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Models for electrostatic drift waves with density variations along magnetic field lines

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[1] Drift waves with vertical magnetic fields in gravitational ionospheres are considered where the unperturbed plasma density is enhanced in a magnetic flux tube. The gravitational field gives rise to an overall decrease of plasma density for increasing altitude. Simple models predict that drift waves with finite vertical wave vector components can increase in amplitude merely due to a conservation of energy density flux of the waves. Field-aligned currents are some of the mechanisms that can give rise to fluctuations that are truly unstable. We suggest a self-consistent generator or "battery" mechanism that in the polar ionospheres can give rise to magnetic field-aligned currents even in the absence of electron precipitation. The free energy here is supplied by steady state electric fields imposed in the direction perpendicular to the magnetic field in the collisional lower parts of the ionosphere or by neutral winds that have similar effects.**Citation:** Garcia, O. E., and H. L. Pécseli (2013), Models for electrostatic drift waves with density variations along magnetic field lines, *Geophys. Res. Lett.*, *40*, 5565–5569, doi:10.1002/2013GL057802.

1. Introduction

[2] In nature as well as many laboratory experiments, we have conditions where the equilibrium plasma density has a gradient *along* as well as *perpendicular* to an externally imposed stationary magnetic field **B**. In the polar ionosphere, in particular, magnetic flux tubes with enhanced plasma density are often associated with auroral precipitation and auroral patches [*Hosokawa et al.*, 2010] where the plasma density has a gradient $\perp$ **B** as well as a decreasing density along magnetic field lines, i.e., increasing altitudes. The case with plasma density gradients perpendicular to **B** has previously been studied, and conditions for electrostatic drift wave instabilities have been established [*Kadomtsev*, 1965]. The basic results are generalized here to allow for a density gradient along the magnetic field as well, using a simple solvable gravitational model. The real and the imaginary parts of the dispersion relation of the waves are modified by this vertical gradient. For kinetic collisionless plasma conditions, we find that the most likely source of enhanced growth rates is a field-aligned electron current. We propose a novel generator (or "battery") mechanism relevant for polar ionospheres.

2. Drift Wave Instability

[3] We consider a horizontally stratified plasma in a gravitational field $\mathbf{g} = -g\widehat{\mathbf{z}}$ and vertical magnetic field $\mathbf{B} = B\widehat{\mathbf{z}}$. The basic equations for the plasma assume cold ions and retain the polarization drift as a correction to the $\mathbf{E} \times \mathbf{B}/B^2$ ion velocity. A steady state solution is found as $\overline{n}(\mathbf{r}_\perp, z) = n_0(\mathbf{r}_\perp)\exp(-z/L_V)$ for the density with a vertical length scale $L_V \equiv \kappa T_e/Mg \equiv C_s^2/g$. For the electrostatic potential, we have $\overline{\phi}(z) = -zMg/e$. With a constant electron temperature T_e, the Brunt-Väisälä frequency [*Yeh and Liu*, 1972] is $\Omega_{\rm BV} = g/C_s$, where $C_s = \sqrt{\kappa T_e/M}$ is the sound speed for cold ions, introducing M as the ion mass. For relevant conditions, we can have $C_s \approx 500$–1000 m s^{-1}, giving $\Omega_{\rm BV} \approx 0.01$–$0.02$ s^{-1}. We have $n_0(\mathbf{r}_\perp)$ accounting for the density enhancement in the magnetic flux tube, while the exponential term $\exp(-z/L_V)$ gives the overall density decrease for increasing altitude z representing a balance between the gravitational force on the ions and the constant vertical ambipolar electric field component $\overline{\mathbf{E}} = (Mg/e)\widehat{\mathbf{z}}$ resulting from the electron pressure. The gravitational field is here used as giving a solvable, yet representative, model for general mechanisms that give rise to plasma density gradients along magnetic field lines. For the ionospheric F region, we can use an approximation with an exponentially decreasing plasma density for altitudes above the F maximum.

[4] Starting from the linearized ion continuity and ion momentum equations with cold ions, $T_i \approx 0$, we obtain after some straightforward algebra an equation for the ion dynamics that gives a dynamic linear relation between the ion density n_i with $\eta \equiv n_i/\overline{n}$ and the electrostatic potential ϕ as

$$\frac{\partial^2}{\partial t^2}\eta - \frac{1}{B\Omega_{\rm ci}}\frac{\partial^2 \nabla_\perp^2 \phi}{\partial t^2} - \left[\frac{\partial \nabla_\perp \phi/\partial t \times \mathbf{B}}{B^2} + \frac{1}{B\Omega_{\rm ci}}\frac{\partial^2 \nabla_\perp \phi}{\partial t^2}\right]\cdot \nabla_\perp \ln n_0 - \frac{e}{M}\frac{\partial \phi}{\partial z}\frac{\partial}{\partial z}\ln \overline{n} - \frac{e}{M}\frac{\partial^2 \phi}{\partial z^2} = 0\,. \quad (1)$$

The set of equations are closed by the assumption of quasineutrality and an equation relating ϕ and η obtained from the electron dynamic equations [*Chen*, 1984]. The simplest such relation assumes electrons to be isothermally Boltzmann distributed at all times. For a plane wave solution $\exp(-i(\omega t - \mathbf{k}\cdot\mathbf{r}))$, we find the dispersion relation $\omega = \omega(\mathbf{k})$ in the form

$$\omega^2\left(1 + k_\perp^2 a_i^2\right) - C_s\omega(a_i\mathbf{k}_\perp)\times\widehat{\mathbf{z}}\cdot\nabla_\perp \ln n_0 - igk_z - C_s^2 k_z^2 = 0\,, \quad (2)$$

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with $a_i^2 \equiv \kappa T_e/(eB\Omega_{ci}) = C_s^2/\Omega_{ci}^2$. We took $\mathbf{k}_\perp \perp \nabla_\perp \overline{n}$. The correction due to the ion polarization drift is found in the $(1 + k_\perp^2 a_i^2)$ multiplier on ω^2. The z variation cancels in the drift frequency defined as $\omega^* \equiv k_\perp(\kappa T_e/eB)|\nabla \ln n_0(\mathbf{r}_\perp)|$, making it independent of altitude. With this standard expression [*Kadomtsev*, 1965] for the drift frequency ω^*, we find

$$\omega(k_\perp, k_z) = \frac{\omega^* \pm \sqrt{\omega^{*2} + 4\left(1 + a_i^2 k_\perp^2\right)\left(C_s^2 k_z^2 + igk_z\right)}}{2\left(1 + a_i^2 k_\perp^2\right)},$$

where we assume $\mathbf{k}$ to be real. We recognize the accelerated and decelerated ion sound waves [*Kadomtsev*, 1965], generalized here by an imaginary part originating from the **B**-parallel density gradient. For upward propagating waves, the relation (2) has solutions which grow exponentially with time. Waves propagating downward ($\omega/k_z < 0$) are damped. For $k_z = 0$, we have $\omega = \omega^*$, while for $k_\perp = 0$, we have $\omega^* = 0$ and the acoustic plasma gravity waves with $\omega = \pm\sqrt{C_s^2 k_z^2 + igk_z}$. The two wave modes are coupled when both $k_\perp \neq 0$ and $k_z \neq 0$.

[5] An initial value problem (with real **k**) as outlined before is relevant for instance for numerical simulations, but for physical applications, for instance in a laboratory plasma, a boundary condition is more appropriate, where now the applied frequency is real and a complex wave vector accounts for spatial damping or growth of the waves. With the boundary condition $\exp(-i\omega t + ik_\perp x)$ at $z = 0$ also this problem can be solved for $k_z(\omega, k_\perp)$ giving

$$k_z(\omega, k_\perp) = -\frac{ig \pm \sqrt{4C_s^2\left(\omega^2\left(1 + a_i^2 k_\perp^2\right) - \omega^*\omega\right) - g^2}}{2C_s^2}, \quad (3)$$

which can be written as

$$k_z L_V = -\frac{i}{2} \pm \frac{1}{2}\sqrt{4(\omega^2(1 + k_\perp^2 a_i^2) - \omega\omega^*)/\Omega_{BV}^2 - 1}.$$

The normalized result (3) is illustrated in Figure 1. The result contains a ratio of length scales that is here assigned a numerical value $\ell/L_V = 0.2$ where $\ell^{-1} \sim \mathrm{d}\ln n_0/\mathrm{d}z$. For $\omega^2(1 + k_\perp^2 a_i^2) - \omega\omega^* < \Omega_{BV}^2/4$, we find a stopband, where the wave has no spatial oscillations along the vertical z direction, only an exponential form that does not correspond to propagating waves with real phase velocity.

[6] The observed wave growth cannot properly be called an instability [*Parkinson and Schindler*, 1969; *Dysthe et al.*, 1975; *D'Angelo et al.*, 1975]. The amplitude increase with increasing altitude z is merely a consequence of the decreasing plasma density: For fluctuating plasma velocities $\mathbf{u}$, the vertical component of the wave energy density flux is to lowest order $\overline{n}(\mathbf{r}_\perp, z)Mu^2C_s$. The sound velocity C_s is independent of plasma density, so to have a constant vertical energy flux, we must have u^2 increasing when $\overline{n}$ is decreasing and vice versa. For $\omega^2(1 + k_\perp^2 a_i^2) - \omega\omega^* > \Omega_{BV}^2/4$, we thus have $\Im\{k_z L_V\} = \frac{1}{2}$, in which case the energy flux density $C_s\overline{n}Mu^2$ is independent of z as stated. Since this is not a proper instability driven by free energy, we use the term "geometric growth." An observer will, however, find that the fluctuation level increases with altitude and might interpret this as an instability. When the wavelength becomes smaller than the vertical length scale L_V, the relative growth rate $\Im\{\omega\}/\Re\{\omega\}$ becomes small, and the waves can be assumed to propagate with the local dispersion relation, corresponding to the appropriate altitude z.

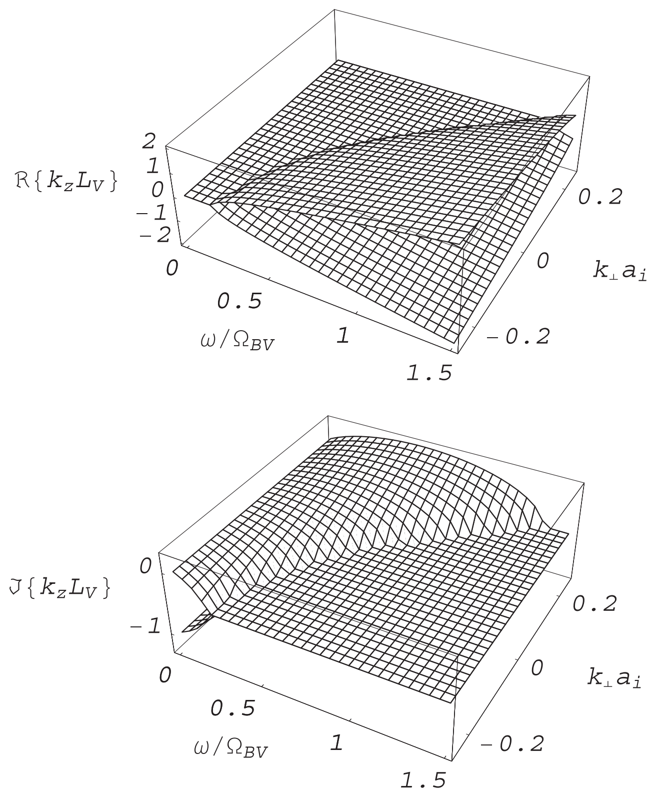

Figure 1. Normalized dispersion relation for real applied frequency ω/Ω_{BV} and complex wave number $k_z L_V$, where $\Re\{\}$ and $\Im\{\}$ denote real and imaginary parts, respectively, of the term in the angular brackets. Negative $\Im\{k_z\}$ corresponds to spatial growth in the positive z direction.

[7] Allowing for a nonvanishing ion temperature, $T_i \ll T_e$, we find in the quasi-neutral limit that ion Landau damping with $g = 0$ gives an imaginary part of the frequency so that the ratio of the real and imaginary parts are constant for varying real wave number, when an initial value problem is considered. For a similar boundary value problem, the ratio of imaginary and real wave numbers is approximately constant for varying real frequency. For the geometric growth discussed for the initial value problem with real wave numbers, we found that the imaginary part of the frequency approached a constant level for large k_z, i.e., $\Im\{\omega\}/\Re\{\omega\} \to 0$ for $k_z \to \infty$. It was demonstrated by *Parkinson and Schindler* [1969] that for stable conditions, the ion Landau damping will dominate for large k_z.

[8] To generate a substantial perturbation consistent with observations [*Greenwald et al.*, 2002; *Hosokawa et al.*, 2010], free energy is needed to drive an instability. Several sources of energy can be recognized as relevant for the present problem. Here we focus on electron kinetic effects. These contributions to the instabilities are important for conditions where the growth rates are comparable to or exceeding characteristic ion collision frequencies. For more general cases, modification due to collisions needs to be included.

[9] In order to include electron kinetic effects, we relax the condition on Boltzmann-distributed electrons and use a drift kinetic electron model [*Kadomtsev*, 1965].

$$\frac{\partial f}{\partial t} + \nabla_\perp \cdot (\mathbf{u}_{\mathbf{E}\times\mathbf{B}} f) + u_\parallel \frac{\partial f}{\partial z} + \frac{e}{m}\frac{\partial \phi}{\partial z}\frac{\partial f}{\partial u_\parallel} = 0, \quad (4)$$

with m being the electron mass, $\mathbf{u}_{\mathbf{E}\times\mathbf{B}} \equiv -\nabla_\perp \phi \times \mathbf{B}/B^2$, and $f = f(z, \mathbf{r}_\perp, u_\parallel, t)$ is here the electron gyrocenter distribution function. The polarization drift is ignored for the electrons. Since the magnetic moment μ_m is constant, within the relevant approximations, μ_m does not appear explicitly in (4).

[10] We find a steady state velocity distribution $\bar{f}$ from

$$-\frac{\nabla_\perp \bar{\phi} \times \mathbf{B}}{B^2} \cdot \nabla_\perp \bar{f} + u_\parallel \frac{\partial}{\partial z} \bar{f} + \frac{e}{m} \frac{\partial \bar{\phi}}{\partial z} \frac{\partial}{\partial u_\parallel} \bar{f} = 0 . \tag{5}$$

We anticipated a constant vertical electric field $\overline{\mathbf{E}} = (Mg/e)\widehat{\mathbf{z}}$ to compensate the (constant) gravitational force on the ions. For steady state, we have $\nabla_\perp \bar{\phi} = 0$. We readily obtain

$$\bar{f}(x, z, u_\parallel) \equiv n_0(x) \exp(-zMg/\kappa T_e) F(u_\parallel) \equiv \bar{n}(z) F(u_\parallel) , \tag{6}$$

where $F(u_\parallel)$ is a Maxwellian with normalization $\int_{-\infty}^{\infty} F(u_\parallel) \mathrm{d}u_\parallel = 1$. Deviations from this form will in general imply modifications of the steady state. Such changes are generally small. We here used a local model for the modifications of the growth rate of the drift wave instability as caused by deviations from strictly isothermally Boltzmann-distributed electrons and considered again the initial value problem with real $\mathbf{k}$. The local model is applicable when $\omega > \Omega_{\mathrm{BV}}$ giving $k_z L_V > 1$, see Figure 1 for $\Re\{k_z L_V\}$.

[11] For a slowly drifting Maxwellian electron velocity distribution with a drift velocity $U \ll u_{T_e}$, where $u_{T_e} = \sqrt{\kappa T_e/m}$ is the electron thermal velocity, we obtain by (4) a relation between plasma density and potential in the form

$$\frac{n_e}{\bar{n}} = \frac{e\phi}{\kappa T_e} \left[\frac{\omega - Uk_z - \omega^*}{k_z u_{T_e}} Z\left(\frac{\omega - Uk_z}{k_z u_{T_e}} \right) + 1 \right] , \tag{7}$$

where Z is the plasma dispersion function. In this kinetic model, (7) is replacing the assumption of Boltzmann-distributed electrons. In, for instance, laboratory experiments, it has been demonstrated that drift waves can be made unstable by currents along the magnetic field lines [*Hatakeyama et al.*, 2011]. To estimate the growth rate under the present conditions, we again assume quasi-neutrality $n_e \approx n_i = n$ and combine (1) and (7) to get

$$\frac{\omega^*}{\omega} - C_s^2 \left(\frac{k_\perp^2}{\Omega_{\mathrm{ci}}^2} - \frac{k_z^2}{\omega^2} \right) + i \frac{k_z g}{\omega^2} = 1 + \frac{\omega - Uk_z - \omega^*}{k_z u_{T_e}} Z\left(\frac{\omega - Uk_z}{k_z u_{T_e}} \right) . \tag{8}$$

[12] The result is local in the sense that the electron drift velocity U is taken to be constant. We have two limiting cases: $\omega \approx 0$ for the slow sound mode and $\omega \approx \omega^*$ for the fast mode. For small ω/k_z and $U \ll u_{T_e}$, we can approximate $Z \approx i\sqrt{\pi}$. For small k_z, we find the previous results with real $\mathbf{k}$ and complex ω for the slow mode (where $\omega \approx 0$) with negligible kinetic effects, while the fast mode (where $\omega \approx \omega^*/(1 + (k_\perp a_i)^2)$) becomes

$$\omega \approx \frac{\omega^*}{1 + (k_\perp a_i)^2} + \frac{C_s^2 k_z^2}{\omega^*} \left(1 + (k_\perp a_i)^2\right) + i \left(\frac{k_z g}{\omega^*} \left(1 + (k_\perp a_i)^2\right) + \omega^* \sqrt{\pi} \, \frac{Uk_z \left(1 + (k_\perp a_i)^2\right) + \omega^* (k_\perp a_i)^2}{k_z u_{T_e} \left(1 + (k_\perp a_i)^2\right)^2} \right) . \tag{9}$$

[13] Kinetic effects give rise to instabilities even when $U = 0$, but the growth rate $\Im\{\omega\}$ is then small and scales with $(k_\perp a_i)^2$. Significant growth rates can be obtained even for moderate bulk electron flow velocities, where waves are unstable in the direction determined by the condition $Uk_z > 0$. The growth rate increases with U with no threshold. The contribution from the kinetic instability, i.e., the second term in the parenthesis in (9), will dominate the first geometric growth term, when $U > u_{T_e} k_z g/(\omega^*)^2$, in the limit of small $(k_\perp a_i)^2$.

[14] We have similar, but more lengthy, results for the boundary value problem with real applied frequency ω and complex k_z as in Figure 1. The stopband is recovered also with these kinetic effects, but the vertical growth rate receives a contribution from a "true" instability.

3. Proposal for a Battery Mechanism

[15] A complete model has to account for electron current generators. Currents can for instance be caused by diffuse auroral electron precipitation [*Ossakow and Chaturvedi*, 1979], but these will generally not be localized at the density gradients. Here we focus on a different and more relevant current-generating mechanism, originating from steady state horizontal electric fields $\mathbf{E}_0$ imposed perpendicular to $\mathbf{B}$ in the ionospheric E region, demonstrating that a magnetic flux tube with enhanced density extending through E and F regions will be unstable with $\mathbf{E}_0 \neq 0$. We introduced the notation $\mathbf{E}_0$ to distinguish this horizontal electric field from the vertical $\overline{\mathbf{E}}$. The electron and ion mobilities are different in the E region since $\omega_{\mathrm{ce}} \gg \nu_{\mathrm{en}}$ while $\Omega_{\mathrm{ci}} \leq \nu_{\mathrm{in}}$ in terms of electron and ion collision frequencies (ν_{en} and ν_{in}) with the neutral background gas. These neutral collision frequencies decrease rapidly with altitude. For altitudes above some 120–130 km, the electrons and ions are both drifting with essentially the same $\mathbf{E}_0 \times \mathbf{B}/B^2$ velocity, see for instance the illustration by *Dyrud et al.* [2006, Figure 3]. We model the E region as a collisional horizontal "slab" of thickness d with $\omega_{\mathrm{ce}} \gg \nu_{\mathrm{en}}$ while $\Omega_{\mathrm{ci}} \leq \nu_{\mathrm{in}}$. For higher altitudes above approximately 120 km, we ignore collisions all together: As what concerns the steady state electron-ion velocity difference, this is a good approximation [*Dyrud et al.*, 2006].

[16] The electrons flow approximately at the $\mathbf{E}_0 \times \mathbf{B}/B^2$ velocity, but the neutral drag on the ions implies that electron and ion steady state drifts differ to give the electrojet current [*Primdahl and Spangslev*, 1977; *Kelley*, 1989]. The model for the current generation is also understood in the $\mathbf{E}_0 \times \mathbf{B}$ moving frame, where the neutral component is in motion. The localized density enhancement can also be moving. To interpret the free energy driving the current as an electric field $\mathbf{E}_0$ or a neutral wind is merely a question of choosing the frame of reference. With the present approximations, the electrojet current density is $J_0 \approx -en_0 E_0/B$ giving a net current $I_0 \approx -den_0 E_0/B$ per length unit along $\mathbf{E}_0$.

[17] If we have a local plasma density enhancement with a steady state density gradient perpendicular to $\mathbf{B}$ with $\nabla_\perp n_0 \parallel \mathbf{E}_0 \times \mathbf{B}$ in the rest frame as in Figure 2, we have a local enhancement of the net current in that region since $\mathbf{E}_0$ is imposed externally to give a constant velocity and the number of charge carriers is locally enhanced. The electron current $I_0 + I_1$ in the enhanced density region ($n_0 + \Delta n$ between the two intervals a and b in Figure 2) is not compensated at the boundaries of the magnetic flux tube with enhanced plasma density: In regions a and b, the current $\mathbf{I}_1$ therefore has to expand along the vertical magnetic field lines as illus-

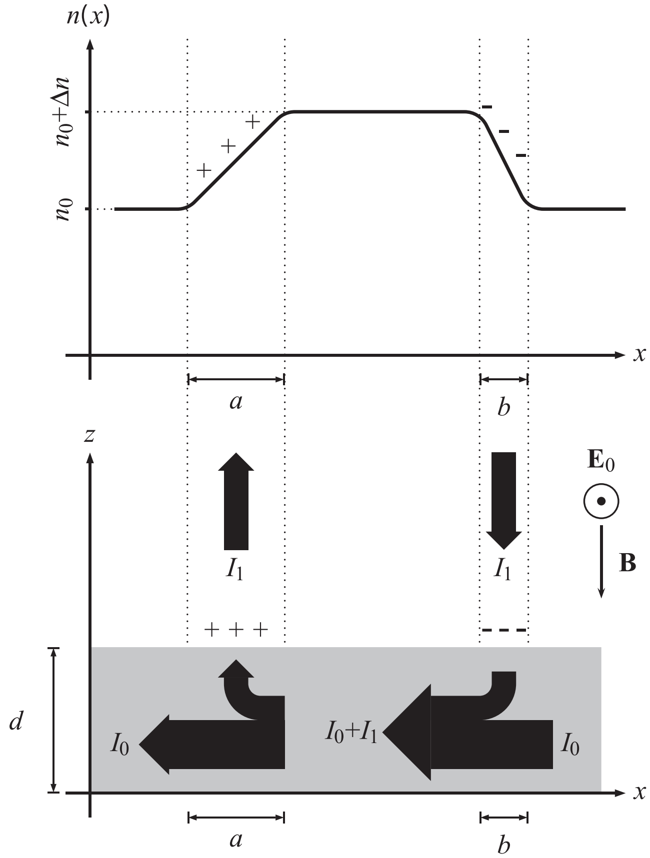

Figure 2. Illustration of a cut through a magnetic flux tube with enhanced plasma density. (top) Details of the geometry and density variations; (bottom) illustration of the currents generated by a steady state electric field $\mathbf{E}_0 \perp \mathbf{B}$. The thickness of the *E* region is $d \approx 10$–20 km.

trated in Figure 2. It cannot escape downward into the *D* region because of the high collisionality there with a corresponding low electron mobility. In the vertical direction along magnetic field lines into the *F* region, we can have a current propagating [*Primdahl and Spangslev*, 1977; *Primdahl et al.*, 1987]. This current will flow along magnetic field lines perpendicular to $\nabla_\perp n_0$ and give rise to unstable drift waves.

[18] The current I_1 in Figure 2 is carried by the electrons, both in the *E* and *F* regions. With a horizontal density variation, we have an asymmetry between the part facing the $\mathbf{E}_0 \times \mathbf{B}$ drift and the opposite side. Due to the abundance of electrons in the *E* region, a quasi steady state is achieved rapidly where electrons are flowing from the high plasma density *E* region into the lower density in the *F* region. On the side facing the $\mathbf{E}_0 \times \mathbf{B}$ drift in Figure 2 (region *a*), the electrons have to flow from a lower plasma density into the larger density in the *E* region. In this latter case, stationary conditions will need longer time to be established. In the *b* region in Figure 2, the electron drift will enhance upward traveling low-frequency waves as described by (9); in region *a*, unstable waves are propagating in the opposite direction. Standard models for the current convective instability in the *F* region [*Ossakow and Chaturvedi*, 1979] do not distinguish the direction of the electron flow.

[19] The conservation of net current ($I_0 + I_1$) through the cut between regions in Figure 2 (*a* and *b*) will act as an amplification for current densities and thereby average electron flow velocities. For regions outside the flux tube with enhanced plasma density, we have a net electrojet current $I_0 = J_0 d = en_0 dE_0/B$ per unit length in the direction perpendicular to $\nabla_\perp n_0$, so by Kirchhoff's laws, we have for instance at the region *b* in Figure 2 where J_1 is the current density, that $ed\Delta n E_0/B \approx J_1 b = be(n_0 + \Delta n)U$, giving the vertical electron drift velocity estimate $U \approx (\Delta n/(n_0 + \Delta n))(d/b)E_0/B$, so that U increases linearly with E_0 with no threshold. In the ionospheric *E* region, we often have $E_0 \geq 20$ mV/m [*Kelley*, 1989] which gives $E_0/B \approx C_s$, so we can argue that substantial electron drifts can be achieved by this mechanism. Taking a reference case of $\ell/L_V = 0.1$ and $U \approx C_s$, we find that the kinetic effects contribute with a growth rate of $\Im\{\omega\}/\Omega_{\rm BV} \approx 0.1$ to the geometric growth over a wide wave number range. The results are sensitive to the parameter values: smaller ℓ/L_V generally leads to larger dominant frequencies and higher growth rates.

4. Conclusion

[20] The analysis outlined in this study has several verifiable features that distinguishes it from, for instance, the current convective instability [*Ossakow and Chaturvedi*, 1979] that assumes a fully collisional plasma. These instabilities are often invoked for explaining observations of irregularities in the ionospheric *E* and *F* regions [*Greenwald et al.*, 2002; *Hosokawa et al.*, 2010]. While most studies of convective drift instabilities [*Simon*, 1963; *Fejer et al.*, 1984] rely on steady state electric fields having a component along $\nabla_\perp n_0$ so that $\mathbf{E}_0 \cdot \nabla n_0 > 0$, our generator model is based on steady state electric fields $\mathbf{E}_0 \perp \nabla_\perp n_0(\mathbf{r}_\perp)$ and operates self consistently with the density enhancement in a magnetic flux tube that forms the basis for a universal drift wave instability. The model predicts a difference in the characteristics of the fluctuations on the density gradient facing the electrojet current as compared to the one on the opposite side. To the lowest order, the growth rate is directly proportional to the electron flow velocity as indicated by (9), where our previous arguments give U proportional to E_0/B. The analysis used a simplified gravitational model that can be solved directly but the basic physical arguments for the imaginary part of (2) are robust and apply also for cases where density gradients along magnetic fields are caused by sinks and sources [*D'Angelo et al.*, 1975]. A complete description of the battery mechanism discussed in this communication requires modeling the current closure, which for ionospheric conditions implies a study of the full inhomogeneous magnetic field configuration [*Primdahl and Spangslev*, 1977, 1983]. This analysis is outside the scope of the present study.

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