Relations Between Sámi and Western Ways of Sorting and Organizing Elements in a Set

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Abstract

This paper describes three examples of Sámi mathematics. These examples, which are associated with different cultural contexts, show similarities to what Western mathematics calls 'combinatorics'. All examples are cultural practices that show descriptive use of mathematical knowledge. These are not mathematical problems and thus there is no need to find an exact number of possibilities. We describe the cultural practices by connecting them to the framework 'Cultural symmetry'. Two of the practices, the Sámi braiding ruvden and the traditional reindeer ear marking, show how sorting and organizing items results in a variety of labels that provide further information. The third practice, the use of birccut in the Sámi board game Sáhkku, displays many possibilities for how a player could move the pieces and develop strategies. Based on our findings, we suggest that combinatorics is important for prospective Sámi mathematics curriculum.

Keywords: Sámi traditional knowledge, combinatorics, Indigenous, cultural symmetry

Oktavuođat gaskkal sámi ja oarjemáilmmi vugiid sirret ja organiseret áđaid

Abstrákta

Dát artihkal čilge golbma ovdamearkka mii sámi matematihkka lea. Dát ovdamearkkat vižžon iešguđet osiin sámi kultuvrras gullet dasa maid oarjemáilmmi matematihkkarat gohčodit kombinatorihkkan. Buot ovdamearkkat čájehit matematihkalaš gelbbolašvuođa anus kultuvrralaš práksisas. Dát eai leat matemáhtalaš čuolmmat, ja nu ii leat dárbu gávdnat galle vejolašvuođa leat. Moai čilgejetne kultuvrralaš práksisiid "Kultuvrralaš symmetriija" rámmavuogádagain. Guokte ovdamearkka, ruvden ja bealljemerken, čájehit mot oažžu eanet diehtu juogaman áššis dainna go sortere ja organisere áššiid ja nu oažžu ovdán oktasašvuođaid. Goalmmát práksis, Sáhkku spealu birccut, čájeha ges ollu vejolašvuođaid mot spealli sáhttá sirdit olbmáid ja nu strategiijaid ráhkadit alcces. Munno suokkardallan čájeha ahte kombinatorihkka lea dehálaš boahttevaš sámi matematihkkaoahppoplánai.

Čoavddasánit: *Sámi árbevirolaš máhttu, kombinatorihkka, eamiálbmot, kultuvrralaš symmetriija*

Introduction

Three different approaches to identification of what Sámi (Indigenous) mathematics is and might be, show similarities with what Western mathematics calls 'combinatorics'. Examples of practices from three different Sámi cultural contexts provide insight into how 'combinatorics' emerge. The practices are: i) the Sámi braiding *ruvden* (Fyhn et al., 2014; 2015; 2017), ii) the traditional marking of reindeer ears (Steinfjell, 2021) and iii) rolling the *birccut* (dice) in the Sámi board game *Sáhkku* (Fyhn, 2020). The Sámi are an Indigenous people of the Arctic who inhabit Sápmi: northern parts of Norway, Sweden and Finland, and the Kola Peninsula of Russia. There are a total of ten different Sámi languages. Religion and other cultural expressions vary between the different Sámi areas. This paper focuses on North Sámi contexts; the North Sámi language area covers the northern parts of Norway, Sweden and Finland. There is no Sámi mathematics curriculum and the schools are lacking Sámi culture based teaching material. Jannok Nutti's (2013) study from Sweden revealed that no Sámi culture-based mathematics teaching takes place, because the Sámi teachers must adapt their teaching to the local culture.

School mathematics in Western societies is often decontextualized, and the teaching of ethnomathematical issues in majority languages like English and French is rarely problematised (Trinick et al., 2016). As a consequence, activities are taught regardless of the culture and the language in which they are embedded. Battiste (2015) points out that Indigenous peoples experience tensions created by a Eurocentric education system that has taught them to distrust their Indigenous knowledge systems. Our study reveals combinatorical competence embedded in Sámi knowledge systems.

Combinatorics is a branch of [Western] mathematics that is used for studying enumeration, combination and permutation of sets of elements and the mathematical relations that characterize their properties (Weisstein, 2022b). However, there is no Sámi word for combinatorics.

The first two practices presented herein involve sorting and organizing elements according to the sets in which they belong; *ruvden* cords show an individual's home area and familial relationships, and reindeer ear markings show who is the owner of an animal. The third practice considers how to calculate risks for where to move your pieces in a board game. In this paper, we analyze our previous publications that concern these cultural practices. The first author, professor II at the Sámi University of Applied Sciences, has cooperated with Sámi mathematics teachers and researchers for more than a decade. The second author is both North- and South Sámi. She holds a master's degree in mathematics education and 60 credits of *duodji* (Sámi handicraft). She has cultural knowledge of traditional ear marking. She is a general teacher with broad working experiences in addition to research experience. The aim of this paper is to contribute to a prospective Sámi mathematics curriculum by providing examples of Sámi mathematical reasoning. Our research question is, how does the sorting and organizing of elements in Sámi cultural practices relate to Western mathematics?

To illuminate this research question, we use the framework 'cultural symmetry' (Trinick et al., 2016; Meaney, Trinick, et al., 2022) for discussing relations between 'combinatorics' and the sorting and organizing of set elements in each cultural practice. The framework is based on the point that language, cultural practices, and mathematics, must be balanced if mathematics education is to contribute to decolonizing the education process. The second author's background is crucial for the quality of the analysis; her language skills are at mother tongue level and in addition she knows how the researched contexts are embedded in the culture. The framework includes three steps. In Step 1 we identify how sorting and organizing set elements is valued within Sámi traditions and expressed in Sámi language. Step 2 concerns exploration of different perspectives on cultural traditions and practices. In Step 3 we discuss how mathematics can add value to Sámi ways of sorting and organizing set elements, without diminishing the cultural understanding.

In 2007, Norway ratified the UNESCO (2003) convention of safeguarding intangible cultural heritage. By intangible cultural heritage, the convention means practices, representations, expressions, knowledge, and skills – as well as the instruments, objects, artifacts and cultural spaces associated therewith – that communities, groups and, in some cases, individuals recognize as part of their cultural heritage.

> This intangible cultural heritage, transmitted from generation to generation, is constantly recreated by communities and groups in response to their environment, their interaction with

> nature and their history, and provides them with a sense of identity and continuity, thus promoting respect for cultural diversity and human creativity. (p. 5)

In the implementation of the UNESCO convention, Norway will place a special focus on Indigenous people and national minorities (Ministry of culture and equality, 2021). The three cultural practices discussed in this paper belong to Sámi intangible cultural heritage.

Step 1: Cultural Understanding and Values

In this section we identify and value the three cultural practices expressed in Sámi language.

Ruvden

Sámi handicraft, *duodji* (North Sámi)/*duedtie* (South Sámi) is anchored in Sámi culture and Sámi values. According to Dunfjeld Aagård (1989), ornamentation is a central part of a *duedtie* product, because the product is not considered as complete before it has ornamentations or other decoration with symbolic colors. Sámi ornamentation has a double function, it is an aesthetical expression as well as a way of communication. Ornamentation communicates ethical attitudes and Sámi ways of being in addition to injunctions and prohibitions regarding nature and culture (Dunfjeld, 2001/2006).

The intangible cultural heritage includes how to perform the braiding process, how to teach younger generations about the braidings and the choice of colors, and the use of *ruvden* cords, typically where you need a round shaped cord – to fasten clothing and fur shoes, for closing leather and fabric bags, etcetera. Details of the use of colors provide information about the individual who wears it. This applies to all homemade clothes. A trained eye can spot where an individual is from, what family/clan they belong to, and often who has made the garment.

When a group of researchers (two Sámi and one non-Sámi) and Sámi teachers investigated how to approach *ruvden* as a basis for teaching mathematics (Fyhn, et al, 2017), two Sámi teachers presented their approaches at the first meeting. A narrative that focused on behavioral rules for visiting your neighbor's *lávvu* was an allegory for how to perform the braiding (Steinfjell, 2013; Fyhn et al., 2017). A *lávvu* is a traditional Sámi dwelling, like a Native American tipi. When braiding, each hand represent a *lávvu,* while each woolen thread represent a family member. Using narratives as a means for child upbringing is common in Sámi traditional child rearing (Balto, 2005). Use of narratives as a teaching tool is a Sámi value. Another teacher provided a systematic, detailed and stepwise presentation of the *ruvden* procedure.

Reindeer Earmarking

Reindeer earmarking is used to clearly identify who a reindeer belongs to. It is not acceptable to assign a unique random mark to a baby child, because earmarking shows the child's familial relationships. The calf's earmarking will share similarities with either the (child's) mother or father's earmark. The earmarks also have an aesthetic dimension. A cut is supposed to be cut in a manner that makes it easy to see and recognize, or *čábbát merkejuvvon*. *Čáppa mearka,* a beautiful earmark, is highly valued. Some marks are easier to remember and recognize, like the mark in Figure 1. It is almost symmetrical with few cuts. This is an old earmark that has been passed down through generations. The mark belonged to the owners' great-great-grandfather, and could be even older. The earmarking is considered beautiful and special.

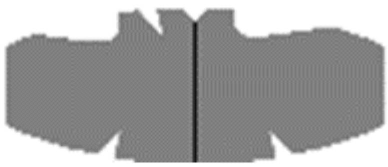

Figure 1: A beautiful reindeer ear mark. Screenshot from reinmerker.no

A beautiful earmark is clear and neatly cut. The markings should not be too coarsely cut, because the ear might bleed for a longer time than necessary. One aspect of the mark is that it should not be easy to alter later on, meaning that no one should be able to easily re-mark your reindeer to steal it. Lastly, marks are inherited and a family will have a few main marks that are easily differentiated; a baby cannot receive a mark that is very different from its family's. Children are taught to mark their own calves from a young age, and are expected to learn their families' marks. A child that is interested in reindeer husbandry will be given more responsibilities as s(he) grows older, and both marking and knowing earmarks are important competencies to master.

The Sámi use a small sharp knife to cut the ear markings. At large, the marks are somewhat similar across Sápmi, but have small differences from region to region. The differences are not substantial, due to the fact that the techniques and marking tools are the same all over Sápmi. The reindeer calves' ears have an elliptical shape with pointy ear tips. Figure 2 shows how the ear parts are organized[1]. The illustration is from a South Sámi publication.

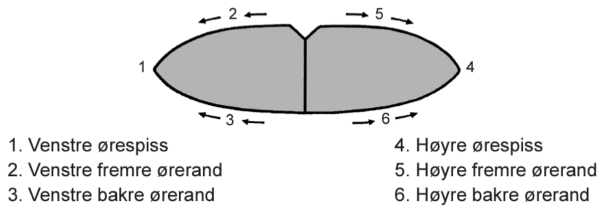

Figure 2: Forklaring av reinmerker. [Explanation of reindeer marks]. Reprinted from *Mïerhkh* (p. 13), by Aajege, 2011, Aajege. Saemien gïele- jïh maahtoejarnge. Reprinted with permission.

The calf is to be marked typically between one to four months old. The type of markings are different at the tip of the ear compared to the types that can be made on the side of the ear. Figure 1 shows the personal mark of the second author's son. The illustration shows both ears, the left ear's tip is to the left and the right ear's tip is to the right. Both ears are cut straight over, (*gaskat* in Sámi). The left and right ears are almost similar, but the left ear has one v-cut in addition to the other two cuts. The upper part of the left ear has two types of cuts, the right ear only one. This mark has only three types of cuts in total.

The tip of the ear can have three types of cuts: a straight cut either straight over or angled somewhat to the front or to the back, a curved cut angled front or back, and a cut that makes the ear tip split in half. The front and back sides of the ear can similarly have a rounded cut or v-like cut in a range of widths. The width of the cut depends largely on how many smaller cuts the mark has on that side. If you have only one v-shaped cut in the front of the left ear, it can be slightly broader than if you need to put three v-shaped cuts there. It is possible to have both v- and u-shaped cuts in the same position. In addition, one can make a small hole in the ear. These types can also be

[1] 1. Venstre ørespiss = left ear tip, 2. Venstre fremre ørerand = left front ear edge, 3. Venstre bakre ørerand = left back ear edge. Silimar for 4-6, where 'Høyre' = Right.

combined, so you can have a straight cut in the tip of the ear, with a small v-shaped cut inside of it and so on. These cut options accommodate a huge range of possible marks. For further information: Download the app 'MerkeAppen' or check reinmerker.no for marks in the Norwegian part of Sápmi.

Sáhkku

The traditional Sámi board game *Sáhkku* is a newcomer in Sámi culture, compared to *ruvden* and ear marking. In Schefferus' book Lapponia from 1673, the priest Olaus Sirma mentions a dice game that Sámi used to play with stakes (Berg-Nordlie, 2019). This game might be *Sáhkku*. The word *Sáhkku* means 'fine' or 'mulct' (Nielsen, 1962/1979). For various reasons, the game almost went extinct, but *Sáhkku* has been revived in recent years. The game is a relative to Backgammon and other board games that have travelled from areas further south and developed into new variations. A related game, Daldøs, is and was played in Coastal areas of Denmark and in Jæren in Southern Norway. This shows that these board games gained foothold there and in Sápmi, but not in central parts of Denmark-Norway, where policy makers traditionally live.

Sáhkku is a game for two players. Each player has a given number of soldiers; this can provide associations with Backgammon pieces/checkers or Chess pawns (Fyhn, 2020). In addition, each player has a queen. There is one king in *Sáhkku* and the king belongs to the player who last conquered it. There are different rules for the game in different areas. Berg-Nordlie (2018; 2019) describes the rules of the game for those who are interested. Figure 3 shows the start position. The black pieces are *nissonat* (women) because a single deep cut is a traditional way of shaping the female pieces. The white pieces are *dievddut* (men).

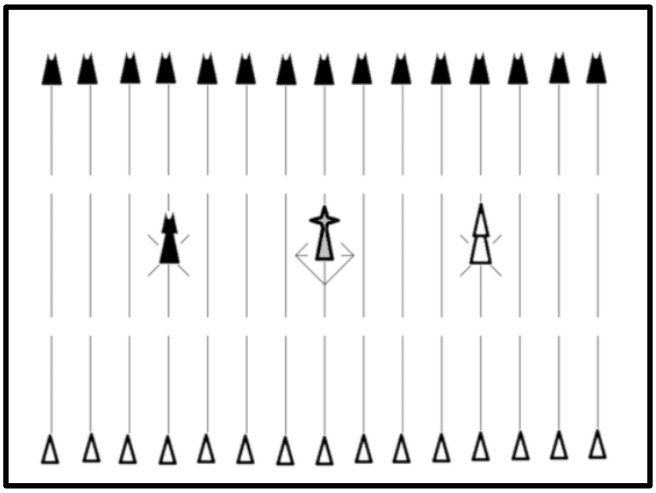

Figure 3. ***Sáhkku*** **board with pieces. Drawing: Mikkel Berg-Nordlie. Reprinted from "*Sáhkku*" by M. Berg-Nordlie, 2019.** ***Reaidu*****. Reprinted with permission.**

Step 2: Different Perspectives of the Cultural Practices

Here we explore different perspectives of the three cultural practices.

The Organization of Colors and Threads in *Ruvden*

Regarding the two teacher's presentations of *ruvden* referred in Step 1, some of them argued that maybe the narrative presentation would work for primary school children, but they preferred to use a stepwise introduction for lower secondary students (Fyhn et al., 2015). A Sámi researcher showed her schoolwork where the braiding procedure was represented by drawings of cords in different colors. She also explained how various numbers of threads were used for making cords for various purposes.

Two students participated in a video about *ruvden* and mathematics (Fyhn et al. 2014). They presented the four *ruvden* cords in Figure 4, and one cord that was not *ruvden*. The girls distinguish between the verb *ruvdet* and the noun *ruvden* and explain that the *ruvden* cords are round in shape. They use the term *ruvden* for braiding with both four and eight threads; this is an example of Sámi generalization.

Figure 4: Four *ruvden* cords made by the students Ronja Kristine Gaup and Ann Kristina Somby Gaino. Printed with permission.

When you *ruvdet* with two threads in one color and two threads in a contrasting color, you have two possible outcomes depending on the organization of the cords, the rightmost cord in Figure 4 is an example of this. When you *ruvdet* with four different colors, you have three different outcomes, depending on what combination of colors you have in each hand. The leftmost cord is *ruvdet* by white, yellow, red and blue. The lower and upper parts of this cord provide two different alternatives.

Marking of Reindeer Ears

A reindeer herder needs her/his mark to be easily differentiated from other marks in a moving herd. At a minimum, (s)he needs to be able to differentiate her/his own family's reindeer from other families'. Siblings can have earmarks where the only difference is the number of v-shaped marks in the front of the left ear. To differentiate these two marks, you would likely need to grab and hold the reindeer still so you can stroke the ear to feel with your fingers whose reindeer it is. This is both strenuous and time consuming, and is not necessary to do very often, since one family's reindeer are together all year long. In the summer, the herd is very big and spread over large areas in the pastures. This requires binoculars to see whose reindeer are present. In the winter, the herd is separated into smaller herds with a few families' reindeer. It is only when the reindeer are gathered in the pen or lassoed that you can stroke the ear to distinguish exactly who's reindeer it is.

The organisation NOAH, *For Animals' rights*, has critisized the traditional ear marking of reindeer calfs (Utsi & Aas, 2022). A veterenarian in NOAH argues that this tradition is against animal welfare because the calves' ears are cut without anesthetia. She does not suggest how to provide anesthetia to the calves in a careful way. A board member of Norwegian Reindeer Sámi' National youth organisation, argues against her view: "You will need much physical contact with the reindeer before you can find out who the owner is. That is a tougher strain for the reindeer than cutting its ear once and use binoculars the rest of its life to identify the owner" (author's translation). This gap between Indigenous cultural practice and the major society's misinterpretation of a culture they do not know exists in other parts of the world too. Meaney, Fyhn, et al. (2022) document and discuss more examples.

Sákku Birccut

Exploration of the game caused that we focused on the three *birccut*, dice. *Sáhkku* is also called *bircun,* which means dice rolling (Berg-Nordlie, 2019). You roll the three *birccut* simoultaneously and a piece can move the number of steps that one or more *birccut* shows. Figure 5 shows that each *bircu* has twelve side surfaces. Opposed to ordinary dice, a *Sáhkku bircu* can land on only four of its sides. This means that rolling one *bircu* has four possible outcomes: 'Blank', II, III and X. 'Blank' is similar to zero. II means 2 and III means 3. The *Sáhkku* symbol, X, can be used in three ways: i) Start one piece, ii) move one piece one step, and iii) roll the *bircu* once again in hope for a better outcome.

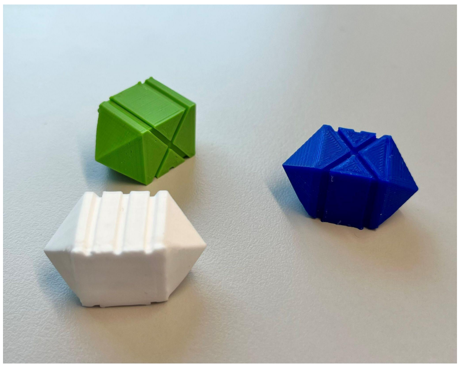

Figure 5: *Sáhkku birccut.* Photo: Anne Birgitte Fyhn

Step 3: How Western Mathematics Might Add Values

The first investigations of *ruvden* revealed perspectives of how algebra provided insight into *ruvdet* braiding with a large number of threads and why the number of threads had to be divisible by four (Fyhn et al., 2015), while Fyhn et al.'s (2017) study revealed how combinatorics added insight into how to get an overview of how threads in different colors provide different *ruvden* cords.

From a Sámi cultural perspective, each of the three practices show how you can express a large number of possibilities. The purpose is not to calculate the exact number of possibilities. For *ruvden* and ear markings, the purpose is to express belonging and relationship in a clear way that is easy to grasp. From the perspective of Western mathematics, all three cultural practices can be denoted as combinatorics. The further text focuses on how each cultural practice relates to combinatorics.

Combinatorics – Ancient Chinese and Hindu Mathematics

Combinatorics is an old form of mathematics, according to Biggs (1979) its first descriptions stem from more than 1000 years BC. The Egyptian Rhind papyrus from about 1650 BC is one of the oldest surviving mathematical manuscripts (Biggs, 1979). One of its problems deals with the summation of a series of powers of 7. Alongside the numbers hieroglyphs are also shown, which might be translated like this:

Houses	7
Cats	49
Mice	343
Wheat	2401
Hekat	16807
	19607

The Rhind papyrus was 'found' in 1858, and the interpretation of this problem was a mystery for some years until it was realized that Fibonacci (ca 1170 - 1250) had described a similar problem centuries earlier. There is no way of proving that Fibonacci's problem stems from this old papyrus, but the similarity is obvious. Fibonacci is known as the mathematician who introduced combinatorics to Europe.

The Chinese Book of Changes, or *I Ching*, from 700 BC describes how to order the two symbols Yin and Yang in strings of three or six symbols (Biggs, 1979). According to Western mathematics the strings are ordered outcomes with repetition and the number of strings are $2^3 = 8$ and $2^6 = 64$ respectively. The general expression for *r* different symbols in strings of totally *n* elements is n^r. The oldest Hindu description of combinatorics stems from around 600 BC, from the book Bishnagratna. This book presents a discussion of the various kinds of taste which can be made by combining six basic qualities: sweet, acid, saline, pungent, bitter, and astringent. In Western mathematical terminology, this is denoted as the binomial coefficient; the number of subsets of size *r* which can be formed from a set of *n* items, or the number of combinations without repetition of *n* things taken *r* at a time.

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

According to Biggs, there are reasons to believe that the Hindus around 600 AD knew the rule for calculating the binomial coefficient.

The above examples, together with the point that the ancient Greeks hardly contributed to the history of combinatorics, show that combinatorics is a relative newcomer into Western mathematics. The Sámi approach to combinatorics, opposed to the examples from the literature, is interpreted to reveal mathematics as a descriptive way of thinking that allows for many distinguishable possibilities.

Relations Between *Ruvden* and Combinatorics

As for *ruvden*, there is a huge range of mathematical possibilities with yarn in different colors. For *ruvden* with four threads in the same color, there is only one braiding pattern to make. If there are two colors, you have two possible solutions, both with two threads in each color. The alternative of three threads in one color and the fourth in a different color is not used in *ruvden*. When the number of colors is three, you can make six possible cords. Different positions of the threads will not necessarily provide different cords, because of the cyclic repetition of colors. In total, you can make 31 different cords with access to yarn in four different colors. You can organize and list all possible combinations and count them if there is a need for knowing the exact number of possible combinations. However, for practical daily use, there are more options than needed. Some of the mathematical options will not be used because the cord will look strange or different than the cords conventionally do. For example, three threads of one color and a single thread of another color would result in a cord with a dotted line down one of the sides, which is not standard for *ruvden.*

The Western mathematical approach of explaining these potential combinations is the number of ways of picking r unordered outcomes (without repetition) from n possibilities (Weisstein, 2022a), without repetition. The expression for this is the binomial coefficient, presented in section 4.1. This means that if you have yarn in red, blue, white and yellow and you want to pick two different colors, then you pick 2 colors from 4 possible choices, without repetition. In this case, $n = 4$ and $r = 2$, and you have 6 possibilities. As shown in Figure 4, you have two possible options for *ruvden* with two threads in each color. The binomial coefficient is not needed if you want to list the number of cords you can *ruvdet* with four threads. However, if the number of threads increases to eight, the binomial coefficient can be useful in the case that you need to know the number of options. In modern Sámi design that uses traditional *duodji*-items, it can be useful to know the range of possibilities.

Even if the number of possibilities is high with four threads, many *duojárat* (*duodji* craftsmen) choose to use eight threads. Not necessarily to use more colors, but to be able to use thinner yarn so that the result looks more elegant.

Relations Between Traditional Ear Marking and Combinatorics

The mathematics interpretation of the highest possible number of distinctly different earmarks, is combinatorics. There are six positions where you can put several markings, see Figure 2. This produces an overwhelming amount of different earmarks. However, many of these are not in use, due to the preference for marks to have few cuts. The earmarking must be easy and fast to cut and easy to spot for the owner, while also considering animal welfare.

If there are many reindeer owners in the same area, the number of ear marks increases. The possibilities of ear marks are very substantial. First of all, you have the six positions of the ear where *a* stands for how many cuts can be in the tip of the ear, and *b* stands for how many different cuts can be on the four sides of the ear.

$$p = a_1 \times b_2 \times b_3 \times a_4 \times b_5 \times b_6$$

With $a = 9$ and $b = 5$, then $p = 50\ 625$. In the language of Western mathematics, the total number of possible cuts is “ordered outcomes with repetition. However, the number of possible cuts is not the same in all Sámi areas, because regional traditions vary. Some cuts have the option to be repeated or combined with other cuts, so the possibilities are greater. An expression for all the possibilities would also have to distinguish between wider cuts that can have one narrow cut beside them, and narrow cuts that can be in groups of up to three at positions 2, 3, 5 or 6. The wider cuts can also have additional cuts inside them. Figure 6 shows a *gaskat* with two *sárggaldat* in it.

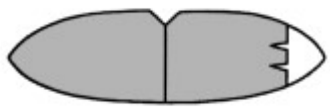

Figure 6: Andre merker [Other marks]. Reprinted from *Mïerhkh* (p. 11), by Aajege, 2011, Aajege. Saemien gïele-jïh maahtoejarnge. Reprinted with permission.

Relations Between Rolling *Sáhkku Birccut* and Combinatorics

Fyhn (2020) describes three issues in *Sáhkku* that are relevant for mathematics: a) the use of strategies and strategic reasoning, b) problem solving and c) combinatorics. The strategies are aspects of solving different problems that occur during the game. Among other things, *Sáhkku* can offer combinatorics problems for those who are interested. Rolling two ordinary dice, for instance in Backgammon, results in 6 x 6 = 36 different outcomes. Rolling three *Sáhkku birccut* offers 4 x 4 x 4 = 64 possible outcomes, but the outcome X, III and II is the same as III, X and II for instance. In Western mathematics, this is 'unordered outcomes with repetition', as described by the Chinese 600 B. C. Table 1 presents the 20 different outcomes of rolling three *Sáhkku birccut*.

X X X	0 0 0	II II II	III III III	X 0 II
X X 0	0 0 X	II II X	III III X	X 0 III
X X II	0 0 II	II II 0	III III 0	X II III
X X III	0 0 III	II II III	III III II	0 II III

Table 1: Different outcomes of rolling three *Sáhkku birccut*. Reprinted from "*Sáhkku* - et spill for Fagfornyelsens kjerneelementer" by A. B. Fyhn, 2020, *Tangenten – tidsskrift for matematikkundervisning, 31*(3), p. 16. Reprinted with permission.

Finding the number of *different* outcomes of three *birccut* is a non routine problem for lower secondary students. They have to practice trial and error to come up with a strategy for solving the problem, because they do not have access to any known formula to apply to this situation. The threshold for trying to solve the problem is low, because you do not need much mathematical knowledge to give it a try. The Western mathematical approach in this case asks for the number of ways of selecting *r* objects from a set of *n* elements, with repetition. The solution is

$$\frac{(n+r-1)!}{r!\,(n-1)!} = \binom{n+r-1}{r}$$

For the situation of selecting 3 birccut outcomes from 4 possibilities, *r* is 3 while *n* is 4 and the solution is 20.

Discussion and Conclusion

An important outcome from our study is that the descriptive perspective of 'combinatorics' that appears in *ruvden* and traditional reindeer ear markings, differs from the curriculum interpretation of combinatorics as a problem solving issue. Dunfjeld (2001/2006) describes the use of South Sámi triangular engravings, *gulmien borth*: "By varying combinations of triangular engravings, each combination can be interpreted as a sign or a symbol that is representing something else" (p. 79, authors' translation). She points at how varying combinations allows for new expressions. For the combination of structures, lines and ornaments, she has listed 21 + 12 combined structures. Her presentation indicates more of a problem solving approach than the three cultural practices in our study.

Table 2 shows how the Western mathematics concept cominatorics can add value to Sámi cultural reasoning. The three three examples in our study represent different categories of combinatorics. This categorization works as a tool that highlights and reveals a way of reasoning that is embedded in Sámi culture.

	Ordered Outcomes	Unordered Outcomes
with repetition	The total number of ear markings	The number of different *birccut* combinations
without repetition		The number of color combinations in *ruvden*

Table 2. Categorisation of the combinatorics examples

One main subject area in the previous mathematics curriculum (Ministry of Education and Research, [KD], 2013), is 'Statistics, probability and combinatorics'. The Norwegian government's white paper (KD, 2016) about renewal of school subjects, pointed at a need for reducing the number of knowledge areas in all subjects. The aim was a curriculum with focus on in depth learning. A stronger focus on combinatorics might have caused a more positive attitude towards Sámi mathematics. This is in contrast to the new curriculum, where the knowledge area's new name is reduced to 'Statistics and probability' (KD, 2019). The word 'combinatorics' does not appear in the mathematics curriculum for grades 1- 10.

The only appearances of probability and combinatorics in the present curriculum are in the competence aims for year 5 and year 9. After year 5, the pupil is expected to be able to "discuss randomness and probability in games and practical situations and relate this to fractions" (p. 9). After year 9 there are two aims, "calculate and assess probability in statistics and games", and "simulate outcomes in random trials and calculate the probability that something will happen by using programming" (p. 14). Our interpretation of the curriculum's focus on games as a central issue in probability is that playing *Sáhkku* is highly relevant for implementing Sámi cultural contexts into mathematics education.

The removal of combinatorics from the national curriculum's subject areas shows how the present curriculum increases the obstacles for including Sámi cultural practices like *ruvden* and traditional reindeer ear marking. The present curriculum is an argument for a) creating a separate Sámi mathematics curriculum, as researchers have argued for during the last decades and b) including Sámi perspectives of combinatorics into the national mathematics curriculum. Based on our findings, we suggest that 'combinatorics' should be allocated significant space when the Sámi mathematics curriculum is created.

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