Coherence through inquiry based mathematics education

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SUM is a four-year research and developmental project with the aim of contributing to coherence in children's and students' motivation for, activities in, and learning of mathematics throughout the educational system from kindergarten to higher education. The concept of inquiry is key in the project, and it involves the implementation of different types of theories and methods related to inquiry based mathematics teaching (*IBMT*) *at three systemic levels:* (1) *The students' inquiry in and with mathematics.* (2) *The teachers' use of inquiry based mathematics teaching as a means for supporting the students' learning and inquiry into their own practice across a particular transition.* (3) *Inquiry into the interplay between the development of teaching practice and research in IBMT. In this paper, based on the project design and preliminary findings from the implementation, we discuss the project as an implementation of theory at these three levels, with a particular focus on level* 2.

Keywords: Professional development, teacher collaboration, inquiry, mathematics.

Introduction

In this paper, we present and discuss how the design of a four-year professional development project called SUM can facilitate the implementation of research findings related to Inquiry Based Mathematics Teaching (IBMT) as a means for creating better coherence across important transitions in the educational system. Strategically, the project focuses on five transitions in the educational system where particular challenges with developing and retaining students' motivation for and learning of mathematics are evident in teaching practice and in research. The principles behind the design of the project are based on research findings in the mathematics education literature specifically related to IBMT. Therefore, this paper can be seen as a case study on how professional developmental projects can be a vehicle for implementation of theories in the practice of mathematics teaching.

The project design is based on a broad understanding of IBMT going back to the educational philosophy of Dewey (Artigue & Blomhøj, 2013). More specifically, in SUM we conceptualize and operationalize the inquiry concept at three different systemic levels developed by Jaworski (2004, p. 24): (1) the children's and students' inquiry in and with mathematics; (2) the teachers' inquiry into their own practice and their use of IBMT as a means for supporting students as learners; (3) inquiry into the interplay between the development of teaching practice and research in IBMT. These three levels of inquiry have formed the project design and structured the application. In that sense, as a research and professional development project, SUM is an implementation of a general theory of inquiry also used in other projects (Bjuland & Jaworski, 2009).

Coherence is another key concept in the project. In SUM, the fundamental premise and proposition is that we can help create and sustain coherence at each of the three levels through inquiry. However, as with the term inquiry, the term coherence takes different meanings at each of the three systemic levels. At level 1, the focus is on children's and students' motivation for, activities in, and learning of mathematics. The term coherence here refers to how students can make sense of mathematics, maintain their joy of, and develop a deep understanding of mathematics. In other words, coherence means that students' experience mathematics as a meaningful, relevant, and consistent subject. However, we cannot expect students to develop these desirable characteristics on their own. Students need opportunities to develop productive habits and deep conceptual knowledge for creating and maintaining a joy of and to make sense of mathematics (Schoenfeld, 2014). This leads us to level 2.

At level 2, coherence refers to the teacher's practices and their knowledge and beliefs about IBMT and challenges related to students' learning in general, and across the particular transitions in the system of mathematics teaching in particular. For students to experience mathematics as a meaningful endeavor both in general and across transitions, teachers have to, among many things, create a learning environment that involves inquiry through investigative activities such as experiments, problem solving, and modelling. The aim of the project is that the teachers can develop mathematical and didactical competences, through inquiry into their own practice and IBMT, for a practice of teaching that provide students with these types of learning environments.

At level 3, the focus is on the coherence between the development of practices of teaching and research in mathematics education. Nearly all professional practitioners, including teachers, experience a gap between theory and practice (Schön, 1983). This dilemma is also closely related to what Nilsen (2015) calls determinant frameworks. Determinant frameworks describe and help identify types of determinants, which act as barriers and enablers that influence implementation outcomes. Determining barriers and enablers, both at a systemic and individual level, is a key element in creating coherence between research in IBMT and development of teaching practice

At each of the three systemic levels, coherence also refers to the issues related to academic transitions in mathematics. As students move through the school system, the mathematics instruction, opportunities to engage, expectations, mathematical concepts and ideas, classroom norms etc. are all likely to change – in particular in transitions between key academic levels. These transitions are a potential risk to students' academic progress, unless they are managed well (McGee, Ward, Gibbons & Harlow, 2003).

The basic premise of the SUM project, and central claim of this paper, is that IBMT has a potential for meeting these demands at the three systemic levels. In this paper we address the following research question with a particular emphasize on the implementation of inquiry at level 2:

In what way does the design of the SUM project support the implementation of IBMT as a means for creating better coherence?

To answer this question, we will explain how research related to IBMT has been implemented in the design of the project, and report on some preliminary findings that show how this design of the SUM project has helped create better coherence. In this paper we explain how inquiry is implemented – i.e. conceptualized and operationalized – in the design of the project at each of the three systemic levels, and how it can help create better coherence across academic transitions. Due to the limited scope of this paper, we then focus more closely on one of the systemic levels, and provide an example of how theory related to IBMT has been implemented at level 2. We highlight in particular a 3-phased didactical model that has been developed to help organize and design IBMT lessons. Finally, we present some preliminary findings from the project, and discuss how this implementation of inquiry at level 2 in the design of the project has helped teachers incorporate investigative activities in their own practice – and thus helped create a better coherence between teachers' practice of teaching and the intentions for the development of students' beliefs about mathematics and their mathematical learning.

The organizational structure of SUM

However, a quick description of the project is needed. SUM is the acronym in Norwegian for "Sammenheng gjennom Undersøkende Matematikkundervisning". In English, the title is "Coherence through inquiry based mathematics teaching". The project is based at the Artic University of Norway in Tromsø (UiT), and financed by The Norwegian Research Council for the period 2017- 2021. The SUM project is organized around five transitions in the educational system where students typically experience a discontinuity in their mathematics teaching:

Kindergarten (T1) → Primary school (T2) → Middle school (T3) →

Lower secondary (T4) → Upper secondary (T5) → University

The organizational structure of SUM is in accordance with what we know about effective professional development (PD). Research has shown that effective high-quality PD possesses a robust content focus, features active learning, is collaborative and job embedded, aligned with relevant curricula and policies, and provides sufficient learning time for participants (Desimone, 2009).

The SUM project has a robust content focus that is also aligned with current curriculum reforms in Norway. An important part of the work in the transition groups is to identify challenges related to the transition in question. These challenges are related to the learning of key mathematical concepts such as for example the number line as a model for natural, whole and rational numbers at the transitions T1-T3, or on how to support the coherence and progression across a transition with regard to particular mathematical competences such as for example mathematical reasoning or modelling competence. Such competences and deep learning of key concepts are playing a dominant role in an ongoing curriculum reform for mathematics teaching from primary to upper secondary level in Norway (Kunnskapsdepartementet, 2016).

The SUM project also has a clear focus on active learning, job-embedded collaboration, and sufficient learning time. For each of the mentioned transitions, a group of 8-14 mathematics teachers from school and/or kindergarten (as for T1) is formed. There are at least two teachers from each participating school. Each group is led by two or three mathematics educators from the project team at the university. Currently, around 55 teachers are participating in the project. For three consecutive school years each group work together on identifying and discussing challenges regarding students' learning of mathematics across a particular transition. In collaboration and with support from the group leaders the teachers develop, implement and evaluate IBMT with the explicit aim of helping the students to overcome the identified challenges and to contribute to a better coherence across the transition in focus for the group.

It should be mentioned that SUM includes a particular focus on IBMT as a means for developing a cultural responsive approach for supporting the coherence in mathematics teaching for Sámi children. One of the transition groups consist of teachers at a Sámi school working at both T1 and T2.

Implementing IBMT in the SUM project

As mentioned, the inquiry concept is coming into play differently at the three systemic levels. At level 1 and 2 it is in the form of IBMT, while at level 3 it is inquiry as part of the research process. The implementation of inquiry at each of the three systemic levels consists of a further conceptualization and operationalization, which is discussed below.

At level 1, inquiry means that students are learning mathematics through exploration in tasks and problems in the classroom. The aim is to help students develop and maintain the joy of, motivation for, and coherence in their learning. For students to experience mathematics as a meaningful endeavor, they need to, among other things, struggle with "important mathematics". The term struggle does not mean that students should waste time on extreme levels of challenges that lead to needless frustration. Struggle, in this context, means that students should expend effort, explore, investigate and make sense of problems and situations that are not immediately apparent (Hiebert & Grouws, 2007). This view of learning as a result of reflective inquiry and meaningful struggle is a key aspect of IBMT. In the SUM project, we put special emphasis on IBMT as a means for developing and maintaining the students' interests for and joy of working with mathematics, both in general and across the transitions. Through problem oriented activities the students should have the opportunity to experience the joy of solving problems, which they to begin with find challenging.

At level 2, inquiry means that teachers explore and reflect on the design and implementation of tasks, problems and activity in classrooms. The teachers use inquiry as a tool to explore teaching, alongside researchers and other teachers who offer both theoretical and practical support. The teachers also develop their practice through successive cycles of inquiry, working in their own classroom, interpreting a design they have produced in collaboration with researchers and other teachers (Jaworski, 2004). Through this inquiry into their own practice and IBMT, the aim of the SUM project is that the teachers can develop mathematical and didactical competences for a practice of teaching that provide students with learning opportunities that help them develop and maintain the joy of, motivation for, and coherence in their learning of mathematics – with a particular focus on challenges related to academic transitions. In the transition groups we present and discuss with the teachers a number of different activities spanning the options of IBMT at that particular transition, Together with the didactical structure presented below, these examples and the related discussion help the teachers in developing their own inquiry based activities.

At level 3, the focus is on how to facilitate interplay between developments of practice and research of IBMT. Here, inquiry is a tool used by both researchers and teachers for understanding the relationship between theory and practice. This is primarily accomplished through co-learning partnerships, in which researchers and practitioners are both participants in processes of using inquiry as a tool for understanding the relationship between theory and practice. Through a close collaboration in designing, implementing, and reflecting on IBMT, both teachers and researchers can develop a better understanding of each other's' worlds (Jaworski, 2004). The SUM project is designed for providing exactly such opportunities for collaboration over three school years between teachers representing different grade levels, and in some cases also different institutions, and researchers in mathematics education. In close collaboration the groups will go through the process of designing, testing and evaluating inquiry based courses three times each school year and nine times during the project. In that sense the design of SUM builds on what is known about developing IBMT in interplay with research (Boaler, 2008; Artigue & Blomhøj, 2013).

Implementing IBMT at level 2: The teachers' development of IBMT in their practice

In the beginning, the work in the transition groups focus on level 2. The teachers need support for seeing IBMT as a didactical means for engaging the students in formulating and solving mathematical problems and using mathematics to describe and analyze real life situations. In addition, it is important for the teachers to see that such activities can motivate students and, at the same time, help them understand mathematical concepts and ideas that are essential for their mathematical learning across a particular transition.

The teachers need support for developing and teaching IBMT in their own classes. To that effect at the first seminar in the transition group, the teachers were introduced to a 3-phased didactical model for structuring inquiry based activities or courses. The model was exemplified with different types of inquiry based activities and courses relevant for the particular transition. The examples variated with respect to duration from one-lesson activities to 5-8 lessons courses, the degree of freedom given to the students, and the degree to which the activities was stirred by a mathematical focus or by an extra mathematical problem or situation.

The 3-phased didactical model for structuring IBMT includes these elements:

Phase1: *Setting the scene for the students' inquiry work.* This phase could involve: telling a story, refer to or create student experiences to motivate the inquiry work; establishing a challenge or a problem in a context that makes sense for the students; creating classroom dialogues about the meaning of the situation; motivating the activity or problem—how could this be fun or interesting and important for life and/or mathematics?; establishing and communicating the didactic environment for the students' work, i.e. the temporal and practical conditions of the work; and presenting and arguing for the product requirements and assessment format.

Phase 2: *The students' independent (of the teacher) investigative work.* In this phase the students' should have: sufficient time, freedom, resources and support for their investigative work; support through dialogues with the dominant questions to groups or individual students being: What are you thinking?, How did you find out?, Why is it right?, What if ..?

Phase 3: *Supporting the students' learning of mathematics through sheared reflections in class.* In this phase the teacher should: let the students shear their experiences and results and related reflections; organize the students' presentation of their working process, products and results; facilitate classroom dialogues focused on the mathematical elements in the work; help systematizing the results for the class; pinpoint key ideas, concepts and methods in the students' work; try to build sheared mathematical knowledge in the class rooted in the students' work; use multiple mathematical representations and make connections to the students' previous mathematical knowledge.

This 3-phased model provide a good starting point for the collaboration among the teachers and between teachers and the researchers in the transition groups. The model was presented in the transition groups together with lists of essential student and teacher activities in each of the three phases. In addition, in each group some examples of inquiry based activities relevant for the transition in question were presented and discussed. The examples variated from relatively closed mathematical or practical problems over systems of connected problems forming a landscape of investigation to more open thematic investigations and modelling activities as illustrated in (Artigue & Blomhøj, 2013). Together, these elements supported the teachers in developing and planning inquiry based activities in their own teaching. At the transition groups meetings, the model structured the discussions and made it possible to focus on specific issues of IBMT at the transition in question. For instance, at the first meeting, the focus was on establishing a challenge or a problem in a context that makes sense for the students, and to set scene for the students' inquiry activities. This is in line with several other development projects on IBMT (see vol. 45, issue 6 of ZDM, 2013), where this or similar models has shown to be instrumental for the teachers' operationalization of IBMT.

A short example illustrates how the teachers used the 3-phased model. In this example, taken from upper secondary school, both the teacher and the students had little experience with IBMT. The students had been working on quadratic equations for a few weeks, and the teacher wanted to use IBMT in order for the students to get a better understanding of what each term in the equations meant. In one of the lessons, the teacher first set the scene by showing the students a visual proof of the identity $(a+b)^2 = a^2 + 2ab + b^2$. She then asked the students to make a visual proof of $(a-b)^2 = a^2 + 2ab + b^2$. During the second phase, the students' investigative work, she tried to provide constructive feedback that led the students in a productive direction. At the end of the lesson, the students were asked to present their solutions, and their findings were tied into how they could solve quadratic equations by completing the square.

This example illustrates how one of the teachers used the 3-phase model to implement, gradually, IBMT into her own teaching. Although this particular lesson is a structured and strongly guided example of IBMT, it is an example of how the 3-phased model helped the teachers to gradually include IBMT in their own teaching. The teacher expressed that the 3-phased model had helped her plan and implement IBMT when she first joined the project, as it had reduced the larger and more complex task of developing IBMT lessons into smaller sub-tasks in each of the 3 phases. In particular, the 3-phased model had helped clarify her role during an IBMT lesson.

Research context and methods

The main research goal of the SUM project is to contribute to the further theoretical development of IBMT and researching its potentials and limitations for overcoming challenges in the practice of mathematics teaching related in particular to the five transitions. In order to fulfill this and other sub-objectives, several methods of data collection have been, and will be, employed. In this paper we report on some preliminary findings specifically related to the implementation of IBMT at level 2. More detailed analyses will be presented in later papers, with more specific scopes. The findings in this paper are based on preliminary analyses of focus group interviews and recordings of transition group meetings throughout the first year in transition group five (TG5).

TG5 consists of six mathematics teachers from three upper secondary schools, three mathematics instructors who teach mathematics at tertiary level, and two mathematics education researchers. Each meeting of TG5 during the first year of the project was recorded. The participants of TG5 was also interviewed in focus groups at the end of the first year of the project. These transition group meeting recordings and focus group interviews covered a large set of subjects and themes, but for the purpose of this paper we focused only on statements and discussions related to the teachers' exploration and reflections on the design and implementation of IBMT in their classrooms.

Both the transition group meeting recordings and focus group interviews were first transcribed, and then analyzed using an inductive qualitative content approach. For each interview and transition group meeting, we extracted all text components that captured some aspects of the design and implementation of IBMT in the teachers' own classrooms. We then formed categories, across the interviews and transition group meetings, which summarized the teachers' experiences and reflections. In the next section we report on some of the key findings from this analysis.

Findings

We found that the 3-phased model, and the design of the SUM project in terms of inquiry at level 2, in general helped the teachers design and implement IBMT in their own lessons. There were several reasons for this, most of which were tied to specific challenges in designing or implementing IBMT.

One of the strongest findings was related to the design of IBMT. All of the teachers said they had little experience with open-ended activities and problems, and therefore found it difficult to design such activities themselves. One teacher said for instance that she "didn't know how to come up with problems and activities like the examples they were given in TG5". Other teachers expressed similar thoughts, and said it was difficult coming up with good tasks or activities that were open-ended, relevant for the content-specific syllabus, and also interesting for the students.

There were also several issues related to the implementation of IBMT lessons and tasks. The teachers found it slightly uncomfortable to give up control of the lessons, and allow the students to investigate and explore more freely. A majority of the teachers said that it was difficult to let students work on tasks and problems, without correcting the students. Three of the teachers even said that this lack of clear instructions and corrections could lead to students developing content-related misconceptions. They explained further that misconceptions had to be dealt with early and resolutely. Another challenge, was the large difference in terms of skills and knowledge between the students. Some students struggled with even the most basic aspects of the IBMT lessons, while other students solved the more difficult tasks with a fair bit of ease.

However, in spite of the challenges related to the design and implementation of IBMT, all of the teachers in TG5 expressed that the 3-phased didactical model alleviated to some extent these challenges. The teachers agreed that it provided them with a more structured approach for designing IBMT for their own practice. Instead of tackling a large and intimidating challenge of designing full lessons of IBMT, the 3-phased model reduced a large and complex task to several smaller less complex tasks. Together with the examples of IBMT provided in the TG5 meetings, the 3-phased model simplified a large and abstract task into a set of smaller and more concrete sub-tasks. Furthermore, the teachers said that the 3-phased model helped them remind them about their and their students' role at each phase of IBMT. As one teacher said, "it was a nice reminder of what she should do and what the students should do, and it gave a nice partition of the lesson".

We also noticed that the teachers seemed to find the collaborative and job-embedded structure, and inquiry-focus, of SUM to be meaningful and productive. The teachers said that developing, implementing, and reflecting on IBMT, in collaboration with other teachers, provided them with a better understanding of what IBMT is and how IBMT lessons can be developed. The teachers expressed in particular that implementing IBMT in their own classes, made the activities more important and relevant as the teachers had an "ownership" of the lessons. Furthermore, a majority of the teachers said that discussing both the model and the examples of IBMT in the TG5 meetings helped them to better understand the key characteristics of IBMT and the purpose of it.

Summing up in relation to our research question, at level 2, the 3-phased model with concrete examples and the organization for the work in the transition groups seem to be instrumental for implementing IBMT in order to help creating better coherence in the participating teachers' practice across the respective transitions. Within implementation theory the 3-phased model for IBMT together with the organization of the transition groups can be considered as an innovation for implementing IBMT in the practice of mathematics teaching. Rogers (2003) identify three levels of knowledge, which has to be developed among practitioners—here teachers—as a necessary condition for a successful implementation of an innovation in relation a problematic in an existing practice. Concretized in relation to the SUM, these are (1) knowledge of and experiences with challenges regarding the students' motivation and learning in mathematics teaching across the transitions; (2) knowledge about how to use the 3-phased model for planning, conducting and evaluating IBMT; (3) knowledge about and experiences with cases where the 3-phased model has been effective as a didactical means for supporting students' investigative activities and their learning of mathematical concepts and methods. So far, we have some evidence that the transition group meetings have enabled the teachers to shear experiences with and to reflect about challenges regarding coherence in the students' motivation for and learning of mathematics across the transition in question. The 3-phased model has been instrumental for structuring the presentations and discussions of IBMT in the transition groups. Also, the model has shown to be a tool for the teachers in designing and implementing IBMT in their own practice. It is still to be seen if the teachers will reach the third level of knowledge and experience IBMT as an effective didactical means for improving the students' motivation for and learning of mathematics across the transitions.

Large developmental projects running over several years and with funds for establishing close collaboration between selected teachers and researchers is one very important way in which theories can come into play in the development of teaching practice – as support by Rogers (2003). Therefore, it is relevant to analyze the role and function of theories from mathematics education research in the design of such research based developmental projects. In this paper, we have analyzed how elements of theory related to IBMT come into play in the SUM project at the systemic level focusing on the teachers' experimental practice and their related reflections.

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