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Study of Virtual Inertia and its impacts on Power Oscillations

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ABSTRACT

This study explores the changing dynamics of power systems in the context of the global shift to renewable energy, focusing on inertia and its influence on frequency stability. Traditional stability mechanisms, centered around rotating masses like synchronous generators, face challenges due to the integration of renewables and frequency converters, resulting in a reduction in overall system inertia. The report includes a detailed literature review, specifically the understanding of the role of inertia in traditional power systems and its connection to frequency response. Focusing on the methods and research work mentioned in the paper "Adaptive Virtual Inertia Control Strategy of VSG for Micro-Grid Based on Improved Bang-Bang Control Strategy." The research methodology involves simulation studies demonstrating methods to estimate inertia and evaluating the potential of synthetic inertia from Virtual synchronous generator to enhance frequency stability, particularly in systems with reduced inertia.

The work done in this thesis is divided into two phases. In the initial phase the focus was to do a literature review, selection of a relevant published article for the implementation of Virtual Inertia. After the selection of the research article "Adaptive Virtual Inertia Control Strategy of VSG for Micro-Grid Based on Improved Bang-Bang Control Strategy," the proposed model in the paper, small signal model was implemented which was only possible by comprehensively understanding the working of Virtual Synchronous Generators. In the later phase, after implementation of the small signal model, a micro grid model is implemented that too from the base research article to get as close to the real-world scenario as possible. These implementations done using MATLAB helped in understanding the issue that can be faced in the absence of virtual inertia and what method can be used to mitigate these issues. The results for both the implementations are discussed in detail emphasizing the importance of virtual inertia and how Bang-Bang control strategy can be used to minimize the frequency deviation.

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Abbreviations

VSG	Virtual synchronous generator
IBR	Inverter Based Resources
PWM	Pulse Width modulation
IBB	Improved Bang-Bang
SSM	Small Signal Model
EMF	Electromagnetic Force
THD	Total Harmonic Distortion
RL	Resistance Inductor
LCL	Inductor Capacitor Inductor
SPWM	Synchronized Pulse Width Modulation
DC	Direct Current
AC	Alternating Current

1. Introduction:

The excess of renewable energy sources has revolutionized the landscape of power generation, with inverter-based systems playing a pivotal role in facilitating the integration of renewable resources into the grid. Among these systems, the Virtual Synchronous Generator (VSG), equipped with inverters, stands out as a promising solution for emulating the behavior of conventional synchronous generators while harnessing the benefits of renewable energy technologies. Central to the operation of VSGs connected to the grid is the imperative of frequency regulation. Variations in renewable energy generation, influenced by factors such as weather conditions and load, can induce frequency deviations, potentially compromising grid stability and reliability. Effectively mitigating these deviations is essential for ensuring seamless grid operation and maximizing the utilization of renewable energy resources.

This study focuses on application of bang-bang control techniques tailored specifically for reducing frequency deviation in Virtual Synchronous Generators with inverters connected to the grid as in [5]. Through a comprehensive blend of theoretical analysis, and simulation studies, we aim to highlight the impact of virtual inertia on the power oscillations and evaluate the efficacy of this approach in fortifying grid stability by reducing the deviation in the frequency.

1.1. Background and motivation

The escalating demand for clean energy has promoted the use of Renewable energy sources, replacing traditional energy sources in today's power networks. Particularly solar and wind energy have grown in popularity, replacing more traditional energy sources like coal and nuclear. In 2009, the European Union made a long-term commitment to achieve 100% renewable energy by 2050, with a target of producing around 33% of electricity from renewable sources by 2030. [1]

With the increasing popularity of various renewable energy sources, integrating them into power networks presents a unique problem. In contrast to conventional power generation, which is based on inertia-producing synchronous generators, solar and wind power mostly use inverter-based resources (IBRs). The dynamics of power system stability are radically changed by the introduction of IBRs, which is made possible by power electronic inverters. The lack rotational mass in IBRs indicates a deviation from the traditional model based on the equations of motion for rotating masses, and thus calls for an important reconsideration of methods to maintain frequency stability in the presence of fluctuating load disturbances.

Consider inertia as a power system stabilizing force. It is comparable to the capacity of large rotating gears in power plants to react fast in the event of an imbalance between the quantity of energy produced and consumed. These rotating devices can supply energy to the grid or withdraw energy from it in response to changes in the power balance to maintain a steady frequency, or rhythm. Less system inertia, or stabilizing force, means that the power grid is more sensitive to abrupt variations in the amount of power produced or consumed and responds to them faster.

Inverters that connect renewable energy sources to the power grid are less equipped to withstand abrupt changes in power flow. When a significant event occurs, such as an abrupt power outage, this can cause problems with voltage and frequency in microgrids.[2]

Several solutions have been proposed to address the issue of decreasing inertia in power systems. One easy method is to employ many generators with spinning components running at a power lower than maximum, which causes them to function as though they have greater inertia collectively, even when a lot of renewable energy is being produced [3]. This may be costly even when it works, as it necessitates operating additional generators that may not be generating as much energy as they might.

1.2. Problem Statement:

Integrating Virtual Synchronous Generators (VSGs) with inverters into modern power grids lacks inherent mechanical inertia, leading to frequency deviations. The challenge is to develop a Bang-Bang controller to adjust VSG inertia swiftly and accurately to regulate the frequency, ensuring grid stability amidst fluctuating renewable energy sources. In addressing this challenge, the application of the "Improved Bang-Bang control" emerges as a compelling strategy.[5] The Bang-Bang controller plays a crucial role in stabilizing grid frequency by dynamically adjusting the inertia of the Virtual Synchronous Generator (VSG). In power systems, inertia is vital for resisting changes in speed and maintaining grid stability. However, VSGs, lacking mechanical components, rely on power electronic converters. To replicate the stabilizing effect of inertia, the Bang-Bang controller controls VSG parameters. When grid frequency deviates, the controller adjusts the VSG's effective inertia accordingly. Increasing inertia slows down speed changes when frequency decreases, while decreasing inertia enables quicker responses while frequency increases. This dynamic adjustment helps mitigate frequency deviation, ensuring grid stability and reliable operation, especially in systems with high penetrations of renewable energy sources.

1.3. Study objectives:

The objective of this study is as under:

1. To identify the effect of load change on the frequency of VSG in the grid connected renewable energy-based power system.
2. Analyze the improved Bang-Bang control strategy to control the virtual inertia of the VSG to regular the grid frequency.
3. Implementation of the improved Bang-Bang control strategy on the small signal model, transfer function-based simulation, and Grid connected inverter system.

1.4. Section of the thesis:

The thesis comprises of various sections discussing literature, simulations, and their results.

- **Introduction:** This section has discussed the background of the frequency deviation problem in grid connected renewable energy sources. The problem is summarized in the form of a problem statement. This section also provides a research objective of this thesis along with a brief overview of the section of thesis.
- **Literature review:** This section provides a detailed overview of the inertia of the power system. This section also discusses the impact of inertia on the stability of the system and comprises the challenges in the modern power landscape specifically related to grid connected renewable energy sources.
- **Methodology:** This section discussed the ideal, transfer function based, analysis and full grid connected system. This section also discussed the small signal model and micro grid system.
- **Fundamental of VSG:** This section provides the modelling of a virtual synchronous generator. This section also provides a detailed description of the active and reactive power loop.
- **Control strategy of virtual inertia:** This section provides a detailed description of the improved Bang-Bang control strategy.
- **Implementation of small signal model:** This section includes the Simulink implementation of the SSM, transfer function-based system, with and without the Improved Bang-Bang control.

- **Implementation of micro grid:** This section includes the Simulink implementation of the full-fledged grid connected system with and without the IBB control strategy.
- **Result and Discussion:** This section includes the simulation result of the small signal and micro grid system and includes discussion about the obtained results.
- **Conclusion:** This part concludes the thesis by summarizing the work done in this thesis.

2. Literature Review

2.1 Inertia of the Power System

Power system inertia is defined as the ability of a power system to resist changes in system frequency, primarily attributed to the resistance provided by rotating masses, such as synchronous generators. In conventional power systems, the stability and reliability of frequency responses are closely tied to the amount of kinetic energy stored in these rotating masses. Understanding power system inertia is fundamental for maintaining the dynamic balance necessary for uninterrupted power supply. A significant body of literature has explored the intrinsic relationship between power system inertia and conventional power generation.

Power system inertia is a bit like when you drive a car or ride a bike – it is about how things keep moving even after you stop pushing them. For example, when a driver takes their foot off the gas pedal, the car still moves due to inertia. This idea also applies to spinning objects, like wheels or power generators, which have what we call rotational inertia.

In the power grid, inertia means the energy stored in generators that are spinning. Think of it like the collective power from many generators working together. These generators all spin at the same speed, which we call synchronization. It is like they are all in tune, working together to keep the power grid stable.

Imagine these generators connected by invisible forces, like chains shown in Figure 1. These "chains" represent the forces that hold the generators together. So, all the individual generators, when they are online and spinning, team up to keep the power grid steady.

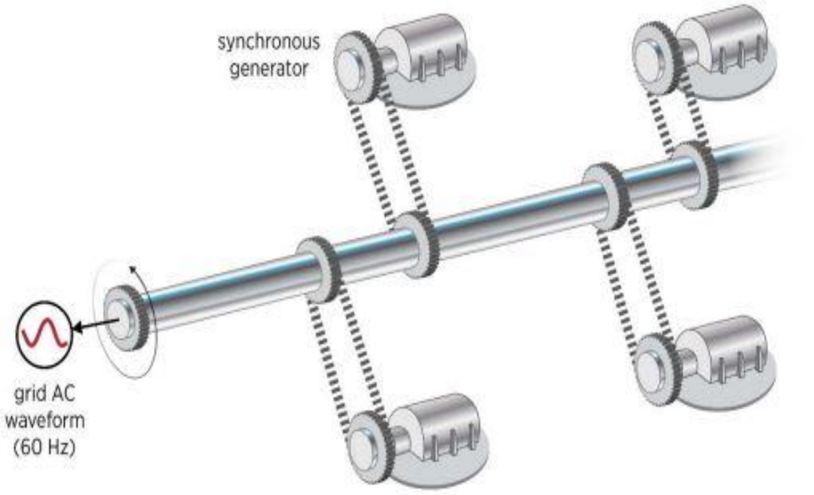

Fig 2: Depiction of typical power system inertia. [4]

Power system inertia is crucial because it helps the power system handle changes in how fast things are moving, especially when we use things like synchronous generators. These generators, with their spinning parts, play a key role in providing inertia. People have studied how much energy these spinning parts contribute and how it affects the overall stability of the power system.

Another important thing about inertia is how it directly affects how steady the power frequency is. Many studies have investigated how the level of inertia relates to the power system's ability to resist and bounce back from changes in frequency. Understanding this connection is essential for predicting and managing the effects when the system's inertia changes.

In a power system, the primary source of inertia is derived from conventional power plants' generators and turbines. These components, synchronized with the system, have a direct connection between their mechanical rotational speed (ωg) and the electrical angular frequency (ω_e). Consequently, the movement of individual generators can be described in terms of this coupling.

$$\frac{dJ_{sg}.w_e}{dt} = T_m - T_e \qquad (1)$$

T_e and T_m represent the electrical and mechanical torque, while Jsg quantifies the collective moment of inertia arising from the generator and turbine, adjusted to the electrical angular frequency, and factoring in the number of pole pairs. In power system engineering, the conventional practice involves representing the swing equation in terms of power rather than torque.

$$\frac{d\frac{J_{sg}.w_e^2}{2}}{dt} = P_m - P_e \qquad (2)$$

The derivative on the left side of Equation (2) represents the rate of change of kinetic energy stored in the turbine and generator. Typically, this energy, proportional to its power rating, is denoted as the inertia constant (H). In other words, it signifies the time, measured in seconds, that a generator can deliver nominal power solely based on the kinetic energy stored in the rotating mass.

2.2 Impact of Inertia on Frequency Stability

In both riding a bicycle and running a power grid, inertia plays a pivotal role. When you are on a bicycle, inertia allows you to stop pedaling and coast without tipping over. Similarly, in the power grid, inertia gives the system operator a chance to manage power plant failures, which are called contingencies. This is because inertia resists sudden changes in frequency, giving time for other systems to react and balance the supply and demand of power.

Grid operators in Norway might face financial penalties if they do not keep the frequency within specific limits. One big challenge for them is keeping the grid frequency stable after a contingency event, like when a large power plant fails suddenly. In this situation, the grid can slow down, and the frequency starts decreasing, like how a car slows down when the driver eases off the gas pedal. Similarly, when the grid frequency falls below a set level (49.7 Hz in most of the Norway), a part of the customer load is disconnected. This is called under frequency load shedding (UFLS) and is done to balance the remaining load with the remaining power generation. The same kind of disconnection happens if the frequency goes too high (e.g., 50.3 Hz).

2.3 Challenges in the Modern Landscape

The integration of renewable energy sources, especially those connected through frequency converters, challenges the conventional paradigm of relying on synchronous generators for inertia. Many modern generating plants prioritize operating independently from the system frequency, resulting in a decrease in overall system inertia. This reduction in inertia poses significant challenges to frequency stability, prompting the need for a reevaluation of conventional approaches.

To address these challenges, a promising strategy involves the introduction of virtual inertia. Virtual inertia technologies emulate the stabilizing effects traditionally provided by the rotating masses of synchronous generators. By implementing advanced control systems and algorithms, virtual inertia can be synthetically created within renewable energy systems, compensating for the reduction in physical inertia. In [5], an adaptive virtual inertia control strategy is implemented for Virtual Synchronous Generators (VSG) in a microgrid, incorporating an enhanced bang-bang control approach. To bolster frequency stability during the steady state, specific steady-state frequency intervals and inertia values are defined. The virtual inertia dynamically adjusts to the maximum or minimum values based on the frequency change rate and deviation when surpassing the steady-state interval. This innovative approach will be discussed and implemented in this thesis which enhances the ability of renewable sources to contribute to frequency stability, mitigating the impact of reduced system inertia.

3. Methodology

This chapter will cover the methods and tools that were used to achieve the objectives. The objective is to implement an electrical system to assess the frequency fluctuations subject to change in demand power. The main methodology discussed in this thesis will only be focused on the methods discussed in [5]. The topology of the electrical system considered to assess the frequency fluctuations subject to indirect change in the inertia is below in the figure.

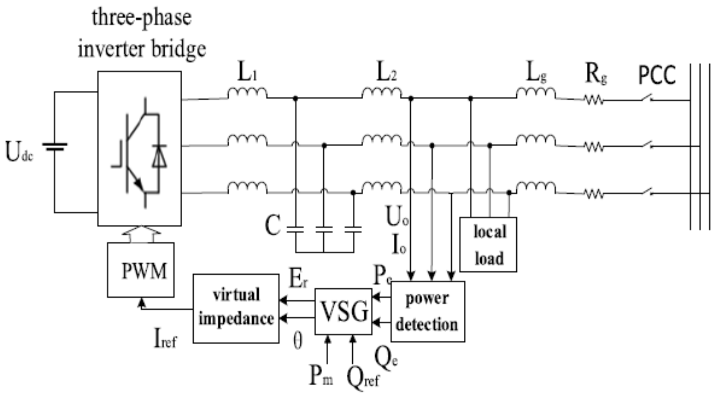

Fig 2: Simple topology of the electrical system [5]

This system consists of the DC source, Renewable energy source, followed by an inverter, an LCL filter, Local load, and finally connected to the grid.[5] Since the inverter required PWM signals to operate. So, a technique should be employed to generate the required signal.

To do so, the power will be measured on the output of the LCL filter using the following relation.[6]

$$P = v_{o\alpha} i_{g\alpha} + v_{o\beta} i_{g\beta} \qquad (3)$$

$$Q = v_{o\beta} i_{g\alpha} - v_{o\alpha} i_{g\beta} \qquad (4)$$

Here $v_{o\alpha}$ and $v_{o\beta}$ are the capacitor voltage, LCL filter, in the $\alpha\beta$ frame. $i_{g\alpha}$ and $i_{g\beta}$ are the injected grid current in the $\alpha\beta$ frame.

The active and reactive power measured, along with some reference reactive power and mechanical power, should be then input to the Virtual synchronous generator block. This synchronous generator block will generate the electromotive force E_r and rotational radian frequency ω to mimic the real operation of the synchronous generator.[5] The modelling of the Virtual synchronous generator will be discussed in the later part.

The electromotive force (E_r) and the rotational radian frequency (ω) will injected to a virtual impedance block to generate the required three phase sine wave that will be considered as reference to generate the required PWM using the SPWM technique, signals to operate the inverter.

The control logic from Figure 2 is clearly shown below:

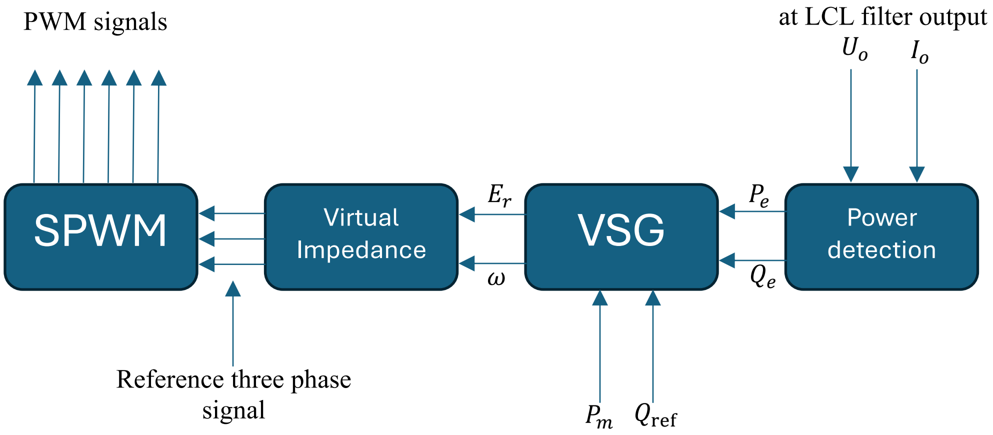

Fig 3: Control strategy for inverter operation

The control system will control the operation of the inverter to generate the three phase AC signals, and then the three-phase signal will be fed to the LCL filter to eliminate the high frequency harmonics and reduce the total harmonics distortion (THD). Notice that the reference AC signals are generated considering the dynamics of the virtual synchronous generator (VSG). So, when the frequency is changed, the frequency of the reference signals will also change accordingly. And hence, the frequency of the LCL output voltage will also change accordingly. So, the effect of the frequency deviation can be assessed using the above topology.[5]

To get the most out of this research, the system will be analyzed using two topologies. This 1st one is by considering a simple circuit of impedance $R + jX$. So, this analysis will be like considering the ideal scenario that will be mostly based on equations and system dynamics. This topology can be called a small signal model (SSM). The 2nd topology is to consider a completed power system connected with the grid as discussed above. This research thesis was done in two phases, in the first phase the small signal model was implemented while in the later phase the focus was the second topology along with the results and discussions.

3.1. Small signal Model

A simple RL circuit connected with a voltage source is considered to perform analytical analysis of the frequency deviation.[6] The SSM is totally based on transfer function and block diagram. A detailed implementation will be discussed below.

3.2. Micro grid model with inverter

As mentioned in [5], A DC voltage connected with the inverter followed by an LCL filter, local loads, and then connected with the grid to perform the analysis by considering practical realization. For this analysis, a completed power system shown in Fig 2 and Fig 3 will be implemented to show how its effect in the real power system.

4. Fundamental of VSG

A Virtual Synchronous Generator (VSG) is a sophisticated solution within power systems engineering aimed at replicating the essential functions of conventional synchronous generators using modern power electronics. In traditional power grids, synchronous generators play a pivotal role by providing stability and inertia, crucial for maintaining grid reliability. However, with the increasing integration of renewable energy sources such as solar and wind, which inherently lack these stability features, the need for alternative technologies like VSGs has become evident.[16]

VSGs function by manipulating inverters or other power electronic devices to emulate the behavior of synchronous generators. They regulate parameters like voltage and frequency, akin to their traditional counterparts, to stabilize the grid amidst fluctuating renewable energy inputs and varying demand patterns. This emulation is pivotal for grid stability, ensuring that power supply remains dependable even as the energy mix shifts towards renewables.[10]

What sets VSGs apart is their adaptability and responsiveness. Unlike synchronous generators, which are fixed in their characteristics, VSGs can be dynamically controlled to swiftly respond to grid fluctuations. This agility enables them to effectively counteract disturbances and maintain grid stability in real-time, a capability highly sought after in modern power systems striving for resilience and sustainability.[13]

VSGs represent a fusion of traditional power generation principles with innovative power electronics, offering a flexible and effective solution for integrating renewable energy while ensuring grid stability. Their ability to mimic the behavior of synchronous generators makes them invaluable assets in the transition towards a more sustainable and reliable energy future.

The model of the virtual synchronous generator can be divided into two parts. The active power loop and the reactive power loop. The active power loop obviously maintains the angular velocity to deliver the required active power. And the reactive power loop adjusts the change in the potential difference to fulfill the requirement of the reactive power considering a reference for the reactive power. [5]

Both the power loops are discussed below in detail.

4.1 Active power loop

The active power loop of a synchronous generator stands as a cornerstone in the realm of efficient electrical power generation and distribution. It commences with a prime mover, typically a turbine fueled by steam, water, or wind, imparting mechanical energy to the generator's rotor. This rotor, housing a field winding or permanent magnets, revolves to generate a rotating magnetic field. This field dynamically interacts with the stationary armature winding housed in the stator, thereby inducing an electromotive force (EMF) within the stator windings. Essential to this process is the excitation system, which ensures the rotor maintains a constant magnetic field, vital for inducing voltage in the stator windings. The resultant induced voltage is then conveyed to the electrical grid through the generator's output terminals, constituting the active power output. Control over this output is facilitated by modulating the excitation current to the rotor. Furthermore, in certain configurations, a governor system is employed to regulate the prime mover's speed, ensuring synchronization of the generated power with the grid frequency for stable operation. The active power loop orchestrates the intricate conversion of mechanical energy into electrical energy, seamlessly integrating synchronous generators into the broader power grid infrastructure.

For simplicity, a synchronous generator with the number of poles equal to 1 is considered. The rotor motion equation, for the purpose to simulate the active power loop, of the synchronous generator is given below.[5]

$$J\frac{d\omega}{dt} = \frac{P_m}{\omega_N} - \frac{P_e}{\omega_N} - D_p(\omega - \omega_N) \qquad (5)$$

$$\frac{d\theta}{dt} = \omega \qquad (6)$$

Here:

J = Rotational inertia
ω = Rotor angular velocity
P_m = Mechanical Power
P_e = Electromagnetic power

ω_N = Rated angular velcoity
D_p = Damping coefficient
θ = Power angle

The block diagram based on the above differential equation is shown below in the figure.[5]

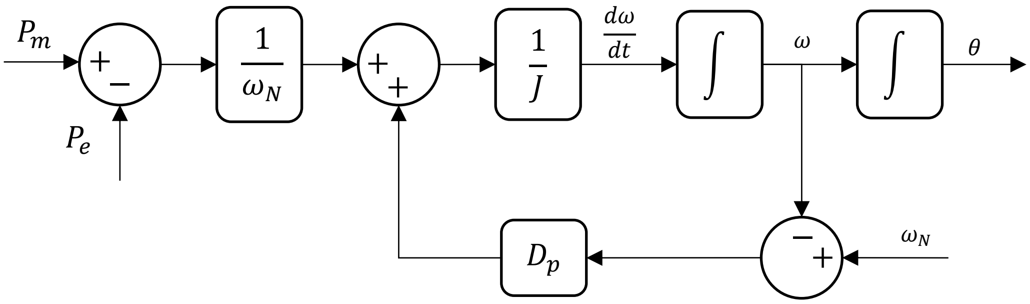

Fig 4: The active power loop of the VSG

To complete the active power loop, a secondary frequency regulator is needed. An integrator is considered for this purpose because it eliminates the steady state error. So, an integrator is added to the damping link as shown in the following figures.[5]

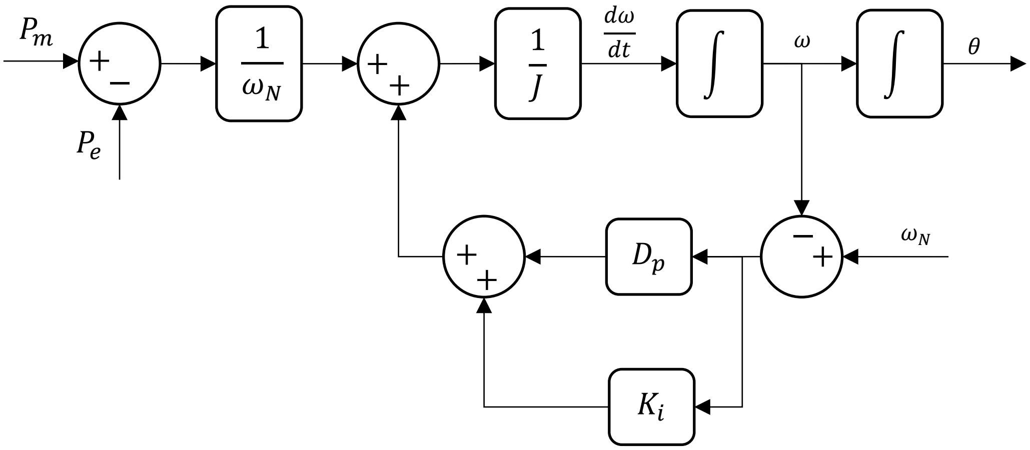

Fig 5: The Active power loop of the VSG with the frequency regulator [5]

4.2 Reactive power loop

The reactive power loop of a synchronous generator plays a vital role in managing the system's voltage and supporting the transmission of electrical power. Beginning with the rotor's rotation driven by the prime mover, which could be a turbine powered by various sources, the generator generates a rotating magnetic field. This field interacts with the stator windings, inducing an electromotive force (EMF) within them. However, in the reactive power loop, the focus shifts to the excitation system's role. This system ensures precise control over the generator's reactive power output by adjusting the strength of the magnetic field in the rotor. By regulating this magnetic field, the excitation system enables the generator to produce or absorb reactive power as needed, thereby stabilizing voltage levels within the power system. Through the output terminals, the generator can inject or absorb reactive power into the electrical grid, supporting voltage regulation and ensuring the system's overall stability. In conjunction with voltage control devices and reactive power compensation equipment, the reactive power loop of a synchronous generator contributes significantly to maintaining grid reliability and power quality.

In summary, the reactive power loop is responsible for the voltage regulation to improve the quality of the power transmission. The reactive power loop can be defined by the following differential equation.[5]

$$K\frac{dE_r}{dt} = Q_{\mathrm{ref}} - Q_e - D_q(U_o - U_n) \qquad (7)$$

Here:

$K =$ Reactive power voltage inertia coefficient
$E_r =$ Virtual Electrmotive force
$Q_{ref} =$ Reference Reactive power
$D_q =$ Reactive power voltage droop coefficient
$Q_e =$ Reactive power output
$U_o =$ Output voltage amplitude
$U_n =$ Rated voltage amlitude

The block diagram of the reactive power loop based on the above differential equation is shown below in the figure.

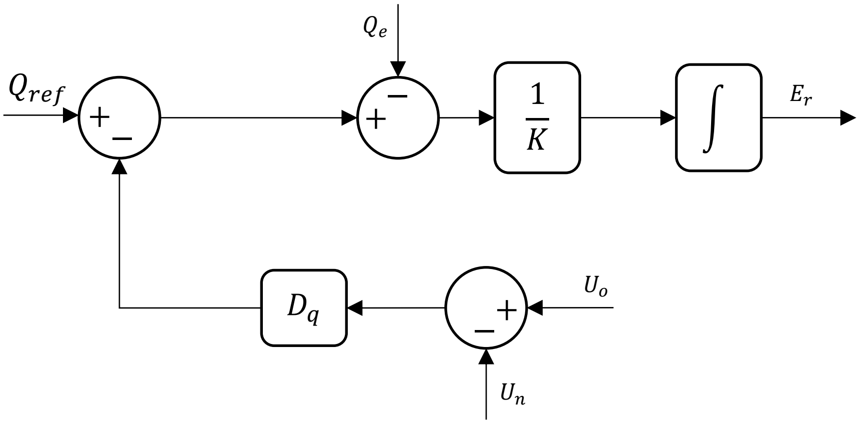

Fig 6: Reactive Power loop [5]

5. Control strategy of virtual inertia

The virtual inertia of the VSG fluctuates between maximum and minimum subject to 2 factors. The first one is the rate of change on the frequency $\frac{d\omega}{dt}$ and second is the frequency deviation $\Delta\omega$. With this strategy, the fluctuation between the maximum and minimum is overly sensitive because of the rate of change of frequency. This may cause fluctuating inertia in the steady state. Because a very fluctuation in frequency may occur in the steady state. This will impact the stability of the system indirectly.[5]

To address this issue, a steady state interval is set to avoid the fluctuation in the steady state. So, if the frequency is within the steady state interval. No, fluctuation in the virtual inertia will occur and hence it will keep the system stable for small disturbance in the steady state. If the frequency exceeds the steady state, the virtual inertia will fluctuate adaptively in maximum and minimum values.

To achieve this, the virtual inertia will fluctuate based on the product of the rate of change of frequency and frequency deviation, and check whether the frequency deviation is not within the steady state interval. if the frequency deviation is within the steady state interval. The VSG will consider the steady virtual inertia J_s. To prevent the system from having a large frequency deviation, this system is disturbed by the minimum virtual inertia. This adaptive technique is known as the improved Bang-Bang control strategy. The mathematical form of the improved Bang-Bang strategy is.[5]

$$J = \begin{cases} J_{max} & \Delta\omega \dfrac{d\omega}{dt} > 0 \cap |\Delta\omega| > 2\pi f_s \\ J_{min} & \Delta\omega \dfrac{d\omega}{dt} \leq 0 \cap |\Delta\omega| > 2\pi f_s \\ J_s & |\Delta\omega| \leq 2\pi f_s \end{cases}$$

6. Implementation of small signal model

6.1 Modelling

For the sack of simplicity, the equivalent model of the micro grid is considered. The equivalent model of the VSG is just a simple RL circuit with voltage source as shown below:

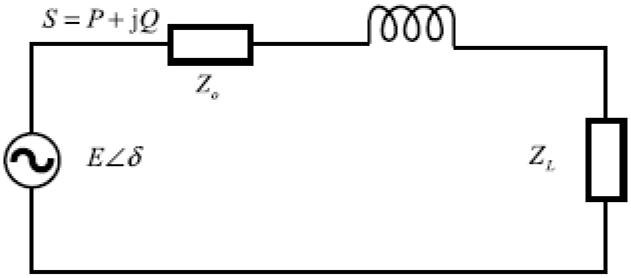

Fig 7: Equivalent circuit of inverter under micro grid [5]

In the figure $E\angle\delta$ is an AC voltage deliver power to the circuit. Active power is consumed by the resistance and reactive power is consumed by the reactance of the circuit. Let us assume that equivalent impedance of the circuit is $R + jX$. So, the following relation can calculate the active and Reactive power.

$$P = \frac{RE^2(\cos^2\delta - \sin^2\delta) + 2XE^2\sin\delta\cos\delta}{R^2 + X^2} \tag{8}$$

$$Q = \frac{2RE^2 \sin\delta \cos\delta - XE^2(\cos^2\delta - \sin^2\delta)}{R^2 + X^2} \tag{9}$$

Keeping the impedance of the circuit constant, like load is constant for some time span. The Active and Reactive power is dependent on the emf E_r and power angle δ.

To analyze the deviation in the frequency, a step change in the load will be considered. So, the power flow in the circuit will be change and hence it will change the emf E_r and Power angle δ. So, lets define a relation to evaluate the change in the active and reactive powers, change in the emf E_r, and change in the power angle δ.

To achieve this, let us assume that system is running on static value of emf E_s and static value of the power angle δ_s. Considering this static operation point. The active and reactive power flow at this static point can be calculated as:

$$P_s = \frac{RE_s^2(\cos^2\delta_s - \sin^2\delta_s) + 2XE_s^2 \sin\delta_s \cos\delta_s}{R^2 + X^2} \tag{10}$$

$$Q_s = \frac{2RE_s^2 \sin\delta_s \cos\delta_s - XE_s^2(\cos^2\delta_s - \sin^2\delta_s)}{R^2 + X^2} \tag{11}$$

Let us define the change in the active power, change in reactive power, change in emf, and change in the delta as below:

$$\Delta P = P - P_s \tag{12}$$

$$\Delta Q = Q - Q_s \tag{13}$$

$$\Delta E = E - E_s \tag{14}$$

$$\Delta\delta = \delta - \delta_s \tag{15}$$

Utilizing equation 8 to equation 15 results in the following relations.

$$\Delta P = K_{pf}\Delta\delta + K_{pe}\Delta E \tag{16}$$

$$\Delta Q = K_{qf}\Delta\delta + K_{qe}\Delta E \qquad (17)$$

Here:

$$K_{pf} = \frac{2XE_s^2\cos 2\delta_s - 2RE_s^2\sin 2\delta_S}{R^2 + X^2} \qquad (18)$$

$$K_{pe} = \frac{2RE_s\cos 2\delta_s + 2XE_s\sin 2\delta_S}{R^2 + X^2} \qquad (19)$$

$$K_{qf} = \frac{2RE_s^2\cos 2\delta_s + 2XE_s^2\sin 2\delta_S}{R^2 + X^2} \qquad (20)$$

$$K_{pf} = \frac{2RE_s\cos 2\delta_s - 2XE_s\sin 2\delta_S}{R^2 + X^2} \qquad (21)$$

Considering the equations (16) and (17), a block diagram can be created as:

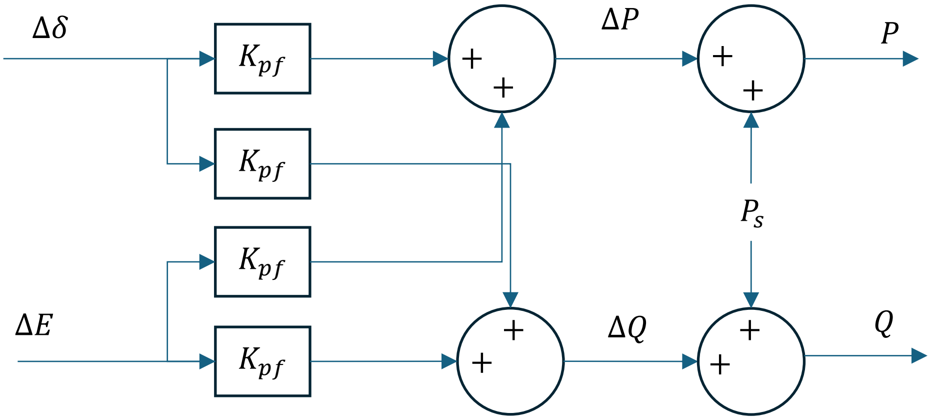

Fig 8: Equivalent block diagram of simple RL circuit (VSG) [5]

Notice that the system requires 2 inputs. One is ΔE and other is $\Delta\delta$. To get these inputs, Equation (5), the dynamic of the VSG, should be considered as:

$$J\frac{d\omega}{dt} = \frac{P_m}{\omega_N} - \frac{P_e}{\omega_N} - D_p(\omega - \omega_N)$$

Define $\Delta\omega = \omega - \omega_N$ and $\Delta P_m = P_m - P_e$ Substitute to get:

$$J\frac{d\omega}{dt} = \frac{\Delta P_m}{\omega_N} - D_p\Delta\omega$$

Find the transfer function by considering the ΔP_m as input and $\Delta\omega$ as output. The transfer function can be written as:

$$\frac{\Delta\omega}{\Delta P_m} = \frac{1}{\omega_N}\left(\frac{1}{Js + D_p}\right)$$

Considering the secondar frequency regulator. The transfer function based on the active power loop, shown in the Fig 5, will be:

$$\frac{\Delta\omega}{\Delta P_m} = \frac{1}{\omega_N}\left(\frac{1}{Js + D_p + \frac{k_i}{s}}\right) \qquad (22)$$

For the change in the emf, consider the equation (7) of the reactive power loop as below:

$$K\frac{dE_r}{dt} = Q_{\mathrm{ref}} - Q_e - D_q(U_o - U_n)$$

For the small signal model, $U_o = E$ and $U_n = E_s$. So, $\Delta E = U_o - U_n$, and define $\Delta Q_r = Q_{ref} - Q_e$. Substitute to get:

$$K\frac{dE_r}{dt} = \Delta Q_{\mathrm{r}} - D_q\Delta E$$

Take the Laplace transform to get the transfer function as:

$$\frac{\Delta Q_r}{\Delta E} = \frac{1}{Ks + D_q} \qquad (23)$$

Extend the block diagram in the Fig 8 by including the equation (22) and (23) as:

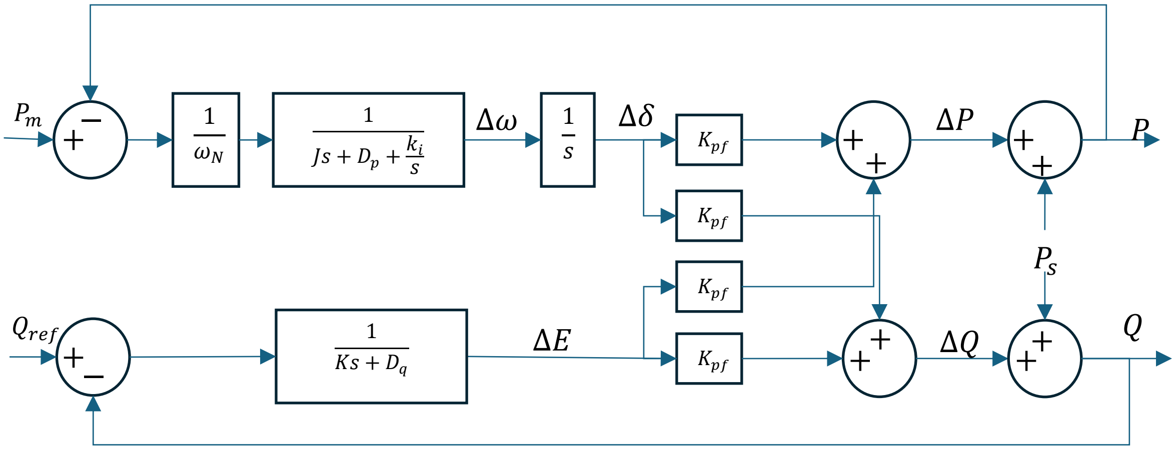

Fig 9: Small signal model of the virtual synchronous generator (VSG) [5]

Notice that deviation in the frequency occurs only in the active power loop. Let us find the close loop transfer function to get an idea about the frequency deviation subject to step change in active power drawn by the load connected.

The open loop transfer function will be:

$$G_{op} = \left(\frac{1}{\omega_N}\right)\left(\frac{1}{Js + D_p + \frac{k_i}{s}}\right)\left(\frac{1}{s}\right)(K_{pf})$$

$$G_{op} = \frac{K_{pf}}{J\omega_N s^2 + D_p\omega_N s + \omega_N k_i} \tag{24}$$

Since the system is unity feedback. So, the close loop transfer function will be calculated as:

$$G_{cl} = \frac{G_{op}}{1 + G_{op}}$$

Substitute to get:

$$G_{cl} = \frac{\frac{K_{pf}}{J\omega_N}}{s^2 + \frac{D_p}{J}s + \frac{k_i}{J} + \frac{K_{pf}}{J\omega_N}} \tag{25}$$

Notice that the system resembles the standard 2nd order system. And the characteristics of the system can be calculated by comparing the denominator with $s^2 + 2\zeta\omega_{Na}s + \omega_{Na}^2$. Here ζ and ω_{Na} are the damping ratio and natural frequency, respectively.

The natural radian frequency is: [5]

$$\omega_{Na} = \sqrt{\frac{k_i}{J} + \frac{K_{pf}}{J\omega_N}} \qquad (26)$$

The damping ratio is: [5]

$$\zeta = \frac{D_p}{2J\sqrt{\frac{k_i}{J} + \frac{K_{pf}}{J\omega_N}}} \qquad (27)$$

The amplification factor can be evaluated as: [5]

$$K_p = \frac{K_{pf}/\omega_N}{k_i + \frac{K_{pf}}{\omega_N}} \qquad [28]$$

The settling time can evaluate as: [5]

$$T_s = \frac{4}{\zeta\omega_{Na}} \qquad (29)$$

6.2 Simulink Model without Adaptive control

The Simulink Model based on the block diagram in the Fig 9 is shown below:

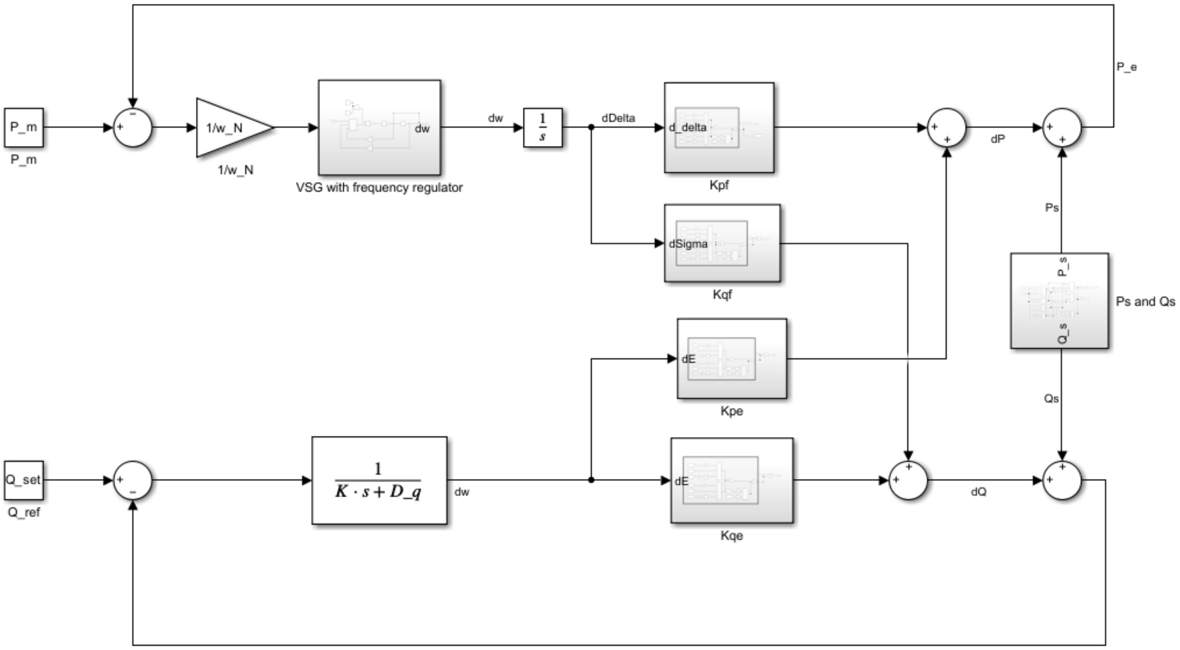

Fig 10: Simulink Model of the Small signal

Notice that model exactly matches the Block diagram in figure 9. There are 6 subsystems defined for this model for the purpose of clarity. The figure of each of the models is shown below.

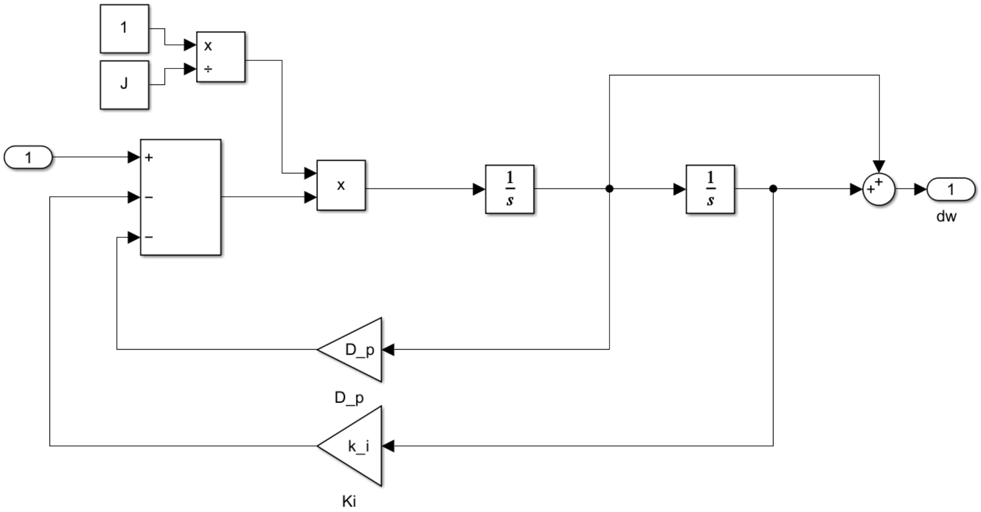

Fig 11: VSG with frequency regulator

This block can also be implemented using the simple transfer function block available in the SIMULINK but the latter for the adaptive control strategy, changing the inertia value based on the bang-bang control will not be possible. This Simulink mode is based on the following transfer function.

$$\frac{1}{Js + D_p + \frac{k_i}{s}}$$

Which is defined in the block diagram shown in Fig 9. The following Simulink Model is based on the equation (18), (19), (20), and (21).

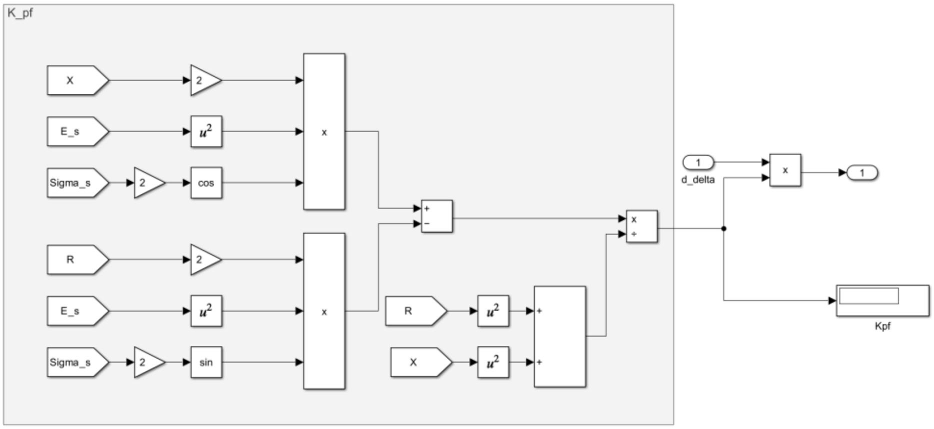

Fig 12: SIMULINK Model for the K_{pf}

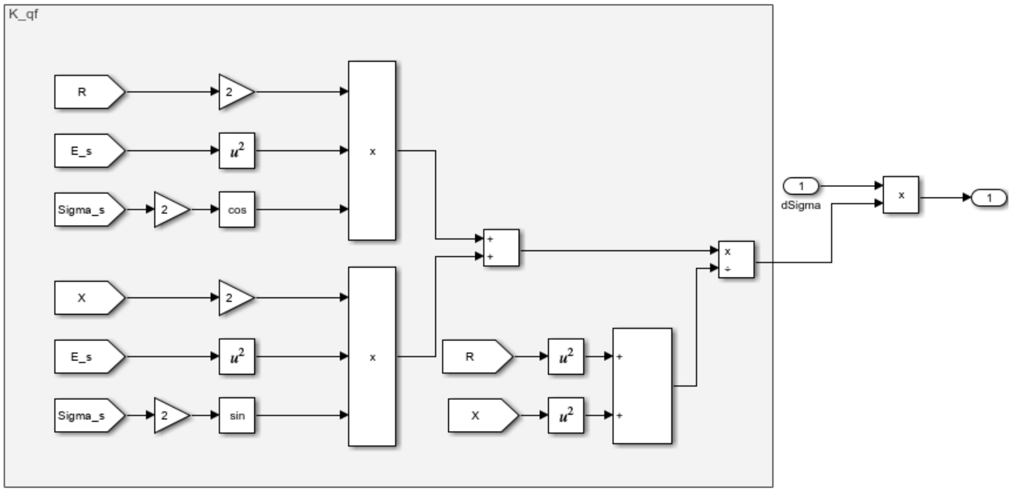

Fig 13: SIMULINK Model for the K_{qf}

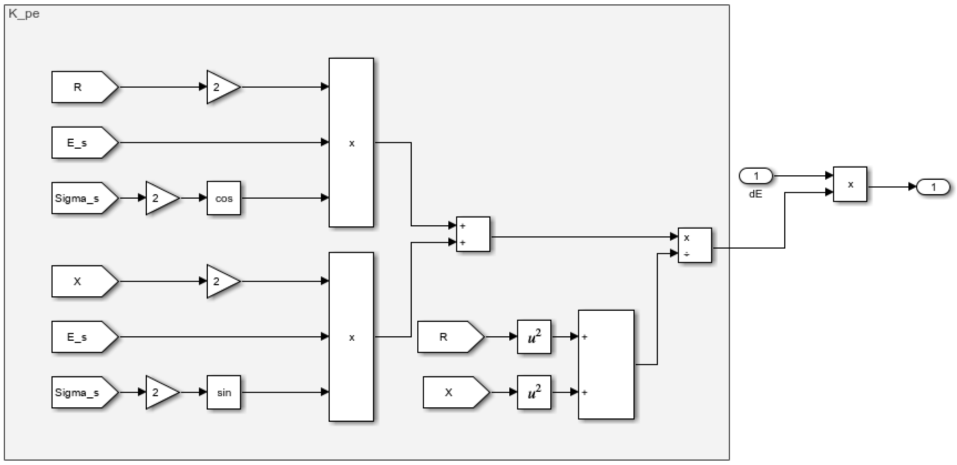

Fig 4 SIMULINK Model for the K_{pe}

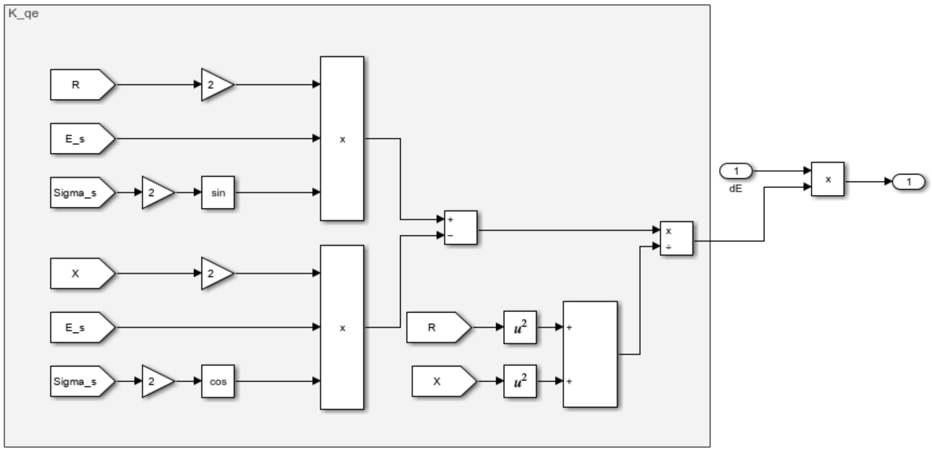

Fig 15: SIMULINK Model for the K_{qe}

The Simulink model for the "P_s and Q_s" is based on equation (10) and (11). The subsystem P_s and Q_s is shown below in the figure.

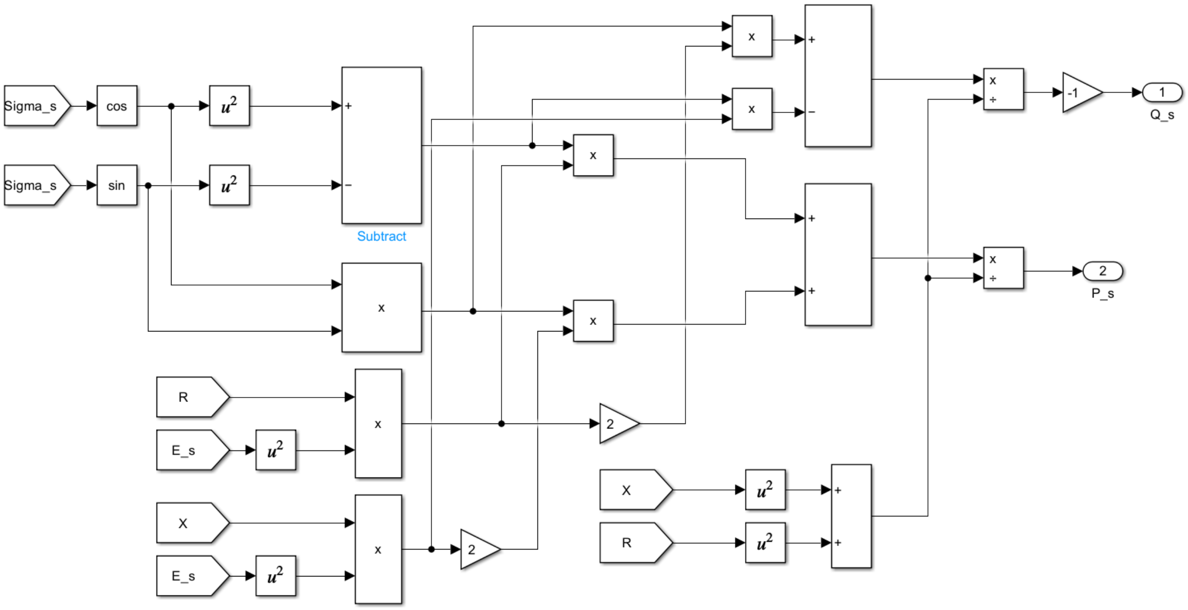

Fig 16: SIMULINK Mode for the evaluation of P_s and Q_s

6.3 Simulink Model with adaptive control

The SIMULINK model will be exactly same as in Part B, except the subsystem "VSG with the frequency regulator." This block will be changed to account for the inertia change based on the Bang-Bang adaptive control strategy. The adaptive control strategy is defined in the following equation.

$$J = \begin{cases} J_{max} & \Delta\omega \dfrac{d\omega}{dt} > 0 \ \cap |\Delta\omega| > 2\pi f_s \\ J_{min} & \Delta\omega \dfrac{d\omega}{dt} \leq 0 \ \cap |\Delta\omega| > 2\pi f_s \\ J_s & |\Delta\omega| \leq 2\pi f_s \end{cases} \tag{30}$$

The SIMULINK model of the VSG is shown below in the figure.

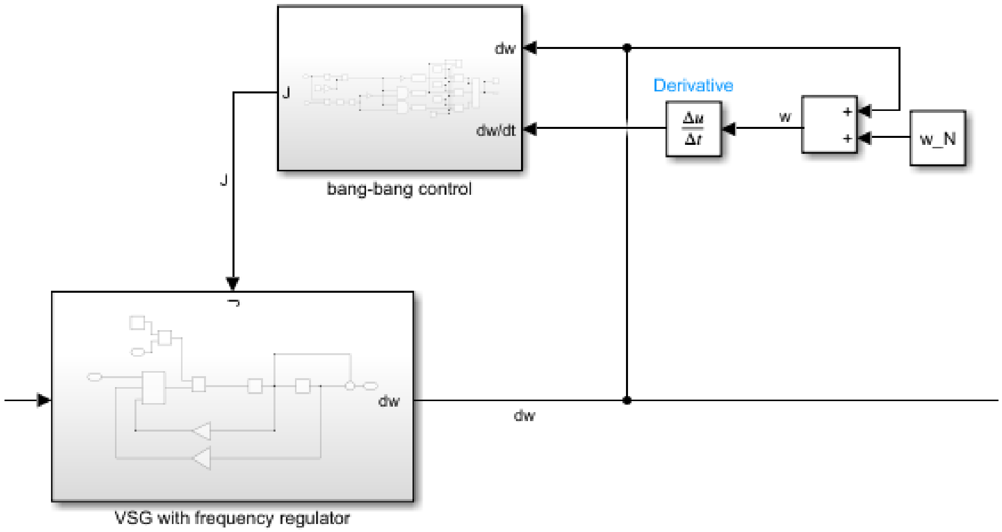

Fig 17: VSG connected with the Bang-Bang control block.

The subsystem “Bang-Bang control” block is based on the equation (24). The Simulink model is shown below in the figure.

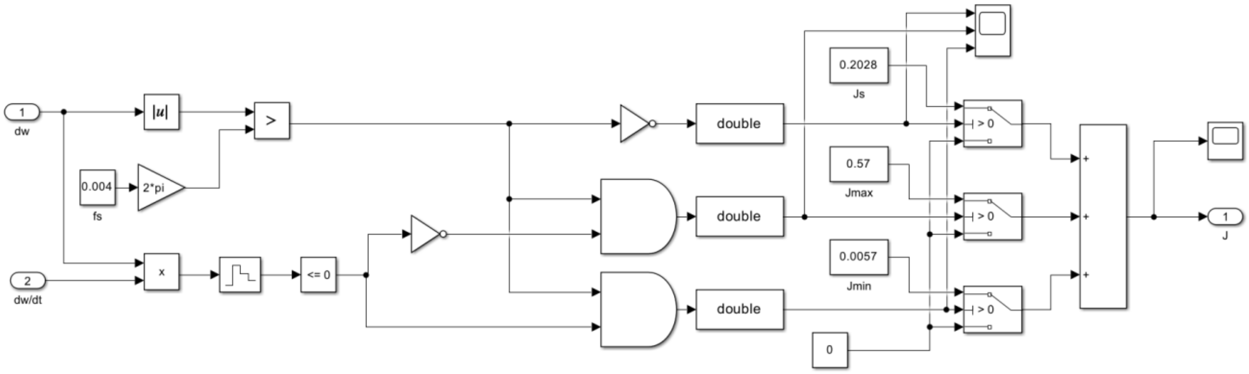

Fig 18: Bang-Bang control block

7. Implementation of the micro grid

As stated in Fig 2 and 3, the micro grid consists of DC source, Inverter, LCL filter, Point of coupling (PCC), Grid, Power detection module, VSG, Virtual impedance, and PWM module. The SIMULINK model of the micro grid is shown below in the figure.

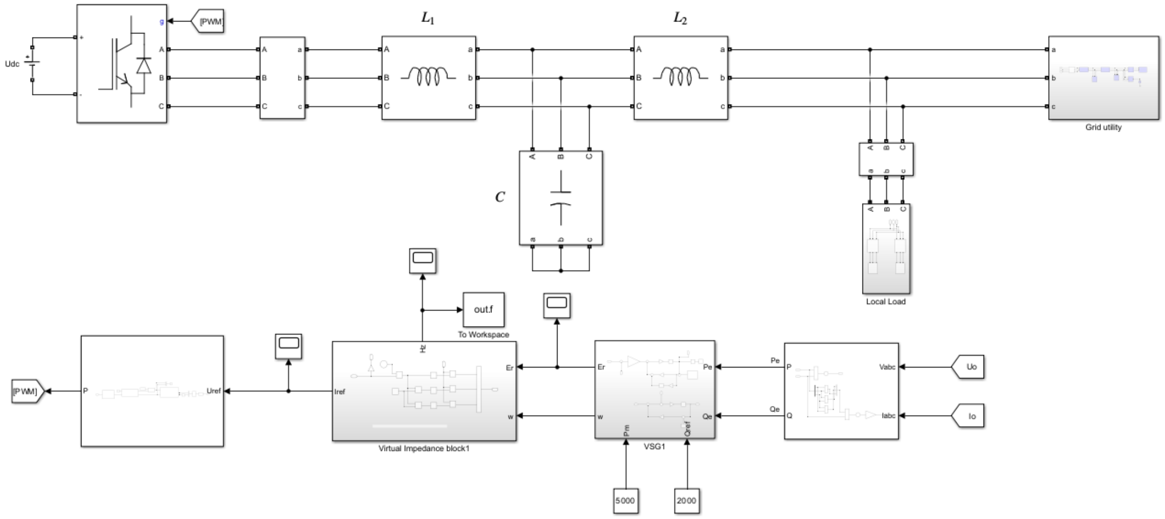

Fig 19: SIMULINK model of the micro grid with VSG

The power detection measured the power flow to the load. The measured active and reactive power is injected to the VSG module along with the mechanical power and reference reactive power. The VSG module output the emf E and radian angular frequency ω which fed to the virtual impedance module to generate the reference three-phase signal for the PWM module to generate the PWM signals to drive the inverter.

The system consists of 3 subsystems named as “Grid utility”, “VSG”, and “Virtual impedance module”. The Simulink model for each is shown below in the figure.

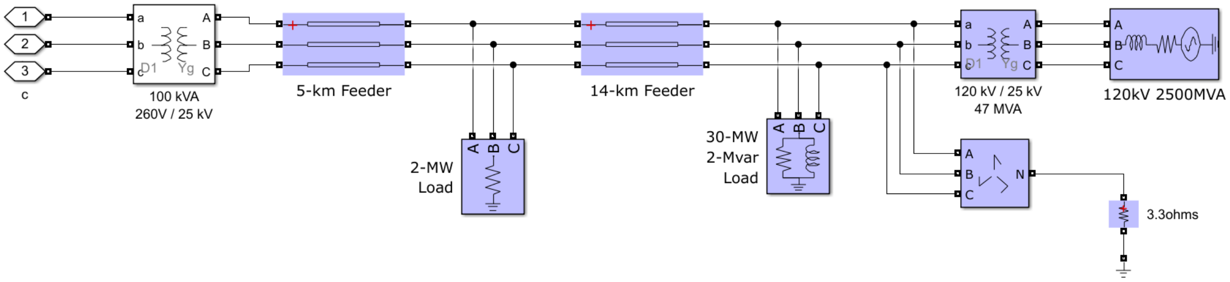

Fig 20: Grid utility

The grid, short for electrical grid or power grid, is a complex network of interconnected power generation, transmission, and distribution systems that deliver electricity from power plants to consumers. It consists of power plants, transformers, transmission lines, substations, and distribution lines, working together to ensure a reliable and constant supply of electricity to homes, businesses, and industries. The grid enables the efficient transfer of electricity over long distances and facilitates the integration of various sources of energy, including fossil fuels, nuclear, and renewable sources like solar and wind. Its resilience and adaptability are crucial for maintaining stability in the face of changing demand, weather events, and other challenges, making it a cornerstone of modern society's infrastructure.

The grid utility has been taken from the MATLAB built in example set. Ideally, a three-phase voltage source can select with the amplitude equal to the grid voltage. This purpose of the grid utility is considering a practical realization. The desire to consider scenario as much as possible.

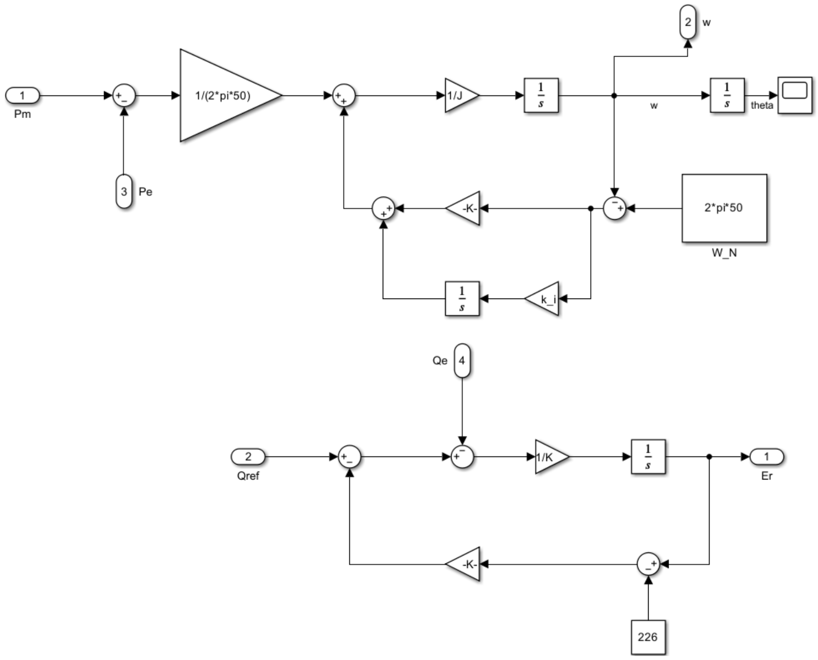

Fig 21: VSG module for the micro grid

The above VSG model is based on the differential equations defined in the equation (5) and (7) along with secondary frequency regulator.

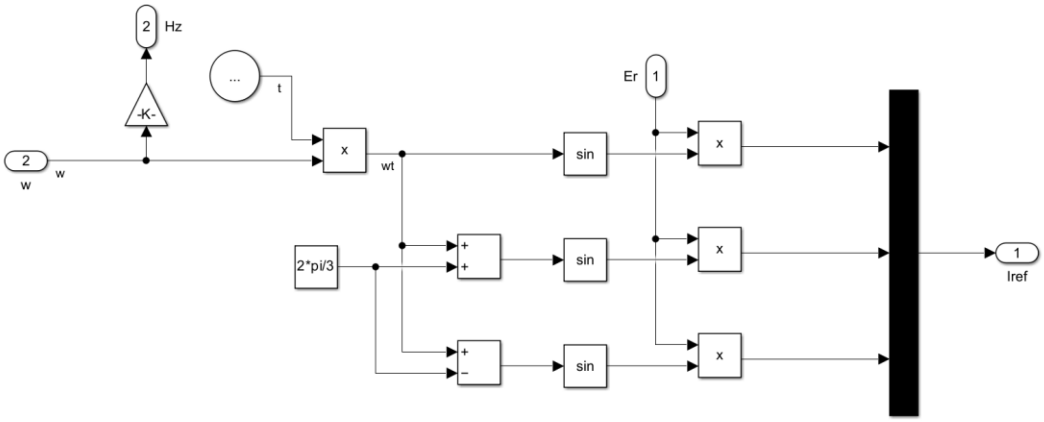

Fig 22: Virtual impedance module for the micro grid

Virtual impedance is a technique used in power electronics to mimic the behavior of traditional impedance in power systems. It is applied through control algorithms or circuit configurations in devices like inverters, enhancing stability and grid integration, particularly in renewable energy systems.

Since the VSG module provide the amplitude of the emf E and rotational angular velocity ω. Using this info and three phase signals with 120-degree phase, does the job of the virtual impedance.

8. Result and Discussion

This system is simulated by the considering that the load connected will be initially of $5kW$ and $2kW$. At 1 sec, the active load changes to $10kW$ keeping the reactive load constant. At 1.5s, the active load change back to the $5kW$ keeping the reactive load constant. This scenario is considered for both the simulation, without the Bang-Bang control and with the Bang-Bang control. The constant parameters of the system are defined below in the table. [5]

Parameter	Value	Parameter	Value
U_{dc}	800V	δ_s	0.05
D_p	5 Nm. s/rad	P_m	5kW
D_q	100	Q_{ref}	2kVAR
E_s	226V	f_N	50Hz
k_i	780	C	$800\mu F$
L_1	$3.2mH$	L_2	$800\mu H$
K	10	J_s	0.2028
J_{max}	0.57	J_{min}	0.0057
f_s	0.004		

Table 1: Parameters settings for VSG algorithm. [5]

In the simulation with the bang-bang control, the virtual inertia is constant and equal to J_s. And the inertia will change to max and min base on the Bang-Bang control strategy. All these values and conditions were the same as presented in [5]. But in [5], the test results of 4 strategies were discussed but due to shortage of time only two of them were implemented and compared in this thesis.

8.1 Small Signal Model simulation result

Since the small signal model is based on the transfer function. So, the only way to change the load is to change the value of $R + jX$. Initially, the load is 5kW and 2kVar. So, find such combination of R and X, such that the active power is 5kW and reactive power is 2kVar. Consider the equations (8) and (9) to achieve this objective as:

$$P = \frac{RE^2(\cos^2\delta - \sin^2\delta) + 2XE^2\sin\delta\cos\delta}{R^2 + X^2}$$

$$Q = \frac{2RE^2 \sin\delta\cos\delta - XE^2(\cos^2\delta - \sin^2\delta)}{R^2 + X^2}$$

Substitute $P = 5000$, $Q = 2000$, $E = E_s = 226$, and $\delta_s = 0.05$ and solve the equation to get the value of R and X. The calculated values of the R and X are below.

$$X = 4.384 \qquad R = 8.41$$

For the $P = 10kW$ and rest values are the same. The required values of the R and X are as below.

$$X = 1.468 \qquad R = 4.788$$

To simulate the defined scenario, the value of R and X changes from 8.41 and 4.38 to 4.788 and 1.468 respectively at 1sec. Similarly, the values of R and X will be back to the initial one at 1.5sec.

The simulation results are shown below in the figures.

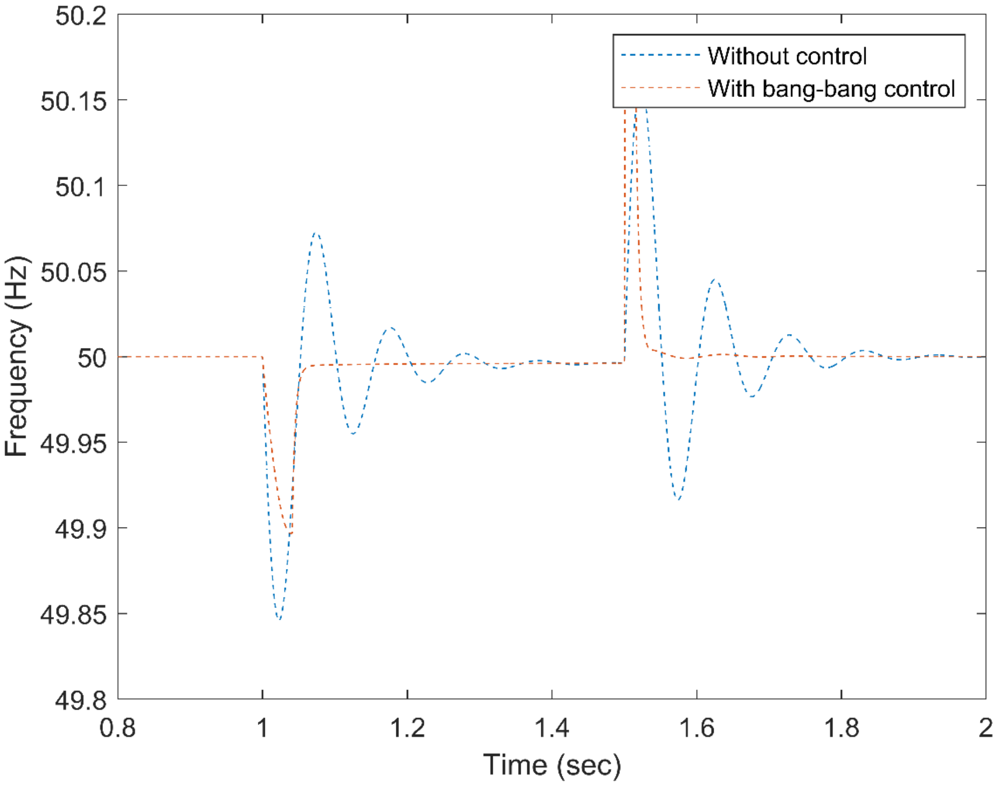

Fig 23: Frequency deviation, in small signal model, with and without control

Notice that without bang-bang control, the settling time is about 0.4 sec with much overshoot. Frequency deviation lasts for a noticeably brief time for the case of Bang-Bang control. The settling time is reduced by 75% and hence the settling time is about 0.1 sec. These results are like the one shown in [5].

The active power load is first increased by 5kW at 1sec and decreases by 5kW at 1.5 sec. But the overshot for both cases is much higher at 1.5 sec compared to 1 sec. The overshot at 1.5 sec for the Bang-Bang control is a little higher than without control but a significant decrease in the settling time is obviously an improvement. One of the key features is that increasing load causes decrease in the frequency and decrease in the load cause increase in the frequency.

The other plots of the simulation are shown below.

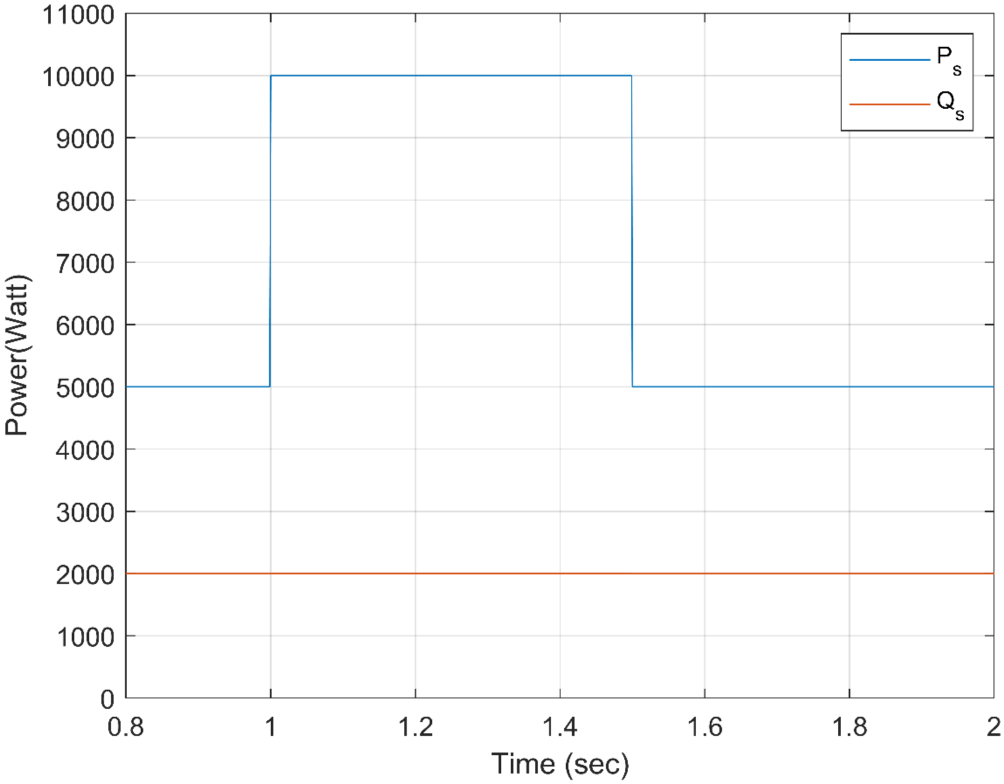

Fig 24: P_s and Q_s at static emf E_s and δ_s

As stated, that active power is increased by 5kW at 1 sec and decrease by 5kW at 1.5 sec. The actual active power and reactive power delivered to the load is shown below in the figure.

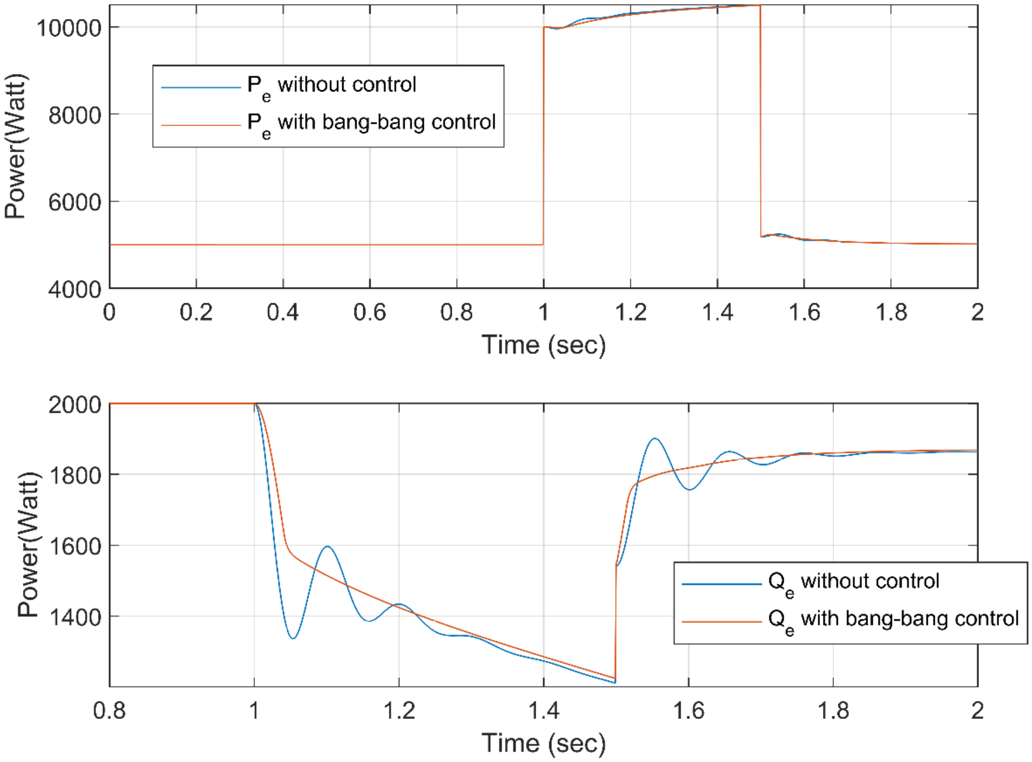

Fig 25: The Active and Reactive Power (P_e and Q_e) for both the cases

Notice that active power P_e or P is somehow same for both the cases. But change in the reactive power is considerable. The drop in the reactive power is significant even for a minute deviation in the frequency. This shows how crucial is to maintain the frequency of the system. This oscillation in the reactive power is eliminated by the Bang-Bange controller. Load demand reactive of 2kVar but due to deviation in the frequency, the system is not able to provide it. This effect is minuted but this is because of the 0.17Hz change in the frequency. So, it is crucial to stabilize the frequency as early as possible to fullfill the requirement of the reactive of the system to increase relaibility and power quality of the system.

Change in the Electromotive force ΔE is shown below in the following figure.

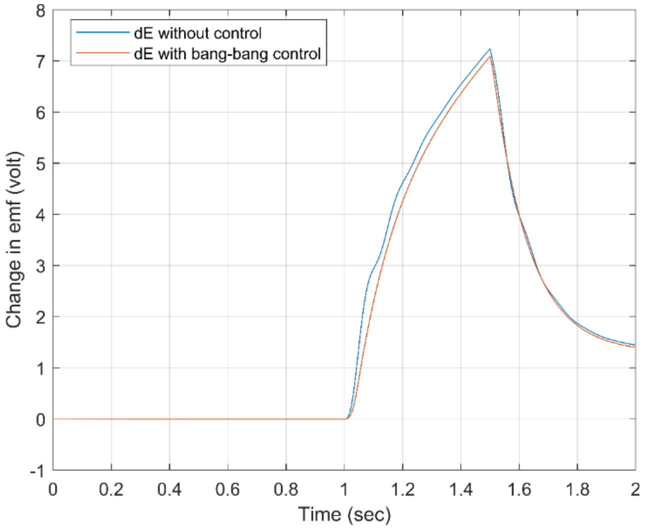

Fig 26: ΔE for both the cases

The change in the electromotive force ΔE is increase when the load is increases and vice versa. The response is like a first order system with a time constant of about 0.15 sec. For this observation, it also concluded that electromotive force E has an inverse relation with load change. So, when the load decrease, the emf E will increases and vice versa.

Since virtual inertia is constant for the 1st case. But for this 2nd case, the inertia can jump between maximum and minimum level as shown in the following figure.

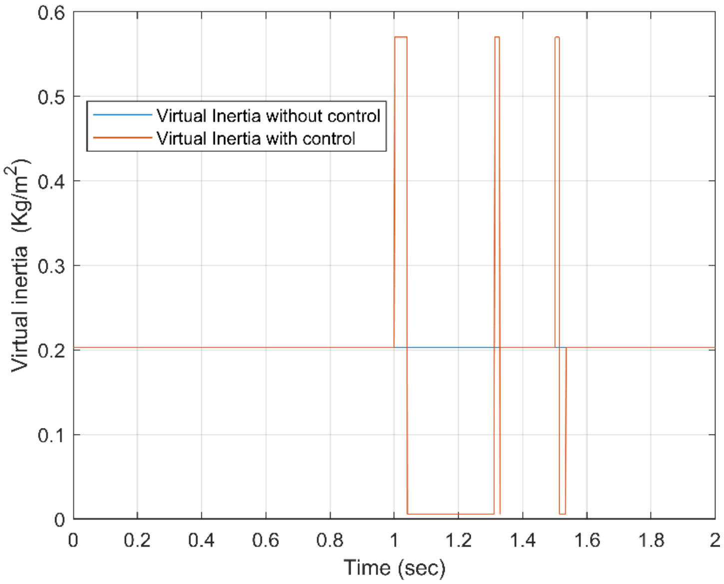

Fig 27: Virtual inertia for both the cases

Notice that the inertia jumps just 4 to 5 times and attains the best performance on minimizing the frequency deviation duration. This shows that optimized working of the Bang-Bang control strategy.

8.2 Micro grid simulations results

Since the VSG generator is connected to the grid right from the initial. So, in the transient state, the system is connected to the grid. Let us analyze the frequency deviation for the case IBB control strategy. The required figure is below:

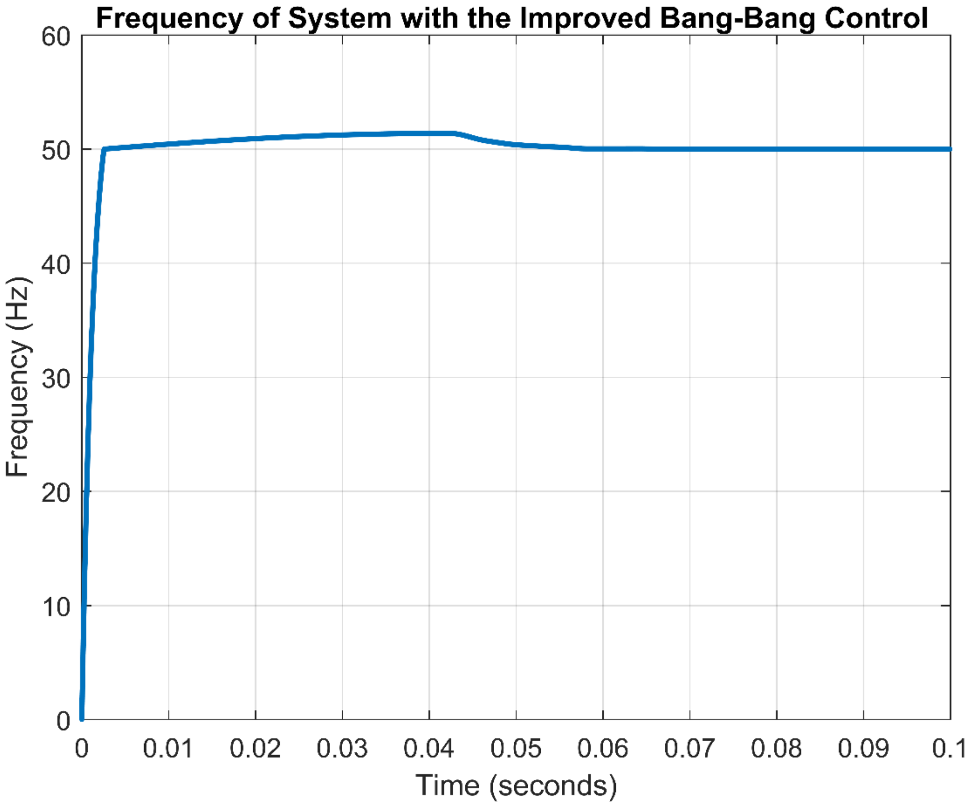

Fig 28: Transient state of the frequency deviation

Notice that the frequency deviation in transient state, for the case of the IBB control strategy, is quite less and converges to 50Hz within 50ms while in the other case it was unstable. Since the focus of this research is to analyze the frequency subject to load change and propose adaptive controller to overcome the deviation. The steady state frequency deviation is shown below in the figure.

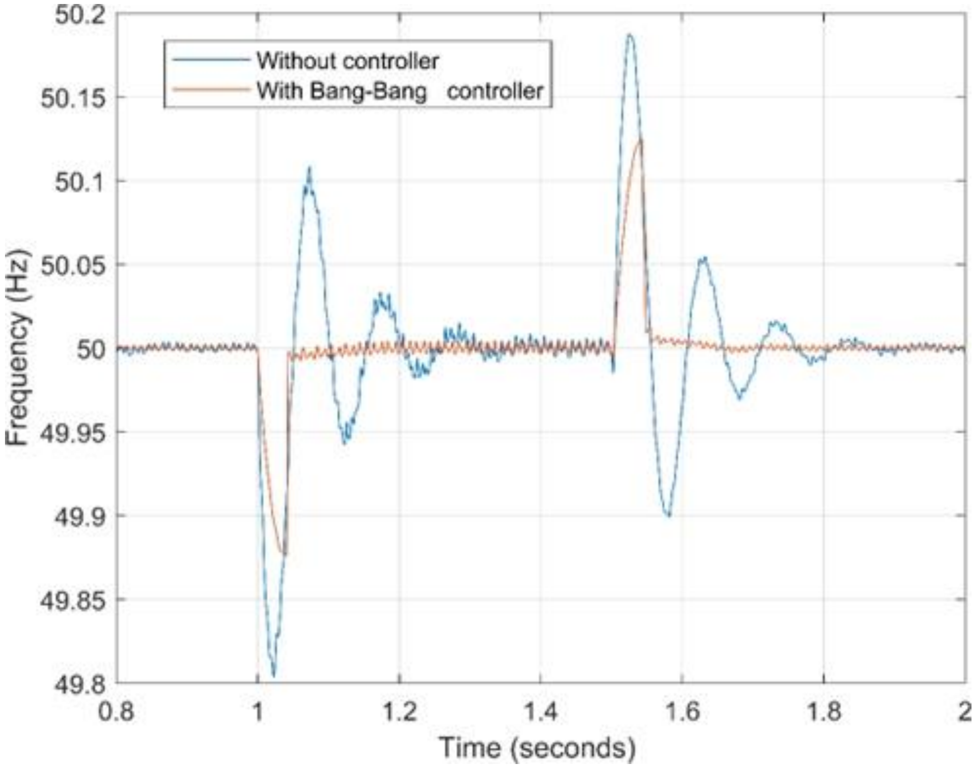

Fig 29: Frequency deviation subject to load change

The response resembles the small signal model. Hence it verifies that there was no issue in the simulation. Although there is some noise that may occur because of the other harmonics other than fundamental frequency. The LCL filter filters most output of it but still the THD is not 0. Hence it reflects the practical scenario. Surprisingly, the peaks of the frequency for the Bang-Bang case are much smaller. The settling time is reduced by more than 75%.

The output three phase voltage is shown below in the figure.

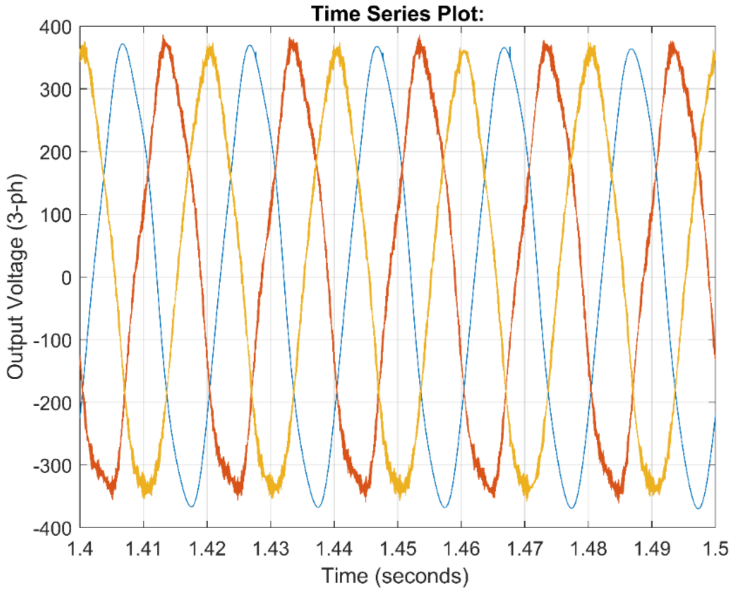

Fig 30: Output 3-phase voltage waveforms

Notice that 3-phase voltage waveforms have some harmonics and hence this is the reason of noise in the frequency deviation. Considering a practical scenario, this much of little noise is acceptable.

The amplitude of the emf generated by the VSG block is shown below in the figure.

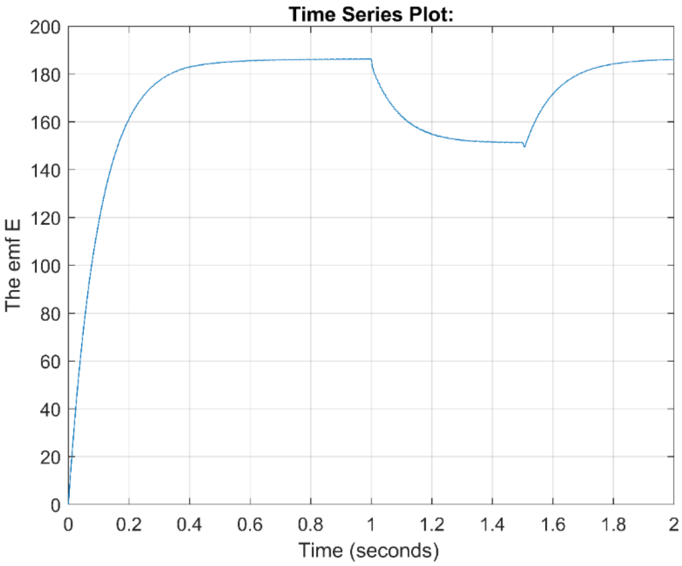

Fig 31: Emf *E* generated by VSG block in micro grid system.

It can be seen from the plot that emf E decreases from 1s. This is because of the increase in load and 1.5sec the emf is increased because of the decrease in the load. The reference signal is generated by the virtual impedance block for the purpose of generating the required PWM signal for the inverter. This reference 3-phase signal is shown below in the figure.

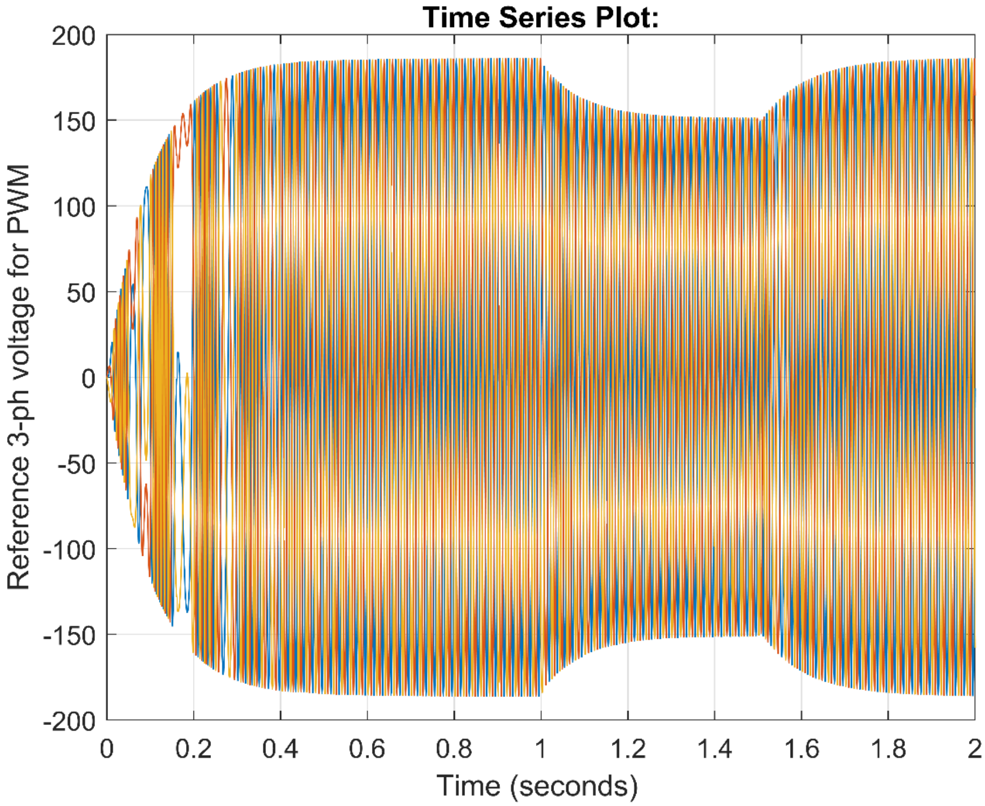

Fig 32: Reference signal for PWM generation.

Notice the transient state, the signal not resembling the 3-phase signal. This is because of the frequency deviation in the transient that start from 0Hz and range up to about 80Hz and approach the steady state at 50Hz which is the required fundamental frequency. For frequency near to 50Hz, the signal is exactly like a 3-phase wave form. A zoom view of the reference signal is shown below.

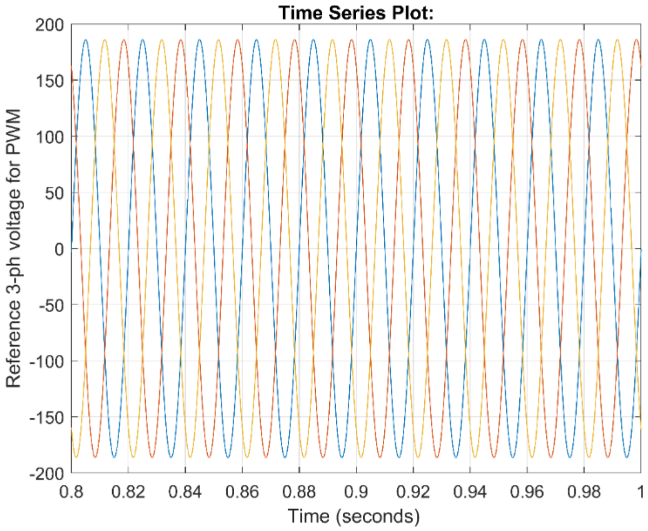

Fig 33: Zoom view of the Reference signal for the PWM generation

The Bang-Bang controller changes the inertia of the virtual synchronous generator to overcome the deviation in the frequency as shown above in the figures. So, now let us see how the Bang-Bang controller changes the inertia for the micro grid system. The following figures shows the inertia of the virtual generator.

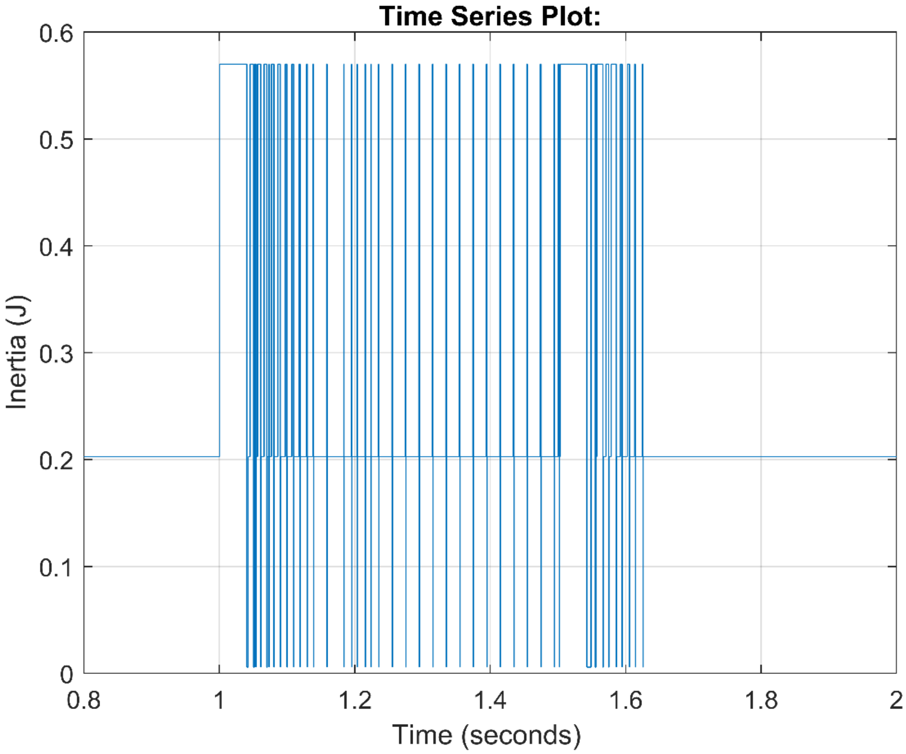

Fig 34: Inertia of the VSG for the micro grid for the Bang-Bange controller

Notice that jumping from minimum to maximum inertia is much higher compared with the small signal model. And this is because of the distortion in the frequency deviation. Notice that near 1 sec and 1.5 sec, where the load is changed, one inertia state lasts for long time compared to intermediate one. These intermediate jumps look like an impulse that appear because of the wrong decision taken by the Bang-Bang controller. This issue can be fixed by increasing the steady state interval f_s. So, it prevents the controller from fluctuation in the steady state.

9. Conclusion

Changing the load and observing its effect on the frequency and its impact on the overall system shows the importance of inertia in the power systems. To conclude, we can say that introducing virtual inertia in the system can improve the response but comparatively having dynamic inertia can improve the response further. Analyzing the SSM and the micro grid, with and without the Bang-Bang controller, concluded that with the improved Bang-Bang controller, the settling time is reduced to about 75% with cost of exceedingly small overshoot. The oscillation is eliminated with the Bang-Bang controller. With the frequency deviation, the active power increases that causes a decrease in the reactive power. The change in power is incredibly significant subject to frequency deviation. With 0.15Hz maximum frequency deviation, the reactive power is decreased by about 600Var. That is why maintaining the frequency for the power system is very crucial. And hence the Bang-Bang is a strategy to do so. The virtual inertia fluctuates more in microgrid compared to the small signal model. Because the frequency deviation in steady state exceeds the define steady state interval. It also concluded that the frequency deviation behaves like a second order system with underdamped response.

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APPENDIX

The MATLAB responsible to generate all the plots is:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%      Author: Muhammad Ibrar      %%%
%%% This script required 4 simulink  %%%
%%%            model                 %%%
close all; clear; clc
%% Simulate the simulink model as

% simulate the small signal models as
SM = sim("SSM_Without_BangBang.slx");
SM_BB = sim("SSM_With_BangBang.slx");

% simulate the micro grid
MG = sim("MG_Without_BangBang.slx");
MG_BB = sim("MG_With_BangBang.slx");

%% Create the figure 23 as
figure;
plot(SM.t, SM.f, "--");
hold on
plot(SM_BB.t, SM_BB.f, "--");
xlim([0.8 2]);
xlabel("Time (sec)");
ylabel("Frequency (Hz)")
legend("Without control", "With Bang-Bang control")


%% Create the figure 24 as
figure;
plot(SM.t, SM.Ps, "--");
hold on
plot(SM.t, SM.Qs, "--");
xlim([0.8 2]);
ylim([0 11000])
xlabel("Time (sec)");
ylabel("Power (Watt)")
legend("P_s", "Q_s")

%% Create figure 25
figure;
subplot(2,1,1);
plot(SM.t, SM.Pe)
hold on
```

```
plot(SM_BB.t,SM_BB.Pe);
xlim([0.8 2])
xlabel("Time (sec)");
ylabel("Power (W)");
l = legend("P_e without control",...
            "P_e with Bang-Bang control");
l.Location = "best";

subplot(2,1,2);
plot(SM.t, SM.Qe)
hold on
plot(SM_BB.t,SM_BB.Qe);
xlim([0.8 2])
xlabel("Time (sec)");
ylabel("Power (VaR)");
l = legend("Q_e without control",...
            "Q_e with Bang-Bang control");
l.Location = "best";

%% Figure 26
figure;
plot(SM.t, SM.dE);
hold on
plot(SM_BB.t, SM_BB.dE)
xlabel("Time (sec)")
ylabel("Change in emf (volt)")
l = legend("dE without control",...
    "dE with bang-bang control");
l.Location = "best";
grid on


%% Figure 27
figure;
plot(SM.t,0.2028*ones(length(SM.t),1));
hold on
plot(SM_BB.t,SM_BB.J)
xlabel("Time (sec)")
ylabel("Virtual inertia (Kg/m^2)")
grid on
l = legend("Virtual inertia without control", ...
            "Virtual inertia with control");
l.Location = "best";

%% Figure 28
figure;
```

```
plot(MG.tout, MG.f);
hold on
plot(MG_BB.tout, MG_BB.f);
xlim([0 0.6]);
xlabel("Time (seconds) ")
ylabel("Frequency (Hz) ")
legend("With controller", ...
        "With Bang-Bang controller")
grid on

%% Figure 29
figure;
plot(MG.tout, MG.f);
hold on
plot(MG_BB.tout, MG_BB.f);
xlim([0.8 2]);
ylim([49.8 50.2])
xlabel("Time (seconds) ")
ylabel("Frequency (Hz) ")
l = legend("With controller", ...
        "With Bang-Bang controller");
l.Location = "best";
grid on

%% Figure 30
figure
plot(MG.Uo);
xlim([1.4 1.5])
xlabel("Time (seconds) ");
ylabel("Output Voltage (3-ph)")
grid on

%% Figure 31
figure;
plot(MG.E);
xlabel("Time (seconds) ");
ylabel("The emf E")
grid on

%% Figure 32
figure;
plot(MG.Uref);
xlabel("Time (seconds) ");
ylabel("Reference 3-ph voltage for the PWM")
grid on
```

```
%% Figure 33
figure;
plot(MG.Uref);
xlim([0.8 1])
xlabel("Time (seconds) ");
ylabel("Reference 3-ph voltage for the PWM")
grid on

%% Figure 34
figure;
plot(MG_BB.J);
xlim([0.8 2])
xlabel("Time (seconds) ")
ylabel("Inertia (J)")
grid on
```

The MATLAB script responsible for calculating the value R and X for certain active and reactive power is below:

```
%% Define the constant parameters
E_s = 226;
Sigma_s = 0.05;
P = 10000;
Q = -2000;

syms R X

eq1 = P == (R * E_s ^ 2 * ((cos(Sigma_s)) ^2 - (sin(Sigma_s)) ^ 2) +
...
            2 * X * E_s ^ 2 * cos(Sigma_s) * sin(Sigma_s)) /...
            (R ^ 2 + X ^ 2);

eq2 = Q == (2 * R * E_s ^ 2 * cos(Sigma_s) * sin(Sigma_s) - ...
            X * E_s ^ 2 * ((cos(Sigma_s)) ^2 - (sin(Sigma_s)) ^ 2))
/...
            (R ^ 2 + X ^ 2);

[R,X] = solve(eq1,eq2,R,X);
R = vpa(R);
X = vpa(X);
disp(R)
disp(X)
```