Redirecting, progressing and focusing actions – a framework for describing how teachers use students' comments to work with mathematics

Ove Gunnar Drageset

University of Tromsø, Norway

ove.drageset@uit.no

Abstract

In order to describe and analyse teachers' orchestrating of classroom discourse, detailed descriptions of teachers' comments and questions are critical. The purpose of this article is to suggest new concepts that enable us to describe in detail how teachers use or do not use students' comments to work with the mathematical content. Five teachers from upper primary school (grades five to seven, students aged ten to thirteen) were studied. Beginning with the analysis of a pattern where the teacher gives a confirmation followed by a question that indicates a rejection, their practices form the basis for the development of thirteen categories of teacher comments. These categories are then grouped into redirecting, progressing and focusing actions. The categories and their groupings shed light on tools and techniques which these teachers use to make student strategies visible, to make students justify, apply and assess, to ensure progress towards a conclusion, or to redirect the students into alternative approaches. These findings can help us develop in the direction of a more profound understanding of how communication affects learning.

***Keywords**: Communication – Teachers' comments – Redirecting – Progressing – Focusing – Funnelling*

1. Introduction

It is fair to assume that there are elements or qualities in the communication between a teacher and his or her students that matter for the learning of mathematics. This article deals with how teachers respond to student comments, and it tries to establish a framework which will enable us to describe this issue in detail. The following example will set the scene. The excerpt is from a lesson about fractions in grade five (children aged ten and eleven). The teacher asks how it can be that the fractions one half, two fourths and 25/50 are equal, and the below conversation follows:

SC: If you remove the line in the middle (of the drawing of 2/4) then it is the same.
T: Yes, but what has happened with this? 25/50. How…what do I do mathematically to get this? (SS)
SS: Divide it into more pieces
T: I divide it into more pieces, yes. But what if I have to calculate it? (SR)
SR: You have to check that the numerator is half of the denominator.

In both comments, the teacher first confirms that the student answers correctly and then indicates that there is more to it by saying 'but' and asking a question. This observation, that one teacher sometimes used a confirmation followed by a question indicating a form of rejection, initiated this study of teacher comments and the role they play in communication.

2. Communication

2.1 Conversation

Conversation analysis is an approach within the social sciences that 'aims to describe, analyse and understand talk as a basic and constitutive feature of human social life' (Sidnell, 2010, p. 1). This tradition focuses on close observation of the world, studying interaction on its own terms as interaction and not only as a window through which we can view other processes (Hutchby & Wooffitt, 1998; Sidnell, 2010). Other approaches study interaction in order to understand the culture (anthropology), to understand the individual (psychology) or to characterize participants and the world they live in (sociology) (Sidnell, 2010). Conversation analysis developed from the hypothesis that ordinary talk is a structurally organized and ordered phenomenon, and one important aim of conversation analysis is to describe such organization and order and how the participants accomplish it (Hutchby & Wooffitt, 1998). In a conversation people take turns of speaking and the default option is to speak one at a time (Sidnell, 2010). But even if the turns are sequentially organized it is not possible to characterize a conversation as a series of individual actions, instead it is a social practice where each turn is thoroughly dependent on previous turns and individual contributions cannot be understood in isolation from each other (Linell, 1998). According to Sacks, Schegloff, and Jefferson (1974), turns are the most fundamental feature of conversation, and it requires some sort of organization to manage the contributions from various people. Within this system of turns participants monitor a turn-at-talk to find out when the turn is approaching completion and ‘target points of possible completion as places at which to begin their own talk’ (Sidnell, 2010, p. 56). There are also two kinds of turn-allocation techniques; those where the current speaker selects the next speaker and those where next turn is allocated by self-selection (Sacks et al., 1974). When responding it is normally possible to give different types of responses (for example acceptance or rejection) but there are also usually one or a few responses that are preferred to others. Sidnell (2010) exemplifies this by saying that the preferred response to a dinner invitation is to accept. By giving the preferred response there is no need for an explanation, but if one rejects the dinner invitation this requires an accompanying explanation. A somewhat similar concept to preference is that of relevance (Linell, 1998), which describes that some responses are more relevant than others. Both preference and relevance also points out that the choice of one certain response or question has an impact on later turns. The result is that a dialogue is a joint construction ‘made possible by the reciprocally and mutually coordinated actions and interactions by different actors’ (Linell, 1998, p. 86).

2.2 Patterns and concepts describing classroom communication

‘Developing mathematical understanding requires that students have the opportunity to present problem solutions, make conjectures, talk about a variety of mathematical representations, explain their solution processes, prove why solutions work, and make explicit generalizations’

(Franke, Kazemi, & Battey, 2007, p. 230)

To develop mathematical understanding in the manner described by Franke et al (2007), the classroom discourse should be characterized by rich opportunities for students to contribute. However, quite often the classroom discourse is dominated by teacher talk, in a discourse pattern described as IRE (initiation-response-evaluation) where the teacher initiates the questions, the students respond to them, and the teacher evaluates the response (Cazden, 2001). IRE is seen as ‘the default option – doing what the system is set to do ‘naturally’ unless someone makes a deliberate change’ (Cazden, p. 31). It is also argued that within this pattern the students are normally engaged in a procedure-bound discourse, such as calculating answers and memorizing procedures, and with little emphasis on ‘students explaining their thinking, working publicly through an incorrect idea, making a conjecture, or coming to consensus about a mathematical idea’ (Franke et al., 2007, p. 231). However, Wells argues that IRE is treated too undifferentiated ‘as if all the occasions when it occurs are essentially similar’ (1993, p. 3). Using examples from the classroom, Wells (1993) illustrates how much variation is hidden within the IRE pattern and that this variation also includes qualitatively different initiatives, responses and evaluations. This indicates that within IRE there might be teachers dominating, but there also might be room for student contributions beyond answering teachers’ questions and beyond evaluations limited to correct or in-correct.

Brendefur and Frykholm (2000) label a discourse pattern similar to a teacher-dominated version of IRE *uni-directional communication*. This means that the teacher dominates the discussions ‘by lecturing, asking closed questions, and allowing few opportunities for students to communicate their strategies, ideas, and thinking' (Brendefur & Frykholm, 2000, p. 126).

In a study of how children think in different classroom cultures Wood, Williams and McNeal (2006) refer to four cultures. The first two cultures, namely *conventional textbook* and *conventional problem solving*, refers to classroom cultures dominated by the teacher. The major interaction pattern in the *conventional textbook* culture is IRE, while the major interaction pattern in the *conventional problem solving* culture is the teacher giving hints. Such hints 'essentially removed the mathematical challenge or complexity of the problem' (Wood et al., 2006, p. 234). The third classroom culture is the *strategy-reporting classroom* where the students report strategies and can sometimes be asked (by the teacher) to provide more information about how they solved the problem. The fourth is the *inquiry/argument classroom* where the goal of sharing is for others listeners to ask questions for further clarification and understanding. Often, these discussions include a challenge or a disagreement from a student or the teacher. In this way the students are trained in justification and assessment, and this might help them develop robust mathematical arguments and reasoning.

Both *strategy-reporting* and *enquiry/argument* seem to encourage more discussions than the IRE-pattern. However, even though an increased level of discourse is positively related to student learning we know that just getting students to talk is not enough (Franke et al., 2007). Merely making your thinking available to others is insufficient because too much is normally unsaid. The manner in which we make our thoughts available seems to be crucial (Kieran, 2002). Consequently, details matter, or in the words of Franke et al:

'One of the most powerful pedagogical moves a teacher can make is one that supports making detail explicit in mathematical talk, in both explanations given and questions asked'

(Franke et al., 2007, p. 232)

In addition to making details explicit, it is also important for teachers to structure the discourse around the mathematical ideas. Franke et al (2007) suggest scaffolding, monitoring and facilitating the discourse.

During what Stein, Engle, Smith and Hughes (2008) call the first generation with respect to mathematical discussions in the classroom, focus was on the use of cognitively demanding tasks, encouragement of productive interactions, and letting the students feel that their contributions were listened to and valued. Even though mathematical discussions always have existed in some classrooms, this focus came high on the agenda as a result of the first NCTM standards (NCTM, 1989). However, little attention was directed towards how teachers can guide the class towards worthwhile mathematics, and many teachers had the impression that guidance should be avoided (Stein et al., 2008). The result could be that the students took turns sharing their solution strategies without any filtering or highlighting.

'In short, providing students with cognitively demanding tasks with which to engage and then conducting 'show and tell' discussions cannot be counted on to move an entire class forward mathematically'

(Stein et al., 2008, p. 319)

Brendefur and Frykholm (2000) label this *contributive communication*, where 'the conversation is limited to assistance or sharing, often with little or no deep thought' Brendefur and Frykholm (2000, p. 127). It is also typical that the conversations are corrective in nature and that the teacher is the mathematical authority. However, this is also an important step forward from *uni-directional communication*, since in *contributive communication* the students are allowed to articulate solution strategies.

The second generation practice 're-asserts the critical role of the teacher in guiding mathematical discussions' (Stein et al., 2008, p. 320). The hallmark is that the teacher actively uses students' ideas and work to lead them toward more powerful, efficient and accurate mathematical thinking. Ball uses the term *show and tell* as an example of the same:

'For the lesson to be more than a drawn out "show and tell" of the different methods requires the composition of a mathematical discussion that takes up and uses the individual contributions ... making available one child's thinking for the rest of the class to work on.'

(Ball, 2001, p. 20)

Ball here emphasises an active use of students' contributions. This is similar to what Brendefur and Frykholm (2000) call *reflective communication*, where the intention of sharing ideas is to deepen mathematical understanding. This is done by providing opportunities 'to reflect on the relationships within the mathematical topics by focusing on other students' and the teacher's ideas, insights, and strategies' (Brendefur & Frykholm, 2000, p. 148). Here, focus is no longer on transmitting information but instead on generating meaning through a dialogic discourse.

Brendefur and Frykholm (2000) also present a fourth level of communication labelled *instructive communication* where the teacher participates closely along with the students and the progression is 'altered to build upon and deepen the students' present understanding of the mathematics at hand' (Brendefur & Frykholm, 2000, p. 148).

However, even though there is increasing agreement that students' contributions must play an important role in classroom communication there is a need to understand how this can be achieved. Carpenter, Fennema, Franke, Levi and Empson (1999) suggest using a careful selection and sequencing of student strategies. Stein et al (2008) suggest a similar strategy as part of a model that specifies five key practices in order for a teacher to use student responses more effectively in discussions:

1. *anticipating likely student responses to cognitively demanding mathematical tasks*

2. *monitoring students' responses to the tasks during the explore phase*
3. *selecting particular students to present their mathematical responses during the discuss-and-summarize phase*
4. *purposefully sequencing the student responses that will be displayed*
5. *helping the class make mathematical connections between different students' responses and between students' responses and key ideas*

(Stein et al., 2008, p. 321)

This model may move attention away from learning mathematical content independently of student thinking. Instead, attention is directed towards how students' thinking about mathematical content can be used to create reflection and learning. Such a strategy will also give the teacher regular access to students' ideas and the details that support them. This is essential knowledge for teaching and learning in mathematics (Franke et al., 2007).

Fraivillig, Murphy and Fuson (1999) and Cengiz, Kline and Grant (2011) report studies of how teachers actively use the students' ideas to lead them towards more powerful, efficient and accurate mathematical thinking and in which situations this occurs. Fraivillig et al (1999) present a framework called 'Advancing children's thinking' (ACT) based on an in-depth analysis of one skilful first grade teacher. The framework has three components: *eliciting children's solution methods*, *supporting children's conceptual understanding*, and *extending children's mathematical thinking*. While the *eliciting* and *supporting* components focus on the assessment and facilitation of mathematics with which the students are familiar, the *extending* component is focused on the further development of the students' thinking. Each of these components is defined by several categories of instructional techniques, for example 'encourage elaboration', 'remind student of conceptually similar situations' and 'demonstrate teacher-selected solution methods'.

In a recent development of ACT, Cengiz et al (2011) studied six experienced elementary teachers to find out more about what *extending students' mathematical thinking* looks like. They find extension in three types of episodes. The first is *encouraging mathematical reflection* by using multiple solutions or encouraging students to consider whether the solution or claim is reasonable or valid. The second is *going beyond initial solution methods* by pushing the student to try alternative methods or promoting the use of more effective methods. The third is *encouraging mathematical reasoning* by encouraging students to justify their solutions or claims. When Cengiz et al (2011) looked for the instructional actions used by the teachers to *extend the students' mathematical thinking*, the three categories from Fraivillig et al (1999) re-emerged. This means that *eliciting*, *supporting* and *extending* were used as actions in the three different types of episodes. Cengiz et al (2011) described instructional actions that exemplified what *eliciting*, *supporting* and *extending* actions looked like. Examples of such instructional actions are to 'provide reasoning for a claim', 'use same method for new problems' and 'reminding students of the goal of the discussion, the problem, or other information'.

Alrø and Skovsmose (2002) introduce the notion of inquiry co-operation as a particular form of student-teacher interaction when exploring a landscape of investigation. As part of the inquiry-cooperation model they identify eight *communicative features*: Getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging and evaluating. These features were present both in the student-student interaction and in the teacher-student interaction. *Getting in contact* means tuning in to the co-participant and his or her perspectives, which is a precondition for cooperation. *Locating* is a process where perspectives are expressed and made visible, often connected to 'what if'-questions. *Identifying* is often connected to why-questions in an attempt to crystallise mathematical ideas. *Advocating* means stating what you think and also being open to an examination of your understanding, or more generally examining statements and suggestions before rejecting or accepting them. *Thinking aloud* means to make thinking public and thereby accessible to collective inquiry. *Reformulating* is to repeat what has just been said with a different emphasis or a slight change in words or tone. *Challenging* describes attempts to try to move the discussion in a new direction or to question gained knowledge or fixed perspectives. *Evaluating* can come in the form of correction of mistakes, critique, advice, support and in many other forms.

Teachers' orchestrating also includes actions other than those described by Stein (2008) and Fraivillig (1999). Corrections, for example, are necessary in some situations. Alrø and Skovsmose (2002) differentiate between explicit and implicit corrections. An explicit correction is a clear expression that something is wrong and needs to be corrected. An implicit correction can have several forms, for example a question or highlighting of parts of the solution to be inspected more thoroughly. Corrections without argumentation are based on authority. This authority is often the teacher, but it can also be the textbook or mathematics itself.

Mortimer and Scott (2003) suggest a model to describe a teacher's communicative approach, or how the teacher works with the students to develop ideas. This model includes two dimensions, the dialogic – authoritative dimension and the interactive – non-interactive dimension. *Dialogic communication* refers to a situation where more than one point of view is paid attention to while *authoritative communication* is where the attention is focused on only one point of view and 'there is no exploration of different ideas' (Mortimer & Scott, 2003, p. 34). *Interactive communication* means that other people are allowed to participate while *non-interactive communication* excludes other people from participating. The result of combining these two dimensions is that there are four possible communicative approaches. The first is the approach that is both *dialogic and interactive,* where several points of view are paid attention to and people are allowed to participate actively. The second is the *non-interactive and dialogic* approach where several points of view are paid attention to but without allowing others to participate. This could occur when a teacher presents several points of view and discusses these without allowing students to participate actively. The third is the *interactive and authoritative* approach where the participants are allowed to participate but only one point of view is paid attention to. The fourth is the *non-interactive and authoritative* approach where only one point of view is attended to and no other people are allowed to participate.

	INTERACTIVE	NON-INTERACTIVE
DIALOGIC	**A** Interactive / Dialogic	**B** Non-interactive / Dialogic
AUTHORITATIVE	**C** Interactive / Authoritative	**D** Non-interactive / Authoritative

Figure 1: Communicative approach (Mortimer & Scott, 2003, p. 35)

This model offered by Mortimer and Scott (2003) enables us to describe sequences of communication, for example concerning a task or a question, related to the two dimensions. It might also be used to describe different types of appropriation processes. But this model also leaves some challenges as such a sequence rarely is either interactive or non-interactive and rarely is either dialogic or authoritative, but quite often something in-between.

2.3 Reduction of complexity

Sometimes, when a teacher is confronted with repeated failure from students, he or she provides more and more information in order to help. Gradually the teacher takes responsibility for the essential part of the work. When the target knowledge disappears completely, Brousseau (1997) describes it as *the Topaze effect*. It is also characteristic that 'the answer that the student must give is determined in advance; the teacher chooses questions to which this answer can be given' (1997, p. 25).

Another possible label for this phenomenon is *guided algorithmic reasoning* (Lithner, 2008). In *guided algorithmic reasoning* 'all strategy choices that are problematic for the reasoner are made by a guide, who provides no predictive argumentation' (Lithner, 2008, p. 264) and the remaining routine transformations are executed without verificative argumentation. Predictive arguments are related to why the chosen strategy will solve the task, while verificative arguments are related to why the strategy solved the task.

A third way of describing this phenomenon is *funnelling* (Wood, 1998). A teacher's questions *funnel* the conversation when the teacher does most of the intellectual work and 'the student's thinking is focused on trying to figure out the response the teacher wants instead of thinking mathematically himself' (Wood, 1998, p. 172). The alternative is to ask questions so that students' attention is *focused* on important mathematical ideas and place the responsibility of the intellectual work on the students (Stein et al., 2008). *Funnelling* and *focusing* are not only described as different types of teacher questions, but as two different communication patterns reflecting different beliefs. The *funnelling* pattern 'conveys a view that the mathematics to be learned rests solely within the authority of the teacher' (Wood, 1998, p. 175).

As a result, the students will think of the learning of mathematics as 'determining a set of procedures that the teacher already knows and that it is their obligation to learn' (Wood, 1998, p. 175). The *focusing* pattern is found in classrooms where 'the teacher expects the students to think about mathematics, to figure out things for themselves, and to discuss the ideas with others' (Wood, 1998, p. 176). The students are also responsible for explaining their methods to their classmates, and the classmates are expected to ask questions for clarification and justification. The teacher's role is to focus the students' attention and then to make them responsible for solving the problem. In this way the teacher communicates that 'what counts as mathematics in her class are the meanings and understandings that the children have constructed themselves' (Wood, 1998, p. 176)

Altogether the *Topaze effect*, *guided algorithmic reasoning* and *funnelling* describe a phenomenon where the teacher does the main work. On the way the teacher might reduce the complexity for the student in such a way that the task changes into something different, and the teacher might also change the students' focus of thinking from mathematics to qualified guessing about what the teacher wants to hear.

Even though such reduction of complexity arguably can be negative for the learning outcome of students, it is also possible that such actions can play productive roles in classroom discourse. One attempt of describing qualities of classroom discourse as something more than the sum of single comments is *appropriation* (Newman, 1990), describing a process of teachers' interactive support for a student's new interpretation. When a student has to learn something entirely new, such as the subtraction algorithm or drawing a picture, he or she has to learn both the procedures and the overall structure and purpose of the new activity (Newman, 1990). The process involves an expert and a novice and is defined to be going on inside the zone of proximal development (Vygotskij, 1978). The novice makes some action and the expert *appropriates* it using an interactive process. Newman (1990) exemplifies the *appropriation* process by telling about a small group of three year old children that are drawing on paper. These children are just beginning to learn to draw pictures and are not creating representations of things intentionally. When a child announced that he or she had finished the drawing the teacher typically asked what the picture was displaying. The child might answer that was is a moon or two mountains on two orange circles, and even if the answers seems fairly arbitrary the teacher's process of asking, gesturing, follow-up questions and selective acknowledgements directed the discussion toward the teacher's interpretation of the picture as a representation of something. By participating in several such discussions a child might anticipate what will come and begin to produce drawings that represent something that is possible to talk about. In this way, the interaction with the expert gives the child an idea of the overall structure and purpose of the activity. Newman (1990) argues that when a novice has a good understanding of the goal and for example just learns a new procedure for finding a solution, direct instruction might work well. But when the novice do not know the structure or purpose an *appropriation* process may work successfully as long as the novice is able to make some action that can be *appropriated*.

An *appropriation* process often includes actions that in isolation can be labelled as *funnelling*, *teacher-dominated communication* or *IRE*, but as part of the *appropriation* process these actions might be both beneficial and necessary. Mercer and Littleton (2007) argue that instead of looking at the amount of questions a teacher asks one should look at the function of these questions. There are different types of questions, and their function can only be judged as part of their dialogic context. It is often observed that questions require the children to guess the answer the teacher has in mind or that questions often are closed with only one correct answer, but other types of questions encourage children to make their thoughts, reasons and knowledge explicit, some types act as models of useful ways to formulate questions (e.g. when the teacher asks why repeatedly the students will start asking why in their internal conversations), and others again provides opportunities for students to make longer contributions (Mercer & Littleton, 2007). This means that, in order to understand classroom communication, it is both necessary to study single question and its function on one hand and the larger picture, such as for example an *appropriation* process, on the other hand.

3. The purpose of the study

Many of the studies in this review have developed tools for characterising teaching practices, such as Wood's (1998) funnelling and focusing, Brendefur and Frykholm's (2000) four levels of communication and the communicative approach by Mortimer and Scott (2003). While these concepts have explanatory power in the study of entire practices, the limitation lies in the lack of detail. Other studies, such as 'Advancing children's thinking' (Fraivillig et al., 1999), its further development by Cengiz et al (2011) and the inquiry co-operation model described by Alrø and Skovsmose (2002) are slightly different, as these studies characterise elements found in teaching without describing an entire practice. The instructional techniques (Fraivillig et al., 1999), the instructional actions (Cengiz et al., 2011) and the eight communicative features (Alrø & Skovsmose, 2002) are concepts that enable us to describe single teacher comments at a level of detail which is not possible using more general concepts such as funnelling and focusing. Detailed descriptions are critical for researchers to be able to describe and analyze teachers' communication in depth. It is also crucial for professional development as teachers have little use for general advice.

The study reported in this article emerged from the discovery of and inspection into how a teacher included both a confirmation and a rejection in some comments. As a consequence of the attention to single teacher comments this study will follow the path of characterizing elements of teaching and not the entire practices. The attention gradually developed into a more general study of how teachers use students' comments to work with mathematical content.

The purpose of this article is to suggest new concepts that may enable us to give a more detailed description of how teachers orchestrates the classroom as a response to student comments and related to content. Consequently, the research question is: How can teachers' comments be categorized related to how the teachers' use and not use students' comments (suggestions, answers, questions) to work with, and make the students' work with, the mathematical content?

Mercer and Sams (2006) argues that there are two main kinds of interaction in which spoken language can be related to the learning of mathematics; teacher-led interaction with pupils and peer group interaction. This article is limited to the teacher-led interaction with pupils as this is sufficient to answer the research question. Even though only the teacher comments will be categorized this will be done related to the dialogic context and not in isolation. This also means that it is the conversation is studied on its own terms as conversation and not as a window through which we can view other processes.

4. Description of the data and the process

This study is part of a project in which 356 teachers completed a test and a questionnaire from which two knowledge constructs ('common content knowledge' and 'specialized content knowledge') and two belief constructs ('rules' and 'reasoning') were established (Drageset, 2009, 2010). Five general teachers with diverse profiles were selected for further study. In order to describe the teaching of these five teachers their practices were studied. These five teachers teach in upper primary school (grades five to seven, students aged ten to thirteen). All their mathematics teaching for one week was filmed from the start of the topic of fractions. The camera followed the teacher, who also wore a microphone which captured all the dialogues in which the teacher participated. The filmed lessons were divided into segments that lasted from two to ten minutes, typically including a section where one task is solved, one student or group is helped or a new method is introduced.

The analysis was conducted in three phases. The first phase started when it was discovered rather coincidentally that several teacher comments in a segment were a combination of a confirmation and a question (see the section "Correcting questions" below). While trying to describe and understand this practice, and also looking for similar comments, an attention developed. Then the other comments in the segment were analysed, one at a time. The description was not done in isolation, but instead the comments were described as part of the dialogue. Similar comments were put together and characterized as groups that later developed into categories. After repeating this for segments from all teachers, the number of different categories was getting so large that it was difficult to retain an overview. At the same time, several categories were obviously closely related. In the second phase, all the categories from the different segments were put together and compared. This resulted in some being re-defined and some being merged. In the third phase all the remaining segments were coded using the categories established from phases one and two. While the first two phases included ten segments, the third phase added 115 new segments to the data. During this work two new categories were created, two categories re-defined (one broader and one narrower) and several categories had smaller adjustments. In total, more than 1800 teacher comments were used in the development of the categories.

During these three phases particular comments, groups of similar comments and initial categories were regularly brought into discussions in a local research group consisting of five to ten researchers and teacher educators within the field of mathematics education. These discussions were an important part of the process, and feedback and disagreement resulted in changed names, changed or sharper definitions, and merging and splitting of categories. Also, ideas of initial categories were suggested to external groups of researchers and substantial discussions related to these were important for the development of the framework.

5. Findings

In this section thirteen categories will be presented and the next section they will be grouped. This allows the reader to first assess each category and then assess the connections that form the groups. An overview of the categories and the groups is found in the next section.

5.1 Correcting questions

As mentioned above, the discovery of several teacher comments that were a combination of a rejection and a question initiated this analysis. During the process of trying to understand this phenomenon and looking for similar comments, a category started to emerge. One example of this is given in the introduction, where the teacher on several occasions responds by an approval followed by 'but' and a question. Another example is when a student comes up with a suggestion and the teacher tries to get him to use another approach:

SJ: Seven fourteenths.
T: Yes, seven fourteenths can be one half but how many tenths is it that is one half?

Both these examples include a confirmation followed by 'but' and a question. This signals that even though the answer is acceptable it is not the one that the teacher wants.

The comments in this category typically include a question from the teacher to redirect the student towards another approach. The questions therefore act as corrections. Quite often the student gets some sort of confirmation first, but sometimes the questions come directly without any comment on the student's suggestion.

5.2 Advising a new strategy

Another group of comments was put together because these comments all included explicit advice to the students about changing their strategies. On one occasion, the students are trying to divide three circular pizzas with different toppings equally among four persons. One student takes one slice from the first pizza and then another slice from the same pizza before he is interrupted by the teacher:

T: Yes but only take… imagine thinking that you are only taking your share.

The student then starts to take one slice from each pizza and finds that each person will have one slice from each pizza, a total of three slices.

In this category, the teacher typically redirects the students by advising an alternative approach or way of thinking. Sometimes the student's strategy might have resulted in a correct answer while at other times it is obviously a wrong strategy.

5.3 Put aside

One of the first groups that were developed contained comments where the teacher put aside or rejected student comments. An example of this category of comments occurs when one of the teachers has drawn two pizzas on the blackboard and divided each pizza into eight slices. All the slices in one pizza and seven slices from the second are marked. The numerator fifteen is found, and this conversation follows:

T: What should we put under the fraction line?
SI: Fifteen sixteenths.
T: Sixteenths? (has lifted the hand to write as the student answers, but lowers it and turns back to the students while saying sixteenths)
Another student: Eights.

By repeating 'sixteenths' as a question and turning back to the student the teacher clearly signals that the answer is wrong. Also the tone of voice signals the same. It is an implicit correction, and the students understand this and eights is suggested instead. The teachers put aside students' comments in different ways, and both implicit and explicit corrections are found. Also, at least once a comment were put aside even if it was regarded as correct because the teacher wanted to check whether the other students also understood.

It is typical of the comments in this category that the teacher put aside a comment or suggestion without providing any help; it is just put aside. Sometimes the answer is wrong and at other times it just seems that the teacher wants to follow another path.

5.4 Demonstration

Most of the teacher comments in this data are short and part of a dialogue between the teacher and the students. However, one category emerged from a group of longer comments. One example is found at the end of a long exercise with several smaller tasks. The group has arrived at the calculation 8/8+7/8, illustrated by two pizzas:

SH: Seven eights.
T: Then this is seven eights (writes 7/8). Okay. Then we said that this equals one whole (points at 8/8). Then it becomes one whole (writes =1 and an arrow from 8/8 to 1) plus seven eights (writes +7/8). Do you agree? That it is one whole plus the seven here (points to the second pizza with seven slices marked). And that is written like this, one whole and seven eights (writes =1 7/8). We read it like this: One whole and seven eights. And we remove the addition sign.

In this case the teacher demonstrates the rest of the solution without involving or asking the students (The teacher asks 'do you agree' but talks on without waiting for an answer).

In the comments in this category, the teacher typically demonstrates several steps or the entire solution process as a monologue. Sometimes this monologue is broken by the teacher asking for confirmation by asking whether the students agree or understand. Even though most comments in this category are long, a few are also quite short as the teacher rapidly demonstrates how to solve the task.

5.5 Simplification

Sometimes the teacher changes or adds information to a task in a comment, as in the following example:

T: And then I will give one fifth of it (40 kroner) to… to my little brother, how much is one fifth of this? … then I have to have forty kroner, then I probably have to take forty coins and then I have to divide them into five equal piles and how much money would that be? Can you manage to solve that?

By adding the information that they need to take forty coins and divide them into five equal piles, the teacher adds information that makes the task easier. The crux of this task is to understand what one fifth of forty means and transform this knowledge into a method for solving the problem. It is fairly easy to take forty coins and divide them into five equal piles.

A second example is when a teacher asks how much two fifths and three fifths are:

S: Ten.
T: TWO fifths and THREE fifths, how many fifths is that? (emphasises TWO and THREE)
S: Ten.
T: If you have two fifths here (holds up two fingers) and three fifths there (holds up three fingers on the other hand), how many fingers do you see?

The student's answer seems strange, but can be explained by the task and the illustration. Each fifth is two orbs, so altogether there are ten orbs. The answer gives the teacher an opportunity to check whether others think in the same way and to try to resolve the misunderstanding. Instead, the teacher first chooses to repeat the question with more emphasis and then to provide more information to help the student answer correctly. These questions act as corrections because the student's suggestion is met by a new question without any comment on the first answer offered by the student (correcting question), but they also make the problem easier by changing the task the student has to solve from fifths as pairs of orbs to simply counting fingers.

In this category the teacher typically simplifies the task by changing or adding information, giving hints or telling the students what to do to solve it. The teacher is leading or pulling the student towards the solution. It often seems that this involvement is meant to ensure the progress of the class and sometimes these comments appear to come as a consequence of a halted progress. Many of the simplification comments could also be characterized as hints.

5.6 Closed progress details

Quite often the teachers attend to details, and this happens in various ways. One such example is from work with fractions equivalent to one half.

SC: (Draws two vertical lines to divide the square into sixteen parts)
T: And then it is … what did you say this becomes then? (SS)
SS: Sixteenths.
T: Yes, sixteenths. How many… wait a moment SC… How many sixteenths did you say it is?
SS: Eight sixteenths.

First, the teacher asks what the denominator becomes (pointing to a former suggestion). After getting the suggestion of sixteenths, the teacher confirms this and asks how many sixteenths. These questions play the role of moving the process forward one step at a time. In an example from another teacher we can see a similar pattern:

T: How many fifths do we have on this side (points to the right side of the long vertical line) (SI)
SI: Three fifths.
T: We have three fifths (writes 3/5 to the right of the long vertical line). If we look at the whole thing, how much is it then? Two fifths and three fifths, how many fifths is that? (SA)

Instead of asking about the final answer, the teacher splits it up into several smaller tasks and asks for answers to each of these. One aim of this strategy might be to ensure that every student is able to follow the line of thought by following them through every important step. The result is that the teacher takes control of the process and probably reduces the complexity of the task for the students, as they do not need to see the whole picture.

Closed progress details are about how (many, large, much, big, to do it) and what (it becomes, shall we write, is, to do). Details regarding process answers are asked for by how many, how large, how much, how big, what it becomes, what shall we write and what is. Details regarding the process itself are asked for by how to do it, what to do. Questions about how and what brings the process forward, always asking for the next step in the stair. Such a process flows fine as long as the students answer correctly on each process detail.

In this category, questions typically request details needed to move the process forward. These details can be process answers (one step at a time) or details about how the process should go on to reach the answer. The questions are related to calculations, answers and clarifying details about the process. These questions typically have only one correct or desired response, which is quite often easy to find.

5.7 Open progress initiatives

Sometimes the teachers initiate an open progress. One such comment is the following:

T: Yes. One out of four parts. Then the fraction is one fourth. Can you find other fractions that are one fourth?

The result is several answers that are correct. And then the teacher asks:

T: Is there any rule for this then? Or can we, can we see a pattern in this?

The question does of course indicate that there is at least one pattern to find. But more importantly, the teacher does not indicate what this or these patterns are. The result is a question where the teacher initiates progress but still leaves it at least partly open to the students to choose or suggest which path to follow. Another similar example is the following:

T: But then there are such tasks where you should… find out which is the largest out of for example seven tenths (writes 7/10 on the blackboard) and for example sixty-five hundredths (writes 65/100 on the blackboard). How can we manage to find out this? What do you think, SA?

Here the teacher asks how we can find the answer, not what the correct answer is. This results in two different suggestions from the students that are both appreciated by the teacher.

The comments in this category are typically open questions with several possible answers, including questions about how to do, how to think, how to solve and how to generalize patterns. The comments are also aimed at moving the process forward, but without pointing out the direction.

5.8 Enlighten details

Another category is based on teacher comments which halt the progress and request that the students focus on a detail. Below is an example of such a comment:

SM: I thought that … (points at the fraction, interrupted).
T: Yes. And what was it that the numerator shows us?
SM: It is … (hesitates)
T: It shows us…
SM: How … how many there are.
T: How many there are. And what does the denominator show us?
SM: How many it is divided into.
T: How many it is divided into. Correct. Uh-huh.

In the above example the teacher seems to try to expose the meaning behind the concepts. In this case, short and incomplete answers are accepted. The meaning could be expressed more thoroughly, but the questions request some sort of explanation of the concepts. Such requests for meaning and explanation come in different forms. Below are two further examples:

SE: Er … they can take one each.
T: What 'one each'?

SA: (Draws a vertical line that divides the square into two equal parts)
T: Then you draw a line, and that means that…

Comments in this category typically request that the students stop and explain what something means or how something happens. The effect is that the detail is brought into focus. Such explanations can be necessary for other students to follow the line of thought, for the teacher to understand how the student thinks, or to check whether the student knows or understands.

5.9 Justification

On other occasions the teachers request more thorough explanations, as in the below examples:

SE: Five six … five sixths.
T: You have found that five sixths is larger than one half (writes 5/6). Why do you know with certainty that this is larger than one half?

ST: Ten twelfths.
T: Why is ten twelfths larger than one half? (writes 10/12)

SJ: It is seven … seven plus seven is fourteen and there are seven tenths.
T: Yes, but why do you know that it is more than one half?

Requests for justification are typical of this category, such as questions about why the answer found or the method used is correct. In these cases, the teachers are not satisfied with just the correct answer or with a presentation of what the student did to arrive at the answer.

5.10 Apply to similar problems

One category emerged from a group of questions which the teachers seemed to invent on the spot. In one example the students have found several fractions equivalent to one half. The teacher then comes up with a new challenge for them:

T: I know you talked about a rule. Can you find a rule where it is easy to see such number-value-fractions; how we can make it? (SK)
SK: The denominator is the double of the numerator.
T: Yes. The denominator, that is the one that is below, is the double of the numerator. Numerator and denominator (pointing at each). Yes. Great.
T: But if I invent another fraction that is for example thirty-four (writes 34) can we then find... er... that the numerator is thirty-four, can we then find a fraction that is equal to one half? (SE)
SE: Sixty-eight.
T: Bravo. Sixty-eight (writes it below 34 with a line in between to make it a fraction). Because of, SE, what is the reason for this answer?

First, the teacher asks about a rule that one student comes up with quite immediately. This is both confirmed and appreciated by the teacher. But then the teacher invents a fraction where the denominator is missing. This question seems to be invented on the spot to check whether the students can apply the rule to a new and different problem. In another example a teacher is supervising problem solving from the textbook:

SG: One fourth out of twelve is three and one fourth out of thirty-six is nine.
T: Yes. What will three fourths of thirty-six be then?

The last question is not from the textbook; it is a new task invented by the teacher.

The comments in this category typically serve as tests of whether the student can apply the knowledge he or she just demonstrated by on a new and related problem. Sometimes this leads to a wrong answer, which also gives the teacher important information and the opportunity to address the problem.

5.11 Request assessment from other students

The standard solution when these five teachers are given an answer or a suggestion by a student is to assess it immediately, for example through a correcting question or by approving it and moving on. In a few cases the teachers requests other students to assess a student answer or suggestion as in the following example:

S: Three sixteenths.
T: Three sixteenths, is that correct?
Several students: Yes.

Sometimes the teacher asks whether the students are certain that the answer is correct, whether they agree and whether they understand. It is also interesting to observe that such requests only appear when the answer suggested by the student is correct.

In this category, the teacher typically leaves the assessment to other students. This might be a strategy to check whether they are paying attention or whether they are able to follow the line of thought. The danger of applying this strategy only when the proposed answer is correct is that the students may understand this and consequently do not need to think mathematically to agree.

5.12 Recap

One category is based on comments that terminate the task, discussion or line of thought. In this example a student suggests a method to find out whether 7/9 is more or less than one half:

SR: Er … at seven and nine … I think in another way. I think in a way that (teacher writes 7/9)… seven … er … for example seven … er is seven plus seven that is … that is … er… fourteen, and then it is not fourteen below in the denominator.
T: But you said that seven ninths is larger than one half. Wasn't that what you said?
SR: Yes.
T: Yes. But then you started with fourteenths. How many fourteenths did you find in the pink cloud?
SR: Er … no that wasn't what I answered. If you add seven … if you add seven then … if the denominator had been fourteen then it would have been one half. But it is not fourteen, it is nine, it is smaller than fourteen. So it is more than one half.
T: Okay. Then ... but … SR says that seven ninths is more than one half because she thinks that if she had one half she would have had to have seven fourteenths, and nine is smaller than fourteen. You have fewer pieces to divide it into. Uh-huh.

The teacher does not seem to understand the approach in the beginning and tries to redirect the approach by using correcting questions. But in the last comment the teacher repeats what the student suggests in a more streamlined form. It seems that the teacher repeats it in order to make it easier for others to follow the line of thought. In another example a teacher asks for fractions that are equivalent to one half. One student suggests four eights and has drawn a figure to illustrate the equivalency:

T: Can you show that this is…what is it that is four eights here?
SI: These are the eights (pointing) and so four are coloured.
T: Yes. One, two, three, four, five, six, seven, eight (pointing at each part while counting) and four of these are coloured. Or cross-hatched four. Yes, then you have equal value. Exactly the same.

In the last comment the teacher repeats the students' answer but puts some extra emphasis on it by counting each part. In this way the teacher makes a point out of why four eights is equivalent to one half.

What these two examples have in common is that the teacher recaps the solutions. In the first example the teacher streamlines the solution, while in the second the teacher put some extra emphasis on the equivalency.

The comments in the recap category typically pull together information, clarify and point out what is important. Recap is also used as a confirmation when the teacher repeats a student's answer and ends the dialogue, sometimes slightly altering or adding information to the answer to clarify the thinking behind it or the reason why it is correct.

5.13 Notice

One category emerged from comments aimed at emphasizing or pointing out important elements during a dialogue. In one example the students are choosing fractions (from a group of fractions) which are larger than one half:

SE: Because three … is one half and … (interrupted)
T: Yes. Because three sixths is one half (writes 3/6=1/2). Right? When we have the half in the numerator of the denominator then we have one half. Then we can find all that have more than the half in the numerator … SR

In the last comment the teacher interrupts a student to emphasize when a fraction is equivalent to one half. The teacher adds information to make the idea clearer. In another example the following happens:

SR: I found this too, that I had to divide nine by three, and then this became three then, and then… and then I just counted three, three, three, three...
T: OK. But three eights; is that the same as nine? Do we agree on that?

Here the teacher stops the progress and points out what they already know. Even though this comment is a question, it serves to tell the students what is important and has to be noticed. Such comments might be meant as help for the student talking or as support for other students to keep track.

It is typical for the comments in this category that the teacher tells the students to notice some important detail. The teacher often slightly changes the statement or adds new information to make the point clearer, or reminds them of information or process answers on which they have agreed earlier in the solution process. It seems that the purpose is to support the students by pointing out important elements which they should use in their solution process, or important aspects to notice which they should understand or use in the future.

6. Grouping of the categories

In the process of developing categories, they were put together in groups of related categories to investigate whether any categories should be merged or reorganized. The groups of categories were then described and named.

The categories 'correcting questions' and 'advising a new strategy' are related because both categories describes teacher actions which redirect the students' approach. Also the comments in the category 'put aside' are redirecting, since the teacher uses these to direct attention to something else.

The categories 'demonstration', 'simplification', 'closed progress details' and 'open progress initiatives' are related because they describe actions that help the process move forward. 'Demonstration' describes actions where the teacher does all the work, for example by solving a task without help from the students. 'Simplification' describes actions where the teacher reduces the complexity by changing a task, adding information or telling the students what to do at each step until the correct answer has been reached. The category 'closed progress details' describes actions that help the students move one step further in the solution process, for example by dividing the task into steps and asking one question for each step. For each question the teacher accepts only one answer. The category 'open progress initiatives' describes actions where the teacher initiates a progress without limiting the possible responses to only one. These categories form a group called 'progress actions' because they all serve to move the process forward.

The categories 'enlighten details', 'justification', 'apply to similar problems' and 'request assessment from other students' all describe teacher requests for student input. Instead of accepting an answer or suggestion and going on, the teacher asks the students to stop and enlighten a detail, justify an answer or choice of procedure, to apply the knowledge to solve a new and related task or to assess a suggestion from a fellow student. The 'recap' and 'notice' categories describe two different manners in which the teachers point out important information. Recap is pointing out through summing up, and at the same time closing the dialogue. Notice is pointing out during problem solving through helping the student see what is important and useful in the specific case. Both requests for student input and pointing out information are teacher actions that focus on details.

Even though some of the categories seem to be general, it is important to remember that this framework was developed from analyzing mathematical discourses and the mathematical content had a profound influence on both the categorization process and the resulting framework.

1. **REDIRECTING ACTIONS**
    a. Put aside
    b. Advising a new strategy
    c. Correcting questions

2. **PROGRESSING ACTIONS**
    a. Demonstration
    b. Simplification
    c. Closed progress details
    d. Open progress initiatives

3. **FOCUSING ACTIONS**
    a. Requests for student input
        i. Enlighten details
        ii. Justification
        iii. Apply to similar problems
        iv. Request assessment from other students
    b. Pointing out
        i. Recap
        ii. Notice

Figure 2. Redirecting, progressing and focusing actions.

This article presents the categories by defining what is typical for each category in order to describe the idea or concept as clearly as possible. This does not mean that all comments of these five practices are easy to place in one single category. Without going into a longer discussion of this challenge, a few examples can be mentioned. One example is when teachers point out important elements to be remembered or used during the solution process (notice) this action sometimes also seems to work as a simplification. Another example is that it is sometimes difficult to say when a teacher presents the process and solution to a task essentially alone (demonstration) and when the teacher uses information from the discussion to recap. Considering such problems of where the border goes were important during the categorization process as these often sharpened definitions and sometimes gave rise to new categories.

7. Discussion

In this section the redirecting, progressing and focusing framework will be discussed related to relevant research and established concepts presented in section 2. See figure 3 in the end of the section for an overview.

The focusing actions are examples of how teachers use students' ideas to go deeper into the details of the content (enlighten details and justification), check understanding (apply to similar problems and request assessment from other students) and point out important details (recap and notice). These actions have the potential to lead students towards more powerful, efficient and accurate mathematical thinking. Knowledge about such actions can provide teachers with tools to create what Brendefur and Frykholm (2000) call reflective communication. Knowledge of such actions is important in order to advance from contributive communication and 'show and tell'. We know that just making thinking available is not enough; it is the way we make our thoughts available that matters (Kieran, 2002), and making details explicit is one of the most powerful strategies a teacher can use (Franke et al., 2007). The first four (enlighten details, justification, apply to similar problems, request assessment from other students) are arguably important elements in Wood's (1998) definition of focusing which emphasizes that the students themselves think mathematically and explain their ideas to others. Furthermore, to request that students enlighten, justify, apply and assess can provide a window into students' thinking and in this way give the teachers access to knowledge that is essential for teaching mathematics (Franke et al., 2007). On the other hand, it is also possible to ask for justifications so often that the students lose track of the direction or the goal of the activity. It is also possible to point out so many important elements that the amount of information gets confusing or too large.

Redirecting and progressing actions can be seen as the two main elements of funnelling (Wood, 1998). Redirecting actions are about getting the student to change to the correct or desired approach and progressing actions are about moving the process forward. Combined, these actions might result in the teacher dominating the process and the students' participation reduced to figuring out the response that the teacher wants instead of thinking mathematically. Used in this way, redirecting and progressing actions will thwart discussions by only allowing desired comments. Also, a teacher might hinder reflection and understanding of important details if he or she too quickly demonstrates, simplifies or asks for closed progress details. On the other hand, redirecting actions might be used to put aside suggestions without too much discussion to keep the class concentrated and in order not to lose the line of thought. Advising a new strategy could also solve a situation where the students are getting nowhere, and correcting questions might work well as a way to redirect the approach during problem solving without taking the responsibility away from the students. Also, moving the process forward is sometimes necessary to arrive somewhere within a given time frame. Knowledge of redirecting actions can equip teachers with tools to redirect or put aside comments whenever necessary while knowledge of different progress actions can equip a teacher with tools that might be helpful to get a halted process to move forward.

Another aspect of the redirecting and progressing actions can also be observed in the data. Simplification often includes new information which changes and reduces the complexity of the task. In these cases it seems that the teacher asks whatever question necessary to get the desired answer from the student. The result is a Topaze effect (Brousseau & Balacheff, 1997). At other times the teacher seems to make all the important strategy choices, as in Lithner's (2008) guided algorithmic reasoning. This occurs often with closed progress details, advising a new strategy and demonstrating, and sometimes with correcting questions. The result is a reduction of complexity as the student solves all the easy tasks, often calculations, while the teacher controls the process and the strategy choices.

One characteristic of the data in this study is that no situation can be described as enquiry where the students are responsible for the process. In fact, all situations can be described as belonging to the IRE pattern as the teachers always intervene and comment on every student comment. This also means that the teacher distributes the turns when he or she speaks while the teacher always takes the next turn when a student speaks. The students are given the responsibility for answering single questions when they are selected for the next turn, but they are never expected to take responsibility for entire solution processes. The thirteen categories in three groups also illustrates how diverse patterns and practices can be characterized as IRE, which supports the findings of Wells (1993) that within

IRE there are room for large variations. This lack of detail also means that IRE and other widely used concepts such as funnelling and focusing (Wood, 1998), the four levels of communication (Brendefur & Frykholm, 2000), and the model to describe a teacher's communicative approach (Mortimer & Scott, 2003) might be used to describe entire practices but are not useful when studying shorter sequences of communication in more depth. Instead there is a need for more detailed frameworks describing elements in the classroom discourse. Mercer and Littleton (2007) argues that there are different types of questions, such as questions that require the children to guess the answer the teacher has in mind, questions that are closed with only one correct answer, questions that encourages children to make their thoughts, reason and knowledge explicit, and questions that provides opportunities for students to make longer contributions. The redirecting, progressing and focusing framework can be seen as further detailing and organizing the different types of questions observed by Mercer and Littleton (2007). A majority of the teacher comments in these five classrooms are questions, but also includes other types of teacher comments. The redirecting, progressing and focusing framework also has similarities with Advancing children's thinking (Cengiz et al., 2011; Fraivillig et al., 1999) and the eight communicative features (Alrø & Skovsmose, 2002) as all these describes elements in the classroom discourse without trying to characterize entire practices.

Details are also critical when trying to understand longer sequences or entire practices. For example, the redirecting, progressing and focusing framework can be seen as a detailing of different types of Wood's (1998) funnelling and focusing (see figure 3). Another example is that the redirecting actions (put aside, advising a new strategy, correcting questions) might play important parts in an appropriation process together with demonstration, simplification, notice and recap. These all have in common that the teacher actively tries to guide the student during the solution process. In this way, the redirecting, progressing and focusing framework can give names to some of the actions the teacher uses in an appropriation process. That may enable us to identify and describe different types of appropriation processes. A third example is that all thirteen categories are examples of the IRE pattern, and using these might also enable us to describe different types of IRE patterns.

In figure 2 the categories are organized in three groups regarding how the different types of teacher comments affect the progress towards an answer. However, it is possible to organize the categories related to other dimensions. One such example is to distinguish according to where the intellectual authority is located. The students are given the responsibility in different ways when the teacher uses open progress details and the four requests for student input (enlighten detail, justification, apply to similar problems, request assessment from other students). On the other hand, the teacher takes the responsibility in different ways when using the focusing actions of recap and notice, when using the progressing actions of demonstration, simplification and closed progress details, and when using redirecting actions. It is also possible to relate the categories to mathematical distinctions such as the mathematical competencies of Niss and Højgaard

Jensen (2002). For example, it seems likely that a teacher that frequently uses justification will support the development of reasoning competencies for the students, and frequent use of enlighten details probably will support the development of problem solving competencies because this action requests a student to explain the thinking process behind the solution. But the question of how these actions might contribute to develop students' mathematical knowledge is far from trivial and cannot be judged by simply counting the quantity of each type of teacher comments. Instead, studying longer sequences to understand how these different types of teacher comments contribute to the learning process, for example by studying their role in an appropriation process, might be more productive. In an appropriation process it is possible that actions where the teacher keeps the intellectual authority, such as advising a new strategy, demonstration and notice, might give important contributions and support to students struggle to develop problem solving or reasoning competencies.

The examples of the use of the redirecting, progressing and focusing framework together with IRE, focusing and funnelling, appropriation processes and mathematical distinctions all points out that in order to understand the larger picture it is important to be able to describe and understand the detailed building blocks.

Figure 3 has been created in order to help the reader to understand where the categories and actions are related to other concepts from the literature review of section 2 and the discussion in this section.

<table>
<tr><td colspan="2" rowspan="3">REDIRECTING ACTIONS</td><td>Put aside</td><td>Implicit and explicit corrections</td><td rowspan="3">Challenge students</td><td rowspan="6">Teacher is the intellectual authority</td><td rowspan="6">Funnelling</td><td rowspan="13">IRE</td></tr>
<tr><td>Advising new strategy</td><td></td></tr>
<tr><td>Correcting questions</td><td>Corrections</td></tr>
<tr><td colspan="2" rowspan="4">PROGRESSING ACTIONS</td><td>Demonstration</td><td>Make details explicit</td><td></td></tr>
<tr><td>Simplification</td><td>Hint - Topaze effect</td><td></td></tr>
<tr><td>Closed progress details</td><td>Guided algorithmic reasoning</td><td></td></tr>
<tr><td>Open progress details</td><td></td><td rowspan="5">Access to student thinking</td><td rowspan="5">Student is the intellectual authority</td><td></td></tr>
<tr><td rowspan="6">FOCUSING ACTIONS</td><td rowspan="4">Requests for student input</td><td>Enlighten details</td><td>Make details explicit</td><td rowspan="6">Focusing</td></tr>
<tr><td>Justification</td><td>Make details explicit - Identify – Encourage reasoning</td></tr>
<tr><td>Apply to similar problems</td><td></td></tr>
<tr><td>Request assessment from other students</td><td></td></tr>
<tr><td rowspan="2">Pointing out</td><td>Recap</td><td>Make details explicit</td><td></td><td rowspan="2">Teacher is the intellectual authority</td></tr>
<tr><td>Notice</td><td>Make details explicit</td><td>Reminding students</td></tr>
</table>

Figure 3. Redirecting, progressing and focusing actions related to other concepts mentioned in this article. References has been omitted in order to make the table readable, and are instead listed here: IRE (Cazden, 2001), funnelling and focusing (Wood, 1998), access to student thinking (Franke et al., 2007), challenge students (Alrø & Skovsmose, 2002), implicit and explicit corrections (Alrø & Skovsmose, 2002), make details explicit (Franke et al., 2007), hint (Wood et al., 2006), Topaze effect (Brousseau & Balacheff, 1997), guided algorithmic reasoning (Lithner, 2008), reminding students (Cengiz et al., 2011), identify (Alrø & Skovsmose, 2002), encourage reasoning (Cengiz et al., 2011).

8. Conclusion

The search for new concepts to describe teacher comments has provided a framework of thirteen categories which fall into three superordinate groups. Through these categories it is possible to describe all relevant teacher comments in the data, and the grouping of the categories adds a more general dimension by describing three types of actions.

These categories and their grouping shed light on tools and techniques which these teachers use to make students' strategies visible, to make students justify choices or results, apply methods or rules on similar problems and assess each other's responses, to ensure a progress which moves towards a conclusion, or to redirect the students into alternative approaches. Sometimes these actions provide insight into students' thinking, while at other times they help the student reach a mathematical result. It is also a tool that can be used to describe how teachers use or do not use student contributions to work with, create reflection on and learn mathematics. These findings can help us develop in the direction of a more profound understanding of how communication affects learning, either by studying a few comments in depth or by using these concepts to study processes or detailing existing concepts such as funnelling and focusing.

Two main limitations are worth mentioning. One is that only the mathematics teaching of five teachers was studied, for a short period. Another is that the data are characterized by the IRE pattern and no sign of student-led enquiry is found. It is probable that a similar study of other teachers where there is more student-led enquiry or other patterns of turn-taking and turn-allocation would result in additional or broader categories.

There are several possible ways to develop or use this framework, and five ideas will be mentioned briefly here. One approach might be to develop these categories and actions through further studies of other teachers in other cultures. Another approach might be to study how combinations of these categories occur and whether they form patterns. Such patterns might have explanatory power beyond the study of single comments, for example by studying how a teacher uses different actions or categories as part of an appropriation process when the students have to learn something entirely new. As mentioned earlier, this might also result in the description of different types of appropriation processes. A third approach might be to study how different beliefs and varying levels of knowledge are related to the practice described using the redirecting, progressing and focusing framework. This is planned for a later article, where this framework will be connected to the measured beliefs (rules and reasoning) and knowledge (common content knowledge and specialized content knowledge) of the teachers in order to evaluate whether or how the quantitative descriptions of knowledge and beliefs are related to the qualitative description of the teachers' practice. A fourth approach might be to study the role preference (Sidnell, 2010) or relevance (Linell, 1998) of a student response has for the teachers' choice of turn. It is possible that closed progress details are more frequent as a response to student answers that are preferred, while teachers' might have a tendency to use redirecting actions or requests for students to enlighten details and justify following a less preferred response. A fifth approach might be to develop a similar categorization of the student comments. Then it will be possible to study relations between different types of student comments and different types of teacher comments and get insight into the larger picture of for example an appropriation process.

Literature

Alrø, Helle, & Skovsmose, Ole. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Dordrecht: Kluwer Academic Publishers.

Ball, Deborah Loewenberg. (2001). Teaching, with respect to mathematics and students. In T. L. Wood, B. S. Nelson & J. Warfield (Eds.), *Beyond classical pedagogy: teaching elementary school mathematics* (pp. 11-22). Mahwah, N.J.: L. Erlbaum Associates.

Brendefur, Jonathan, & Frykholm, Jeffrey. (2000). Promoting mathematical communication in the classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education, 3*(2), 125-153. doi: 10.1023/a:1009947032694

Brousseau, Guy, & Balacheff, Nicolas. (1997). *Theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Carpenter, Thomas P., Fennema, Elisabeth, Franke, Megan Loef, Levi, Linda, & Empson, Susan B. (1999). *Children's mathematics: cognitively guided instruction*. Portsmouth, N.H.: Heinemann.

Cazden, Courtney B. (2001). *Classroom discourse: the language of teaching and learning*. Portsmouth: Heinemann.

Cengiz, Nesrin, Kline, Kate, & Grant, Theresa. (2011). Extending students' mathematical thinking during whole-group discussions. *Journal of Mathematics Teacher Education, 15*(5), 1-20. doi: 10.1007/s10857-011-9179-7

Drageset, Ove Gunnar. (2009). Exploring Mathematical Knowledge for Teaching. In M. Tzekaki, M. Kaldrimidou & C. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematical Education* (Vol. 2, pp. 473-480).

Drageset, Ove Gunnar. (2010). The interplay between the beliefs and the knowledge of mathematics teachers. *Mathematics Teacher Education and Development Journal, 12*(1).

Fraivillig, Judith L., Murphy, Lauren A., & Fuson, Karen C. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education, 30*(2), 148.

Franke, Megan Loef, Kazemi, Elham, & Battey, Daniel. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256): NCTM.

Hutchby, Ian, & Wooffitt, Robin. (1998). *Conversation analysis : principles, practices and applications*. Cambridge: Polity Press.

Kieran, Carolyn. (2002). The mathematical discourse of 13 year old partnered problem solving and its relation to the mathematics that emerges. . In C. Kieran, E. A. Forman & A. Sfard (Eds.), *Learning discourse: discursive approaches to research in mathematics education*. Dordrecht, The Netherlands: Kluwer.

Linell, Per. (1998). *Approaching dialogue. Talk, interaction and contexts in dialogical perspectives.* Amsterdam: John Benjamins.

Lithner, Johan. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*(3), 255-276.

Mercer, Neil, & Littleton, Karen. (2007). *Dialogue and the development of children's thinking : a sociocultural approach*. London: Routledge.
Mortimer, Eduardo Fleury, & Scott, Phil. (2003). *Meaning making in secondary science classrooms*. Buckingham: Open University Press.
NCTM. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.
Newman, Denis. (1990). Cognition, computing, and cooperation. In S. P. Robertson, W. Zachary & J. B. Black (Eds.), *Cognition, computing, and cooperation* (pp. 84-94). Norwood, N.J.: Ablex Publ. Corp.
Niss, Mogens, & Højgaard Jensen, Tomas. (2002). *Kompetencer og matematiklæring: Ideer og inspiration til udvikling af matematikundervisning i Danmark*. København: Undervisningsministeriet.
Sacks, Harvey, Schegloff, Emanuel A., & Jefferson, Gail. (1974). A Simplest Systematics for the Organization of Turn-Taking for Conversation. *Language, 50*(4), 696-735.
Sidnell, Jack. (2010). *Conversation Analysis. An Introduction.* . Chichester, West Sussex, United Kingdom: Wiley-Blackwell.
Stein, Mary Kay, Engle, Randi A., Smith, Margaret S., & Hughes, Elizabeth K. (2008). Orchestrating Productive Mathematical Discussions: Five Practices for Helping Teachers Move Beyond Show and Tell. *Mathematical Thinking and Learning, 10*(4), 313 - 340.
Vygotskij, Lev Semenovič. (1978). *Mind in society : the development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner & E. Souberman Eds.). Cambridge, Mass.: Harvard University Press.
Wells, Gordon. (1993). Reevaluating the IRF sequence: A proposal for the articulation of theories of activity and discourse for the analysis of teaching and learning in the classroom. *Linguistics and Education, 5*(1), 1-37.
Wood, Terry. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167-178). Reston, Virginia: National council of teachers of mathematics.
Wood, Terry, Williams, Gaye, & McNeal, Betsy. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education, 37*(3), 222-255.