Resilience of a transportation network: Importance of vulnerable nodes

Behrooz Ashrafi[a], Masoud Naseri[a,b], Javad Barabady[a,c]
[a] UiT, The Arctic university of Norway, Tromsø, Norway, *behrooz.ashrafi@uit.no*
[b] *masoud.naseri@uit.no*
[c] *javad.barabady@uit.no*

Transportation Infrastructure is one of key critical infrastructures which are crucial for a well-functioning modern society. This study proposes a resilience-oriented framework for analyzing and ultimately mitigating risk and improving resilience for land transportation networks with a focus on vulnerable nodes. The framework uses network theory and considers the concept of independent pathways, network topology, redundancies in the network, reliability of roads, importance of hub nodes and vulnerable nodes. A network score is developed based on the aforementioned criterion of the network which will be used for comparing different failure scenarios under extreme weather events hazard type. The focus of this paper is the importance of vulnerable nodes, and it tries to highlight that throughout the paper. A numerical example has been utilized to illustrate the use of this framework.

Keywords: Resilience assessment, Transportation infrastructures, Network analysis, Risk mitigation

1. INTRODUCTION

Modern Societies rely on critical infrastructures (CIs) for their daily functions. Transportation infrastructure (TI) is one of the most crucial of these CIs which is essential for a functioning modern society. Along with the normal usage and importance of TIs, most CI sectors are dependent on TIs to function effectively; Also, in case of a crisis (natural or man-made) TIs are essential for post-crisis recovery.

Resilience engineering is gradually moving towards becoming one of the major tools in CI management. Resilience can be defined as the ability of a system to resist disruptions and adapt to them and in case of a failure recover from them quickly. Resilience concept can be divided into four elements: I) robustness/redundancy – the ability of a system to avoid and mitigate perturbations while maintaining an acceptable service level. II) rapidity – the ability of a system to quickly recover from to the base service level after suffering from a perturbation. III) resourcefulness – the ability of a system to continue to detect and analyse perturbations to provide adequate measures. IV) adaptability – the ability of a system to emerge stronger and learn from disruptions. [1-3]

TI assessment has been tackled with different modelling approaches in the available literature; these methodologies can be categorized into graph theoretical and network analysis approaches, big data approaches, simulation models, and optimization models.[*] [4]

I) Graph theoretical and network analysis approaches – modelling the transportation networks (TNs) as a complex network with cities or intersections as nodes and roads between them as corresponding links. There are different perspectives for using network analysis for analysing a TN, some are solely looking at the connectivity aspect of the networks, for example, Zhang et al. [5] discovered four different network topologies based on their connectivity and ranked them for best to worst resilience. Some are looking into various network types and their effect on network resilience, for example Hartmann [6] discovered that reducing the network diameter might increase the resilience of TNs by analysing different network types. Ip et al. [7] used the concept of reliable passageways for analysing the

[*] Note that there might be some models that fall out of this categorization, and some models might combine two or more of these approaches together.

connectivity of origin-destination (O-D) pairs and based on that formulates resilience of each node and whole network's resilience. Zhang et al. [8] used the concept of independent pathways (pathways that do not share any common road links) to formulate a performance measure for TNs and used it to find the TN's resilience.
II) Big data approaches; Chan et al. [9] used the concept of lost service day to formulate the resilience, Donavan et al. [10] utilized the probe vehicle numbers (e.g. taxies) to analyse the resilience of TNs in a city. Cox et al. [11] measured the difference between passenger behaviour before and after a terrorist attack to measure TNs resilience.
III) Simulation models; Alipour et al. [12] used the concept of change in capacity of the aging bridges to assess the seismic resilience of TNs. Mostafavi et al. [13] looked at the sea level rise and its effects on long term resilience of TNs.
IV) Optimization models. Patil et al. [14] minimized the network cost while maximizing the capacity degradation of each link before network failure.

A variety of different metrics is being utilized for modelling TNs as seen in the short literature review done above. These metrics have two main types, generic or functional. Generic metrics describe the network structure, e.g. degree centrality, betweenness centrality, connectivity, etc. Functional metrics describe the attributes of a TN, e.g. path length, travel time, traffic flow, path reliability, population density of nodes/cities, network topology, etc. [15]

The aim of this paper is to build a resilience-oriented framework of TNs while focusing on the robustness/redundancy aspect of resilience is presented. This framework is based on graph theory and network analysis and takes into account the concept of independent pathways (i.e., a set of all the pathways connecting each node-pair that do not share any identical edge), network topology, redundancies in the network, traffic flow, reliabilities of edges, vulnerabilities in nodes (nodes that have fewer outgoing edges relative to the other nodes) and the importance of hub nodes (also critical nodes). This paper argues that in resilience assessment for transportation networks, node vulnerabilities is crucial and needs to be considered with extra emphasis when analysing the network. Last but not least, this framework will utilize a score system for comparing different failure scenarios (edge failure combinations) to discover the most critical elements (nodes and edges) in the network. The scoring system is employed for the whole network and the individual nodes. A hypothetical network consisting of 30 nodes subjected to extreme weather hazards will be utilized for a numerical example in this study.

2. METHODOLOGY

The main purpose of any TI is to handle traffic from any point of origin to any destination. The resilience of these TIs can be summed up to them being able to carry out their purpose in case of any disruptions. However, as discussed in section 2, previous studies mainly focused on the whole TN in their resilience assessment, which will cause vulnerable nodes (VNs) to have little to no effect on the total resilience of the TNs. This paper considers the effect of VNs into TNs resilience. VNs are particularly of interest to this paper because in case of a disturbance in any of their outgoing edges, their overall connection to the network decreases drastically.

This paper defines a resilience-oriented framework of TNs by extending the concept (i.e. weighted number of independent pathways or WIPW) suggested by Zhang et al [8]. A pathway between any two nodes in a network, consists of multiple edges that in TNs means roads, that are connected in series. Independent pathways (IPWs) are a set of all pathways connecting each node-pair that do not share any identical edges. the number of IPWs are limited compared to the number of all possible pathways between node-pairs in a network which makes this concept particularly useful in networks with a large number of nodes due to calculation times. The process for finding IPWs in a network is discussed later in the paper.

2.1 Network construction

By using the terminology of graph theory [16], the TN can be defined by $G = (V, A)$ in which $V = \{1,2, \dots n\}$ is a set of nodes that shows either road intersections (if the network is for modelling a city TNs) or cities (if the network is for modelling national or international TNs) and $A = \{1,2 \dots m\}$ is a set of edges that corresponds to roads connecting these nodes. This paper will use the concept of adjacency matrix for building the network. Adjacency matrix (A) shows the connectivity of all nodes with respect to each other ($a_{ij} = \{0\ for\ no\ connection\ or\ 1\ for\ connected\}$), in a 2D array. Which will be denoted by a matrix of ones and zeros.

$$A = \begin{bmatrix} 0 & \cdots & a_{ij} \\ \vdots & \ddots & \vdots \\ a_{ij} & \cdots & 0 \end{bmatrix} \tag{1}$$

2.2 Vulnerable nodes (VNs)

There are instances in a network when a nodes will get isolated from the main network. This is of particular importance for nodes that have relative low degrees with respect to other nodes. For assessing resilience of TNs, it is imperative to consider vulnerable and remote nodes. This metric will help to consider the remote areas in a transportation network. The model is using node degree to find the VNs in the network; however, the resulting metric contributes to edge importance weights. The model will assign scores which will signify the importance of each edge with respect to the degree of the node its connected to, the lower the node degree, the connecting edges of that node become more important. For calculating the scores, there is a need to construct an empirical cumulative distribution function (ECDF) from all node degrees in the network, and the nodes that fall under the 10th percentile in the empirical distribution function will be considered to be VNs in the network. Note that, usually since the nodes in the network won't exceed a certain amount, the 10th percentile of the ECDF won't fall exactly on a certain node degree number, in that case, the model will consider the closest node degree number to the 10th percentile in ECDF to be the threshold for being a VN. Any node degree number that is smaller than the threshold number will assumed to be a VN. When the VNs have been identified, based on the value of the highest node degree in the VNs, a score system will be assigned to them as shown in table 1:

Table 1: Edge scores with respect to node vulnerabilities

Node degree	Edge/Edges score
1	n
2	n-1
…	…
n-1	2
n	1
n+	0

A situation may arise in this scoring system that an edge has several different scores based on its connection to several VNs. In this case, that edge will have a score of the more important category (e.g. if an edge have both 3 and 4 scores, it will be assigned a 4 in the final result.)

2.3 IPW calculation algorithm

The process of finding IPWs in a network is explained in algorithm 1. This algorithm has been adapted from Zhang et al [8]:

Algorithm 1 (IPW finding algorithm) [8]

Input	TNs adjacency matrix for construction of the network
Output	IPWs between all node-pairs in the network
Step 1	Finding all node-pairs in the network
Step 2	Finding the shortest path between node-pair (i,j) using Dijkstra's algorithm [17]
Step 3	Storing the shortest path $m_k(i,j)$
Step 4	Removing the edges in $m_k(i,j)$ from the network
Step 5	$\begin{cases} \text{no path can be found between node} - \text{pair } (i,j) \rightarrow \text{end} \\ \text{otherwise} \rightarrow \text{return to step 2} \end{cases}$
Step 6	Repeat steps 2-5 for all node-pairs in the network

2.4 Framework formulation

Using the concept of IPWs [8], an overall score can be defined to analyse the performance of the network with an emphasis on VNs. This paper calls this score "Network Score" (NS), the network score is showing the weighted number of IPWs between all node-pairs in the network. NS can be formulated as shown in equation 2:

$$NS(G) = \sum_{i=1}^{n} w_i h_i \tag{2}$$

in which w_i is the weight of each individual node in $V = \{1,2, \dots n\}$ which will be explained later on, and h_i is the normalized number of IPWs between node i and all other nodes except for i which can be seen in equations 3 and 4:

$$h_i = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} A(i,j) \tag{3}$$

$$A(i,j) = \sum_{k=1}^{k_{(i,j)}} w'_k(i,j). R_k(i,j) \tag{4}$$

in which $k_{(i,j)}$ is the number of IPWs between node-pair (i,j), $w'_k(i,j)$ is the weight of the kth IPW between node-pair (i,j) (this model will use $m_k(i,j)$ for showing the kth IPW between node-pair (i,j) , and $R_k(i,j)$ is the reliability of the kth IPW between node-pair (i,j). Since each IPW is a set of connected edges in series the total reliability of any IPW can be calculated by multiplying the reliability of all edges in that IPW (this paper shows the reliability of each edge by r_e and the reliability of edges are assumed to be constant) which can be found in equation 5:

$$R_k(i,j) = \prod r_e \ \forall \ edge \ in \ kth \ IPW \tag{5}$$

w_i which has been introduced in equation 2 is proportional to the degree centrality of each node. Degree centrality is a way of ranking nodes in any network based on the number of edges connected to them. In a TN, considering degree centrality as a weighting index helps the model to identify the critical nodes (nodes that have the most degree and therefore are critical to the networks connectivity). In resilience assessment of TNs, nodes with high degree centrality are crucial since a disruption in one of these nodes can disrupt the whole network. In this paper, Degree centrality of each node is shown by C_D^i and it can be calculated by equation 6 [18]:

$$C_D^i = \frac{degree \ of \ node \ i}{n-1} \tag{6}$$

in which, "degree of node i is the number of outgoing edges from node i, and $n-1$ is the total number of possible outgoing edges which is the number of total nodes minus one. With this index, the model can now calculate the weight of each node by equation 7:

$$w_i = \frac{C_D^i}{\sum_i^n C_D^i} \tag{7}$$

note that $\sum_{i}^{n} w_i = 1$.

$w'_k(i,j)$ which has been introduced in equation 4 and is the weight of each IPW, is related to two different metrics that are related to edges of the graph: 1) length of each edge 2) vulnerability of nodes factor in each edge. The model needs to calculate each of these measures separately. Let's start with the length of each edge, which needs to first be transformed into length of each IPW:

$$L_{m_k}(i,j) = \sum L_e \; \forall \; edge \; in \; kth \; IPW \tag{8}$$

in which, $L_{m_k}(i,j)$ is the length of kth IPW, and L_e is the length of each respective edge in $m_k(i,j)$. importance of length of $m_k(i,j)$ is negatively correlated to the value of $L_{m_k}(i,j)$, lower values for $L_{m_k}(i,j)$ represents better scores for $m_k(i,j)$. To be able to use this as a metric in the weights of IPWs it needs to be normalized. The normalized version of length of $m_k(i,j)$ can be found in equation 9:

$$L^n_{m_k}(i,j) = \frac{\frac{1}{L_{m_k}(i,j)}}{\sum_{k=1}^{k(i,j)} \frac{1}{L_{m_k}(i,j)}} * K(i,j) \tag{9}$$

in which $L^n_{m_k}(i,j)$ is the normalized length of $m_k(i,j)$, and $K(i,j)$ is the total number of IPWs between node-pair (i,j). Now to formulate the VN factor in edges, this paper will present the score of each edge by I_e, equation 10 shows the VN factor in $m_k(i,j)$:

$$I_{m_k}(i,j) = \sum I_e \; \forall \; edge \; in \; kth \; IPW \tag{10}$$

in which $I_{m_k}(i,j)$ is the VN factor of $m_k(i,j)$. The normalized version of VN factor of $m_k(i,j)$ is shown in equation 11:

$$I^n_{m_k}(i,j) = \frac{I_{m_k}(i,j)}{\sum_{k=1}^{k(i,j)} I_{m_k}(i,j)} * \; K(i,j) \tag{11}$$

Finally, the weight of each IPW ($w'_k(i,j)$) can be calculated by combining the normalized version of VN factor of $m_k(i,j)$ and normalized length of $m_k(i,j)$ and it is shown in equation 12:

$$w'_k(i,j) = \; x * L^n_{m_k}(i,j) + (1-x) * I^n_{m_k}(i,j) \tag{12}$$

in which x is a decision variable, and is a number between 0 to 1 and can be manipulated based on the needs of the decision makers.

2.5 Failure scenarios

The failure scenarios that this paper uses are based on extreme weather events which in this case the paper is talking about severe icing on roads which makes transportation on some roads of the TN impossible. The failure will be binary (i.e., a road is either failed completely or is operational there is no limited operationality defined). Failure scenarios are constructed on a basis of semi-random edge removal which is linked to the reliability of each edge (i.e., the removal of edges are randomized based on their reliability) using Monte-Carlo simulation.

2.6 Resilience enhancement strategies

This paper knowingly will ignore the cost-benefit analysis for enhancing the typical overall network resilience in favour of enhancing the vulnerable nodes and with that enhances the network resilience in a different way in other words, this paper will consider an unlimited resources scenario. There are many ways of enhancing the pre-disaster network resilience in a transportation network this paper will discuss two possible solution scenarios. note that, these scenarios can be combined given the unlimited resources assumption.

1. Changing the networks topology:
   Adding new construction plans (adding new edges to the network) for enhancing the vulnerable nodes.

2. Improving the reliability of roads:
   This strategy is about strengthening the roads for the specific hazard scenario of extreme weather events. Which can be done by adding modifications and repairing the existing roads. There are three ways of achieving this

2.7 Illustrative numerical example

In this section, A numerical example will be presented to illustrate the role of the network score metric. The network consists of 30 nodes and 45 edges that represent a TN which is illustrated in Figure 1.

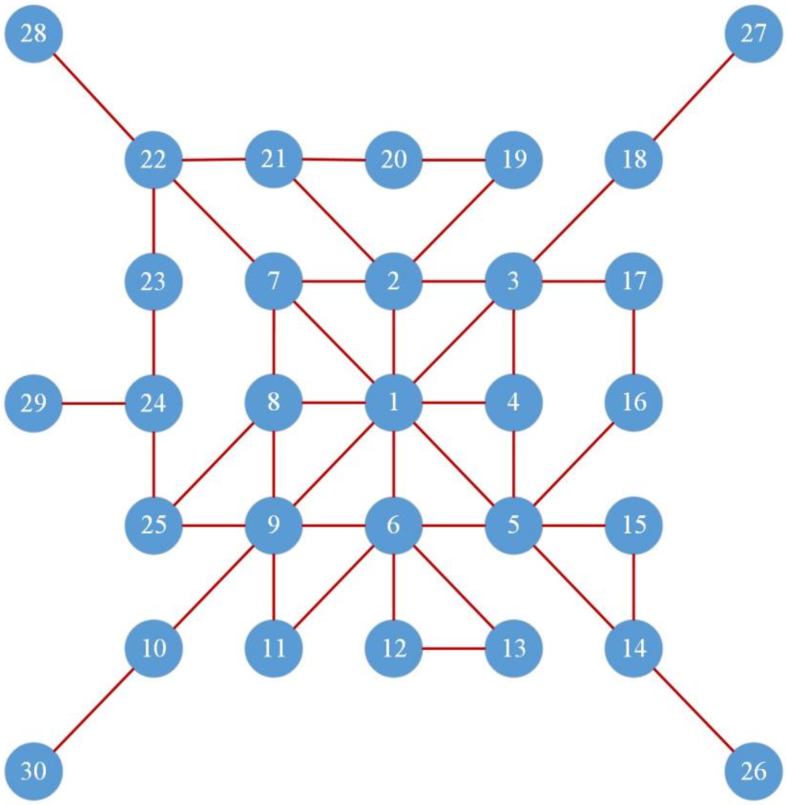

Figure 1 Hypothetical TN's Illustration

Table 2 VN factor calculations

Node degree	$P(x \leq Node\ Degree)$	VN factor for outgoing edges
1	0.167	1
2	0.533	0
3	0.7	0
4	0.8	0
5	0.867	0
6	0.967	0
7	0.967	0
8	1	0

The reliability of each edge is based on the failure scenario of extreme weather events which means the numbers provided in table 2 are based on reliability of edges against this specific hazard. The reliability of each edge is considered to be a normal distribution with a mean value as shown in table 3 for each respective edge. VN factor for this network has been calculated based on the empirical cumulative distribution function of all the node degrees in the network, as explained in section 3.2. Table 2 shows the respective probabilities based on the ECDF of the node degrees for each node degree number. Based on table 2 only nodes with node degrees one or less are considered to be VNs in this network; thereafter, with respect to table 1, the VN factor for edges can be calculated. Nodes 26, 27, 28, 29 and 30 are the VNs and respectively edges (22,28), (18,27), (14,26), (10,30) and (24,29) are edges that are affected by the VN factor. The attributes for all edges can be found in Table 3.

Before analysing the network in its real operational condition, this study looks at this network under different operational condition these conditions are described in four scenarios that are:

1. Perfect reliability for all roads, No VN factor:
   In this scenario, the reliability of all roads in the network is set to perfect condition (0.999) and the decision variable x is set to one. Which means the VN factor is removed from the formula in this scenario the model will produce $NS = 1.2435$ which means there are 1.2435 average IPWs between all node-pairs under this perfect condition.
2. Perfect reliability for all roads, VN factor included:
   In this scenario, again the reliability of all roads in the network is set to perfect condition, however, the decision variable is set to $x = 0.5$. which means both length and VN factor are considered equally as important. The model will give $NS = 0.7095$ which indicated a huge decline in NS after introducing the VN factor into it. Note that this number is not representative of the average number of IPWs between all node-pairs in the network.
3. Real reliability for all roads, No VN factor:
   In this scenario, the reliability of all roads has been set to their actual number as stated in table 3, and the decision variable is set to $x = 1$, which again means that VN factor has been eliminated from the formula. For these conditions the model produces $NS = 0.6092$ which again is representative of the number of average IPWs between all node-pairs, under real conditions this time.
4. Real reliability for all roads, VN factor included:
   In this scenario, like the previous one, the reliability of all roads has been set to their actual numbers; However, the decision variable is set to $x = 0.5$. Under these conditions the model produces $NS = 0.3354$ which again shows a significant decline compared to the previous one with no VN factor.

The model will use scenario 4 as its base of calculations. In this scenario, $NS = 0.3354$ is considered the baseline network score. The model now considers an extreme precipitation event in wintertime which results in freezing rain on roads and renders some of the roads out of service. Some of the roads will be eliminated based on their reliability to this specific hazard type. For achieving this a semi-random removal of roads with respect to their reliability has been implemented in the system. Fig. 2 shows the histogram of NS after using 300000 Monte-Carlo simulations under this specific hazard type, the mean network score is $mean(NS) = 0.1464$ which is less than half of the baseline NS of the network. This shows that the network is susceptible to this specific hazard type and in case of an extreme precipitation event, this network will lose half its functionality while focusing on the VNs.

Table 3 attributes of edges in the hypothetical TN

Edge	Reliability	VN factor	Length (Km)
(1,2)	0.73	0	10
(1,3)	0.83	0	9.7
(1,4)	0.72	0	9.6
(1,5)	0.69	0	10.4
(1,6)	0.79	0	4
(1,7)	0.75	0	6
(1,8)	0.69	0	7
(1,9)	0.67	0	8.2
(2,3)	0.65	0	9.5
(2,7)	0.71	0	8.9
(2,19)	0.80	0	7.3
(2,21)	0.81	0	10.8
(3,4)	0.89	0	4.58
(3,17)	0.75	0	5.69
(3,18)	0.80	0	7.45
(4,5)	0.79	0	6.57
(5,6)	0.61	0	9.24
(5,14)	0.78	0	7.54
(5,15)	0.77	0	7.16
(5,16)	0.89	0	6.32
(6,9)	0.71	0	5.96
(6,11)	0.89	0	8.29
(6,12)	0.70	0	7.46
(6,13)	0.84	0	6.38
(7,8)	0.85	0	4.85
(7,22)	0.77	0	8.42
(8,9)	0.82	0	11.58
(8,25)	0.55	0	10.43
(9,10)	0.88	0	9.28
(9,11)	0.82	0	8.79
(9,25)	0.76	0	5.17
(10,30)	0.66	1	6.84
(12,13)	0.84	0	7.14
(14,15)	0.75	0	10.74
(14,26)	0.76	1	8.11
(16,17)	0.59	0	7.12
(18,27)	0.85	1	8.55
(19,20)	0.74	0	6.99
(20,21)	0.83	0	6.5
(21,22)	0.77	0	8.5
(22,23)	0.82	0	5.5
(22,28)	0.81	1	10.5
(23,24)	0.76	0	7.6
(24,25)	0.74	0	5.6
(24,29)	0.70	1	5.64

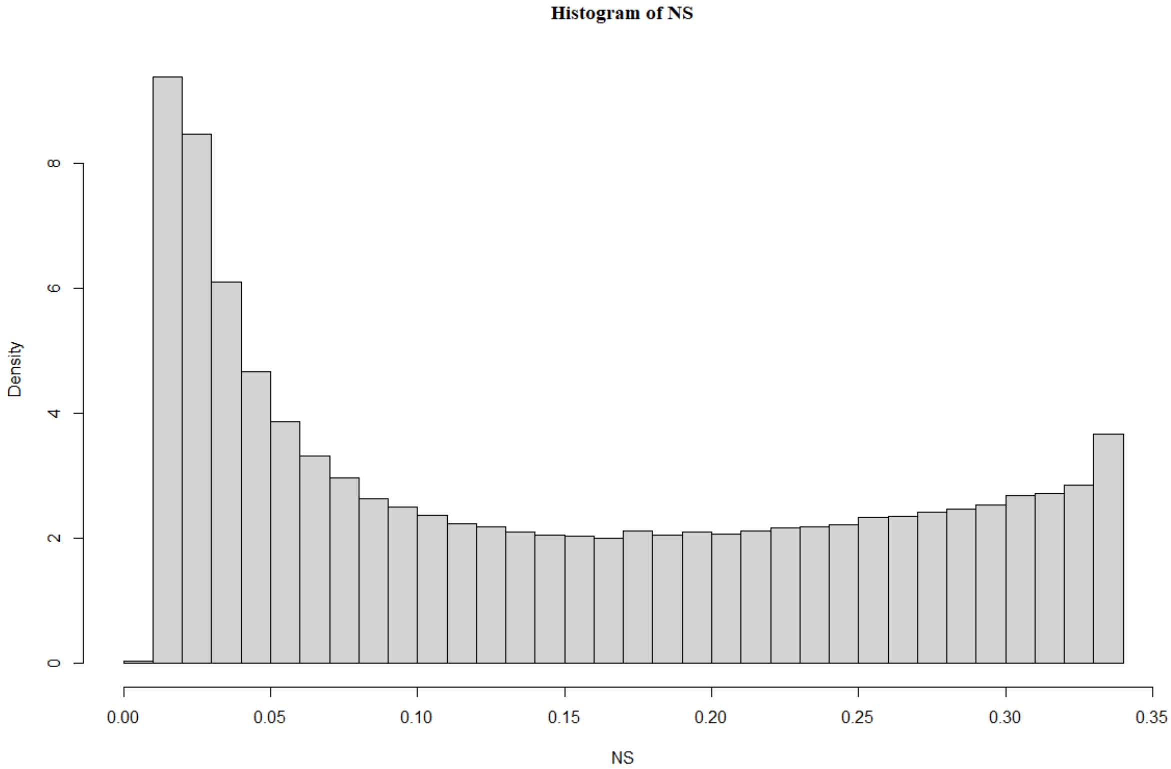

Figure 2 Histogram of Network Score (NS)

Resilience enhancement strategy one (changing the topology of the network):

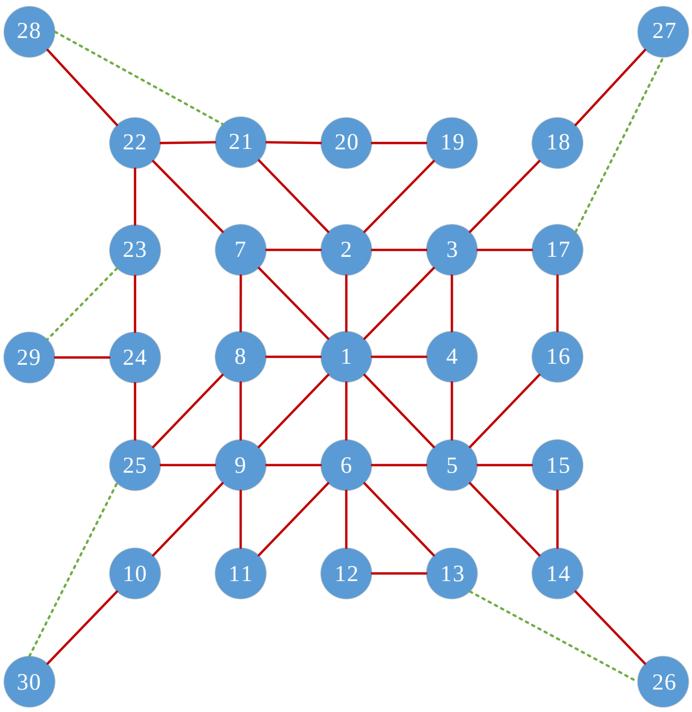

Figure 3 retrofitted network

This resilience enhancement strategy requires proposing new construction plans for enhancing the overall network's resilience with respect to VNs. This paper proposes adding 5 new edges to the network; Figure 3 shows the retrofitted network. Edges (13,26), (17,27), (21,28), (23,29) and (25,30) has been added to the network to eliminate the vulnerable nodes factor entirely. With these modifications, the baseline network score will increase up to $NS = 0.6811$ which is more than twice the baseline NS for the network before the retrofit operations. Also, by imposing the retrofitted network to the same failure scenario of semi-random edge removals with respect to extreme weather events, mean network score for this retrofitted network post-disruption will be $mean(NS) = 0.2994$ which is more than twice the mean NS for the original network before retrofit operations.

Resilience enhancement strategy two (Improving the reliability of roads):

This resilience enhancement strategy can be done in two different ways. The decision makers can either improve the reliability of all the roads by a small amount; or, to improve the reliability of certain important roads that contribute the most to NS. In the scope of this paper, the roads that lead to the VNs are the most critical roads since in case they are removed from the network, the VNs will become isolated from the main body of the network. This paper proposes to increase the reliability of the edges (14,26), (18,27), (22,28), (24,29) and (10,30) which will decrease the probability of isolation for the VNs, the reliability of this roads with the planned modifications and repairs will be close to as good as new conditions which is 0.99. The reliabilities of these edges and the improved ones are shown in table 4.

Table 4 Improved reliability data

Edge	Original Reliability	Improved Reliability
(14,26)	0.76	0.99
(18,27)	0.85	0.99
(22,28)	0.81	0.99
(24,29)	0.7	0.99
(10,30)	0.66	0.99

This resilience enhancement scenario will lead to a baseline NS of $NS = 0.357$ which is a 6.5% increase from the original network's NS. This slight increase in NS shows that, a small, directed enhancement of edge reliabilities can prove very effective in the overall NS of the network. Also, by imposing the enhanced network to the same failure scenario of semi-random edge removals with respect to extreme weather events, mean network score for this enhanced network post-disruption will be $mean(NS) = 0.152$, which is almost a 4% increase in the mean NS compared to the original network.

3. CONCLUSION

This paper introduced a framework for analyzing the resilience of a network before and after a disruption with an emphasis on vulnerable nodes and based on extreme weather events. The framework uses network theory and considers the concept of independent pathways, network topology, redundancies in the network, reliability of roads, importance of hub nodes and vulnerable nodes. The main objective of this framework is to integrate the vulnerable nodes into the resilience assessment of transportation networks, which is achieved by introducing a new metric vulnerable nodes factor that is utilized in the importance of the edges in the network. The framework allow for studying different scenarios to see the effects of different changes in the baseline value for NS; this shows that we'll the vulnerable nodes factor introduced by this paper will reduce the effective amount of IPWs between all node-pairs significantly, in the numerical example it can be seen that NS has decreased by 81%. There are two different resilience enhancement strategies has been proposed by this paper I) Changing the topology of the network II) Improving the reliability of the roads. In the numerical example illustrated by this paper, the first strategy proved to be more effective since by adding new roads connecting the

VNs to the main body of network, the VN factor was eliminated from the network, resulting in both baseline and mean NS being doubled. The secondary resilience enhancement strategy which increased the reliability of roads affected by VN factor, however, increased the baseline NS by 6.5% and mean NS by 4%; the secondary strategy is only recommended if implementation of the first strategy is either not physically possible or economically unreasonable. This study introduced a way for the marginal and vulnerable nodes to be entered into the discourse of transportation networks resilience analysis.

References

[1] M. Bruneau *et al.*, "A framework to quantitatively assess and enhance the seismic resilience of communities," *Earthquake spectra*, vol. 19, no. 4, pp. 733-752, 2003.

[2] S. Hosseini, K. Barker, and J. E. Ramirez-Marquez, "A review of definitions and measures of system resilience," *Reliability Engineering & System Safety,* vol. 145, pp. 47-61, 2016.

[3] D. Rehak, P. Senovsky, M. Hromada, and T. Lovecek, "Complex approach to assessing resilience of critical infrastructure elements," *International journal of critical infrastructure protection*, vol. 25, pp. 125-138, 2019.

[4] W. Liu and Z. Song, "Review of studies on the resilience of urban critical infrastructure networks," *Reliability Engineering & System Safety,* vol. 193, p. 106617, 2020.

[5] X. Zhang, E. Miller-Hooks, and K. Denny, "Assessing the role of network topology in transportation network resilience," *Journal of Transport Geography,* vol. 46, pp. 35-45, 2015.

[6] A. K. Hartmann, "Large-deviation properties of resilience of transportation networks," *The European Physical Journal B,* vol. 87, no. 5, pp. 1-10, 2014.

[7] W. H. Ip and D. Wang, "Resilience and friability of transportation networks: evaluation, analysis and optimization," *IEEE Systems Journal*, vol. 5, no. 2, pp. 189-198, 2011.

[8] W. Zhang and N. Wang, "Resilience-based risk mitigation for road networks," *Structural Safety,* vol. 62, pp. 57-65, 2016.

[9] R. Chan and J. L. Schofer, "Measuring transportation system resilience: Response of rail transit to weather disruptions," *Natural Hazards Review,* vol. 17, no. 1, p. 05015004, 2016.

[10] B. Donovan and D. B. Work, "Empirically quantifying city-scale transportation system resilience to extreme events," *Transportation Research Part C: Emerging Technologies,* vol. 79, pp. 333-346, 2017.

[11] A. Cox, F. Prager, and A. Rose, "Transportation security and the role of resilience: A foundation for operational metrics," *Transport policy,* vol. 18, no. 2, pp. 307-317, 2011.

[12] A. Alipour and B. Shafei, "Seismic resilience of transportation networks with deteriorating components," *Journal of Structural Engineering,* vol. 142, no. 8, p. C4015015, 2016.

[13] M. Batouli and A. Mostafavi, "Multiagent simulation for complex adaptive modeling of road infrastructure resilience to sea-level rise," *Computer-Aided Civil and Infrastructure Engineering,* vol. 33, no. 5, pp. 393-410, 2018.

[14] G. R. Patil and B. Bhavathrathan, "Effect of traffic demand variation on road network resilience," *Advances in complex systems,* vol. 19, no. 01n02, p. 1650003, 2016.

[15] B. A. Jafino, J. Kwakkel, and A. Verbraeck, "Transport network criticality metrics: a comparative analysis and a guideline for selection," *Transport Reviews,* vol. 40, no. 2, pp. 241-264, 2020.

[16] D. B. West, *Introduction to graph theory*. Prentice hall Upper Saddle River, 2001.

[17] E. W. Dijkstra, "A note on two problems in connexion with graphs," *Numerische mathematik,* vol. 1, no. 1, pp. 269-271, 1959.

[18] V. Latora, V. Nicosia, and G. Russo, *Complex networks: principles, methods and applications*. Cambridge University Press, 2017.