Relative consumption of housing: Marginal saving subsidies and income taxes as a second-best policy?\textsuperscript{a,}\textsuperscript{*}

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\textbf{Abstract}

This paper analyzes whether marginal taxation of labor and capital income are useful second-best instruments for internalizing the externalities caused by conspicuous housing consumption, when the government is unable to implement a first best corrective tax on housing wealth. The rationale for studying income taxation in this particular context is that first best taxes on housing wealth may be infeasible (at least in a shorter time perspective), while income taxes indirectly affect both the level and composition of accumulated wealth. We show that a suboptimally low tax on housing wealth provides an incentive for the government to subsidize financial saving and tax labor income at the margin.

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\textsuperscript{c} Academic work on conspicuous consumption dates back at least to Veblen (1899), although the associated policy implications were briefly touched upon already by Mill (1848). An economic theory of relative consumption was first presented in Duesenberry (1949).

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driven by the desire to signal status or wealth, one would expect that clearly visible goods are more positional than less visible goods, i.e., that the utility of consuming visible goods to a larger extent is driven by preferences for relative consumption. Evidence from survey–experimenting studies suggests that this is also the case, since visible goods such as houses and cars have been found to be more positional than other, less visible goods (Alpizar et al., 2005; Solnick and Hemenway, 2005; Carlsson et al., 2007). Calculations presented in Alpizar et al. (2005) show that the degree of positionality (the extent to which relative consumption matters compared to absolute consumption) for housing is substantial: on average, about 50 per cent of the utility gain of additional expenditures on housing may be due to increased relative consumption. Therefore, individuals’ choices of housing seem to impose substantial externalities on other people.

If housing, at least in part, represents conspicuous consumption, a first best policy would be to tax housing wealth such that the externality that each individual imposes on other people becomes fully internalized. Yet, although taxes on housing wealth are used in many countries, the tax rates are often quite low; at least by comparison with the magnitude of the positional externality mentioned above. This argument will be substantiated below, where we show that our model combined with empirical evidence of relative consumption concerns would imply an annual tax on housing wealth of between 2 and 3 per cent of the market value under reasonable assumptions. However, in many countries (Denmark being a notable exception) property taxes are substantially below this rate. In addition, taxes on owner-occupied housing are in practice often lower than official tax rates imply. This is so since mortgage interest is often deductible from taxable income and since imputed rents go untaxed. This implies that capital in terms of housing is preferentially treated in comparison to other types of capital (e.g., Gervais, 2002). One reason for setting such low tax rates might be that property taxes are politically controversial, and homeowners constitute an influential group in society. In Sweden, for instance, the Homeowners’ Association was formed partly for the purpose of collective action against taxation of housing wealth and most likely contributed to the significant reduction in effective tax rates during the latest decade. Also, since taxes on housing wealth (or property in general) are often local or regional, the policy incentives implicit in such taxes may not correctly reflect positional externalities; at least not if the consumption comparisons go beyond the local or regional jurisdiction.

Therefore, if an optimal corrective tax on housing wealth is not feasible, it is important to consider other instruments to correct for the externalities caused by conspicuous consumption of housing. In this paper, we focus on an optimal mix of labor and capital income taxation, which is defined conditional on the existing tax on housing wealth. A key role of real estate is to fund future consumption, suggesting that a marginal saving subsidy (negative marginal capital income tax) provides an incentive for individuals to save more in financial assets and spend less on housing. In turn, this policy counteracts the positional consumption externality that housing gives rise to. One may thus conjecture that marginal saving subsidies/taxes are key second best instruments for internalizing externalities associated with durable goods consumption. Yet, since such subsidies/taxes are still only indirect instruments for correction, there is room for other instruments as well, and we consider an optimal labor income tax alongside the savings-oriented policy described above. The idea here is that positional concerns may lead individuals to increase their labor supply, and a marginal labor income tax policy can be designed to counteract this incentive. Therefore, the mixture of labor and capital income taxes constitutes an interesting combination, since it will affect both the level and composition of accumulated wealth.

Our study contributes to a large literature on tax and other policy responses to consumption positionality and to some extent to the ample literature on taxation of owner-occupied housing. The latter has shown that preferential tax treatment of owner-occupied housing causes over consumption of housing services (see, e.g., Skinner, 1996; Gervais, 2002). However, to the best of our knowledge, this literature disregards externalities caused by positional preferences. The literature dealing with optimal policy responses to positional externalities focuses almost exclusively on (a) positional concerns for non-durable goods, and (b) typically also on model economies with one single private consumption good (in addition to leisure). In other

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2 See also Zahirovic-Herbert and Chatterjee (2011), who find that people are willing to pay more for homes with a name attached to it, and Patacchini and Venanzoni (2014), who find significant peer-effects on the demand for housing quality in USA.

3 In Sweden, for instance, the tax on housing property is 0.75 per cent of the value of the property up to a maximum limit (where the value attached to each property by the tax authority is typically lower than the market value). The corresponding rate is between 0.2 and 0.7 per cent in Norway, where the municipalities freely decide on the implementation (about 30 per cent of the municipalities did not implement such a tax in 2009). Denmark applies a system with two rates: 1 per cent if the market value of the property is less than 3 million DKK and 3 per cent otherwise. The corresponding tax in Finland is 0.32–0.75 per cent of the market value. In Germany, the tax rate varies between 0.26 and 0.35 per cent. Great Britain also applies a zero rate except for homes with very high market values. Property taxes in the U.S. are based on the market value and the rates vary between states (although the tax is formally collected at the local level); in California, the maximum rate is 1 per cent (The Swedish Homeowners’ Association, http://www.villaagarna.se/; Wikipedia, https://wikipedia.org; Germany Trade and Invest, http://www.gtaf.de/GTAI/Navigation/EN/Invest/Investment-guide/The-tax-system/taxation-of-property.html; International living, http://internationalliving.com/real-estate/countries/france/taxes/; http://internationalliving.com/real-estate/countries/spain/taxes/; Properties in Europe, http://www.properties-in-europe.com/info_italy_tax.htm; http://soumi.f).

4 Recent evidence suggests that social reference groups are not formed solely based on the local environment; possibly due to technological developments of social media and the Internet. For instance, Becchetti et al. (2010) found that the importance of social comparisons between countries has increased over time, and Clark and Senik (2010) found that that Internet access is positively correlated with relative consumption concerns.

5 Goerke and Hellesen (2013) show in a theoretical model that individuals under certain conditions supply more labor if they are concerned with their relative consumption than they would in an undistorted economy without relative concerns. Empirical evidence pointing in this direction is presented by Bowles and Park (2005). They consider a model where individuals derive utility from their own consumption relative to the consumption of a reference group with higher income, and show empirically that increased inequality is associated with a larger number of work hours.

words, most previous work does not distinguish between positional and non-positional goods, and there is surprisingly little research on how tax or other policy instruments ought to respond to relative concerns for durable consumption. One important exception from category (b) is Eckerstorfer and Wendner (2013) dealing with optimal linear commodity taxation in a two-good economy, where one of the goods is positional and the other non-positional. Their main contribution is to examine tax policy implications of non-atmospheric and asymmetric consumption externalities, and by studying the joint tax policy implications of relative consumption concerns and inequality-aversion.7 Yet, their study neither addresses the implications of relative consumption of durable goods, nor the optimal mix of labor and capital income taxation when a flexible direct instrument to target the externality is unavailable, which are the key-ingredients of the present paper. To our knowledge, the only earlier study dealing with policy implications of relative concerns for durable goods is Aronsson and Mannberg (2013). They consider an overlapping generations model where each consumer lives for three periods and analyze the joint tax policy implications of positional concerns for housing and a self-control problem caused by quasi-hyperbolic discounting. Their contribution is to show how the optimal mix of taxes on housing wealth and capital income varies over the individual life-cycle, as well as how it depends on whether consumers have naïve or sophisticated attitudes to their self-control problems.

The present paper contributes to the literature in primarily two ways. First, we analyze indirect instruments to correct for the externalities caused by conspicuous consumption when a direct instrument is not available; a scenario of clear practical relevance for reasons discussed above. Our setting also means that we extend the study of optimal income taxation in dynamic economies where the consumers have positional preferences beyond the standard framework with only one private, non-durable consumption good (in which case income taxes constitute direct instruments for correction).8 As such, our study takes a different direction than the paper by Eckerstorfer and Wendner (2013), which focuses on optimal commodity taxation in a model-economy with positional and non-positional goods. Second, there is not much research on the tax policy implications of positional durable goods; the only earlier study that we are aware of is Aronsson and Mannberg (2013) referred to above. We supplement their study by considering (i) a different policy problem, (ii) another mix of tax instruments, and (iii) by examining a second best optimal tax policy. As such, the present paper’s focus on durable consumption also contributes to the literature on a more general level.

In Section 2, we present the benchmark model and briefly describe how a government may implement a first best optimum through marginal taxation of housing wealth. Section 3 assumes that the possibility to tax housing wealth is restricted and examines the optimal second best taxation of labor and capital income. The results show that if the marginal tax on housing wealth falls short of the value of the marginal positional externality, the second best optimal policy features a marginal saving subsidy and a marginal labor income tax. In Section 4, we extend the analysis by considering two issues that the benchmark model does not address: non-separability between leisure and housing, and bequest motives. Section 5 summarizes and concludes the paper.

2. A benchmark model

The model presented here aims at capturing two aspects of housing: (1) the possibility that individuals derive utility from their relative consumption compared to referent others, and (2) that the individual invests in housing wealth when young and consumes this housing wealth along with other accumulated wealth when old. To accomplish this task in the simplest possible way, we abstract from the price formation process in assuming that the supply of housing units is infinitely elastic in each period, and that the individual can sell any remaining units of the house when old. As such, and by analogy to a large literature on optimal taxation, we assume that the production technology (in our case in both the durable and non-durable goods sectors) is such that the producer and factor prices are fixed in each period, although not necessarily constant over time.9

The economy is represented by an overlapping generations model, in which individuals live for two periods; they work in the first and are retired in the second. Each individual derives utility from the consumption of two goods: a durable good referred to as housing and a non-durable good. Housing is assumed to be a positional good in the sense that individuals derive utility from their relative consumption compared to referent others (in addition to the utility gain of the absolute

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7 Among other things, they show that if the externality is non-atmospheric (meaning, in their study, that individuals differ in their contribution to the externality), the principle of targeting is no longer fully applicable such that both commodity taxes typically are used to correct for the externality caused by the consumption of the positional good. Eckerstorfer (2014) extends the analysis to a problem of optimal mixed taxation under asymmetric information based on a static model with two non-durable consumption goods (in addition to the labor–leisure choice), where the government decides on an optimal nonlinear income tax and a set of linear commodity taxes.


9 Our abstraction from the price formation process and from location issues does not mean that we think these issues to be unimportant. However, in this paper, we are primarily interested in analyzing tax policy to counteract positional externalities. As long as the consumers have preferences for their relative consumption of housing (which is the basic assumption on which our paper rests), the policy incentives characterized below would also be present in a more general framework (although possibly combined with other policy incentives depending on the functioning of the housing market). The model presented below is analytically convenient and allows us to identify basic mechanisms behind a second best optimal policy designed to correct for positional consumption externalities.
consumption), while the non-durable good is completely non-positional.\footnote{The assumption that the non-durable good is completely non-positional is a simplification, yet of little practical importance as long as non-durables are less positional than durable goods.} Individuals also derive disutility from work effort. To simplify the notation (since we are focusing on efficiency aspects of public policy), we also assume that individuals of the same generation are identical and normalize the number of individuals per generation to one.

An individual entering the economy at the beginning of period \( t \) to be referred to as generation \( t \) is young (in the sense of being active in the labor market) in period \( t \) and old in period \( t + 1 \). Let \( c_{0,t} \) and \( c_{1,t+1} \) denote the consumption of the non-durable good by the young and old generation \( t \), respectively, \( l_t \) denotes labor supply, while \( h_t \) denotes housing consumption. The lifetime utility function facing an individual of generation \( t \) is given by

\[
U_t = U(c_{0,t}, I_t, h_t, h_t, c_{1,t+1}) = u_0(c_{0,t}) + \nu(l_t) + \phi(h_t) + \Phi(h_t - h_t) + u_1(c_{1,t+1})
\]

Eq. (1) means that the individual derives utility both from his/her absolute consumption, \( h_t \), and relative consumption, \( h_t - h_t \), of housing, as well as from his/her absolute consumption of the non-durable good and use of leisure. Although the housing choice is made during the first period, the utility derived from housing can be interpreted as referring to both periods of life. There are several possible ways of conceptualizing a unit of housing, and one can (at least) think of \( h_t \) as either a purely quantitative measure (e.g., the number of square meters) or in terms of units of quality of housing (e.g., finishes and fixtures).\footnote{We are grateful to a referee for pointing out different possible interpretations.} Even if both interpretations may be relevant from the perspective of relative concerns, which is the key issue in our paper, we prefer the latter (quality oriented) interpretation primarily because our model lacks a spatial dimension. The variable \( h_t \) denotes the reference consumption in period \( t \). We assume that the functions \( u_0(\cdot), \phi(\cdot), \Phi(\cdot), \) and \( u_1(\cdot) \) are increasing and strictly concave in their respective arguments, while \( \nu(\cdot) \) is decreasing and strictly concave. All goods are normal. Also, the individual is assumed to act as an atomistic agent and treat \( h_t \) as exogenous. The separable structure of Eq. (1) is maintained throughout the analysis in Sections 2 and 3 and will allow us to sign key comparative statics (see Eqs. (16a) and (16b)). However, to simplify the presentation (and avoid unnecessary notation in the formulas), the first order conditions will, nevertheless, be written in terms of the general function \( U(\cdot) \).

Let \( y_t = w_t h_t \) denote labor income, \( w_t \) the wage rate, \( s_t \) saving, \( r_t \) the interest rate, and \( P_t \) the price per unit of housing in period \( t \). Before imposing any restriction on the possibility to tax housing wealth, the individual’s budget constraint can be written as follows:

\[
c_{0,t} = y_t - s_t - P_t h_t - T_0(P_t h_t) - T_0(y_t) \tag{2}
\]

\[
c_{1,t+1} = s_t(1 + r_{t+1}) - T_1(s_t r_{t+1}) + P_{t+1} h_t (1 - \delta) \tag{3}
\]

where \( \delta \) denotes the rate of depreciation of a house, \( T_0(\cdot) \) is a tax on housing wealth paid when young, \( T_1(\cdot) \) a labor income tax paid when young, and \( T_1(\cdot) \) a capital income tax paid when old. When young, the individual consumes (and invests in) housing, consumes the non-durable good, and saves on the capital market; when old, he/she uses the financial wealth and housing wealth for non-durable consumption. The labor and capital income tax functions are assumed to be nonlinear such that the government may carry out any redistribution between generations on a lump-sum basis (if it wishes to do so). As will be described in greater detail below, this enables us to concentrate on corrective aspects of marginal taxation.

An individual of generation \( t \) chooses housing, non-durable consumption, and work hours to maximize the utility given by Eq. (1) subject to the budget constraint in Eqs. (2) and (3), while treating \( w_t, r_{t+1}, P_t, P_{t+1}, \) and \( h_t \) as exogenous. By substituting the budget constraint into the objective function to replace the consumption of non-durable goods, we obtain a decision-problem in \( s_t, h_t, \) and \( l_t \) with the first order conditions:

\[
s_t : -u_t c_{0,t} + u_t c_{1,t} (1 - r_{t+1}) = 0 \tag{4a}
\]

\[
h_t : -u_t c_{0,t} [P_t + P_t T_0'(P_t h_t)] + u_t c_{1,t} P_{t+1} (1 - \delta) = 0 \tag{4b}
\]

\[
h_t : t c_{0,t} w_t (1 - T_0(y_t)) + u_t l_t = 0 \tag{4c}
\]

where \( u_t c_{0,t} = \partial u_t / \partial c_{0,t}, u_t c_{1,t} = \partial u_t / \partial c_{1,t} \), and \( u_t h_t = \partial u_t / \partial h_t \). Denote partial derivatives of the utility function, while \( T_0'(P_t h_t) \) and \( T_1'(s_t r_{t+1}) \) are the marginal tax rates on housing wealth, labor income, and capital income, respectively. Note also that the old consumer makes no active decision other than selling the house: he/she then spends the
remaining wealth for consumption of the non-durable good. For purposes of comparison, in Section 4 we consider a case where generation t instead leaves the house as a bequest to the next generation.

2.1. Optimal tax policy in the benchmark model

The government is first mover vis-à-vis the private sector and aims at correcting the positional externalities. We assume a general social welfare function that is increasing in the lifetime utility faced by each generation

\[ W = W(U_0, U_1, \ldots). \]  

Note that nonlinear taxation of labor income and capital income accompanied by a flexible tax on housing wealth will allow the government to perfectly control the hours of work, the consumption of durable and non-durable goods, and the capital stock in the economy. Therefore, we follow convention in the literature on optimal nonlinear and mixed taxation in writing the social decision-problem as a direct decision-problem in terms of work hours, private consumption, and the capital stock, instead of in terms of parameters of the tax functions (such as intercepts and marginal tax rates).\(^{14}\) The optimal marginal tax rates can then be derived through a comparison of social and private first order conditions. The resource constraint for the economy as a whole in period t can be written as\(^{15}\)

\[ y_t + k_t(1 + r_t) - P_t(h_t - h_{t-1}(1 - \delta)) - c_{0,t} - c_{1,t} - k_{t+1} = 0 \]  

where \(k_t\) is the physical capital stock defined such that \(s_{t-1} = k_t\) for all \(t\). The resource allocation preferred by the government can be derived by choosing \(c_{0,t}, h_t, c_{1,t}, h_t\) and \(k_t\) for all \(t\) to maximize the social welfare function subject to the resource constraint and \(h_t = h_t\) for all \(t\). The latter follows because the individuals are identical, and the (externality correcting) government recognizes how the reference consumption is determined. The Lagrangean corresponding to this decision-problem is given by

\[ L = W + \sum_t \lambda_t \left[ y_t + k_t(1 + r_t) - P_t(h_t - h_{t-1}(1 - \delta)) - c_{0,t} - c_{1,t} - k_{t+1} \right] \]  

where \(\lambda_t\) is the Lagrange multiplier of the resource constraint in period \(t\). Define

\[ \text{MRS}^t_{h,0} = \frac{U_{t,h}}{U_{t,0}} > 0, \quad \text{MRS}^t_{c,0} = \frac{U_{t,c}}{U_{t,0}} < 0, \quad \text{MRS}^t_{c,c} = \frac{U_{t,c}}{U_{t,c}} > 0 \]

to be the marginal rates of substitution between housing and first period consumption; between work effort and first period consumption; and between the first and second period consumption, respectively, for an individual of generation \(t\). The social first order conditions can then be written as follows:

\[ \text{MRS}^t_{h,0} = (1 + r_{t+1}) \]

\[ \text{MRS}^t_{h,c} + \frac{U_{t,h}}{U_{t,c}} - P_t + \frac{1}{\text{MRS}^t_{c,c}} P_{t+1}(1 - \delta) = 0 \]

\[ -\text{MRS}^t_{c,c} = w_t \]  

Now, let \(\Delta_t = h_t - \tilde{h}_t\), and notice from Eq. (1) that \(U_{t,h} = -\partial \Phi(\Delta_t)/\partial \Delta_t = -\Phi_{t,\Delta}\), and \(U_{t,c} = \Phi_{t,h} + \Phi_{t,c}\Delta\) where \(\Phi_{t,h} = \partial \Phi(h_t)/\partial h_t\). We can then define the degree of positionality for housing in period \(t\) as the fraction of the overall utility gain from an additional dollar spent on housing that is due to increased relative consumption, i.e.,

\[ \alpha_t = -\frac{U_{t,h}}{U_{t,h}} \frac{\Phi_{t,\Delta}}{\Phi_{t,h} + \Phi_{t,\Delta}} \in (0, 1). \]

The degree of positionality is also interpretable as the marginal externality per unit of housing consumption. As such, it will play a key role in the analysis of marginal tax policy below. The first best optimal tax policy can then be summarized as follows based on Eqs. (4), (8) and (9):

**Observation 1.** The optimal tax policy in the benchmark model satisfies \(T_0'(w_t h_t) = 0\), \(T_1'(s_{0,t} r_{t+1}) = 0\) and \(P_t \Gamma_{0}(P_t h_t) = \alpha_t \text{MRS}^t_{h,0}\) for all \(t\).

Clearly, since the government of the benchmark model has access to a flexible tax on housing wealth, it may use this tax to fully internalize the externality caused by conspicuous housing consumption. This explains why the marginal labor and capital income tax rates are equal to zero: since there are no other distortions than the externality caused by each consumer’s desire to have more units of housing than other people, and since there is no need to distort behavior just to raise revenue,

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14 See, e.g., Stiglitz (1982).
15 Note that the analysis presupposes that \(h_t \geq h_{t-1}(1 - \delta)\); otherwise, old consumers would not be able to consume all their housing wealth.
the only reason for taxing housing wealth in this model is to internalize the positional externality. Consequently, the policy rule for this tax depends on how each individual’s housing choice affects the well-being of other people (as reflected in the degree of positionality). It does not explicitly depend on other aspects of the housing market such as the change in the price over the individual life-cycle. 

The left hand side of the tax formula for housing measures the marginal tax on housing wealth times the price paid per unit of housing in period $t$, i.e., the tax payment for an additional unit of housing, which is set equal to the degree of positionality times the marginal willingness to pay for housing in period $t$. Therefore, if $\alpha_t$ is in the neighborhood of 0.5, as suggested by empirical evidence presented in the introduction, about 50 per cent of the consumer’s marginal willingness to pay for housing represents social waste and should be taxed away. To exemplify and give some substance to the argument made in the introduction that existing taxes on housing wealth often fall short of the marginal positional externality, we assume that $P_t = P_{t+1} / (1 + \tau_{t+1})$ (in which case there is no “bubble-component” in the price), and that $\delta$ is between 2 and 4 per cent. If we further assume that an individual owns a house for 30 years, our model implies that the optimal yearly tax on housing wealth is between 1.3 and 2.7 per cent if $\alpha_t = 0.4$, and between 2 and 4 per cent if $\alpha_t = 0.5$. Except for Denmark, the countries surveyed above (see footnote 3) apply much lower rates.

An interesting special case of our model arises when $\delta = 1$, in which the home is no longer interpretable as a durable asset to the individual. This special case can be interpreted such that the individuals rent their homes instead of owning them, also meaning that the optimal tax on housing wealth ought to be re-interpreted as an ordinary commodity tax attached to the rent payment. As can be seen from Observation 1, since the optimal marginal tax on housing wealth does not depend directly on $\delta$, and provided that the relative concerns are only associated with the quality of the home (as we assume above), the optimal policy rule takes exactly the same form irrespective of whether individuals own or rent their homes. The intuition is, of course, that the externality each individual imposes on other people is independent of ownership. We show in the next section that this qualitative result also carries over to the case where a flexible tax on housing wealth (or commodity tax on rented homes) is not available to the government; yet with a slightly modified interpretation since one of the policy rules depends explicitly on $\delta$ in that case.

3. Suboptimal marginal taxation of housing wealth

Suppose that the government is unable to implement the optimal tax on housing wealth described in the previous section. The tax is now given by $T_0(P_t, h_t) = t_t P_t h_t$, in which $t_t$ is a nonnegative tax rate such that $t_t \leq \bar{t}_t$, where $\bar{t}_t$ denotes an upper limit. We can interpret the upper limit as reflecting the resistance against taxes on housing property described in the introduction. This formulation is also convenient as it encompasses an economy without taxation of housing wealth (in which $\bar{t}_t = 0$ for all $t$) and the first best optimal tax policy characterized above (where the constraint does not bind) as special cases. In this section, we assume that the constraint is binding, such that the tax rate on housing wealth is fixed. The question is then whether marginal taxation/subsidization of labor and capital income may be useful as supplementary instruments to correct for positional externalities.

To address this question, it is convenient to model the individual’s decision-problem in two stages. In the first stage, we choose $h_t$ to maximize lifetime utility in Eq. (1) subject to the following budget constraint:

\[
\begin{align*}
& b_{0,t} = c_{0,t} + P_t h_t (1 + \tau_t) \\
& b_{1,t+1} = c_{1,t+1} - P_{t+1} h_t (1 - \delta)
\end{align*}
\]

(10a)

(10b)

where $b_{0,t}$ and $b_{1,t+1}$ are treated as fixed incomes in the first and second period of life for generation $t$. This gives the first order condition

\[
-U_{t,h_t} P_t (1 + \tau_t) + U_{t,h_t} + U_{t,h_t} P_{t+1} (1 - \delta) = 0
\]

(11)

which implicitly defines the individual’s demand for housing as a function of $P_t (1 + \tau_t)$, $P_{t+1} (1 - \delta)$, $b_{0,t}$, $b_{1,t+1}$ and $\bar{h}_t$, i.e.,

\[
\begin{align*}
& h_t = \bar{h}_t \left( P_t (1 + \tau_t), P_{t+1} (1 - \delta), b_{0,t}, b_{1,t+1}, \bar{h}_t \right).
\end{align*}
\]

(12)

Notice that Eq. (12) is interpretable as a conditional demand function in the sense of measuring this demand conditional on the individual’s income and saving (which are part of $b_{0,t}$ and $b_{1,t+1}$). By substituting Eqs. (10a), (10b) and (12) into Eq. (1), the corresponding conditional indirect utility function can be written as

\[
\begin{align*}
& V_t = V \left( P_t (1 + \tau_t), P_{t+1} (1 - \delta), b_{0,t}, b_{1,t+1}, h_t, \bar{h}_t \right) = U \left( b_{0,t} - P_t (1 + \tau_t) h_t, \bar{h}_t, \bar{h}_t, b_{1,t+1} + P_{t+1} \bar{h}_t (1 - \delta) \right)
\end{align*}
\]

(13)

Although price changes over the individual life-cycle do not lead to a change in the policy rule for the corrective tax, they may, nevertheless, affect its level via the demand for housing. For instance, a change in the selling price may influence the corrective tax either through the degree of positionality or the marginal willingness to pay for housing (or both).

Harding et al. (2007) estimate the gross annual rate of depreciation to 3 per cent for the typical home in the U.S. and the depreciation net of maintenance to 2 per cent. A slightly higher opportunity cost might be motivated from commuting costs and other time-costs associated with a typical home.

This interpretation does not depend on the assumption that the individual only pays rent once. Even if we were to introduce a rent payment in the second period, this commodity tax would still be designed such that the marginal externality is taxed away.
where $h_t$ is given by Eq. (12). In the second stage, we derive the savings and labor supply behavior by choosing $s_t$ and $l_t$ to maximize the indirect utility function given by Eq. (13) subject to

$$b_{0,t} = y_t - s_t - T_0(y_t)$$  \hspace{1cm} (14a)

$$b_{1,t+1} = s_t(1 + r_{t+1}) - T_1(s_t r_{t+1}).$$  \hspace{1cm} (14b)

The first order conditions for this problem can be written as

$$-V_{t,b_t} + V_{t,b_t} [(1 + r_{t+1}) - T'_1(s_t r_{t+1})] r_{t+1} = 0$$  \hspace{1cm} (15a)

$$V_{t,b_t} w_t [1 - T'_0(y_t)] + V_{t,c} = 0$$  \hspace{1cm} (15b)

in which $V_{t,b_t} = \partial V_t/\partial b_{0,t}$, $V_{t,b_t} = \partial V_t/\partial b_{1,t+1}$ and $V_{t,c} = \partial V_t/\partial l_t$.

3.1. Second best optimal income taxation

As explained above, the savings and labor supply choices by generation $t$ affect the conditional demand for housing via $b_{0,t}$ and $b_{1,t+1}$. It is, therefore, instructive to begin by deriving comparative statics of the housing demand with respect to $b_{0,t}$ and $b_{1,t+1}$. Since the government aims at internalizing the positional externality and incorporates it into its decision-problem that $h_t = h_t$, we differentiate Eq. (11) with respect to $h_t$, $b_{0,t}$ and $b_{1,t+1}$, while using $h_t = h_t$. This gives

$$\frac{dh_t}{db_{0,t}} = \frac{U_{t,c}Q_t}{\Theta} = 0$$ \hspace{1cm} (16a)

$$\frac{dh_t}{db_{1,t+1}} = -\frac{U_{t,c}P_{t+1}(1 - \delta)}{\Theta} < 0$$ \hspace{1cm} (16b)

where $U_{t,c}Q_t < 0$, $U_{t,c}P_t < 0$ and

$$\Theta = U_{t,c}Q_t (P_t(1 + r_t))^2 + U_{t,h} + U_{t,c}P_t(1 - \delta)^2 + U_{t,h} < 0.$$  

Double sub-script (e.g., $c_tQ_t$) denotes second order partial derivative.

With a fixed tax on housing, the social decision-problem is a second best problem, which can be written as

$$\max_{b_{0,t},b_{1,t+1},k_t,l_t} W(V_0, V_1, \ldots ,) \hspace{1cm} (17)$$

s.t. : $y_t - b_{0,t} - k_t(1 + r_t) - \tau_t h_t - b_{1,t} - k_{t+1} = 0 \text{ for all } t$

where the resource constraint is here expressed in terms of the “savings-adjusted” measures of disposable income given in Eqs. (14a) and (14b). The resource constraint is derived by using $\tau_t P_t h_t + T_0(y_t) + T_1(s_{t-1} r_{t}) = 0$ and $s_{t-1} = k_t$ in combination with Eqs. (14a) and (14b).\footnote{With general, nonlinear taxes on labor and capital income, the government is able to control the hours of work, disposable income, and the capital stock, although it no longer has a direct instrument to control the housing choice. Instead, since $r_t$ is fixed, the government is only able to influence the revenue from the housing wealth tax indirectly via the effects that the labor and capital income taxes have on $h_t$ (through $b_0$ and $b_1$), which explains the resource constraint in Eq. (17). In a way similar to the analysis in Section 2, therefore, we have still written the resource constraint in terms of the direct decision-variables. An alternative would be to write the resource constraint as in Eq. (6) above, which can be derived by substituting Eqs. (10a) and (10b) into Eq. (17), and then use that $c_t$ and $s_t$ are also functions of $b_0$ and $b_1$. The latter approach (yet modified to suit their model) was taken by Edwards et al. (1994) in their study on optimal mixed taxation. It does not matter for the results derived below which option is chosen.} By using $\lambda_t$ to denote the Lagrange multiplier of the resource constraint in period $t$ (as before), the social first order conditions for $b_{0,t}$, $b_{1,t+1}$, $k_t$, and $l_t$ (the conditions characterizing an optimal allocation for generation $t$) become

$$b_{0,t} : \frac{\partial W}{\partial V_t} \left( V_{t,b_t} + V_{t,h} \frac{dh_t}{db_{0,t}} \right) + \lambda_t \left( -1 + \tau_t P_t \frac{dh_t}{db_{0,t}} \right) = 0$$ \hspace{1cm} (18a)

$$b_{1,t+1} : \frac{\partial W}{\partial V_t} \left( V_{t,b_t} + V_{t,h} \frac{dh_t}{db_{1,t+1}} \right) + \lambda_t \tau_t P_t \frac{dh_t}{db_{1,t+1}} - \lambda_{t+1} = 0$$ \hspace{1cm} (18b)

$$k_{t+1} : \lambda_{t+1}(1 + r_{t+1}) - \lambda_t = 0$$ \hspace{1cm} (18c)

$$l_t : \frac{\partial W}{\partial V_t} V_{t,l} + \lambda_t w_t = 0.$$ \hspace{1cm} (18d)

By using Eqs. (18a)–(18d) together with $(1 + r_{t+1}) - V_{t,b_t}/V_{t,k_t} = T'_1(s_t r_{t+1}) r_{t+1}$ and $w_t + V_{t,l}/V_{t,b_t} = w_t T'_0(y_t)$ from the private first order conditions given in Eqs. (15a) and (15b), respectively, we can derive the following result:
**Proposition 1.** If the marginal tax on housing wealth is fixed at \( \tau_t \) for all \( t \), the optimal marginal income tax rates will take the form

\[
T_1'(s_t r_{t+1}) r_{t+1} = (1 + r_{t+1}) \left( \frac{\Pi_t}{1 + r_{t+1}} \alpha t \text{MRS}_{h,c_0}^t - \tau_t P_t \right) \left( \frac{\text{MRS}_{c_0,c_1}^t}{\text{db}_{1,t+1}} - \frac{\text{dh}_t}{\text{db}_{0,t}} \right)
\]

\[
w_t T_0'(y_t) = -\text{MRS}_{t,c_0}^t \left( \frac{\Pi_t}{1 + r_{t+1}} \alpha t \text{MRS}_{h,c_0}^t - \tau_t P_t \right) \frac{\text{dh}_t}{\text{db}_{0,t}}
\]

for all \( t \), where \( \Pi_t = (\partial W / \partial \lambda_t)(V_t,b_t,\lambda_{t+1}) > 0 \).

**Proof.** See Appendix.

In each tax formula in Proposition 1, the second term within square brackets on the right hand side is proportional to the actual (and possibly suboptimal) tax on housing wealth, while the first term reflects the value of the marginal externality. Before we interpret the results in greater detail, note that Proposition 1 contains two useful special cases. First, if \( \tau_t = 0 \), Proposition 1 and Eqs. (16a) and (16b) together imply \( T_1'(s_t r_{t+1}) < 0 \) and \( T_0'(y_t) > 0 \). Second, the first best policy summarized by Observation 1 also follows as a special case of the more general result derived in the proposition: in a first best resource allocation where \( P_t \tau_t = \alpha t \text{MRS}_{h,c_0}^t \) and \( \Pi_t = 1 + r_{t+1} \), we obtain \( T_1'(s_t r_{t+1}) = T_0'(y_t) = 0 \). These two special cases also suggest the following more general corollary to Proposition 1:

**Corollary 1.** If \( P_t \tau_t < \alpha t \text{MRS}_{h,c_0}^t \), Proposition 1 implies \( T_1'(s_t r_{t+1}) < 0 \) and \( T_0'(y_t) > 0 \).

**Proof.** See Appendix.

Corollary 1 means that if the housing wealth tax per unit of housing falls short of the marginal externality, in which case this tax is too low to achieve full externality correction, there is an incentive for the government to counteract the externality further through a marginal saving subsidy and marginal labor income tax policy. The intuition is that increased saving in financial wealth leads to less investment in housing, i.e., financial saving becomes a more attractive means of funding future consumption at the individual level. As a consequence, the positional consumption externality attached to housing decreases. Similarly, reduced labor income reduces the resources available for all types of wealth accumulation.

The multiplier \( \text{MRS}_{c_0,c_1}^t (\text{dh}_t/\text{db}_{1,t+1}) - \text{dh}_t/\text{db}_{0,t} < 0 \) in the marginal capital income tax formula in Proposition 1 appears because the capital income subsidy constitutes an indirect instrument to reduce the housing wealth, and works through the effect of savings on the housing demand. As such, the more an increase in savings reduces the demand for housing (through a decrease in \( \text{dh}_t/\text{db}_{0,t} \)), the more effective will be the savings subsidy as an instrument to reduce the positional externality. On the other hand, if \( \text{dh}_t/\text{db}_{0,t} \) and \( \text{dh}_t/\text{db}_{1,t+1} \) are close to zero, the savings subsidy may not be a useful instrument to influence the accumulation of housing wealth, in which case the subsidy would be close to zero or not used at all. The multiplier in the marginal labor income tax formula, \( \text{dh}_t/\text{db}_{0,t} > 0 \), has an analogous interpretation.

Finally, let us briefly return to the special case discussed in Section 2 where \( \delta = 1 \), which is interpretable to mean that individuals rent their homes instead of owning them. Note that this special case gives the same qualitative results as those derived in Proposition 1 and Corollary 1. We should still subsidize the saving and tax the labor income at the margin under the same general conditions to counteract the positional consumption externality. However, while the policy rule for marginal labor income taxation would be identical to that derived in the proposition, since this formula does not depend directly on \( \delta \), the marginal saving subsidy is likely to be smaller if individuals rent their homes because \( \text{dh}_t/\text{db}_{1,t+1} = 0 \) if \( \delta = 1 \). The intuition is that the home is no longer a durable good through which individuals may fund future consumption and, therefore, not an alternative to financial wealth for the individual. In that case, the marginal saving subsidy works much like the marginal labor income tax in the sense of just reducing the amount of resources available for consumption in the first period. This suggests to us that the role of marginal saving subsidies is weaker if individuals rent their homes compared to when they own them.

4. Extensions

Results always depend on the assumptions made, and the reference model examined above is no exception. In this section, we briefly consider two relevant extensions of the analysis set out in Sections 2 and 3 by examining the implications of (i) non-separability between housing and leisure and (ii) a bequest motive such that generation \( t \) leaves the home as a bequest to generation \( t + 1 \). These extensions serve as a sensitivity analysis to check whether the qualitative results of the benchmark model hold in slightly different settings.

\[\text{If } P_t \tau_t > \alpha t \text{MRS}_{h,c_0}^t, \text{ Proposition 1 instead implies that } T_1'(s_t r_{t+1}) > 0 \text{ and } T_0'(y_t) < 0. \text{ However, this would imply a scenario where the government sets the housing tax higher than the value of the marginal externality. This seems unrealistic since the government in this case could choose the first best optimal tax instead of the second best solution. We therefore do not consider this scenario in our analysis.}\]
4.1. Non-separability between leisure and housing

By generalizing the utility function of the benchmark model to include non-separability between leisure and housing, Eq. (1) can be rewritten as

\[ U_t = U(c_{0,t}, l_t, h_t, c_{1,t+1}) = u_0(c_{0,t}) + \psi(l_t, h_t) + \phi(h_t) + u_1(c_{1,t+1}) \]  

(19)

where the sub-utility function \( \psi(\cdot) \) replaces \( \chi(\cdot) \) and \( \phi(\cdot) \) in Eq. (1); the other parts are the same as before. As such, we assume that \( \psi(\cdot) \) is decreasing in \( l \) and increasing in \( h \).

This change of functional form has no implication for the first best policy rule presented in Section 2. If the degree of positionality is properly redefined to reflect Eq. (19), i.e., \( \alpha_t = -U_{l,h}/U_{h,h} = \Phi_{t,\Delta}/(\psi_{t,\Delta} + \Phi_{t,\Delta}) \) where \( \psi_{t,\Delta} = \partial \psi(l_t, h_t)/\partial h_t \), the first best policy can be written in exactly the same format as in Observation 1. Therefore, if the government is equipped with a flexible tax on housing wealth, the best it can do is still to fully internalize the externality through this direct instrument, in which case there is no need to distort the saving behavior or labor–leisure choice of the consumers.

The second best policy presented in Section 3 is modified. When the utility function is given by Eq. (19) instead of Eq. (1), generation \( t \)'s conditional demand for housing will also depend on the hours of work, \( l_t \), i.e., Eq. (12) extends to

\[ h_t = h_t(P_t(1 + \tau_t), P_{t+1}(1 - \delta), b_{0,t}, b_{1,t+1}, l_t, \bar{h}_t). \]  

(20)

It is straightforward to derive \( dh_t/dl_t < 0(>0) \) if \( \psi_{t,lb} = \partial^2 \psi(l_t, h_t)/\partial l_t \partial h_t < 0(>0) \), which is interpretable to mean that housing is complementary with (substitutable for) leisure. The comparative statics with respect to \( b_{0,t} \) and \( b_{1,t+1} \) are still given by Eqs. (16a) and (16b), respectively. As such, it is still desirable to subsidize saving at the margin under the condition described in Corollary 1, and the optimal marginal saving subsidy takes the same form as in Proposition 1. However, the optimal marginal labor income tax rate changes such that the formula in Proposition 1 is now replaced by

\[ w_t T_0(y_t) = \left[ \frac{\Pi_t}{1 + \Pi_{t+1}} \alpha_t \text{MRS}_{h,c_0}^t - \tau_t P_t \right] \left( \frac{dh_t}{dl_t} - \frac{\text{MRS}_{l,c_0}^t dh_t}{db_{0,t}} \right). \]  

(21)

Even if \( \alpha_t \text{MRS}_{h,c_0}^t > \tau_t P_t \), the right hand side of Eq. (21) can be either positive or negative. If leisure and housing are substitutes, or if they are weak enough complements such that \( dh_t/dl_t - \text{MRS}_{l,c_0}^t dh_t/db_{0,t} > 0 \), it is still optimal to tax labor earnings at the margin, whereas the policy turns into a marginal subsidy if leisure and housing are strong enough complements to imply \( dh_t/dl_t - \text{MRS}_{l,c_0}^t dh_t/db_{0,t} < 0 \).

4.2. Bequest motives

In the benchmark model examined in Sections 2 and 3, we assumed that individuals sell their houses when becoming old and then spend all remaining wealth on non-durable consumption. Yet, a durable good may not only be used to fund future consumption of the current owner, it may also provide a source of wealth for future generations. An alternative formulation of the model would thus be to assume that old individuals give their homes as a bequest to the next generation. Allowing for altruism of this type makes the analysis substantially more complex, and we therefore only briefly discuss the implications of bequests in a highly stylized case.\(^{21}\)

The important implication of altruism from our perspective is that relative consumption concerns for housing will lead to intergenerational externalities. This aspect can be captured in a simplified model where only one arbitrary generation \( t \) leaves the home as a bequest to generation \( t + 1 \), while all other generations are assumed to behave in the same way as in Sections 2 and 3. Such a framework makes it possible to examine the policy implications of altruism without having to alter the time-separable structure in other parts of the model.\(^{22}\) In this setting, generation \( t + 1 \) may increase its housing consumption through an investment in the inherited home. Our model then implies that generation \( t \) directly affects the positional externality in period \( t + 1 \), \( h_{t+1} \), via its choice of housing. Also, to avoid the extra complications that would follow if generation \( t \) and the government prefer different intergenerational tradeoffs, we have assumed that the social welfare function is given by a discounted sum of generational utilities, and that generation \( t \) discounts the utility of generation \( t + 1 \) using the same discount rate as the government.

If generation \( t \) treats the decision-variables of generation \( t + 1 \) as exogenous, it is straightforward to show that the first best policy presented in Observation 1 continues to apply. The intuition is that generation \( t \) effectively faces the marginal tax on housing wealth both in period \( t \) and \( t + 1 \), and the externality it imposes on generation \( t + 1 \) is proportional to the externality that each individual of generation \( t + 1 \) imposes on other people of the same generation. Therefore, the intergenerational externality caused by generation \( t \) is internalized through the marginal tax on housing wealth in period \( t + 1 \), while the

\(^{21}\) Calculations and proofs for the stylized bequest case are available from the authors upon request.

\(^{22}\) An implication of altruism is that the bequest-giving generation’s first order condition for housing becomes dependent on the next generation’s decision-variables, which complicates the analysis considerably. To avoid most of these complications, while still being able to address the intergenerational externality mentioned above, we assume that all generations other than \( t \) sell the house during the second period of life.
remaining within-generation externality is internalized through the marginal tax on housing wealth in period $t$. This means, in turn, that the optimal marginal labor and capital income taxes are equal to zero.

In case the marginal tax on housing wealth is predetermined (the scenario examined in Section 3), the second best optimal tax policy implemented for generation $t$ generally differs from that of the benchmark model. More specifically, the marginal savings and labor income taxes implemented for generation $t$ contain two analogous components: one referring to the discrepancy between the marginal positional externality and the marginal tax on housing wealth in period $t$ and the other to the corresponding discrepancy in period $t+1$. Only the first component was present in the benchmark model, which is seen from Proposition 1. The intuition is, of course, that the tax policy is now designed to correct both for within-generation and between-generation externalities. As a consequence, the results are more difficult to interpret here. However, if the marginal tax on housing wealth is sufficiently small compared to the marginal positional externality in period $t+1$, the second best optimal policy includes a marginal saving subsidy and marginal labor income tax for generation $t$ under the same conditions as in Corollary 1, in which case the qualitative result from the benchmark model remains valid.

5. Conclusions

This paper deals with the policy implications of relative consumption concerns for durable goods, exemplified by housing, where each individual's consumption choice imposes negative positional externalities on other people. Without any other distortion, a first best policy would in this case be to tax housing wealth such that the marginal tax exactly corresponds to the value of the marginal externality. However, taxes on housing wealth are politically controversial, and the tax rates observed in many countries are much lower than a rate that would internalize such a positionality externality if calculated based on empirical evidence. We therefore consider a second-best solution where marginal taxes (or subsidies) on savings and labor income are used to supplement the tax on housing wealth. Our analysis shows that as long as the existing marginal tax on housing wealth falls short of the marginal externality that relative housing consumption gives rise to, a second-best optimal solution is achieved through a mix of marginal saving subsidies and marginal labor income taxes.

There are several issues worth further examination. First, although our analysis briefly addresses altruism and intergenerational transfer of housing, additional questions remain to be addressed. For instance, if taxes on housing wealth cannot be used to their full potential (a scenario addressed both in Sections 3 and 4), taxes on inheritance or gifts also constitute potential second best instruments through which to correct for positional externalities. This is clearly an interesting topic for future research. Second, if the tax on housing wealth is restricted, a consumption tax or subsidy on the non-positional good may serve as a complement to the income tax policy examined above. Such an extension is interesting in its own right, and would also allow us to integrate the analysis carried out here with earlier work referred to in the introduction.23 Third, although our model distinguishes between home ownership and renting (where renting is equivalent to the special case where the rate of depreciation is equal to one), it does not allow for a choice between these two possibilities. An obvious extension would be to consider a model where each individual can either buy or rent a home, and allow such choices to be associated with different relative concerns. This kind of analysis would (most likely) necessitate a discrete choice model and is clearly complex enough to motivate its own paper.

Finally, we have assumed away that conspicuous consumption may influence prices; an issue of potential importance for understanding the housing market. This simplification is of no major concern for the qualitative results derived in the present paper, since the efficiency costs of relative consumption would be driven by the mechanisms laid out above also in a more general model. Price formation is, nevertheless, important for our understanding of wealth accumulation and distribution more generally and, therefore, a relevant topic to address in future research. It also opens up for studying the interesting case where individuals are positional in terms of housing wealth (or possibly both in terms of housing attributes and wealth). In our model, where the price is fixed, it does not matter for the qualitative results whether individuals are positional in terms of the attributes (or quantity), $h$, or in terms of housing wealth, $Ph$. In a more general model with an endogenous housing price, on the other hand, this distinction will most likely matter, and may possibly strengthen the role of the marginal saving subsidy and marginal labor income tax examined above, as the reduced demand for housing is likely to contribute to a lower price as well.

Appendix.

Proof of Proposition 1. By using Eq. (9), Eqs. (18a) and (18b) can be rewritten as follows:

$$
\frac{\partial W}{\partial V_t^{1,b_0}} = \frac{\partial W}{\partial V_t} V_t^{1,b_0} \alpha_t MRS_{h,c_0} \frac{dh_t}{db_{0,t}} + \lambda_t \left( 1 - \tau_t P_t \frac{dh_t}{db_{0,t}} \right)
$$

(1A)

23 We have done some preliminary analysis here. More precisely, we have analyzed whether the introduction of a small tax or subsidy on the non-positional good consumed in the first period may lead to higher welfare, given that the marginal saving subsidy and marginal labor income tax are optimally chosen (as described in Proposition 1). If the marginal tax on housing wealth falls short of the marginal positional externality, we found that introducing a small tax on the non-positional good leads to higher welfare if it decreases the compensated demand for housing. These calculations are available from the authors upon request.
\[ \frac{\partial W}{\partial V_t} V_{t,b_1} = \frac{\partial W}{\partial V_t} V_{t,b_0} \alpha_t \text{MRS}_{h,c_0}^{r,c} \frac{d\theta_t}{db_{t+1}} - \lambda_t \tau_t P_t \frac{d\theta_t}{db_{t+1}} + \lambda_{t+1} \]  

(Eq. A2)

Eqs. (A1), (A2) and (18c) can be used to derive

\[ \lambda_{t+1} \left[ (1 + r_{t+1}) - \frac{V_{t,b_0}}{V_{t,b_1}} \right] = \frac{\partial W}{\partial V_t} V_{t,b_0} \alpha_t \text{MRS}_{h,c_0}^{r,c} \left( \frac{V_{t,b_0}}{V_{t,b_1}} \frac{d\theta_t}{db_{t+1}} - \frac{d\theta_t}{db_{t+1}} \right) - \lambda_t \tau_t P_t \left( \frac{V_{t,b_0}}{V_{t,b_1}} \frac{d\theta_t}{db_{t+1}} - \frac{d\theta_t}{db_{t+1}} \right) \]  

(A3)

while Eqs. (A1) and (18d) can be combined such that

\[ \lambda_t \left[ W_t + V_{t,l} \frac{d\theta_t}{db_{t+1}} \right] = -V_{t,b} \frac{\partial W}{\partial \alpha_t} V_{t,b_0} \alpha_t \text{MRS}_{h,c_0}^{r,c} - \lambda_t \tau_t P_t \frac{d\theta_t}{db_{t+1}} \]  

(A4)

Substituting \((1 + r_{t+1}) - \frac{V_{t,b_0}}{V_{t,b_1}} = T_1(1 + r_{t+1})/r_{t+1} \) into Eq. (A3), and \(w_t + V_{t,l}/V_{t,b} = w_t T_0(y_t)\) into Eq. (A4), while using \(\text{MRS}_{c_0}^{r,c} = \frac{U_t}{U_t} = \frac{V_{t,b}}{V_{t,b}} + \text{MRS}_{c_0}^{r,c} = \frac{U_t}{U_t} = \frac{V_{t,b}}{V_{t,b}} + \), and \(\text{MRS} = (\partial W/\partial V_t)(V_{t,b_0}/\lambda_{t+1})\), we obtain the marginal income tax rates in Proposition 1.

**Proof of Corollary 1.** As can be seen from the two marginal tax formulas in Proposition 1, a sufficient condition for Corollary 1 to hold is that \(\alpha_t \text{MRS}_{h,c_0}^{r,c} \geq \tau_t P_t\) implies \(\Pi_t \geq 1 + r_{t+1}\). By using \(\lambda_t = (1 + r_{t+1}) = \lambda_t\) from Eq. (18c) and \(\Pi_t = (\partial W/\partial V_t)(V_{t,b_0}/\lambda_{t+1}) > 0\), the first order condition for \(b_{t+1}\) as given by Eq. (18a) can be rewritten as

\[ \Pi_t \left( 1 - \alpha_t \text{MRS}_{h,c_0}^{r,c} \frac{d\theta_t}{db_{t+1}} \right) = (1 + r_{t+1}) \left( 1 - \tau_t P_t \frac{d\theta_t}{db_{t+1}} \right) \]  

(A5)

By using Eq. (10a), we can derive

\[ 1 - \tau_t P_t \left( \frac{d\theta_t}{db_{t+1}} \right) = -\frac{d\theta_t}{db_{t+1}} + P_t \left( \frac{d\theta_t}{db_{t+1}} \right) > 0 \]

since both goods are normal, meaning that \(1 - \alpha_t \text{MRS}_{h,c_0}^{r,c} \frac{d\theta_t}{db_{t+1}} > 0\) from Eq. (A5). Therefore, by reorganizing Eq. (A5) such that

\[ \frac{\Pi_t}{1 + r_{t+1}} = \frac{1 - \tau_t P_t (d\theta_t/db_{t+1})}{1 - \alpha_t \text{MRS}_{h,c_0}^{r,c} (d\theta_t/db_{t+1})} \]

(A6)

we can see that \(\alpha_t \text{MRS}_{h,c_0}^{r,c} > \tau_t P_t\) implies \(\Pi_t > 1 + r_{t+1}\). \(\square\)

**References**


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