Regulation of a vertically differentiated duopoly

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Abstract

This paper focuses on the optimal quality regulation of vertically differentiated duopolies in the presence of asymmetric information. In the model presented there are cross-effects on the information rent. Contrary to standard single-agent models, the production levels are distorted in favour of the most efficient firm, whose production level is increased under asymmetric information relative to full information. The first-best outcomes can only be achieved if both firms are of the most efficient types. The optimal degree of vertical differentiation is also discussed. Furthermore, some extensions to the model are examined (the presence of cost complementarity, quality as complements etc.).

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1 Introduction

In a situation of increasing competition, many firms will attempt to differentiate their products from those of their competitors to earn higher levels of profits. In the market for (tele)communications services, one way of doing this would be to vary the degree of functionality (or quality) of a service. With the introduction of digital technology, providers of telecommunications services have a wide range of options on how to do this. The old copper wires previously used for analogue telephony only, can now be used to supply, e.g., digital telephony, ISDN (Integrated Service Digital Network) or ADSL (Asymmetric Digital Subscriber Line). All of which no doubt will increase the value of the subscription to consumers.

It is reasonable to assume that some kind of regulation will remain in this market for some time still as efficient competition establishes itself. One type of regulation which would be reasonable to investigate more closely is the regulation of quality. A frequent (unwanted) consequence of competition is the degradation of quality. This fear is certainly well-founded if the industry is regulated by price-caps, as price-caps normally do not give benefits to firms for increasing the quality of the product they sell.[1] Quality of access to services such as telecommunications is an important aspect of the communications industry, and is often emphasised in the political debate. It is therefore not unlikely that some sort of regulation of quality will be considered for this sector, to ensure that the benefits of competition does not come at the expense of the quality offered.

Here I focus on the case where a regulatory body induces the two firms to choose certain levels of quality, using a direct revelation mechanism, which implies that the extent of vertical differentiation is in fact induced through the regulation mechanism. I show that the regulator may find it welfare improving to induce the firms to supply different levels of quality. In a (tele)communications setting, this could amount to supplying different speeds of connection, or different choices of technology.[2] These

[1]Following the deregulation of the UK telecommunications industry quality did not fall, but did not increase as much as would be expected taking into account the technical advances which were made (Armstrong, Cowan and Vickers, 1994).

[2]We will see that the most efficient firm (with the lowest unit cost of producing *quantity*) is

issues are closely related to the literature on (strategic) investments in R&D, but the majority of this literature considers situations with full information.[3] In the R&D literature the external effect imposed on rival firms is (usually) positive provided that the degree of spillover between firms is sufficiently high.

An alternative interpretation of the model could be the *regulation of advertising*. In such a case, one could assume that the firms produce goods which are identical physically, and the firms use advertising to attempt to distinguish their product from that of the competitor and thereby capturing market shares. Such advertising expenditure is socially wasteful. One concern could be that opening up a sector to free competition results in, at best, no positive effect on welfare (or even a loss of welfare) due to excess spending on advertising, and a regulator may want to limit such behaviour. In such a setting the result that the most efficient firm is allowed a higher level of the regulated variable (here, advertising expenditure) is interpreted as a measure to allow this firm to capture a larger share of the market.

The main goal of this paper is to provide a starting point for further analysis of the regulation of multi-agent settings, where the agents (firms) produce vertically differentiated products. The level of quality is the means for differentiating the products. An analysis of multiproduct monopolies would have similar features as the model presented below, but there is an important difference. In a model of the optimal regulation of a multiproduct monopoly the incentive correction has the standard property that allocations are not distorted for the most efficient type, commonly known as "no distortion at the top". However, in the model of the optimal regulation of single-product duopolies, this is not generally the case. The result of "no distortion at the top" requires either that both firms are of the most efficient type, or that firms are equally inefficient. Thus, the standard properties of single-agent models does not necessarily carry over to multi-agent settings.

Optimal regulation of duopolies under asymmetric information is not very well

induced to supply high quality access (e.g., broadband, and possibly wireless access). The other firm, which has a higher unit cost, is induced to supply access of lower quality (access with lower capacity and speed).

[3] D'Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992) consider the strategic incentives to invest in R&D.

covered in the literature. Furthermore, the literature on regulation and quality concerns is relatively scarce. Other examples of regulation and quality can be found in Laffont and Tirole (1993: chapter 4), Lewis and Sappington (1991), Auriol (1992, 1998), and Wolinsky (1997). Both Laffont and Tirole (1993) and Lewis and Sappington (1991) considers the regulation of a monopoly. Wolinsky (1997) presents a model where the quality choices of the firms cannot be controlled by the regulator, and considers the regulation of prices and market shares. The results he obtains are similar to some of the results in this paper (those obtained in the absence of cost complementarity and for a simple demand and cost structure specification). My work is closer to the paper by Auriol (1998), but there are some notable differences.

My model considers the regulation of two firms who produce a single product each, in the absence of strategic interaction, whose cost characteristics are independently distributed. The regulator designs a menu of contracts for the firms, specifying quality and transfers as functions of reported types. The qualities of the two firms are assumed to be substitutes, which corresponds to the case where firms invests in quality to differentiate their products. The two firms' products are thus assumed to be vertically differentiated. The regulator is unable to observe total costs, and each firm has private information about its level of efficiency. Auriol (1998) considers a setting which is similar to mine, but where qualities are complements. The products are supplied through a common network by two firms. The only things that matters in the gross surplus function is total quality and whether both firms produce at the optimum (diversity increases consumer surplus). This implies that there is a public good aspect to quality; if one of the firms invests in quality, both the investing firm's customers and the customers served by the other firm benefit from the increased quality. This, together with unverifiable quality, introduces a problem of free-riding in quality provision which is absent in my initial model.[4] This point is, however, elaborated in the section on extensions to the basic

[4]The reason for this is of course that quality is observable, or verifiable, to the regulator in my model. If quality is unverifiable to the regulator, the solution would be interior in the general case. That is, both firms supply positive levels of quality. Furthermore, by rephrasing the model to analyse qualities as complements, free-riding is a solution only if there is cost complementarity

model. Furthermore, in Auriol's model quantity is regulated directly, whereas I consider the regulation of qualities. The choice of quality levels affect the production levels, which indicates that the regulation in my model is an indirect regulation of quantities. Contrary to the model by Auriol, the quality provision and information rent are not separated. In Auriol (1998), quantity is regulated and is independent of the level of quality provided. This is not so in my model. The information rent depends on the production levels, and the production levels are affected by the (regulated) quality levels. Thus, the level of quality affects the information rent. The implication is that the regulator may choose to distort quality levels to affect the firms' incentives. Furthermore, I obtain conditions which characterise the optimal degree of vertical differentiation. I consider a more general cost function than does Auriol (1992, 1998) which allows for cost complementarity, and I show that cost complementarity may, or may not, decrease the marginal information cost.

Olsen (1993) considers a model of multiagent regulation, where the (R&D) actions of the firms are substitute activites. He obtains the result that only if one firm is close to being maximally efficient and the other firm is less efficient, then the maximally efficient firm produces more than its full information level. If firms' efficiency parameters are identically distributed, but not equally inefficient, nothing can be said about the relations between full and asymmetric information production levels. However, in my model, production levels are distorted in favour of the relatively more efficient firm even for ex ante identical firms when total demand is exogenous.

There may be several reasons for there being more than one operator in a market that previously has been characterised as a natural monopoly (involving large fixed costs). First of all, there may be a yardstick effect, in which competition among firms with correlated costs reduces the information rent necessary to induce truthful revelation.[5] Secondly, the natural monopoly argument may lose some of its validity in some sectors (due to technological innovations). The telecommunications sector

between quantity and quality. This is briefly discussed in the extensions to the basic model.

[5]Auriol and Laffont (1993) consider, among other things, the yardstick effect as a reason for socially valuable duplication of fixed costs.

is one in which possibly only the local access network is still considered a natural monopoly.[6] Thirdly, and possibly most importantly, by introducing competition one may, at a later stage, be able to abolish regulations altogether. Thus, competition may be seen as a substitute for regulation. However, even if the main reason for allowing new firms into a market is to be able to use competition as a substitute for regulation, there may still be scope for some sort of regulation. The reason for this is that unregulated competition does not generally result in socially optimal outcomes.

The paper is organised as follows: In section 2, the model is set out. In section 3, the optimal solution under full information is analysed, section 4 examines the requirements for implementation, and section 5 analyses the optimal solution under asymmetric information. In section 6, some extensions to the basic model is considered.

2 The model

The model consists of three basic elements: (1) The demand side and consumer welfare, (2) the firms' choice of quality, or functionality, and (3) the welfare maximising regulator. The market is of a fixed size, and regulating the firms affects the distribution of market shares. Prices are exogenous to the model, whereas the level of quality is endogenous. The assumption of an exogenous price is obviously a simplification. A more complete regulatory mechanism where prices, in addition to the quality levels, are included would be preferred. However, including prices in the regulatory mechanism would not change the basic results of the model (and will in some situations even strengthen the results).

The quality variable could be interpreted as the quality of access. For instance, whether the line is analog, or digital (the speed of the connection), or simply what type of digital technology should be adopted. At the top-end, one could think

[6] Hansen (1996) finds evidence of natural monopoly when analysing the cost structure of Telenor. In his model, if competition somehow reduces slack (by only a small amount), thus making the firms produce more efficiently, competition may be welfare enhancing.

of wireless broadband links, while at the bottom of the scale we would find fixed analog connections. Firms have private information with respect to some efficiency parameter, β. The regulator is assumed to be a benevolent maximiser of welfare; that is, he has no agenda of his own, and is purely interested in maximising a utilitarian welfare function.

2.1 Consumers

The demand functions are in general given by $q_i = q_i\left(s, \overline{\theta}\right)$, for $i = 1, 2$, where $s = (s_i, s_j)$ is the vector of qualities. The vector of qualities could for instance be interpreted as the quality offered for access, as offered by the two firms operating in the market. The market is of fixed size $\overline{\theta}$, which implies that $q_i = \overline{\theta} - q_j$. The demand functions have the following properties:

Assumption 1 *Demand is increasing in own quality, decreasing in the other firm's quality, and the marginal effect of quality changes on demand is decreasing in the other firm's quality.*

Mathematically, $\partial q_i/\partial s_i > 0$, $\partial q_i/\partial s_j < 0$, $\partial^2 q_i/\partial s_i \partial s_j < 0$ for all (s_i, s_j). In addition, demand increases with the size of the market.

Consumers' net surplus is in general given by: $S = S\left(s, \bar{\theta}\right)$, which implies that net surplus is a function of qualities and the size of the market. Consumers' net surplus has the following characteristics:

Assumption 2 *Consumers' net surplus is increasing and concave with respect to quality.*

The conditions$\frac{\partial S}{\partial s_i} > 0$, $\frac{\partial S}{\partial s_j} > 0$, $\frac{\partial^2 S}{\partial s_i^2} < 0$, and $\frac{\partial^2 S}{\partial s_i \partial s_j} < 0$, together with $\frac{\partial^2 S}{\partial s_1^2}\frac{\partial^2 S}{\partial s_2^2} - \left[\frac{\partial^2 S}{\partial s_1 \partial s_2}\right]^2 \geq 0$, secures that net surplus is an increasing and concave function on (s_1, s_2).

The assumption $\frac{\partial^2 S}{\partial s_i \partial s_j} < 0$ together with the assumption $\frac{\partial^2 q_i}{\partial s_i \partial s_j} < 0$, ensures that the marginal value of s_i to the regulator decreases in s_j if $|\beta_i - \beta_j|$ is sufficiently small. In the definition of consumers' net surplus, S, the consumers' expenditures for purchasing the products supplied by firms 1 and 2 are taken into account. Thus, the fact that net surplus increases in quality indicates that the consumers prefer higher levels of quality for given prices.

As noted, prices are exogenous and equal for both firms even if quality offered by the two firms are different. An explanation for why some consumers still buy from the firm providing the product with the lowest quality could be that there is an unmodelled horisontal differentiation aspect. Thus, a consumer may prefer the product from the low quality provider because it is closest to his preferred product. An alternative explanation may be that some consumers for some reason are locked-in (at least in the short run), and may continue to buy a product of inferior quality even if there is a higher quality product available at an identical price.

2.2 Firms

The firms operate as profit maximising entities, and provide a single quality product each. Prices are exogenous, and given as $p_i = p > \beta_i \; \forall i$ (see note above). Firm i provides functionality level s_i, where $s_i \geq \underline{s} \geq 0$, for $i = 1, 2$. The lower bound on the level of quality can be thought of as a minimum quality standard. The firms are said to be vertically differentiated if $s_i \neq s_j$, for $i \neq j$.

As for firms' information about each other's costs, it is assumed that each firm knows only its own efficiency level. The firms' efficiency parameters are perceived to be (independently) distributed according to common knowledge distributions by the principal/regulator and the (other) firms/agents.[7]

[7]The assumption of "ignorance" makes the analysis non-trivial. If firms have complete information about each other's efficiency parameters, the regulator could potentially force each firm to report their true type by constructing a mechanism which results in an infinitely high penalty to the firms if their reports do not coincide. Each firm is in such a mechanism assumed to report the other firm's efficiency parameter in addition to its own. See Moore (1992) for more on this. Of course, the realism of such a regulatory mechanism could be questioned. The assumption that firms' efficiency parameters are *independently* distributed means that yardstick competition is not

The firm maximises the following objective function:

$$\max_{\widehat{\beta}_i} \pi_i = E_{\beta_j}\left[(p-\beta_i)\,q_i\left(s\left(\widehat{\beta}_i,\beta_j\right),\bar{\theta}\right)+t_i\left(\widehat{\beta}_i,\beta_j\right)-\psi^i\left(s_i\left(\widehat{\beta}_i,\beta_j\right)\right)\right] \tag{1}$$

where $i=1,2$. E denotes the expectations operator. Expectations are taken over firm j's efficiency parameter, since firm i does not know firm j's type. The transfer from the regulator to firm i is given by t_i, and β_i denotes unit (and marginal) costs. β_i and β_j are assumed to be independently distributed. This would be the case if the two firms utilise different technologies (e.g., wireless vs. fixed-line technologies), which are subjected to independent technology shocks. Marginal cost of producing one unit of good i is initially assumed to be independent of the level of quality.[8] $\psi^i(s_i)$ defines firm i's costs of undertaking quality enhancing investments. The cost function is a cost element that is independent of quantity produced, or can be interpreted as investment costs, and is an increasing and convex function.

Total costs for firm i are thus defined as[9]

$$TC_i = \beta_i E_{\beta_j} q_i\left(s,\bar{\theta}\right)+E_{\beta_j}\psi^i(s_i)$$

Assumption 3 *Marginal cost of quantity increases with inefficiency.*

Assumption 3 is the familiar single-crossing condition, which in this case is $-\frac{\partial^2 TC_i}{\partial\beta_i\partial q_i}=-1<0$, for $i=1,2$. Total costs are assumed to be unobservable to the regulator.

an issue.

[8]See section 6 for the analysis of a case with a more general cost function.

[9]This cost structure bears resemblance to the model presented by Lewis and Sappington (1991).

2.3 The regulator

The regulator is assumed to be benevolent (that is, he has no agenda of his own), and maximises a utilitarian welfare function. By assuming that there are positive shadow costs of public funds (the approach used in Laffont and Tirole, 1993), there is a trade-off between information rents to the firms and economic efficiency.

The regulator's objective function is given as:

$$W = S + \pi_1 + \pi_2 - (1 + \lambda)(t_1 + t_2)$$

The regulator maximises a utilitarian welfare function, but takes into account the welfare loss of transfers due to shadow costs of public funds. By inserting for transfers and simplifying (using the fact that $q_1 + q_2 = \overline{\theta}$), we get the welfare function (2):

$$\begin{aligned} W\left(p, s, \lambda, \pi_1, \pi_2, \bar{\theta}\right) &= S\left(s, \bar{\theta}\right) - (1 + \lambda)\left[\psi^1(s_1) + \psi^2(s_2)\right] \\ &+ (1 + \lambda)\left[p\bar{\theta} - \beta_1 q_1\left(s, \bar{\theta}\right) - \beta_2 q_2\left(s, \bar{\theta}\right)\right] - \lambda\pi_1 - \lambda\pi_2 \end{aligned} \tag{2}$$

where s is the vector $(s_1(\beta_1, \beta_2), s_2(\beta_1, \beta_2))$.

Assumption 4 *The welfare function is increasing and concave on* (s_1, s_2).

In the full information case, the regulator maximises W subject to the participation constraints only. In the case of asymmetric information, the regulator knows only the support and distribution of the efficiency parameters, β_1 and β_2, and will thus have to maximise expected welfare, with expectations taken over β_1 and β_2. These (for the regulator) random variables have the (cumulative) distribution $F(\beta_i)$, with corresponding density function, which is assumed to be strictly positive over the relevant range, with $\beta_i \in \left[\underline{\beta}, \bar{\beta}\right]$. Furthermore, $F\left(\underline{\beta}_i\right) = 0$ and $F\left(\overline{\beta}_i\right) = 1$, for $i = 1, 2$. The efficiency parameters are assumed to be independently distributed, and the monotone hazard rate property is assumed to hold (assumption 5):

Assumption 5 $F(\beta_i)/f(\beta_i)$ *is non-decreasing.*

The regulator maximises the welfare function under the restrictions that the firms report their true types (the incentive compatibility constraints), and that firms choose to participate voluntarily (the participation constraints).

3 Optimal regulation under symmetric information

The main purpose of this section is to provide the full information benchmark of duopoly regulation of qualities. Below, I will compare optimal policies under symmetric and asymmetric information.

In the case of full information, the regulator can instruct the firms to implement whatever qualities he finds to be optimal. He has, however, to consider the fact that the firms may not wish to participate. By maximising the welfare function (eqn. (2)), subject to the participation constraints that each firm is secured a non-negative level of profit in expectation terms, $E_{\beta_j}\pi_i(\beta_i,\beta_j) \geq 0$, or $E_{\beta_j}\pi_i\left(\bar{\beta}_i,\beta_j\right) \geq 0$,[10] we obtain the optimal quality for firm i, s_i^{FI}, as defined by equation (3):

$$\frac{\partial \psi^i}{\partial s_i} + (\beta_i - \beta_j)\frac{\partial q_i}{\partial s_i} = \frac{1}{1+\lambda}\frac{\partial S}{\partial s_i} \tag{3}$$

for $i = 1, 2$, and $i \neq j$ (by using the fact that the market size is fixed). Quality is set such that the marginal cost of providing quality is equal to the sum of the weighted marginal (net) consumer surplus and price-cost margins (weighted by the marginal effect on demand from a change in quality). An unregulated firm i would ignore the effect of a change in the level of quality on consumers' net surplus and

[10] A note to the formulation of the participation constraints: Since the incentive compatibility condition for firm i is strictly decreasing in the (in)efficiency parameter, β_i, for any given β_j, it suffices to require that the participation constraint for the least efficient type is satisfied. Then, the participation constraint is satisfied for any type. In order for this to be true, the regulator needs to design a mechanism such that the least efficient type gets a profit of zero for any report made by the other firm, since firm i's profit is strictly increasing in the other firm's (in)efficiency parameter.

the cross-effect on firm j's profit.[11] Since the outcome of an unregulated duopoly competing in qualities is not generally identical to the socially optimal quality levels, there may be some scope for the regulation of qualities.[12] There are external effects due to investments that the unregulated firms cannot internalise. There are positive external effects on consumers' surplus from investing in quality. In addition, the firm that invests in quality imposes a negative external effect on the other firm from the business stealing effect of the investments. Due to the opposing signs of the external effects, the unregulated quality levels may be both too high or too low compared to the socially optimal levels.

If firms' investment costs are identical, an exogenously given market size, and a fully covered market (such that $\partial q_i/\partial s_i = -\partial q_j/\partial s_i$), the most efficient firm always provides the higher level of quality. Thus, we have the following result:

Lemma 1 *If $\psi^i(s_i) = \psi^j(s_j)$ and $\partial q_i/\partial s_i = -\partial q_j/\partial s_i$ for $\forall i, j$, $i \neq j$, then for $\beta_i < \beta_j$ we have $s_i^{FI} > s_j^{FI}$.*

Proof. Compare eqn.(3) for firms i and j. Using the assumptions on ψ, S, and q_i, we observe that $s_i^{FI} = s_j^{FI}$ only if $\beta_i = \beta_j$. It can be shown that in order to satisfy the first-order conditions defined by eqn. (3), $\beta_i < \beta_j$ must imply $s_i^{FI} > s_j^{FI}$. If this is not the case, the first-order conditions with respect to s_i and s_j cannot hold. ■

4 Implementation

A set of contracts is incentive compatible if a set of first- and second-order conditions are met. Local incentive compatibility (by using the first-order condition) requires that the state variables (here, profits) vary in a certain manner with the efficiency parameter. The second-order conditions ensure that the local optimum is also a global optimum.

[11] An unregulated profit maximising firm would choose its level of quality after the following rule: $\frac{\partial \psi^i}{\partial s_i} = (p - \beta_i) \frac{\partial q_i}{\partial s_i}$, which generally differs from the socially optimal quality level.

[12] If social costs of public funds approaches infinity, optimal quality for the least efficient of the two firms is the minimum quality whereas the more efficient firm produces a positive level of quality.

I assume that the regulator designs a (direct) revelation contract of the form (utilising the Revelation Principle); $M_i = \left\{ s_i(\hat{\beta}_i, \hat{\beta}_j), t_i(\hat{\beta}_i, \hat{\beta}_j) \right\}$, where $\hat{\beta}_i$ is firm i's report of its efficiency parameter to the regulator. Let $\pi_i(\beta_i) \equiv E_{\beta_j}[\pi_i(\beta_i, \beta_j)]$, where E is the expectations operator.[13] In order for the regulator to maximise welfare under asymmetric information, the following conditions must be met:

(IC) $E_{\beta_j}[\pi_i(\beta_i, \beta_j)] \geq E_{\beta_j}\left[\pi_i\left(\hat{\beta}_i, \beta_j\right)\right]$, for all $\left(\beta_i, \hat{\beta}_i\right), \forall i$,
and

(PC) $E_{\beta_j}\pi_i(\beta_i, \beta_j) \geq 0$, for all $\beta_i, \forall i$.

The requirements for implementation of incentive compatible contracts are summarised in lemma 2 (local incentive compatibility) and proposition 3 (second-order conditions):

Lemma 2 *When quality is verifiable to the regulator, local incentive compatibility requires (using the envelope theorem):*

$$\frac{d\pi_i}{d\beta_i} = \frac{\partial \pi_i}{\partial \beta_i} = -E_{\beta_j}[q_i(s_i(\beta_i, \beta_j), s_j(\beta_i, \beta_j))] < 0 \tag{4}$$

Both firms will earn information rents, except for firms of the least efficient types, $\overline{\beta}$.[14] The reason that both firm 1 and 2 earn rents is that the firms' efficiency parameters are stochastically independent. Any information the regulator may have on either firm is useless for the purpose of rent extraction. The information rent for firm i is given by equation (5):

$$\pi_i(\beta_i) = \int_{\beta_i}^{\overline{\beta}} E_{\beta_j} q_i\left(s_i\left(\widetilde{\beta}_i, \beta_j\right), s_j\left(\widetilde{\beta}_i, \beta_j\right)\right) d\tilde{\beta}_i + E_{\beta_j}\pi_i\left(\overline{\beta}_i, \beta_j\right) \tag{5}$$

for $i, j = 1, 2$.

[13] Let $E_{\beta_i}[x] \equiv \int_{\underline{\beta}}^{\overline{\beta}} x f(\beta_i)\, d\beta_i$.

[14] The regulator designs the contracts such that if a firm reports the highest β -value, he gets a profit of zero no matter what the other firm reports.

The first element on the right-hand side is the information rent a firm of type (efficiency level) β_i earns (with expectations taken over firm j types). The information rent is, of course, positive since the integrand is positive. Since rents to firms are costly, the profit to firm i of type $\overline{\beta}$ is set equal to zero; i.e., $E_{\beta_j}\pi_i\left(\overline{\beta}_i, \beta_j\right) = 0$.

The information rent is increasing in a firm's own quality level, but decreasing in the other firm's quality level. This can be seen by differentiating the information rent expression with respect to s_i and s_j, respectively:

$$\frac{\partial \pi_i}{\partial s_i} = \left[\int_{\beta_i}^{\overline{\beta}} E_{\beta_j} \frac{\partial q_i}{\partial s_i} d\widetilde{\beta}_i\right] > 0$$

$$\frac{\partial \pi_i}{\partial s_j} = \left[\int_{\beta_i}^{\overline{\beta}} E_{\beta_j} \frac{\partial q_i}{\partial s_j} d\widetilde{\beta}_i\right] < 0$$

From assumption 1 we know that the partial derivative of q_i with respect to s_i is positive, and with respect to s_j negative. All other things equal, distorting firm i's quality downwards and firm j's quality upwards, reduces the information rent to firm i since it reduces the quantity firm i produces. This implies that it is less tempting for firm i (of any given level of efficiency) to imitate less efficient types. Thus, the revelation process is made cheaper for the regulator.[15] Note that there are two different aspects to changing the level of information rent a firm earns. First, there is the issue of distorting the production levels for less efficient types for a given firm (the standard result in single-agent models). Second, we must consider incentives between the two firms. This amounts to awarding the relatively more efficient of the two firms a higher production level. Given that firm j is of the most efficient type, distorting downwards the quality levels of all but the most efficient type of firm i implies that firm i's incentives to imitate less efficient types is weakened. This is so for two reasons: First (ignoring the other firm), reducing the quality and thereby the quantity of less efficient types of firm i makes it less profitable for a more efficient type to imitate less efficient type. Second, by imitating a less

[15]However, since qualities (and hence quantities) are substitutes, firm j's production level, and hence information rent, is increased.

efficient type, his level of quality and thereby quantity, is further reduced because of the comparison between the two firms. Thus, because there are two firms in the market, the incentives to portray oneself as less efficient are weakened further relative to a monopoly situation.

Note that a more efficient firm is a firm which produces *quantity, and* not necessarily quality, at a lower cost. Since the total size of the market is assumed to be fixed, and fully covered, the fact that the distortion in quality levels (and market share) in favour of the more efficient firm implies that total production is made at a lower cost.

We have seen that the information rent of any given firm depends on the level of quality of both firms. These cross-effects which affect the information rent may be termed *fiscal externalities.* The provision of quality is therefore, unlike the result in Auriol (1998), not separated from the rent extraction. The reason for this is due to the difference in the regulation mechanism. In my model the production level of each firm depends on the quality levels since quantity is not regulated directly. Quantity is regulated indirectly through qualities, whereas Auriol (1998) considers direct regulation of quantities.

The second-order sufficient conditions for incentive compatibility are given in proposition 1:

Proposition 1 *Given the assumption on single-crossing of cost curves, A.3, sufficient conditions for incentive compatibility on the vector of control variables, s, are: (i)* $\frac{\partial s_i}{\partial \beta_i} \leq 0$*, and (ii)* $\frac{\partial s_i}{\partial \beta_j} \geq 0$ *for* $i, j = 1, 2$*, and* $i \neq j$*.*

The proof of Proposition 1 is in appendix 1.

This implies that quality, s, must be non-increasing in the firm's own inefficiency parameter, β, and must be non-decreasing in the other firm's (in)efficiency parameter. This is satisfied under assumptions 1, 5, and if the virtual surplus function (expression (2) inserted for the informational costs) is concave in quality.

5 Optimal regulation under asymmetric information

In the case of asymmetric information, I assume that the regulator knows only the distribution and support of the random variables β_1 and β_2 , and therefore maximises expected welfare. Furthermore, the regulator needs to induce the firms to reveal whatever private information they may have. By utilising the Revelation Principle (e.g., Myerson, 1979), the regulator may formulate a direct mechanism in which each firm will choose to reveal their types, provided that the transfer function is constructed in such a manner that the mechanism is incentive compatible.

5.1 Optimal policies

The regulator's maximisation problem is the following:

$$\max_{\{s_1,s_2\}} \int_{\beta_1} \int_{\beta_2} W\left(s_1\left(\beta_1,\beta_2\right), s_2\left(\beta_1,\beta_2\right), \lambda, p, \pi_1, \pi_2\right) dF\left(\beta_1\right) dF\left(\beta_2\right) \tag{6}$$

subject to

$$\frac{d\pi_1}{d\beta_1} = -E_{\beta_2}\left[q_1(s_1\left(\beta_1,\beta_2\right), s_2\left(\beta_1,\beta_2\right), \bar{\theta})\right] \tag{IC1}$$

$$\frac{d\pi_2}{d\beta_2} = -E_{\beta_1}\left[q_2(s_1\left(\beta_1,\beta_2\right), s_2\left(\beta_1,\beta_2\right), \bar{\theta})\right] \tag{IC2}$$

$$E_{\beta_2}\pi_1\left(\overline{\beta}_1, \beta_j\right) \geq 0 \tag{PC1}$$

$$E_{\beta_1}\pi_2\left(\overline{\beta}_2, \beta_j\right) \geq 0 \tag{PC2}$$

Since transfers are costly, we can safely assume binding participation constraints for the least efficient firms (for both firms 1 and 2). Integrating by parts the incentive constraints, taking into account that the participation constraints bind at type $\overline{\beta}$ for both firms, and inserting into the welfare function (2), we obtain the virtual surplus function. Let s denote the vector $(s_1(\beta_1,\beta_2), s_2(\beta_1,\beta_2))$:

$$\begin{aligned} VS &= \int_{\beta_1}\int_{\beta_2} \{S(s,\bar{\theta}) + (1+\lambda)\left[p\bar{\theta} - \beta_1 q_1(s,\bar{\theta}) - \beta_2 q_2(s,\bar{\theta})\right] \\ &\quad - (1+\lambda)\left[\psi^1(s_1) + \psi^2(s_2)\right] \\ &\quad -\lambda\left[q_1\left(s,\overline{\theta}\right)\frac{F(\beta_1)}{f(\beta_1)} + q_2\left(s,\overline{\theta}\right)\frac{F(\beta_2)}{f(\beta_2)}\right]\} dF(\beta_1)\, dF(\beta_2) \end{aligned} \tag{7}$$

Maximising the expression (7) with respect to s_i, we obtain the formula for optimal quality, s_i^{AI}, for $i = 1,2$, and $i \neq j$, given by:

$$\frac{\partial \psi^i}{\partial s_i} + (\beta_i - \beta_j)\frac{\partial q_i}{\partial s_i} = \frac{1}{1+\lambda}\frac{\partial S}{\partial s_i} - \frac{\lambda}{1+\lambda}\left[\frac{\partial q_i}{\partial s_i}\frac{F(\beta_i)}{f(\beta_i)} + \frac{\partial q_j}{\partial s_i}\frac{F(\beta_j)}{f(\beta_j)}\right] \tag{8}$$

Note here that the formulas for optimal quality for the asymmetric information case are identical to the full information formulas, except for the incentive correction component. The right-hand side of the formula for optimal s_1 is reduced by the (incentive correction) term $\frac{\lambda}{1+\lambda}\left[\frac{\partial q_1}{\partial s_1}\frac{F(\beta_1)}{f(\beta_1)} + \frac{\partial q_2}{\partial s_1}\frac{F(\beta_2)}{f(\beta_2)}\right]$. For firm 1, we have that the quality level, s_1 , should be distorted downwards if this expression is positive, since the quality investment function is increasing and convex.

By assuming that the whole market is covered, we have $\frac{\partial q_i}{\partial s_i} = -\frac{\partial q_j}{\partial s_i}$. This implies that the increase in demand for the investing firm is fully offset by the reduction in the other firm's demand. Then the incentive correction terms for s_1^{AI} and s_2^{AI} are reduced to expressions (9) and (10), respectively:

$$-\frac{\lambda}{1+\lambda}\left[\frac{F(\beta_1)}{f(\beta_1)} - \frac{F(\beta_2)}{f(\beta_2)}\right]\frac{\partial q_1}{\partial s_1} \tag{9}$$

$$\frac{\lambda}{1+\lambda}\left[\frac{F(\beta_1)}{f(\beta_1)} - \frac{F(\beta_2)}{f(\beta_2)}\right]\frac{\partial q_2}{\partial s_2} \tag{10}$$

If the sign of the bracketed term is assumed to be positive,[16] then because of the convexity of the quality investment functions, optimal quality for firm 1 is reduced

[16]The sign is positive if $\beta_1 \geq \beta_2$ when distributions are identical using assumption 5.

under asymmetric information. For firm 2 the exact opposite result holds; under asymmetric information; optimal s_2 is increased.

A note to the relevance of actual efficiency levels: When the principal is to implement the optimal asymmetric information policies, he knows the true value of the efficiency parameters. The reason is that he has already devised an incentive compatible scheme, which induces firms to reveal their true types.

For the model presented here (with full market coverage), we have the following results:

Proposition 2 *If firm i is less efficient than firm j, i.e., $\beta_i > \beta_j$, then we have the following relationships: (1) $s_i^{FI} \geq s_i^{AI}$, and (2) $s_j^{FI} \leq s_j^{AI}$.*

Corollary *Assume that $\beta_i \neq \beta_j$. Since the most efficient firm always provides a higher level of quality under full information, the optimal policy under asymmetric information leads to more differentiated products than in the full information case.*

This result is quite obvious. From lemma 1 we know that the most efficient firm always provides the higher level of quality under full information. The economic intuition behind this result is that an increase in the degree of differentiation distorts the division of the market in favour of the most efficient producer. Increasing the quality level of the most efficient of the two firms and reducing it for the other firm increases the production level of the most efficient and reduces the production level of the other firm (see assumption 1).

Reducing the quality level of the less efficient firm and increasing it for the more efficient firm implies that the most efficient firm's production level increases (and the production level of the other firm decreases). Thus, contrary to standard one-agent models, production levels under asymmetric information is higher for the most efficient firm relative to the full information solution.[17] The explanation for this is that a departure from the full information solutions distorts the market shares in favour of the firm with the highest level of efficiency. Increasing the most efficient firm's

[17]In standard one-agent models, production is distorted downwards to reduce information rent payments. This, however, is not the case here. The reason is that it is the relative efficiency levels (between the two firms) which matters.

market share implies that the total production is made more cheaply, since efficiency is related to the production of quantity (and not necessarily the cost of producing quality). This result correspond to Olsen (1993), in which the most efficient agent is required to have a higher R&D output in the asymmetric information case. The reason for this result is, since actions are substitutes in his paper, "..the principal's need to balance the gains from coordination of agents' outputs against the costs associated with giving the agents rents.." (Olsen, 1993:p.535). In my paper it is the fact that qualities are substitutes which gives rise to the similarity. This implies that quantities/outputs are substitutes, and thus the analogy to the R&D result is clear. However, there is a difference in the results. In my model, contrary to Olsen (1993), the result that the production is distorted in favour of the more efficient firm is obtained when firms' probability distributions are identical.[18] This result is obtained by assuming that total demand is given, and that the whole market is served (or more precisely, that any increase in the demand for the investing firm's product is exactly offset by the reduction in demand for the other firm's product). Thus, if production is distorted in favour of the most efficient firm in the asymmetric information case, this firm must also increase its production relative to the full information case.

Since qualities are substitutes in the regulator's welfare function, the increase in the necessary total information rents to the firms is balanced by the increase in welfare. Optimal policies call for increasing the quality of the most efficient of the two firms and reducing the quality of the other firm - thus the most efficient of the two increases production and the less efficient reduces production, with the result that information rents to the less efficient of the two is reduced, whereas the information rents to the more efficient is increased. On the other hand, if qualities are complements the regulator would attempt to increase both firms' qualities to increase welfare, but this would also result in an increase in both firms' productions and information rents. Thus, the optimal policy in such a case would call for reductions

[18] In my model there is also the result that first-best levels are obtained if firms' distributions are identical and firms are equally inefficient. In this case, the incentive correction terms vanish. This is also different from Olsen (1993: p.536).

in production under asymmetric information (see McAfee and McMillan, 1991).[19]

If we consider the case where $\beta_1 < \beta_2$, then the optimal quality for firm 1 is increased, while firm 2's quality is decreased (again compared to the full information solution). This is in accordance with the monotonicity condition, which requires that quality be non-increasing in the parameter β. This implies that market shares are distorted in favour of firm 1; i.e., the firm with the highest actual level of efficiency. This is an illustration rather than a rigorous proof of how the optimal policies under asymmetric information in fact comply with the monotonicity conditions.

5.2 First best market shares under asymmetric information

From the analysis above we have the subsequent result:

Proposition 3 *Contrary to standard results in single-agent regulation models, first-best solutions (for market shares) are only obtained if both firms are of the most efficient types, or if firms are equally inefficient.*

The incentive correction terms for optimal qualities under asymmetric information, eqns. (9) and (10), will vanish only if the firms' efficiency parameters are identical.[20] In such a case, even if the optimally regulated qualities (and indirectly, market shares) results in first-best levels, the outcome is still second-best because of the shadow costs of transfers. Generally, the specifics of a first-best solution depends on the distributions, the specification of the cost function, and the demand structure. A sufficient condition for a first-best solution (in the general case where we allow for firms' efficiency parameters being drawn from different distributions) is that both firms are of the most efficient type.

[19]An increase in quantity may, in the case of complements, be a result of increased qualities, ceteris paribus. Such an increase in quantity results in increased information rent payments to both firms (since both firms' production levels are increased). Thus the cost of providing quality to consumers is increased. E.g., for a given level of quality for firm 2, and increase in the quality level of firm 1 would increase the production levels of both firm 1 and firm 2. This would increase the informational costs of production. Thus optimal policy would be to reduce both firms' qualities, and thus production levels.

[20]For a similar result in a different setting, I refer the reader to Osmundsen (1997).

If firm i is of the least efficient type, $\beta_i = \overline{\beta}_i$, and firm j's efficiency parameter is $\beta_j \in [\underline{\beta}, \overline{\beta})$, then the market shares are distorted in favour of firm j. Observe that in accordance with conventional wisdom, the most efficient of the two firms earns a higher rent, for identical distributions. However, if we allow for non-identical distributions, firms of equal efficiency levels may earn different information rents.[21] This is due to the fact that the inverse hazard rates are different for a given level of efficiency, which again leads to a distortion in the market shares relative to the full information solution.

6 Extensions to the basic model

6.1 Cost complementarity and optimal regulation of quality

In some cases, it may be natural to assume that there is cost complementarity between, for instance, network traffic and quality of a service (where quality may be interpreted as content, or functionality). For the case of telecommunications, it may be reasonable to assume that the owner of a network has lower costs for the provision of content, or functionality.[22] It has also been argued that there is a link between the provision of high-speed internet connections and size of the network. Higher-speed is here interpreted as higher quality. Having a large network and a large customer base implies that more of the traffic can move via fewer network links, and thus the speed of the traffic can increase. Thus, the variant of the model presented in this section could be interpreted as analysing the regulation of quality in the internet infrastructure. However, it should be noted that the Internet, as such, remains an unregulated industry, so the regulation is here interpreted purely as the regulation of the underlying infrastructure (much of the Internet traffic is transported over telecommunications networks, which currently are subjected to regulation).

Let us assume that each firm has its own network, with traffic level indicated by

[21] The exception is if both firms are of the least efficient types, in which case none of them earn any information rents.

[22] A similar situation can be found in the software industry, where a supplier of both the operating system and applications may have a cost advantage over a firm which only supplies applications.

the size of q.[23] The firm with the highest production level then has a cost advantage in the provision of quality (interpreted as content or functionality). Below, I will examine the effects of economies of scope on the optimal regulatory policy for two alternative specifications of the cost function.

The model is augmented only on the firm's cost side to incorporate the concept of economies of scope. One possible cost structure which allows for the idea of cost complementarity is the following:

$$TC_i = c_i\left(\beta_i, s_i\right) q_i\left(s, \bar{\theta}\right) + \psi^i\left(s_i\right) \tag{11}$$

where

$$\frac{\partial c_i}{\partial \beta_i} > 0, \frac{\partial c_i}{\partial s_i} < 0, \frac{\partial^2 c_i}{\partial \beta_i \partial s_i} \leq 0$$

Note that the only difference to the model presented in section 2.2 is with respect to marginal costs. Here, marginal cost is a function of both the efficiency parameter, β, and the level of quality. To have cost complementarity, we need marginal production costs to be decreasing in the level of quality, or equivalently, marginal costs of quality to be decreasing in the production level.[24] This could for instance be the case if we assume that there are learning effects in production, such that higher production levels lowers the marginal costs of providing quality. Another interpretation could be that a large customer base, or equivalently a high level of traffic, makes it possible for the firm to maintain a R&D department, which again may be able to reduce the cost of functionality enhancing activities.

6.1.1 Symmetric information

For optimal qualities, we need to take into account that the marginal cost of producing depends on the level of quality (still assuming a fixed market size):

$$\frac{\partial \psi^i}{\partial s_i} = \frac{1}{1+\lambda}\frac{\partial S}{\partial s_i} + \left(c_j - c_i\right)\frac{\partial q_i}{\partial s_i} - \frac{\partial c_i}{\partial s_i} q_i\left(s, \bar{\theta}\right) \tag{12}$$

for $i, j = 1, 2$ and $i \neq j$. Compared to the full information qualities in section 3, the only difference is the additional term $-\left(\frac{\partial c_i}{\partial s_i}\right) q_i$, which is positive if there is

[23] I assume that the networks are interconnected.

[24] By assuming $\frac{\partial c_i}{\partial s_i} < 0$, there is cost complementarity between q and s.

cost complementarity between quantity and quality.[25] Thus, all other things equal, cost complementarity increases the optimal quality under full information.

6.1.2 Asymmetric information

The presence of cost complementarity affects the information rent expression. Therefore, the incentive correction term is affected. The formula for optimal quality changes somewhat, and is given by:

$$\begin{array}{l}\frac{\partial \psi^i}{\partial s_i} = \frac{1}{1+\lambda}\frac{\partial S}{\partial s_i} + (c_j - c_i)\frac{\partial q_i}{\partial s_i} - \frac{\partial c_i}{\partial s_i} q_i \\ -\frac{\lambda}{1+\lambda}\left[\left(\frac{\partial c_i}{\partial \beta_i}\frac{\partial q_i}{\partial s_i} + \frac{\partial^2 c_i}{\partial \beta_i \partial s_i} q_i\right)\frac{F(\beta_i)}{f(\beta_i)} + \frac{\partial c_j}{\partial \beta_j}\frac{\partial q_j}{\partial s_i}\frac{F(\beta_j)}{f(\beta_j)}\right]\end{array} \quad (13)$$

for $i, j = 1, 2$ and $i \neq j$. The incentive correction term has an additional element, $\left(\frac{\partial^2 c_i}{\partial \beta_i \partial s_i}\right) q_i$, which is assumed to be less than, or equal to zero.[26] This implies that the marginal information cost (the incentive correction term) of providing quality is reduced when cost complementarity is introduced. Thus, optimal quality is increased (or decreased less) compared to the solution in section 5. The intuition behind this result is that cost complementarity, and thus the quality levels, affects the value of having private information. It is the private information about the cost difference which determines the information rent given to the firms. For the case where firm i is less efficient than firm j (i.e., $\beta_i > \beta_j$), an increase in firm i's quality level in effect reduces the real cost difference in providing quality.[27] By increasing firm i's optimal level of quality, the regulator is able to reduce the value of private information to firm i, and therefore reduce the information rent payment to this firm. This, of course, reduces the (virtual) cost of providing quality, and optimal quality can be increased relative to the solution in section 5.

[25] Obviously, marginal costs are also affected. If a firm supplies a positive level of quality, then, if there is cost complementarity, marginal cost is reduced.

[26] If c is linear in β and s, e.g., $c = \beta - s$, this term is equal to zero.

[27] The regulator is assumed to know the structure of the cost function, and thus he knows whether there is cost complementarity or not.

6.2 Quality as complements

The model analysed above can easily be transformed into a model where the quality of access mandated by the regulator (or chosen by the firms) are complements. We need to modify the assumptions on the demand functions and consumers' surplus in the following manner:

Assumption 1'

Quantity is increasing in both firms' quality, and marginal effect on demand of increasing a firm's own quality is increasing in the other firm's quality.

Assumption 2'

The marginal effect of increasing quality of one firm on consumers' net surplus is increasing in the other firm's quality.

In such a model, the result with respect to optimal quality is qualitatively different from the substitutes case. The following proposition summarises the result in the complements case:

Proposition 4

When individual quality contributions are verifiable to the regulator and in the absence of cost complementarity, the optimal level of quality is reduced under asymmetric information relative to the full information case. Cost complementarity makes the result more ambiguous.

The proof is omitted, but can be obtained from the author.

The intuition behind this result is that increases in quality for a given firm raises the (marginal) consumers' net surplus (with respect to the other firm's quality), but it also increases the information rent necessary for truthful revelation to both firms. An increase in quality by firm i raises firm i's demand and also has positive spill-over effects on firm j's demand. Thus, since a firm's information rent is increasing in quantity, this leads to a higher cost of quality provision. Thus, the socially optimal level of quality is reduced. However, if there is cost complementarity and this effect is sufficiently strong, this may "finance" the additional information rents to firms 1 and 2 and make it optimal to increase quality.

6.3 Unverifiable quality

The unverifiability of qualities may have several justifications. There may be many aspects of quality which is not readily observed by anyone but the user, and these may be aspects which are difficult or expensive to ascertain for the regulator (see the discussion in Laffont and Tirole, 1993: chapter 4). In this respect, we may see the complete quality as a function of both observable and unobservable features. Based on the observable features, the regulator is assumed to be able to set a minimum quality standard. This would mean that the quality investments made by the firms affect the network quality with respect to the unobservable features. Since it is assumed that quality is the only regulatory instrument, the case of unverifiable quality is in reality the unregulated case (with the restriction that prices are exogenously given). I will analyse both qualities as substitutes and as complements in separate subsections below.

The problem of optimal contracts for teams, when the action variables (here, qualities) are complements has recently been analysed by Auriol (1998) and McAfee and McMillan (1991).[28] In Auriol's model, unverifiability results in a free-rider problem in quality provision, whereas in McAfee and McMillan there is no such problem.[29] The model by McAfee and McMillan suggests "that the source of team problem is not the unobservability of team members' efforts or abilities per se" (McAfee and McMillan, 1991: p.571). They suggest that features such as risk aversion, or collusion may be the source of such inefficiencies.

In my model, there are two main results when qualities are complements and verifiable. First, optimal quality is distorted downwards under asymmetric information (relative to the full information case) for both firms, as both firms' incentive correction terms are unambiguously negative - a result which differs from Auriol (1998).

[28] Holmstrom (1982) is the seminal paper on the problem of team production.

[29] It should be noted that McAfee and McMillan does not consider quality provision, but their models is a more general team model where the actions of the team members are complements.

In her model, optimal quality is the same under both symmetric and asymmetric information for verifiable quality. The quantity level determines the information rent in both the present model and her model, but since quantity (in addition to quality) is regulated directly in Auriol (1998) this implies that there is no effect on the information rent of distorting the quality levels in her model. Second, in the absence of cost complementarity and if prices are insensitive to the quality levels, there is no problem of free-riding in quality provision. However, by assuming that prices do change with the level of quality, free-riding is a problem - similar to the result of Auriol (1998). The result of no free-riding corresponds to the result of McAfee and McMillan (1991). Thus, in my model the unobservability of firms' actions is not necessarily the source of the problem of free-riding. Free-riding is, however, a problem in my model if either cost complementarity is present, and/or prices are sensitive to the quality level. Either of these two factors in effect introduce an advantage of having a large market share, or asymmetry between firms. Thus, the reason for free-riding being a problem in my model is that firms' payoffs for identical levels of quality are different if their market shares differ. Cost complementarity implies that a firm enjoys a cost advantage in the provision of quality if it has a larger market share than its competitor. Price sensitivity to the quality level has an impact on the marginal revenue of changing quality. The more sensitive prices are to the level of quality, the higher is the potential for increasing profits by undertaking quality enhancing investments. Marginal profit of quality is higher for the firm with the larger market share. The analysis of optimal quality choice by the firms resembles the traditional analysis of monopoly pricing. First of all we have the effect of changing the quality level through the effect on demand which in reality is a second-order effect. Increased quality raises demand, and at given prices and costs, revenue is increased. This effect would disappear if prices are determined optimally. Then there is the *direct* effect which works through the price and cost effects. Increased quality raises the price the product can be sold for, and similarly, in the presence of cost complementarity, reduces the marginal cost of providing the product at a given level of quality.

To simplify the representation, I assume that the efficiency parameters are common knowledge among the firms and symmetric (unless otherwise stated). I assume that the firms maximise their profit functions given by equation (6), and choose their respective levels of quality simultaneously. We consequently look for Nash equilibria in the quality game.

6.3.1 Qualities as complements

In the substitutes case there is no problem of free-riding in quality investments, whereas when qualities are complements this may be a problem. This analysis is summarised in propositions 5 and 6. Propositions 5 and 6 assume that prices are exogenously given. This implies that quality investments has no effect on the firms' revenue except through the quantity effect. It may also affect their costs due if there is cost complementarities. To focus on the effect of cost complementarities, I choose to ignore asymmetries due to differences in the efficiency levels (i.e., I assume that $\beta_i = \beta_j$ except where stated otherwise).

One may then ask why firms choose to provide additional quality if such an investment has no effect on prices and which increases costs? Increasing quality does increase the quantity of the investing firm, but whether the firm will choose to invest in quality will depend on the cost structure and the profit-margin. If the firm chooses to invest when the price is fixed, it will certainly do so if the price is increasing in quality.[30]

Let us assume that the first-order condition is binding for some interior value of s. Let $\widetilde{s}_i$ be defined by equation (14) when there is no cost complementarity:

$$(p_i - \beta_i) \frac{\partial q_i (p, s_i + s_j)}{\partial s_i} - \frac{\partial \psi}{\partial s_i} = 0 \tag{14}$$

for some $\widetilde{s}_i \in (\underline{s}, \overline{s})$ for a firm with marginal cost β_i. If the price-cost margin is sufficiently high relative to the marginal investment costs, an interior solution will

[30] It is reasonable to expect that prices are increasing in quality in the absence of any restrictions on prices.

exist. If there exists such a $\widetilde{s}_i$, then both firms provide positive levels of quality.

If there is cost complementarity (of the particular type considered here), the profit maximising choice of quality is given by s_i^+ and is defined by equation (15):

$$(p_i - c_i(\beta_i, s_i)) \frac{\partial q_i}{\partial s_i} - \frac{\partial c_i}{\partial s_i} q_i - \frac{\partial \psi}{\partial s_i} = 0 \tag{15}$$

where $\partial c_i / \partial s_i < 0$. Observe that the (symmetric) level of quality provided when there is no cost complementarity will be lower than the (symmetric) level provided if there is cost complementarity; i.e., $\widetilde{s}_i \leq s_i^+$.

Proposition 5

Assume that individual quality contributions are unverifiable and firms are equally efficient ex ante (i.e., $\beta_1 = \beta_2$). If prices are exogenous and there are no cost complementarities, then there is no problem of free-riding in quality provision. Both firms provide a positive level of quality unless the price is sufficiently low, or the investment cost sufficiently convex.

Proposition 6

If there is cost complementarity and quality is unregulated, then the problem of free-riding in quality provision is introduced. The firm with the largest market share will be the sole provider of quality.

The proof is found in appendix 2.

The intuition behind the result of Proposition 6 is that cost complementarity between quantity (or network capacity) and quality gives rise to a cost advantage for the firm with the largest market share.[31] This factor indicates that there is an advantage in having a large market share. Optimal quality is then increasing in the market share, and it is the firm with the largest market share which invests in

[31] In addition, if prices are sensitive to quality changes the cost of increasing quality is partly offset by the increase in price.

quality, whereas the firm with the smallest market share free-rides on the investment made by the other firm.

The result in proposition 5 can be explained by observing that the absence of cost complementarity makes firms symmetric. The level of optimal total quality is the same for both firms, since there are no cost advantage of having high production levels. The quality levels are then determined by the outcome of the quality game between the two firms, an equilibrium which is symmetric and in which both firms provide half of the optimal total quality each. If prices are subjected to price-cap regulation, we observe that we may have investments in quality even though firms face a binding price-cap. To see how quality affects a firm's marginal profitability, observe that the marginal profit (with respect to quality) increases with the quantity produced, since $\frac{\partial^2 \pi_i}{\partial q_i \partial s_i} = -\frac{\partial c_i}{\partial s_i} > 0$. Thus, for each extra unit produced by firm i, marginal profit with respect to quality is increased by $-\partial c_i / \partial s_i$. Thus, cost complementarity implies that for a given level of quality, firm i enjoys higher marginal profit (at this level of quality) than firm j if firm i has a higher market share than firm j; that is, if $q_i > q_j$. Firm i will produce a higher output in equilibrium (i.e., $q_i > q_j$) if, for instance, firm i is more efficient than firm j ($\beta_i < \beta_j$).

What about the effect on optimal (total) quality of changes in the efficiency parameter, β? If firms' efficiency parameters are different, this also affects the optimal level of quality in a similar way as both cost complementarity and quality sensitive prices, since different values of β_i and β_j introduces asymmetry between the firms. If $\frac{ds_i^*}{d\beta_i} < 0$, more inefficient firms has a lower optimal (total) quality. This inequality holds in the absence of cost complementarity. When there is cost complementarity, the inequality may be reversed, since higher level of quality reduces the marginal cost of producing the product. A larger market share increases the potential for having $\frac{ds_i^*}{d\beta_i} > 0$.[32] Thus, for a large enough market share for the most inefficient firm, the effect may be that it is the provider of the highest level of quality whereas the most efficient firm free-rides on the less efficient firm's investment in quality. However, if firms are symmetric the equilibrium of the game will be

[32] The sign of the inequality is determined by the sign of the following expression: $\frac{\partial^2 \pi_i}{\partial \beta_i \partial s_i} = -\frac{\partial c_i}{\partial \beta_i}\left(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial p_i} p_i'\right) - \frac{\partial^2 c_i}{\partial \beta_i \partial s_i} q_i \lesseqgtr 0$.

symmetric in terms of quantities. Consequently, to justify the more inefficient firm having a larger market share than the more efficient firm we need to make some *ad hoc* assumptions about, e.g., one firm being an incumbent with an installed base.

It could also happen in the case of observable quality that the less efficient firm provides the highest level of quality if, for instance, the market share of the less efficient firm is sufficiently large. In the observable case the socially optimal level of quality is still higher than the outcome of the unregulated game. Furthermore, it is generally the case that both firms provide positive levels of quality when the regulator can design a contract contingent upon the quality levels, unless the marginal (quality) investment cost is very large.

6.3.2 Qualities as substitutes

The case of verifiable quality in the substitute case is considered in detail in the analysis of the basic model. We have seen that with *ex ante* symmetric firms (i.e., the efficiency parameters are drawn from identical distributions), the most efficient firm provides a higher level of quality. The firms provide positive levels of quality (or rather, is instructed to provide positive levels of quality), if the marginal cost of investing in quality and the shadow cost of public funds is not too large.

For the case of unverifiable quality, however, the solution may depend on whether there is cost complementarity.[33] In the absence of cost complementarity, it may be the case that both firms provide the same (positive) level of quality, or that both firms decides on no quality investment. In any case, the absence of cost complementarity leads to symmetric Nash equilibria.[34] Thus, we have the following proposition:

Proposition 7

Assume unverifiable quality and equally efficient firms. When qualities are substitutes, and in the absence of cost complementarity, then there is no vertical dif-

[33]The price sensitivity to quality changes and differences in efficiency may also affect the solution. However, in the absence of these two effects, the solution depends critically on whether there is cost complementarity.

[34]This should be no surprise, since firms (at a given level of efficiency) are symmetric in the absence of cost complementarity and if prices are insensitive to quality changes.

ferentiation between the networks. Either both firms provide the same positive level of quality, or both make no quality investments. There is therefore no problem of free-riding on the other's investments.

The proof is provided in appendix 3.

When allowing for cost complementarity of the particular type considered here, the optimal level of quality for any given firm is increasing in the market share. A larger market share (corresponds to a large q) implies a cost advantage in quality provision, which again may lead the firm to increasing its level of quality whereas the firm with the smaller market share reduces his. Thus, in the presence of cost complementarity (and/or price sensitivity to quality changes) the effect is that the degree of vertical differentiation is increased. This is similar to the effect of cost complementarity in the case of observable quality.

Furthermore, the unregulated choice of quality is in general different from the socially optimal quality level, but the bias of the distortion is difficult to ascertain in general. The reason for this is that the unregulated firm cannot internalise all the external effects from the investments. The investing firm is able to internalise is the demand effect (on its own demand), but is not able to internalise the (negative) external effect on the other firm nor the (positive) external effect on consumers' surplus.

7 Summary

The situation considered in this paper is the situation where a benevolent regulatory agency is able to affect (through the regulation mechanism) the access quality which the two firms offer to end-users. Assuming that firms' efficiency levels are different, we have seen that the regulator should induce firms to produce access of different quality; i.e., that some degree of (vertical) differentiation is indeed socially optimal in the setting of this model. The presence of asymmetric information necessitates even more differentiation in order to sufficiently distort the market shares in favour of the most efficient of the two firms, which implies that the more efficient firm produces more under asymmetric information. The primary reason for the optimal degree of differentiation to depend on the information which is available to the regulator is related to the cross-effects on information rents. Another reason is that both firms' efficiency parameters are drawn from identical distributions. Allowing for different distributions may modify the results. It is then not necessarily the case that the term correcting for asymmetric information unambiguously distorts optimal quality in favour of the most efficient of the two firms.

It is, of course, important for the regulator to know both the cost structure and how the level of quality impacts on the demand side. In the extensions to the basic model, I allow for variations in the model. The results there highlight the importance of knowing the structure of both demand and costs for optimal regulatory policy.

8 Appendices

Appendix 1

Proof of Proposition 1

I follow the standard approach of, e.g., Guesnerie and Laffont (1984) when clarifying the requirements for implementation under asymmetric information. However, there are some changes since expectations must be taken into account.

Firm i's profit function, when taking into account that firms have private information, and for a more general cost function, is given by (taking expectations over the other firm's efficiency parameter)

$$
\begin{aligned}
E_{\beta_j}\left[\pi_i\left(\hat{\beta}_i,\beta_i,\beta_j\right)\right] &= E_{\beta_j}\Big[(p-\beta_i)\,q_i\left(s_i\left(\widehat{\beta}_i,\beta_j\right),s_j\left(\widehat{\beta}_i,\beta_j\right)\right) \\
&\quad + t_i(\hat{\beta}_i,\beta_j)-\psi^i\left(s_i(\hat{\beta}_i,\beta_j)\right)\Big]
\end{aligned}
$$

for $i,j=1,2$, and $i\neq j$.

Firm i chooses its report to maximise profits, and the report must satisfy the following first- and second-order conditions:

(1) $\frac{\partial E_{\beta_j}\left[\pi_i\left(\left(\hat{\beta}_i,\beta_i,\beta_j\right)\right)\right]}{\partial\hat{\beta}_i}=0$

(2) $\frac{\partial^2 E_{\beta_j}\left[\pi_i\left(\left(\hat{\beta}_i,\beta_i,\beta_j\right)\right)\right]}{\partial \hat{\beta}_i^2} \leq 0$

Differentiating (1) with respect to the true efficiency parameter, β_i, and noting that at the optimum the reported type is equal to actual type (this is the property of incentive compatibility), the second-order condition can be rewritten as:

$$\frac{\partial^2 E_{\beta_j}\left[\pi_i\left(\left(\beta_i,\beta_j\right)\right)\right]}{\partial s_i \partial \beta_i}\frac{\partial s_i}{\partial \beta_i}+\frac{\partial^2 E_{\beta_j}\left[\pi_i\left(\left(\beta_i,\beta_j\right)\right)\right]}{\partial s_j \partial \beta_i}\frac{\partial s_j}{\partial \beta_i} \geq 0 \qquad (*)$$

We can show that $\frac{\partial^2 E_{\beta_j}\left[\pi_i\left(\left(\beta_i,\beta_j\right)\right)\right]}{\partial s_i \partial \beta_i} = -E_{\beta_j}\left[\frac{\partial q_i\left(\beta_i,\beta_j\right)}{\partial s_i}\right]$, which is assumed to be negative, and $\frac{\partial^2 E_{\beta_j}\left[\pi_i\left(\left(\beta_i,\beta_j\right)\right)\right]}{\partial s_j \partial \beta_i} = -E_{\beta_j}\left[\frac{\partial q_i\left(\beta_i,\beta_j\right)}{\partial s_j}\right]$, which is assume to be positive. Furthermore, by symmetry we have that $\frac{\partial s_j}{\partial \beta_i} = \frac{\partial s_i}{\partial \beta_j}$. Thus, the inequality (*) is satisfied for the following conditions:

(i) $\frac{\partial s_i}{\partial \beta_i} \leq 0$

(ii) $\frac{\partial s_i}{\partial \beta_j} \geq 0$

Thus, inequalities (i) and (ii) are sufficient conditions for implementation. These inequalities will be utilised as (the equivalent of the) second order conditions of incentive compatibility in the section on optimal regulation under asymmetric information. The standard procedure is to ignore these constraints ex ante, and checking whether they are satisfied ex post to avoid unnecessary messy calculations. If the monotonicity conditions are not satisfied, optimal policies may be characterised by partial pooling contracts.

Appendix 2

(proof of Propositions 5 and 6. The proof is adapted from Auriol, 1998)

The firms chooses a level of quality which maximises profits. Optimal (total) quality for firm i, s_i^* , is given by the first-order condition: $\frac{\partial \pi_i}{\partial s_i} = 0$, or $(p_i - c_i)\left(\frac{\partial q_i(p,s_i+s_j)}{\partial s_i} + \frac{\partial q_i}{\partial p_i}p_i'\right) + \left(p_i' - \frac{\partial c_i}{\partial s_i}\right)q_i - \partial\psi/\partial s_i = 0$. Qualities are complements, and what matters is total quality, not which firm supplies the quality. s_i^* defines the level of total quality which maximises firm i's profit, and can be supplied by firm i alone, by firm j alone, or as a team effort by both firms. Firm i's best response is given by: $s_i(s_j) = s_i^* - s_j$, for $s_j \leq s_i^*$, and $s_i(s_j) = 0$ otherwise. Implicit differentiation of the first-order condition with respect to q_i yields:

$\frac{ds_i^*}{dq_i} = -\left(\frac{\partial^2\pi_i}{\partial q_i \partial s_i}\right) / \left(\frac{\partial^2\pi_i}{\partial s_i^2}\right)$. The denominator (the second-order condition) is assumed to be negative. The sign of $\frac{ds_i^*}{dq_i}$ is determined by the sign of the numerator: $\frac{\partial^2\pi_i}{\partial q_i \partial s_i} = p_i' - \frac{\partial c_i}{\partial s_i} \geq 0$. In the absence of cost complementarity (CC) and if prices are insensitive to quality changes (that is, if $p_i' = 0$), $\frac{ds_i^*}{dq_i} = 0$, and for CC, or if prices respond to quality changes, $\frac{ds_i^*}{dq_i} > 0$. Thus, if there is CC, the optimal (total) quality is increasing in the market share and otherwise not (assuming $p_i' = 0$). For CC or for quality sensitive prices, $q_i > q_j$ implies $s_i^* > s_j^*$. Thus, the unique Nash equilibrium in the quality game corresponds to Auriol (1998): For $q_i > q_j$, $s_i = s_i^*$ and $s_j = 0$. Thus, free-riding in quality provision is present. In the absence of CC (and for $p_i' = 0$), optimal quality for both firms is independent of the market shares; $s_i^* = s_j^* = s^*$. Best responses for firm i and j are given by: $s_i(s_j) = s^* - s_j$ and $s_j(s_i) = s^* - s_i$. Thus, the equilibrium strategy is to have $s_i = s_j$, which amounts to: $s_i(s_j) = \frac{1}{2}s^*$ for i, j.

Appendix 3

(proof of proposition 7)

Let $s_i^+ = \arg\max\left[(p_i(s_i) - c_i(\beta_i, s_i))\, q_i - \psi(s_i)\right]$ define the profit maximising quality choice for firm i. In the absence of cost complementarity and with unresponsive prices, firms are symmetric if $\beta_i = \beta_j$. This implies that at the symmetric Nash equilibrium $s_i^+ = s_j^+ = s^+$; that is, the optimal level of quality is identical for both firms $i = 1, 2$. Whether the firms provide additional quality (in excess of the minimum quality standard) depends on the sign of the first-order condition at $\underline{s}$: $s_i = s_j = \underline{s}$ if $\frac{\partial \pi_i}{\partial s_i} \leq 0$ at $s_i = \underline{s}$, and $s_i = s_j > \underline{s}$ if $\frac{\partial \pi_i}{\partial s_i} > 0$ at $s_i = \underline{s}$. For a price p sufficiently close to marginal cost β_i, or for sufficiently high marginal investment cost, s_i^+ is equal to zero for $i = 1, 2$.

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