Do Incumbents Have Incentives to Degrade Interconnection Quality in the Internet?

by

Øystein Foros, Hans Jarle Kind and Jan Yngve Sand

No. 10/03, December 2003

Department of Economics and Management
Norwegian College of Fishery Science
University of Tromsø
Norway
Do Incumbents Have Incentives to Degrade Interconnection Quality in the Internet?\textsuperscript{1}

\textbf{Øystein Foros}
Norwegian School of Economics and Business Administration
oystein.foros@nhh.no

\textbf{Hans Jarle Kind}
Norwegian School of Economics and Business Administration
hans.kind@snf.no

\textbf{Jan Yngve Sand}\textsuperscript{2}
University of Tromso
Jan.Sand@nfh.uit.no

\textbf{JEL classification:} L11, L13, L96

\textbf{Keywords:} Network externalities, competition, interconnection

\textbf{Abstract:} In this paper we analyze the interconnection incentives for firms that have an installed base of customers and that also compete for new customers. We show that the small firm may be harmed in the competition for new customers if the customers in the installed bases are charged a high price, since this makes the large firm more aggressive. It is also shown that the price charged to the installed base customers affects the incentives that determine interconnection quality. A high price to the installed base may, or may not, make a high interconnection quality likely.

\textsuperscript{1}We are indebted to Pinar Dogan, Lars Sørgard, and participants at EARIE (Madrid September 2002) for helpful discussions and comments. Furthermore, we thank Telenor for financial support through its research program at Institute for Research in Economics and Business Administration (SNF).

\textsuperscript{2}Corresponding author: University of Tromso, Department of Economics and Management, NFH, N-9037 Tromso, Norway. Tel. +47 776 45540, fax. +47 776 46020, e-mail: Jan.Sand@nfh.uit.no.
1 Introduction

The Internet consists of a number of sub-networks that are not necessarily seamlessly connected. Thus, the customers’ willingness to pay for being connected to a particular sub-network depends both on this network’s intrinsic quality and on the interconnection quality with other sub-networks. The interconnection quality may therefore become an important strategic variable for competing firms that each control one sub-network. If rational consumers expect that one network for some reason will be larger than another, the seminal paper by Katz and Shapiro (1985) shows that the owner of the larger network will have lower incentives to improve the interconnection quality than has the owner of the small network. The reason for this is that there will de facto be a quality differentiation between the two networks that favors the large network if the interconnection quality is poor.

Firms within the Internet and the telecommunication industry normally have installed bases of customers that they serve at the same time as they compete over new customers. A hot topic has been whether a firm with a relatively large installed base has incentives to degrade the interconnection quality towards its smaller rivals. This issue was raised during the AOL-Time Warner merger and the MCI-WorldCom merger.3

Crémer, Rey and Tirole (2000) use a modified version of the Katz and Shapiro (1985) model to analyze a dominant firm’s incentives to degrade quality to rival firms. They assume that the merged firm will be in possession of a relatively large base of installed customers, and demonstrate how this may give the firm an incentive to reduce the interconnection quality towards a smaller rival. Furthermore, they show that the large firm’s incentive to maintain a high interconnection quality is decreasing in the size difference between the installed bases of the two firms.

3Rubinfeld and Singer (2001) and Crémer et al. (2000) analyze this question for the AOL/TimeWarner merger and the MCI/WorldCom merger, respectively, while Foros, Kind and Sørgard (2002) analyze the interplay between telecommunication incumbents, global IBPs and regional ISPs. The Internet backbone market is also analyzed by Besen et al. (2001), Milgrom et al. (2000), and Laffont et al. (2001a, 2001b). A recent overview of the market structure and regulation of the Internet is given by Cave and Mason (2001).
Crémer et al. assume that, other things equal, the utility new customers’ get from network subscription is increasing in the total network size and the interconnection quality. Hence, new customers are charged a price which is increasing in the total network size and in the interconnection quality. In contrast, the price paid by customers in the installed bases is assumed to be fixed in their model. The main focus in our paper is on the analysis of how interconnection incentives for large firms are affected by changes in the revenue collected from the installed base customers. More specifically, how does an income structure where customers in the installed bases are being charged a price that is increasing in the total network size and in the interconnection quality affect the interconnection incentives for the larger firm?

On the one hand, we may expect that the larger firm will have relatively strong incentives to improve the interconnection quality, because this makes it possible to charge higher prices from customers in its installed base. In line with this, we show that the larger firm may be willing to set a high interconnection quality even if it means that it captures a lower number of new customers than would be the case with a poor interconnection quality.

On the other hand, the larger firm gains a competitive advantage if it sets a low interconnection quality. We demonstrate that this effect is more likely to dominate the larger the difference between the installed bases and the higher the price the firm can charge each customer in the installed base. We further show that, for any given interconnection quality, the total number of new customers served by the firms depends only on the total size of the installed bases and not on whether the bases are asymmetric in size. However, an asymmetry in the size of the installed bases may affect the interconnection incentives, and this will in turn influence the market size. Moreover, the smaller firm may be harmed in the competition for new customers if there is an increase in the price that is being charged from the customers in the installed base. The reason for this is that a high price to installed base customers makes the larger firm more aggressive in the market for new customers.

In Crémer et al. (2000) and Katz and Shapiro (1985) the degree of vertical differentiation between the large and the small provider is a function of the interconnection quality. The larger firm has a quality advantage only when the inter-
connection quality is imperfect. In our basic model we have a similar assumption, and analogous to Crémer et al. we assume that the interconnection quality is the same with respect to the installed base segment and the new competitive market. In an extension of our basic model, we in contrast assume that the interconnection quality to the installed base is set before the interconnection quality to the competitive market. Hence, the large provider needs not to reduce the interconnection quality towards the rival’s new customers in order to have a quality advantage. We show that in this context, we may have perfect interconnection quality in the new market even if the interconnection quality in the installed base segment is imperfect. Furthermore, we show that, if the interconnection quality is perfect for the installed base customers, the larger firm always choose to set perfect interconnection quality in the new market too.

The existence of an installed base seems realistic for the markets we have in mind. When a firm like AOL Time Warner enters a regional market in Europe, for instance, they compete with a regional ISP. AOL Time Warner’s customer base in the USA may be seen as an installed base or a clientele. Obviously, AOL Time Warner may gain a competitive advantage by offering the regional European ISP a low interconnection quality with the customers that AOL Time Warner has in the USA. However, it is likely that AOL Time Warner’s income from American customers also depends on the interconnection quality with European Internet users that are connected to regional ISPs. Typically, the revenue from the installed base customers will be higher if there are more people with whom they can have high quality communication. Intuitively, the gain from the installed base from a high interconnection quality may well offset the loss due to reduced competitiveness in the new market.

The installed base segment may also be seen as the market for the basic service, while the new competitive segment is the market for emerging services. One example is the question of interconnection quality (or interoperability) between AOL’s instant-messaging service and rivals (e.g. Yahoo and Microsoft) instant-messaging services. When firms like Yahoo and Microsoft began offering instant-messaging AOL had a large installed base, and AOL tried to reduce the degree of intercon-
nection towards the new rivals (see Faulhaber, 2002). In this context there seems realistic to assume that there is one interconnection variable for the existing text-based instant-messaging services. This variable is set before the interconnection variable for emerging instant-messaging services (that requires broadband). As a condition for the merger between AOL and Time Warner FCC imposed the condition that AOL have to offer interconnection with other providers of advanced instant-messaging services. However, no condition was imposed on the interconnection agreements for existing text-based instant-messaging services. Faulhaber (2002) argue that even if the interconnection quality between advanced instant-messaging services are perfect, AOL will have an advantage from degrading the interconnection to its large installed base. The reason is that also advanced instant-messaging services will probably use the same directory of "buddy lists" (Names and Presence Directory (NPD)). Hence, as low interconnection quality for existing text-based instant-messaging services will give AOL an advantage also in the market for advanced instant-messaging services (even if the interconnection quality for new services is perfect).

As another example, consider the market for broadband access to residential users. The two main alternatives are offered by telecommunication incumbents (who upgrade their copper network to handle DSL) and by cable-TV providers. In Europe the coverage of the telecommunication network is much larger than that of the cable-TV networks. Hence, we have a duopoly in some regions (typically in urban areas), while we have a monopoly controlled by the telecommunication incumbent in other regions (rural areas). Suppose that there are strong network effects such that the reservation price of a customer increases with the number of broadband users and with the interconnection quality between DSL and the cable-TV network. Since existing broadband users in rural areas have no alternative access possibilities, they can be seen as an installed base or a clientele for the telecommunication incumbent. The presence of a clientele implies that the incumbent has the ability to create a competitive advantage over the cable-TV providers in urban areas if it degrades the interconnection quality. The degradation may take place by reducing the data flow capacity between the networks, such that, for instance, an interactive
videoconference between people in rural and urban areas is possible only if all parties subscribe to the incumbent’s services. However, degrading the interconnection quality reduces the reservation price to the customers in the monopoly area, and this may well dominate the competition effect.\footnote{The same feature is found in the mobile networks, where the incumbent controlling a full coverage network may degrade the interconnection quality to smaller entrants. This will most likely become an important topic when new firms enter the mobile market with third generation mobile networks in Europe (UMTS).}

The rest of the paper is organized as follows. First, we present the basic model. Second, we focus on the main features of the market equilibrium for a given interconnection quality. Third, we analyze the incentives of a firm with a large installed base to degrade interconnection quality towards a smaller rival. Fourth, we have an extension of the basic model where interconnection is set independently for the installed base and the new customers, respectively. Finally, we make some concluding remarks.

\section{The model}

Suppose that two firms compete in a Cournot fashion, choosing the quantities $q_1$ and $q_2$ simultaneously.\footnote{Crémé et al. (2000) argue that Cournot gives a realistic description of the competition in the Internet backbone market. Faulhaber and Hogendorn (2000) show that the conditions in Kreps and Scheinkman (1983) are fulfilled in the broadband access market. Hence, they analyze a capacity constrained price game as a one-stage Cournot game. Foros and Hansen (2001) analyze the incentives to be compatible if the downstream firms compete à là Hotelling. In a model without installed bases they show that the firms choose to be completely compatible in order to reduce the competitive pressure.} Firm $i$ has an installed base $\beta_i$ of customers, and without loss of generality we will assume that firm 1 possibly has a larger installed base than firm 2, i.e., $\beta_1 \geq \beta_2$. The installed base of firm 1 may be customers living in an area not covered by the network of firm 2 and vice versa. The total number of installed base customers is equal to $\beta \equiv \beta_1 + \beta_2$. We assume that the contracts with the installed base customers are such that the revenue from the installed base increases both with the number of users (the total network size) and with the interconnection...
Let \( s_i \) denote the perceived quality of network \( i \). The inverse demand curve of firm \( i \) is given by

\[
p_i = 1 + s_i - q_i - q_j.
\]

The quality \( s_i \) of the service is given by:

\[
s_i = vN_i
\]

The term \( N_i \equiv \beta_i + q_i + \theta(\beta_j + q_j) \) is the \textit{quality-adjusted} total network size; other things equal, it is increasing in the interconnection quality \( \theta \) and in the total number of new and existing customers in the two networks. The interconnection quality between the two networks is measured by the parameter \( \theta \in [0, 1] \); there is no interconnection if \( \theta = 0 \), and perfect interconnection if \( \theta = 1 \). The parameter \( v \) may be interpreted as a measure of network effects; the higher the value of \( v \) the more important is the total network size for the customers.

The equilibrium price in the competitive segment is then given by:

\[
p_i = 1 - q_i - q_j + s_i = 1 + v(\beta_i + \theta \beta_j) - (1 - v)q_i - (1 - \theta v)q_j
\]

This is analogous to Crémer, Rey and Tirole (2000).

The cost of connecting one additional customer is \( c \), where \( c \in [0, 1] \). Throughout we assume that the cost of increasing the interconnection quality \( \theta \) is equal to zero.

The profit for the firms is:

\[
\pi_i = (p_i - c)q_i + \pi_i^\beta,
\]

where the last term is the profit from the installed base \( \beta_i \). More specifically, we assume that the profit from the installed base is given by

\[
\pi_i^\beta = \beta_i(wN_i).
\]

The variable \( w \) is the price that each customer in the installed base is charged by network owner \( i \). Since we will not be focusing on the contracts that the networks
have with customers in their installed bases, we will treat $w$ as an exogenously given parameter.\footnote{In the context of broadband access, $wN_i$ may be seen as a discounted monthly fee that depends on the quality-adjusted network size.}

Throughout we make the following assumptions:

**Assumption 1:** The equilibrium interconnection quality is equal to the level chosen by the firm that values interconnection the least, and there are no access prices paid for interconnection.

**Assumption 2:** The firms can price discriminate between new customers and installed base customers, and the installed base customers cannot switch or resign from a provider as a result of degraded interconnection quality or higher subscription price.

Assumption 1 is similar to Crémer et al., and they motivate the limited possibilities to charge an access price between networks in the Internet backbone market by problems with writing complete contracts. An alternative interpretation can be found by looking at the broadband access market and the mobile market, where incumbents are subject to regulation that limits free determination of access prices.

Assumption 2 is realistic in contexts where firms like AOL Time Warner enter a new country. Then the providers may price discriminate between customers in different national markets, since the installed base customers cannot buy the service in another country. Obviously, some installed base customers may resign when the price is high, but that is not considered in the model since the price charged to the installed base customers is exogenous. The installed base customers are assumed to have positive levels of utility of being connected to the network for any permissible prices.\footnote{The assumption that installed base customers cannot resign or switch may be problematic if $w$ becomes very large, but in Assumption 3 we restrict the magnitude of $w$ to avoid such a problem.}

In the following we will consider a two-stage game. In the first stage the firms set the interconnection quality, and in the second stage they choose quantities simul-
taneously. We will first characterize the properties of the second stage, before we proceed to analyze the question of whether the firms may have incentives to degrade the interconnection quality.

2.1 Cournot competition

The first-order condition for firm $i$ with respect to quantity gives the following reaction function for firm $i$:

$$q_i(q_j) = \frac{1 - c + v(\beta_i + \theta \beta_j) + w\beta_i - (1 - \theta v)q_i}{2(1 - v)}$$

(2)

In order to ensure stability for $\theta \in [0, 1]$ we need to assume that the importance of connectivity to new users (or the importance of the network effect) is sufficiently small; more specifically, $v < 1/2$ (see assumption 3 below). Note that neither $w$ nor the size of the installed bases affects the slope of the reaction curves, but an increase in $w$ or in $\beta_i$ shifts the reaction curve $q_i(q_j)$ upwards. In particular, we may end up in a monopoly equilibrium if $w$ or the difference $(\beta_1 - \beta_2)$ is sufficiently large.\(^8\)

Throughout the paper we make the following assumption:

**Assumption 3:** We assume that $v < 1/2$ and $w \leq v$.

The first part of Assumption 3 ensures that the equilibrium is stable, as noted above, whereas the latter part is sufficient to ensure that the smaller firm will always produce a positive quantity in equilibrium. If $w \leq v$, the value to the firms of each new customer exceeds that of each customer in the installed base. If $w$ is sufficiently larger than $v$, the firms’ main focus may turn towards the installed base customers where the larger firm has an absolute advantage. Consequently, in the latter case we may end up in a situation where the larger firm is the sole producer.

Solving equation (2) for the two firms we find the equilibrium quantities:

$$q_i^* = \frac{1}{2} \left( \frac{2(1 - c) + v(1 + \theta)\beta}{2(1 - v) + (1 - \theta v)} + \frac{v(1 - \theta)\Delta_1}{2(1 - v) - (1 - \theta v)} \right) + \frac{2(1 - v)\beta_1 - (1 - \theta v)\beta_2 w}{4(1 - v)^2 - (1 - \theta v)^2}$$

(3)

\(^8\)Since $\beta = \beta_1 + \beta_2$ is fixed, a larger $\beta_j$ implies that $\beta_i$ is smaller. Equation (2) therefore shows that a larger $\beta_j$ shifts the reaction curve $q_i(q_j)$ downwards.
\[ q_2^* = \frac{1}{2} \left( \frac{2(1 - c) + v(1 + \theta)\beta}{2(1 - v) + (1 - \theta v)} - \frac{v(1 - \theta)\Delta_i}{2(1 - v) - (1 - \theta v)} \right) + \frac{2(1 - v)\beta_2 - (1 - \theta v)\beta_1}{4(1 - v)^2 - (1 - \theta v)^2} w \]

where \( \beta \equiv \beta_1 + \beta_2 \) is the total installed base, and \( \Delta_i \equiv \beta_i - \beta_j \) (for \( i, j = 1, 2 \), and \( i \neq j \)) is the difference in installed bases of the two firms.

The first term in the bracket of equations (3) and (4) shows that both firms tend to have a higher output the larger is the total installed bases. This simply reflects the fact that larger installed bases make the networks more attractive for unattached customers. However, the firm with the larger installed base will have a competitive advantage if \( \theta < 1 \). Therefore, the second term in the bracket is positive for firm 1 and negative for firm 2.

The third term in equations (3) and (4) shows how the quantities depend on \( w \) and the size of the installed bases, \( \beta_1 \) and \( \beta_2 \). This term is unambiguously positive for firm 1, but is negative for firm 2 if \( 2(1 - v)\beta_2 - (1 - \theta v)\beta_1 < 0 \).

Adding (3) and (4) we find that total quantity \( Q^* \equiv q_1^* + q_2^* \) is:

\[ Q^* = \frac{2(1 - c) + v(1 + \theta)\beta}{2(1 - v) + (1 - \theta v)} + \frac{2(1 - v) - (1 - \theta v)}{4(1 - v)^2 - (1 - \theta v)^2} w \beta. \]

Since \( \partial Q^*/\partial w > 0 \), we thus have:

**Proposition 1** A larger price \( w \) charged to the installed base customers implies that

(i) the number of new customers served by the larger Firm 1 increases \((\partial q_1^*/\partial w > 0)\).

(ii) the number of new customers served by the smaller Firm 2 increases if and only if \( \beta_2/\beta_1 > \bar{\beta} \equiv (1 - \theta v) / [2(1 - v)] \).

(iii) the total number of new customers served by the two firms increases \((\partial Q^*/\partial w > 0)\).

The intuition behind Proposition 1 is that the incentive to increase the network size in order to generate higher profit from the installed base is increasing in the price \( w \) that is paid by these customers. Both firms will therefore tend to be more aggressive in the end-user market the higher the value of \( w \). However, if the installed base advantage of the larger firm is sufficiently pronounced, we see from equation (4) that an increase in \( w \) actually reduces firm 2’s output, \( q_2 \). In particular, for \( \beta_2 \) close to 0 the smaller firm’s output will always decrease when \( w \) increases. This not
only implies that the more aggressive behavior of firm 1 subsequent to an increase in \( w \) reduces the number of new customers captured by the smaller firm; but also that it may reduce the total profit of firm 2. This is most easily seen for \( \beta_2 = 0 \), in which case \( \pi_2 = (1 - v)q_2^2 \).

From equation (5) we further see that

**Proposition 2** For any given level of \( \theta \), the total quantity \( Q^* \) depends positively on the total size of the installed base (\( \beta \)) and is independent of the difference in installed bases between the firms (\( \beta_i - \beta_j \)).

This result is in line with Bergstrom and Varian (1985), who show that given certain conditions total quantity in a Cournot game is independent of the individual agents’ characteristics.

### 2.1.1 The relationship between the interconnection quality and output

In order to see how improved interconnection quality affects output, we first note from equation (5) that

\[
\frac{dQ^*}{d\theta} = 2(1-c) + (3-(v-w))\beta \frac{\beta}{(3-2v-\theta v)^2} v > 0.
\]

Improved interconnection quality will thus unambiguously increase total quantity. The reason for this is simply that an increase in \( \theta \) implies that the total quality-adjusted network size increases, and this makes it more attractive for new customers to connect to the networks.

To see how improved interconnection quality affects the output of each single firm, we differentiate equations (3) and (4) with respect to \( \theta \) to find:

\[
\frac{dq_i^*}{d\theta} = \frac{1}{2} v \left[ \frac{\Delta_i (1-v)}{(2(1-v) - (1-\theta v))^2} + \frac{2(1-c) + (3-v)\beta}{(2(1-v) + (1-\theta v))^2} \right] - \frac{w\Delta_i}{(2(1-v) - (1-\theta v))^2} + \frac{w\beta}{(2(1-v) + (1-\theta v))^2}.
\]  

The two first elements in (6) are identical to Crémer et al. (2000), whereas the latter two elements explain how installed base profit influences the solution. Changing the quality of interconnection has the following effects on the firms’ equilibrium outputs:
The first term is the quality differentiation effect; an improved interconnection quality reduces the competitive advantage of the large firm. This term is negative for firm 1 and positive for firm 2.

The second term is the demand expansion effect; an improved interconnection quality increases all consumers’ willingness to pay. This effect is positive for both firms, and indicates that both firms will capture a larger number of new customers if $\theta$ increases.

The third and fourth terms are the installed base effects, which in essence strengthen the quality differentiation effect and the demand expansion effect. On the one hand, the presence of the installed base makes it less profitable for the larger firm to increase the quality of interconnection when the degree of differentiation is high, since a large $\Delta_1$ will result in a lower equilibrium output by the smaller firm (the larger the disadvantage of the smaller firm the more negatively affected is the smaller firm’s equilibrium output). This results in a lower profit from the installed base, all other things equal. On the other hand, it is more profitable for the larger firm to increase $\theta$ when the total level of the installed base, $\beta$, is large, since this tends towards a higher output by the smaller firm.\(^9\)

We can thus conclude:\(^{10}\)

**Lemma 1** When the profit from the installed base depends on the quality-adjusted network size we have that:

i) The smaller firm’s equilibrium output is increasing in the interconnection quality ($dq_1^*/d\theta \geq 0$).

ii) The larger firm’s equilibrium output may be increasing or decreasing in the interconnection quality ($dq_2^*/d\theta \leq 0$).

iii) The total equilibrium output is increasing in the interconnection quality ($dQ^*/d\theta > 0$).

\(^9\)Of course, the increase in the magnitude of the installed base can be a result of increasing only the larger (smaller) firm’s installed base, in which case the degree of differentiation will also be higher (lower).

\(^{10}\)See Appendix A.1 for a discussion of necessary and sufficient conditions for $dq_1^*/d\theta$ to be negative.
2.2 Incentives for quality degradation?

We will now analyze the firms’ incentives to increase \( \theta \). The interconnection quality is assumed to be in the interval \( \theta \in [0, 1] \), which ensures that all the comparative statics are valid for all permissible \( \theta \). The equilibrium profit may be written as:

\[
\pi_i = (1 - v)(q_i^*)^2 + \beta_i w(\beta_i + q_i^* + \theta(q_j^* + \beta_j))
\]  

(7)

Differentiating the equilibrium profit in (7) with respect to \( \theta \) we can identify three different effects of improved interconnection quality:

\[
\frac{d\pi_i}{d\theta} = 2(1 - v)(q_i^*) \frac{dq_i^*}{d\theta} + \beta_i w(q_j^* + \beta_j) + \beta_i w \left( \frac{dq_i^*}{d\theta} + \frac{dq_j^*}{d\theta} \right)
\]  

(8)

The first term is similar to Crémer et al. (2000), and is the effect on the profit from the new customers when the interconnection quality improves. This term is positive for firm \( i \) if and only if the firm captures new customers when \( \theta \) increases. The second and third terms relate to the installed base effect. The second term is always positive, and is the increase in profit from firm \( i \)’s installed base when \( \theta \) increases for a given number of new and locked-in customers of firm \( j \). When the quality of interconnection improves, the number of new customers is affected. This will have an impact on the profit captured from the installed base, and is described in the third term.

For the smaller firm all three terms in equation (8) are positive (see Appendix A.2). Hence, the profit for firm 2 is increasing in \( \theta \), so that it prefers perfect interconnection quality. In contrast, for firm 1 we may have \( d\pi_1/d\theta < 0 \) since term 1 and term 3 in equation (8) may be negative. Consequently, given Assumption 1 it is the larger firm’s choice of interconnection quality that determines which interconnection quality will prevail.

In Appendix A.2 we show the profit function for firm 1 is convex in \( \theta \), and we have the following result:\(^{11}\)

\(^{11}\)If we introduce costs associated with increasing interconnection quality, we may have an interior solution with respect to quality if these costs are convex enough. However, this does not change our results qualitatively.
Lemma 2 Firm 2 will always prefer to have complete interconnection quality, while firm 1 chooses θ = 0 or θ = 1.

From equation (8) we see that \(d\pi_1/d\theta\) is strictly positive if \(dq_1^*/d\theta = 0\). By continuity, it then follows that there exists some interval where \(d\pi_1/d\theta\) is positive even if \(dq_1^*/d\theta < 0\):

Proposition 3 Assume that the profit from the installed base is affected by the quality-adjusted network size \(N_i\). An improvement of the interconnection quality may then increase the profit level of the larger firm even if its number of unattached customers falls (i.e., \(d\pi_1^*/d\theta > 0\) even if \(dq_1^*/d\theta < 0\)).

This is in contrast to the case analyzed by Crémer et al. (2000) where the profit from the installed base is not affected by \(N_i\). Then \(dq_1^*/d\theta < 0\) is a sufficient and necessary condition to ensure \(d\pi_1^*/d\theta < 0\).

The intuition behind the result in Proposition 3 is the following: When the profit from the installed base is no longer assumed to be constant, the installed base effect implies that the profit from increased willingness to pay by the locked-in users will make it less profitable to degrade the interconnection quality. In this case, improved interconnection quality has an additional positive effect on profits, which together with the demand expansion effect may dominate the quality differentiation effect.

The importance of the profit from the installed base, valued through \(w\), will have an impact on the firms’ incentives to choose high quality connectivity.\(^{12}\) When \(w\) increases, the profit from the installed base increases if we keep output constant. However, the magnitude of \(w\) will also affect the equilibrium output. By differentiating (6) with respect to \(w\) we find how the basic trade-off between the degree of differentiation and the total installed base affects the marginal profitability of increasing the interconnection quality:

\[
\frac{\partial^2 q_1^*}{\partial \theta \partial w} = \frac{1}{2} w \left[ \frac{-\Delta_1}{((2(1-v)) - (1 - \theta v))^2} + \frac{\beta}{((2(1-v)) + (1 - \theta v))^2} \right]
\]

\(^{12}\)From Lemma 2 we know that firm 2 will always prefer maximum interconnection quality, so it suffices to examine firm 1’s incentives to choose high quality when \(w\) changes.
We see that when the difference between the firms’ installed bases is low (i.e., for low values of $\Delta_1$) it becomes less likely that the bigger firm will choose $\theta = 0$ when $w$ increases, while for high values of $\Delta_1$ (i.e., $\Delta_1$ close to $\beta$), it becomes more likely that firm 1 will choose $\theta = 0$ when $w$ increases. Thus, when the installed bases of the firms are of similar sizes, the demand expansion effect (and not the quality differentiation effect) becomes more important for the investing firms. The demand expansion effect is positive for both firms, but the quality differentiation effect may be negative for the larger firm. Consequently, when the difference between the installed bases is small, the firms will want to make as much out of the demand expansion effect as possible, which entails setting the interconnection quality as high as possible (i.e., $\theta = 1$).

This is summarized in the following proposition:

**Proposition 4** When the difference in installed base between the firms is low compared to the total installed base (i.e., $\Delta_1$ low compared to $\beta$), a larger price $w$ charged to the installed base lowers firm 1’s incentive to degrade the interconnection quality. In contrast, when $\Delta_1$ is close to $\beta$, a large $w$ increases firm 1’s incentive to set $\theta = 0$.

If $\theta = 1$ we can use equations (3) and (4) to find

$$q_1 = \frac{1}{3} \left( \frac{1 - c + v\beta}{1 - v} \right) + \frac{2\beta_1 - \beta_2}{3(1 - v)}w$$

$$q_2 = \frac{1}{3} \left( \frac{1 - c + v\beta}{1 - v} \right) + \frac{2\beta_2 - \beta_1}{3(1 - v)}w,$$

from which it follows that

**Proposition 5** When there is perfect interconnection $\theta = 1$ we have that:

(i) When the profit from the installed base is independent of the quality-adjusted total network size $N_i$ the firms will be symmetric in the market for new customers, $q_1^* = q_2^*$, even if $\beta_1 > \beta_2$ (Crémer et al., 2000).

(ii) When the profit from the installed base depends on the quality-adjusted total network size $N_i$ the firms will be asymmetric in the market for new customers, $q_1^* > q_2^*$, also with perfect interconnection.
The intuition behind this result is as follows: The competitive advantage stemming from having a large installed base falls as the interconnectivity improves, and the networks have the same quality when $\theta = 1$. However, the larger firm always have relatively strong incentives to be aggressive in the market for new customers if $w > 0$. This is due to the fact that this firm, at the margin, always has a higher profit from the installed base by capturing a new customer compared to its smaller rival. Therefore $q_1^* > q_2^*$ even at $\theta = 1$ if the profit from the installed base depends on the quality-adjusted network size.

The result in Proposition 5 is to some extent a consequence of Assumption 2, namely that the firms can price discriminate between new customers and installed base customers. In contrast, if the firms cannot price discriminate they will be less aggressive in the market for new customers. The reason for this is that an aggressive behavior (i.e., a low price) tends to reduce the income from the installed base (see Schmalensee, 1983).

2.3 Installed base interconnection and interconnection for new customers are set sequentially

In contrast to the basic model we now assume the following three stage game. At stage 1 the interconnection quality with respect to the installed bases ($\hat{\theta}$) is set. At stage 2 the interconnection quality with respect to the new competitive market ($\theta$) is set. Finally, at stage 3 we have Cournot competition. The the quality-adjusted total network size is now $N_i \equiv \beta_i + q_i + \hat{\theta} \beta_j + \theta q_j$. The qualitively results in this section do not depend on the effect of the installed base profit. Hence, for the sake of simplicity, we assume that the profit is $\pi_i = (1 - v)(q_i^*)^2$.

The Cournot equilibrium in stage 3 is then given by:

$$q_i^* = \frac{1}{2} \left( \frac{2(1 - c) + v(1 + \hat{\theta}) \beta}{2(1 - v) + (1 - \theta v)} + \frac{v(1 - \hat{\theta}) \Delta_i}{2(1 - v) - (1 - \theta v)} \right)$$

(9)

Since $q^*$ is convex in $\theta$, firm 1 sets $\theta = 1$ or $\theta = 0$ at stage 2. We define $\hat{\theta} \theta$ as a strategy for firm 1. For instance, if $\hat{\theta} \theta = 10$, firm 1 sets $\theta = 1$ at stage 2 and $\hat{\theta} = 0$.  

15
at stage 1. The necessary and sufficient condition to ensure that firm 1 sets \( \theta = 1 \) at stage 2 is given by the following:

\[
q_1^{\hat{\theta}} - q_1^{0\hat{\theta}} = \frac{1}{2} v \left( \frac{2(1 - c)}{3(1 - v)(3 - 2v)} + \frac{v(1 + \hat{\theta})\beta}{3(1 - v)(3 - 2v)} - \frac{v(1 - \hat{\theta})\Delta_1}{(1 - v)(1 - 2v)} \right) \geq 0 \tag{10}
\]

From equation (10) we have the following result:

**Proposition 6** If the installed base interconnection is high (\( \hat{\theta} \) close to 1), firm 1 will set perfect interconnection quality in the new market (\( \theta = 1 \))

Recall from the introduction that FCC imposed as a condition for the AOL merger that AOL offer perfect interconnection quality for new advanced instant-messaging services (\( \theta = 1 \) in the present context), while they did not impose a condition on the interconnection quality for existing text-based instant-messaging (\( \beta \) in the present context). However, from Proposition 6 we see that a firm will ensure high interconnection quality in the new market if the interconnection quality for the installed base is high.

We define \( \Delta_1^{\hat{\theta}-0\hat{\theta}} \) as a critical value of \( \Delta_1 \) that ensure that \( q_1^{\hat{\theta}} - q_1^{0\hat{\theta}} = 0 \). If \( \Delta_1 \leq \Delta_1^{\hat{\theta}-0\hat{\theta}} \) we have \( q_1^{\hat{\theta}} - q_1^{0\hat{\theta}} \geq 0 \), and the stage 2 equilibrium is \( \theta^* = 1 \). In contrast, if \( \Delta_1 > \Delta_1^{\hat{\theta}-0\hat{\theta}} \) we have \( q_1^{\hat{\theta}} - q_1^{0\hat{\theta}} < 0 \), and the stage 2 equilibrium is \( \theta^* = 0 \). From equation (10) we have:

\[
\Delta_1^{\hat{\theta}-0\hat{\theta}} = \left[ \frac{1 - 2v}{3v(1 - \hat{\theta})(3 - 2v)} \right] \left[ 2(1 - c) + v(1 + \hat{\theta})\beta \right] \tag{11}
\]

Since \( d\Delta_1^{\hat{\theta}-0\hat{\theta}}/d\hat{\theta} > 0 \), the term \( \Delta_1^{\hat{\theta}-0\hat{\theta}} \) is lowest for \( \hat{\theta} = 0 \). If we insert for \( \hat{\theta} = 0 \) into equation (11) we find that:

\[
\Delta_1^{10-00} = \left[ \frac{1 - 2v}{3v(3 - 2v)} \right] \left[ 2(1 - c) + v\beta \right] \tag{12}
\]

When \( \Delta_1 \leq \Delta_1^{10-00} \), firm 1 sets \( \theta^* = 1 \) at stage 2 independent of \( \hat{\theta} \) at stage 1. If we insert for \( \Delta_1 = \beta \) into equation (12) we find that firm 1 sets \( \theta^* = 1 \) at stage 2 for all \( \hat{\theta} \in [0, 1] \) when \( (1 - c) \geq \beta \). This result may be summarized in the following Proposition:
Proposition 7  For low values of $\beta$, i.e. $\beta \leq (1 - c)$, firm 1 sets $\theta^* = 1$ at stage 2 independent of the degree of vertical differentiation $\Delta_1(1 - \hat{\theta})$ where $\Delta_1 \in [0, \beta]$ and $\hat{\theta} \in [0, 1]$.  

The intuition behind this result is that for low values of $\beta$, the installed base is small compared to the new competitive market. Therefore, the degree of vertical differentiation will be so low also when $\hat{\theta} = 0$ that it is optimal to set $\theta = 1$.

3 Concluding remarks

In this paper we have shown that the incentives of an incumbent with a smaller rival to degrade the interconnection quality depend on the total network size and on the price charged to the installed base customers. If the price charged to the installed base is high, and the difference between the installed bases is small, the incumbent will have relatively strong incentives to set a high interconnection quality. In contrast, if the incumbent has close to the entire installed base, a high price charged to the installed base makes it more likely that the minimum interconnection quality is chosen. Furthermore, an improvement in the interconnection quality may increase the profit level of the larger firm even if its number of unattached customers falls.

In the paper we have also shown that a smaller firm may be harmed if the customers in the installed bases are being charged a high price, because this will make the larger firm more aggressive in the competition for new customers. However, it should be noted that few customers are truly locked-in. Some firms may find it profitable to enter the monopolized segment to capture the installed base customers if the price charged by the monopolist to those customers becomes sufficiently high. It seems reasonable to assume that the price charged to the installed base will depend on the likelihood of new firms entering to compete for the installed base customers. If the entry barriers in the competition for customers in the installed base are sufficiently high, new entry is less likely and the price charged to the installed base may be high.
Interconnection quality will have an impact on the welfare of consumers, but exactly how consumers’ surplus will be affected by a change in the interconnection quality depends on the output effect of such a change. If both firms’ outputs increase when quality increases, we can say that consumers will be better off. Consequently, interconnection quality is likely to be an important parameter to regulators. Our model suggests that in order to ensure a high interconnection quality a regulator should make an effort to encourage entry into the installed base segments. This will cancel out any asymmetry that might exist between firms in terms of the level of the installed bases, and will make it more difficult to charge a high price in the installed base segment.

4 Appendix

A.1 Necessary and sufficient conditions for $dq_1^*/d\theta < 0$

Differentiating (6) with respect to $\theta$ we find

$$\frac{d^2q_1^*}{d\theta^2} = \left[ \frac{(1 - v - w) \Delta_1}{(1 - (2 - \theta) v)^3} + \frac{2 (1 - c) + (3 - v + w) \beta}{(3 - (2 + \theta) v)^3} \right] v^2 > 0, \quad (A.1.1)$$

which means that $dq_1^*/d\theta$ is more likely to be negative at $\theta = 0$ than at $\theta = 1$. In particular, this means that a necessary condition for $q_1^*$ to be negatively affected by improved interconnection quality is that $dq_1^*/d\theta|_{\theta=0} < 0$, while a sufficient condition is that $dq_1^*/d\theta|_{\theta=1} < 0$.

For $\theta = 0$ we find

$$\left. \frac{dq_1^*}{d\theta} \right|_{\theta=0} < 0 \text{ if } \Delta_1 > \hat{\Delta}_1 = \frac{2 (1 - c) + (3 - v + w) \beta}{(3 - 2v)^2 (1 - v - w)} (1 - 2v)^2,$$

where $d\hat{\Delta}_1/dw > 0$ and $d\hat{\Delta}_1/d\beta > 0$. Similarly, we find

$$\left. \frac{dq_1^*}{d\theta} \right|_{\theta=1} < 0 \text{ if } \Delta_1 > \hat{\Delta}_1 = \frac{2 (1 - c) + (3 - v + w) \beta}{9 (1 - v - w)},$$

where we also have that $d\hat{\Delta}_1/dw > 0$ and $d\hat{\Delta}_1/d\beta > 0$.

A.2 Convexity of the profit functions

18
Differentiating the equilibrium profit, (7), with respect to interconnection quality we obtain the following expression:

$$\frac{d\pi_i}{d\theta} = \left[ 2(1 - v)(q_i^*) \frac{dq_i^*}{d\theta} \right] + \left[ \beta_i w(q_i^* + \beta_j) \right] + \left[ \beta_i w \left( \frac{dq_i^*}{d\theta} + \theta \frac{d^2 q_i^*}{d\theta^2} \right) \right]$$  \hspace{1cm} (A.2.1)

By observing equation (6) we see that $$\frac{dq_i^*}{d\theta}$$ is always increasing in $$\theta$$, and firm 2 prefers perfect interconnection quality.

For firm 1, the first term in (A.2.1) is negative if $$\frac{dq_i^*}{d\theta} < 0$$. The second term is always positive. The sign on the third term is ambiguous for firm 1. It is obvious that $$\frac{dq_i^*}{d\theta} \geq 0$$ (see A.1) is a sufficient condition for ensuring that firm 1’s profit is increasing in $$\theta$$ for all permissible values of $$\theta$$.

The second order condition for firm i is given by:

$$\frac{d^2\pi_i}{d\theta^2} = \left[ 2(1 - v) \left( \frac{dq_i^*}{d\theta} \right)^2 + q_i^* \frac{d^2 q_i^*}{d\theta^2} \right] + \left[ \beta_i w \left( \frac{d^2 q_i^*}{d\theta^2} + \theta \frac{d^2 q_i^*}{d\theta^2} + 2 \frac{d^2 q_i^*}{d\theta^2} \right) \right]$$  \hspace{1cm} (A.2.2)

To determine the sign on the second-order derivative on firm i’s profit, we need to determine the sign of expression (A.1.1). Examining (A.1.1), we see that $$\frac{d^2 q_i^*}{d\theta^2} \geq 0$$ since $$\Delta_1 \equiv \beta_1 - \beta_2 \geq 0$$ and $$v < 1/2$$, whereas the sign on $$\frac{d^2 q_i^*}{d\theta^2}$$ is ambiguous. Define the difference $$\Delta_{socq} \equiv \frac{d^2 q_i^*}{d\theta^2} - \frac{d^2 q_i^*}{d\theta^2}$$. It can be shown that $$\Delta_{socq} = -2v^2(\beta_1 - \beta_2)(v - w - 1) / ((2(1 - v)) - (1 - \theta v))^3 \geq 0$$. Since (A.1.1) is positive for firm 1, the conditions $$\Delta_{socq} \geq 0$$ and $$\theta \leq 1$$ imply that $$\pi_1$$ is convex in $$\theta$$. Hence, the firm 1 will choose $$\theta = 0$$ if $$\frac{\partial \pi_1}{\partial \theta} < 0$$, and $$\theta = 1$$ if $$\frac{\partial \pi_1}{\partial \theta} \geq 0$$.

5 References


