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An algorithmic review of the technological progress and milestones in resource-constrained project planning

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A B S T R A C T

Engineering, procurement, and construction projects are time-intensive and subject to resource constraints. Modern project planning software requires optimization algorithms to schedule tasks while considering resource availability. A comprehensive review of the optimization algorithms used in project planning has not yet been conducted. This study seeks to bridge the gap through an algorithmic review of the Resource-constrained Project Scheduling Problems (RCPSPs) literature and investigates the following research questions: What are the milestones on the main development trajectory of optimization algorithms for solving RCPSPs? How might this influence future advancements in the field? To answer these questions, the Main Path Analysis (MPA) method is employed to review the development trajectory and milestones from over 1100 project scheduling articles published between 1980 and 2024. Cluster Analysis (CA) complements the investigations by identifying the prevalent research themes, mathematical features, and solution algorithms. Recommendations for future research directions, supported by the systematic review, conclude the study. This review provides a reference for project management researchers focused on industrial applications of project scheduling problems.

A R T I C L E   I N F O

Keywords:
Project management;
Task scheduling;
Resource constraint;
Multi-objective optimization;
Evolutionary computation;
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Metaheuristics;
Main path analysis;
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1. Introduction

Modern project planning must account for the rapid changes in the resources [1]. Traditionally, project managers use planning tools based on the Critical Path Method (CPM) and the Program Evaluation and Review Technique (PERT) to schedule tasks and predict project progress, milestones, and completion times. These methods are predominantly based on operative and predictive models [2]; they overlook resource availability, requiring schedules to be adjusted each time a task is delayed due to the lack of resources. As the scope and complexity of projects increase, such tools, although powerful, cannot provide dependable solutions that are both optimal and robust. Mathematical models and solution algorithms are required for optimal project planning, taking into account various practical constraints. The Resource-Constrained Project Scheduling

Problem (RCPSP) is an advanced planning alternative that optimizes resource allocation while sequencing project tasks.

The primary investigation shows that research on project scheduling has grown steadily since 1995, reaching a peak of 107 published articles in 2021. The early works focused on developing priority rules for resource-constrained project scheduling [3]. More recent studies evolved towards developing advanced heuristics and metaheuristics. Among the seminal works, Debels *et al.* [4] developed a hybrid Scatter Search/Electromagnetism algorithm for project scheduling. Kolisch and Hartmann [5] contributed an experimental analysis of heuristic algorithms for resource-constrained project scheduling. Van Peteghem and Vanhoucke [6] developed a Genetic Algorithm for solving preemptive and non-preemptive multi-mode RCPSPs. Most recently, several constructive heuristic algorithms were developed by Nekoueian *et al.* [7] for selecting and scheduling alternative subgraphs in resource-constrained projects. Melchiors *et al.* [8] conducted an experimental analysis comparing the performance of priority rules for dynamic stochastic RCPSPs. Servranckx *et al.* [9] proposed a Genetic Algorithm integrated with a Boolean satisfiability solver to solve RCPSP with alternative subgraphs.

Identifying the main development trajectory of RCPSP sheds light on scientific progress and milestones in project planning. There are several reviews of the literature on project scheduling, three of which are recent and relevant. Gomez *et al.* [10] reviewed multi-project scheduling problems as one of the several variants of RCPSP. Aghileh *et al.* [11] contributed a more focused literature review focusing on multi-project scheduling problems under uncertainty and resource flexibility. These articles investigated specific variants of RCPSP. The most relevant study, Hartmann *et al.* [12] surveyed the prominent extensions of RCPSP. They considered a theoretical lens, focusing solely on mathematical features; *solution methods* for solving RCPSP were not investigated. Moreover, their review *scope* covered only articles published between 2010 and 2020. Most importantly, the existing surveys are based on a traditional literature reviews, i.e. *manual reading* of articles, which is *subjective* and cannot trigger big-picture thinking.

To the authors' best knowledge, the present study is the first algorithmic review (see [13]) of the project planning literature using mathematical methods. Main Path Analysis (MPA) and Cluster Analysis (CA) are used to analyze the literature published between 1980 and 2024. The objective is to explore the following research questions: What are the milestones on the development trajectory of RCPSP solution algorithms? How might this influence future advances in the field?

This article continues in three more sections. The research methods are first described in Section 2. MPA results are presented in Section 3, followed by discussions on the outcomes of CA and keyword analysis. This research concludes in Section 4, where directions for future studies are suggested.

2. Research framework

The Web of Science (WoS) database serves as the data collection source in this study. WoS comprises six index databases, namely Emerging Sources Citation Index (ESCI), Science Citation Index Expanded (SCIE), Social Sciences Citation Index (SSCI), Arts & Humanities Citation Index (A&HCI), Book Citation Index (BCI), and Conference Proceedings Citation Index (CPCI). Compared to other databases such as Google Scholar and Scopus, WoS strictly controls the quality of its collections, and only includes peer-reviewed documents of high quality.

Step 1. *Data collection*

The keyword TS = (Resource-Constrained Project Scheduling) was considered the primary term for the search. "AND" and "*" operators were used for keyword searches to reduce the chances of missing relevant documents: TS = (Resource-Constrained) AND TS = (project*) AND TS = (scheduling*). The search was conducted on April 25, 2024, covering the period FPY = (1980-2023). Initial investigations showed that 1,264 documents fall within the defined research scope considering the articles' titles, abstracts, and keywords. The manual screening was then conducted to remove irrelevant records from the analysis; 1,189 documents remained in the database. Next, 32 retrospective articles were excluded to ensure their high citation count would not influence the path analysis procedure. Finally, the main path analysis software, MainPath 480, identified 42 isolated documents that neither cited other articles nor were cited by others; these were also removed from the database. A total of 1,130 documents were considered for further investigations.

The accuracy index Precision in Eq. 1, and the Digital Object Identifier percentage DOI in Eq. 2 were used to check whether the database is representative of the entire literature; values greater than 0.7 are considered a green flag for MPA and CA analysis [14, 15]. The former index compares the number of articles before and after the removal of isolated points. In this study, the numbers of articles before and after excluding isolated points were 1,174 and 1,130, respectively. The search accuracy was 0.96, which is pretty high. Additionally, a high DOI percentage confirms the representativeness of the database. The total citation count of the topic is 43,550 while the DOI Total is 32,301 times, accounting for 0.74, which is acceptable and indicates that the database can be used for further analysis.

$$\text{Precision} = \frac{\text{Network Size}}{\text{Number of Articles in the Original Database}} \times 100\,\% \tag{1}$$

$$\text{DOI ratio} = \frac{\text{DOI Total}}{\text{Citation Record}} \times 100\,\% \tag{2}$$

Step 2. *Data processing using MPA, CA, and keyword analysis*

The database must be converted into a citation network to establish the basis for data processing. This network categorizes documents into *source* (where knowledge dissemination begins), *intermediate* (where knowledge is transferred from one node to another), or *sink* (where knowledge dissemination ends). The citation relationships within the network are represented by directed arrows, indicating knowledge diffusion.

The analysis continues by calculating the weights of the arrows, taking into account all possible citation chains from source to sink. MPA uses these weights to identify the backbone of the citation network. Search Path Count (SPC; from all source nodes to all sink nodes of the network), Search Path Link Count (SPLC; all the ancestors of a tail node for a specific link), or Search Path Node Pair (SPNP; all the ancestors and descendants of a specific link) methods can be used to calculate and assign weights to the network links. SPLC best represents the knowledge dissemination process in academic literature [16] and is therefore used in the analysis. The weighted network forms the basis for identifying the main path and the key development branches.

Using the weighted network as input, the MPA function of MainPath480 considers all possible citation chains and selects the one with the largest overall SPLC as the main path. The Global Main Path setting is used to identify the major citation chain over the entire literature period, i.e. from sink to source nodes. The Key-route Main Path is further used to identify the main path while including top-cited references; this study uses Key-route 10, which ensures that the top ten contributions are included in the analysis.

The same database is also used for the data-driven categorization of articles using the CA [17] function of MainPath480. A three-step procedure organizes articles based on their similarities (the shortest path between all the node pairs, calculating edge credit using Eq. 3, and removing the edge(s) with the highest score to isolate groups of articles). Additionally, similar terms are merged to identify the cluster containing the most frequently used words.

$$\text{Edge\_Credit} = \left(1 + \sum \text{Incoming Edge Credit}\right) \times \frac{\text{Score of Destination}}{\text{Score of Start}} \tag{3}$$

Step 3. *Literature content analysis*

The literature content analysis will explore the advances in solution algorithms for solving RCPSPs. A three-field notation, $\alpha|\beta|\gamma$ is used to characterize the reviewed RCPSPs; the problem characteristics are treated as an influencing factor in the advances in project scheduling algorithms. In the notation system, α specifies the process type; β identifies the practical features and characteristics considered in the studied RCPSP; and γ represents the objective function. Tables 1-3 define the notations used in the three sections, respectively.

Table 1 Process type-related notations, α

Notation	Definition
PS	General project scheduling.
MPS	Multi-mode project scheduling.
$PS_{m,\sigma,\rho}$	General project scheduling considering m resources, σ units of available resources, each activity requiring at most ρ units of the resources.
$MPS_{m,\sigma,\rho;\mu,\tau,\omega}$	Multi-mode project scheduling with m renewable and μ non-renewable resources; σ and τ units of renewable and non-renewable resources with each activity requiring at most ρ and ω units of the renewable and non-renewable resources, respectively.
$\alpha_1 = 0$	No resource types are considered.
$\alpha_1 = 1$	One resource type is considered.
α_1=m	The number of resource types is equal to *m.*
$\alpha_2 = 0$	Absence of any resource type specification.
$\alpha_2 = 1$	Renewable resources; availability is specified for a time unit.
$\alpha_2 = T$	Non-renewable resources, the availability of which is specified for the entire project horizon *T.*
$\alpha_2 = 1T$	Both renewable and non-renewable resources are considered.
$\alpha_2 = \upsilon$	Partially (non-)renewable resources the availability of which is renewed in specific periods.
$\alpha_3 = 0$	(Partially) Renewable resources available in constant amounts.
$\alpha_3 = \upsilon a$	(Partially) Renewable resources available in variable amounts.
$\alpha_3 = \tilde{a}$	Stochastic resource availability with constant value over time.
$\alpha_3 = \upsilon\tilde{a}$	Stochastic resource availability with variable values over time.
$MPS_{m,\sigma,\rho;\mu,\tau,\omega} - MR$	RCPSP with Multiple Routes.
$PS_{m,\sigma,\rho} - MOP - DC$	Integrated RCPSP and material ordering with discounted cash flows.
$PS_{m,\sigma,\rho} - PS$	RCPSP with a model-endogenous decision on the project structure.
$MS - PS_{m,\sigma,\rho}$	Multi-Skill Resource-Constrained Project Scheduling Problem.
$MPS_{m,\sigma,\rho;\mu,\tau,\omega} - CS$	Multi-mode Integrated RCPSP with Contractor Selection.
$M - MPS_{m,\sigma,\rho;\mu,\tau,\omega}$	Multi-objective Multi-mode RCPSP.
$MMPS_{m,\sigma,\rho;\mu,\tau,\omega}$	Multi-mode, Multi-project RCPSP.
$MS - MPS_{m,\sigma,\rho;\mu,\tau,\omega}$	Multi-skill Multi-modal RCPSP.
$MPPS_{m,\sigma,\rho}$	Resource-constrained multi-project scheduling problem.
$CCPS_{m,\sigma,\rho}$	Resource-constrained project scheduling with critical chain.

Table 2 Process characteristics-related notations, β

Notation	Definition
$p_j = 1$	All processing times are equal to one.
$p_j = sto$	Stochastic processing times.
d	Deadline for project duration.
$prec$	Precedence constraints between activities.
$chain, intree, outtree, tree$	Precedence relations between activities are specified.
$temp$	General temporal constraints, given the minimum and maximum start-start time lag between activities.
$\beta_1 = 0$	No preemption is allowed.
$\beta_1 = pmtn$	Preemptions of the preempt-resume type are allowed.
$\beta_1 = pmtn - rep$	Preemptions of the preempt-repeat type are allowed.
$\beta_2 = 0$	No precedence constraints.
$\beta_2 = cpm$	Strict finish-start precedence constraints with zero time lag, as used in the basic PERT/CPM model.
$\beta_2 = min$	Precedence diagramming constraints of the types start-start,finish-start, start-finish, and finish-finish with minimal time lags.
$\beta_2 = gpr$	Generalized precedence relations of the types start-start, finish-start, start-finish, and finish-finish with both minimal and maximal time lags.
$\beta_2 = prob$	The activity network is of probabilistic type where the evolution of the corresponding project is not determined in advance.

Table 2 (Continuation)

$\beta_3 = 0$	All ready times are zero.
$\beta_3 = \rho_j$	Ready times differ per activity.
$\beta_4 = 0$	Activities have arbitrary integer durations.
$\beta_4 = cont$	Activities have arbitrary continuous durations.
$\beta_4 = (d_j = d)$	All activities have a duration equal to d units.
$\beta_4 = \tilde{d}_j$	The activity durations are stochastic.
$\beta_5 = 0$	No deadlines are assumed in the system.
$\beta_5 = \delta_j$	Deadlines are imposed on activities.
$\beta_5 = \delta_n$	A project deadline is imposed.
$\beta_6 = 0$	Constant discrete resource requirements.
$\beta_6 = vr$	Variable discrete resource requirements.
$\beta_6 = \tilde{r}$	Stochastic constant discrete resource requirements.
$\beta_6 = v\tilde{r}$	Stochastic discrete variable resource requirements.
$\beta_6 = disc$	The requirements are a discrete function of the activity duration.
$\beta_6 = cont$	The requirements are a continuous function of the activity duration.
$\beta_6 = int$	The requirements are expressed as an intensity or rate function.
$\beta_7 = 0$	Activities must be performed in a single execution mode.
$\beta_7 = mu$	Activities have multiple prespecified execution modes.
$\beta_7 = id$	Activities are subject to mode identity constraints.
$\beta_8 = 0$	No cash flows are specified in the project scheduling problem.
$\beta_8 = c_j$	Activities have an arbitrary cash flow.
$\beta_8 = \tilde{c}_j$	Cash flows are stochastic.
$\beta_8 = c_j^+$	Activities have an associated positive cash flow.
$\beta_8 = per$	Periodic cash flows are specified for the project.
$\beta_8 = sched$	Both the amount and the timing of the cash flows are determined.
$\beta_9 = 0$	No change-over (transportation) times.
$\beta_9 = s_{jk}$	Sequence-dependent change-over times.

Table 3 Objective function-related notations, γ

Notation	Definition
$\sum c_j^F \beta^{c_j}$	Net present value.
$\sum c_k f(r_k(S,t))$	Resource leveling.
$\sum c_k \max r_k (S,t)$	Resource investment.
$\gamma = reg$	The performance measure is any early completion (regular) measure.
$\gamma = nonreg$	The performance measure is any free completion (non-regular) measure.
$\gamma = C_{max}$	Minimize the project makespan.
$\gamma = \bar{F}$	Minimize the average flow time across all sub-projects or activities.
$\gamma = L_{max}$	Minimize the project lateness.
$\gamma = T_{max}$	Minimize the project tardiness.
$\gamma = earlv/tardv$	Minimize the weighted earliness-tardiness of the project.
$\gamma = n_T$	Minimize the number of tardy activities.
$\gamma = \sum sq.dev.$	The sum of squared deviations of the resource requirements from the average.
$\gamma = av$	Minimize the resource availabilities to meet the project deadline.
$\gamma = rac$	Minimize the resource availability costs.
$\gamma = curve$	Determine the complete time/cost trade-off curve.
$\gamma = npv$	Maximize the net present value of the project.
$\gamma = E[\cdot]$	Optimize the expected value of a performance measure.
$\gamma = cdf$	Determines the cumulative density function of the project realization date.
$\gamma = ci$	Determines the criticality index of an activity or a path.
$\gamma = mci$	Determines the most critical path(s) or activities based on the criticality index.
$\gamma = multi$	Different objectives are weighted or combined.
$\gamma = multicrit$	Multi-criteria functions.
$\gamma = EPD$	Minimizes the expected project duration.
$\gamma = EC_{max}$	Expected makespan.
$\gamma = CF$	Minimize the carbon footprints.
$\gamma = TC$	Minimize the Total cost.
$\gamma = TD$	Minimize the Total Duration.

Step 4. *Dissemination*

The outputs of MainPath480 consist solely of character results. The results were further processed using Pajek 5.18 (*http://mrvar.fdv.uni-lj.si/pajek/*) and VOSviewer 1.6.18 [18] for dissemination. A satellite-like view, which illustrated the major knowledge diffusion paths is used to visualize the findings.

3. Results and discussions

3.1 Main development trajectory of solution algorithms

A total of 27 articles formed the main development trajectory of RCPSPs. The contributions predominantly fall under at least one of the following categories: new solution algorithms or optimization models, new datasets, and benchmarking or organizing the available methods. The trajectory is shown in Fig. 1, which is followed by an elaboration on the scientific progress and milestones.

Fig. 1 The main development trajectory of RCPSPs

As the starting point of the main development trajectory, Bell and Park [19] studied $PS_{m,\sigma,\rho}|prec|C_{max}$ and developed the A-STAR search method to solve it, which operates by identifying active nodes that violate resource restrictions and prioritizing them to reduce resource conflicts. Bell and Han [20] proposed a new heuristic to solve $PS_{m,\sigma,\rho}||C_{max}$. The algorithm first generates an initial solution and uses a procedure inspired by the Hill Climbing method to improve the solution. They used Patterson's dataset to compare their algorithm with six traditional heuristic rules. Sampson and Weiss [21] developed several local search techniques to solve $PS_{m,\sigma,\rho}||C_{max}$ and used Patterson's 110 datasets to test their performance, comparing it with the as-is situation in a hypothetical problem.

Kolisch *et al.* [22] developed a new dataset for the problems of $PS_{m,\sigma,\rho}||C_{max}$, $PS_{m,\sigma,\rho}||tardv$, and a multi-objective variant of the two. Experimental analysis showed that some of the small-scale instances from their test bank can be solved in polynomial time under certain circumstances. Kolisch and Drexl [23] developed an Adaptive Search Procedure with new sorting rules and random search techniques for $MPS_{m,\sigma,\rho;\mu,\tau,\omega}||C_{max}$. They used the 308 instances of Gen-instances to show that the algorithm can effectively limit the solution space and compared it with four other heuristics coded using different programming languages. Kolisch and Sprecher [24] referred to the ProGen system to develop the new database, Project Scheduling Problem Library (PSPLIB) for RCPSPs. Their platform generated instances of different sizes for minimizing the overall project completion time and considers the number of activities, single-mode and multi-mode activity execution modes, activity duration, and resource constraints. Brucker *et al.* [25] developed a new Branch and Bound algorithm for solving $PS_{m,\sigma,\rho}|\beta_6 = {}^o, prec|C_{max}$. Kolisch's PSPLIB test bank was used to test the algorithm comparing it with the best solutions found by Demeulemeester and Herroelen [26].

Hartmann [27] developed a Competitive Genetic Algorithm for $PS_{m,\sigma,\rho}||C_{max}$ and used the PSPLIB database to compare its performance with other variants of the Genetic Algorithm. Hartmann and Kolisch [28] compared the Genetic Algorithm, Tabu Search, and Simulated Annealing for $PS_{m,\sigma,\rho}||C_{max}$ and the PSPLIB dataset. They analyzed the impact of different sorting rules, computational mechanisms, problem size, and resource limitations on algorithm performance. Hartmann [29] introduced the self-adapting Genetic Algorithm to solve $PS_{m,\sigma,\rho}||C_{max}$ and used the PSPLIB database to compare its performance with Simulated Annealing, Tabu Search, and several variants of the Genetic Algorithm.

Valls *et al.* [30] proposed a Four-Phase-Based procedure that uses the Convex Search Algorithm to generate the initial solution and the Homogeneous Interval Algorithm to improve the initial solution. The algorithm was compared with state-of-the-art heuristics using randomly generated instances for solving $PS_{m,\sigma,\rho}||C_{max}$. *Valls et al.* [31] proposed a heuristic with several justification techniques for solving $PS_{m,\sigma,\rho}|prec|C_{max}$. They used the ProGen test bank to compare their algorithm with 22 different heuristics. Kolisch and Hartmann [5] used the PSPLIB test bank to evaluate many of the algorithms commonly used to solve RCPSPs and analyzed their characteristics to speculate on the reasons for their performance.

Vanhoucke and Debels [32] explored the impact of practical characteristics, such as activity duration, task splitting, and fast-tracking on the total lead time and improved the Branch and Bound method to solve RCPSPs. Van Peteghem and Vanhoucke [6] proposed the bi-population Genetic Algorithm to address $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|\beta_1 = pmtn - rep|C_{max}$. They used the PSPLIB database to evaluate the solution algorithm's performance with and without preemption. Coelho and Vanhoucke [33] introduced the AND-OR network for the project structure of $MPS_{m,\sigma,\rho;\mu,\tau,\omega}, \alpha_2 = 1T||C_{max}$ and compared hybrid algorithms based on Tabu Search and Non-dominated Sorting Genetic Algorithms (NSGA) -I and -II using the PSPLIB database. Zamani [34] proposed a Magnet-Based Genetic Algorithm to solve $MPS_{m,\sigma,\rho;\mu,\tau,\omega}, \alpha_2 = 1T||C_{max}$ and used the PSPLIB database to compare their algorithm with baseline Genetic Algorithms.

Cheng *et al.* [35] proposed the Fuzzy Clustering Chaotic-based Differential Evolution to minimize the project duration in $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$ and used a case study from the construction sector to demonstrate the practical implications of implementing their optimization method. Tran *et al.* [36] developed a new hybrid solution method based on the Artificial Bee Colony and Differential Evolution algorithms to solve $PS_{m,\sigma,\rho}|prec|C_{max}$. They used PSPLIB as a benchmark set to compare their algorithm with improved versions of the Genetic Algorithm, Particle Swarm Optimization, Artificial Bee Colony and Differential Evolution algorithms. Sonmez and Gürel [37] developed the Harmony Search Algorithm to solve $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$ and used PSPLIB to evaluate its performance against two improved Genetic Algorithms, as well as Particle Swarm and Ant Colony Optimization algorithms. Tao and Dong [38] studied the problem of $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|curve$ and developed an improved hybrid method based on the Tabu Search and NSGA-II to solve it. They used the PSPLIB to compare the solution algorithm's performance with two basic versions of NSGA-II.

Birjandi and Mousavi [39] proposed a Cluster-based Tabu Search integrated with the Particle Swarm Optimization algorithm to solve $MPS - MR_{m,\sigma,\rho;\mu,\tau,\omega}|prec|curve$. The new algorithm was evaluated using random test instances and compared with basic versions of Particle Swarm Optimization and Genetic Algorithms. Chakrabortty *et al.* [40] developed Variable Neighborhood Search-based Local Search Heuristic algorithms to solve $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j, \beta_6 =^o|C_{max}$ and $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j, \beta_6 = vr|C_{max}$. They considered six different priority rules and analyzed the algorithms' performance using random test instances in dynamic environments.

Asadujjaman *et al.* [41] developed the Immune Genetic Algorithm for the problem of $PS_{m,\sigma,\rho}|prec, \beta_5 = \delta_n|npv$ and benchmarked its performance against the Lagrangian relaxation-based forward-backward improvement heuristic and the Scatter Search using the database from [42]. Asadujjaman *et al.* [43] proposed $PS_{m,\sigma,\rho} - MOP - DC|prec, \beta_5 = \delta_n|npv, pf$, which integrated an RCPSP with a material ordering problem. The author used the Immune Genetic Algorithm to solve the problem considering random test instances and comparing the results with those of the basic Genetic and Immune algorithms. Asadujjaman *et al.* [44] proposed a multi-operator version of the Immune Genetic Algorithm to solve $PS_{m,\sigma,\rho}|prec, \beta_4 = cont|npv$. They used 17,280 different instances from the database of Vanhoucke [42] to evaluate and compare the algorithm's performance against the Branch & Cut algorithm as the baseline.

The last study on the main development trajectory, Rahman *et al.* [45], proposed the Genetic Algorithm-based Memetic Algorithm to solve $PS_{m,\sigma,\rho}|prec|TC, CF$, considering Carbon Footprints as one of the optimization objectives. They developed a new test set to compare the performance of their algorithm with the basic Genetic and Memetic algorithms, as well as the basic and enhanced versions of NSGA-II.

Overall, early studies developed exhaustive solution methods and decision trees suitable for solving small-scale problems with low computational complexity. Studies evolved after 2006, incorporating practical constraints to bridge the gap between scheduling theory and practice. The precedence condition is a prime example. The optimization models and algorithms shifted from single-objective to multi-objective optimization after 2016. Initial works focused on minimizing project completion time, but later studies considered optimization objectives such as Resource Utilization Rate, total project cost, net present value, and environmental factors.

3.2 Key branches in the development of solution methods

Different values for the key-route parameter, i.e. 5, 10, 15, 20, 25, 30, 35, 40, and 45, are considered to find the most reasonable setting for the literature analysis. The key-route value of 30 is deemed suitable, resulting in a total of 51 articles: 2 sources, 2 sinks, and 47 intermediate nodes, including the articles on the main path. The key branches are shown in Fig. 2, followed by a review of the articles that emerged from the main path.

The first source node, Patterson *et al.* [46], proposed a Backtracking Algorithm to solve $PS_{m,\sigma,\rho}|prec, \alpha_2 = 1, \alpha_2 = T|C_{max}$ and confirmed its efficiency through numerical experiments while considering different problem characteristics. The second source, Bell and Park [19], is present on the main development path; this study was later cited by Bell and Han [20], and was followed by Sampson and Weiss [21], both of which are main path articles. Sprecher *et al.* [47] explored the active, semi-active, and non-delay schedules in the context of RCPSPs. They used small illustrative examples to explain the applicability and implications of their method. This article was later cited by Kolisch and Drexl [23], which is considered a main path article. Kolisch and Drexl [48] explored $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec, \alpha_2 = 1, \alpha_2 = T|C_{max}$ and proposed a new local search technique to find feasible solutions as well as a neighborhood search method to improve it. The authors used the test bank generated by ProGen to compare results with two general non-preemptive algorithms developed by Drexl and Gruenewald [49].

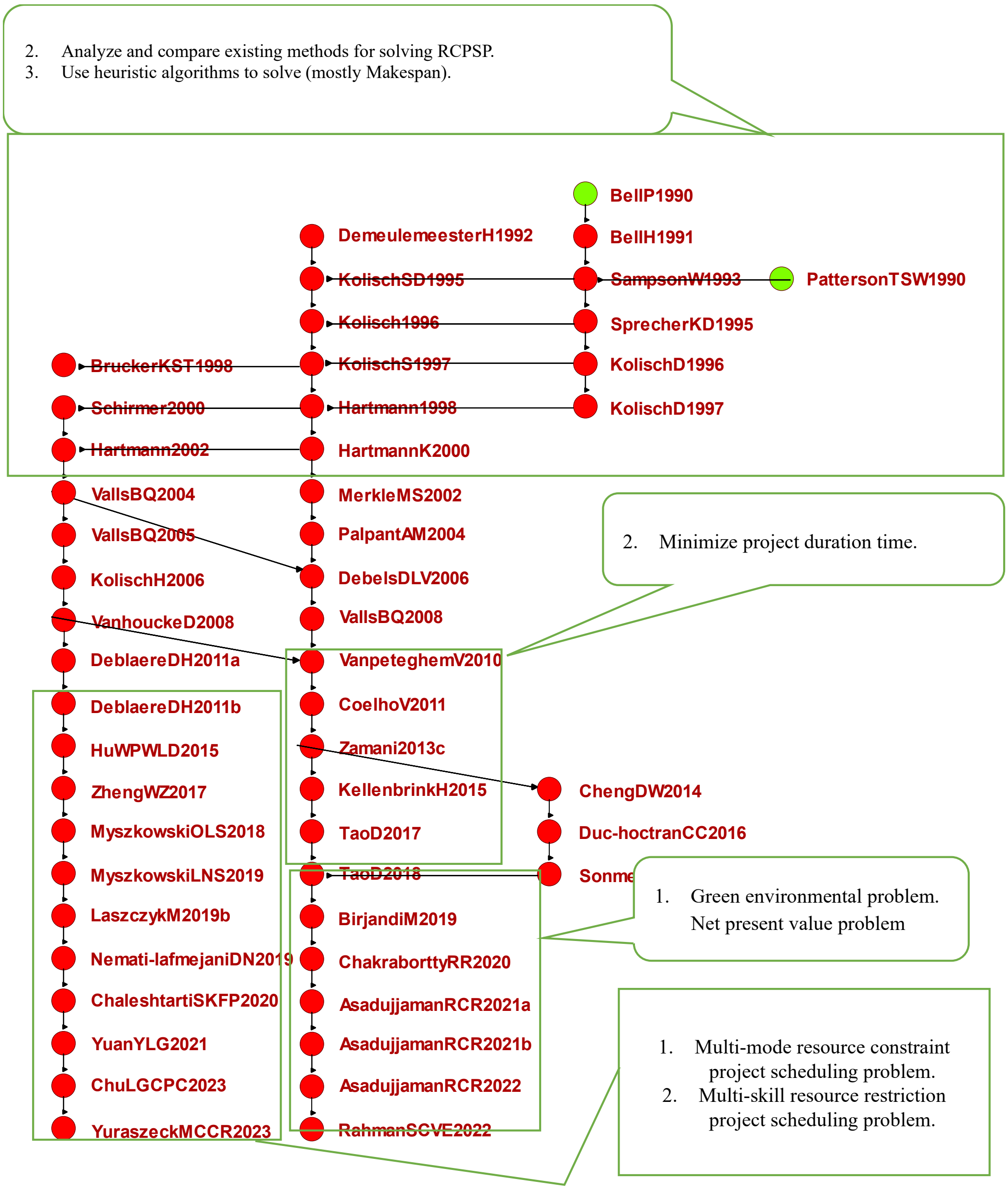


Fig. 2 The key development branches from the main path

The middle path constitutes many references from the main path, i.e. Kolisch *et al.* [22], Kolisch and Sprecher [24], Hartmann [27], and Hartmann and Kolisch [28]. Demeulemeester and Herroelen [50] developed a new Branch-and-Bound method suitable for addressing PERT/CPM problems with preconditions and solving $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec, \beta_1 = ^o|C_{max}$; they used Patterson's test bank to evaluate its performance, comparing it with the Branch-and-Bound method developed by Stinson *et al.* [51]. Kolisch [52] investigated the limitations of the serial and parallel scheduling methods; they put forward new strategies that were tested using the PSPLIB databases in comparison to well-known priority rules, including the most total successors, latest start time, latest finish time, minimum slack, and the greatest rank position weight.

Merkle *et al.* [53] developed an Ant Colony Optimization algorithm to solve $PS_{m,\sigma,\rho}|prec|C_{max}$. This algorithm was tested on the PSPLIB instances and compared with the lower bounds using a critical path heuristic. Palpant *et al.* [54] developed the neighborhood search algorithm with exact resolution of sub-problems to solve $PS_{m,\sigma,\rho}|prec|C_{max}$ and considered Patterson's 110 *easy* instances, along with some other datasets, to evaluate its performance by comparing the results with the best-known solutions at that time. Debels *et al.* [4] introduced the hybrid Scatter

Search Electromagnetism algorithm to solve $PS_{m,\sigma,\rho}|prec|C_{max}$ and used PSPLIB to showcase its superiority over self-adaptive and robust Genetic Algorithms, Tabu Search, and Simulated Annealing. Valls *et al.* [55] proposed a hybrid Genetic Algorithm to solve $PS_{m,\sigma,\rho}|prec|C_{max}$. They used ProGen's instances and performed extensive comparisons with several versions of the Genetic Algorithm, Tabu Search, and solution methods based on network decomposition.

After the studies of Van Peteghem and Vanhoucke [6], Coelho and Vanhoucke [33], and Zamani [34], which are considered on the main path, Kellenbrink and Helber [56] explored the problem of model-endogenous decisions on the project structure in RCPSPs, and developed a novel Genetic Algorithm with a module that adjusts the activity structure of inter-project activities to improve the solutions; the authors extended the ProGen generator and compared the results with those of CPLEX. Tao and Dong [57] introduced a new AND-OR network, proposed a new mathematical formulation for $PS_{m,\sigma,\rho}|prec|C_{max}$, and adjusted the Simulated Annealing algorithm to solve the problem. They designed a new instance set to evaluate the algorithm's performance, comparing it with an exact solver and several variants of the same algorithm. These studies also considered stochastic activity restrictions. The study of Zamani [34] was continued along the main path by Cheng *et al.* [35], Tran *et al.* [36], and Sonmez and Gürel [37]. The next seven articles were reviewed in the main path section, namely: Tao and Dong [38], Birjandi and Mousavi [39], Chakrabortty *et al.* [40], Asadujjaman *et al.* [41], Asadujjaman *et al.* [43], Asadujjaman *et al.* [44], and Rahman *et al.* [45]. These articles focused on green operational factors and multi-skill RCPSPs.

The left path in Fig. 2 includes Brucker *et al.* [25], Hartmann [29], Valls *et al.* [30], Valls *et al.* [31], Kolisch and Hartmann [5], and Vanhoucke and Debels [32], all of which are part of the main path. Schirmer [58] incorporated the Case-Based Reasoning, which is a learning method, into adaptive search algorithms to solve $PS_{m,\sigma,\rho}|prec,\alpha_2 = 1, \alpha_2 = T|C_{max}$. The authors focused on developing a mechanism for selecting algorithms based on the problem characteristics and used PSPLIB to confirm the practicability of the method.

Deblaere *et al.* [59] proposed a new execution strategy for the stochastic RCPSPs, $PS_{m,\sigma,\rho}|prec, \alpha_3 = v\tilde{a}|\gamma = E[\cdot]$. They used PSPLIB to compare their Simulation-based Descent Algorithm with the general heuristic developed by Van de Vonder et al. [60]. Deblaere *et al.* [61] explored $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$ and extended the Iterative Deepening A-STAR algorithm that is equipped with a module to repair the disrupted schedules. They considered the PSPLIB project scheduling library to compare the algorithm with different dominance rules as well as different search strategies, including the regular Branch-and-Bound and one integrated with Tabu Search.

Hu *et al.* [62] proposed an Outer-Inner Fuzzy Cellular Automaton for the Dynamic Uncertainty Multi-Project Scheduling Problem, $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = curve, \gamma = RUR$. They considered the PSPLIB database for performance comparison with a Bee Swarm Optimization that utilizes forward-backward interchange and a hybrid Genetic Algorithm. Zheng *et al.* [63] developed the Teaching–Learning-Based Optimization algorithm to solve $MS - PS_{m,\sigma,\rho}|prec|C_{max}$. They used the iMOPSE database to compare the results obtained by their method with those from the Hybrid Ant Colony Optimization algorithm. Myszkowski *et al.* [64] developed a Hybrid Differential Evolution and Greedy Algorithm for $MS - PS_{m,\sigma,\rho}|prec|C_{max}$ and considered the iMOPSE database to compare its performance with Hybrid Ant Colony Optimization algorithm that utilizes Greedy Randomized Adaptive Search Procedure (GRASP). Myszkowski *et al.* [65] introduced the iMOPSE platform for the $S - PS_{m,\sigma,\rho}$ variant of RCPSPs. The software includes test instances, visualization tools, and other material. Laszczyk and Myszkowski [66] developed the Non-dominated Tournament Genetic Algorithm to solve $MS - PS_{m,\sigma,\rho}|prec|\gamma = curve$. The authors considered the iMOPSE dataset to evaluate the effectiveness of the algorithms, comparing the results with those of NSGA-II and Differential Evolution hybridized with Greedy Algorithm.

Nemati-Lafmejani *et al.* [67] developed dual-objective optimization methods, NSGA-II, and Multi-Objective Particle Swarm Optimization to solve $MPS_{m,\sigma,\rho;\mu,\tau,\omega} - CS|prec|\gamma = curve, \gamma = TD$, and compared them using the PSPLIB test set. Chaleshtarti *et al.* [68] developed a Hybrid

Genetic and Lagrangian Relaxation Algorithm to solve $PS_{m,\sigma,\rho}, \alpha_2 = T\left|prec\right|C_{max}$. Yuan *et al.* [69] developed a Hybrid Cooperative Co-evolution Algorithm to solve $M - MPS_{m,\sigma,\rho;\mu,\tau,\omega}\left|prec\right|\gamma = curve$. The algorithm performed exceptionally well in terms of robustness when compared to the Genetic Algorithm, Particle Swarm Optimization, and hybrid multi-objective version of the Estimation of Distribution Algorithm. Chu *et al.* [70] introduced a metaheuristic recommendation model to address $MPS_{m,\sigma,\rho;\mu,\tau,\omega}\left|prec\right|C_{max}$, which can select the most suitable solution algorithm based on the problem characteristics. The authors used PSPLIB as the testbed to evaluate the performance of the developed solution method, considering various practical scenarios. Yuraszeck *et al.* [71] proposed a new constraint programming model to address the problem of $M - MPS_{m,\sigma,\rho;\mu,\tau,\omega}\left|prec\right|C_{max}$. The authors considered a total of 321 test instances from five earlier studies and compared the results with the best-found lower bounds reported by Dauzère-Pérès *et al.* [72].

Overall, the key branch articles focused primarily on developing case-specific project scheduling variants. Multi-mode RCPSPs and the differentiation between renewable and non-renewable resources became prevalent in the key branches. The most recent studies explored multi-skill RCPSPs and project scheduling that considers green factors. Hybridization played a significant role in key development branches of RCPSP algorithms.

3.3 Major thematics

Fig. 3 provides an overlay view of the most frequently occurring keywords over time. Larger nodes represent words with higher frequencies of occurrence. The closer the connection between the two nodes is, the stronger their correlation. Nodes with more recent occurrences appear in brighter colors, with yellow indicating the most recent ones. This visualizes the shift in focus within the literature over time.

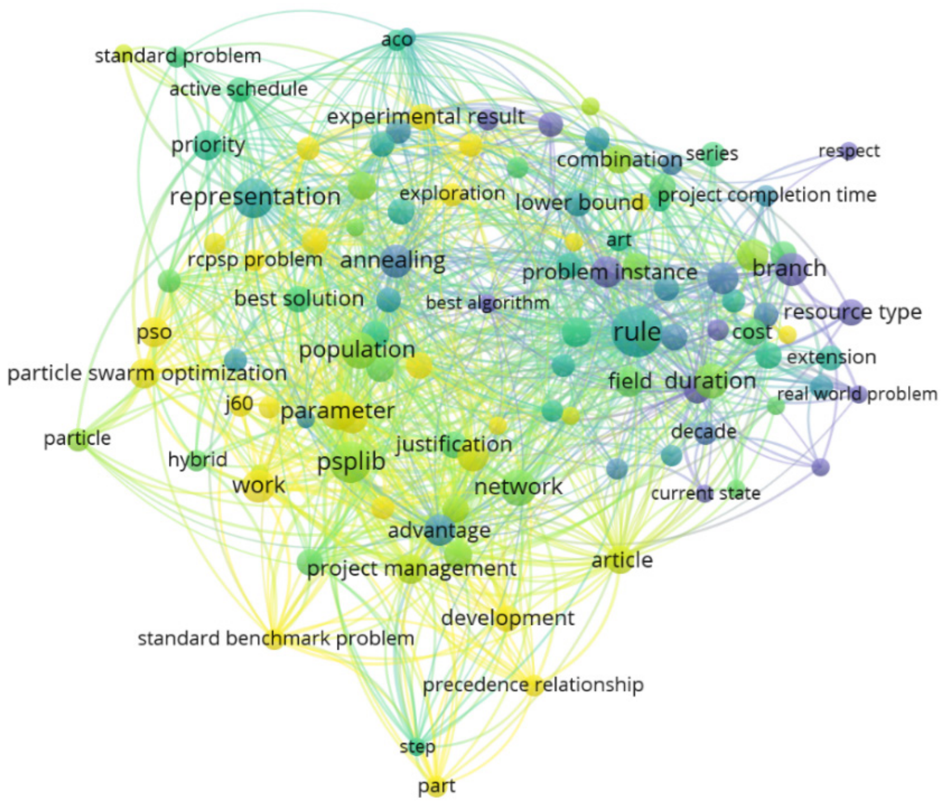
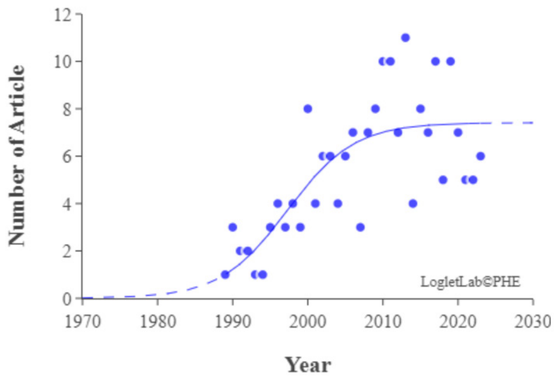
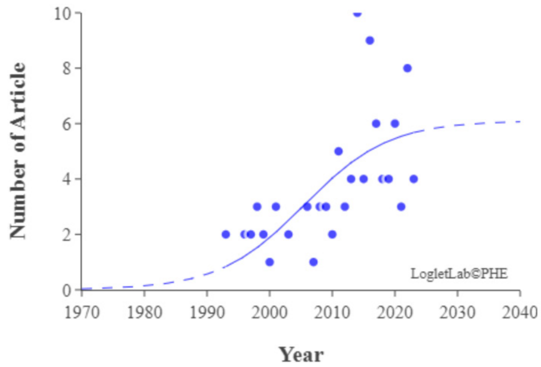
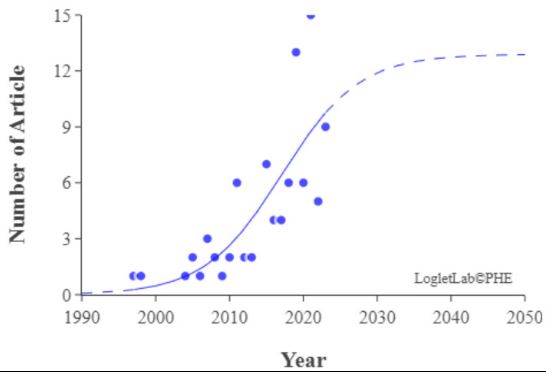


Fig. 3 Overlay visualization of the keywords network

CA indicates that the literature on RCPSPs can be divided into 23 themes. The top three themes will be considered for further analysis. Each theme is identified based on the proportion of keyword occurrences in the associated cluster. The first theme is *priority rules and heuristic* *algorithms*, which include 191 articles. This research theme experienced significant growth between 1995 and 2015. Methods for modeling and solving *multi-mode RCPSPs*, with 99 articles, constitute the second major research theme. The cluster began in 1993 and continued to grow until 2017. The first two clusters are currently in the saturation stage. Algorithms for *RCPSPs with random activity durations* constitute the third major theme, highlighting its practical relevance with 93 articles. This research theme received recognition only after 2010 and is now in the final years of the growth stage. The growth forecast trends and keyword proportions of these clusters are presented in Table 4. This is followed by a brief analysis of the seminal literature within each cluster.

Table 4 Major research themes

Cluster / Count	Keyword – Occurrence Ratio	Growth trend
Cluster A: *priority rules and heuristic algorithms* /191	Rule – 0.178 PSPLIB – 0.115 Representation – 0.109 Parameter – 0.104 Network – 0.094 Duration – 0.083	
Cluster B: *multi-mode RCPSPs* /99	Experiment – 0.263 Computational result – 0.172 Parameter – 0.162 Multiple execution mode – 0.152 Type – 0.152 Efficiency – 0.132	
Cluster C: *RCPSPs with random activity durations* /93	Baseline schedule – 0.28 Stochastic resource – 0.24 Robustness – 0.2 Cost – 0.19 Computational experiment – 0.17 Heuristic – 0.17	

3.3.1 Priority rules and heuristics for RCPSPs

Fig. 4 illustrates the primary development trajectory within the first cluster. Among the 23 articles recognized as significant development progress in developing priority rules and heuristic algorithms, only four articles are absent from the main path and the key development branches.

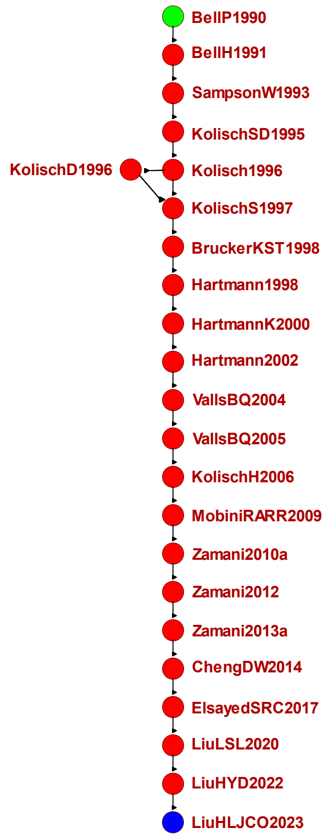


Fig. 4 Progress in the development of priority rules and heuristics for RCPSPs

From the remaining articles, Mobini *et al.* [73] improved the Scatter Search algorithm to solve $PS_{m,\sigma,\rho}|prec|C_{max}$. Using the PSPLIB database, the algorithm was compared with the basic Scatter Search, the hybrid of Genetic Algorithm and Tabu Search with path relinking, and several other versions of Genetic Algorithms. Zamani [74] developed an Accelerating Two-Layer Anchor Search algorithm to solve $PS_{m,\sigma,\rho}|prec|C_{max}$, and used the PSPLIB database to compare its performance with the Local Search algorithm integrated with subproblem exact resolution. Zamani [75] introduced the Polarized Adaptive Scheduling Scheme for $PS_{m,\sigma,\rho}|prec|C_{max}$, which dynamically adjusts the priority of activities in the project. He used the PSPLIB database and Local Search with subproblem exact resolution to evaluate the performance of the new scheme. Elsayed *et al.* [76] proposed a consolidated optimization algorithm, which combines GA and the Multi-operator Differential Evolution to solve $PS_{m,\sigma,\rho}|prec|C_{max}$. Using the PSPLIB databases, they showed that their algorithm outperforms 11 algorithms, including the decomposition-based Genetic Algorithm, the hybrid of Ant Colony Optimization and Scatter Search, Particle Swarm Optimization, Bee Colony Algorithm, and an improved version of the Shuffled Frog-leaping Algorithm.

This research theme dominated scientific progress in the early development stages of RCPSPs. These studies are mainly focused on testing the developed rules and heuristics on simple RCPSPs and establishing the foundations for solution modeling and representation.

3.3.2 Multi-mode resource-constrained project scheduling algorithms

Fig. 5 shows the main development trajectory of solution algorithms to solve multi-mode RCPSPs. In contrast to the first theme, this group of studies did not contribute much to the main development trajectory. The progress in this specific category shows the wide real-world applications of multi-mode RCPSPs. Considering the increase in the scale and complexity of problems, the articles under this theme are focused on improving the efficiency of solution algorithms and calibrating the parameters.

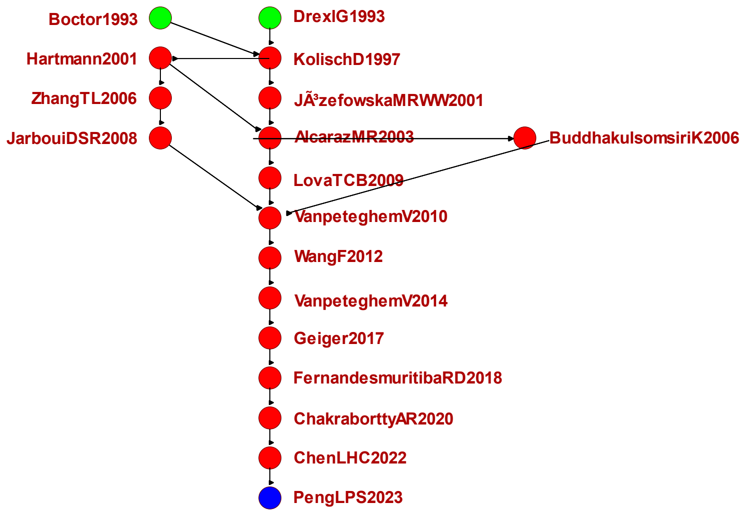


Fig. 5 Progress in multi-mode resource-constrained project scheduling algorithms

Boctor [77] proposed several effective heuristic rules for $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec, \alpha_2 = 1T|C_{max}$. He randomly generated 240 test problems to compare them with 18 other heuristics. Drexl and Gruenewald [49] proposed a novel stochastic scheduling method for the general class of non-preemptive RCPSPs. They used randomly generated test problems to compare their method with the min LF method, which was the state-of-the-art deterministic search method at the time. Kolisch and Drexl [48] cited both source nodes and is recognized as one of the key branch articles. Hartmann [78] extended the Genetic Algorithm to optimize $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$. Their experiments confirmed that the improved algorithm outperforms both Simulated Annealing and Branch and Bound algorithms. Józefowska *et al.* [79] introduced a new Simulated Annealing algorithm to optimize $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$, and used PSPLIB to confirm its effectiveness with and without a penalty function. The results were compared with the Genetic Algorithm developed by Hartmann. Alcaraz *et al.* [80] proposed the Multi-Mode Two-Point Forward-Backward Crossover Genetic Algorithm for $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$. The algorithm performed better than those developed by Hartmann [78], Kolisch and Drexl [48], and the Genetic Algorithm developed by Ozdamar [81].

Buddhakulsomsiri and Kim [82] improved Hartmann's Branch and Bound algorithm and adjusted it so that the activities can be split in $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$. The algorithm was used to compare the project performance in various practical scenarios. Zhang *et al.* [83] proposed the Multimode Particle Swarm Optimization to solve $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$, and compared the algorithm with the current best algorithms such as SA, BB, and so on. The results confirm its superior performance. Jarboui *et al.* [84] extended the Particle Swarm Optimization algorithm with a new local search method for $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$. They used PSPLIB to test it and showed that the algorithm outperforms the Simulated Annealing Algorithm, and the algorithm proposed by ZhangTL2006. Lova *et al.* [85] proposed a multi-mode Hybrid Genetic Algorithm for $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|C_{max}$. They considered the instances generated by the project generator ProGen to compare their algorithm with the Simple Genetic Algorithm as a baseline, as well as those developed by Hartmann [78], Kolisch and Drexl [48], Ozdamar [81], and Bouleimen and Lecocq [86]. Van Peteghem an Vanhoucke [6] proposed the Bi-Population Genetic Algorithm for solving $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec, \beta_1 = pmtn, \beta_1 = pmtn - rep|C_{max}$. The algorithm outperformed the Hybrid Genetic Algorithms developed by Lova *et al.* [85] and Alcaraz *et al.* [80], the extended Particle Swarm Optimization of Jarboui *et al.* [84], and the Hybrid Scatter Search of Ranjbar *et al.* [87].

Wang and Fang [88] adapted the Estimation of Distribution Algorithm to optimize $MPS_{m,\sigma,\rho;\mu,\tau,\omega}|prec|\gamma = \sum sq.dev$. Experimental results based on PSPLIB instances showed that the average percentage deviation of this algorithm is smaller than that of the Simulated Annealing of Józefowska *et al.* [79], the Genetic Algorithms of Alcaraz *et al.* [80], Van Peteghem and Vanhoucke [6], the Hybrid Scatter Search of Ranjbar *et al.* [87], the Artificial Immune System of [89], and the Hybrid Genetic Algorithm of Lova *et al.* [85]. Vanpeteghem and Vanhoucke [90] introduced the Multi-Mode Library, which is a new database for Multi-mode RCPSPs. They compared MMLIB and PSPLIB using different metaheuristics to verify the new instances. Geiger [91] studied $MMPS_{m,\sigma,\rho;\mu,\tau,\omega}, \alpha_2 = 1T\left|prec\right|Cmax_{max}$. He combined Variable Neighborhood Search with the Iterated Local Search algorithm for a Multi-Threaded Local Search Algorithm to solve it. Using the MMLIB database, it was shown that the new algorithm outperforms several state-of-the-art metaheuristics, including those developed by Hartmann [78], Lova *et al.* [85], VanpeteghemV2010, and the Differential Evolution algorithm by Damak *et al.* [92].

Fernandes Muritiba *et al.* [93] proposed a Path-Relinking algorithm to explore the solution spaces of $MPS_{m,\sigma,\rho;\mu,\tau,\omega}\left|prec, \alpha_2 = 1T\right|C_{max}$. Experimental results confirmed that its performance is better than the competing methods proposed earlier, such as the Genetic Algorithms of Lova et al. [85], Vanpeteghem and Vanhoucke [6], the Differential Evolution algorithm by Damak *et al.* [92], and the Estimation of Distribution Algorithms of Wang and Fang [88], among others. Chakrabortty *et al.* [94] proposed a modified Variable Neighborhood Search Heuristic algorithm for $MMPS_{m,\sigma,\rho;\mu,\tau,\omega}, \alpha_2 = 1T\left|prec, \beta_1 = ^o\right|C_{max}$. They used PSPLIB to confirm its effectiveness against adopted versions of the Simulated Annealing, Genetic Algorithm, Particle Swarm Optimization and its discrete variant, Differential Evolutionary Algorithm, Estimation of Distribution Algorithm, and Ant Colony Optimization algorithm. Chen *et al.* [95] developed a Hybrid Genetic Algorithm that combines different priority rules, as well as the series and parallel scheduling methods to solve $MMPS_{m,\sigma,\rho;\mu,\tau,\omega}, \alpha_2 = 1T\left|prec\right|C_{max}$. The experimental results show that this algorithm is particularly effective in reducing the construction period while evaluating the integration with various priority rules. Finally, Peng *et al.* [96] introduced the Hybrid Quantum Particle Swarm Optimization to solve $MS - MPS_{m,\sigma,\rho;\mu,\tau,\omega},, \alpha_2 = 1T\left|prec\right|C_{max}$. The authors used PSPLIB to test, and the experimental results show that the algorithm provides more accuracy and better convergence than the baseline Particle Swarm Optimization algorithm.

Genetic Algorithms have been the cornerstone of the developments in multi-mode resource-constrained project scheduling. Nearly all seminal studies under this research theme included different versions of the Genetic Algorithms in their numerical experiments.

3.3.3 Integrating stochasticity into RCPSPs

The last major theme constitutes the integration of stochasticity into RCPSPs. Fig. 6 shows the backbone of scientific progress in this research theme, where the studies are predominantly concerned with uncertain time parameters and resource availability. Developing robust solution methods to minimize the expected project completion time and makespan has been at the center of attention in the development of this theme.

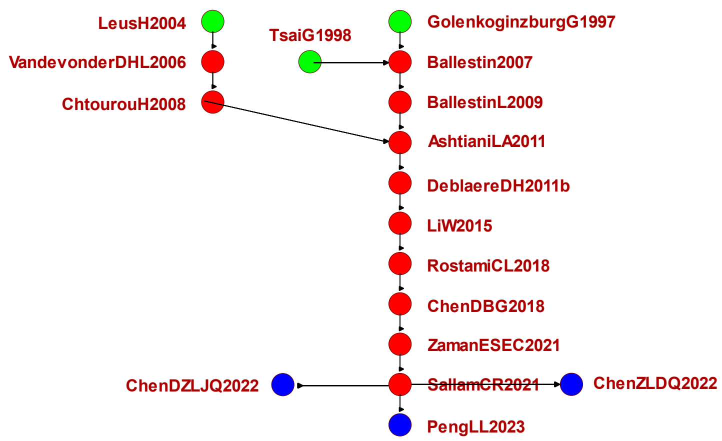


Fig. 6 Scientific progress in the integration of stochasticity into RCPSPs

Golenko-Ginzburg and Gonik [97], Tsai and Gemmill [98], and Leus and Herroelen [99] are the source articles in the development trajectory of this research theme. Golenko-Ginzburg and Gonik [97] proposed a heuristic method for $PS_{m,\sigma,\rho}|\beta_4 = \tilde{d}_j|\gamma = EPD$ and tested it by considering various possibility terms. Tsai and Gemmill [98] developed a Tabu Search algorithm to enhance the computational performance when solving $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|C_{max}$. They tested the method by considering various combinations of heuristics. Leus and Herroelen [99] proposed a resource allocation model for $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|C_{max}$. They used the RanGen platform to generate test instances and improved the Branch and Bound algorithm developed by Demeulemeester and Herroelen [50] to test their method, comparing it with insertion techniques for both online and offline RCPSPs.

Vandevonder *et al.* [100] developed a new heuristic method to solve $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|C_{max}$. The instances used in their numerical experiment were generated using RanGen software to analyze the impact of the weighting parameter, the number of activities, the order strength, buffer sizes, and the resource constraints. Ballestín [101] developed an adaptive Genetic Algorithm to study $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|C_{max}$. The performance of the developed algorithm was compared with Simulated Annealing and Tabu Search algorithms using the ProGen-generated question bank. Chtourou and Haouari [102] developed the Two-Stage Priority Rule-Based Algorithm for investigating $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = av$, mentioning the stability of the project schedule as the main motivation. They conducted a simulation analysis based on the instances of Kolisch *et al.* [103]. Ballestín and Leus [104] developed the GRASP algorithm to solve $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = EPD$. Using instances from PSPLIB, they compared this algorithm with the Genetic Algorithm developed in an earlier study, Ballestín [101]. Ashtiani *et al.* [105] introduced a novel scheduling policy, namely the Preprocessor Polic for $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = C_{max}$. Using instances generated by the ProGen data generator and RanGen, they compared this policy with an activity-based (priority) policy using Genetic Algorithms.

Li and Womer [106] developed an Approximate Dynamic Programming algorithm with Hybrid Backward and Forward Approximation for $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = C_{max}$. This type of solution algorithm is particularly useful for scheduling highly uncertain and variable projects. The authors tested their method using Patterson's benchmark and PSPLIB instances and compared the algorithm with the GRASP algorithm developed by Ballestín and Leus [104]. Rostami *et al.* [107] proposed a new strategy and a new two-stage heuristic algorithm for $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = C_{max}$. Instances from the PSPLIB library were used to compare the two-phase metaheuristic procedure with the Estimation of Distribution algorithm by Wang and Fang [88], the Genetic Algorithm of Ashtiani *et al.* [105], as well as the GRASP method of Ballestín and Leus [104]. Chen *et al.* [108] developed five new priority rules and compared them with twelve from the literature while solving $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = C_{max}$. They considered instances from PSPLIB with three different probability distribution functions for activity durations. Zaman *et al.* [109] developed the Scenario-based Combined Optimization Algorithm by integrating the Multi-operator Genetic Algorithm with the Multi-operator Differential Evolution to solve $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = C_{max}$. The algorithm was compared with Genetic Algorithm of Ballestín [101], GRASP of Ballestín and Leus [104], the Two-phase Genetic Algorithm of Ashtiani *et al.* [105], and the Estimation of Distribution Algorithm of Fang *et al.* [110], while using PSPLIB instances. Sallam *et al.* [111] developed a Reinforcement Learning-Based Multi-Method Approach, which is a hybrid of the Multi-Objective Genetic Algorithm and Differential Evolution, to solve $PS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|\gamma = C_{max}$. They considered three different industrial cases along with instances from the PSPLIB to showcase its effectiveness compared with the enhanced local search heuristic of Chakrabortty *et al.* [94], the Genetic Algorithm of Ballestín [101], the GRASP of Ballestín and Leus [104], the Two-phase Genetic Algorithm of Ashtiani *et al.* [105], and the Approximate dynamic programming of Li and Womer [106], and the optimization algorithm developed by Zaman *et al.* [109]. This is a seminal work on applications of machine learning in project scheduling.

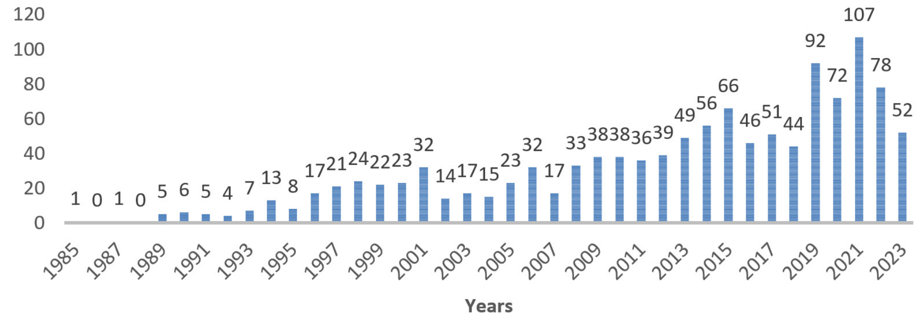
Chen *et al.* [112] proposed a Hierarchical Hybrid Filtering Genetic Programming method that regulates the scope and attributes of priority rules to solve $MPPS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|C_{max}$. They used the test instances from PSPLIB and considered a set of traditional priority rules to showcase the superiority of their algorithm. Chen *et al.* [113] developed a Hyper-heuristic-based Two-Stage Genetic Programming Framework to solve $MPPS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|C_{max}$. They considered a benchmark of 1,000 instances based on PSPLIB to compare their method, using SPEA2 and NSGA-II as the evaluation methods, as well as the traditional priority rules as the baseline. Peng *et al.* [114] introduced the Proactive-Reactive Scheduling Algorithm, which combines a novel scheduling strategy with CPM to study $CCPS_{m,\sigma,\rho}|prec, \beta_4 = \tilde{d}_j|C_{max}$. They used the datasets J30, J60, and J120 from the PSPLIB to evaluate their algorithm against the Approximate Dynamic Programming algorithm of Li and Womer [106] under various settings.

Overall, studies under this cluster paid special attention to establishing a balance between computational stability, efficiency, and solution quality, especially when dealing with the randomness of activity durations, interruptions, and other uncertainty issues. Robust solution algorithms were at the center of scientific progress under this research theme.

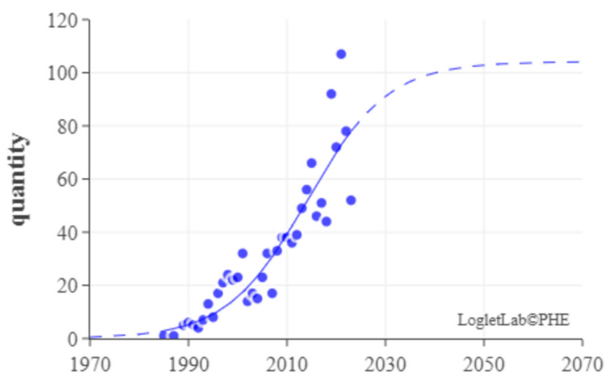
4. Concluding remarks and future research

This study comprehensively reviewed the resource-constrained project scheduling articles published between 1980 and 2024. The development paths and milestones in knowledge dissemination in project planning research were analyzed systematically. It was found that the impactful research directions gradually shifted from basic scheduling models and priority rules to multi-modal, multi-objective problems considering practical research constraints. The most recent focus shift of RCPSPs is towards dynamic scheduling. Practical aspects of resource uncertainty and multi-project coordination have also gained attention in recent years.

Research on RCPSPs has risen steadily, reaching a record high in published articles in 2021 (see Fig. 7a). Investigating the past development trajectory, including the maximum records, the starting point of the growth curve, and the turning point from growth to saturation indicates continuing growth until the late 2030s (see Fig. 7b).



a) The historical trend on the scientific progress of RCPSPs



b) An estimation of the future growth

Fig. 7 Historical trends and an estimation of the future development trend using a Logistic model

The less tangible patterns of development in the literature on RCPSPs were also explored. It was found that solution methods based on network structure analysis and dynamic programming approaches have received limited attention and therefore require further development. Overall, project planning requires a fresh multidisciplinary perspective to enrich task scheduling and extend mathematical applications by incorporating real-life needs and features into RCPSPs. More specific suggestions follow for future research directions.

- Process mining and dynamic scheduling. This allows for real-time adjustment of tasks, considering changes in both tangible parameters and intangible operational patterns.
- Changes in resource availability driven by machine failures, maintenance, and employee shiftwork require more attention. Additionally, resource conflicts and multi-agent systems in multi-project management require new coordination strategies.
- Standard datasets considering multiple heterogeneous projects and projects with dynamic resource allocation are needed to provide a more diverse test platform for advances in solution algorithms.
- Learning from large datasets to improve the search in solution spaces. Machine learning-based solution algorithms are required to balanced computational efficiency and solution quality.
- Interactive multi-objective optimization schemes to balance conflicting objectives in real-time. Minimizing carbon footprints in large-scale projects is a prime example of an objective that conflicts with financial considerations.
- Finally, we think that some of RCPSPs features reviewed in this article can inspire new directions for future research in the production scheduling literature. Taking the dual-resource-constrained flexible job-shop scheduling problem [115] as an example, differentiating between renewable and non-renewable resources may be of interest to be able to account for additional practical features in the optimization process.

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