



Artificial Thawing of Frozen Ground: A Review

Huong Phan Thu Ngo¹; Svein-Erik Sveen²; and Eirik Gjerløw³

Abstract: Understanding the freezing and thawing processes in porous media such as soils is important, especially in regions experiencing seasonal frost or permafrost. These processes have a wide range of implications as diverse as how to maintain the structural integrity of roads, railways, pipelines, and buildings, to when to plant seeds during the growth season. Thawing of frozen ground is the opposite process of ground freezing but has not received nearly as much attention as the latter in research studies or field experiments. Accurately predicting thaw depth or thaw rate is a challenging task. Many mathematical models have been proposed to describe the thawing process, with different perspectives and complexity. This paper provides an overview of historical modeling efforts made for predicting heat and mass transfer during thawing. Assumptions and premises for each model are discussed, as well as limitations and some applications. In addition, this paper reviews historical and modern approaches to thawing of frozen ground in cold regions, lists pros and cons of each method, and gives examples of applications. The review shows the need for further research and more accurate models, specifically for predicting thaw depth and thaw rates in frozen ground subjected to artificial thawing. DOI: [10.1061/\(ASCE\)CR.1943-5495.0000280](https://doi.org/10.1061/(ASCE)CR.1943-5495.0000280). This work is made available under the terms of the Creative Commons Attribution 4.0 International license, <https://creativecommons.org/licenses/by/4.0/>.

Author keywords: Artificial thawing; Frozen ground; Heat and mass transfer.

Introduction

Access to efficient methods for thawing of frozen ground is important for people living and working in cold regions. Municipalities, utilities, and contractors need to be able to establish and maintain infrastructure during the cold season. Artificial thawing allows for groundwork and excavations throughout the year, a more even distribution of the workforce, and thus a reduction in the need for seasonal layoffs. In recent years, frozen ground-related issues have gained actuality with the increased focus on oil and gas extraction in the Arctic and concerns about present and future effects of global warming.

Frozen ground is generally divided into seasonally frozen ground and permafrost, where the latter is defined as frozen ground conditions for at least 2 consecutive years (Brown et al. 1998). These exist in both the Arctic and the Antarctic as well as in high altitude areas. Approximately 57% of the exposed land area on the northern hemisphere consists of some form of frozen ground (intermittently and seasonally frozen ground plus permafrost) and close to 26% consists of permafrost regions (Zhang et al. 2003). In cold regions, the ground can be frozen from several months to the whole year. These regions also have other issues associated with the thawing processes such as thaw settlements and thermokarst, and in recent years, there have been many examples of damage to roads and buildings because of thawing permafrost (Hjort et al. 2018; Wang et al. 2020). Owing to global warming, it is expected that thawing of permafrost will continue in the future

(Vaks et al. 2020) and that some permafrost regions may turn into seasonally frozen ground. Furthermore, deteriorating permafrost also increases the rate of global warming owing to methane emissions from melting tundra (Strauss et al. 2017).

In seasonally frozen soils, the ground freezes in winter and thaws naturally during the summer because of solar radiation and an increase in the air temperature. Consequently, construction work is usually carried out during summer and autumn. Artificial thawing of frozen soil during the cold season, using an auxiliary heat source (Sveen and Sorensen 2013), allows for construction work to be conducted all year round. Although various approaches have been tried in the past, more sophisticated thawing methods have been developed in recent years. When comparing the two methods, natural thawing is obviously less expensive, but the many benefits of artificial thawing outweigh such drawbacks.

When considering both natural and artificial thawing, there are many different variables that influence the progress and the soil. One of the most influential parameters is grain size distribution, as it influences physical properties such as porosity, permeability, and capillarity, which in turn also affects the capacity of the soil to hold water and ultimately also the thaw rate. Experiments by Sveen et al. (2020) demonstrate that frost-susceptible soils, such as silty sand with more fine material (clay and silt) and higher water content, require more time to thaw compared with coarser soils, such as gravelly sand, under similar conditions.

Along with the development of methods for artificial thawing of frozen ground, researchers have attempted mathematical modeling of heat and fluid flow occurring during the process. To optimize artificial thawing, it is necessary to take into consideration the heat and mass transfer occurring in the soil pore structure. The pore fluid flow depends on several physical parameters such as enthalpy, volume of water, volume of ice, freezable (unbound) water, permeability, and temperature gradient. The diffusion model is often used to describe the mass transfer occurring during phase changes in initially frozen ground (Harlan 1973). However, deviations between analytical solutions and experiments suggest that the classic diffusion model does not accurately describe the actual flow motion during artificial thawing of frozen soil (Newman 1995). Obviously, the actual fluid flow is a complex transport process,

¹Dept. of Building, Energy and Material Technology, UiT-Arctic Univ. of Norway, Narvik 8514, Norway (corresponding author). ORCID: <https://orcid.org/0000-0002-1566-9802>. Email: huong.p.ngo@uit.no

²Dept. of Building, Energy and Material Technology, UiT-Arctic Univ. of Norway, Narvik 8514, Norway.

³Dept. of Building, Energy and Material Technology, UiT-Arctic Univ. of Norway, Narvik 8514, Norway. ORCID: <https://orcid.org/0000-0002-2404-1996>

Note. This manuscript was submitted on June 13, 2021; approved on April 11, 2022; published online on July 27, 2022. Discussion period open until December 27, 2022; separate discussions must be submitted for individual papers. This paper is part of the *Journal of Cold Regions Engineering*, © ASCE, ISSN 0887-381X.

where physical properties of different soils have a strong influence. Therefore, numerous models with various degrees of complexity have been proposed in recent decades.

The first part of this paper gives an overview of artificial thawing methods that have been tried since the early 1900s by considering their configuration, operation principle, and pros and cons. Comparison and suitable application scopes of the methods are also discussed, together with tools for measuring frost and thaw depths. Then, mathematical models for heat and mass transfer that have been proposed in various studies are reviewed, ranging from simple to complex models. Finally, conclusions are drawn, and further research is suggested.

Methodology

Thawing of frozen ground has attracted the attention of scientists for decades, and studies on this topic span from calculation methods, process optimization, and physical transient mechanisms, to in situ measurement methods. In this paper, we aim to summarize and synthesize the main findings of studies related to the thawing of frozen soils. To provide a comprehensive overview of artificial thawing of frozen ground, the methodology for this review includes determining targeted research questions, identifying eligibility criteria, figuring out appropriate boundaries for the review, and selecting what data to extract from the literature (Snyder 2019). Owing to the rather limited number of studies relating to thawing of frozen soils, no time frame was applied in the literature searches. Literature searches were conducted using Scopus, Web of Science, and library databases, including textbooks, peer-reviewed journal articles, conference papers, reports, and dissertations mainly written in English but also a few in Norwegian. The strategy for literature searches has been to apply search terms such as “artificial thawing,” “thawing soils,” and “thawing frozen ground,” and then collecting articles that have abstracts, titles, or keywords related to the selected search terms. They were then screened to and categorized into: (1) practical applications of artificial thawing of frozen soils; (2) optimization of thawing frozen soils; (3) heat and mass transfer in thawing soils; and (4) models relating to heat and mass transfer in thawing soils.

This following section presents a review of artificial thawing of frozen ground, starting with a description of artificial thawing and methods applied in this field. The next section outlines central mathematical models describing the thawing process and subsequent parameters. Finally, the results are summarized, and conclusions drawn with suggestions for future work.

Artificial Thawing

Unfrozen soil is a three-phase material consisting of a mixture of solids (mineral particles), liquid (water), and gas (air). In its frozen state, unbound pore water is frozen, thus adding ice as a fourth phase to the mix (Jumikis 1979). The ice phase can manifest in several ways, such as soil particle coatings, small lenses, large inclusions, or massive deposits. When frozen soils start thawing, the ice phase disappears, gradually transforming into liquid water that absorbs latent heat in the process (Andersland and Ladanyi 2004). During the phase transition, the soil skeleton will have to adapt to a new equilibrium void ratio. Migration of water and soil consolidation lead to well-known effects such as reduced bearing capacity (Ryabets and Kirzhner 2003; Han et al. 2014; Hou et al. 2016), and thaw settlements (Wang et al. 1999; Ming et al. 2012; Ozgan et al. 2015).

As opposed to natural (solar) thawing, artificial thawing involves utilizing an auxiliary heat source to initiate and expedite the process.

Historically, various thawing techniques have been tried to facilitate excavation and foundation work in cold regions. During the gold rush to Alaska and northwestern Canada in the late 1800s, prospectors lit fires to thaw gold-rich alluvial sand and gravel deposits (Beistline 1963). With the mechanization of mining operations in the early 1900s, other thawing techniques came into favor. Esch (2004) outlines commonly used methods, referred to in the following section, which can be classified by either the direction of heat distribution (surficial or radial), or by the type of heat source.

Traditional and Modern Thawing Methods

Hot- and cold-water thawing are basically alike, but the former requires access to a heat exchanger or boiler, making it more expensive to use. These methods were commonly used in dredging operations during the 1930s and 1940s in Alaska. Cold-water thawing took place in the warm season and relied on access to large amounts of water from nearby rivers or creeks. Both methods required predrilling vertical boreholes and was thus mainly used for radial thawing (Jumikis 1979), and in some cases also for removing fines by flushing the ground surface.

Steam thawing has the same requirements as hot-water thawing and was mainly used as an open lower-end jet system for radial thawing. Although it is more expensive compared with other methods, it is also efficient and has the added benefit of being applicable all year round. A modified version of this method has been used for the past two decades in Norway to thaw subdrains beneath roads during winter (Reitan 2013).

Various forms of electric thawing have been attempted over the years. Early efforts involved inserting electrical resistance elements into boreholes (radial thawing), imposing an alternating current between the electrodes, and using the soil's resistance to generate a Joule-heating effect. The method was considered most suitable for silt and clay soils (Jumikis 1985), but it was apparently rarely used. Another method, based on electrically heated blankets laid on the soil surface, was experimentally tested during the construction of the Trans-Alaskan Pipeline from 1974 to 1977 (Esch 2004).

Esch (1982, 1984) also refers to additional methods relying on solar radiation as a heat source, although these are normally considered natural or passive thawing techniques. Nonetheless, net energy flow into the ground can be increased by various modifications to the surface, such as stripping away all surface cover and shade-causing vegetation. This approach is termed “active” solar thawing but is a slow and laborious process compared with other methods.

In his doctoral thesis, Sveen (2017) continues Esch's and others' work, focusing on innovative ways of utilizing traditional methods for artificial thawing of seasonally frozen soil. The performance characteristics of modern hydronic-based systems were examined through a series of full-scale thawing experiments, supported by numerical simulations. His study showed that hot-water systems adapted for field-use are both very effective and versatile when used for surficial thawing, achieving thaw rates up to approximately 40 cm in 24 h in gravelly sand. These types of systems have other benefits such as the ability to simultaneously thaw areas up to 400 m², using just a single standalone unit, thus requiring less manpower and supervision compared with traditional methods. Another and more consequential advantage is that the approach permits year-round operations, thus extending the period for excavations and groundwork into the cold season. This is of great importance to municipalities, utilities, and contractors in cold regions, giving access to subsurface infrastructure and allowing for establishing or maintaining building foundations, roads, railways, pipelines, and so on throughout the year.

Determining Thaw Depth

Frozen ground started to become more of an issue in the early 1900s with the advent of the automobile. However, it was not until the 1940s—when new road systems were rapidly constructed to meet a growing increase in number of vehicles—that concerns about frost action and thaw settlements became real issues to be dealt with. At the time there were few studies on thaw duration or how to determine thaw depth in initially frozen soils, which are important prerequisites for frozen ground engineering in general. Over the last five decades, this has changed, where the topic has received more attention from scientists and engineers. As a result, today there are a plethora of methods and instruments available for determining thaw depth in frozen soils (Jumikis 1985; Lindroth et al. 1995; Chen and Horino 1998; Hirsch et al. 2002; Bradford et al. 2005; Harms and Jones 2012; Zeinali et al. 2020). These include manual methods such as soil sampling, temperature measurements, and the use of time or frequency domain reflectometry (TDR or FDR) and ground-penetrating radar to determine moisture and ice content.

Manually determining thaw depth is divided into so-called “direct” and “indirect” methods. An example of a straightforward direct method is using a penetrometer, where a pointed rod is driven into partly thawed soil and the distance it penetrates is measured. This method is suitable for measuring thaw depths of about 0.2 to 0.6 m depending on the type of soil, water content of the thawed zone, ice content of the frozen zone, and the rod thickness (Brenton and Donald 2005; Iwata et al. 2012). Another is using a sampling probe to extract a soil sample and locate the interface between the frozen and the thawed zone. The probe is a handheld tool with a core bit, extension rods, and a handle. The core bit is drilled manually into the soil and can penetrate several meters into the soil with low ice content. Partly mechanized versions such as various power hammers (Cobra, Pico) powered by gasoline engines can be used to ease the drilling process.

A simple and inexpensive method to indirectly determine frost and thaw depths is the use of so-called “frost tubes,” initially introduced back in 1957 by the Swedish geologist R. Gandahl (Gandahl and Bergau 1957). The tube is installed vertically into the soil for manually observing frost and thaw depths. It contains a PVC outer guide and an inner flexible acrylic tube. The inner tube is filled with a solution of methylene blue dye and sealed at the top. The methylene solution changes its color from blue to colorless when frozen. Frost or thaw depth is then determined by pulling the inner tube out of the outer pipe and measure the length of the partition at the top. Other, but considerably more expensive, indirect methods include soil temperature measurements. Soil temperature is often used as an indicator when assessing thaw depth and has been used

for several decades. Typically, thaw depth is determined based on the location of the 0°C isotherm, although this can be misleading in some situations. For example, accurate representation of soil temperature depends on the resolution and accuracy of the monitoring device and sensors used, methodology (e.g., thermocouples versus resistance transducers), and their ability to quickly respond to changes in temperature caused by, for example, solar radiation, varying air temperatures, and precipitation. Moreover, factors such as salinity will lower the thawing point, resulting in the soil thawing at the subzero temperatures (Sveen and Sorensen 2013).

Prediction Models

Our understanding of the mechanisms driving the thawing process, which factors are involved, and to what extent, is still lacking (Rankinen et al. 2004). Obviously, the temperature difference between ambient air and frozen soil is a key factor when studying heat transfer during natural thawing. Moreover, soil type and texture, gradation, porosity, and ice content are examples of factors that affect not only the heat transfer occurring during the process but also the mass transport during phase change. For instance, thaw rates are higher in coarse and sandy soils compared with fine graded soils such as clay. Another factor is presence of organic matter, which influences the bulk density of soil, and thus its thermal conductivity (Oelke et al. 2003). Our understanding of these processes is further complicated by the introduction of an artificial heat source, causing many of the traditional models for predicting the resulting thaw depth to fall short (Sveen 2017).

In contrast to the relatively sparse amount of studies concerning artificial ground thawing, there is an abundance of studies and mathematical models covering frost action and artificial ground freezing, that is, the reverse process (Alzoubi et al. 2017; Blanchard and Fremont 1985; Bronfenbrener 2009; Bronfenbrener and Bronfenbrener 2010; Brown and Payne 1990; Mackay et al. 1992; Naaktgeboren 2006; Nixon 1990; Qi et al. 2020; Rouabhi et al. 2018; Yan et al. 2019; Yokoo et al. 2005; Zhou et al. 2009, 2021). However, many such models cannot be directly applied for accurately predicting thaw depth.

There are several mechanisms at work during thawing of frozen soil, such as phase change, release of latent heat, and changes in thermophysical properties. Fig. 1 depicts the thawing process occurring in an initially frozen soil column [Fig. 1(a)]. The soil temperature close to the surface is controlled by the energy exchange between the soil and air, which in turn govern the heat propagation within the rest of the soil column [Fig. 1(b)] (Sveen 2017). This is a very complex process by itself, and more so since some mechanisms occur

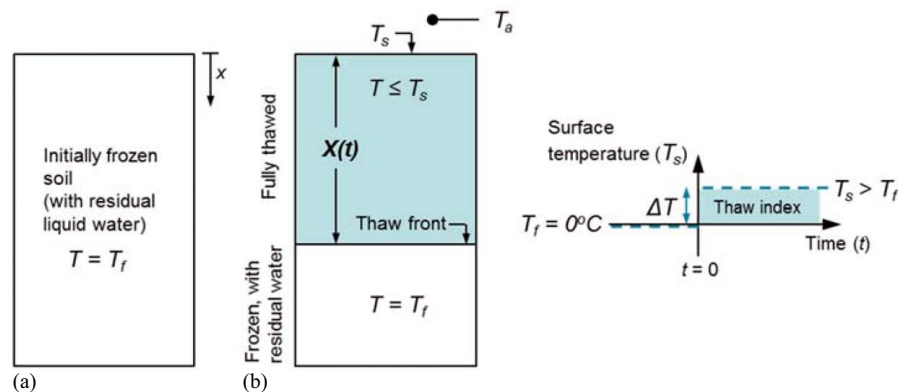


Fig. 1. Newmann problem (moving boundary): (a) initial conditions: frozen, thermally uniform, semi-infinite soil column; and (b) during thawing: exposure to a step-increase in the temperature at the surface results in the thaw front moving downwards.

simultaneously and interact with each other. Mathematical modeling can be applied to determine temperature profiles and estimate the location of the interface between thawed and frozen soil. The next section outlines the development and efforts made in modeling the thawing process in frozen soils.

Analytical Models

The first attempt to solve frost or thaw depth in frozen soil is credited with Stefan in the late 1800s (Jumikis 1966). His model is also the simplest model for predicting the depth of frost or thaw penetration in soil. For the simple case of pure heat transfer in a frozen soil, variations in water content due to heat and mass transfer are considered minor and can be ignored, then the following Fourier heat conduction equation applies (Andersland and Ladanyi 2004):

$$\rho c \frac{T}{t} = \nabla(k \nabla T) + q \quad (1)$$

If the heat conductivity of soil is constant, the Eq. (1) can be expressed as

$$\frac{1}{\alpha} \frac{T}{t} = \frac{\partial^2 T}{\partial x^2} + \frac{q}{k} \quad (2)$$

where k = soil thermal conductivity; t = time; T = temperature of the soil; q = internal heat source; and x = direction of the heat transfer.

$$\alpha = \frac{k}{\rho \cdot c} \quad (3)$$

is the thermal diffusivity of soil; ρ = density of the soil; c = specific capacity of the soil; and k = soil thermal conductivity.

To solve the differential equation of heat conduction [Eq. (2)], it is necessary to determine the uniqueness conditions of the solutions. The solving conditions consists of the initial and boundary conditions. The initial conditions are temperature distribution at the beginning of the process, while the boundary conditions describe heat exchange at the surface of the soil.

For the simplest case, the initial condition for the freezing or thawing process is a uniform temperature distribution throughout the entire volume of the soil represented as

$$T = T_{ini} \text{ for time } t = 0 \quad (4)$$

whereas the boundary conditions comprise three types, as explained in the following text.

The first type is the Dirichlet boundary condition, where it is assumed that the entire boundary ($x = 0$) is subjected to a constant temperature. This can be represented as

$$T(0, t) = T_s \text{ at the location } x = 0 \quad (5)$$

The second type is the Neumann boundary condition, where the heat flux at the boundary is given as

$$\dot{q} = -k \left. \frac{\partial T}{\partial x} \right|_{x=0} \quad (6)$$

The third type is the Fourier boundary condition, based on the heat flow balance at the boundary between convective heat flux and the heat through the boundary, which can be represented a

$$h[T_\infty - T(0, t)] = -k \left. \frac{\partial T}{\partial x} \right|_{x=0} \quad (7)$$

where h = convection heat transfer coefficient.

To solve the problem of locating the interface between the frozen and unfrozen region, the concept of a moving phase transition boundary is applied. This means that the rate in change of latent heat at time t is equal to the heat flux across the interface. The moving boundary of one-dimensional (1D) heat transfer is represented by

$$k_u \left. \frac{\partial T}{\partial x} \right|_X - k_f \left. \frac{\partial T}{\partial x} \right|_X = -\rho L \frac{dX}{dt} \quad (8)$$

where X = distance from the ground surface to the thaw or frost front; k_u and k_f = thermal conductivity of the unfrozen and frozen soils, respectively; and L = latent heat of fusion.

When both the phase transition and latent heat release occur at a given temperature, this is known as the Stefan problem. Exact analytical solutions can be applied to predict the frost or thaw depth of frozen soil as a function of time. Analytical models are available for a few simple cases, and despite the fact that their practicality is limited, they can be used for validating numerical calculations.

Among the analytical models, Stefan's model is a simple, common approach for determining thawing and freezing depths. The Stefan equation was derived for soil by assuming that the latent heat of soil moisture is the only heat that must be removed during freezing or thawing soil, and the volumetric heat used for changing temperature to the freezing point is neglected. With these assumptions, the Stefan equation can be written as

$$L \frac{dX}{dt} = k_f \left(\frac{v_s}{X} \right) \quad (9)$$

where X = distance from the ground surface to the thaw or frost front; k_u and k_f = thermal conductivity of the unfrozen and frozen soils, respectively; L = latent heat of fusion; and v_s = difference between the temperature of the ground surface and the freezing point of the soil moisture.

Integration of Eq. (9) gives an equation for determining thaw or frost depth as

$$X(t) = \sqrt{\frac{2kI(t)}{L\theta\rho}} \quad (10)$$

where θ = volumetric ice content; and $I(t)$ = ground surface thawing or frost index ($^{\circ}\text{C}$), and it is the time integral of the ground surface temperature during thawing or freezing, $I(t) = T_s dt$.

Another common analytical solution was proposed by Kudryavtsev et al. (1977). The maximum annual thaw depth by Kudryavtsev's model is calculated as

$$X = \frac{2(A_s - T_z) \sqrt{\frac{\lambda_T \cdot P_{sn} \cdot C_T}{\pi}} + \frac{(2A_z \cdot C_T \cdot Z_c + L \cdot Z_c) L \sqrt{\frac{\lambda \cdot P_{sn}}{\pi \cdot C_T}}}{2A_z \cdot C_T \cdot Z_c + L \cdot Z + (2A_z \cdot C_T + L) \sqrt{\frac{\lambda \cdot P_{sn}}{\pi \cdot C_T}}}}{2A_z \cdot C_T + L} \quad (11)$$

where the annual temperature amplitude A_z at depth Z is expressed as

$$A_z = \frac{A_s - T_z}{\ln\left(\frac{A_s + \frac{L}{2C_T}}{T_z + \frac{L}{2C_T}}\right)} - \frac{L}{2C_T} \quad (12)$$

$$Z_c = \frac{2(A_s - T_z)\sqrt{\frac{\lambda \cdot P_{sn} \cdot C_T}{\pi}}}{2A_z \cdot C_T + L} \quad (13)$$

and where the mean annual temperature at depth Z can be written as

$$T_z = \frac{0.5T_s(\lambda_F - \lambda_T) + \frac{A_s(\lambda_F - \lambda_T)}{\pi} \left[\frac{T_s}{A_s} \arcsin \frac{T_s}{A_s} + \sqrt{1 - \frac{\pi^2}{A_s^2}} \right]}{\lambda^*} \quad (14)$$

$$\lambda^* = \begin{cases} \lambda_F, & \text{if numerator} < 0 \\ \lambda_T, & \text{if numerator} > 0 \end{cases}$$

where A_s = annual temperature amplitude at the soil surface; T_z = mean annual temperature at depth Z ; λ_F = frozen thermal conductivity; λ_T = thawing thermal conductivity; C_T = thawing volumetric heat capacity; and P_{sn} = annual period.

Empirical and Semiempirical Models

Considering a semi-infinite domain of frozen soil as described in Fig. 1. Let the frozen soil be subjected to a surface temperature of T_s (higher than thawing temperature). The temperature of the soil region near the surface increases its temperature to the thawing temperature and then starts thawing from the surface location to a depth $X(t)$ at time t , assuming that soil properties of frozen and thawed soils are homogeneous and temperature independent. In addition, the latent heat is also assumed to be released at a thawing point of 0°C. Then the thaw depth at time t can be determined as described by Nixon and McRoberts (1973) as

$$X = \alpha\sqrt{t} \quad (15)$$

where X = thaw depth; and α = constant determined as a root of the transcendental equation.

Nixon and McRoberts (1973) proposed a semiempirical equation for determining α that is more accurate and can be shown as

$$\frac{\alpha}{2\sqrt{k_u}} = \sqrt{\frac{Ste}{2} \left(1 - \frac{Ste}{8}\right)} \quad (16)$$

where Ste = Stefan number.

The proposed equation is much simpler than the original equation, and the accuracy compared with the exact solution and Stefan solution. The relationship between the dimensionless thaw parameter and Stefan number of three different solutions for the cases that temperatures are close to the melting point are shown in Fig. 2. As can be seen, the proposed solution almost coincides with the exact solution, hence the proposed solution can be applied to predict α . For the case where the Stefan number is small (less than 0.1), the Stefan equation may be used with negligible error.

Woo (1976) presented in his study semiempirical equation for estimating thaw depth, which is a simplified version of the Stefan

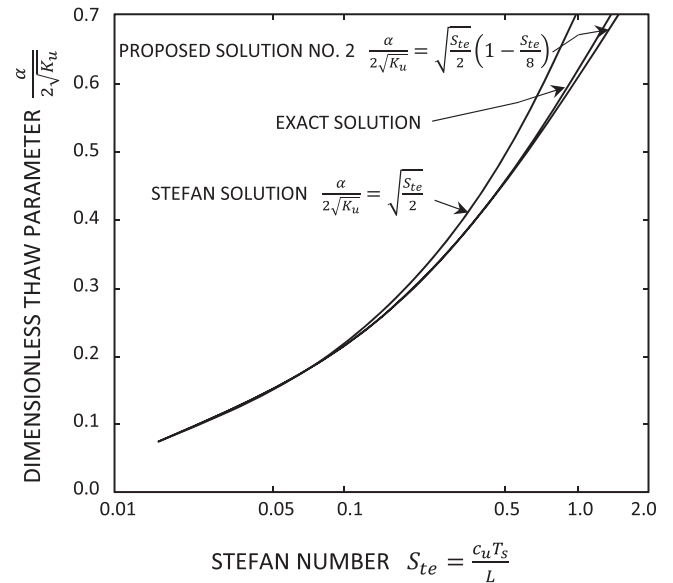


Fig. 2. Solution for 1D thawing when soil temperatures are close to zero. (Reprinted with permission from Nixon and McRoberts 1973.)

equation and be represented as

$$X = \beta\sqrt{t} \quad (17)$$

where β = empirical coefficient that is about 0.07 to 0.15, depending on the soil type and moisture content. Using this equation does not require any soil parameters; however, the empirical coefficient must be calibrated with thaw depths observed in situ. This limits the application of the method. Zhang et al. (2019) utilized in situ data to determine coefficient β for five various types of frozen soil in Canada and concluded that the resulting thaw depths by Eq. (17) correspond reasonably well what was observed in situ.

Numerical Models

Coupled Mass and Heat Transfer Model

Jumikis (1966) and Hoekstra (1966) confirmed that in addition to heat transfer, freezing and thawing in soils depend significantly on mass transfer processes. Harlan (1973) was one of the first researchers who investigated coupled heat and mass transfer in frozen soils. Simultaneous transfer of heat and mass in a partially frozen porous medium consists of two systems of equations demonstrating the interrelationships between the laws of fluid and heat flow; the continuity equations of mass and energy; and the characteristics of the fluids and the medium. These equations cannot be solved analytically, and subsequently, the numerical technique has to be used.

With the assumption that the effect of vapor transfer on water transport is small and negligible, the mass transfer equation of a 1D steady or unsteady flow in a saturated or partially saturated heterogeneous porous medium subject to freezing or thawing can be expressed as

$$\frac{\partial}{\partial x} \left[\rho_l K(x, T, r) \frac{\partial \Phi}{\partial x} \right] = \frac{\partial(\rho_l \theta_l)}{\partial t} + \Delta S \quad (18)$$

where x = position coordinate; t = time; ρ_l = density of liquid fraction; θ_l = volumetric liquid fraction; K = effective hydraulic conductivity; Φ = total head; r = matrix or capillary pressure head; and ΔS = change in ice per unit volume per unit time.

With the assumption that the convection heat transfer associated with the gas phase movement is minor and its effect on heat transfer is insignificant, the 1D steady or nonsteady convection and conduction heat transfer equation becomes

$$\frac{\partial}{\partial x} \left[\lambda(x, T, t) \frac{\partial T}{\partial x} \right] - c_l \rho_l \frac{\partial (v_x T)}{\partial x} = \frac{\partial (\bar{c} \rho T)}{\partial t} \quad (19)$$

where λ = thermal conductivity; c_l = bulk specific heat of water; v_x = fluid flow velocity in x -direction; and $\bar{c} \rho$ = apparent volumetric specific heat.

However, the model proposed by Harlan (1973) was only valid for the experiments he carried out on Yoho Clay soil, a low-porosity soil, but it showed errors when modeling freezing in Del Monte Sand due to the omission of vapor transport. The laboratory data and analytical results did not agree. Therefore, it is difficult to comment on the validity of that assumption. Many researchers have followed this and used Harlan's model, omitting the effect of vapor transfer.

Wilson (1990) studied evaporative flux and developed a coupled heat and mass transfer model for nonfreezing soil. Based on this model, Newman (1995) expanded the transport equation to be able to predict transport in frozen soil during thawing and freezing processes.

In the Newman model, moisture transport in freezing soil is calculated by

$$\frac{\partial \theta_l}{\partial t} + \frac{\rho_i}{\rho_l} \frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial x} \left[k_w \frac{\partial}{\partial x} \left(\frac{\psi}{\rho_l} + x \right) \right] + \frac{1}{\rho_l} \frac{\partial}{\partial x} \left(D_1 \frac{\partial \psi}{\partial x} + D_2 \frac{\partial T}{\partial x} \right) \quad (20)$$

where ψ = soil matric suction; k_w = coefficient of permeability.

The heat transfer equation has been modified to include latent heat released or absorbed during the phase transition process of the liquid phase and the solid phase. The resulting equation is

$$c_l \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) - L_v \frac{\partial}{\partial x} \left(D_1 \frac{\partial \psi}{\partial x} + D_2 \frac{\partial T}{\partial x} \right) + L_l \rho_l \frac{\partial \theta_l}{\partial t} \quad (21)$$

where

$$D_1 = D_v \left(\frac{W}{\rho_w RT} \right) P_v \quad (22)$$

$$D_2 = D_v \left(\frac{h_r \partial P_{vs}}{\partial T} - \frac{P_v \psi W}{\rho_w RT^2} \right) \quad (23)$$

where D_v = coefficient of vapor diffusion in porous soil; h_r = relative humidity in the soil; P_v = partial pressure of water vapor; P_{vs} = saturated water vapor pressure; R = universal gas constant; and W = molecular mass of water vapor.

Newman's proposed theory mentioned previously was verified with a laboratory modeling program. This program was performed by simulating soil freezing using a high water content silica powder in the unfrozen zone. The result confirmed the capabilities of Newman's theory and the numerical model in describing heat and mass transfer in unsaturated frozen soils.

Numerical Solution

Analytical and empirical algorithms are practical and comparatively accurate when the heat transfer parameters are constant and there is no heat generation in the ground. In order to model thawing processes and include the various thermophysical properties of the soil, we need to use numerical methods. Numerical methods that have been used in modeling freezing and thawing processes in soils comprise finite-difference, finite-element, and finite-

volume methods. One of the most important advantages of numerical methods is that freezing and thawing can take place over a range of temperatures, not just at the sharp phase change interface assumed in analytical solutions. This allows for the flow of ground water at subzero temperatures. In addition, numerical approaches can address other complexities; for example, coupled heat and moisture transport, complicated temperature boundary conditions, soil heterogeneity, and time-varying thermal properties (Walvoord and Kurylyk 2016).

In addition to the analytical and empirical models, many numerical models have been studied and enhanced to simulate the thawing process of initially frozen soils. Early on, studies on numerical models of thawing frozen soils were based on the hydraulic method and soil water freezing characteristics. Harlan (1973) is one of the pioneers in modeling research on heat and mass transfer with freezing and thawing. These models assume that the fluid movement mechanism in partially frozen soils is similar to that in unsaturated soils. The simultaneous heat and water model (SHAW) was developed based on Harlan's model and is a robust model for simulating soil freezing and thawing (Hayhoe 1994). Other efforts on numerical models of freezing and thawing soils were made by Giakoumakis (1994). Heat and mass transfer in frozen soils were solved simultaneously in a 1D model that predicts accurately both temperature and total water content profiles during freezing and thawing. In the models mentioned previously, heat and fluid transition in the frozen zone were solved either separately or combined into a single equation and the phase change was usually taken care of using different iterative numerical procedures. Engelmarm and Svensson (1993) presented a novel numerical method for describing the phase change process of freezing and thawing soils. Their method is insensitive to sudden temperature change at the boundary related to the final frost front position and total accumulated moisture content there. In previous studies, vapor transport was usually neglected in numerical models due to its insignificant contribution compared with liquid transfer. To reduce uncertainty due to the phase transition of vapor–water–ice in numerical iterations, Liang et al. (2020) developed a new numerical model based on coupled thermal and hydrological processes. In the new model, the mass transfer process is governed by the vapor flow without considering the liquid water flow. In order to simulate large-scale freeze–thaw experiments, Shoop and Bigl (1997) presented a coupled heat and moisture transfer model for predicting soil moisture variations. The authors modified the flow potential to account for three phases: water, air, and ice. Numerical models are increasingly being developed and diversified with different levels of complexity, but there is a lack of physical models that can be used for validating them.

Summary and Conclusion

Historically, people living and working in cold regions have had to learn to deal with seasonal frost and frozen ground conditions. Some adaptations have been made to sustain life under harsh conditions, while others have developed out of necessity. During the last two decades, frozen ground engineering has developed rapidly. In recent years, the topic has gained actuality with the increased focus on oil and gas extraction in the Arctic and concerns about global warming. Deteriorating permafrost and resulting methane emissions have heightened the attention about frozen ground issues in general.

As shown by, for example, Balossi Restelli et al. (2016), Yuan and Yang (2016), and Rouabhi et al. (2018), artificial ground freezing techniques are well documented and referred to in literature. The same cannot be said about the opposite process, which is the

main focus of this review. Esch (2004) and others outlined traditional approaches to artificial thawing, supporting the impression that there has been little or no development in this particular field of frozen ground engineering since the mechanization of mining operations in the early 1900s.

However, modern systems have been shown to perform well in some circumstances: Reitan (2013) showed that modernized steam-based systems are very efficient for removing ice in subdrains beneath roadbeds during winter. Another is Sveen (2017), who examined the performance characteristics of modern hot water-based systems utilizing flexible pipes distributed across the frozen ground surface. He showed that since its introduction in the United States and Canada in 1996, this approach has also become the preferred method for thawing of frozen ground in northern Europe. Although he pointed out that artificial thawing has become efficient, allows for comparatively larger areas to be thawed, makes it easier to move to adjacent areas for continuous operation, and is less demanding in terms of labor and equipment, more research is still needed in order to accurately predict thaw rates in various types of soils.

The physical process of the frozen and unfrozen soil during thawing are not well understood, therefore the methods for analyzing thawing process of the frozen soil are varied. They can be categorized into analytical, empirical, and numerical models.

The Stefan model was the simplest analytical solution and derived under many assumptions, including no lateral heat transfer, constant moisture content, constant thermal conductivity, neglecting soil volumetric capacity as well as heat advection (Walvoord and Kurylyk 2016). However, a number of studies have been conducted recently relaxing the limitations of the Stefan equation by modifying the equation to accommodate for temporal soil moisture variations (Hayashi et al. 2007), spatially changing moisture content and thermal properties of the soils (Kurylyk 2015), two-dimensional freeze and thaw algorithm (Gao et al. 2016; Woo et al. 2004), advection mechanism (Kurylyk et al. 2014), and the effect of lateral heat transfer (Kurylyk et al. 2016). Because the Stefan equation is simple and flexible, it has been incorporated in many hydrological and land surface models (Yi et al. 2006; Carey and Woo 2005; Li and Koike 2003).

The analytical solution proposed by Kudryavtsev et al. (1977) outperforms the original Stefan equation since it takes into account the soil thawing delay due to soil heat capacity. Numerous studies have been done to validate the Kudryavtsev et al. (1977) model by experimental data from the North Slope of Alaska. The results showed that this model estimates the thaw depth more accurately than the Stefan model (Walvoord and Kurylyk 2016). Nelson et al. (1999) and Romanovsky and Osterkamp (1997) presented an alternative analytical solution for calculating the vertical freeze–thaw depth of soils. Their method considered phase-dependent thermal conductivity of snow and soil. A hydrology model, known as Hydrograph, applied their analytical algorithm to simulate active layers of frozen ground (Lebedeva et al. 2014).

Harlan (1973) turned into one of the pioneers in studying the coupled heat and mass transfer model of frozen soils. He developed the model by assuming that vapor transport has a negligible effect on mass transfer. Many researchers after Harlan followed him and ignored the effect of vapor transfer in their models. However, several studies showed that vapor transport has a significant effect on heat and mass transfer mechanisms and should not be ignored. Philip and de Vries (1957) proposed a heat and mass transfer equation for frozen soils that took into account the effect of vapor transport.

Semiempirical and empirical models are time-consuming and error-prone due to undertaking such as inaccurate chart reading, and they prevent calculating automatically by computer software

(Martin 2007). Moreover, these models require in situ test data for input parameters. Numerous analytical models have been developed for describing the thawing process in frozen soil, and most of them neglect convection. Convection takes place by both the movement of molecules (diffusion) and heat transfer through bulk motion of fluid flow (advection). Most of the models used to predict heat and mass transfer in frozen ground seem to neglect the effects of convection. Harlan (1973) and Luthin and Guymon (1974) added a convection term in their models, while Nixon (1975) and Taylor and Luthin (1978) showed that convective heat transfer was two to three orders of magnitude lower than conduction, and was therefore omitted. Jame and Norum (1980) applied the Harlan (1973) model without the convective term to model freezing of a fine silica powder within 72 h and obtained pretty reasonable results. Flerchinger (1987) considered the convection term in his analysis of the freeze–thaw process. Tao and Gray (1994) also took into account convection in their model to determine the penetration of snowmelt into frozen ground. The inclusion or omission of the convection term in modeling moisture transport in frozen soils appears to be mainly dependent on the boundary conditions of the system and the soil permeability. In general, when modeling highly permeable soils, convection mechanism should be taken into account, especially where there is capacity for a large amount of moisture fluxes.

Predicting thaw depth of initially frozen soil is a challenging research topic because thermophysical properties of the frozen and unfrozen soils change with temperature and because the mechanisms occurring during thawing interact with each other. Numerous calculation models and solutions with different approaches have been proposed and developed to simulate the thawing process. Analytical models are simple to apply, but they were developed based on several assumptions, which limit their application and scope. Empirical and semiempirical models were derived from analytical or numerical solutions. They are more flexible and suitable for specific conditions with empirical constants. However, empirical models need to be validated with in situ test data. Numerical models are the most flexible of all as they can simulate thawing under various initial conditions. Nevertheless, they required a large number of calculations and take time. Numerical solutions are often implemented into a software, but the required software is not always available and free. Compared with numerical models, analytical and empirical models are more useful, simple, and available to engineers.

Simply put, thawing has often been considered as the reverse of the freezing process, and it has received much less attention in the literature than freezing. A majority of studies on frozen ground are about freezing problems, while only few studies have discussed thawing, in particular artificial thawing. Since thawing problems have received less attention there have been limited developments in methods for thawing frozen soils described in scientific literature until recently. Traditional methods seem to be outdated and ineffective, and several studies related to thawing methods have shown the remarkable advantages of modern thawing methods over traditional ones. Therefore, more research on improving thawing methods is recommended to create thawing systems that have not only high efficiency but also economic benefits. This development will increase demand for thawing experiments, which are currently sparse, especially full-scale ones. Compared with laboratory work, full-scale experiments reduce scaling errors and related uncertainties that may occur during testing. However, full-scale experiments on thawing soils are still limited with regards to soil types. Additional full-scale experiments should be carried out in the future on various types of soils and with different initial conditions. Data obtained

from full-scale experiments, if published, will be a valuable data source for researchers seeking to validate their thawing models.

Building computational models is always challenging work, especially due to the limited amounts of high-quality data accessible and is even more difficult for thawing soils. However, this should not discourage us from continuing modeling efforts on thawing processes. Although, thawing can be considered the reverse of freezing, they are two different processes, and it is necessary to separate the two and develop specialized models taking into consideration factors affecting the thawing process. Further modifications to thawing models should include vapor transport and convection heat transfer, especially in modeling high-permeability soils. In addition, others factors also need to be considered, such as modeling water migration after the thawing period due to changes in thawed soil properties, the rate of thawing, and soil types. It is important to continue modeling efforts of heat and mass transfer occurring in frozen soils based on previous work, evaluate effects and factors via experiments, and implement these into models that can describe the thawing process more realistically.

Data Availability Statement

No data, models, or code were generated or used during the study.

Acknowledgments

The authors acknowledge the Department of Building, Energy and Material Technology, The Arctic University of Norway. The publication fee for this article was covered by the publication fund of UiT-The Arctic University of Norway.

References

- Alzoubi, M. A., A. P. Sasmito, A. Madiseh, and F. P. Hassani. 2017. "Intermittent freezing concept for energy saving in artificial ground freezing systems." In Vol. 142 of *Proc., 9th Int. Conf. on Applied Energy*, 3920–3925. Netherlands: Elsevier.
- Andersland, O. B., and B. Ladanyi. 2004. *Frozen ground engineering*. 2nd ed. Chichester, UK: Wiley.
- Balossi Restelli, A., E. Rovetto, and A. & Pettinaroli. 2016. "Combined ground freezing application for the excavation of connection tunnels for Centrum Nauki Kopernik Station-Warsaw Underground Line II." In Vol. 3 of *Proc., ITA-AITES World Tunnel Congress 2016*, 2099–2108. Englewood, CO: Society for Mining, Metallurgy and Exploration.
- Beistline, E. H. 1963. "Placer mining in frozen ground." In *Proc., Int. Conf. on Permafrost*, 463–467. Washington, DC: National Academy of Sciences–National Research Council.
- Blanchard, D., and M. Fremond. 1985. "Frost heave of frozen soils." *C. R. Acad. Sci. Ser. II* 300: 637–642.
- Bradford, J. H., J. P. McNamara, W. Bowden, and M. N. Gooseff. 2005. "Measuring thaw depth beneath peat-lined arctic streams using ground-penetrating radar." *Hydrol. Processes* 19 (14): 2689–2699. <https://doi.org/10.1002/hyp.5781>.
- Brenton, S. S., and K. M. Donald. 2005. "Frost depth." *Micrometeorol. Agric. Syst.* 47: 155–177.
- Bronfenbrener, L. 2009. "The modelling of the freezing process in fine-grained porous media: Application to the frost heave estimation." *Cold Reg. Sci. Technol.* 56 (2–3): 120–134. <https://doi.org/10.1016/j.coldregions.2008.11.004>.
- Bronfenbrener, L., and R. Bronfenbrener. 2010. "Modeling frost heave in freezing soils." *Cold Reg. Sci. Technol.* 61 (1): 43–64. <https://doi.org/10.1016/j.coldregions.2009.12.007>.
- Brown, J., O. J. J. Ferrians, J. A. Heginbottom, and E. S. Melnikov. 1998. *Circum-Arctic map of permafrost and ground-ice conditions*. Boulder, CO: National Snow and Ice Data Center/World Data Center for Glaciology. Digital Media.
- Brown, S. C., and D. Payne. 1990. "Frost action in clay soils. I. A temperature-step and equilibrate differential scanning calorimeter technique for unfrozen water content determinations below 0°C." *J. Soil Sci.* 41 (4): 535–546. <https://doi.org/10.1111/j.1365-2389.1990.tb00224.x>.
- Carey, S. K., and M. K. Woo. 2005. "Freezing of subarctic hillslopes, Wolf Creek basin, Yukon, Canada." *Arctic Antarct. Alp. Res.* 37 (1): 1–10. [https://doi.org/10.1657/1523-0430\(2005\)037\[0001:FOSHWJC\]2.0.CO;2](https://doi.org/10.1657/1523-0430(2005)037[0001:FOSHWJC]2.0.CO;2).
- Chen, X., and H. Horino. 1998. "Three methods including TDR method to measure frozen or thawed depth of soil in field." *Permafrost Actions Nat. Artif. Cooling* 1998: 104–111.
- Engelmark, H., and U. Svensson. 1993. "Numerical modelling of phase change in freezing and thawing unsaturated soil." *Hydrol. Res.* 24 (2–3): 95–110. <https://doi.org/10.2166/nh.1993.0016>.
- Esch, D. C. 1982. *Permafrost prethawing by surface modification*. Rep. No. FHWA-AK-RD-83-23. Juneau, AK: Alaska Dept. of Transportation.
- Esch, D. C. 1984. *Surface modifications for thawing of permafrost*. Rep. No. FHWA-AK-RD-85-10. Juneau, AK: Alaska Dept. of Transportation.
- Esch, D. C. 2004. "Thermal analysis, construction, and monitoring methods for frozen ground." Chap. 7 in *Thawing techniques for frozen ground*, edited by D. C. Esch, 239–257. Reston, VA: ASCE.
- Flerchinger, G. N. 1987. "Simultaneous heat and water model of snow-residue-soil system." Ph.D. thesis, Dept. of Engineering Science, Washington State Univ.
- Gandahl, R., and W. Bergau. 1957. "Methods for measuring the frozen zone in soil." In Vol. 88 of *Proc., 4th Int. Conf. on Soil Mechanics and Foundation Engineering*. Oxford: Butterworths Scientific Publications.
- Gao, J. Q., Z. H. Xie, A. W. Wang, and Z. D. Luo. 2016. "Numerical simulation based on two-directional freeze and thaw algorithm for thermal diffusion model." *Appl. Math. Mech.* 37 (11): 1467–1478. <https://doi.org/10.1007/s10483-016-2106-8>.
- Giakoumakis, S. G. 1994. "A model for predicting coupled heat and mass transfers in unsaturated partially frozen soil." *Int. J. Heat Fluid Flow* 15 (2): 163–171. [https://doi.org/10.1016/0142-727X\(94\)90071-X](https://doi.org/10.1016/0142-727X(94)90071-X).
- Han, X., Z. L. Zhao, and Z. R. Gao. 2014. "Experimental research on the characteristics of soil strength freeze-thaw weakening in Songhua river songpu bank." In *Advances in civil and structural engineering III*, edited by Y. Huang, Pts 1–4, 501–504, 403–409. Zurich, Switzerland: Trans Tech Publications.
- Harlan, R. L. 1973. "Analysis of coupled heat-fluid transport in partially frozen soil." *Water Resour. Res.* 9 (5): 1314–1323. <https://doi.org/10.1029/WR009i005p01314>.
- Harms, T. K., and J. B. Jones. 2012. "Thaw depth determines reaction and transport of inorganic nitrogen in valley bottom permafrost soils." *Global Change Biol.* 18 (9): 2958–2968. <https://doi.org/10.1111/j.1365-2486.2012.02731.x>.
- Hayashi, M., N. Goeller, W. L. Quinton, and N. Wright. 2007. "A simple heat-conduction method for simulating the frost-table depth in hydrological models." *Hydrol. Processes* 21 (19): 2610–2622. <https://doi.org/10.1002/hyp.6792>.
- Hayhoe, H. N. 1994. "Field testing of simulated soil freezing and thawing by the SHAW model." *Can. Agric. Eng.* 36: 279–285.
- Hirsch, A. I., S. E. Trumbore, and M. L. Goulden. 2002. "Direct measurement of the deep soil respiration accompanying seasonal thawing of a boreal forest soil." *J. Geophys. Res. Atmos.* 107 (D23): WFX2-1–WFX2-10.
- Hjort, J., O. Karjalainen, J. Aalto, S. Westermann, V. E. Romanovsky, F. E. Nelson, B. Etzelmüller, and M. Luoto. 2018. "Degrading permafrost puts Arctic infrastructure at risk by mid-century." *Nat. Commun.* 9: 5147. <https://doi.org/10.1038/s41467-018-07557-4>.
- Hoekstra, P. 1966. "Moisture movement in soils under temperature gradients with the cold-side temperature below freezing." *Water Resour. Res.* 2 (2): 241–250. <https://doi.org/10.1029/WR002i002p00241>.
- Hou, F., Q. M. Li, E. L. Liu, C. Zhou, M. K. Liao, H. W. Luo, and X. Y. Liu. 2016. "A fractional creep constitutive model for frozen soil in consideration of the strengthening and weakening effects." *Adv. Mater. Sci. Eng.* 2016: 5740292.

- Iwata, Y., T. Hirota, T. Suzuki, and K. Kuwao. 2012. "Comparison of soil frost and thaw depths measured using frost tubes and other methods." *Cold Reg. Sci. Technol.* 71: 111–117. <https://doi.org/10.1016/j.coldregions.2011.10.010>.
- Jame, Y. W., and D. I. Norum. 1980. "Heat and moisture transfer in a freezing unsaturated porous medium." *Water Resour. Res.* 12: 513–522.
- Jumikis, A. R. 1966. *Thermal soil mechanics*. New Brunswick, NJ: Rutgers University Press.
- Jumikis, A. R. 1979. "Some aspects of artificial thawing of frozen soils." *Eng. Geol.* 13 (1–4): 287–297. [https://doi.org/10.1016/0013-7952\(79\)90038-3](https://doi.org/10.1016/0013-7952(79)90038-3).
- Jumikis, A. R. 1985. "Electrical thawing of frozen soils." In Vol. 2 of *Proc., Permafrost 4th Int. Conf.*, 333–337. Washington, DC: National Academy Press.
- Kudryavtsev, V. A., L. S. Garagulya, K. A. K. Yeva, and V. G. Melamed. 1977. *Fundamentals of frost forecasting in geological engineering investigations*. Hanover, NH: Cold Regions Research and Engineering Laboratory.
- Kurylyk, B. L. 2015. "Discussion of "A simple thaw-freeze algorithm for a multi-layered soil using the stefan equation" by Xie and gough (2013)." *Permafrost Periglacial Processes* 26 (2): 200–206. <https://doi.org/10.1002/ppp.1834>.
- Kurylyk, B. L., M. Hayashi, W. L. Quinton, J. M. Mckenzie, and C. I. Voss. 2016. "Influence of vertical and lateral heat transfer on permafrost thaw, peatland landscape transition, and groundwater flow." *Water Resour. Res.* 52 (2): 1286–1305. <https://doi.org/10.1002/2015WR018057>.
- Kurylyk, B. L., K. T. B. MacQuarrie, and J. M. McKenzie. 2014. "Climate change impacts on groundwater and soil temperatures in cold and temperate regions: Implications, mathematical theory, and emerging simulation tools." *Earth Sci. Rev.* 138: 313–334. <https://doi.org/10.1016/j.earscirev.2014.06.006>.
- Lebedeva, L., O. Semenova, and T. Vinogradova. 2014. "Simulation of active layer dynamics, Upper Kolyma, Russia, using the hydrograph hydrological model." *Permafrost Periglacial Processes* 25 (4): 270–280. <https://doi.org/10.1002/ppp.1821>.
- Li, X., and T. Koike. 2003. "Frozen soil parameterization in SiB2 and its validation with GAME-Tibet observations." *Cold Reg. Sci. Technol.* 36 (1–3): 165–182. [https://doi.org/10.1016/S0165-232X\(03\)00009-0](https://doi.org/10.1016/S0165-232X(03)00009-0).
- Liang, S. H., J. D. Teng, F. Shan, and S. Zhang. 2020. "A numerical model of vapour transfer and phase change in unsaturated freezing soils." *Adv. Civ. Eng.* 2020: 8874919.
- Lindroth, D. P., W. R. Berglund, and C. F. Wingquist. 1995. "Microwave thawing of frozen soils and gravels." *J. Cold Reg. Eng.* 9 (2): 53–63. [https://doi.org/10.1061/\(ASCE\)0887-381X\(1995\)9:2\(53\)](https://doi.org/10.1061/(ASCE)0887-381X(1995)9:2(53)).
- Luthin, J. N., and G. L. Guymon. 1974. "Soil moisture-vegetation temperature relationships in central Alaska." *J. Hydrol.* 23 (3–4): 233–246.
- Mackay, M. H., D. K. Hein, and J. J. Emery. 1992. "Evaluation of frost action mitigation procedures for highly frost-susceptible soils." In *Proc., 37th Annual Conf. Canadian Technical Asphalt Association*, 91–109. Washington, DC: Transportation Research Board.
- Martin, B. P. 2007. "The application of semiempirical methods in drug design." Ph.D. thesis, Dept. of Chemistry, Univ. of Florida.
- Ming, F., D. Q. Li, and K. Zhang. 2012. "Theoretical study on thaw settlement of saturated frozen soil." In *Progress in industrial and civil engineering*, edited by J. Liang, X. Wu, W. Yang, and W. Chen, Pts. 1–5, 204–208, 155–162. Zurich, Switzerland: Trans Tech Publications.
- Naaktgeboren, N. M. 2006. "Artificial ground freezing: How to model and calculate the frost-heave?." In *Geotechnical aspects of underground construction in soft ground*, edited by M. Elshafie, G. Viggiani, and R. Mair, 419–424. London: Routledge.
- Nelson, F. E., N. I. Shiklomanov, and G. R. Mueller. 1999. "Variability of active-layer thickness at multiple spatial scales, north-central Alaska, U.S.A.." *Arctic, Antarct. Alpine Res.* 31 (2): 179–186. <https://doi.org/10.1080/15230430.1999.12003295>.
- Newman, G. P. 1995. "Heat and mass transfer in unsaturated soils during freezing." Ph.D. thesis, Dept. of Civil Engineering, Univ. of Saskatchewan.
- Nixon, J. F. 1975. "The role of convective heat transport in the thawing of frozen soils." *Can. Geotech. J.* 12 (3): 425–429. <https://doi.org/10.1139/t75-046>.
- Nixon, J. F., and E. C. McRoberts. 1973. "A study of some factors affecting the thawing of frozen soils." *Can. Geotech. J.* 10 (3): 439–452. <https://doi.org/10.1139/t73-037>.
- Nixon, J. F. D. 1990. "A new concept of frost-heave characteristics of soils, by Otto J. Svec." *Cold Reg. Sci. Technol.* 18 (2): 217–218. [https://doi.org/10.1016/0165-232X\(90\)90010-T](https://doi.org/10.1016/0165-232X(90)90010-T).
- Oelke, C., T. J. Zhang, M. C. Serreze, and R. L. Armstrong. 2003. "Regional-scale modeling of soil freeze/thaw over the Arctic drainage basin." *J. Geophys. Res.* 108: ACL 9-1–ACL 9-19. <https://doi.org/10.1029/2002JD002722>.
- Ozgan, E., S. Serin, S. Erturk, and I. Vural. 2015. "Effects of freezing and thawing on the consolidation settlement of soils." *Soil Mech. Found. Eng.* 52 (5): 247–253. <https://doi.org/10.1007/s11204-015-9336-6>.
- Philip, J. R., and D. A. De Vries. 1957. "Moisture movement in porous materials under temperature gradients." *Trans. Am. Geophys. Union* 38 (2): 222–232. <https://doi.org/10.1029/TR038i002p00222>.
- Qi, Y., J. X. Zhang, H. Yang, and Y. W. Song. 2020. "Application of artificial ground freezing technology in modern urban underground engineering." *Adv. Mater. Sci. Eng.* 2020: 1619721.
- Rankinen, K., T. Karvonen, and D. Butterfield. 2004. "A simple model for predicting soil temperature in snow-covered and seasonally frozen soil: Model description and testing." *Hydrol. Earth Syst. Sci.* 8 (4): 706–716. <https://doi.org/10.5194/hess-8-706-2004>.
- Reitan, K. M. 2013. *Alternative methods for Ice thawing in sub-drains and ditches*. [In Norwegian.] Rep. No. 184. Oslo, Norway: Norwegian Public Roads Administration.
- Romanovsky, V. E., and T. E. Osterkamp. 1997. "Thawing of the active layer on the coastal plain of the Alaskan Arctic." *Permafrost Periglacial Processes* 8 (1): 1–22.
- Rouabhi, A., E. Jahangir, and H. Tounsi. 2018. "Modeling heat and mass transfer during ground freezing taking into account the salinity of the saturating fluid." *Int. J. Heat Mass Transfer* 120: 523–533. <https://doi.org/10.1016/j.ijheatmasstransfer.2017.12.065>.
- Ryabets, N., and F. Kirzhner. 2003. "Weakening of frozen soils by means of ultra-high frequency energy." *Cold Reg. Sci. Technol.* 36 (1–3): 115–128. [https://doi.org/10.1016/S0165-232X\(03\)00002-8](https://doi.org/10.1016/S0165-232X(03)00002-8).
- Shoop, S. A., and S. R. Bigl. 1997. "Moisture migration during freeze and thaw of unsaturated soils: Modeling and large scale experiments." *Cold Reg. Sci. Technol.* 25 (1): 33–45. [https://doi.org/10.1016/S0165-232X\(96\)00015-8](https://doi.org/10.1016/S0165-232X(96)00015-8).
- Snyder, H. 2019. "Literature review as a research methodology: An overview and guidelines." *J. Bus. Res.* 104: 333–339. <https://doi.org/10.1016/j.jbusres.2019.07.039>.
- Strauss, J., et al. 2017. "Deep Yedoma permafrost: A synthesis of depositional characteristics and carbon vulnerability." *Earth Sci. Rev.* 172: 75–86. <https://doi.org/10.1016/j.earscirev.2017.07.007>.
- Sveen, S. E. 2017. "Artificial thawing of seasonally frozen ground: performance characteristics of hydronic based thawing." Ph.D. thesis, Dept. of Energy and Process Engineering, Norwegian Univ. of Science and Technology (NTNU).
- Sveen, S. E., H. T. Nguyen, and B. R. Sorensen. 2020. "Soil moisture variations in frozen ground subjected to hydronic heating." *J. Cold Reg. Eng.* 34 (4): 04020025. [https://doi.org/10.1061/\(ASCE\)CR.1943-5495.0000231](https://doi.org/10.1061/(ASCE)CR.1943-5495.0000231).
- Sveen, S. E., and B. R. Sorensen. 2013. "Establishment and instrumentation of a full scale laboratory for thermal and hygroscopic investigations of soil behavior in cold climates." In *Measurement technology and its application*, edited by P. Yarlagadda and Y.-H. Kim, Pts 1 and 2, 239–240, 827–835. Guangzhou, China: Trans Tech Publications.
- Tao, Y. X., and D. M. Gray. 1994. "Prediction of snowmelt infiltration into frozen soils." *Numer. Heat Transfer, Part A* 26 (6): 643–665. <https://doi.org/10.1080/10407789408956015>.
- Taylor, G. S., and J. N. Luthin. 1978. "A model for coupled heat and moisture transfer during soil freezing." *Can. Geotech. J.* 15 (4): 548–555. <https://doi.org/10.1139/t78-058>.
- Vaks, A., A. J. Mason, S. F. M. Breitenbach, A. M. Kononov, A. V. Osinzev, M. Rosensaft, A. Borshevsky, O. S. Gutareva, and G. M. Henderson. 2020. "Palaeoclimate evidence of vulnerable permafrost during times of low sea ice." *Nature* 577 (7789): 221–225. <https://doi.org/10.1038/s41586-019-1880-1>.

- Walvoord, M. A., and B. L. Kurylyk. 2016. "Hydrologic impacts of thawing permafrost-A review." *Vadose Zone J.* 15 (6): vzj2016.01.0010. <https://doi.org/10.2136/vzj2016.01.0010>.
- Wang, J. P., W. S. Wang, and T. S. Shi. 1999. "Development and application of a testing device for study of frost heave and thawing settlement of artificially frozen soil." *Min. Sci. Technol.* 99: 439–442.
- Wang, S. J., F. J. Niu, J. B. Chen, and Y. H. Dong. 2020. "Permafrost research in China related to express highway construction." *Permafrost Periglacial Processes* 31 (3): 406–416. <https://doi.org/10.1002/ppp.2053>.
- Wilson, G. W. 1990. "Soil evaporative fluxes for geotechnical engineering problems." Ph.D. thesis, Dept. of Civil Engineering, Univ. of Saskatchewan.
- Woo, M. K. 1976. "Hydrology of a small Canadian High Arctic basin during the snowmelt period." *Catena* 3 (2): 155–168. [https://doi.org/10.1016/0341-8162\(76\)90007-2](https://doi.org/10.1016/0341-8162(76)90007-2).
- Woo, M. K., M. A. Arain, M. Mollinga, and S. Yi. 2004. "A two-directional freeze and thaw algorithm for hydrologic and land surface modelling." *Geophys. Res. Lett.* 31: L12501.
- Yan, Q. X., W. Wu, C. Zhang, S. Q. Ma, and Y. P. Li. 2019. "Monitoring and evaluation of artificial ground freezing in metro tunnel construction-A case study." *KSCE J. Civ. Eng.* 23 (5): 2359–2370. <https://doi.org/10.1007/s12205-019-1478-z>.
- Yi, S. H., M. A. Arain, and M. K. Woo. 2006. "Modifications of a land surface scheme for improved simulation of ground freeze-thaw in northern environments." *Geophys. Res. Lett.* 33: L13501.
- Yokoo, M., M. Shibasaki, H. Yoshida, H. Souma, A. Ousaka, K. Kusano, and K. Horii. 2005. "Prediction and improvement of artificial ground freezing." In Vol. 2 of *Proc., ASME Fluids Engineering Division Summer Conf.*, 369–372. New York: ASME.
- Yuan, Y. H., and P. Yang. 2016. "Research on the application of artificial freezing technology inside the tunnel with subsurface excavation." *J Railway Eng. Soc.* 33 (9): 82–86 and 103.
- Zeinali, A., T. Edeskär, and J. Laue. 2020. "Mechanism of thawing." *Cogent Engineering* 7 (1): 1716438. <https://doi.org/10.1080/23311916.2020.1716438>.
- Zhang, T., R. G. Barry, K. Knowles, F. Ling, and R. L. Armstrong. 2003. "Distribution of seasonally and perennially frozen ground in the Northern Hemisphere." *Permafrost* 1 and 2: 1289–1294.
- Zhang, Y., A. Chipanshi, B. Daneshfar, L. Koiter, C. Champagne, A. Davidson, G. Reichert, and F. Bédard. 2019. "Effect of using crop specific masks on earth observation based crop yield forecasting across Canada." *Remote Sens. Appl.: Soc. Environ.* 13: 121–137. <https://doi.org/10.1016/j.rsase.2018.10.002>.
- Zhou, G. Q., Y. Z. Zou, and C. Boley. 2009. "Decreasing frost heaves of artificial frozen soils by intermission freezing." *Bautechnik* 86 (9): 566–573. <https://doi.org/10.1002/bate.200910057>.
- Zhou, J., W. Q. Zhao, and Y. Q. Tang. 2021. "Practical prediction method on frost heave of soft clay in artificial ground freezing with field experiment." *Tunnelling Underground Space Technol.* 107: 103647. <https://doi.org/10.1016/j.tust.2020.103647>.