

**Volume 32, Issue 3****Vertical integration through Rubinstein bargaining**

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**Abstract**

We consider a vertical structure in which an upstream manufacturer bargains with a downstream retailer over the price of an intermediate good. In an alternating offers framework, we show that when the managers of the firms can choose their response time in the negotiation that the solution conforms either to the non-integrated or fully integrated structure from standard models of successive monopoly.

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## 1. Introduction

As suggested by Tirole (1988), vertical relationships between upstream firms and downstream firms are often much richer and more complex than those between a firm and the consumers. It is well known that the simple relationship between a manufacturer and a retailer creates a vertical externality which can be avoided by vertical integration by eliminating the double price distortion when each firm sets a mark-up over its cost at each stage of production. This paper considers the conditions under which the choice of the market structure, non-integrated or fully integrated, can be the result of a bargaining process over the wholesale price between the managers of the two firms. Our focus is on Rubinstein alternating offers bargaining in the vertical structure. Traditionally, the time interval between two consecutive offers is fixed and exogenous. Several authors have relaxed this assumption, supposing that there may be a "waiting time" and a response time before a counteroffer can be made (Perry and Reny, 1993 and Sakovics, 1993). The former exemplify the waiting time by considering a company manager who might have to discuss offers with the company president at fixed (weekly) meetings, whilst a response time occurs since offers may be complex and take time to understand and digest. Following Muthoo (1999), we consider that players can have unequal response time, defined as the amount of time it takes a player to make a counteroffer after rejecting the offer of his opponent. The contribution of this paper is that these response times are chosen strategically. In particular, we calculate the intermediate price that arises depending upon relative response times and who is the first to propose a deal. The choice of the response time between offers as a strategic variable in a Rubinstein model with incomplete information has been used in Admati and Perry (1987) and Cramton (1992) as a mechanism for players to signal their reservation prices, and by Colby (1995) to signal surplus size in bargaining over agricultural property rights.<sup>1</sup> Here we show that the use of response time by managers as a strategic variable allows them in effect to choose the market structure in which they wish to operate.

Section 2 presents the analysis of the successive monopoly that we examine, and Section 3 considers the managers' optimal choice of their response time. Section 4 exemplifies the analysis for a linear demand and Section 5 concludes.

## 2. A simple vertical structure

A single manufacturer produces an intermediate good at a constant unit cost,  $c < 1$  selling it to a single downstream retailer who bears no retailing cost for simplicity. The retailer has a monopoly on a technology that transforms one unit of the intermediate good into one unit of the final good.  $p_w$  denotes the wholesale price of the intermediate good,  $p$  the consumer price and  $q$  denotes the quantity bought by the retailer. Consumer demand is  $q = D(p)$  with  $\frac{dD}{dp} < 0$ . The payoff functions of the manufacturer ( $m$ ) and the retailer ( $r$ ) can be written as:

$$\pi^m = (p_w - c) D(p) \quad (1)$$

$$\pi^r = (p - p_w) D(p) \quad (2)$$

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<sup>1</sup>Also other phenomena related to response time have been investigated. Examples include the determination of bargaining power in wage bargaining (Cahuc et al., 2006), and the effect of response time on the perception of the outcome in marketing channels (Srivastava and Oza, 2006).

where the terms  $(p_w - c)$  and  $(p - p_w)$  measure the successive mark-ups. Given the wholesale price, the retailer chooses the final price of the good to maximize profit in the downstream market; the first-order condition for this choice is

$$\frac{\partial \pi^r}{\partial p} = D(p) \left( 1 - \varepsilon_p^D \left( \frac{p_w}{p} \right) \right) = 0 \quad (3)$$

where  $\varepsilon_p^D = \frac{dD}{dp} \frac{p}{D(p)} < 0$  is the price elasticity of demand. Denote the solution to (3) by  $p^*(p_w)$ .

Traditionally, if the firms are independent, the manufacturer would typically choose the wholesale price to maximize  $\pi^m$ , taking account of the effect that any decision would have in the downstream market. The first-order condition for the optimal choice is

$$\frac{\partial \pi^m}{\partial p_w} = D(p^*) \left( 1 + \varepsilon_p^D \left( \frac{p_w - c}{p^*} \right) \frac{dp^*}{dp_w} \right) = 0. \quad (4)$$

Denote the solution to (4) by  $p_w^{mi}$  (non-integrated). If the two firms are vertically integrated, the wholesale price is set internally at marginal cost,  $p_w^i = c$ , to take out as much surplus in the downstream market as possible. A bargaining procedure that introduces the idea of response time into the negotiation follows Rubinstein (1982) in which the wholesale price would be determined by successive rounds of offer and counteroffer until agreement is reached. Following Muthoo (1999, p193), we allow players to have unequal response time, denoted by  $\Delta_i$   $i = r, m$ , that are defined as the amount of time it takes a player to make a counteroffer after rejecting the offer of his opponent. To isolate the impact of these response times, we assume that the players discount future utilities at a common rate  $d > 0$ , and define  $\delta_i \equiv \exp(-d\Delta_i)$  as a measure of the cost to player  $i$  of rejecting an offer. Hence a short response time of player  $i$  ( $\Delta_i \rightarrow 0$ ) means that he responds quickly, implying that the cost of rejecting an offer is low ( $\delta_i \rightarrow 1$ ). In most of the analysis, we use the notation in terms of  $\delta_i$  ( $0 < \delta_r, \delta_m < 1$ ) for simplicity of exposition.<sup>2</sup>

With  $t = 0, 1, \dots$  as the time index, the discounted payoff functions, taking account of the optimal price in the consumer market, are

$$\begin{aligned} \pi^m(p_w, t) &= \delta_m^t (p_w - c) D(p^*(p_w)) \\ \pi^r(p_w, t) &= \delta_r^t (p^*(p_w) - p_w) D(p^*(p_w)). \end{aligned}$$

The manufacturer and retailer make offers and counteroffers over the wholesale price, where  $p_w^{(j)}$  denotes the offer made by  $j = m, r$ . The subgame perfect Nash equilibrium offers solve the two indifference conditions:

$$\begin{aligned} \pi^r(p_w^{(m)}, 0) &= \pi^r(p_w^{(r)}, 1) \\ \pi^m(p_w^{(r)}, 0) &= \pi^m(p_w^{(m)}, 1) \end{aligned}$$

so that each firm is indifferent between accepting the current offer of his opponent, and

<sup>2</sup>A property of the Rubinstein solution is that it approaches the Nash result as  $\delta_r \rightarrow 1$  and  $\delta_m \rightarrow 1$ . See Binmore *et al.* (1986).

making a counteroffer in the next period. Specifically, the offers satisfy

$$(p^*(p_w^{(m)}) - p_w^{(m)}) D(p^*(p_w^{(m)})) = \delta_r (p^*(p_w^{(r)}) - p_w^{(r)}) D(p^*(p_w^{(r)})) \tag{5}$$

$$(p_w^{(r)} - c) D(p^*(p_w^{(r)})) = \delta_m (p_w^{(m)} - c) D(p^*(p_w^{(m)})) \tag{6}$$

with solutions  $p_w^{(r)}(\delta_r, \delta_m)$  if the retailer makes the first offer and  $p_w^{(m)}(\delta_r, \delta_m)$  if the negotiations are started by the manufacturer.<sup>3</sup>

### 3. Optimal response times

The intermediate price in the Rubinstein solution depends on the two firms' response times. We now consider the case in which each negotiator can choose his response time  $\Delta_i \in (0, \infty)$ ,  $i = m, r$  leading to an implied value of  $\delta_i \in (0, 1)$ ,  $i = m, r$ . We have in mind a three stage game in which at stage 1, the retailer and the manufacturer simultaneously choose their response times; at stage 2 the negotiators play the Rubinstein bargaining game, and at stage 3 production decisions are made and profits reaped. Optimal actions at stage 2 are given by  $p_w^{(r)}(\delta_r, \delta_m)$  and  $p_w^{(m)}(\delta_r, \delta_m)$ , and at stage 3 the retailer sets  $p^*(p_w^{(j)})$  where  $j$  is the identity of the firm that leads the negotiation at stage 2.

At stage 1, the Nash equilibrium is characterized by choices  $\delta_r^*$  and  $\delta_m^*$  such that

$$\pi^m(\delta_r^*, \delta_m^*) \geq \pi^m(\delta_r^*, \delta_m) \tag{7}$$

$$\pi^r(\delta_r^*, \delta_m^*) \geq \pi^r(\delta_r, \delta_m^*). \tag{8}$$

Suppose that the manufacturer leads the negotiations. Maximizing its payoff by choice of  $\delta_m$  implies

$$\frac{\partial \pi^m}{\partial \delta_m} = D(p^*) \left( 1 + \varepsilon_p^D \left( \frac{p_w^{(m)} - c}{p^*} \right) \frac{dp^*}{dp_w^{(m)}} \right) \frac{dp_w^{(m)}}{d\delta_m} = 0. \tag{9}$$

Notice the similarity between (9) and (4); essentially, the manufacturer would choose a response time with the aim of enforcing the price  $p_w^{ni}$  since this maximizes his payoff. Whether this can be achieved as an equilibrium is dependent upon whether this choice, and the response of the retailer satisfy (7) and (8) simultaneously.

If the retailer leads the negotiations, it would prefer to have as low a price as possible in order to secure profit for itself. This implies that the retailer prefers to set its price at  $c$ , and would choose his response time such that this is enforced.

### 4. An example - linear demand

Suppose that  $D(p) = 1 - p$ . The following solutions obtain for the non-integrated and integrated solutions.

	$p_w$	$p^*$	$\pi^m$	$\pi^r$	$\pi^m + \pi^r$
Non-integrated	$\frac{1+c}{2}$	$\frac{3+c}{4}$	$\frac{(1-c)^2}{8}$	$\frac{(1-c)^2}{16}$	$\frac{3(1-c)^2}{16}$
Integrated	$c$	$\frac{1+c}{2}$			$\frac{(1-c)^2}{4}$

<sup>3</sup>The dependence of these expressions on  $c$  is suppressed for ease of notation.

Given the retailer's monopoly price  $p = \frac{1+p_w}{2}$ , (1) and (2) are in the linear case:

$$\pi^m = \frac{1}{2}(p_w - c)(1 - p_w) \tag{10}$$

$$\pi^r = \frac{1}{4}(1 - p_w)^2. \tag{11}$$

Consider now the Rubinstein solution. The indifference conditions in (5) and (6) are given by:

$$\begin{aligned} (1 - p_w^{(m)})^2 &= \delta_r (1 - p_w^{(r)})^2 \\ (p_w^{(r)} - c)(1 - p_w^{(r)}) &= \delta_m (p_w^{(m)} - c)(1 - p_w^{(m)}). \end{aligned}$$

This system gives the solution<sup>4</sup>:

$$p_w^{(m)} = \frac{1}{1 - \delta_r \delta_m} \left( \sqrt{\delta_r} (1 - \delta_m \sqrt{\delta_r}) c + 1 - \sqrt{\delta_r} \right) \tag{12}$$

$$p_w^{(r)} = \frac{1}{1 - \delta_r \delta_m} \left( (1 - \delta_m \sqrt{\delta_r}) c + \delta_m (\sqrt{\delta_r} - \delta_r) \right) \tag{13}$$

where  $p_w^{(m)} > p_w^{(r)}$ . The equilibrium price (12) proposed by the manufacturer  $p_w^{(m)}$  decreases in  $\delta_r$  and increases in  $\delta_m$

$$\frac{\partial p_w^{(m)}}{\partial \delta_r} = -\frac{(1 + \delta_m (\delta_r - 2\sqrt{\delta_r}))}{2\sqrt{\delta_r} (1 - \delta_m \delta_r)^2} (1 - c) < 0 \tag{14}$$

$$\frac{\partial p_w^{(m)}}{\partial \delta_m} = \frac{\delta_r (1 - \sqrt{\delta_r})}{(1 - \delta_m \delta_r)^2} (1 - c) > 0. \tag{15}$$

Given the definition of  $\delta_i$ , it is immediate that  $\frac{\partial p_w^{(m)}}{\partial \Delta_r} > 0$ ,  $\frac{\partial p_w^{(m)}}{\partial \Delta_m} < 0$ . The equilibrium price (13) proposed by the retailer  $p_w^{(r)}$  increases in  $\delta_m$  (decreases in the response time  $\Delta_m$ )

$$\frac{\partial p_w^{(r)}}{\partial \delta_m} = \frac{\sqrt{\delta_r} (1 - \sqrt{\delta_r})}{(1 - \delta_m \delta_r)^2} (1 - c) > 0. \tag{16}$$

The effect on  $p_w^{(r)}$  of a decrease in  $\Delta_r$ , leading to an increase in  $\delta_r$  is ambiguous. We have

$$\frac{\partial p_w^{(r)}}{\partial \delta_r} = \frac{1}{2\sqrt{\delta_r}} \delta_m \frac{(\delta_r \delta_m - 2\sqrt{\delta_r} + 1)}{(1 - \delta_r \delta_m)^2} (1 - c) < 0 \text{ if } \delta_m < \frac{2\sqrt{\delta_r} - 1}{\delta_r} \tag{17}$$

Note that whatever the value of  $\delta_m$ ,  $p_w^{(r)} \rightarrow c$  when  $\delta_r \rightarrow 0$  or  $\delta_r \rightarrow 1$ .

#### 4.1 The retailer is the proposer

The derived impact of the response time on the equilibrium payoff of the retailer (11) is given by the expression for  $j = m, r$ :  $\frac{\partial \pi^r}{\partial \delta_j} = -\frac{1}{2} (1 - p_w^{(r)}) \frac{\partial p_w^{(r)}}{\partial \delta_j}$ . Since  $\partial p_w^{(r)} / \partial \delta_m > 0$

<sup>4</sup>This system has two solutions. In line with the discussion earlier, we keep the one which converges to the Nash solution when  $\delta_r = \delta_m = \delta \rightarrow 1$  (Hoel, 1986).

from (16), the equilibrium payoff of the retailer decreases in  $\delta_m$ . For a given value of  $\delta_m$ ,  $\partial p_w^{(r)}/\partial \delta_r$  in (17) is first positive, and becomes negative as  $\delta_r > \frac{(2-\delta_m-2\sqrt{1-\delta_m})}{\delta_m^2}$ .<sup>5</sup> Hence,  $\partial \pi^r/\partial \delta_r$  is negative for small values of  $\delta_r$  and positive for larger values so that the retailer would prefer to choose extreme response times in the negotiation. As noted above, both  $\delta_r \rightarrow 0$  and  $\delta_r \rightarrow 1$  result in an equilibrium intermediate price of  $p_w^{(r)} = c$  when the retailer is the first proposer. When  $\delta_r \rightarrow 0$ , the counteroffer made by the manufacturer in the subgame perfect equilibrium is  $p_w^{(m)} = 1$  so that there is maximum distance between the offers, and  $p_w^{(m)} = c$  when  $\delta_r \rightarrow 1$  so that each firm makes the same offer.

Inserting the equilibrium price offered by the retailer (13) into the firms' payoff functions (10) and (11) gives the following:

$$\pi^r(p_w^{(r)}(\delta_r, \delta_m)) = \frac{1}{4} \left( \frac{1 - \sqrt{\delta_r \delta_m}}{1 - \delta_r \delta_m} \right)^2 (1 - c)^2 \tag{18}$$

$$\pi^m(p_w^{(r)}(\delta_r, \delta_m)) = \frac{1}{2} \left( \frac{\delta_m \sqrt{\delta_r} (1 - \sqrt{\delta_r}) (1 - \sqrt{\delta_r \delta_m})}{(1 - \delta_r \delta_m)^2} \right) (1 - c)^2 \tag{19}$$

Note that  $\pi^m(p_w^{(r)}(0, \delta_m)) = \pi^m(p_w^{(r)}(1, \delta_m)) = 0$ , so that the choice of  $\delta_m$  made by the manufacturer does not influence the payoffs in equilibrium. Also, the retailer gets the payoff he could expect in an integrated structure  $\pi^r(p_w^{(r)}(0, \delta_m)) = \pi^r(p_w^{(r)}(1, \delta_m)) = \frac{1}{4} (1 - c)^2$ .

Proposition 1 summarizes.

**Proposition 1** *When the retailer leads the negotiations, it prefers to choose extreme response times  $\Delta_r \rightarrow 0$  (or equivalently  $\delta_r \rightarrow 1$ ) and  $\Delta_r \rightarrow \infty$  (or equivalently  $\delta_r \rightarrow 0$ ). The retailer captures all the surplus as in the integrated structure and the wholesale price gets pressed towards the unit cost of the intermediate good.*

#### 4.2 The manufacturer is the proposer

When the manufacturer makes the first proposal, the equilibrium wholesale price is given by  $p_w^{(m)}$ . Inserting the optimal price (12) in (11) and (10) gives the following payoffs:

$$\pi^r(p_w^{(m)}(\delta_r, \delta_m)) = \frac{1}{4} \left( \frac{\sqrt{\delta_r} (1 - \sqrt{\delta_r \delta_m})}{(1 - \delta_r \delta_m)} \right)^2 (1 - c)^2 \tag{20}$$

$$\pi^m(p_w^{(m)}(\delta_r, \delta_m)) = \frac{1}{2} \left( \frac{\sqrt{\delta_r} (1 - \sqrt{\delta_r}) (1 - \delta_m \sqrt{\delta_r})}{(1 - \delta_r \delta_m)^2} \right) (1 - c)^2 \tag{21}$$

For  $\delta_r > \frac{1}{4}$  the profit of the manufacturer (21) reaches a maximum at  $\delta_m = \frac{2\sqrt{\delta_r}-1}{\delta_r}$ . When the manufacturer leads the negotiations then it will choose  $\delta_m \rightarrow 0$  if  $\delta_r < \frac{1}{4}$  and  $\delta_m = \frac{2\sqrt{\delta_r}-1}{\delta_r}$  if  $\delta_r > \frac{1}{4}$ . In contrast to the case in which the retailer is the leader, the leading manager this time cannot capture all of the surplus. The optimal reaction of the retailer will be the following: if  $\delta_m = 0$  then from (20) the retailer gets  $\pi^r = \frac{\delta_r}{4} (1 - c)^2$  and when  $\delta_m = \frac{2\sqrt{\delta_r}-1}{\delta_r}$  then  $\pi^r = \frac{1}{16} (1 - c)^2$ . Hence, the retailer is indifferent between all

<sup>5</sup>This is the equivalent expression to (17).

the response times such that  $\delta_r \in [\frac{1}{4}, 1)$ . Proposition 2 summarizes this result, recasting it in terms of the equilibrium response times.

**Proposition 2** *When the manufacturer leads the negotiations, he chooses the response time  $\Delta_m = -\frac{1}{d} \ln \left( \frac{2\sqrt{\exp(-d\Delta_r)-1}}{\exp(-d\Delta_r)} \right)$  and the retailer is indifferent between all response times  $\Delta_r \in (0, \frac{-\ln \frac{1}{4}}{d}]$ . The retailer and the manufacturer get what they can expect in the non-integrated structure :  $\pi^r = \frac{1}{16} (1-c)^2$  and  $\pi^m = \frac{1}{8} (1-c)^2$  The price of the intermediate good is  $\frac{1}{2} (1+c)$ .*

Thus, the manufacturer optimally chooses an intermediate response time. A short response time increases the wholesale price, and will also increase the final price to the consumers, limiting demand.

Notice that the response times are chosen optimally here whilst the discount rate  $d$ , is held fixed. It is possible to get to the same result with a fixed response time for each firm and different discount rates. If we fix the common response time at  $\Delta$ , then the non-integrated structure (Proposition 2) is replicated if the manufacturer makes the first offer and has a discount factor which equates  $\delta_m = \frac{2\sqrt{\delta_r-1}}{\delta_r}$ , i.e.  $\exp(-d_m\Delta) = \frac{2\sqrt{\exp(-d_r\Delta)-1}}{\exp(-d_r\Delta)}$ , yielding  $d_m = -\frac{1}{\Delta} \ln \left( \frac{2\sqrt{\exp(-d_r\Delta)-1}}{\exp(-d_r\Delta)} \right)$ . Any combination of discount factors that satisfies this equation results in the same prices and profits as the integrated structure.<sup>6</sup> An integrated structure is replicated if the retailer leads negotiations (Proposition 1) and has a discount rate of  $d_r \rightarrow 0$  (implying  $\delta_r \rightarrow 1$ ) or  $d_r \rightarrow \infty$  (implying  $\delta_r \rightarrow \infty$ ).

## 5. Conclusion

Our starting point has been the remark by Tirole (1988) that vertical relationships between upstream firms and downstream firms are often complex. A standard textbook treatment of the issue is to investigate different structures in this setting, comparing the non-integrated and the integrated cases in order to show that the firms have an incentive to integrate in order to maximize industry profit. We propose a bargaining solution for the resolution of the interaction between an upstream supplier and a downstream manufacturer. We show that the endogenous choice of personal response time provides to the negotiators a way to choose the market structure in which they operate. In this way, the non-integrated and integrated structures in a vertical relationship can arise endogenously depending upon who proposes the wholesale price, with response times determined as part of the equilibrium.

In the Rubinstein model, there is an advantage to being the proposer; hence when the retailer makes the first proposal, it would prefer to press the intermediate price as far down towards marginal cost as possible. This is accomplished by setting a very short response time (implying a large value of future gains), or a very long response time (implying that future gains are worth little). In the former case, the retailer can start with an initial offer at the manufacturer's marginal cost. This would give the manufacturer a profit of zero, but the fact that the retailer discounts future gains little means that the manufacturer cannot make a counteroffer other than at this level that would be accepted by the opponent. The same is true when the retailer has a long response time, since the

<sup>6</sup>Note that we are not suggesting that discount rates are chosen optimally here, so that we do not have to account for the "best reply" of the retailer.

value of any counteroffer to it will be close to zero. This effectively gives the retailer the bargaining power, enforcing a similar result to that of vertical integration.

The intuition is similar when the manufacturer leads the negotiations; it prefers to set the monopoly price for its intermediate product, and chooses the response time accordingly. Once this response time and price is set, the retailer cannot increase its own profit by offering any other acceptable price, enforcing the successive monopoly solution.

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