum range aims to maximise the trade-off between power consumption and ground distance covered [8]. This airspeed is relevant for missions that require the sUAS to fly as far as possible on one battery load, such as an A-B mission or an A-B-A mission. As described by [11] the speed for maximum range occurs at $(P_r/v_\infty)_{min}$ and can be found by substitution of Eq. 2 and 4 into 6, resulting in:

$$\left(\frac{P_r}{v_\infty}\right)_{min} = W \left(\frac{C_D}{C_L}\right)_{min} \tag{8}$$

Since the aircraft's weight is considered constant, this resulting expression shows that the maximum range is achieved by flying at the minimum ratio between C_D and C_L, commonly known as $(C_L/C_D)_{max}$. The corresponding airspeed can be found by substituting the value for C_L into Eq. 4. Under the standard conditions, as described in the section for the required power, the P31015 has a maximum range of 214km, at a speed of 25.6 meters per second.

III. Effective battery capacity

A typical (but inaccurate) way to determine the flight time of a battery-powered aircraft is by dividing the specified battery capacity by the current draw. Often it is assumed that a battery with a capacity (C) of 2Ah, while being discharged at a rate of 2A, is expected give a flight time of one hour. Similarly the flight time is often incorrectly assumed to be reduced to half an hour when the battery is discharged at 4A. In contrast to this method, a higher current draw reduces the battery's available capacity [2][3][14]. This behaviour can be assigned to the so-called Peukert-effect. In [2] it is proposed that when accounting for the Peukert-effect the discharge time (t) is described by:

$$t = \frac{R_t}{i^n} \left(\frac{C}{R_t} \right)^n \tag{9}$$

Where R_t is the battery hour rating in hours, and i the battery discharge current in Amperes. Here n is the Peukert-constant, which is a discharge parameter that depends on the battery type, and battery-specific factors, such as temperature, age and cycles runned [15]. Proposed by [2] the total battery output power (P_B) is then expressed by:

$$P_B = V \frac{C}{R_t} \left(\frac{R_t}{t} \right)^n \tag{10}$$

Where V is the battery voltage. By considering the total battery capacity to be invariant, and instead modelling the effective power consumption to be increased, the range and endurance can be determined by solving Eq. 10 for different airspeeds [2][3]. Consequently, the aircraft's maximum endurance (E_{max}, in hours) and maximum range (R_{max}, in kilometers) can be determined through:

$$E_{max} = \left(\frac{V \times C\, \eta_p}{\sqrt{\frac{2W^3}{\rho_\infty S} \left(\frac{C_D^2}{C_L^3} \right)_{min}}} \right)^n R_t^{1-n} \tag{11}$$

$$R_{max} = \left(\frac{V \times C\, \eta_p}{W \left(\frac{C_D}{C_L} \right)_{min}} \right)^n \left(\sqrt{\frac{2W}{\rho_\infty\, S\, C_L}} \right)^{1-n} \cdot {R_t}^{1-n} \cdot 3.6 \tag{12}$$

The P31015 shall be equipped with a LiPo battery pack with a capacity of 977Ah, of which the specific model is yet to be determined. Due to a lack of data the Peukert-constant n of the battery is therefore assumed to have a value of 1.05, corresponding to the typical value for a lithium-polymer battery pack found in [3]. This observation corresponds to a 2012 study by Omar [16] where it was found that the Peukert-constant for Lithium-ion based batteries typically vary between 1 and 1.09. The value R_t is the discharge time over which the capacity was determined. It is assumed that the capacity of the battery used in this study was determined over one hour, thus giving a constant value of R_t of 1.0. Similarly, for this study the battery voltage during discharge is assumed to be invariable, as these effects on the mission performance are usually limited, as found in [2].

IV. Mission Parameters

This paper investigates the mission performance characteristics (stall speed, endurance and range) during the cruise phase of a mission. The typical parameters that a user often changes prior to a mission are the cargo capacity, flight altitude and airspeed. This paper aims to give a better insight in the impact of the change in these parameters on the mission performance of the aircraft through a sensitivity analysis. There are several interdependent relationships between these performance characteristics. The effects on the mission performance shall determined by analysing the shift of the power curve as the mission parameters change. These effects shall be discussed individually in the following sections.

A. *Effects of changing Weight*

Although a battery-powered sUAS typically has a fixed airframe weight, the choice of cargo can cause a change in total aircraft weight. To determine how the P_r versus v_∞ curve shifts, it is assumed that with changing total weight the altitude and aircraft configuration remain constant. Deriving from Eq. 4 and 6 the expression for the power curve can be reduced to:

$$P_r = constant_1 \sqrt{W^3} \tag{13}$$

$$v_\infty = constant_2 \sqrt{W} \tag{14}$$

As described by [11], with increasing weight the required power increases with $\sqrt{W^3}$, while the airspeed increases with $\sqrt{W}$. However, when also including the Peukert effect, as determined through by Eq. 10, the power curve shifts as illustrated in Fig 3. This figure shows the power curve at a total weight increase of 50%. Also this figure illustrates the aircraft's performance sensitivity to changes in weight on v_{stall}, E_{max} and R_{max}. In addition Table I lists the sensitivity to weight by showing the corresponding performance parameters to a fraction of the original total aircraft mass of 17.5kg (W_f in %).

TABLE I. RESULTING PERFORMANCE AT VARYING WEIGHT (W_f AS FRACTION OF 17.5KG)

W_f	v_{stall} (ms^{-1})	E_{max} (hr)	R_{max} (km)
80%	12.3	3.7	272.0
100%	13.8	2.57	214.0
120%	15.1	1.9	175.9
140%	16.3	1.51	149.1

Through Eq. 11 and 12 an expression can be given for the sensitivity of the flight performance to the weight by:

- The stall speed is influenced by a factor of $\sqrt{W}$. Note that the aircraft's stall speed is not influenced by the Peukert effect. Thus the new stall speed can no longer be read directly from this resulting power curve.
- The maximum endurance is influenced by a factor of $W^{\frac{-3n}{2}}$
- The maximum range is influenced by a factor of $W^{\frac{1-3n}{2}}$.

An important remark is that the increase in weight is considered to be due to increased cargo

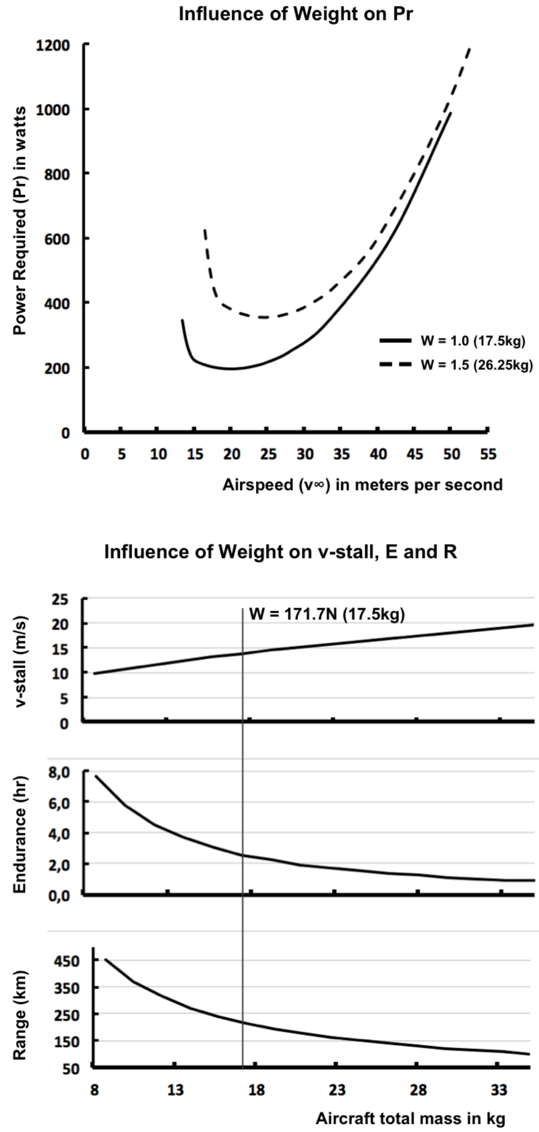

Fig. 3. Influence of Weight on P_r, $v_s tall$, E and R (n=1.05, $\rho = 1.225$)

weight. Alternatively the weight increase can be caused by an additional battery. In [2] and [17] a study was performed on the effects of the battery weight fraction on the in-flight performance in. However this study shall continue to focus solely on the influence of increased cargo load.

B. Effects of changing Altitude

The influence of altitude on the performance parameters are evaluated similarly to that of the weight. To determine the shift of the power curve with increased altitude, it is assumed that the aircraft weight and configuration remain constant. As described by [11] Eq. 4 and 6 can be reduced to:

$$P_r = constant_3 \sqrt{\frac{1}{\rho}} \tag{15}$$

$$v_\infty = constant_4 \sqrt{\frac{1}{\rho}} \tag{16}$$

Eq. 15 and 16 demonstrate that with increasing altitude both the required power and the airspeed shall increase by $\sqrt{\frac{1}{\rho}}$. Similarly to the weight analysis, when also including the Peukert effect, the power curve shifts as illustrated in Fig 4. This illustration shows the shift of the power curve to an altitude of 3.0km, corresponding to ρ_∞ = 0.909 under ISA conditions.

In aircraft designs where the effects of ρ_∞ on C_L and C_D are small, or where the change of altitude is relatively small, the effects of altitude on the flight performance can be approximated through:

- The stall speed is influenced approximately by a factor of $\sqrt{\frac{1}{\rho}}$, as previously described by [11]. Note that similarly to an increase in weight, the aircraft's stall speed is not influenced by the Peukert effect.
- The maximum endurance is influenced approximately by a factor of $\rho^{\frac{n}{2}}$
- The maximum range is influenced approximately by a factor of $\rho^{\frac{n-1}{2}}$, as previously described by [2]

Fig. 4 shows the approximated values for the aircraft's mission performance as a result to change in pressure altitude. In addition Table II lists this sensitivity by listing the approximated performance parameters corresponding to a change in air density (ρ_∞) from zero to three kilometers. For the determination of the air density the ISA model was applied consistently.

TABLE II. RESULTING PERFORMANCE AT VARYING ALTITUDES (ISA)

h in km	v_{stall} (ms^{-1})	E_{max} (hr)	R_{max} (km)
$0km$	13.8	2.6	214.0
$1km$	14.5	2.4	213.5
$2km$	15.2	2.3	213.0
$3km$	16.0	2.20	212.4

Note also that, as mentioned before, the theoretical model presented in this paper assumes an invariant propulsion efficiency. As described in [15] the temperature of the battery influences its available capacity. As the temperature may drop with increasing altitude this could ultimately influence the range and endurance of the aircraft as the battery cools down.

C. Effects of changing Airspeed

By deviating from the speed $v_{E_{max}}$ or $v_{R_{max}}$, the aircraft will no longer follow the optimal cruise speeds for respectively maximum endurance and maximum range. The aircraft's range becomes solely a function of the achievable endurance, multiplied by the corresponding airspeed. The aircraft's stall speed remains unchanged. For level and horizontal flight the required value of C_L changes in relation to a change in speed. Through the aerodynamic model of the aircraft the required airspeed for sustained level flight can be determined through Eq. 4. Then, through Eq. 11 the endurance can be determined for varying airspeeds. Note that this expression shall now produce the value for E instead of E_{max}. This is because the expression no longer utilizes the maximum C_L^3/C_D^2 ratio of the aerodynamic model, but simply the C_L^3 and C_D^2 values that correspond to that specific airspeed.

Fig. 5 shows the aircraft's mission performance sensitivity to changes in airspeed expressed in E and R. Table III shows the new performance parameters corresponding to a change in airspeed ranging van 20m/s to 40m/s kilometers.

V. LIMITATIONS AND FUTURE WORK

As this paper presents an analytical case study, the validity of the proposed method is yet to

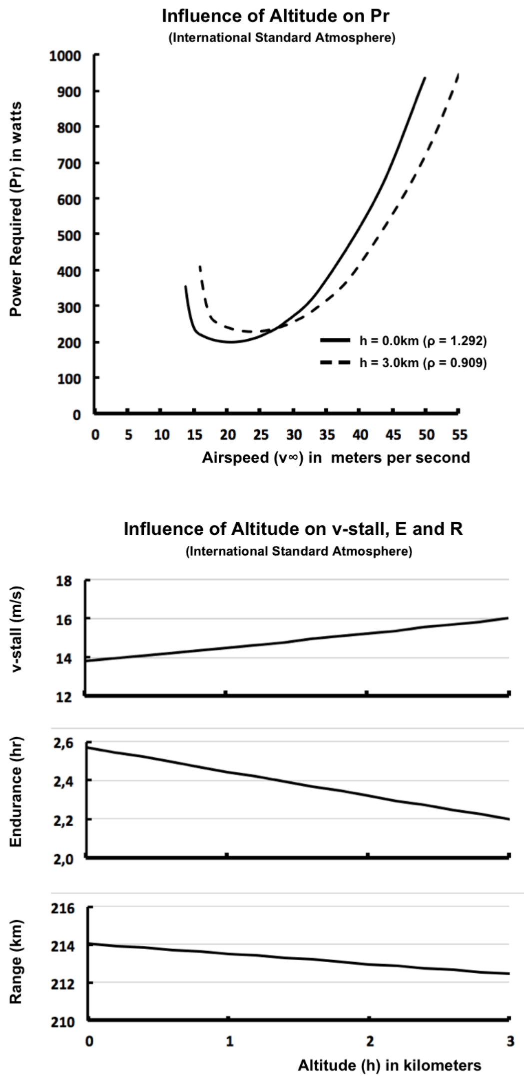

Fig. 4. Influence of Altitude on P_r, v_{stall}, E and R (n=1.05)

TABLE III. RESULTING PERFORMANCE AT VARYING AIRSPEEDS (INTERPOLATED VALUES)

v_∞ in ms^{-1}	Endurance (hr)	Range (km)
20	2.5	187.5
30	1.89	202.7
40	1.02	141.59

be demonstrated through experimental test flights. Since the study was conducted on a conceptual design, it was performed with the assumption of an invariable propulsion efficiency, thus neglecting variances in battery temperature, voltage discharge effects and propeller efficiency. Anyone considering using these presented methods with the purpose of determining the performance of an sUAS, ought to know the aircraft's complete aerodynamic model and propulsion efficiency parameters in order to obtain the correct results.

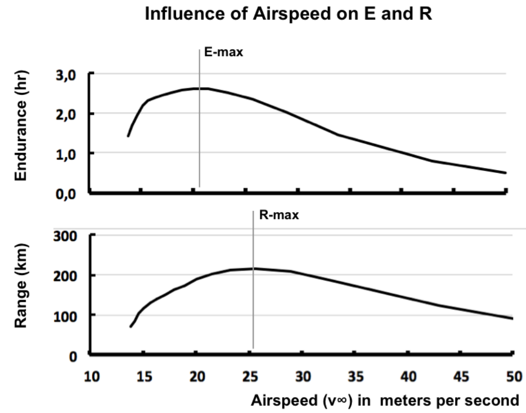

Fig. 5. Influence of Airspeed on E and R (n=1.05)

VI. CONCLUSION

A method has been proposed which quantifies the influence of weight and airspeed on the mission performance of a battery-powered fixed-wing sUAS. In addition a method has been proposed that, with limitations, approximates the influence of a change in altitude on the mission performance parameters of the aircraft.

VII. ACKNOWLEDGEMENT

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