mathematics

MDPI

Article

New Fermatean Fuzzy Distance Metric and Its Utilization in the Assessment of Security Crises using the MCDM Technique

Paul Augustine Ejegwa [1], Manasseh Terna Anum [1], Nasreen Kausar [2], Chukwudi Obinna Nwokoro [3], Nezir Aydin [4,5] and Hao Yu [6,*]

[1] Department of Mathematics, Joseph Sarwuan Tarka University, Makurdi 970101, Nigeria; ejegwa.augustine@uam.edu.ng (P.A.E.); manasseh.anum@uam.edu.ng (M.T.A.)
[2] Department of Mathematics, Faculty of Arts and Science, Yildiz Technical University, Esenler, Istanbul 34220, Türkiye; nkausar@yildiz.edu.tr
[3] Department of Computer Science, Faculty of Science, University of Uyo, Uyo 502101, Nigeria; chukwudi.nwokoro@iaue.edu.ng
[4] College of Science and Engineering, Hamad Bin Khalifa University, Qatar Foundation, Doha 34110, Qatar; naydin@hbku.edu.qa
[5] Department of Industrial Engineering, Yildiz Technical University, Besiktas, Istanbul 34349, Türkiye
[6] Department of Industrial Engineering, Faculty of Engineering Science and Technology, UiT-The Arctic University of Norway, 8514 Narvik, Norway
* Correspondence: hao.yu@uit.no

Abstract: The problem of insecurity is a global phenomenon that has several forms like terrorism, banditry, kidnappings, etc. Insecurity has taken hold in the Sub-Saharan Region of West Africa, especially in Nigeria, for over two decades. Nigeria's security crisis is more pronounced in the Northern Region, with a new wave in the North-Central Region of Nigeria. It is herculean to assess insecurity in the North-Central Region of Nigeria because of the region's fuzzy or imprecise nature of insecurity. This constitutes the rationale for deploying the Fermatean fuzzy technique to assess insecurity due to the capacity of the Fermatean fuzzy scheme to handle imprecision. To this end, a new Fermatean fuzzy distance metric is presented to evaluate insecurity in the North-Central Region of Nigeria using a multi-criteria decision-making technique. To express the logic for creating the new Fermatean fuzzy distance metric, some existing Fermatean fuzzy distance metrics are discussed, along with their drawbacks. The mathematical properties of the new technique are discussed, and the new method is applied computationally to assess insecurity in the North-Central Region of Nigeria. The data for the security assessment are collected via Fermatean fuzzy linguistic variables using the opinions of security experts and analyzed using the technique for order of preference by similarity to ideal solution, which is a commonly used multi-criteria decision-making method. Finally, the numerical validity of the new technique is expressed with comparative results, and the finding shows the benefit of the new distance approach over the existing methodologies. The outcome of the work will provide reliable traveling advisories for safe voyages within the region.

Keywords: Fermatean fuzzy distance metric; Fermatean fuzzy sets; security crises; travel advisories; security assessment; multi-criteria decision-making technique; North-Central Region of Nigeria

MSC: 20N20, 03E72, 47S40

Citation: Ejegwa, P.A.; Anum, M.T.; Kausar, N.; Nwokoro, C.O.; Aydin, N.; Yu, H. New Fermatean Fuzzy Distance Metric and Its Utilization in the Assessment of Security Crises using the MCDM Technique. *Mathematics* **2024**, *12*, 3214. https://doi.org/10.3390/math12203214

Academic Editors: Rajeev Kumar, Suhel Ahmad Khan, Alka Agrawal and Raees Ahmad Khan

Received: 31 August 2024
Revised: 7 October 2024
Accepted: 11 October 2024
Published: 14 October 2024

1. Introduction

1.1. Background of the Study

Security is crucial in safeguarding lives and properties. The term security crisis is the breakdown of law and order resulting in the loss of lives and properties. It comes in different forms like terrorisms, kidnapping for ransom, armed robbery, banditry, militancy, and communal clashes, among other social vices. Everyone strives for security to live a secure life, and as such, substantial efforts are made to guarantee individual safety [1]. Every government has the responsibility to ensure the safety of its citizens' lives and properties through the implementation of various security measures. The implementation of the security measures improves safety, reduces incidents, safeguards properties, and promotes the general welfare of individuals and communities [2]. In all these, the role of the security professionals cannot be overemphasized. Nigeria has been plagued with insecurity for more than two decades. The issue of insecurity in Nigeria is particularly pronounced in Northern Nigeria, especially in the North-Central Region of Nigeria (NCRN), where conflicts continue among ethnic and religious groups, as well as between individuals with ancestral ties to an area and those who have temporarily resided there. Tensions between cattle herders and crop farmers further complicate the social landscape in the NCRN. Okoro [3] emphasized the significant impact of historical grievances and political marginalization in fueling ethnic and religious discord. Some security professionals suspected that the issue of insecurity in the NCRN is due to its topographical terrains and the ungoverned spaces [4,5]. The economic repercussions of these conflicts, including disruptions in agricultural activities leading to food shortages and heightened poverty levels, pose a considerable concern due to the fuzziness of the problem. The employment of security measures is not an exclusive approach to ameliorate the problem of insecurity in the NCRN, but a better understanding of the root cause of the problem is essential.

Katsina [6] presented an approach to understand the causes of security crises in Nigeria and alleged that the crises are deep-rooted in society due to unemployment, poverty, and inequality. However, the work fails to evaluate the security crises but gives directives for the resolution of the problems. In addition, Ifedayo et al. [7] assessed the Nigeria security crises based on descriptive statistics through discrete values without minding the fuzziness of the crises. However, both descriptive statistics and discrete values cannot capture the imprecision and fuzziness of security crises. As a consequence, the work cannot reliably assess security crises due to the setback of descriptive statistics and discrete values. Similarly, van den Berg et al. [8] gave an account on assessing contemporary crises in security studies and safety science.

Moreover, Gizun et al. [9] discussed a method of assessing critical levels of crisis situations based on fuzzy logic and expert approaches. However, this method cannot be trusted due to the limitations of fuzzy logic [10]. Thus, a reliable assessment of the problem of insecurity can only be achieved by the deployment of soft computing tools like intuitionistic fuzzy sets (IFSs), Pythagorean fuzzy sets (PFSs), and Fermatean fuzzy sets (FFSs) due to their knack in handling fuzziness and imprecision.

1.2. Literature Review

Employing approximation techniques, such as fuzzy set theory (FST) [10], remains a promising tool for reducing the fuzziness of uncertain problems and offers a significant approach to addressing insecurity. The FST evaluates the membership degree (MD) of an element in a set, and it is defined within the range [0,1]. Unlike the orthodox set, where a member either belongs to the set or otherwise (with membership values strictly 0 or 1), FST allows for partial membership. Nevertheless, the fundamental problem of FST is that it does integrate the non-membership degree (NMD) of an element in a set and the possibility of hesitancy [11]. It is on this premise that Atanassov established the IFS [12], which aims at combining the MD and NMD such that $1 - MD$ may not be equivalent to NMD. The intuitionistic fuzzy hesitation boundary is indicated by deducting one from the aggregate of MD and NMD. These features above validate the notion of IFSs as an effective tool for mitigating ambiguity and inaccuracy in various real-world scenarios, like medical decisions and diagnosis [13,14], pattern classification [15,16], polling [17], assessment of energy alternatives [18], academics [19], and selection problems [20]. Despite the widespread use of IFS in addressing cases of vagueness, which surpass the traditional fuzzy sets, it cannot handle multiple inputs, complicating decision-making processes and computational tasks where the aggregate of NMD and MD exceeds one.

PFSs were created to overcome the problem of IFSs, especially in accurately and flexibly managing MD and NMD [21]. In PFSs, the square sum of MD and NMD is at most a value of one, including the indeterminacy degree, which confidently represents the level of uncertainty or ambiguity in the MD/NMD. This additional property, with the robust representation of uncertainty than IFSs, made PFSs resourceful in decision-making [22–25], In [26,27], some distance methods for PFSs were constructed to discuss medical diagnosis. Büyüközkan et al. [28] integrated the Choquet integral approach for vertical farming using PFSs, and the significant impact of PFSs has led to the development of similarity measures [29]. Despite the significant successes of PFSs, increased computational complexity when the square sum of MD and NMD exceeds one remains a notable limitation of PFSs.

The Fermatean fuzzy set (FFS), prefaced as a progression from PFSs [30], offers a novel approach to address uncertainty by applying the Fermatean fuzzy principle. This principle incorporates a distinct indeterminacy measure inspired by the Fermat's principle of least time to provide an enlarged intuitive and active representation of uncertainty in decision-making. Additionally, this method incorporates three essential constraints: MD, NMD, and hesitancy. Several practical problems have been explicated based on FFSs. In [31], a decision-making problem was presented for green supplier evaluation using FFSs, and another use of FFSs was presented in [32]. In [33], the COVID-19 assessment laboratory selection was carried out using FFSs, and other uses of FFSs in decision-making were discussed in [34–40]. Similarly, FFSs have been utilized in medicine [41,42] and pattern recognition [43,44].

A comprehensive and nuanced understanding of data is provided by utilizing distance measures between data. This approach captures the richer representation's intricacies and establishes a robust quantitative framework for thorough evaluation. Due to the presence of fuzziness in data analysis, Fermatean fuzzy distance (FFD) has been constructed. Senapati and Yager [45] pioneered the research on the Fermatean fuzzy distance method (FFDM) by proposing an approach for the FFDM, but the cardinality of the sets was ignored, which could affect the distance outcome. Deng and Wang [46] suggested three FFDMs and explored their implementation, but the three parameters of the FFSs were not taken. Ganie [47] created four FFDMs and discussed their application in multi-criteria decision-making (MCDM), but the Fermatean fuzzy hesitation margins (FFHMs) were excluded. In [48], the FFDM in [45] was modified with robust outcomes and used to discuss career placement. Kirisci [49] created an FFDM using cosine similarity with application in MCDM. In [50], a comment was made on the limitation of the approach in [49]. Ganie et al. [51] presented a 3D FFDM and discussed using it in approximate reasoning and classification. In this method, the number of parametric differences is not integrated, which may affect the distance output. Liu [52] presented a 2D FFDM using triangular divergence with application in diagnosis without the effect of FFHMs, which could affect the outcome.

1.3. Motivation and Contributions

A number of decision-making problems have been addressed with the aid of the FFDM. The choice of FFSs over either IFSs or PFSs in assessing security crises is because of the ability of FFSs to curb complex cases of fuzziness in decision-making and assessment. From the review of works on the various methods of the FFDM, the rationale for the construction of a new approach for the FFDM is justifiable because of the drawbacks of the existing approaches of the FFDM. In addition to the stated limitations of the FFDMs in [45–52], none was developed based on Fermatean fuzzy tendency coefficients (FFTCs) of the Fermatean fuzzy numbers (FFNs), which is very significant for a reliable outcome. Although some security experts have researched the problem of insecurity in the NCRN [3–5], no work has be done on the assessment of security crises based on soft computing tools like FFDMs, and hence, the interpretations of the hitherto studies may be biased due to the fuzziness of security crises. Motivated by the security assessment research gap and the drawbacks of the existing FFDMs, this work presents a new FFDM based on the FFTCs and the incorporation of the FFHMs to discuss the assessment of security crisis within the NCRN, which is one of the insecurity hotbeds in Nigeria. This work contributes to knowledge in the following ways:

I. Introducing a new FFDM based on FFTCs and the inclusion of FFHMs.
II. Characterizing the new FFDM to reinforce its satisfaction with the FFD conditions.
III. Based on expert knowledge with Fermatean fuzzy linguistic variables and FFNs, the Fermatean fuzzification of the insecurity situations in the NCRN is obtained.
IV. The assessment of the security crises within the NCRN will determine the most insecure state for reliable travel advice based on the new FFDM using the MCDM technique.
V. The comparative analysis of the explored FFDMs is presented to substantiate the preeminence of the new FFDM.

A brief account of the rest of the article is provided. Section 2 contains the elementary knowledge of PFSs and the exploration of the existing FFDMs. Section 3 discusses the new FFDM and its properties. Section 4 presents the application of the FFDM in assessing insecurity situations using the MCDM technique with comparative analysis, and Section 5 presents the conclusion of the article and suggests research directions.

2. Preliminaries

This section reviews the basics of FFSs and distance measures between FFSs. Throughout the course of this work, let $R = \{r_1, r_2, \dots, r_k\}$ be a finite, nonempty set, where k is the number of elements in R, and $FFS(R)$ is the collection of all FFSs in R.

Definition 1 [30]**.** *A FFS* $\mathfrak{C}$ *in* R *can be described as follows*:

$$\mathfrak{C} = \left\{\left(r_j, \mathfrak{C}_m(r_j), \mathfrak{C}_n(r_j)\right) | r_j \in R\right\},$$

where $\mathfrak{C}_m, \mathfrak{C}_n : R \to [0,1]$*, such that* $0 \le \mathfrak{C}_m^3(r_j) + \mathfrak{C}_n^3(r_j) \le 1$ *for all* $r \in R$*, where* $\mathfrak{C}_m(r_j)$ *is the MD and* $\mathfrak{C}_n(r_j)$ *is the NMD of* r_j *in* $\mathfrak{C}$*. The FFHM, which is denoted as* $\mathfrak{C}_h(r_j)$ *and defined by* $\mathfrak{C}_h(r_j) = \sqrt[3]{1 - \mathfrak{C}_m^3(r_j) - \mathfrak{C}_n^3(r_j)}$*, is the grade of non-determinacy of* $r_j \in R$ *to the set* $\mathfrak{C}$*. Thus,* $\mathfrak{C}_h(r_j) \in [0,1]$ *where* $j = 1, 2, \dots, k$*.*

Next, we present some basic operations on PFSs.

Definition 2 [30]**.** *Presume we have two FFSs,* $\widehat{\mathfrak{C}}$ *and* $\widetilde{\mathfrak{C}}$*, in* R*. Then, we have the following properties:*

I. *Complement;* $\widehat{\mathfrak{C}}^c = \left\{\left(r_j, \widehat{\mathfrak{C}}_n(r_j), \widehat{\mathfrak{C}}_m(r_j)\right) | r_j \in R\right\}$,
II. *Intersection;*

$$\widehat{\mathfrak{C}} \cap \widetilde{\mathfrak{C}} = \left\{\begin{pmatrix} r_j, min\{\widehat{\mathfrak{C}}_m(r_j), \widetilde{\mathfrak{C}}_m(r_j)\}, \\ max\{\widehat{\mathfrak{C}}_n(r_j), \widetilde{\mathfrak{C}}_n(r_j)\} \end{pmatrix} | r \in R\right\},$$

III. *Union;*

$$\widehat{\mathfrak{C}} \cup \widetilde{\mathfrak{C}} = \left\{\begin{pmatrix} r_j, \max\{\widehat{\mathfrak{C}}_m(r_j), \widetilde{\mathfrak{C}}_m(r_j)\}, \\ \min\{\widehat{\mathfrak{C}}_n(r_j), \widetilde{\mathfrak{C}}_n(r_j)\} \end{pmatrix} | r_j \in R\right\},$$

IV. *Inclusion relation;* $\widehat{\mathfrak{C}} \subseteq \widetilde{\mathfrak{C}} \Leftrightarrow \widehat{\mathfrak{C}}_m(r_j) \le \widetilde{\mathfrak{C}}_m(r_j)$ *and* $\widehat{\mathfrak{C}}_n(r_j) \ge \widetilde{\mathfrak{C}}_n(r_j)$ *for all* $r_j \in R$,
V. *Equality;* $\widehat{\mathfrak{C}} = \widetilde{\mathfrak{C}} \Leftrightarrow \widehat{\mathfrak{C}}_m(r_j) = \widetilde{\mathfrak{C}}_m(r_j)$ *and* $\widehat{\mathfrak{C}}_n(r_j) = \widetilde{\mathfrak{C}}_n(r_j)$ *for all* $r_j \in R$,
VI. *Sum;*

$$\widehat{\mathfrak{C}} \oplus \widetilde{\mathfrak{C}} = \left\{ \left(\begin{matrix} r_j, \left(\widehat{\mathfrak{C}}_m^3(r_j) + \widetilde{\mathfrak{C}}_m^3(r_j) - \widehat{\mathfrak{C}}_m^3(r_j)\widetilde{\mathfrak{C}}_m^3(r_j) \right)^{\frac{1}{3}}, \\ \widehat{\mathfrak{C}}_n(r_j)\widetilde{\mathfrak{C}}_n(r_j) \end{matrix} \right) \middle| r_j \in R \right\},$$

VII. *Product;*

$$\widehat{\mathfrak{C}} \otimes \widetilde{\mathfrak{C}} = \left\{ \left(r_j, \widehat{\mathfrak{C}}_m(r_j)\widetilde{\mathfrak{C}}_m(r_j), \quad \left(\begin{matrix} \widehat{\mathfrak{C}}_m^3(r_j) + \widetilde{\mathfrak{C}}_m^3(r_j) - \\ \widehat{\mathfrak{C}}_m^3(r_j)\widetilde{\mathfrak{C}}_m^3(r_j) \end{matrix} \right)^{\frac{1}{3}} \right) \middle| r_j \in R \right\}.$$

Next, before discussing the FFDM, we present the definition of FFD as follows:

Definition 3 [45]. *Supposing* $\mathfrak{C}, \widehat{\mathfrak{C}}$, *and* $\widetilde{\mathfrak{C}}$ *be FFSs in* R. *Then, the FFDM signified by* $\mathcal{D}$ *between the FFSs is a function* $\mathcal{D}: FFS(R) \times FFS(R) \rightarrow [0,1]$ *such that the following are satisfied:*

I. $0 \leq \mathcal{D}(\widehat{\mathfrak{C}}, \widetilde{\mathfrak{C}}) \leq 1$,
II. $\mathcal{D}(\widehat{\mathfrak{C}}, \widehat{\mathfrak{C}}) = 0$, $\mathcal{D}(\widetilde{\mathfrak{C}}, \widetilde{\mathfrak{C}}) = 0$, and $\mathcal{D}(\mathfrak{C}, \mathfrak{C}) = 0$,
III. $\mathcal{D}(\widehat{\mathfrak{C}}, \widetilde{\mathfrak{C}}) = \mathcal{D}(\widetilde{\mathfrak{C}}, \widehat{\mathfrak{C}})$,
IV. $\mathcal{D}(\widehat{\mathfrak{C}}, \mathfrak{C}) + \mathcal{D}(\mathfrak{C}, \widetilde{\mathfrak{C}}) \geq \mathcal{D}(\widehat{\mathfrak{C}}, \widetilde{\mathfrak{C}})$.

As the distance value approaches 0, it shows that the considered FFSs are well related with each other. In terms of comparisons among varied approaches of the FFDM, the approach that yields the least distance value is considered as the most appropriate and reliable method provided all the Fermatean fuzzy parameters are incorporated.

To arrive at a reliable distance output between FFSs, it is necessary to consider the weight of each of the elements of the considered FFSs. In numerous instances, the significance of each individual element of FFSs needs to be considered. In the MCDM problem, each of the typically criteria holds varying levels of significance, necessitating the assignment of separate weights. In terms of applications, the elements are considered as the criteria. Now, we present a formula for the computation of the weight of each of the elements of FFSs.

Definition 4. *Suppose there are distinct PFSs defined in* $R = \{r_1, r_2, \ldots, r_k\}$, *namely,*

$$\widehat{\mathfrak{C}} = \left\{ \left(r_j, \widehat{\mathfrak{C}}_m(r_j), \widehat{\mathfrak{C}}_n(r_j) \right) | r_j \in R \right\} \text{ and } \widetilde{\mathfrak{C}} = \left\{ \left(r_j, \widetilde{\mathfrak{C}}_m(r_j), \widetilde{\mathfrak{C}}_n(r_j) \right) | r_j \in R \right\}.$$

Then, the weights of the elements of R denoted as W_j is defined as follows:

$$W_j = \frac{\frac{3\widehat{\mathfrak{C}}_m^3(r_j) + \widehat{\mathfrak{C}}_n^3(r_j) + \widehat{\mathfrak{C}}_h^3(r_j)}{3} + \frac{3\widetilde{\mathfrak{C}}_m^3(r_j) + \widetilde{\mathfrak{C}}_n^3(r_j) + \widetilde{\mathfrak{C}}_h^3(r_j)}{3}}{\sum_{j=1}^{k} \left(\frac{3\widehat{\mathfrak{C}}_m^3(r_j) + \widehat{\mathfrak{C}}_n^3(r_j) + \widehat{\mathfrak{C}}_h^3(r_j)}{3} + \frac{3\widetilde{\mathfrak{C}}_m^3(r_j) + \widetilde{\mathfrak{C}}_n^3(r_j) + \widetilde{\mathfrak{C}}_h^3(r_j)}{3} \right)} \quad (1)$$

for $W_j \in [0,1]$ and $\sum_{j=1}^{k} W_j = 1$.

Existing FFDMs

In this section, we outline some approaches of FFDMs and their corresponding weighted forms. Using the FFSs in Definition 3, the existing FFDMs are enumerated. Senapati and Yager [45] presented an approach of FFDM as follows:

$$\mathcal{D}_{SY}(\widehat{\mathfrak{C}}, \widetilde{\mathfrak{C}}) = \sqrt{\frac{1}{2} \sum_{j=1}^{k} \left(\begin{matrix} \left(\widehat{\mathfrak{C}}_m^3(r_j) - \widetilde{\mathfrak{C}}_m^3(r_j) \right)^2 \\ + \left(\widehat{\mathfrak{C}}_n^3(r_j) - \widetilde{\mathfrak{C}}_n^3(r_j) \right)^2 \\ + \left(\widehat{\mathfrak{C}}_h^3(r_j) - \widetilde{\mathfrak{C}}_h^3(r_j) \right)^2 \end{matrix} \right)}, \quad (2)$$

and the weighted form is described as follows:

$$\mathcal{D}_{SY_w}\left(\hat{\mathfrak{T}},\tilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{2}\sum_{j=1}^{k}W_j\left(\begin{array}{c}\left(\hat{\mathfrak{T}}_m^3(r_j)-\tilde{\mathfrak{T}}_m^3(r_j)\right)^2\\+\left(\hat{\mathfrak{T}}_n^3(r_j)-\tilde{\mathfrak{T}}_n^3(r_j)\right)^2\\+\left(\hat{\mathfrak{T}}_h^3(r_j)-\tilde{\mathfrak{T}}_h^3(r_j)\right)^2\end{array}\right)}. \quad (3)$$

In (2), the cardinality of R is ignored, which could affect this distance outcome. Additional approaches for FFDM were developed in [46] and presented as follows:

$$\mathcal{D}_{DW1}\left(\hat{\mathfrak{T}},\tilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{2k}\sum_{j=1}^{k}\left(\begin{array}{c}\frac{\left(\hat{\mathfrak{T}}_m^3(r_j)-\tilde{\mathfrak{T}}_m^3(r_j)\right)^2}{\hat{\mathfrak{T}}_m^3(r_j)+\tilde{\mathfrak{T}}_m^3(r_j)}\\+\frac{\left(\hat{\mathfrak{T}}_n^3(r_j)-\tilde{\mathfrak{T}}_n^3(r_j)\right)^2}{\hat{\mathfrak{T}}_n^3(r_j)+\tilde{\mathfrak{T}}_n^3(r_j)}\end{array}\right)}, \quad (4)$$

$$\mathcal{D}_{DW2}\left(\hat{\mathfrak{T}},\tilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{2k}\sum_{j=1}^{k}\left(\begin{array}{c}\left(\hat{\mathfrak{T}}_m^3(r_j)-\tilde{\mathfrak{T}}_m^3(r_j)\right)^2\\+\left(\hat{\mathfrak{T}}_n^3(r_j)-\tilde{\mathfrak{T}}_n^3(r_j)\right)^2\\+\left(\hat{\mathfrak{T}}_h^3(r_j)-\tilde{\mathfrak{T}}_h^3(r_j)\right)^2\end{array}\right)}, \quad (5)$$

$$\mathcal{D}_{DW3}\left(\hat{\mathfrak{T}},\tilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{2k}\sum_{j=1}^{k}\left(\begin{array}{c}\left(\sqrt{\hat{\mathfrak{T}}_m^3(r_j)}-\sqrt{\tilde{\mathfrak{T}}_m^3(r_j)}\right)^2\\+\left(\sqrt{\hat{\mathfrak{T}}_n^3(r_j)}-\sqrt{\tilde{\mathfrak{T}}_n^3(r_j)}\right)^2\end{array}\right)}. \quad (6)$$

The weighted forms of the measures are expressed as follows:

$$\mathcal{D}_{DW1_w}\left(\hat{\mathfrak{T}},\tilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{2}\sum_{j=1}^{k}W_j\left(\begin{array}{c}\frac{\left(\hat{\mathfrak{T}}_m^3(r_j)-\tilde{\mathfrak{T}}_m^3(r_j)\right)^2}{\hat{\mathfrak{T}}_m^3(r_j)+\tilde{\mathfrak{T}}_m^3(r_j)}\\+\frac{\left(\hat{\mathfrak{T}}_n^3(r_j)-\tilde{\mathfrak{T}}_n^3(r_j)\right)^2}{\hat{\mathfrak{T}}_n^3(r_j)+\tilde{\mathfrak{T}}_n^3(r_j)}\end{array}\right)}, \quad (7)$$

$$\mathcal{D}_{DW2_w}\left(\hat{\mathfrak{T}},\tilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{2}\sum_{j=1}^{k}W_j\left(\begin{array}{c}\left(\hat{\mathfrak{T}}_m^3(r_j)-\tilde{\mathfrak{T}}_m^3(r_j)\right)^2\\+\left(\hat{\mathfrak{T}}_n^3(r_j)-\tilde{\mathfrak{T}}_n^3(r_j)\right)^2\\+\left(\hat{\mathfrak{T}}_h^3(r_j)-\tilde{\mathfrak{T}}_h^3(r_j)\right)^2\end{array}\right)}, \quad (8)$$

$$\mathcal{D}_{DW3_w}\left(\hat{\mathfrak{T}},\tilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{2}\sum_{j=1}^{k}W_j\left(\begin{array}{c}\left(\sqrt{\hat{\mathfrak{T}}_m^3(r_j)}-\sqrt{\tilde{\mathfrak{T}}_m^3(r_j)}\right)^2\\+\left(\sqrt{\hat{\mathfrak{T}}_n^3(r_j)}-\sqrt{\tilde{\mathfrak{T}}_n^3(r_j)}\right)^2\end{array}\right)}. \quad (9)$$

In (4) and (6) and their weighted forms, the FFHMs are not included, and the number of parametric differences is not incorporated in (5) and its weighted form.

In addition, some approaches of FFDM were developed in [47] and presented as follows:

$$\mathcal{D}_{G1}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\frac{1}{k}\sum_{j=1}^{k}\begin{pmatrix}\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\\+\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|\\-\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|\end{pmatrix}, \tag{10}$$

$$\mathcal{D}_{G2}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\frac{1}{k}\sum_{j=1}^{k}min\begin{Bmatrix}1,\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\\+\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|\end{Bmatrix}, \tag{11}$$

$$\mathcal{D}_{G3}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\frac{1}{k}\sum_{j=1}^{k}\left[\frac{\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|+\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|}{1+\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|}\right], \tag{12}$$

$$\mathcal{D}_{G4}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\frac{1}{k}\sum_{j=1}^{k}\left(\frac{\left|\widehat{\mathfrak{T}}_m^3(s_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|+\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|-2\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|}{1-\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\left|\widehat{\mathfrak{T}}_n^3(s_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|}\right). \tag{13}$$

The weighted forms of the measures are expressed as follows:

$$\mathcal{D}_{G1_w}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\sum_{j=1}^{k}W_j\begin{pmatrix}\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\\+\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|\\-\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|\end{pmatrix}, \tag{14}$$

$$\mathcal{D}_{G2_w}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\sum_{j=1}^{k}W_j\left(min\begin{Bmatrix}1,\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\\+\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|\end{Bmatrix}\right), \tag{15}$$

$$\mathcal{D}_{G3_w}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\sum_{j=1}^{k}W_j\left[\frac{\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|+\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|}{1+\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|}\right], \tag{16}$$

$$\mathcal{D}_{G4_w}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\sum_{j=1}^{k}W_j\left(\frac{\left|\widehat{\mathfrak{T}}_m^3(s_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|+\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|-2\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\left|\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|}{1-\left|\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right|\left|\widehat{\mathfrak{T}}_n^3(s_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right|}\right). \tag{17}$$

In (10–13), the FFHMs need to be incorporated; thus, the approaches cannot be reliable.

Onyeke and Ejegwa [48] modified an approach for the FFDM in [45] as follows:

$$\mathcal{D}_{OE}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{3k}\sum_{j=1}^{k}\begin{pmatrix}\left(\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right)^2\\+\left(\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right)^2\\+\left(\widehat{\mathfrak{T}}_h^3(r_j)-\widetilde{\mathfrak{T}}_h^3(r_j)\right)^2\end{pmatrix}}. \tag{18}$$

The weighted form of the method is expressed as follows:

$$\mathcal{D}_{OE_w}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\sqrt{\frac{1}{3}\sum_{j=1}^{k}W_j\begin{pmatrix}\left(\widehat{\mathfrak{T}}_m^3(r_j)-\widetilde{\mathfrak{T}}_m^3(r_j)\right)^2\\+\left(\widehat{\mathfrak{T}}_n^3(r_j)-\widetilde{\mathfrak{T}}_n^3(r_j)\right)^2\\+\left(\widehat{\mathfrak{T}}_h^3(r_j)-\widetilde{\mathfrak{T}}_h^3(r_j)\right)^2\end{pmatrix}}. \tag{19}$$

Another approach of the FFDM was developed in [49] and presented as follows:

$$\mathcal{D}_{K}\left(\widehat{\mathfrak{T}},\widetilde{\mathfrak{T}}\right)=\frac{1-D_1+D_2}{2}, \tag{20}$$

where

$$D_1 = \frac{1}{k}\sum_{j=1}^{k}\left(\frac{\left(\widehat{\mathfrak{C}}_m^3(r_j)\widetilde{\mathfrak{C}}_m^3(r_j)\right)^3+\left(\widehat{\mathfrak{C}}_n^3(r_j)\widetilde{\mathfrak{C}}_n^3(r_j)\right)^3+\left(\widehat{\mathfrak{C}}_h^3(r_j)\widetilde{\mathfrak{C}}_h^3(r_j)\right)^3}{\sqrt[3]{\widehat{\mathfrak{C}}_m^6(r_j)+\widehat{\mathfrak{C}}_n^6(r_j)+\widehat{\mathfrak{C}}_h^6(r_j)}\sqrt[3]{\widetilde{\mathfrak{C}}_m^6(r_j)+\widetilde{\mathfrak{C}}_n^6(r_j)+\widetilde{\mathfrak{C}}_h^6(r_j)}}\right),$$

$$D_2 = \sqrt{\frac{1}{2k}\sum_{j=1}^{k}\begin{pmatrix}\left|\widehat{\mathfrak{C}}_m^3(r_j)-\widetilde{\mathfrak{C}}_m^3(r_j)\right|^2\\+\left|\widehat{\mathfrak{C}}_n^3(r_j)-\widetilde{\mathfrak{C}}_n^3(r_j)\right|^2\\+\left|\widehat{\mathfrak{C}}_h^3(r_j)-\widetilde{\mathfrak{C}}_h^3(r_j)\right|^2\end{pmatrix}}.$$

The weighted form of the method is expressed as follows:

$$\mathcal{D}_{K_W}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right) = \frac{1-D_{W1}+D_{W2}}{2}, \tag{21}$$

where

$$D_{W1} = \sum_{j=1}^{k}W_j\left(\frac{\left(\widehat{\mathfrak{C}}_m^3(r_j)\widetilde{\mathfrak{C}}_m^3(r_j)\right)^3+\left(\widehat{\mathfrak{C}}_n^3(r_j)\widetilde{\mathfrak{C}}_n^3(r_j)\right)^3+\left(\widehat{\mathfrak{C}}_h^3(r_j)\widetilde{\mathfrak{C}}_h^3(r_j)\right)^3}{\sqrt[3]{\widehat{\mathfrak{C}}_m^6(r_j)+\widehat{\mathfrak{C}}_n^6(r_j)+\widehat{\mathfrak{C}}_h^6(r_j)}\sqrt[3]{\widetilde{\mathfrak{C}}_m^6(r_j)+\widetilde{\mathfrak{C}}_n^6(r_j)+\widetilde{\mathfrak{C}}_h^6(r_j)}}\right),$$

$$D_{W2} = \sqrt{\frac{1}{2}\sum_{j=1}^{k}W_j\begin{pmatrix}\left|\widehat{\mathfrak{C}}_m^3(r_j)-\widetilde{\mathfrak{C}}_m^3(r_j)\right|^2\\+\left|\widehat{\mathfrak{C}}_n^3(r_j)-\widetilde{\mathfrak{C}}_n^3(r_j)\right|^2\\+\left|\widehat{\mathfrak{C}}_h^3(r_j)-\widetilde{\mathfrak{C}}_h^3(r_j)\right|^2\end{pmatrix}}.$$

Furthermore, Liu [52] presented another approach of the FFDM as follows:

$$\mathcal{D}_L\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right) = \sqrt{\frac{3}{2k}\sum_{j=1}^{k}\begin{pmatrix}\frac{\left(\widehat{\mathfrak{C}}_m^3(r_j)-\widetilde{\mathfrak{C}}_m^3(r_j)\right)^2}{\widehat{\mathfrak{C}}_m^3(r_j)+\widetilde{\mathfrak{C}}_m^3(r_j)+2}\\+\frac{\left(\widehat{\mathfrak{C}}_n^3(r_j)-\widetilde{\mathfrak{C}}_n^3(r_j)\right)^2}{\widehat{\mathfrak{C}}_n^3(r_j)+\widetilde{\mathfrak{C}}_n^3(r_j)+2}\end{pmatrix}}, \tag{22}$$

The weighted form of the method is expressed as follows:

$$\mathcal{D}_{L_W}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right) = \sqrt{\frac{3}{2}\sum_{j=1}^{k}W_j\begin{pmatrix}\frac{\left(\widehat{\mathfrak{C}}_m^3(r_j)-\widetilde{\mathfrak{C}}_m^3(r_j)\right)^2}{\widehat{\mathfrak{C}}_m^3(r_j)+\widetilde{\mathfrak{C}}_m^3(r_j)+2}\\+\frac{\left(\widehat{\mathfrak{C}}_n^3(r_j)-\widetilde{\mathfrak{C}}_n^3(r_j)\right)^2}{\widehat{\mathfrak{C}}_n^3(r_j)+\widetilde{\mathfrak{C}}_n^3(r_j)+2}\end{pmatrix}}. \tag{23}$$

In this distance method, the value of the FFHM is not considered. This exclusion may affect the distance output.

Finally, another type of the FFDM was developed in [51] and presented as follows:

$$\mathcal{D}_{Ge}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right) = \frac{1}{2k}\sum_{j=1}^{k}\begin{bmatrix}\left|\widehat{\mathfrak{C}}_m^3(r_j)-\widetilde{\mathfrak{C}}_m^3(r_j)\right|\\+\left|\widehat{\mathfrak{C}}_n^3(r_j)-\widetilde{\mathfrak{C}}_n^3(r_j)\right|\\+\left|\widehat{\mathfrak{C}}_h^3(r_j)-\widetilde{\mathfrak{C}}_h^3(r_j)\right|\end{bmatrix}. \tag{24}$$

The weighted form of the method is expressed as follows:

$$\mathcal{D}_{Ge_W}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right) = \frac{1}{2k}\sum_{j=1}^{k}W_j\begin{bmatrix}\left|\widehat{\mathfrak{C}}_m^3(r_j)-\widetilde{\mathfrak{C}}_m^3(r_j)\right|\\+\left|\widehat{\mathfrak{C}}_n^3(r_j)-\widetilde{\mathfrak{C}}_n^3(r_j)\right|\\+\left|\widehat{\mathfrak{C}}_h^3(r_j)-\widetilde{\mathfrak{C}}_h^3(r_j)\right|\end{bmatrix}. \tag{25}$$

In this method, the number of parametric differences is not integrated, which may affect the distance output.

3. New FFDM

In this section, we present the new approach of the FFDM, discuss the properties of the distance method, provide an applicative example of the method in assessing security crises in the NCRN using the MCDM approach, and relate the results of the new FFDM to the current FFDMs. The new approach of the FFDM is the modification and extension of the work of Xie et al. [53] under IFSs. Given two IFSs $\mathfrak{L}$ and $\overline{\mathfrak{L}}$, it is defined as follows:

$$\mathfrak{L} = \left\{\left(r_j, \mathfrak{L}_m(r_j), \mathfrak{L}_n(r_j)\right) | r_j \in R\right\}\ \overline{\mathfrak{L}} = \left\{\left(r_j, \overline{\mathfrak{L}}_m(r_j), \overline{\mathfrak{L}}_n(r_j)\right) | r_j \in R\right\},$$

where $R = \{r_1, r_2, \ldots, r_k\}$. Then, Xie et al.'s approach of distance metric is given as follows:

$$\mathcal{D}_X(\mathfrak{L}, \overline{\mathfrak{L}}) = \frac{1}{k}\sum_{j=1}^{k}\begin{bmatrix} \alpha \frac{|\mathfrak{L}_m(r_j) - \overline{\mathfrak{L}}_m(r_j)|}{1 + \mathfrak{L}_m(r_j) + \overline{\mathfrak{L}}_m(r_j)} + \\ \beta \frac{|\mathfrak{L}_n(r_j) - \overline{\mathfrak{L}}_n(r_j)|}{1 + \mathfrak{L}_n(r_j) + \overline{\mathfrak{L}}_n(r_j)} + \\ \gamma \frac{|\mathfrak{L}_h(r_j) - \overline{\mathfrak{L}}_h(r_j)|}{1 + \mathfrak{L}_h(r_j) + \overline{\mathfrak{L}}_h(r_j)} \end{bmatrix}, \tag{26}$$

where $\alpha, \beta,$ and γ are the tendency coefficients such that $\alpha + \beta + \gamma = 2$ and $\alpha \geq 0$, $\beta \geq 0$, and $\gamma \geq 0$, respectively.

It is observed that the work of Xie et al. [53] was presented in an intuitionistic fuzzy environment, made use of assumed tendency coefficients, and do not include the weights of the elements of the IFSs to enhance reliability of the distance outputs.

Sundry types of FFDMs have been developed by different scholars and reviewed with varying applications. Although most of the approaches do not consider the FFHMs, only some of the distance measures satisfy the properties of a distance function, and none of the methods consider the FFTCs. Hence, we extend the work in [53] to the Fermatean fuzzy setting and modify it by incorporating reliable tendency coefficients and weighted fuzzy values to obtain better results.

Let $\widehat{\mathfrak{C}}$ and $\widetilde{\mathfrak{C}}$ be two FFSs in $R = \{r_1, r_2, \ldots, r_k\}$ defined as follows:

$$\widehat{\mathfrak{C}} = \left\{\left(r_j, \widehat{\mathfrak{C}}_m(r_j), \widehat{\mathfrak{C}}_n(r_j)\right) | r_j \in R\right\} \text{ and } \widetilde{\mathfrak{C}} = \left\{\left(r_j, \widetilde{\mathfrak{C}}_m(r_j), \widetilde{\mathfrak{C}}_n(r_j)\right) | r_j \in R\right\}.$$

Then, the new approach FFDM between $\widehat{\mathfrak{C}}$ and $\widetilde{\mathfrak{C}}$ is given as follows:

$$\mathcal{D}_*(\widehat{\mathfrak{C}}, \widetilde{\mathfrak{C}}) = \frac{1}{k}\sum_{j=1}^{k}\begin{pmatrix} \mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} \frac{|\widehat{\mathfrak{C}}_m^3(r_j) - \widetilde{\mathfrak{C}}_m^3(r_j)|^3}{1 + \widehat{\mathfrak{C}}_m^3(r_j) + \widetilde{\mathfrak{C}}_m^3(r_j)} + \\ \nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} \frac{|\widehat{\mathfrak{C}}_n^3(r_j) - \widetilde{\mathfrak{C}}_n^3(r_j)|^3}{1 + \widehat{\mathfrak{C}}_n^3(r_j) + \widetilde{\mathfrak{C}}_n^3(r_j)} + \\ \pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} \frac{|\widehat{\mathfrak{C}}_h^3(r_j) - \widetilde{\mathfrak{C}}_h^3(r_j)|^3}{1 + \widehat{\mathfrak{C}}_h^3(r_j) + \widetilde{\mathfrak{C}}_h^3(r_j)} \end{pmatrix}, \tag{27}$$

where

$$\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} = \frac{\sum_{j=1}^{k}\left(\widehat{\mathfrak{C}}_m^3(r_j) + \widetilde{\mathfrak{C}}_m^3(r_j)\right)}{k},$$

$$\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} = \frac{\sum_{j=1}^{k}\left(\widehat{\mathfrak{C}}_n^3(r_j) + \widetilde{\mathfrak{C}}_n^3(r_j)\right)}{k},$$

$$\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} = \frac{\sum_{j=1}^{k}\left(\widehat{\mathfrak{C}}_h^3(r_j) + \widetilde{\mathfrak{C}}_h^3(r_j)\right)}{k},$$

for $\mu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \geq 0, \nu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \geq 0$, and $\pi_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \geq 0$ such that $\mu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} + \nu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} + \pi_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} = 2$. The parameters $\mu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}}, \nu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}}$, and $\pi_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}}$ are the tendency coefficients for the FFNs, which embody the grade of support, opposition, and neutrality. The weighted form of (27) is given as follows:

$$\mathcal{D}_{*w}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) = \sum_{j=1}^{k} W_j \begin{pmatrix} \mu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \frac{\left|\hat{\mathfrak{C}}_m^3(r_j)-\tilde{\mathfrak{C}}_m^3(r_j)\right|^3}{1+\hat{\mathfrak{C}}_m^3(r_j)+\tilde{\mathfrak{C}}_m^3(r_j)} + \\ \nu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \frac{\left|\hat{\mathfrak{C}}_n^3(r_j)-\tilde{\mathfrak{C}}_n^3(r_j)\right|^3}{1+\hat{\mathfrak{C}}_n^3(r_j)+\tilde{\mathfrak{C}}_n^3(r_j)} + \\ \pi_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \frac{\left|\hat{\mathfrak{C}}_h^3(r_j)-\tilde{\mathfrak{C}}_h^3(r_j)\right|^3}{1+\hat{\mathfrak{C}}_h^3(r_j)+\tilde{\mathfrak{C}}_h^3(r_j)} \end{pmatrix}, \tag{28}$$

where W_j is the weighted value for $j = 1,2,\ldots,k$ as presented in (1). The modifications made to the work of Xie et al. [53 include the following: extension to a Fermatean fuzzy environment, utilization of real computed tendency coefficients, and the inclusion of the weights of the elements of the FFSs to achieve better performance.

If $k = 1$, then $W_1 = 1$, and (27) and (28) become the following:

$$\mathcal{D}_{**}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) = \begin{pmatrix} \mu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \frac{\left|\hat{\mathfrak{C}}_m^3(r)-\tilde{\mathfrak{C}}_m^3(r)\right|^3}{1+\hat{\mathfrak{C}}_m^3(r)+\tilde{\mathfrak{C}}_m^3(r)} \\ +\nu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \frac{\left|\hat{\mathfrak{C}}_n^3(r)-\tilde{\mathfrak{C}}_n^3(r)\right|^3}{1+\hat{\mathfrak{C}}_n^3(r)+\tilde{\mathfrak{C}}_n^3(r)} + \\ \pi_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} \frac{\left|\hat{\mathfrak{C}}_h^3(r)-\tilde{\mathfrak{C}}_h^3(r)\right|^3}{1+\hat{\mathfrak{C}}_h^3(r)+\tilde{\mathfrak{C}}_h^3(r)} \end{pmatrix}, \tag{29}$$

where $\mu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} = \left(\hat{\mathfrak{C}}_m^3(r) + \tilde{\mathfrak{C}}_m^3(r)\right), \nu_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} = \left(\hat{\mathfrak{C}}_n^3(r) + \tilde{\mathfrak{C}}_n^3(r)\right)$, and $\pi_{\hat{\mathfrak{C}},\tilde{\mathfrak{C}}} = \left(\hat{\mathfrak{C}}_h^3(r) + \tilde{\mathfrak{C}}_h^3(r)\right)$.

Next, we show that $\mathcal{D}_{**}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right)$, $\mathcal{D}_{*}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right)$, and $\mathcal{D}_{*w}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right)$ satisfy the axiomatic characteristics of a distance function.

Theorem 1. *Given three FFSs* $\hat{\mathfrak{C}},\tilde{\mathfrak{C}}$, *and* $\bar{\mathfrak{C}}$ *in* R, *the following hold:*

i. $0 \leq \mathcal{D}_{**}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) \leq 1$,
ii. $\mathcal{D}_{**}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) = 0 \iff \hat{\mathfrak{C}} = \tilde{\mathfrak{C}}$,
iii. $\mathcal{D}_{**}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) = \mathcal{D}_{**}(\tilde{\mathfrak{C}},\hat{\mathfrak{C}})$,
iv. $\mathcal{D}_{**}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) + \mathcal{D}_{**}(\tilde{\mathfrak{C}},\bar{\mathfrak{C}}) \geq \mathcal{D}_{**}(\hat{\mathfrak{C}},\bar{\mathfrak{C}})$.

Proof. Please see the Appendix A. □

Remark 1. *With the information in Theorem 1, we have the following:* $0 \leq \mathcal{D}_{*}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) \leq 1$, $\mathcal{D}_{*}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) = 0 \iff \hat{\mathfrak{C}} = \tilde{\mathfrak{C}}$, $\mathcal{D}_{*}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) = \mathcal{D}_{*}(\tilde{\mathfrak{C}},\hat{\mathfrak{C}})$, *and* $\mathcal{D}_{*}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) + \mathcal{D}_{*}(\tilde{\mathfrak{C}},\bar{\mathfrak{C}}) \geq \mathcal{D}_{*}(\hat{\mathfrak{C}},\bar{\mathfrak{C}})$.

Similarly, the following statements hold: $0 \leq \mathcal{D}_{*w}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) \leq 1$, $\mathcal{D}_{*w}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) = 0 \iff \hat{\mathfrak{C}} = \tilde{\mathfrak{C}}$, $\mathcal{D}_{*w}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) = \mathcal{D}_{*w}(\tilde{\mathfrak{C}},\hat{\mathfrak{C}})$, and $\mathcal{D}_{*w}\left(\hat{\mathfrak{C}},\tilde{\mathfrak{C}}\right) + \mathcal{D}_{*w}(\tilde{\mathfrak{C}},\bar{\mathfrak{C}}) \geq \mathcal{D}_{*w}(\hat{\mathfrak{C}},\bar{\mathfrak{C}})$.

Next, the process of security assessment will be discuss with the aid of the approaches of FFDM based on MCDM method.

4. Application in Assessing the Security Prone States in the NCRN

Nigeria is dealing with an unprecedented wave of diverse but overlapping security challenges, which include abductions, extremist insurgencies, and mayhem, among others. All these have affected practically every part of the nation. The North-Central Region of Nigeria (NCRN) comprises six out of the 36 states in Nigeria together with the Federal Capital Territory (FCT), which is also called Abuja. The NCRN is an insecurity-prone area with several indicators of insecurity. We apply the new FFDM to determine which NCRN state is more prone to insecurity using the approach of the technique for order of preference by similarity to ideal solution (TOPSIS). The TOPSIS is the commonly used MCDM method that ranks and selects the suitable alternative from a group of alternatives. In this work, we use the TOPSIS because of its advantages.

4.1. Case Study

According to the studies in [2–5], the security concerns, namely ethno-religious conflicts/terrorism, farmer-herder clashes, banditry, and kidnapping, struggles/mayhem due to mineral resources, weak/complicit security architecture, porous borders, and political agitations, are the main causes of security crises in the NCRN. The security concerns are presented as a set:

$$\sigma = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7\},$$

where σ_1 stands for ethno-religious conflicts/terrorism, σ_2 stands for farmer-herder clashes due to an open grazing practice (i.e., clashes owing to the use of land, water, and grazing routes), σ_3 stands for banditry and kidnapping due to poverty arising from pervasive material inequalities and corruption, σ_4 stands for struggles/mayhem due to mineral resources, σ_5 stands for weak/complicit security architecture, σ_6 stands for porous borders, and σ_7 stands for political agitations.

The NCRN consists of Benue State, Nassarawa State, Kogi State, Niger State, Plateau State, Kwara State, and the FCT. The study area is the NCRN, which we represent as FFSs signified as follows:

$$\mathfrak{C} = \{\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \mathfrak{C}_4, \mathfrak{C}_5, \mathfrak{C}_6, \mathfrak{C}_7\},$$

which are defined in terms of the insecurity indicators $\sigma = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7\}$, where $\mathfrak{C}_1$ represents Benue State, $\mathfrak{C}_2$ represents Nassarawa State, $\mathfrak{C}_3$ represents Kogi State, $\mathfrak{C}_4$ represents Niger State, $\mathfrak{C}_5$ represents Plateau State, $\mathfrak{C}_6$ represents Kwara State, and $\mathfrak{C}_7$ represents the FCT.

The data for the assessment were collected using a knowledge-based system, where opinions were drawn in terms of linguistic variables from three security experts familiar with the region. The three security experts consulted for the data collection have in-depth insight and knowledge concerning the security crises in the NCRN; thus, consulting more security experts will lead to repeated data with an average that will be similar to the case of the three security experts. That is, if the multiple inputs/opinions in linguistic variables are converted to Fermatean fuzzy numbers (FFNs) and aggregated by taking their mean values, the final data will be similar to the mean aggregate information from the three security experts.

The linguistic variables with respect to the security concerns are given as follows: extremely low; very, very low; very low; low; medium low; medium; medium high; high; very high; very, very high; and extremely high with associated FFNs. The linguistic variables with their associated FFNs are captured in Table 1.

Table 1. Linguistic variables for insecurity evaluation.

Linguistic Variables	FFNs
Extremely low (EL)	(0, 1)
Very, very low (VVL)	(0.1, 0.9)
Very low (VL)	(0.2, 0.8)
Low (L)	(0.3, 0.7)
Medium low (ML)	(0.4, 0.6)
Medium (M)	(0.5, 0.5)
Medium high (MH)	(0.6, 0.4)
High (H)	(0.7, 0.3)
Very high (VH)	(0.8, 0.2)
Very, very high (VVH)	(0.9, 0.1)

Extremely high (EH)	(1, 0)

Three security experts assessed the security situations within the NCRN and gave their expert opinions using linguistic variables as presented in Tables 2–4.

Table 2. Linguistic variables from security expert I.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}_1$	VVH	EH	MH	VL	MH	VVL	EL
$\mathfrak{C}_2$	VH	VVH	ML	MH	M	L	MH
$\mathfrak{C}_3$	VVH	VH	VH	MH	MH	L	H
$\mathfrak{C}_4$	EH	VVH	VH	VH	H	VVH	VL
$\mathfrak{C}_5$	VVH	VVH	MH	VH	VH	L	MH
$\mathfrak{C}_6$	VH	VH	H	M	M	VH	M
$\mathfrak{C}_7$	VH	M	VH	M	M	ML	VVL

Table 3. Linguistic variables from security expert II.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}_1$	VH	VVH	H	L	MH	VL	VL
$\mathfrak{C}_2$	VVH	VH	M	M	L	VL	M
$\mathfrak{C}_3$	VH	H	VH	H	MH	ML	H
$\mathfrak{C}_4$	VVH	VH	H	VH	MH	VH	VL
$\mathfrak{C}_5$	VH	VH	H	H	VH	L	M
$\mathfrak{C}_6$	VVH	VH	MH	M	MH	H	MH
$\mathfrak{C}_7$	VH	MH	H	M	L	ML	VL

Table 4. Linguistic variables from security expert III.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}_1$	VVH	VH	ML	ML	MH	L	L
$\mathfrak{C}_2$	VH	VVH	M	MH	ML	VL	MH
$\mathfrak{C}_3$	VVH	VH	H	VH	H	M	VH
$\mathfrak{C}_4$	VH	H	VH	H	MH	H	VL
$\mathfrak{C}_5$	H	MH	H	MH	H	ML	MH
$\mathfrak{C}_6$	VVH	VH	M	M	MH	MH	H
$\mathfrak{C}_7$	VH	H	MH	MH	ML	L	VL

The linguistic variables from Tables 2–4 were converted to FFNs using the information in Table 1 and the resulting FFNs are displayed in Tables 5–7, respectively.

Table 5. FFNs of linguistic variables provided by security expert I.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}_1$	(0.9, 0.1)	(1.0,0.0)	(0.6, 0.4)	(0.2, 0.8)	(0.6, 0.4)	(0.1, 0.9)	(0.0, 1.0)
$\mathfrak{C}_2$	(0.8, 0.2)	(0.9, 0.1)	(0.4, 0.6)	(0.6, 0.4)	(0.5, 0.5)	(0.3, 0.7)	(0.6, 0.4)
$\mathfrak{C}_3$	(0.9, 0.1)	(0.8, 0.2)	(0.8, 0.2)	(0.6, 0.4)	(0.6, 0.4)	(0.3, 0.7)	(0.7, 0.3)
$\mathfrak{C}_4$	(1.0, 0.0)	(0.9, 0.1)	(0.8, 0.2)	(0.8, 0.2)	(0.7, 0.3)	(0.9, 0.1)	(0.2, 0.8)
$\mathfrak{C}_5$	(0.9, 0.1)	(0.9, 0.1)	(0.6, 0.4)	(0.8, 0.2)	(0.8, 0.2)	(0.3, 0.7)	(0.6, 0.4)
$\mathfrak{C}_6$	(0.8, 0.2)	(0.8, 0.2)	(0.7, 0.3)	(0.5, 0.5)	(0.5, 0.5)	(0.8, 0.2)	(0.5, 0.5)
$\mathfrak{C}_7$	(0.8, 0.2)	(0.5, 0.5)	(0.8, 0.2)	(0.5, 0.5)	(0.5, 0.5)	(0.4, 0.6)	(0.1, 0.9)

Table 6. FFNs of linguistic variables provided by security expert II.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}_1$	(0.8, 0.2)	(0.9, 0.1)	(0.7, 0.3)	(0.3, 0.7)	(0.6, 0.4)	(0.2, 0.8)	(0.2, 0.8)
$\mathfrak{C}_2$	(0.9, 0.1)	(0.8, 0.2)	(0.5, 0.5)	(0.5, 0.5)	(0.3, 0.7)	(0.2, 0.8)	(0.5, 0.5)
$\mathfrak{C}_3$	(0.8, 0.2)	(0.7, 0.3)	(0.8, 0.2)	(0.7, 0.3)	(0.6, 0.4)	(0.4, 0.6)	(0.7, 0.3)
$\mathfrak{C}_4$	(0.9, 0.1)	(0.8, 0.2)	(0.7, 0.3)	(0.8, 0.2)	(0.6, 0.4)	(0.8, 0.2)	(0.2, 0.8)
$\mathfrak{C}_5$	(0.8, 0.2)	(0.8, 0.2)	(0.7, 0.3)	(0.7, 0.3)	(0.8, 0.2)	(0.3, 0.7)	(0.5, 0.5)
$\mathfrak{C}_6$	(0.9, 0.1)	(0.8, 0.2)	(0.6, 0.4)	(0.5, 0.5)	(0.6, 0.4)	(0.7, 0.3)	(0.6, 0.4)
$\mathfrak{C}_7$	(0.8, 0.2)	(0.6, 0.4)	(0.7, 0.3)	(0.5, 0.5)	(0.3, 0.7)	(0.4, 0.6)	(0.2, 0.8)

Table 7. FFNs of linguistic variables provided by security expert III.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}_1$	(0.9, 0.1)	(0.8, 0.2)	(0.4, 0.6)	(0.4, 0.6)	(0.6, 0.4)	(0.3, 0.7)	(0.3, 0.7)
$\mathfrak{C}_2$	(0.8, 0.2)	(0.9, 0.1)	(0.5, 0.5)	(0.6, 0.4)	(0.4, 0.6)	(0.2, 0.8)	(0.6, 0.4)
$\mathfrak{C}_3$	(0.9, 0.1)	(0.8, 0.2)	(0.7, 0.3)	(0.8, 0.2)	(0.7, 0.3)	(0.5, 0.5)	(0.8, 0.2)
$\mathfrak{C}_4$	(0.8, 0.2)	(0.7, 0.3)	(0.8, 0.2)	(0.7, 0.3)	(0.6, 0.4)	(0.7, 0.3)	(0.2, 0.8)
$\mathfrak{C}_5$	(0.7, 0.3)	(0.6, 0.4)	(0.7, 0.3)	(0.6, 0.4)	(0.7, 0.3)	(0.4, 0.6)	(0.6, 0.4)
$\mathfrak{C}_6$	(0.9, 0.1)	(0.8, 0.2)	(0.5, 0.5)	(0.5, 0.5)	(0.6, 0.4)	(0.6, 0.4)	(0.7, 0.3)
$\mathfrak{C}_7$	(0.8, 0.2)	(0.7, 0.3)	(0.6, 0.4)	(0.6, 0.4)	(0.4, 0.6)	(0.3, 0.7)	(0.2, 0.8)

The average equivalence of the FFNs in Tables 5–7 are displayed in Table 8.

Table 8. Average FFNs from the three security experts.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}_1$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}09000\\0.1000\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.3000\\0.7000\end{pmatrix}$	$\begin{pmatrix}0.6000\\0.4000\end{pmatrix}$	$\begin{pmatrix}0.2000\\0.8000\end{pmatrix}$	$\begin{pmatrix}0.1667\\0.8333\end{pmatrix}$
$\mathfrak{C}_2$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.4667\\0.5333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.4000\\0.6000\end{pmatrix}$	$\begin{pmatrix}0.2333\\0.7667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$
$\mathfrak{C}_3$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$	$\begin{pmatrix}0.4000\\0.6000\end{pmatrix}$	$\begin{pmatrix}0.7333\\0.2667\end{pmatrix}$
$\mathfrak{C}_4$	$\begin{pmatrix}0.9000\\0.1000\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.2000\\0.8000\end{pmatrix}$
$\mathfrak{C}_5$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.3333\\0.6667\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$
$\mathfrak{C}_6$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.6000\\0.4000\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.6000\\0.4000\end{pmatrix}$
$\mathfrak{C}_7$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.6000\\0.4000\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.5333\\0.4667\end{pmatrix}$	$\begin{pmatrix}0.4000\\0.6000\end{pmatrix}$	$\begin{pmatrix}0.3667\\0.6333\end{pmatrix}$	$\begin{pmatrix}0.1667\\0.8333\end{pmatrix}$

Now, we determine which of the states, $\mathfrak{C} = \{\mathfrak{C}_1, \mathfrak{C}_2, \mathfrak{C}_3, \mathfrak{C}_4, \mathfrak{C}_5, \mathfrak{C}_6, \mathfrak{C}_7\}$, in the NCRN is the most insecure state using the data in Table 8 based on the TOPSIS technique.

4.2. *TOPSIS Algorithm*

Step I. Formulate the Fermatean fuzzy decision matrix (FFDeM) indicated by $\overline{\overline{\mathfrak{C}}} = \{\mathfrak{C}_i(\sigma_j)\}_{(n\times k)}$ for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, k$.

Step II. Identify the cost criterion (cc) and the benefit criteria (bc) of the security concerns. The cc is the least criterion, and bc represents the other criteria.

Step III. Compute the weights of the security concerns using the weight formula in (1).

Step IV. Find the normalized FFDeM indicated by $\tilde{\overline{\mathfrak{C}}} = \left(\mathfrak{C}_{i_m}(\sigma_j), \mathfrak{C}_i(\sigma_j)\right)_{(n\times k)}$, where σ_j represents the security indicators/concerns, and $\left(\mathfrak{C}_{i_m}(\sigma_j), \mathfrak{C}_{i_n}(\sigma_j)\right)$ for $i = 1, 2, \ldots, n$ are the FFNs. Here, $\tilde{\overline{\mathfrak{C}}}$ is described as follows:

$$\left(\mathfrak{C}_{i_m}(\sigma_j),\mathfrak{C}_{i_n}(\sigma_j)\right)=\begin{cases}\left(\mathfrak{C}_{i_m}(\sigma_j),\mathfrak{C}_{i_n}(\sigma_j)\right) & \text{for bc of } \mathfrak{C}\\ \left(\mathfrak{C}_{i_n}(\sigma_j),\mathfrak{C}_{i_m}(\sigma_j)\right) & \text{for cc of } \mathfrak{C}\end{cases} \tag{30}$$

Step V. Compute the positive ideal solution (PIS), $\mathfrak{C}^+ = \{\mathfrak{C}_1^+, \ldots, \mathfrak{C}_i^+\}$, and negative ideal solution (NIS), $\mathfrak{C}^- = \{\mathfrak{C}_1^-, \ldots, \mathfrak{C}_i^-\}$, which are defined as follows:

$$\left.\begin{array}{l}\mathfrak{C}^+ = \begin{cases}\left(\max\{\mathfrak{C}_{i_m}(\sigma_j)\}, \min\{\mathfrak{C}_{i_n}(\sigma_j)\}\right) \text{ if } \mathfrak{C} \text{ is a bc}\\ \left(\min\{\mathfrak{C}_{i_n}(\sigma_j)\}, \max\{\mathfrak{C}_{i_m}(\sigma_j)\}\right) \text{ if } \mathfrak{C} \text{ is a cc}\end{cases}\\ \mathfrak{C}^- = \begin{cases}\left(\min\{\mathfrak{C}_{i_n}(\sigma_j)\}, \max\{\mathfrak{C}_{i_m}(\sigma_j)\}\right) \text{ if } \mathfrak{C} \text{ is a bc}\\ \left(\max\{\mathfrak{C}_{i_m}(\sigma_j)\}, \min\{\mathfrak{C}_{i_n}(\sigma_j)\}\right) \text{ if } \mathfrak{C} \text{ is a cc}\end{cases}\end{array}\right\} \tag{31}$$

Step VI. Calculate the distance indexes using $\mathcal{D}(\mathfrak{C}^+, \mathfrak{C}_i)$ and $\mathcal{D}(\mathfrak{C}^-, \mathfrak{C}_i)$ based on the new FFDM for $i = 1,2,\ldots,7$.

Step VII. Calculate the closeness coefficient for each NCRN state $\mathfrak{C}_i$ using the following:

$$\mathcal{C}(\mathfrak{C}_i) = \frac{\mathcal{D}(\mathfrak{C}^+,\mathfrak{C}_i)}{\mathcal{D}(\mathfrak{C}^+,\mathfrak{C}_i)+\mathcal{D}(\mathfrak{C}^-,\mathfrak{C}_i)}. \tag{32}$$

Step VIII. Find the ranking/ordering of $\mathcal{C}(\boldsymbol{\mathfrak{C}}_i)$ in ascending order to determine the most insecure state.

4.3. Implementation of the TOPSIS Algorithm

Now, we implement the TOPSIS algorithm. The FFDeM for the NCRN is presented in Table 8. The least security concern in the NCRN is political agitations (σ_7). Here, σ_7 is cc, and the other security concerns are the bc. The weight of the criteria σ_j are computed using the information in Table 8 via (1). The computed weight is given as follows:

$$W = \left\{\begin{array}{c}0.1966, 0.1773, 0.1399, 0.1294,\\ 0.1289, 0.1143, 0.1134\end{array}\right\}.$$

Applying (35) on Table 8 to find the normalized FFDeM, we obtain Table 9.

Table 9. Normalized FFDeM.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}_1$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}09000\\0.1000\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.3000\\0.7000\end{pmatrix}$	$\begin{pmatrix}0.6000\\0.4000\end{pmatrix}$	$\begin{pmatrix}0.2000\\0.8000\end{pmatrix}$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$
$\mathfrak{C}_2$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.4667\\0.5333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.4000\\0.6000\end{pmatrix}$	$\begin{pmatrix}0.2333\\0.7667\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$
$\mathfrak{C}_3$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$	$\begin{pmatrix}0.4000\\0.6000\end{pmatrix}$	$\begin{pmatrix}0.2667\\0.7333\end{pmatrix}$
$\mathfrak{C}_4$	$\begin{pmatrix}0.9000\\0.1000\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.6333\\0.3667\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$
$\mathfrak{C}_5$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.6667\\0.3333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.7667\\0.2333\end{pmatrix}$	$\begin{pmatrix}0.3333\\0.6667\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$
$\mathfrak{C}_6$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.6000\\0.4000\end{pmatrix}$	$\begin{pmatrix}0.5000\\0.5000\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.4000\\0.6000\end{pmatrix}$
$\mathfrak{C}_7$	$\begin{pmatrix}0.8000\\0.2000\end{pmatrix}$	$\begin{pmatrix}0.6000\\0.4000\end{pmatrix}$	$\begin{pmatrix}0.7000\\0.3000\end{pmatrix}$	$\begin{pmatrix}0.5333\\0.4667\end{pmatrix}$	$\begin{pmatrix}0.4000\\0.6000\end{pmatrix}$	$\begin{pmatrix}0.3667\\0.6333\end{pmatrix}$	$\begin{pmatrix}08333\\0.1667\end{pmatrix}$

Applying (36) to the normalized FFDeM in Table 9, we obtain the NIS and PIS in Table 10.

Table 10. NIS and PIS for $\mathfrak{C}_i$.

NCRN	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7
$\mathfrak{C}^+$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}09000\\0.1000\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.3000\\0.7000\end{pmatrix}$	$\begin{pmatrix}0.6000\\0.4000\end{pmatrix}$	$\begin{pmatrix}0.2000\\0.8000\end{pmatrix}$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$
$\mathfrak{C}^-$	$\begin{pmatrix}0.8333\\0.1667\end{pmatrix}$	$\begin{pmatrix}0.8667\\0.1333\end{pmatrix}$	$\begin{pmatrix}0.4667\\0.5333\end{pmatrix}$	$\begin{pmatrix}0.5667\\0.4333\end{pmatrix}$	$\begin{pmatrix}0.4000\\0.6000\end{pmatrix}$	$\begin{pmatrix}0.2333\\0.7667\end{pmatrix}$	$\begin{pmatrix}0.4333\\0.5667\end{pmatrix}$

Next, based on the distance indexes between $\mathfrak{C}_i$ and NIS/PIS using the data in Tables 8 and 10, we obtain the results in Table 11.

Table 11. Distance indexes between $\mathfrak{C}_i$ and NIS/PIS.

Iterations	$\mathcal{D}(\mathfrak{C}^+, \mathfrak{C}_i)$	$\mathcal{D}(\mathfrak{C}^-, \mathfrak{C}_i)$
1	0.0201	0.0286
2	0.0203	0.0144
3	0.0154	0.0098
4	0.0025	0.0342
5	0.0140	0.0127
6	0.0094	0.0114
7	0.0324	0.0129

By applying (37) and using the distance indexes in Table 11, we obtain the closeness coefficients and ranking in Table 12, which are represented in Figure 1.

Table 12. Closeness coefficients and ranking.

States	$C(\mathfrak{C}_i)$	Ranking
$\mathfrak{C}_1$	0.4127	2nd
$\mathfrak{C}_2$	0.5850	5th
$\mathfrak{C}_3$	0.6111	6th
$\mathfrak{C}_4$	0.0681	1st
$\mathfrak{C}_5$	0.5243	4th
$\mathfrak{C}_6$	0.4519	3rd
$\mathfrak{C}_7$	0.7152	7th

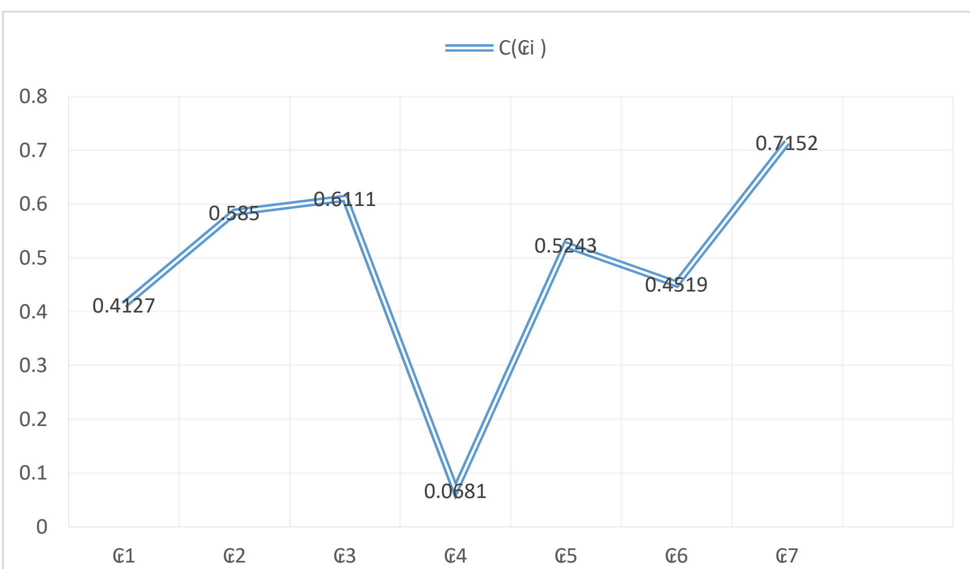

Figure 1. Representation of closeness coefficients.

The information from Table 12 and Figure 1 show that the most insecure state in the NCRN is Niger State symbolized by $\mathfrak{C}_4$, and the FCT represented by $\mathfrak{C}_7$ is the safest place within the region based on comparisons with other states in the region. In addition, the severity of the insecurity problems in the region exhibits the following order: Niger State, Benue State, Kwara State, Plateau State, Nassarawa State, Kogi State, and the FCT. A careful study of the closeness coefficients shows that insecurity in the NCRN is widespread. This expert information will be helpful to inform tourists and travelers.

4.4. Comparison Based on the TOPSIS Technique

Here, we compare the new FFDM with the existing approaches of FFDMs based on the TOPSIS approach in their weighted and non-weighted forms. The purpose of this comparative analysis is to show the advantage of the new FFDM over the existing FFDMs in terms of accuracy of results, fulfilment of the distance conditions, and reliability due to the inclusion of every parameter of the FFSs.

The distance results of $\mathcal{D}(\mathfrak{C}^+, \mathfrak{C}_i)$ and $\mathcal{D}(\mathfrak{C}^-, \mathfrak{C}_i)$ based on the FFDMs are displayed in Tables 13–16 and presented in Figures 2 and 3, respectively.

Table 13. Distances between PIS and the states using non-weighted FFDMs.

FFDMs	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	$\mathfrak{C}_7$
$\mathcal{D}_{SY}$ [45]	0.7378	0.7676	0.6443	0.3019	0.6294	0.5886	0.8161
$\mathcal{D}_{DW1}$ [46]	0.3769	0.3999	0.3163	0.0999	0.2934	0.2742	0.3533
$\mathcal{D}_{L}$ [52]	0.2873	0.2939	0.2358	0.0858	0.2212	0.2002	0.2731
$\mathcal{D}_{DW2}$ [46]	0.2789	0.2901	0.2435	0.1141	0.2379	0.2225	0.3084
$\mathcal{D}_{DW3}$ [46]	0.3209	0.3279	0.2574	0.0719	0.2355	0.2081	0.2792
$\mathcal{D}_{G1}$ [47]	0.1324	0.1209	0.1672	0.0739	0.1621	0.1716	0.1705
$\mathcal{D}_{G2}$ [47]	0.1429	0.1429	0.1429	0.0837	0.1429	0.1429	0.1429
$\mathcal{D}_{G3}$ [47]	0.1390	0.1360	0.1553	0.0783	0.1529	0.1581	0.1534
$\mathcal{D}_{G4}$ [47]	0.1571	0.1608	0.7623	0.0689	0.3809	0.3802	0.0979
$\mathcal{D}_{Ge}$ [51]	0.2419	0.2921	0.2120	0.0759	0.2208	0.2319	0.2946
$\mathcal{D}_{OE}$ [48]	0.2277	0.2369	0.1988	0.0932	0.1942	0.1816	0.2518
$\mathcal{D}_{K}$ [49]	0.3677	0.3620	0.3856	0.4510	0.3884	0.3963	0.3526
$\mathcal{D}_{*}$	0.0201	0.0203	0.0154	0.0025	0.0140	0.0094	0.0324

Table 14. Distances between NIS and the states using non-weighted FFDMs.

FFDMs	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	$\mathfrak{C}_7$
$\mathcal{D}_{SY}$ [45]	0.7821	0.5989	0.6282	0.9700	0.6785	0.6737	0.6888
$\mathcal{D}_{DW1}$ [46]	0.3483	0.2445	0.3207	0.4983	0.3427	0.3290	0.3467
$\mathcal{D}_{L}$ [52]	0.2845	0.1906	0.2277	0.3821	0.2462	0.2431	0.2707
$\mathcal{D}_{DW2}$ [46]	0.2956	0.2264	0.2374	0.3666	0.2565	0.2546	0.2604
$\mathcal{D}_{DW3}$ [46]	0.3026	0.1855	0.2482	0.4284	0.2679	0.2623	0.3007
$\mathcal{D}_{G1}$ [47]	0.1517	0.1473	0.1436	−0.0145	0.1446	0.1398	0.1434
$\mathcal{D}_{G2}$ [47]	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
$\mathcal{D}_{G3}$ [47]	0.1468	0.1458	0.1432	0.1135	0.1436	0.1416	0.1431
$\mathcal{D}_{G4}$ [47]	0.1090	0.1516	0.1412	0.1897	0.1394	0.1486	0.1399
$\mathcal{D}_{Ge}$ [51]	0.2083	0.1745	0.2442	0.3643	0.2492	0.2500	0.1770
$\mathcal{D}_{OE}$ [48]	0.2414	0.1848	0.1939	0.2993	0.2094	0.2079	0.2126

$\mathcal{D}_K$ [49]	0.3592	0.3944	0.3887	0.3228	0.3790	0.3800	0.3771
$\mathcal{D}_*$	0.0286	0.0144	0.0098	0.0324	0.0127	0.0114	0.0129

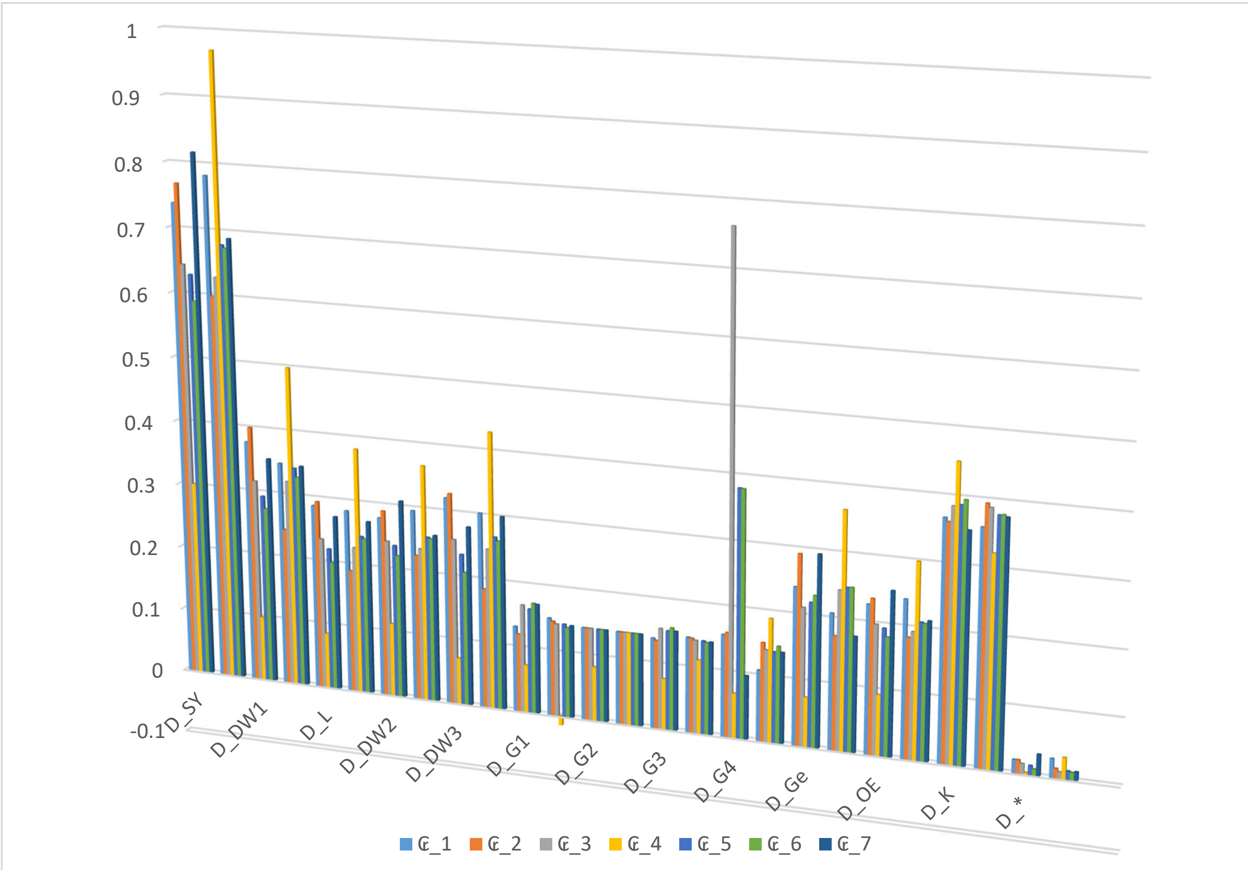

Figure 2. Representation of the distances between states and PIS/NIS using non-weighted FFDM.

Table 15. Distances between PIS and the states using weighted FFDMs.

FFDMs	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	$\mathfrak{C}_7$
$\mathcal{D}_{SY_w}$	0.2590	0.2727	0.2305	0.1173	0.2331	0.2148	0.3124
$\mathcal{D}_{DW1_w}$	0.3493	0.3712	0.2881	0.0995	0.2713	0.2575	0.3430
$\mathcal{D}_{L_w}$	0.2657	0.2731	0.2175	0.0871	0.2089	0.1901	0.2698
$\mathcal{D}_{DW2_w}$	0.2590	0.2727	0.2305	0.1173	0.2331	0.2148	0.3124
$\mathcal{D}_{DW3_w}$	0.2962	0.3030	0.2335	0.0717	0.2164	0.1949	0.2699
$\mathcal{D}_{G1_w}$	0.3078	0.3582	0.2581	0.0831	0.2620	0.2621	0.3635
$\mathcal{D}_{G2_w}$	0.3350	0.3940	0.2730	0.0844	0.2775	0.2768	0.3930
$\mathcal{D}_{G3_w}$	0.3261	0.3804	0.2690	0.0843	0.2732	0.2728	0.3817
$\mathcal{D}_{G4_w}$	0.2885	0.3344	0.2469	0.0820	0.2504	0.2511	0.3441
$\mathcal{D}_{Ge_w}$	0.2165	0.2694	0.1977	0.0772	0.2173	0.2202	0.2962
$\mathcal{D}_{OE_w}$	0.2115	0.2226	0.1882	0.0958	0.1903	0.1754	0.2551
$\mathcal{D}_{K_w}$	0.7284	0.7162	0.7490	0.8359	0.7458	0.7662	0.6771
$\mathcal{D}_{*w}$	0.0201	0.0203	0.0154	0.0025	0.0140	0.0094	0.0324

Table 16. Distances between NIS and the states using weighted FFDMs.

FFDMs	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	$\mathfrak{C}_7$
$\mathcal{D}_{SY_w}$	0.2940	0.2291	0.2320	0.3475	0.2434	0.2340	0.2360
$\mathcal{D}_{DW1_w}$	0.3314	0.2422	0.3111	0.4650	0.3274	0.3274	0.3164
$\mathcal{D}_{L_w}$	0.2729	0.1909	0.2213	0.3563	0.2344	0.2288	0.2453
$\mathcal{D}_{DW2_w}$	0.2940	0.2291	0.2320	0.3475	0.2434	0.2430	0.2360
$\mathcal{D}_{DW3_w}$	0.2837	0.1837	0.2404	0.3966	0.2556	0.2447	0.2727
$\mathcal{D}_{G1_w}$	0.2981	0.2074	0.3111	0.4831	0.3074	0.3075	0.2458
$\mathcal{D}_{G2_w}$	0.3210	0.2170	0.3379	0.5557	0.3335	0.3342	0.2630
$\mathcal{D}_{G3_w}$	0.3138	0.2149	0.3291	0.5181	0.3250	0.3255	0.2585
$\mathcal{D}_{G4_w}$	0.2817	0.1998	0.2921	0.4426	0.2888	0.2886	0.2327
$\mathcal{D}_{Ge_w}$	0.2142	0.1761	0.2379	0.3456	0.2292	0.2379	0.1535
$\mathcal{D}_{OE_w}$	0.2400	0.1871	0.1895	0.2838	0.1988	0.1984	0.1927
$\mathcal{D}_{K_w}$	0.6962	0.7542	0.7475	0.6385	0.7366	0.7422	0.7468
$\mathcal{D}_{*w}$	0.0286	0.0144	0.0098	0.0324	0.0127	0.0114	0.0129

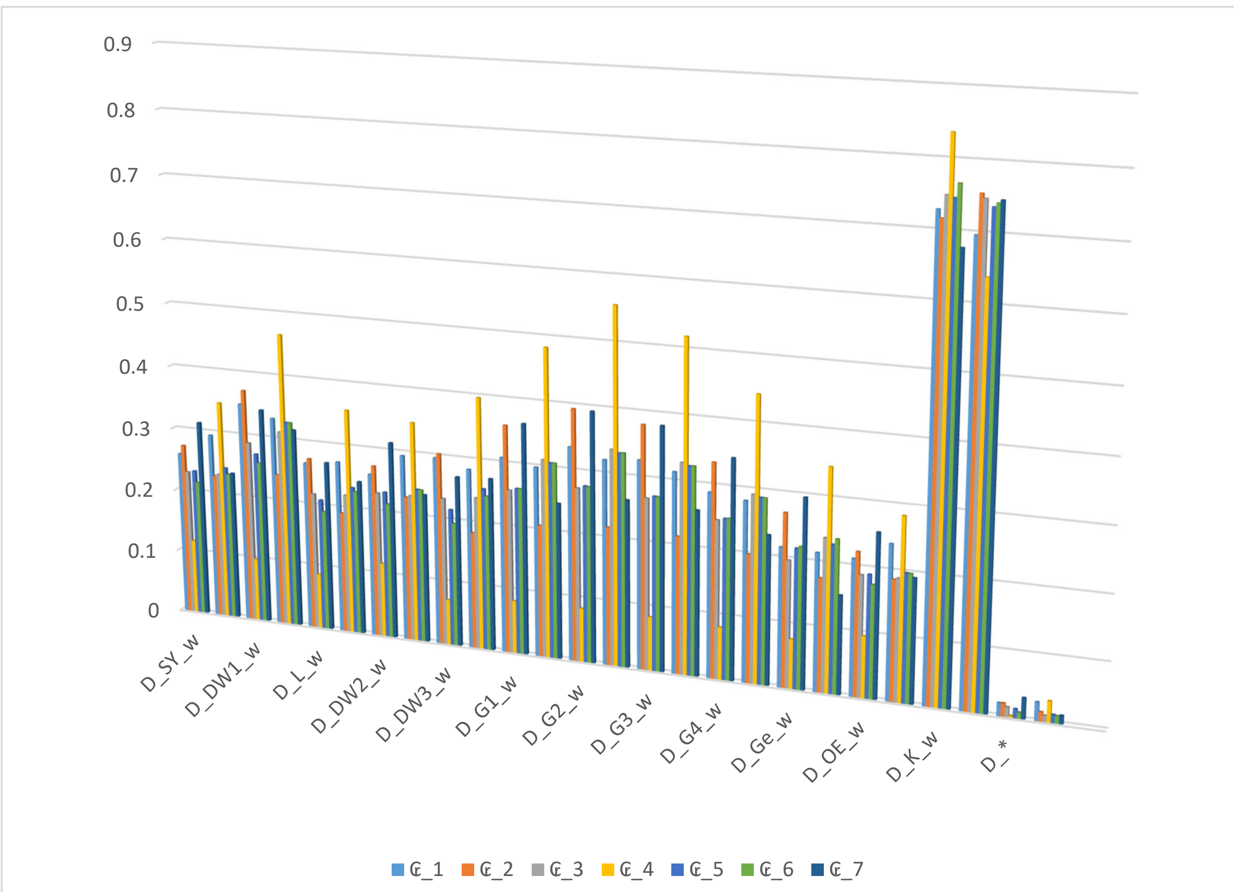

Figure 3. Representation of the distances between states and PIS/NIS using weighted FFDM.

The outcomes in Tables 13–16 and presented in Figures 2 and 3 demonstrate that the new approach gives the most exact results. It is expedient to say that the new FFDM yields the most accurate results because its distance values are the least in comparison with the existing approaches of FFDMs. However, the approach $\mathcal{D}_{G1}$ [47] in Table 14 yields an outcome (i.e.,-0.0145) that is not defined within the range of distance function values.

The closeness coefficients using the varied FFDM approaches are presented in Tables 17 and 18 and pictorially in Figures 4 and 5.

Table 17. Closeness coefficients of non-weighted FFDMs.

FFDMs	$C(\mathfrak{C}_i)$						
	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	$\mathfrak{C}_7$
$\mathcal{D}_{SY}$ [45]	0.4854	0.5617	0.5063	0.2374	0.4812	0.4663	0.5423
$\mathcal{D}_{DW1}$ [46]	0.5197	0.6206	0.4965	0.1670	0.4612	0.4546	0.5047
$\mathcal{D}_{L}$ [52]	0.5024	0.6066	0.5087	0.1834	0.4733	0.4516	0.5022
$\mathcal{D}_{DW2}$ [46]	0.4855	0.5617	0.5063	0.2374	0.4812	0.4664	0.5422
$\mathcal{D}_{DW3}$ [46]	0.5147	0.6387	0.5091	0.1437	0.4678	0.4424	0.4815
$\mathcal{D}_{G1}$ [47]	0.4660	0.4508	0.5380	1.2441	0.5285	0.5510	0.5432
$\mathcal{D}_{G2}$ [47]	0.5000	0.5000	0.5000	0.3694	0.5000	0.5000	0.5000
$\mathcal{D}_{G3}$ [47]	0.4863	0.4826	0.5203	0.4082	0.5157	0.5249	0.5174
$\mathcal{D}_{G4}$ [47]	0.5904	0.5147	0.8727	0.2664	0.7321	0.7190	0.4117
$\mathcal{D}_{Ge}$ [51]	0.5373	0.6260	0.4647	0.1724	0.4698	0.4812	0.6247
$\mathcal{D}_{OE}$ [48]	0.4854	0.5618	0.5062	0.2375	0.4812	0.4662	0.5422
$\mathcal{D}_{K}$ [49]	0.5058	0.4786	0.4980	0.5828	0.5061	0.5105	0.4832
$\mathcal{D}_{*}$	0.4127	0.5850	0.6111	0.0681	0.5243	0.4519	0.7152

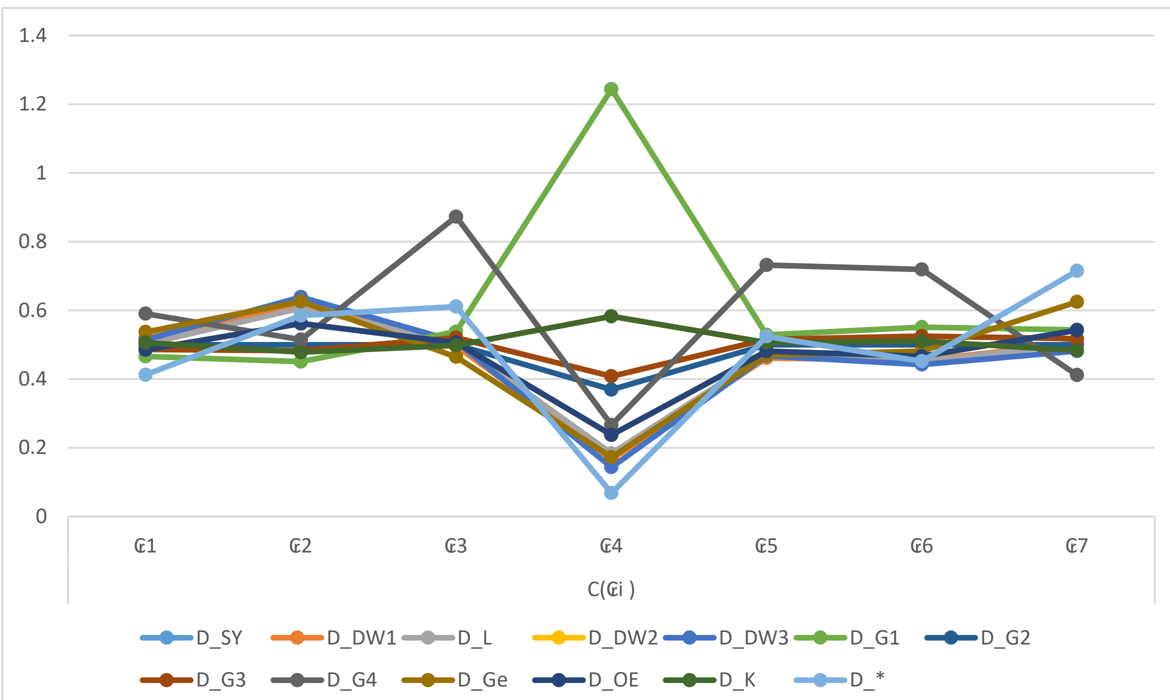

Figure 4. Representation of closeness coefficients using non-weighted FFDMs.

Table 18. Closeness coefficients of weighted FFDMs.

FFDMs	$C(\mathfrak{C}_i)$						
	$\mathfrak{C}_1$	$\mathfrak{C}_2$	$\mathfrak{C}_3$	$\mathfrak{C}_4$	$\mathfrak{C}_5$	$\mathfrak{C}_6$	$\mathfrak{C}_7$
$\mathcal{D}_{SY_w}$	0.4684	0.5434	0.4984	0.2524	0.4892	0.4692	0.5697
$\mathcal{D}_{DW1_w}$	0.5131	0.6051	0.4808	0.1763	0.4531	0.4544	0.5202
$\mathcal{D}_{L_w}$	0.4933	0.5884	0.4955	0.1963	0.4714	0.4536	0.5239
$\mathcal{D}_{DW2_w}$	0.4684	0.5434	0.4984	0.2524	0.4892	0.4692	0.5697
$\mathcal{D}_{DW3_w}$	0.5108	0.6226	0.4927	0.1531	0.4585	0.4434	0.4974

$\mathcal{D}_{G1_w}$	0.5080	0.6333	0.4534	0.1468	0.4601	0.4601	0.5966
$\mathcal{D}_{G2_w}$	0.5107	0.6448	0.4469	0.1319	0.4542	0.4530	0.5991
$\mathcal{D}_{G3_w}$	0.5096	0.6390	0.4498	0.1400	0.4567	0.4560	0.5962
$\mathcal{D}_{G4_w}$	0.5060	0.6260	0.4581	0.1563	0.4644	0.4653	0.5966
$\mathcal{D}_{Ge_w}$	0.5024	0.6044	0.4534	0.1821	0.4867	0.4809	0.6589
$\mathcal{D}_{OE_w}$	0.4739	0.5436	0.4982	0.2523	0.4891	0.4692	0.5697
$\mathcal{D}_{K_w}$	0.5113	0.4871	0.5005	0.5669	0.5031	0.5080	0.4755
$\mathcal{D}_{*w}$	0.4127	0.5850	0.6111	0.0681	0.5243	0.4519	0.7152

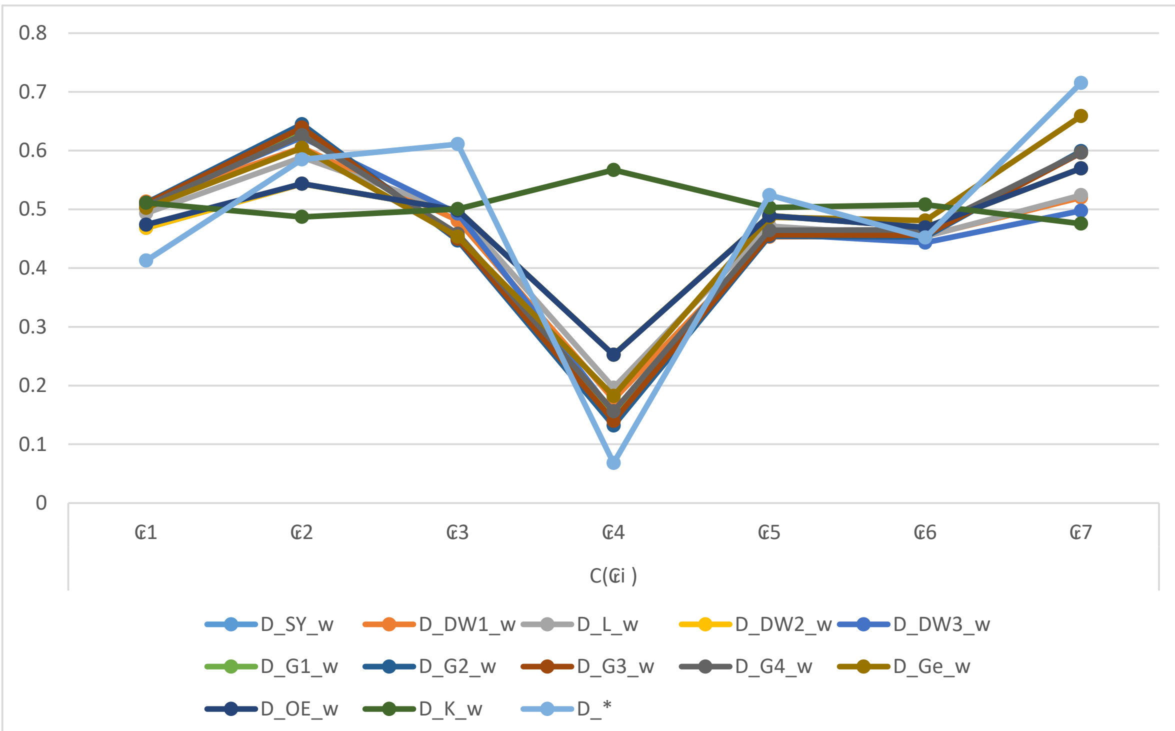

Figure 5. Representation of closeness coefficients using weighted FFDMs.

The ordering of the results in Tables 17 and 18 is presented in Tables 19 and 20, respectively.

Table 19. Comparison information for non-weighted FFDMSs.

FFDMs	Ordering	Decision
$\mathcal{D}_{SY}$ [45]	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_1 \prec \mathfrak{C}_3 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{DW1}$ [46]	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_3 \prec \mathfrak{C}_7 \prec \mathfrak{C}_1 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{L}$ [52]	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_7 \prec \mathfrak{C}_1 \prec \mathfrak{C}_3 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{DW2}$ [46]	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_1 \prec \mathfrak{C}_3 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{DW3}$ [46]	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_7 \prec \mathfrak{C}_3 \prec \mathfrak{C}_1 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{G1}$ [47]	$\mathfrak{C}_4 \prec \mathfrak{C}_2 \prec \mathfrak{C}_1 \prec \mathfrak{C}_5 \prec \mathfrak{C}_7 \prec \mathfrak{C}_3 \prec \mathfrak{C}_6$	$\mathfrak{C}_4$
$\mathcal{D}_{G2}$ [47]	$\mathfrak{C}_4 \prec \mathfrak{C}_2 = \mathfrak{C}_1 = \mathfrak{C}_5 = \mathfrak{C}_7 = \mathfrak{C}_3 = \mathfrak{C}_6$	$\mathfrak{C}_4$
$\mathcal{D}_{G3}$ [47]	$\mathfrak{C}_4 \prec \mathfrak{C}_2 \prec \mathfrak{C}_1 \prec \mathfrak{C}_5 \prec \mathfrak{C}_7 \prec \mathfrak{C}_3 \prec \mathfrak{C}_6$	$\mathfrak{C}_4$
$\mathcal{D}_{G4}$ [47]	$\mathfrak{C}_4 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2 \prec \mathfrak{C}_1 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_3$	$\mathfrak{C}_4$
$\mathcal{D}_{Ge}$ [51]	$\mathfrak{C}_4 \prec \mathfrak{C}_3 \prec \mathfrak{C}_5 \prec \mathfrak{C}_6 \prec \mathfrak{C}_1 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{OE}$ [48]	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_1 \prec \mathfrak{C}_3 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{K}$ [49]	$\mathfrak{C}_2 \prec \mathfrak{C}_7 \prec \mathfrak{C}_3 \prec \mathfrak{C}_1 \prec \mathfrak{C}_5 \prec \mathfrak{C}_6 \prec \mathfrak{C}_4$	$\mathfrak{C}_2$
$\mathcal{D}_{*}$	$\mathfrak{C}_4 \prec \mathfrak{C}_1 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_2 \prec \mathfrak{C}_3 \prec \mathfrak{C}_7$	$\mathfrak{C}_4$

Table 20. Comparison information for weighted FFDMs.

FFDMs	Ordering	Decision
$\mathcal{D}_{SY_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_1 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_3 \prec \mathfrak{C}_2 \prec \mathfrak{C}_7$	$\mathfrak{C}_4$
$\mathcal{D}_{ZW_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_5 \prec \mathfrak{C}_6 \prec \mathfrak{C}_3 \prec \mathfrak{C}_1 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{L_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_1 \prec \mathfrak{C}_3 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{DW1_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_1 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_3 \prec \mathfrak{C}_2 \prec \mathfrak{C}_7$	$\mathfrak{C}_4$
$\mathcal{D}_{DW2_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_3 \prec \mathfrak{C}_7 \prec \mathfrak{C}_1 \prec \mathfrak{C}_7$	$\mathfrak{C}_4$
$\mathcal{D}_{G1_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_3 \prec \mathfrak{C}_6 = \mathfrak{C}_5 \prec \mathfrak{C}_1 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{G2_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_3 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_1 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{G3_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_3 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_1 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{G4_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_3 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_1 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{Ge_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_3 \prec \mathfrak{C}_5 \prec \mathfrak{C}_6 \prec \mathfrak{C}_1 \prec \mathfrak{C}_7 \prec \mathfrak{C}_2$	$\mathfrak{C}_4$
$\mathcal{D}_{OE_w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_6 \prec \mathfrak{C}_1 \prec \mathfrak{C}_5 \prec \mathfrak{C}_3 \prec \mathfrak{C}_2 \prec \mathfrak{C}_7$	$\mathfrak{C}_4$
$\mathcal{D}_{K_w}$	$\mathfrak{C}_7 \prec \mathfrak{C}_2 \prec \mathfrak{C}_3 \prec \mathfrak{C}_5 \prec \mathfrak{C}_6 \prec \mathfrak{C}_1 \prec \mathfrak{C}_4$	$\mathfrak{C}_2$
$\mathcal{D}_{*w}$	$\mathfrak{C}_4 \prec \mathfrak{C}_1 \prec \mathfrak{C}_6 \prec \mathfrak{C}_5 \prec \mathfrak{C}_2 \prec \mathfrak{C}_3 \prec \mathfrak{C}_7$	$\mathfrak{C}_4$

From the information in Tables 19 and 20 and Figures 4 and 5, we see that almost all the FFDM approaches give the same interpretation, pointing to the fact that Niger State is the most insecure state in the NCRN. However, the method in [49] provides a contrary interpretation. This shows the drawback of the cosine-based FFDM. In addition, the methods $\mathcal{D}_{G2}$ and $\mathcal{D}_{G1_W}$ [49] fail to show clear classification and ordering, which further proves their limitations.

5. Conclusions

Security crisis is a topic of public concern because of its unpredictable nature. Thus, the process of assessing security crises is of huge interest to all and sundry. The security crises in the Sub-Saharan Region is widespread, especially in the Northern Region of Nigeria. Therefore, it is crucial to enhance the assessment process of insecurity to facilitate suitable and informed traveling decisions. This study has introduced a novel FFDM based on tendency coefficients and weight values, and the properties of the new method have been carefully demonstrated to validate the technique's effectiveness. The new FFDM is applied to assess the insecurity problem of the NCRN in order to determine the most insecure state and the relatively safest state within the region for the purpose of issuing travel advisories.

The information used for the research is collected based on the Fermatean fuzzy linguistic variables provided by security experts acquainted to the region. A comparative analysis was conducted, and it expressed the superiority of the new FFDM, where the new approach yielded better results that satisfied the distance conditions. From an application point of view, it is observed that the most insecure state within the region is Niger State. The insecurity in the region is ordered according to severity as follows: Niger State, Benue State, Kwara State, Plateau State, Nassarawa State, Kogi State, and the FCT. From the data analysis, we see that almost all the FFDMs give the same interpretation, pointing to the fact that Niger State is truly the most insecure state in the NCRN. Conversely, the method in [47] provides an opposing explanation, and the methods $\mathcal{D}_{G2}$ and $\mathcal{D}_{G1_W}$ in [49] could not show clear classification and ordering. The outcome of the findings will provide traveling advisories for safe journeys within the region. However, assessing other challenging problems would be beneficial in strengthening the resourcefulness of the proposed FFDM in future research. In addition, the utilization of the new FFDM can be investigated in diverse decision-making problems presented in [54–58]. Furthermore, the proposed approach of security crisis assessments can be combined with qualitative and quantitative risk analysis techniques to enhance a robust security assessment in future research. Nonetheless, the new FFDM cannot be deployed to model decision-making cases where the aggregate of MD³ and NMD³ surpasses one. This is the prominent limitation of the new approach.

Author Contributions: Conceptualization, P.A.E. and M.T.A.; methodology, P.A.E., M.T.A. and N.A.; software, P.A.E. and M.T.A.; validation, N.K., N.A., H.Y. and C.O.N.; investigation, N.K., N.A. and H.Y.; resources, C.O.N. and N.K.; data curation, M.T.A.; writing—original draft preparation, P.A.E., M.T.A. and C.O.N.; writing—review and editing, P.A.E. and C.O.N.; visualization, N.K. and H.Y.; supervision, N.A.; project administration, P.A.E.; funding acquisition, H.Y.. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: This work has no associated data. All the data are contained in the work.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Proof of Theorem 1.

Proof:

(i.) It is clear that (29) satisfies $\mathcal{D}_{**}(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}) \geq 0$. Hence, we only need to prove $\mathcal{D}_{**}(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}) \leq 1$. Since $\widehat{\mathfrak{C}}_m(r)$, $\widetilde{\mathfrak{C}}_m(r) \in [0,1]$, $\widehat{\mathfrak{C}}_n(r)$, $\widetilde{\mathfrak{C}}_n(r) \in [0,1]$, and $\widehat{\mathfrak{C}}_h(r)$, $\widetilde{\mathfrak{C}}_h(r) \in [0,1]$, then, the following is noted:

$$-1 \leq \widehat{\mathfrak{C}}_m^3(r) - \widetilde{\mathfrak{C}}_m^3(r) \leq 1,\ \ -1 \leq \widehat{\mathfrak{C}}_n^3(r) - \widetilde{\mathfrak{C}}_n^3(r) \leq 1,$$

$$-1 \leq \widehat{\mathfrak{C}}_h^3(r) - \widetilde{\mathfrak{C}}_h^3(r) \leq 1.$$

Hence,

$$0 \leq \left|\widehat{\mathfrak{C}}_m^3(r) - \widetilde{\mathfrak{C}}_m^3(r)\right|^3 \leq 1,\ 0 \leq \left|\widehat{\mathfrak{C}}_n^3(r) - \widetilde{\mathfrak{C}}_n^3(r)\right|^3 \leq 1,$$

$$0 \leq \left|\widehat{\mathfrak{C}}_h^3(r) - \widetilde{\mathfrak{C}}_h^3(r)\right|^3 \leq 1.$$

We can see the following:

$$0 \leq \left|\widehat{\mathfrak{C}}_m^3(r) - \widetilde{\mathfrak{C}}_m^3(r)\right|^3 \leq \widehat{\mathfrak{C}}_m^3(r) + \widetilde{\mathfrak{C}}_m^3(r),\ 0 \leq \left|\widehat{\mathfrak{C}}_n^3(r) - \widetilde{\mathfrak{C}}_n^3(r)\right|^3 \leq \widehat{\mathfrak{C}}_n^3(r) + \widetilde{\mathfrak{C}}_n^3(r),$$

$$0 \leq \left|\widehat{\mathfrak{C}}_h^3(r) - \widetilde{\mathfrak{C}}_h^3(r)\right|^3 \leq \widehat{\mathfrak{C}}_h^3(r) + \widetilde{\mathfrak{C}}_h^3(r).$$

In addition,

$$0 \leq 2\left|\widehat{\mathfrak{C}}_m^3(r) - \widetilde{\mathfrak{C}}_m^3(r)\right|^3 \leq 1 + \widehat{\mathfrak{C}}_m^3(r) + \widetilde{\mathfrak{C}}_m^3(r), 0 \leq 2\left|\widehat{\mathfrak{C}}_n^3(r) - \widetilde{\mathfrak{C}}_n^3(r)\right|^3 \leq 1 + \widehat{\mathfrak{C}}_n^3(r) + \widetilde{\mathfrak{C}}_n^3(r),$$

$$0 \leq 2\left|\widehat{\mathfrak{C}}_h^3(r) - \widetilde{\mathfrak{C}}_h^3(r)\right|^3 \leq 1 + \widehat{\mathfrak{C}}_h^3(r) + \widetilde{\mathfrak{C}}_h^3(r).$$

Thus, we obtain the following:

$$0 \leq \frac{\left|\widehat{\mathfrak{C}}_m^3(r) - \widetilde{\mathfrak{C}}_m^3(r)\right|^3}{1 + \widehat{\mathfrak{C}}_m^3(r) + \widetilde{\mathfrak{C}}_m^3(r)} \leq \frac{1}{2}, \qquad 0 \leq \frac{\left|\widehat{\mathfrak{C}}_n^3(r) - \widetilde{\mathfrak{C}}_n^3(r)\right|^3}{1 + \widehat{\mathfrak{C}}_n^3(r) + \widetilde{\mathfrak{C}}_n^3(r)} \leq \frac{1}{2},$$

$$0 \leq \frac{\left|\widehat{\mathfrak{C}}_h^3(r) - \widetilde{\mathfrak{C}}_h^3(r)\right|^3}{1 + \widehat{\mathfrak{C}}_h^3(r) + \widetilde{\mathfrak{C}}_h^3(r)} \leq \frac{1}{2}. \tag{A1}$$

Since $\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} \geq 0, \nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} \geq 0, \pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} \geq 0$, and $\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} + \nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} + \pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} = 2$, then the following is obtained:

$$\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_n^3(r)-\widetilde{\mathfrak{C}}_n^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_n^3(r)+\widetilde{\mathfrak{C}}_n^3(r)}+\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_h^3(r)-\widetilde{\mathfrak{C}}_h^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_h^3(r)+\widetilde{\mathfrak{C}}_h^3(r)}$$

$$0\le\frac{1}{2}\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}+\frac{1}{2}\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}+\frac{1}{2}\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}=\frac{1}{2}\left(\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}+\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}+\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\right)=1.$$

Specifically,

$$\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right)=\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}$$
$$+\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_n^3(r)-\widetilde{\mathfrak{C}}_n^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_n^3(r)+\widetilde{\mathfrak{C}}_n^3(r)}+$$
$$\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_h^3(r)-\widetilde{\mathfrak{C}}_h^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_h^3(r)+\widetilde{\mathfrak{C}}_h^3(r)}\le 1.$$

Therefore, $0\le\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right)\le 1$ as required.

(ii.) In particular, suppose $\widehat{\mathfrak{C}}=\widetilde{\mathfrak{C}}$ and $\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}>0$, $\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}>0$, and $\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}>0$. It is clear that the following formula holds for all $\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}$, $\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}$, and $\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}$:

$$\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right)=\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_n^3(r)-\widetilde{\mathfrak{C}}_n^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_n^3(r)+\widetilde{\mathfrak{C}}_n^3(r)}$$

$$+\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_h^3(r)-\widetilde{\mathfrak{C}}_h^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_h^3(r)+\widetilde{\mathfrak{C}}_h^3(r)}=0.$$

On the other hand, according to (29) and $\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\ge 0,\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\ge 0$, and $\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\ge 0$, we obtain the following:

$$\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}\ge 0,\ \ \nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_n^3(r)-\widetilde{\mathfrak{C}}_n^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_n^3(r)+\widetilde{\mathfrak{C}}_n^3(r)}\ge 0,$$

$$\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_h^3(r)-\widetilde{\mathfrak{C}}_h^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_h^3(r)+\widetilde{\mathfrak{C}}_h^3(r)}\ge 0.$$

If $\mathcal{D}_{**}(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}})=0$, we obtain the following according to (29):

$$\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}\ge 0,\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_n^3(r)-\widetilde{\mathfrak{C}}_n^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_n^3(r)+\widetilde{\mathfrak{C}}_n^3(r)}\ge 0,$$

$$\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_h^3(r)-\widetilde{\mathfrak{C}}_h^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_h^3(r)+\widetilde{\mathfrak{C}}_h^3(r)}\ge 0.$$

Therefore, if $\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}>0$, $\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}>0$, and $\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}>0$, then we get $\widehat{\mathfrak{C}}_m^3(r)=\widetilde{\mathfrak{C}}_m^3(r)$, $\widehat{\mathfrak{C}}_n^3(r)=\widetilde{\mathfrak{C}}_n^3(r)$, and $\widehat{\mathfrak{C}}_h^3(r)=\widetilde{\mathfrak{C}}_h^3(r)$. Thus, $\widehat{\mathfrak{C}}=\widetilde{\mathfrak{C}}$, as required.

(iii.) The following holds:

$$\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right)=\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_n^3(r)-\widetilde{\mathfrak{C}}_n^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_n^3(r)+\widetilde{\mathfrak{C}}_n^3(r)}+\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_h^3(r)-\widetilde{\mathfrak{C}}_h^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_h^3(r)+\widetilde{\mathfrak{C}}_h^3(r)},$$

$$=\mu_{\widetilde{\mathfrak{C}},\widehat{\mathfrak{C}}}\frac{\left|\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)\right|^3}{1+\widetilde{\mathfrak{C}}_m^3(r)+\widehat{\mathfrak{C}}_m^3(r)}+\nu_{\widetilde{\mathfrak{C}},\widehat{\mathfrak{C}}}\frac{\left|\widetilde{\mathfrak{C}}_n^3(r)-\widehat{\mathfrak{C}}_n^3(r)\right|^3}{1+\widetilde{\mathfrak{C}}_n^3(r)+\widehat{\mathfrak{C}}_n^3(r)}+\pi_{\widetilde{\mathfrak{C}},\widehat{\mathfrak{C}}}\frac{\left|\widetilde{\mathfrak{C}}_h^3(r)-\widehat{\mathfrak{C}}_h^3(r)\right|^3}{1+\widetilde{\mathfrak{C}}_h^3(r)+\widehat{\mathfrak{C}}_h^3(r)}$$

$$=\mathcal{D}_{**}\left(\widetilde{\mathfrak{C}},\widehat{\mathfrak{C}}\right).$$

Thus, $\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right)=\mathcal{D}_{**}\left(\widetilde{\mathfrak{C}},\widehat{\mathfrak{C}}\right)$, as required.

(vi.) Now, we show that $\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right)+\mathcal{D}_{**}\left(\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}\right)\geq\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\overline{\mathfrak{C}}\right)$ holds. The membership degree compartment of the triangle inequality is presented as follows:

$$\left.\begin{array}{c}\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\dfrac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\mu_{\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}}\dfrac{\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|^3}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}\\ \geq\mu_{\widehat{\mathfrak{C}},\overline{\mathfrak{C}}}\dfrac{\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}\end{array}\right\}. \quad (33)$$

We must show that (A2) is true. Certainly, (A2) holds if $\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}=\mu_{\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}}=\mu_{\widehat{\mathfrak{C}},\overline{\mathfrak{C}}}=0$. Based on the principle of Boolean ring, we have the following:

$$\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|^3=\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|,\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|^3=\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|,$$

$$\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|^3=\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|.$$

Then, (A2) becomes the following:

$$\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\mu_{\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}}\frac{\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}\geq\mu_{\widehat{\mathfrak{C}},\overline{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}. \quad (34)$$

If $\mu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}=\mu_{\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}}=\mu_{\widehat{\mathfrak{C}},\overline{\mathfrak{C}}}>0$, then (A3) can be simplified to the following:

$$\left.\begin{array}{c}\dfrac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\dfrac{\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}\\ \geq\dfrac{\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}\end{array}\right\}. \quad (35)$$

Next, we check whether (A4) holds in the following four cases:

(a) $\widehat{\mathfrak{C}}_m^3(r)\leq\widetilde{\mathfrak{C}}_m^3(r)\leq\overline{\mathfrak{C}}_m^3(r)$,
(b) $\overline{\mathfrak{C}}_m^3(r)\leq\widetilde{\mathfrak{C}}_m^3(r)\leq\widehat{\mathfrak{C}}_m^3(r)$,
(c) $\widetilde{\mathfrak{C}}_m^3(r)\geq max\left\{\widehat{\mathfrak{C}}_m^3(r),\overline{\mathfrak{C}}_m^3(r)\right\}$,
(d) $\widetilde{\mathfrak{C}}_m^3(r)\leq min\left\{\widehat{\mathfrak{C}}_m^3(r),\overline{\mathfrak{C}}_m^3(r)\right\}$.

From (A4), we get the following:

$$\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\frac{\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}-\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}\geq 0.$$

For case (a), it follows that $\widetilde{\mathfrak{C}}_m^3(r)\overline{\mathfrak{C}}_m^3(r)\geq\widetilde{\mathfrak{C}}_m^3(r)\widehat{\mathfrak{C}}_m^3(r)$ and $\widehat{\mathfrak{C}}_m^3(r)\overline{\mathfrak{C}}_m^3(r)\geq\widehat{\mathfrak{C}}_m^3(r)\widetilde{\mathfrak{C}}_m^3(r)$. Thus, the following is obtained:

$$\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\frac{\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}-\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}$$

$$=\frac{\widetilde{\mathfrak{C}}_m^3(r)\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)\overline{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\overline{\mathfrak{C}}_m^3(r)}{\left(1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)\right)\left(1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)\left(1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)}$$

$$+\frac{\widehat{\mathfrak{C}}_m^3(r)\overline{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)\overline{\mathfrak{C}}_m^3(r)}{\left(1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)\right)\left(1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)\left(1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)}$$

$$=\frac{\widetilde{\mathfrak{C}}_m^3(s)\widehat{\mathfrak{C}}_m^3(r)\left[\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)\right]+\widetilde{\mathfrak{C}}_m^3(r)\overline{\mathfrak{C}}_m^3(r)\left[\overline{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right]}{\left(1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)\right)\left(1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)\left(1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)}$$

$$+\frac{\widehat{\mathfrak{C}}_m^3(r)\overline{\mathfrak{C}}_m^3(r)\left[\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right]}{\left(1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)\right)\left(1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)\left(1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)}$$

$$\geq\frac{\widetilde{\mathfrak{C}}_m^3(r)\widehat{\mathfrak{C}}_m^3(r)\left[\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)+\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right]}{\left(1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)\right)\left(1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)\left(1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)\right)}=0.$$

Therefore, (A4) is satisfied for case (a). Since (a) is analogous to (b), (A4) holds for case (b) by the same logic as in (a).

For case (c), we have two situations as follows:

(1) If $\overline{\mathfrak{C}}_m^3(r) \le \widehat{\mathfrak{C}}_m^3(r) \le \widetilde{\mathfrak{C}}_m^3(r)$, then $1 + \widehat{\mathfrak{C}}_m^3(r) + \widetilde{\mathfrak{C}}_m^3(r) \ge 1 + \widetilde{\mathfrak{C}}_m^3(r) + \overline{\mathfrak{C}}_m^3(r)$ and $1 + \widehat{\mathfrak{C}}_m^3(r) + \widetilde{\mathfrak{C}}_m^3(r) \ge 1 + \widehat{\mathfrak{C}}_m^3(r) + \overline{\mathfrak{C}}_m^3(r)$. Thus,

$$\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\frac{\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}-\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}$$

$$=\frac{\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\frac{\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}-\frac{\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}$$

$$=\frac{\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\frac{\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}+\frac{\overline{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}$$

$$\ge\frac{\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}=\frac{2\left(\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)\right)}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}\ge 0.$$

(2) If $\widehat{\mathfrak{C}}_m^3(r) \le \overline{\mathfrak{C}}_m^3(r) \le \widetilde{\mathfrak{C}}_m^3(r)$, then $1 + \widetilde{\mathfrak{C}}_m^3(r) + \overline{\mathfrak{C}}_m^3(r) \ge 1 + \widehat{\mathfrak{C}}_m^3(r) + \widetilde{\mathfrak{C}}_m^3(r)$ and $1 + \widehat{\mathfrak{C}}_m^3(r) + \overline{\mathfrak{C}}_m^3(r) \ge 1 + \widehat{\mathfrak{C}}_m^3(r) + \widetilde{\mathfrak{C}}_m^3(r)$. Therefore, we obtain the following:

$$\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\widetilde{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\frac{\left|\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}-\frac{\left|\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right|}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}$$

$$=\frac{\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\frac{\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}-\frac{\overline{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}$$

$$=\frac{\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)}+\frac{\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}+\frac{\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)}{1+\widehat{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}$$

$$\ge\frac{\widetilde{\mathfrak{C}}_m^3(r)-\widehat{\mathfrak{C}}_m^3(r)+\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)+\widehat{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}=\frac{2\left(\widetilde{\mathfrak{C}}_m^3(r)-\overline{\mathfrak{C}}_m^3(r)\right)}{1+\widetilde{\mathfrak{C}}_m^3(r)+\overline{\mathfrak{C}}_m^3(r)}\ge 0.$$

Hence, (A4) is established under case (c). By following the same logic, (A4) holds for case (d). Thus, (A2) holds, meaning that the triangle inequality holds for the membership degree compartment.

In a similar way, if $\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} = \nu_{\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}} = \nu_{\widehat{\mathfrak{C}},\overline{\mathfrak{C}}} \ge 0$ and $\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}} = \pi_{\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}} = \pi_{\widehat{\mathfrak{C}},\overline{\mathfrak{C}}} \ge 0$, we get the following:

$$\left.\begin{array}{c}\nu_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_n^3(r)-\widetilde{\mathfrak{C}}_n^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_n^3(r)+\widetilde{\mathfrak{C}}_n^3(r)}+\nu_{\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}}\frac{\left|\widetilde{\mathfrak{C}}_n^3(r)-\overline{\mathfrak{C}}_n^3(r)\right|^3}{1+\widetilde{\mathfrak{C}}_n^3(r)+\overline{\mathfrak{C}}_n^3(r)}\\ \ge\nu_{\widehat{\mathfrak{C}},\overline{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_n^3(r)-\overline{\mathfrak{C}}_n^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_n^3(r)+\overline{\mathfrak{C}}_n^3(r)}\end{array}\right\}\tag{36}$$

and

$$\left.\begin{array}{c}\pi_{\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_h^3(r)-\widetilde{\mathfrak{C}}_h^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_h^3(r)+\widetilde{\mathfrak{C}}_h^3(r)}+\pi_{\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}}\frac{\left|\widetilde{\mathfrak{C}}_h^3(r)-\overline{\mathfrak{C}}_h^3(r)\right|^3}{1+\widetilde{\mathfrak{C}}_h^3(r)+\overline{\mathfrak{C}}_h^3(r)}\\ \ge\pi_{\widehat{\mathfrak{C}},\overline{\mathfrak{C}}}\frac{\left|\widehat{\mathfrak{C}}_h^3(r)-\overline{\mathfrak{C}}_h^3(r)\right|^3}{1+\widehat{\mathfrak{C}}_h^3(r)+\overline{\mathfrak{C}}_h^3(r)}\end{array}\right\}.\tag{37}$$

That is, the triangle inequality holds for both the non-membership degree and hesitation margin compartment. Hence, $\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\widetilde{\mathfrak{C}}\right)+\mathcal{D}_{**}\left(\widetilde{\mathfrak{C}},\overline{\mathfrak{C}}\right)\ge\mathcal{D}_{**}\left(\widehat{\mathfrak{C}},\overline{\mathfrak{C}}\right)$ is satisfied since the inequalities (A2), (A5), and (A6) hold. □

References

1. Pranevičienė, B.; Vasiliauskienė, V. Security in crisis situations: The problems of the limits of the human rights**.** *J. Secur. Sustain. Issues* **2018**, *7*, 439–448.
2. Egbefo, D.O.; Salihu, H.A. Internal security crisis in Nigeria: Causes, types, effects and solutions. *Int. J. Arts Humanit.* **2014**, *3*, 176–195.

3. Okoro, J.P. Herdsmen-farmers' conflict: Implication on national development (Nigeria in perspective). *Int. J. Sci. Eng. Res.* **2020**, *11*, 808.
4. Zubairu, N. Rising insecurity in Nigeria: Causes and solution. *J. Stud. Soc. Sci.* **2020**, *19*, 1–11.
5. Jimoh, J.B. Geography and insecurity in northern Nigeria. *Int. J. Res. Geo.* **2020**, *6*, 43–51.
6. Katsina, A.M. Nigeria's security challenges and the crisis of development: Towards a new framework for analysis. *Int. J. Dev. Societ.* **2012**, *1*, 107–116.
7. Ifedayo, T.E.; Akinyemi, O.E.; Azabagun, H.A. Assessment of Nigeria security challenges under President Muhammad Buhari's administration (2015–2022). *Int. J. Peace Conflict Stud*. **2023**, *8*, 39–46.
8. van den Berg, B.; Prins, R.; kuipers, S. Assessing contemporary crises: Aligning safety science and security studies. In *Oxford Research Encyclopedia of Politics*; Oxford University Press: Cary, NC, USA, 2021. https://doi.org/10.1093/acrefore/9780190228637.013.1733.
9. Gizun, A.; Avkurova, Z.; Hriha, V.; Monashnenka, A.; Akatayev, N.; Aleksander, M. Method for the criticality level assessment for crisis situations with parameters fuzzification. In *Advances in Computer Sciencefor Engineering and Education IV. ICCSEEA 2021*; Hu, Z., Petoukhov, S., Dychka, I., He, M., Eds.; Lecture Notes on Data Engineering and Communications Techologies; Springer: Cham, Switzerland, 2021; Volume 83, pp. 147–161.
10. Zadeh, L.A. Fuzzy sets. *Inf. Contr.* **1965**, *8*, 38–353.
11. Ejegwa, P.A.; Kausar, N.; Agba, J.A.; Ugwuh, F.; Ozbilge, E.; Özbilge, E. Determination of medical emergency via new intuitionistic fuzzy correlation measures based on Spearman's correlation coefficient. *AIMS Math*. **2024**, *9*, 15639–15670.
12. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Set Syst*. **1986**, *20*, 87–96.
13. Nwokoro, C.O.; Inyang, U.G.; Eyoh, I.J.; Ejegwa, P.A. Intuitionistic fuzzy approach for predicting maternal outcome. In *Fuzzy Optimization, Decision-Making and Operations Research: Theory and Applications*; Jana, C., Pal, M., Muhiuddin, G., Liu, P., Eds.; Springer International Publishing: Berlin/Heidelberg, Germany, 2023; pp. 399–421.
14. Potwary, J.; Patel, A.; Jana, S.; Mahanta, J. A novel distance measure for intuitionistic fuzzy sets and their application in pattern classification, medical diagnosis, and career determination. *Int. J. Comput. Sci. Math.* **2024**, *19*, 193–205.
15. Anum, M.T.; Zhang, H.; Ejegwa, P.A.; Feng, Y. Tendency coefficient-based weighted distance measure for intuitionistic fuzzy sets with applications. In Proceedings of the 12th International Conference on Intelligent Control and Information Processing (ICICIP 2024), Nanjing, China, 8–10 March 2024; pp. 54–61.
16. Köseoğlu, A. Generalized correlation coefficients of intuitionistic multiplicative sets and their applications to pattern recognition and clustering analysis. *J. Exp. Theor. Artif. Intell*. **2024**. https://doi.org/10.1080/0952813X.2024.2323039.
17. Rajafillah, C.; El Moutaouakil, K.; Patriciu, A.M.; Yahyaouy, A.; Riffi, J. Intuitionistic fuzzy pooling. *Mathematics* **2024**, *12*, 1740.
18. Deveci, K.; Güler, Ö. Ranking intuitionistic fuzzy sets with hypervolume-based approach: An application for multi-criteria assessment of energy alternatives. *Appl. Soft Comput.* **2024**, *150*, 111038.
19. Ejegwa, P.A.; Anum, M.T.; Isife, K.I. A new method of distance measure between intuitionistic fuzzy sets and its application in admission procedure. *J. Uncertain Syst.* **2024**, *17*, 2440005.
20. Mandal, S.; Gazi, K.H.; Salahshour, S.; Mondal, S.P.; Bhattacharya, P.; Saha, A.K. Application of interval valued intuitionistic fuzzy uncertain MCDM methodology for Ph.D supervisor selection problem. *Result Control Optim*. **2024**, *15*, 100411.
21. Yager, R.R. Pythagorean membership grades in multicriteria decision making. *IEEE Trans. Fuzzy Syst.* **2014**, *22*, 958–965.
22. Thakur, P.; Paradowski, B.; Gandotra, N.; Thakur, P.; Saini, N.; Sałabun, W. A study and application analysis exploring Pythagorean fuzzy set distance metrics in decision making. *Information* **2024**, *15*, 28.
23. Ejegwa, P.A.; Zhang, Y.; Li, H.; Feng, Y. Novel measuring techniques with applications in pattern classification and diagnostic process under Pythagorean fuzzy environment. In Proceedings of the 2023 International Conference on New Trends in Computational Intelligence (NTCI), Qingdao, China, 3–5 November 2023; pp. 28–35.
24. Arora, H.D.; Naithani, A. On some new fuzzy entropy measure of Pythagorean fuzzy sets for decision-making based on an extended TOPSIS approach. *J. Manag. Anal.* **2024**, *11*, 87–109.
25. Kirişci, M. Group-based Pythagorean fuzzy soft sets with medical decision-making applications. *J. Exp. Theor. Artif. Intell.* **2024**, *36*, 27–45.
26. Xiao, F.; Ding, W. Divergence measure of Pythagorean fuzzy sets and its application in medical diagnosis. *Appl. Soft Comput.* **2019**, *79*, 254–267.
27. Zhou, Q.; Mo, H.; Deng, Y. A new divergence measure of Pythagorean fuzzy sets based on belief function and its application in medical diagnosis. *Mathematics* **2020**, *8*, 142.
28. Büyüközkan, G.; Göçer, F.; Uztürk, D. A novel Pythagorean fuzzy set integrated Choquet integral approach for vertical farming technology assessment. *Comput. Ind. Eng.* **2021**, *158*, 107384.
29. Zhang, Q.; Hu, J.; Feng, J.; Liu, A.; Li, Y. New similarity measures of Pythagorean fuzzy sets and their applications. *IEEE Access* **2019**, *7*, 138192–138202.
30. Senapati, T.; Yager, R.R. Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods. *Eng. Appl. Artif. Intell*. **2019**, *85*, 112–121.
31. Keshavarz-Ghorabaee, M.; Amiri, M.; Hashemi-Tabatabaei, M.; Zavadskas, E.K.; Kaklauskas, A. A New decision-making approach based on Fermatean fuzzy sets and WASPAS for green construction supplier evaluation. *Mathematics* **2020**, *8*, 2202.
32. Jeevaraj, S. Ordering of interval-valued Fermatean fuzzy sets and its applications. *Expert Syst. Appl*. **2021**, *185*, 115613.

33. Gül, S. Fermatean fuzzy set extensions of SAW, ARAS, and VIKOR with applications in COVID-19 testing laboratory selection problem. *Expert Syst*. **2021**, *38*, e12769.
34. Sahoo, L. Similarity measures for Fermatean fuzzy sets and its applications in group decision-making. *Decis. Sci. Lett*. **2022**, *11*, 167–180.
35. Sindhu, M.S.; Siddique, I.; Ahsan, M.; Jarad, F.; Altunok, T. An approach of decision-making under the framework of Fermatean fuzzy sets. *Math. Probl. Eng.* **2022**, *2022*, 8442123.
36. Rahim, M.; Garg, H.; Amin, F.; Perez-Dominguez, L.; Alkhayyat, A. Improved cosine similarity and distance measures-based TOPSIS method for cubic Fermatean fuzzy sets. *Alex. Eng. J.* **2023**, *73*, 309–319.
37. Alahmadi, R.A.; Ganie, A.H.; Al-Qudah, T.; Khalaf, M.M.; Ganie, A.H. Multi-attribute decision-making based on novel Fermatean fuzzy similarity measure and entropy measure. *Granul. Comput.* **2023**, *8*, 1385–1405.
38. Ejegwa, P.A.; Wanzenke, T.D.; Ogwuche, I.O.; Anum, M.T.; Isife, K.I. A robust correlation coefficient for Fermatean fuzzy sets based on spearman's correlation measure with application to clustering and selection process. *J. Appl. Math. Comput.* **2024**, *70*, 1747–1770.
39. Aruchsamy, R.; Velusamy, I.; Dhandapani, P.B.; Nasiru, S.; Chesneau, C. Modern approach in pattern recognition using circular Fermatean fuzzy similarity measure for decision making with practical applications. *J. Math.* **2024**, *2024*, 6503747.
40. Wang, H.; Tuo, C.; Wang, Z.; Feng, G.; Li, C. Enhancing similarity and distance measurements in Fermatean fuzzy sets: Tanimoto-inspired measures and decision-making applications. *Symmetry* **2024**, *16*, 277.
41. Kirisci, M. Correlation coefficients of Fermatean fuzzy sets with a medical application. *J. Math. Sci. Model.* **2022**, *5*, 16–23.
42. Ejegwa, P.A.; Sarkar, A. Fermatean fuzzy approach of diseases diagnosis based on new correlation coefficient operators. In *Deep Learning in Personalized Healthcare and Decision Support*; Garg, H., Chatterjee, J.M., Eds.; Academic Press: Cambridge, MA, USA, 2023; pp. 23–38.
43. Xu, C.; Shen, J. Multi-criteria decision making and pattern recognition based on similarity measures for Fermatean fuzzy sets. *J. Intell. Fuzzy Syst*. **2021**, *41*, 5847–5863.
44. Ashraf, S.; Attaullah; Naeem, M.; Khan, A.; Rehman, N.; Pandit, M.K. Novel information measures for Fermatean fuzzy sets and their applications to pattern recognition and medical diagnosis. *Comput. Intell. Neurosci.* **2023**, *2023*, 9273239.
45. Senapati, T.; Yager, R.R. Fermatean fuzzy sets. *J. Amb. Intell. Human. Comput*. **2020**, *11*, 663–674.
46. Deng, Z.; Wang, J. New distance measure for Fermatean fuzzy sets and its application. *Int. J. Intell. Syst.* **2022**, *37*, 1903–1930.
47. Ganie, A.H. Multicriteria decision-making based on distance measures and knowledge measures of Fermatean fuzzy sets. *Granul. Comput*. **2022**, *7*, 979–998.
48. Onyeke, I.C.; Ejegwa, P.A. Modified Senapati and Yager's Fermatean fuzzy distance and its application in students' course placement in tertiary institution. In *Real Life Applications of Multiple Criteria Decision Making Techniques in Fuzzy Domain*; Sahoo, L., Senapati, T., Yager, R.R., Eds.; Springer Nature: Singapore, 2023; pp. 237–253.
49. Kirişci, M. New cosine similarity and distance measures for Fermatean fuzzy sets and TOPSIS approach. *Knowl. Inf. Syst.* **2023**, *65*, 855–868.
50. Liu, Z.; Huang, H. Comment on new cosine similarity and distance measures for Fermatean fuzzy sets and TOPSIS approach. *Knowl. Inf. Syst.* **2023**, *65*, 5151–5157.
51. Ganie, A.H.; Gheith, N.E.M.; Al-Qudah, Y.; Ganie, A.H.; Sharma, S.K.; Aqlan, A.M.; Khalaf, M.M. An innovative Fermatean fuzzy distance metric with its application in classification and bidirectional approximate reasoning. *IEEE Access* **2024**, *12*, 4780–4791.
52. Liu, Z. A distance measure of Fermatean fuzzy sets based on triangular divergence and its application in medical diagnosis. *J. Oper. Intell*. **2024**, *2*, 167–178.
53. Xie, J.; Xu, X.; Li, F. Novel intuitionistic fuzzy distance based on tendency and its application in emergency decision-making. *Int. J. Fuzzy Syst.* **2023**, *25*, 2295–2311.
54. Ibrahim, H.Z.; Alshammari, I. n,m-rung orthopair fuzzy sets with applications to multicriteria decision making. *IEEE Access* **2022**, *10*, 99562–99572.
55. Jabeen, K.; Khan, Q.; Ullah, K.; Senapati, T.; Moslem, S. An approach to MADM based on Aczel-Alsina power Bonferroni aggregation operators for q-rung orthopair fuzzy sets. *IEEE Access* **2023**, *11*, 105248–105261.
56. Ali, J.; Naeem, M. Analysis and application of p, q-quasirung orthopair fuzzy Aczel-Alsina aggregation operators in multiple criteria decision-making. *IEEE Access* **2023**, *11*, 49081–49101.
57. Khan, S.; Khan, M.; Khan, M.S.A.; Abdullah, S.; Khan, F. A novel approach toward q-rung orthopair fuzzy rough Dombi aggregation operators and their application to decision-making problems. *IEEE Access* **2023**, *11*, 35770–35783.
58. Sha, L.; Shao, Y. Fermatean hesitant fuzzy Choquet integral aggregation operators. *IEEE Access* **2023**, *11*, 38548–38562.