A Comprehensive Study of Wavelets and Artificial Intelligence Algorithms for SHM and its Application to Concrete Bridges

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Abstract. In this paper, we present a comprehensive study of wavelet theory with a focus on structural damage detection. A new example of the application of operational modal analysis (OMA) techniques to a concrete railway arch bridge located over the Kalix river in Långforsen, Sweden is presented. Results from the OMA techniques are used for finite element model (FEM) updating of this concrete railway arch bridge. Further, a new case study for sensor placement on Herøysund bridge located in Nordland, Norway to conduct OMA is discussed in detail. Moreover, artificial intelligence algorithms that can be useful for addressing the problem of missing data sets in structural health monitoring technologies are reviewed.

Keywords: Structural health monitoring, Vibration analysis, Operational modal analysis, Damage detection, Bridge, Arctic conditions, Finite element model, Signal processing, Wavelet transform, Multi-resolution analysis, Artificial intelligence, Neural network, Wavelet network and Statistical methods.

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INTRODUCTION

As described in the previous article [1], bridge infrastructure in Norway is in a stressed situation due to various challenges. Quite recently, in August 2022, the Tretten bridge catastrophically collapsed in Norway, where a truck and a car became stuck. This bridge is located across the Gudbrandsdalslågen river in the Oyer municipality. Luckily, there have been no causalities (see [2]). In another incident in May 2022, a bridge in Kvænangen municipality, Nord-Troms had serious damage, leading to the closure of a bridge over the important E6 European road [3]. More research is needed in this area so that precautionary measures can be taken to prevent such incidences in the future.

The discussions in the paper [1] gave rise to the notion of writing this paper. The idea to do this paper originated during the discussions in papers [4], [5] and [6], where we emphasized the significance of utilising OMA techniques for structural health monitoring (SHM). In this paper, we do a comprehensive study of wavelet theory with a focus on structural damage detection. Moreover, we present an example where the results from OMA of a concrete railway arch bridge located over the Kalix river in Långforsen are used for FEM updating. In this paper we also present a new case study for sensor placement on Herøysund bridge located in Nordland, Norway to perform OMA.

This paper is organized as follows: In Section 2, some basic definitions of wavelet theory are given and an overview of using wavelets for structural damage detection is described, along with the idea of wavelet networks to resolve some of the challenges that arise in this process. In Section 3, a new example of the application of OMA techniques to a concrete railway arch bridge located over the Kalix river in Långforsen is presented. Results from the OMA techniques are used to develop an FEM-updated model of the bridge. This section also includes a new example for the sensor placement on Herøysund bridge. Further, Section 4 is dedicated to artificial intelligence algorithms that can be useful for addressing the problem of missing data sets in SHM technologies. Finally, in Section 5 we present some concluding remarks.

WAVELET THEORY AND APPLICATIONS

Wavelet transforms are one of the most important transforms that have been developed in order to overcome the shortcomings of the Fourier transforms. The basic concepts of wavelet theory that are crucial for the applications discussed in this paper can be found in (see [7], [8]), and the appendix of the paper [1]. More detailed studies can be found in the book [9]. With the use of the wavelet transform, one can cut data, functions, or operators into different frequency components with a resolution that is matched to their scale. The continuous wavelet transform (CWT) W for a time signal $x(t)$ is defined as follows:

$$W(s,\tau) = \int_{-\infty}^{\infty} x(t)\psi(s,\tau,t)dt, \tag{1}$$

where the wavelet function $\psi(s,\tau,t)$ includes a scaling variable s, a translation variable τ and the time t. Starting with a mother wavelet $\psi_0 = \psi_0(u)$ with the property $\int \psi_0(u)du = 0$, the wavelet building blocks $\psi = \psi(s,\tau,t)$ are defined by (see e.g. [9])

$$\psi(s,\tau,t) = \frac{1}{\sqrt{s}}\psi_0(\frac{t-\tau}{s}). \tag{2}$$

The amplitude of the wavelet function $\psi(s,\tau,t)$ can be controlled by the variation in the scaling variable s, whereas the translation factor τ controls the location of the wavelet function in time. Therefore, depending upon the specific problem, wavelet transforms can be designed accordingly to fit the specific problem.

Since the building blocks ψ builds an orthonormal system (only for the discrete wavelet transform), the general Fourier theory is applicable; in particular, the inverse transform W^{-1} of W exists. This gives the re-construction of the signal $x(t)$ is defined as follows:

$$x(t) = C_\psi \iint_{-\infty}^{\infty} W(s,\tau)\psi(s,\tau,t)dsd\tau. \tag{3}$$

The wavelet transform described above is an integral transform which can be compared with the usual Fourier transform. There is also a discrete wavelet transform, which in a sense, is comparable with the usual Fourier series (for further details, see also Appendix in [1]). Both are very important for various types of applications, e.g. those discussed in this paper.

In the following sub-sections we cover the topics of damage detection methods in structures, data compression, and multi-resolution analysis that are based on the further developments of wavelets, which are relevant to this paper.

Wavelets in structural damage detection

Most of the signals in structural damage detection methods are time-based signals that are recorded by the sensors. Structures vibrate as a result of natural or artificial excitations, such as earthquakes, wind, or artificial excoriations. The output signals from these vibrations, such as accelerations, strains, or displacements, can be recorded. These signals have a non-stationary nature, which means that with time, their features change. But these signals can contain lots of information, which can be useful for stating the health of the structure. Mostly, the signals in the time domain are converted to the frequency domain because damage identification is more effective in the frequency domain (see [10]).

Windowing the signal can provide the time localization with Fourier transforms, but since the window lengths are the same, it does not matter what the frequency component in the signal is. On the other hand, depending on the frequency components, wavelet transforms enable varied time resolutions. In contrast to Fourier Transforms, this results in high accuracy in numerical differentiation and flexible implementation of boundary conditions(see [10]).

Moreover, Fourier sine and cosine functions are not localized in space whereas the wavelet functions are localized, which makes such functions using wavelets "sparse". This feature makes many functions and operators using wavelets “sparse” (see [7]). In applications like data compression, feature detection, and noise removal from temporal signals, wavelets thus become very helpful tools. Wavelets are extremely helpful for non-stationary and non-linear problems that arise in the identification of structural damage since there are no strict limitations on frequency and time resolution.

For example, wavelet transformations are a useful approach for detecting structural degradation (see [11] and [12]). Using wavelet transformations, modal parameters for a structure like mode frequency and modal damping can be calculated (see [13]). The authors of [14] used wavelet transform of the vibration signals from damaged and undamaged structures , and it was possible to detect and locate the damage.

Another study by the authors employed continuous wavelet transform (CWT) and discrete wavelet transform (DWT) techniques to find damage to a tall airport traffic control tower caused by seismic activity. It was found that CWT could detect seismically caused damage even if the vibration signals had noise in them. In contrast, DWT is more effective than CWT at detecting changes in the stiffness of a structure despite being more sensitive to noise.

In the direction of damage detection using wavelets, researchers around the globe are working to develop new methods based on wavelets such as adaptive wavelet transform (AWT) (see [15] and [16]), synchrosqueezed wavelet transform (SWT) (see [17], [18] and [19]), and the empirical wavelet transform (EWT) (see [20]). Brief descriptions of these new wavelet transform methods along with examples of applications on structures can be found in [6].

Moreover, continuous development is needed for the development of new mathematical methods based of Fourier theory that can address the challenges that appear in signal processing for damage detection problems in civil engineering infrastructure (see [21], [22], [23] and [24]) especially under arctic conditions (see [4]). A significant amount of data is produced in the field of structural damage detection from numerous sensors that are used to look for damage. As a result, a lot of data storage space and processing power are needed. We use wavelet theory to address this problem in the subsection that follows.

Wavelets in data compression

The Fourier transform is an integral transform, which means that it is defined as the integral of the transform function over the entire time or space domain (see [6]). As we have seen, the continuous wavelets are integral but consist of other types of building blocks, which gives great advantages for various applications. The wavelet transform is also reversible, which means that the original signal can be reconstructed from the wavelet transform by taking the inverse transform. In addition, the wavelet transform is localized in time and space. This means that the transform is zero outside of a small neighborhood around the point in time or space where the transform is being evaluated. This property is especially useful for compression, since it allows us to focus on the local features of the signal. In contrast, the Fourier transform is non-local since it is defined over the entire time or space domain. As such, wavelets have an inherent sequential property, which makes them useful for handling time series. Another strength is that the wavelet structure can be directly related to the shape of the underlying features of a signal.

Multi-resolution analysis

The DWT described earlier in this section is a multiresolution, band-pass representation of a signal. Wavelet multiresolution analysis(MRA) is a technique that mathematically describes the transition between information density levels. Transitioning between levels reduces or increases the information density. As such, it can be used to reduce the number of measurements required to represent a signal, which basically amounts to compression. Wavelet neural networks (WNN) are a type of artificial neural network (ANN) that uses Wavelet MRA instead of sigmoid activation functions to represent the weights of the network.

As described earlier, wavelets are well known for data compression. Yet for many applications, such as image processing, a multi-resolution representation of the data is required. A multi-resolution representation consists of a set of data representations at different scales, or resolutions. A representation at one scale is generated by low-pass filtering the data at a coarser scale. The multi-resolution representation can then be used to recover the original data by interpolating between the representations.

Wavelets are, as such, a mathematical representation of a signal in which it is represented as a set of basis functions. Each basis function can be defined by a set of coefficients. Each set of coefficients can be obtained by the discrete wavelet transform for the corresponding dyadic dimension (see [25]).

Wavelet networks

Earlier work has shown that wavelet MRA can be used to perform multiscale analysis of time series data. Multi-scale analysis is a set of methods for studying the statistical properties of a signal at different scales. Different scalings are often provided by decomposing a signal into a set of wavelet components. This representation can be used as a replacement for the sigmoid activation functions in neural networks (see [26]).

The basic model of a wavelet network takes form as a three-layer network, with an input, middle (hidden) and output layer. The computing units of the hidden layer are referred to as wavelons, as a contrast to neurons in regular neural networks. These computing units dilate and translate the input variables using the mother wavelet.

The equation $f(x) = \sum_i w_i \phi_{n_i}(x)$ describes how the wavelet MRA decomposition provides the weights w_i for the network. These weights are changed when the network learns, or adapts, to the training data. The wavelet basis is thus modified according to the training data.

The result is that the weights of the network are intrinsically tied to the signal it is representing (see [27]). The output of the network can be mathematically represented as

$$g_\lambda(\mathbf{x};\mathbf{w}) = \hat{y}(\mathbf{x}) = \omega^{[2]}_{\lambda+1} + \sum_{j=1}^{\lambda} \omega^{[2]}_j \cdot \Psi_j(\mathbf{x}) + \sum_{i=1}^{m} \omega^{[0]}_i \cdot \chi_i \tag{4}$$

where $\Psi_j(\mathbf{x})$ is a multidimensional wavelet which is constructed by the product of m scalar wavelets, $\mathbf{x}$ is the input vector, m is the number of input nodes, while λ is the number of hidden units and ω are network weights, or trainable variables. The notations [0] and [2] denote the input and output layers, respectively. For further details, see Figure 1 in [27]. The wavelet network is then trained using back-propagation.

A lot of research is going on in the field of wavelet transforms for damage detection. Vibrational data from accelerometers is used to estimate parameters like mode shapes, mode frequency and modal damping. In SHM of structures, OMA approaches are utilized to identify the modal characteristics of the structure (see [28], [29] and [5]). In the following section, we present a new example of operational modal analysis (OMA) that is used for finite element model (FEM) updating of a concrete railway arch bridge.

APPLICATION OF OMA TECHNIQUES ON CONCRETE BRIDGES

Operational Modal Analysis (OMA) represents a methodology for extracting modal parameters of structures under their natural operating conditions [28]. This technique stands in contrast to traditional experimental modal analysis methods [29], which necessitate external excitation to induce structural vibrations. OMA's reliance on ambient vibrations, such as those from traffic or wind, renders it particularly advantageous for analyzing large-scale or complex structures where controlled excitation is either impractical or economically unfeasible. OMA is based on the assumption that the structure under investigation behaves as a linear, time-invariant system. The ambient vibrations acting as input to the structure are considered stochastic, and the structural response to these inputs is recorded. The primary objective is to deduce the structure's modal parameters—namely, natural frequencies, damping ratios, and mode shapes—from these response measurements.

OMA has found extensive application in the realm of structural health monitoring, where continuous or periodic assessment of a structure's integrity is crucial. It is instrumental in validating structural designs and finite element models, ensuring that the practical behavior of structures aligns with theoretical predictions. Furthermore, OMA facilitates the identification of potential structural deficiencies, guiding maintenance and retrofitting decisions.

In this section we present two case studies of the application of OMA techniques on concrete structures. In the first case study results from the OMA techniques are presented for the Långforsen outside Kalix in Sweden. Further in the second case study an emphasis is made on the sensor placement at Herøysund bridge in Norway for OMA.

Application of OMA on Långforsen bridge in Sweden

In this section, we describe some modal analysis results for a concrete railway arch bridge. The bridge is situated over Långforsen outside Kalix in Sweden. It is a 177 meter long and 60 meter high bridge, built in 1960 (see Figure 1). The bridge's owner, Trafikverket, was interested and wanted to increase the allowed speed and also wanted to increase the maximum allowed axis load from 225 to 300kN. Therefore, vibration measurements were done, as well as measurements of strain and displacements with trains passing at different speeds.

Design and construction

The railway arch bridge over the Kalix River at Långforsen has a length of 177.3 m. The central arch is 89.5 m, while the two side spans are 42 m each. The free spans in the bridge have lengths of 13.0 + 12.8 +12.6 + 87.92 +12.6 + 12.8 + 13.0 m = 164.7 m. The bridge was built in 1960. The arch of the bridge has a reinforced concrete slab via underlying longitudinal and transversal concrete beams that are connected through fixed columns. A reinforced concrete hollow box girder with two hollow chambers makes up the arch. The cross section is highest at the connection to the arch abutment and lowest at the arc's crown. The actual train load corresponds to the locomotive's axle load of 250 kN and a distributed load of 72 kN/m.

Measurement plan

It was unclear whether there was sufficient wind to excite ambient vibrations sufficiently for modal analysis. Therefore, before each measurement, a T43 ra 240 railway engine was driven across the bridge at speeds ranging from 35 to 63 km/h. The engine's weight, which is divided among its eight wheels, is 72 tonnes, while its dynamic weight is 79 tonnes. To have the same linear system (i.e. bridge alone) as in our FE models and to remove nonlinear effects, such as noises from the wheels clattering against the rails and bridge terminals clattering against the foundation, the measurements started after the engine passed the bridge.

During the measuring days, the wind suddenly decreased, which steadily reduced the signal-to-noise ratio (SNR).

FIGURE 1. The concrete arch railway bridge over Kalix river in Långforsen.

Although the SNR was not as good as in earlier measurements from 2009, there was still a clear consistency between the modeled and measured modal data (mode shapes and frequencies).

The Triaxial Colibrys SF 3000L accelerometers were securely fastened to the bridge using expander bolts. Accelerometers were connected with six-wire twisted pair cables to an MGCplus data acquisition system with an ML801B amplifier module. All measurements were made at a sample rate of 1200 Hz. The six-parameter calibration approach described in [30] and [31] was used to calibrate the measurements. Figure 2 shows the measurement plan for the placement of accelerometers over the railway concrete arch bridge.

The Stochastic Subspace Identification (SSI) method was used to carry out modal analysis in the ARTeMIS 4.0 software.

Finite element model updating results

To employ structural control and SHM techniques, the FEM must be highly accurate. The type of FEM used to represent the structural members and the attributes given to these elements both affect how accurate the model is. The FEM of the railway arch bridge over the Kalix River at Långforsen along with its dynamic properties is presented in [32]. The boundary conditions, geometry, or material qualities that change as a material ages are not precisely determined by the finite element model. Depending on the loading conditions, material qualities might cause non-linearity. As a result, the FEM model needs to be calibrated using data from actual structures. The key parameters in the finite element model of the structure are calibrated to minimize the smallest feasible discrepancy between the observed vibrations and the simulated vibrations using a numerical optimization technique called FEM updating [5].

Over the years, a lot of effort has been put into developing various FEM models. Two different bridge model types were created using Abaqus/Brigade in 2011: A comprehensive model with foundations (Type I) and a simplified beam element model where the foundations have been exchanged for springs (Type II). The advantage of the earlier model is that the anticipated results from it could be more reliable and closer to the "actual results," but the drawback is that the scale of the problem is quite large. Type II has 47,438 components compared to Type I, which has 93,910. Moreover, Type II has 282,808 variables in comparison to the 438,800 variables in Type I.

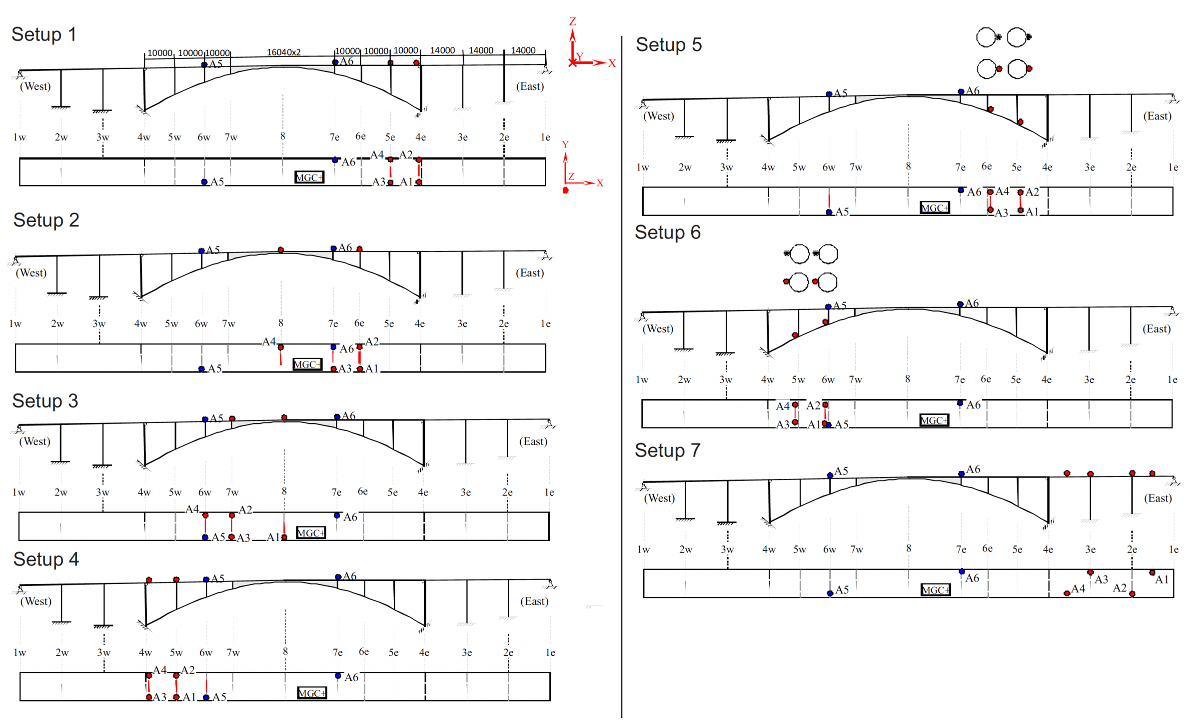

FIGURE 2. Measurement plan showing the placement of accelerometers for the concrete arch railway bridge across the Kalix river in Långforsen.

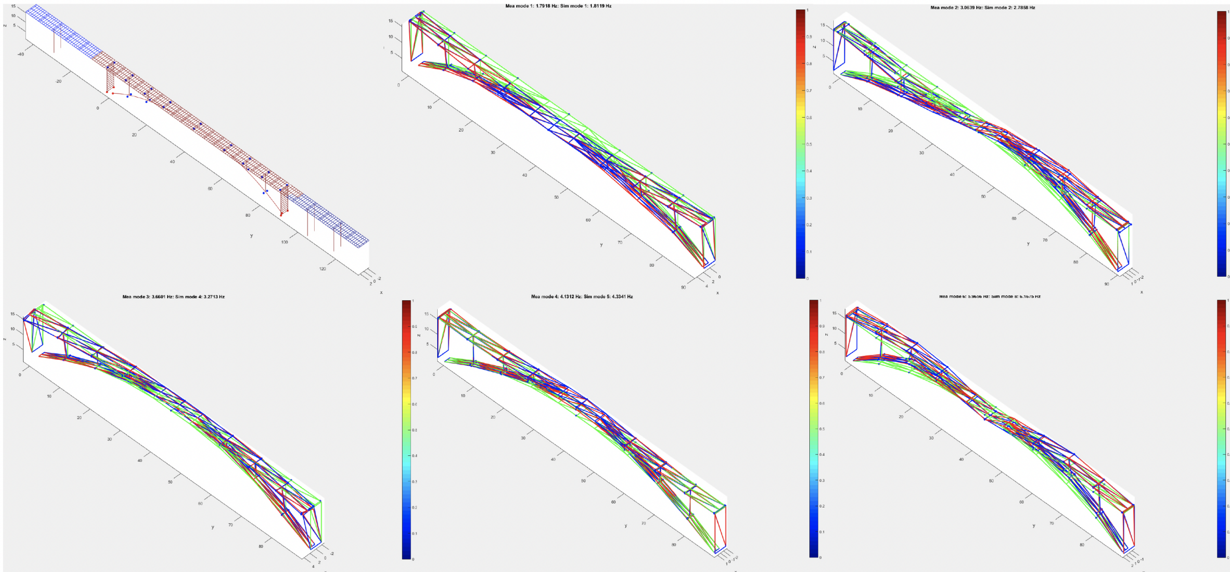

FIGURE 3. FEM updating results for the concrete arch railway bridge over the Kalix river in Långforsen

The Långforsen Bridge's FEM updated model results are shown in Figure 3. It can update the elasticity modulus and boundary conditions (which are modeled as springs) at various locations along the bridge.

The primary issues were that the FEM update did not improve the top right bending mode shape in Figure 3. The top right bending mode shape in Figure 3 has a mode frequency of 2.78 Hz, while the measured mode frequency was 3.06 Hz. The beam element model, which was created to reduce size and increase computation efficiency for updating finite element models and dynamic response simulations, may be one of the possible reasons. However, this model had the downside that the arch could not undergo torsion. Therefore, continued refining of the FEM model and FEM updating software is needed for updating shear modulus with some degrees of freedom [33].

Sensor placement on Herøysund bridge for OMA application

Herøysund Bridge, situated in Nordland Fylkekommune (NFK), Norway, serves as a critical connection between the South-Herøy and North-Herøy islands. Located less than 100 km from the Arctic Circle, this bridge was constructed in 1966 and spans a total length of 154 meters, featuring its longest span at 60 meters and a width of 5.3 meters as shown in Figure 4. The bridge's design includes two girders each reinforced with four post-stressed cables anchored at the ends. Additionally, the bridge incorporates a cast-in-place pressure plate near piers 4 and 5, forming a box-like section in these specific areas.

In 2019, significant structural damages were identified in the Herøysund Bridge, prompting an inspection by the relevant authorities. This inspection led to the decision to undertake restoration efforts. However, during these activities, it was revealed that the bridge had sustained severe damage, limiting the feasibility of a full restoration and consequently reducing its load-bearing capacity. In response to this, in 2020, NFK and the Norwegian Public Road Administration initiated plans to construct a new bridge. This proposed structure, to be located just south of the existing bridge, is projected to cost approximately NOK 300 million (equivalent to over EUR 25 million) and is slated for completion in the summer of 2024. More details about the Herøysund bridge and shell based finite element model can be found in the paper [34].

Dynamic Identification

A critical factor in obtaining reliable estimates of modal parameters is the correct positioning of sensors on the structure. To this end, two numerical strategies have been adopted for the numerical modelling of the structure. A conven-

FIGURE 4. Herøysund Bridge located in Nordland, Norway

tional linear shell-type Finite Element Method (FEM) (FEA ltd, 2022) was compared to a more innovative and less computationally demanding Discrete Macro Element Model (DMEM) method. Without any information about the real material elastic properties and the structural behavior of the two half joints at the ends of the bridge, the models helped to predict the dynamic response of the bridge. The study was focused on obtaining reliable modal shapes.

Figure 5 shows with extension to three modes, the two methods carried out similar results in terms of modal shapes. This preliminary analysis facilitated the identification of a sensor layout comprising a total of 32 axes and 20 measurement points. The main idea here was to have a dense sensor array in order to have better estimation and observation of the modal parameters that is mode shape, mode frequency and modal damping. Better estimations of modal parameters will further help to develop digital twin of the Herøysund bridge with more accuracy.

As Figure 6 below shows the 20 measurement points for detecting as much as possible complete modal shapes. It is worth noting that the central part (the post-tensioned arch bridge) was monitored with a higher level of sensor density if compared to the lateral viaducts. The arch bridge plays a crucial role in the dynamic response of the entire structure and it needed a more accurate identification. Acceleration have been acquired under different level of excitations that will be processed and the results will be presented in the upcoming article.

One of the major problems while doing the analysis of concrete railway arch bridge over the Kalix River at Långforsen and similar projects like steel truss bridges (see [5]) and high-rise buildings (see [6]) was the calibration of sensors with respect to seasonal differences between temperature measurements (winter vs summer measurements). Because of extremely low wind conditions, missing data sets were also a problem. One possible way to deal with such problems could be the use of artificial intelligence, as mentioned in our previous article. In the following section, we will describe this in detail and try to implement similar techniques to deal with the problems in the Herøysund bridge which is being instrumented and monitored.

ARTIFICIAL INTELLIGENCE AND THE PROBLEM OF MISSING DATA

The fields of contemporary machine learning and artificial intelligence are fast-moving fields. New models such as transformers, residual networks (ResNet), and Generative Adversarial Networks (GAN) have been quickly introduced to application fields such as SHM. For more details on recent advances, see [1].

Missing and irregular data are a problem in most fields that use real-life sensor data. This can be caused by sensors that are not synchronized, events that happen irregularly or network latency. In contrast, most machine learning

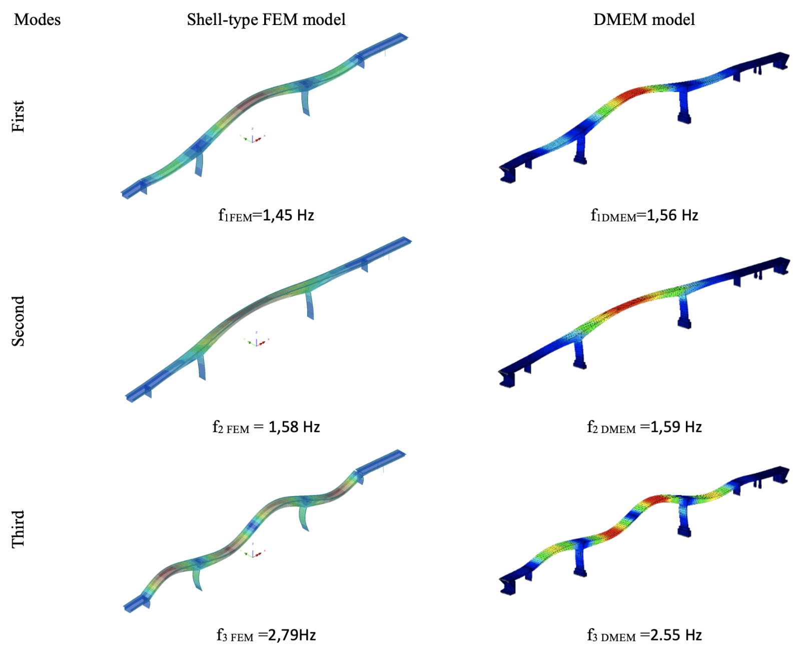

FIGURE 5. Comparison of the results in terms of modal shapes and frequencies of Herøysund Bridge

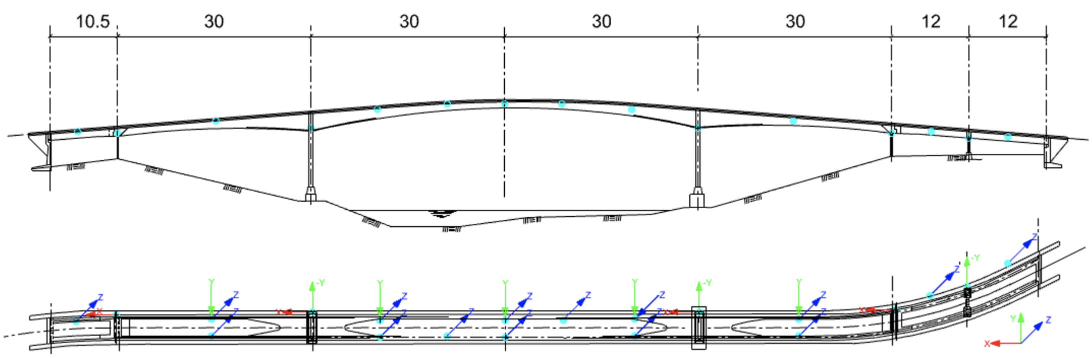

FIGURE 6. Measurement plan of Herøysund Bridge

models expect evenly sampled data. This introduces the need for interpolation or imputation of missing or irregular data. This can, in turn, introduce unwanted bias into the model's latent space.

The SHM and OMA spaces naturally rely on sensor data. As such, the problem is present here (see [35]). In recent years, there has been an influx of new machine learning models trying to handle such data problems. We review those models of special interest for the applications in this paper.

A time series is temporal data collected along the axis of time. This implies that the data is ordered temporally. A time series is usually a real-valued variable. It can be univariate or multivariate, corresponding to one or multiple dimensions, respectively. Time series can be regular or irregular, depending on whether the values of the variables are collected at equal or unequal time gaps. If a time series can be predicted exactly, it is deterministic. If it can only be determined probabilistically, it is stochastic.

Most datasets collected from sensors measuring some real-world phenomena exhibit some form of irregularity. This can have several causes, such as different sample rates, network latency occurring when data is stored, or simply bugs or errors leading to missing or corrupted values. The literature often refers to this as missing or corrupted data.

However, it is clearly a special case of an irregular time series.

Multi-modal data, such as the case with multiple similar or different sensors or data sources, can likewise introduce related problems, such as measurements that are not timed or synchronized. Skewed synchronization can lead to decision-making algorithms misinterpreting correlation or causality between variables or events.

The literature traditionally identifies two distinct strategies for handling irregular time series (see e.g. [36]). Imputation methods, such as interpolation or forward filling, can be used to create a complete time series. This is the more traditional and commonly used approach. Second, and contrarily, are models that inherently handle irregular data.

Statistical methods

Interpolation between irregular time spans assumes that missing data between recorded points behaves predictably. Such an assumption can obviously not always hold. Even small spans of missing data can introduce serious amounts of unpredictable bias. Other common approaches include padding training samples with zero values where data is missing, but this also influences the parameters of an algorithm to varying degrees. The time stamp itself can also be used as an input feature for the machine learning model.

More sophisticated approaches are described in the literature, such as interpolation networks (see [37]). This model uses neural networks to learn the interpolation features for a specific time series in latent space.

Another sophisticated approach is to generate so-called synthetic data. Synthetic data is basically a form of simulated data that can be combined with real-world data. Such data can be advantageous in situations when real data samples are scarce, missing or unavailable. In fact, this approach has been successful in applications such as damage detection in electric grids (see [38]) and self-driving cars (see [39]). In recent years, the aforementioned GAN model has become a benchmark in many fields for generating synthetic data, especially image data augmentation (see [40]). GANs are also fast becoming the preferred generative model for time series (see [41]). Generating data for photovoltaic solar energy generation has also shown to be a very promising method (see [42]).

Gated RNNs

A number of recent developments in gated RNN architectures for time series are reviewed. Gated RNNs, such as LSTM and GRU, still stand as the most resilient and performant models for time series. However, vanilla versions such as LSTM and GRU are not designed to handle irregular data, possibly introducing bias and suboptimality.

t-LSTM

In particular, some architectures are designed to work with irregular time series naturally, without imputation. This design involves modifying the structure of the RNN to accommodate the irregular information supply. In [43], the LSTM memory cell, rather than the forget gate, is modified. The model is thus able to take the time span into consideration, according to the authors, hence the name time-aware LSTM(t-LSTM). The input dynamics of the model are not complex. In addition to the sample itself, another vector includes the elapsed time as a weight transformed by a time decay function.

Phased LSTM

In contrast, a more advanced structural change is presented in [44]. This model, called Phased LSTM, introduces two new oscillatory gates to the vanilla LSTM model, which reduces the number of updates required by the memory cell and hidden state. The authors explicitly state that this architecture is intended to handle irregular sampling rates. The model has been proven to be more effective than vanilla LSTM on long, irregular time series (see [45]).

Bistable recurrent cell

In [46], Vecoven et al. took inspiration from how biological neurons can store information for arbitrary time spans through a process called bistability. This lets the cells themselves remember information from their own past states and inputs. This leads to long-term memory that is potentially very useful for long, sparse, and irregular time series.

ODE-RNN and derivatives

Neural Ordinary Differential Equation(ODE) with an RNN lets the model have continuous-time hidden dynamics. ODE-RNN can naturally handle arbitrary time gaps between observations (see [47]).

The vanishing gradient problem is inherent to the Backpropagation Through Time(BPTT) algorithm, which is used in basic RNNs. As shown in [48], ODE-RNNs also suffer from this shortcoming. The same authors also show that additional gates found in the LSTM can be included to alleviate the problem.

The innovations that came with Neural ODEs (see [47]) are utilized to write the standard GRU as a difference equation (see [49]). This is further combined with a network that updates the current hidden state with incoming observations, making the combined model GRU-ODE-Bayes (see [50]).

Transformer

The performant and efficient Transformer architecture has in recent years outperformed various neural network-based models in many different fields. Transformers have been adopted to model time series and have been shown to be performing in the top tier. As for irregular time series, in [51], a Transformer model was used to compare with their SEFT-Attn model. It was adapted to a classification task by mean-aggregating the final representation, and it was then fed into a Softmax layer to predict classes.

Attention networks

The attention mechanism, usually connected to transformer models, was investigated for irregular time series in [52]. The approach is described as representing the irregular time series at a fixed set of reference points. An encoder-decoder framework is then utilized, such that the encoder produces a fixed-length latent representation over the aforementioned reference points. The decoder then uses the latent space to reconstruct the set of observed time points. The training is done by a Variational Auto-encoder(VAE). The main advantage compared to other cutting-edge techniques is that it gives faster training (1-2 orders of magnitude). This is especially noticeable compared to ODE-RNN methods, since they require an ODE solver.

Set Functions for time series

Recent advances in differentiable set function learning allow for models that are able to handle irregular and unaligned multivariate time series (see [51]). The multivariate time series is encoded as a set. Each data point is treated as having three values; the time it was collected, which variable it belongs to, and its value. As such, the data points lose their sequentiality, but the time information itself is still preserved. Furthermore, the sets are fed into the deep sets architecture with an attention mechanism (see [53]).

CONCLUDING REMARKS

Remark 5.1 The research discussed in this paper is also related to some recent research in more theoretical Fourier analysis [21], [22] and the PhD thesis [54]. We aim to further develop the investigations in this paper as wavelets are unbounded systems.

Remark 5.2 In the forthcoming article, we further aim to develop the theory of wavelet neural networks. We strongly believe that this work will help to address the issue of missing data sets for structural damage detection in structures similar to the concrete railway arch over the Kalix River at Långforsen as discussed in this paper.

Remark 5.3 Several experiments have been conducted on the concrete railway arch over the Kalix River at Långforsen over time. In this paper, we present the results for the finite element model updating in Figure 3. Continuous refining and development of FEM (see [32]) and FEM updating models is needed for updating shear modulus with some degrees of freedom.

Remark 5.4 Sensor placement on Herøysund bridge that is located in Nordland, Norway is described in this paper for performing OMA. In the forthcoming article, we will present our results from the data acquired from this research activity.

Remark 5.5 Multiple promising techniques and models for handling or mitigating missing data have appeared in the AI/ML space recently (see [36]). However, investigating performance requires access to both relevant data and domain-specific knowledge as well as machine learning knowledge. It is thus clear that much work remains in this direction.

Remark 5.6 The research groups at UiT The Arctic University of Norway and Luleå University of Technology Technology are interested in the new research problems that result from this difficult challenge of SHM and OMA in extreme arctic conditions [4] and [1].

Remark 5.7 It is a close connection between wavelet theory and the type of modern Fourier analysis that is described in the new book [55]. We aim to further investigate this fact in relation to the problems described above in a forthcoming paper.

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