Paper IV

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CLIMBING AND ANGLES: A STUDY OF HOW TWO TEACHERS ATTAIN THE INTENTIONS OF A TEACHING EXPERIMENT

ABSTRACT

Based on the results from a previous study (Fyhn, 2006), the students in one Norwegian seventh grade class mathematise their own climbing with respect to angles. The class' mathematics teacher and one trainee mathematics teacher take active part in this three day teaching and climbing experiment (TCE) which is lead by the researcher. In participating in the TCE, the teachers turned out to have different intentions than the researcher. This text describes and analyses how the two participating teachers attain the intentions of the TCE. In addition the text intends to enlighten whether and to what extent the teachers are seduced by the researcher to make their claims.

KEY WORDS

angle, climbing, inductive teaching, intentions of an experiment, mathematising, persuasion, teachers' expectations

INTRODUCTION

Based on how one twelve-year-old girl discovered angles in her climbing narrative, *author* (2006) points out that the climbing discourse can be a possible resource in the teaching of angles in primary school. Thus the girl's class was introduced to climbing as an integrated part of the teaching of angles; the class and two teachers took part in a three day teaching and climbing experiment (TCE) throughout this text. The class and the two teachers spent the first day at the local climbing arena focusing on angles and climbing. The second day was half a day doing follow-up work at school and the third day was one more climbing-and-angles day three months later. This paper presents the TCE's intentions and how these intentions were implemented. The analyses aim to enlighten how the intentions of the TCE are attained by each of the two different teachers who both took part in it.

Innovative and research-based teaching is of little use unless some teachers attain it. Thus the research question is 'How do teachers attain students' mathematising of climbing as an approach to the teaching about angles?' Maybe the informants were simply seduced by the researcher's enthusiasm and thus uncritically claimed that they had attained this approach. That kind of 'manipulation' can be identified by distinguishing between whether the teachers are convinced or persuaded to give their claims. However, as long as the researcher intended to guide the teachers towards attaining the TCE's intentions, some persuasion had to take place.

According to Lakoff & Núñez (2000, p. 365) *"Human mathematics is embodied; it is grounded in bodily experience in the world."* They further claim (ibid.) that angles existed in the early geometry paradigm where space was just the naturally continuous space in which we live our embodied lives. This supports work on the angle concepts in primary school as an integrated part of the students' physical activity.

According to Berthelot & Salin (1998) space can be conceptualised into three different categories:

> microspace (corresponding to the usual prehension relations), mesospace (corresponding to the usual domestic spatial interactions) and macrospace (corresponding to unknown city, maritime or rural spaces...) In consequence, the space representation produced by the usual out-of-school experiences is not naturally homogenous, and is quite different from elementary geometry. (, p 72)

The intention of the TCE is to guide the students to build a bridge between their embodied meso space experiences and school mathematics. And the aim is that the participating teachers attain this intention.

Different Approaches to Angles

Freudenthal (1983, p. 323) *"introduce angle concepts in the plural because there are indeed several ones; various phenomenological approaches lead to various concepts though they* *may be closely connected."* He (ibid.) distinguishes between angle as a static pair of sides, as an enclosed planar or spatial part and as the process of change of direction.

Mitchelmore and White (2000) found that the simplest angle concept was likely to be limited to situations where both the sides of the angle were visible; it is more difficult for children to identify angles in slopes, turns and other contexts where one or both sides of the angle are not visible.

Henderson and Taimina (2005) point out three different perspectives from which we can define angles: as a dynamic notion, as measure and as a geometric shape. Angle as shape refers to what the angle looks like; angle as a visual gestalt.

Krainer (1993) divides angles into four different categories: angle without arc (angle as linked line/knee), angle with arc, angle with arrow (or oriented angle space) and angle with rotation arrow (angle describing the rotation of a ray).

The TCE has five angle categories: angle as static shape, angle as dynamic shape, angle as measure, angle with an arc and finally angle as turn, where one or two of the sides are invisible. The TCE intends to let the teachers experience how the students' conceptions of angle can be based on their experiences from climbing. Figure 1 shows one of the climbing students with bent joints both in her knees, shoulders and elbows.

METHOD

A comparative Case Study

This paper aims to analyse how two different mathematics teachers, Therese and Frode, attain the TCE. Therese is a trainee mathematics teacher, but both she and Frode will be denoted as 'the teachers' throughout this text.

One week before the TCE the students performed a pre-test with tasks that focused on angles and geometry. The results from this test showed that more than half of the participating students failed in a task where they should pick out the largest one and the smallest one among five given angles. This indicated that the students' conceptions of angles needed improvement.

Figure 1. The students' bodily joints shape and reshape different angles while they are climbing. The idea is to let the students identify these angles, describe them and explain how the described angles influence their climbing. The teaching aims to guide the students to use angle as a tool for improved climbing technique.

The Participants

The entire class consisted of 18 students. Two of these made reservations from taking part in the research but these students joined the class as usual during the TCE. Nine girls and four boys in seventh grade participated the first day, the entire class participated the second day, while nine girls and six boys participated in the third day. One boy did not climb and he did not want to try either, however, he edited photos on a computer and was present throughout the climbing.

The researcher designed the TCE and took an active part in it along with the teachers. One non-mathematics trainee and two more grown-ups also took part in day one, to assure that as many students as possible could climb at a time.

Because of Therese's climbing skills she was responsible for the security while the students were climbing. She got some teaching tasks for each day of the project too and she discussed these tasks with the researcher at the end of the TCE days. The students performed the pre-test in one of Frode's lessons and he marked the students' answer to these tasks. Frode took active part in belaying the students while they were climbing. When the students worked in groups, Frode was responsible for organising these groups.

The mathematics trainee teacher, Therese, had the qualifications for teaching mathematics at high school level in Norway. Her father is a professor of physics. One of her brothers has a PhD in Science, the other brother is a graduate engineer and her sister has a PhD in mathematics. Her dialect is quite close to the dialect of the Norwegian capital. Therese is an International Mountain Guide[1]. During her work she has even been guiding visually-impaired students climbing Norwegian mountains. She has no children on her own but she is well educated and experienced in working with youths.

Frode has qualifications for teaching mathematics at high school level; he had completed more than one full year of mathematics studies at the university in addition to his teacher education. He has taught mathematics for 11 years and he also has his own children.

Frode's father is a carpenter and his mother is a cook. His parents have a small goat farm in the countryside in the north of Norway and Frode's dialect is from the northern part of Norway. Frode enjoy taking part in physical leisure activities and he was also partly responsible for his class' lessons in physical education. As for climbing, Frode was familiar with belaying[2] climbers and sometimes he went climbing himself. But climbing was far from being his favourite activity.

The Implemented TCE

At day one, before the climbing started, the day's two focus-words 'climbing' and 'angles' were written in bold letters on a flip over and the students were reminded of this throughout the day. The climbing was top-roping; the rope goes from the climber's harness up to a carabiner in the ceiling and down again to another person. This person is belaying the climber; the belayer keeps the rope to the climber tight at all time in order to prevent the climber from reaching the floor in case of falling.

In the day's last lesson a heavily mathematised activity; a construction of the perpendicular bisection, was performed in the climbing hall, by using a climbing rope, a chalk and some students' bodies. This was an inductive, meso space, enactive representation of a given mathematical concept.

The second day the students shaped angles by using their own bodies, they studied how the rope passed through the belay device and they made drawings from the climbing. The students were split into groups and each group constructed the perpendicular bisection on the floor by use of rope and chalk before they constructed it with ruler and compasses in their books. The aim of this day was to let the students transform their ideas from working with angles in meso space into working with angles in micro space; an approach towards abstraction both by use of words as symbols and through construction by ruler and compasses.

In the period between day two and day three, Frode tried to implement some mathematics into his physical education lessons. Therese and the other trainee presented their impressions from taking part in a researcher's field work, for the rest of their fellow trainees.

On the third day the students were divided into groups and each group had to create one particular climbing route; they decided rules for which holds they were allowed to use. The groups got small pieces of coloured cloth to mark their holds; each group got their own colour. Afterwards the groups would describe how their routes were ascended and these descriptions had to include something about angles. The aim of this day was that the students should reflect upon the consequences of their bodily angles to their climbing.

The Data

Each of the three mornings the teachers wrote down their expectations and in the end of each day they wrote down their impressions. After the two first days in December the teachers were interviewed. In May, they got an e-mail asking their opinions about the use of climbing as basis for teaching about angles in primary school.

In the end of June, each of them was visited by the researcher in order to go through what was written about them so far. The intention was to make sure that their writings were interpreted as correctly as possible; to make sure that the English version of the collected and analysed data reflected their real opinions. However, maybe the teachers' experiences from the TCE made them change their minds, if so there could be some difficulties in validating the data.

In addition to the formal writings some e-mail and sms correspondence took part when the researcher felt a need for contact, but this informal communication is not treated as data.

The use of video in this study could have offered better possibilities to return to what exactly happened. Then there would have been more possibilities of analyses of the data and of restudying details too. But then the researcher's written material would have been an interpretation of what the teachers expressed in these videos. The focus of this research is whether the teachers had attained the intentions of the TCE and video is considered not to be the best tool for getting valid data about this. Most of the data in this research is the teachers' own written material and that makes the analyses close to the data.

The data in this study is:

- The teachers' writings about their expectations to and experiences from each of the three days
- The interviews with each of the teachers after day two
- Frode's e-mail about how he worked out an idea that he talked about in the interview
- Field notes from the presentation Therese and the other trainee held for their fellow trainees
- The teachers e-mail replies to the question *"Can climbing be used as basis for teaching about angles?"*
- The teachers' comments to the researcher's analyses of their writings

THEORETICAL FRAMEWORK

The TCE's model

The TCE's model has three aspects based on the TIMSS framework (Mullis et al., 2005): The intended TCE, the implemented TCE and the attained TCE. These represent, respectively, the teachers' experiences of students' mathematising of climbing with respect to angles; how the students were taught to mathematise their own climbing with respect to angles; and, finally what the teachers think about the climbing approach to angles and whether they have attained the TCE's intentions. The situation that both the teachers have attained the intentions of the TCE is shown in field A in Figure 2.

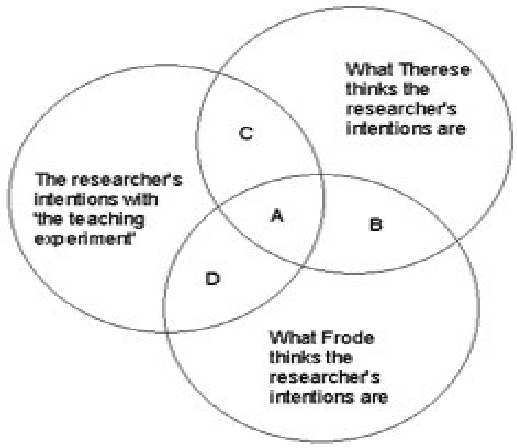

Figure 2. The relations between the researcher's intentions and what the teachers have attained. Fields A and B refer to what both of the teachers think the researcher's intentions are. Field A refers to those of the researcher's intentions that both of the teacher's have attained.

Inductive and Deductive Teaching

In Norway mathematics traditionally is taught deductively. The curriculum of 1987 (KUD, 1987) is interpreted to recommend deductive mathematics teaching even for the lowest grades: *"The subject matter may be introduced at first by the pupils' investigating and experimenting in well prepared learning environments, and/or by the teacher showing and explaining."* (ibid., p. 195, author's translation)

A new curriculum (KUF, 1996a; KUF, 1996b) followed in 1997. The paragraph Approaches to the study of Mathematics focus on learning; the students' experiences and practical work. On the subject of teaching, the paragraph claims

> Mathematics plays an important part in many areas of knowledge, and this is reected in the prominence given in the curriculum to interdisciplinary activities… The teaching of

> mathematics must be attuned to the abilities of individual pupils, who must be given tasks which they find meaningful and are capable of carrying out. (KUF, 1996a, p.154; KUF 1996b)

In this curriculum "practical situations and pupils' own experiences" play an important role throughout elementary school. Despite the claim that students construct their own concepts, this curriculum too can be interpreted to support a deductive approach to mathematics teaching.

The 2006 curriculum (KUD, 2006) focus is that during each of the main stages the students shall aim to achieve some specific competencies in the main mathematics areas. The curriculum's intentions focus on students' achievements and do not concern teaching.

In 1973 Freudenthal (1973, p. 402) warned that *"The deductive structure of traditional geometry has not just been a didactical success."* Many Norwegian mathematics teachers have no theoretical fundament for designing inductive teaching and in addition they lack experience from teaching and learning geometry inductively. What causes trouble for Norwegian schools is that many of these teachers are not aware of their lack of competencies; they are satisfied when their deductive geometry teaching is performed in a practical way because it fulfils the goals in the curriculum.

The TCE has a clear inductive approach to teaching and the Norwegian deductive teaching tradition thus has to be taken into account in the analyses of how the teachers attain the TCE's intentions.

Concretising and Mathematising

The TCE includes both concretising and mathematising and the teachers' respond to these different kinds of teaching will be analysed. Freudenthal (1983) describes Bruner's triad *enactive, iconic, and symbolic*:

> *enactively* the clover leaf knot is a thing that is knotted, *iconically* it is a picture to be looked at, and *symbolically* it is something represented by the word "knot" (ibid., 1983, p. 30)

He further claims that this schema can be useful in work with concept attainment (ibid.).

Concretising is often used deductively in Norwegian mathematics lessons as a tool for explaining something for students who do not understand what is taught. On one occasion in the TCE the entire class started to work inductively with enactive representations, continued with the iconic and ended up with symbolic representations, thus all the students could be able to take part in some of the mathematics activity.

Hans Freudenthal (1983) claims not to teach abstractions by concretising them; when teaching about angles you cannot just start with angles and look around for possible material to concretise this concept. He advised to use the converse approach, to start with searching for a phenomenon that might lead the learner to constitute the understanding of angles (ibid.). He further (ibid., p. 360) points out that *"angles, in contradiction to lengths are being introduced and made explicit in an already heavily mathematised context."* In the TCE the students mathematise their climbing by identifying and describing different angles related to climbing. Furthermore they will explain their climbing moves by use of the described angles.

Teachers' Beliefs

According to Goldin (2002, p. 61) we can distinguish between four sub domains of affective representation in the individual:

Emotions (rapidly changing states of feeling)

Attitudes (moderately stable, involving a balance of affect and cognition)

Beliefs (usually stable and highly cognitive)

Values, *ethics and morals* (deeply held preferences, stable, highly affective as well as cognitive)

The teachers need to *believe* in the students' advantage from the TCE it in order to have attained its intentions. If the teachers just have a positive attitude towards the TCE, there are reasons to believe that these attitudes can change before the teachers will be able to let the intentions influence their own teaching.

According to Lerman (2002) teachers' beliefs are both cognitive and affective, and there is *"a cyclical relationship between changing beliefs and changing practices"* (ibid., p. 235).

Conviction and Acquaintance

According to Russell (1963) you are acquainted with an object when you have a direct cognitive relation to it. To be acquainted with an object means to be aware of sense-data about this object (ibid.). To be acquainted with students' mathematising means to have experienced students' mathematising.

Russell (ibid) furthermore focuses on the cases where we know about an object that fits a description even though we are not acquainted with this object. The TCE cannot fit a definite description; teachers therefore need to become acquainted with it in order to have knowledge about it.

Tranøy (1998) focuses on the difference between conviction and persuasion, *"The distinction between conviction and persuasion touches on important matters, both morally and politically."* (ibid: 66, author's translation). There is a risk that the teachers who participated in the TCE were to a greater or lesser extent persuaded to claim that they believed the TCE to be a valuable way of teaching, which students enjoyed. If so, the result could be that the teachers to a greater or lesser extent attain the TCE's intentions. But the result could just as well be that the teachers just smile politely and keep their old teaching practice.

This is a question about research ethics: in cases where you need to get involved with particularly motivated participants there is a great risk of manipulating them. In such cases all kinds of persuasion need careful consideration. Most likely there exists a non-empty field that represents what both of the teachers think is the TCE's intentions. This field is the union of fields A and B in Figure 2.

One guide for identifying persuasion is to separate the teachers' statements into what they are acquainted with and what they just know by description. If they claim that they believe in something they know just by description, there is a risk that this is something they are persuaded to believe.

In the Gorgias Dialogue (Plato, 1968) Socrates claimed that there are two sorts of persuasion, one which is the source of belief without knowledge, and the other one is of knowledge. Socrates' use of 'knowledge' here is interpreted as 'what you have learned', related to an 'art'. The 'art' in this text refers to the 'art of teaching'. This way of having knowledge about an object is quite similar to what Russel (1963) denotes as to be acquainted with an object; the sort of persuasion which is the source of belief that includes knowledge; the sort of persuasion that leads to conviction.

When they joined the TCE the teachers were acquainted with climbing and with mathematics teaching, but not with mathematics teaching based on climbing

ANALYSES

The Teachers' Attitudes

According to Lerman (2002, p. 234) *"It is in the recognition of conflict between what one wishes to do, or believes oneself to be doing, and the perceived reality of one's teaching that can bring about change."* The two teachers recognised such conflicts and thus they had a positive attitude towards participating in the TCE. However, none of them mentioned inductive teaching and none of them mentioned mathematising. This is interpreted to be that they were not acquainted with any of these.

Frode's school focuses on outdoor schooling; schooling outside the ordinary school-building. His school even offers guiding in how to use outdoor schooling in theory and in practice. Frode wanted to be loyal to his school's aims, but he found that his school integrates the mathematics teaching into the outdoor schooling only for the lowest grades. He felt alone at his school with this concern; if you ask someone who has worked with outdoor schooling, they give you examples like:

> "Take the children to the shore and count stones and pebbles." I think you can do that with students at the lower grades… You have to look ahead, try something… that is where I feel I really need something more. How to use outdoor schooling in mathematics teaching for students at 7th and 8th grade?

Frode had a conflict between his personal goals and what he experienced in his classroom and thus he was ready for a change in teaching practice (Lerman, 2002). He needed new ideas and he had the courage needed to admit it. His emotions were affected and thus he had a positive attitude towards the TCE from the start. Frode is interpreted to wanting to improve his teaching for two reasons. First, with respect to the practical work that ran like a connecting thread throughout the curriculum. Second, with respect to 'outdoor schooling' in the way it was focused at his school.

Therese's statements describe a conflict between her personal goals and her experiences from the mathematics classrooms:

> I became a mathematics trainee teacher just to get the paper that shows I am a teacher. Mathematics is a subject that I had left far behind; actually I am really fed up with it. Now that I have started teaching myself, I find myself in the worst case scenario regarding mathematics. I experience my own teaching as dreadful and boring… This goes deep into my soul… I really do not want to force this upon other people because I think this is not all right.

Therese's emotions and attitude were interpreted to be strongly positive towards the TCE from the very beginning for two reasons. First, she wanted to experience mathematics teaching that was based on students' mastery experiences. Second, she wanted to experience mathematics teaching that differed from the traditional deductive teaching which she was against.

The First Day

Therese believed there was a great potential for angle teaching based on climbing. Frode thought that both the students and the teachers would learn a lot about angles.

None of them mentioned mathematising of climbing at the end of the day: Therese lost the angle focus when she started focusing on belaying and Frode did not mention the word angle in his text. Approximately half of their writings concerned the perpendicular bisection. Thus the writings are interpreted to belong to field B in Figure 2; the teachers had just partly attained the TCE's intentions. Therese was excited and wanted to work more this way. Frode wrote:

> Two students were placed on the floor with some distance between them. The rest of the students would then position themselves in equal distance from both of these two students. Finally we get the perpendicular bisection. A rope was used for control. Afterwards we constructed the perpendicular bisection by use of a chalk and a rope. It was a great and informative day for teachers and students.

Both of the teachers show that the perpendicular bisection made their expectations fulfilled, but their writings are interpreted to belong to field B in Figure 2. Frode had experienced practical mathematics teaching that took place outside the classroom and Therese had experienced mathematics teaching that differed from the traditional deductive teaching she was used to. However, the researcher intended to bring the teachers beyond their own expectations. Through the teachers' experience of students' mathematising of climbing they were guided to field A.

This text aims to enlighten how teachers' acquaintance with mathematising of climbing develops and the intentions of the TCE are interpreted to be attained only if the teachers' writings can be interpreted as belonging to field A in Figure 2.

The aim of the climbing approach to angles was to take into account the fact that more than half of the students failed on a task about comparing the sizes of different angles. The teachers were aware of this, but their writings did not tell anything about how they thought this teaching challenge was taken care of throughout the day.

The Second Day

This day intended to follow up the climbing approach from day one. Because the perpendicular bisection showed to be a very popular activity among the teachers and belaying showed to be a very popular activity among the students, some extra attention were paid towards these two activities. Therese put on a harness, attached herself to a rope by a belay device and asked what to do if she was belaying someone who fell. She asked for angles shaped by the rope and the belay plate. The students did not understand what she meant.

Therese concluded that she should rather have let the students perform this activity themselves, and then more of them probably would have understood what she meant; she suggests a possible approach to field C in Figure 2. If the students had performed the activity themselves they could have done it inductively by repeatedly checking out how different ways to use the device worked out.

Frode was busy doing various other things so he did not write about his expectations and experiences this day. Unfortunately the researcher was not aware of this until afterwards. This is an example of the data's weaknesses and these weaknesses need to be discussed in the analyses. Because Frode did not write anything this day, there could not be pointed out any similarities between his and Therese's writings from this day.

Most of Therese's expectations concerned mathematics. She was curious about how much of the students understanding of angles there would be left from day one. She was curious about how the groups would succeed in the construction of the perpendicular bisection. She ended: *"I believe I will learn about how to work with concepts in the classroom with angles as starting point."*

At the end of the day she wrote that she was satisfied and pleased about how much the students had absorbed about angles:

> A physical approach to angles leads some misconceptions to surface. The students are not sure which angle we refer to. Many of them thought that the angle disappeared when the rope was

> straightened. And that is correct in a way. But I believe they absorbed that the straight rope represents a 180° angle. The students really differ in how fast they understand this. But by this approach I believe that we reached all the students at some level and that all of them have got something from this.

Therese here nicely describes how the students' conceptions of angles were extended because the students' intuitive ideas of angles were challenged while they tried to understand how to belay a fellow climber. Thus the students' mathematising of climbing with respect to angles caused extension of their angle concepts.

The Interviews

Therese appreciated observing the students' growing consciousness about angles in their bodies:

> The students said that, well… there are no angles in our bodies… and then the consciousness-raising that happens throughout such a day… On the second day, when they were asked to perform an acute angle and a right angle by their bodies, then we could see all these different ways to stand and move. That was nice.

When she was asked if she thought that the students would think about angles related to climbing in the future, she answered that they would have to take their 'angle glasses' on. This statement is interpreted to be that she thinks the students can use their climbing bodies as models for angles. Furthermore she wrote:

> Then the natural activity can take its own course but the mathematics is still there. I like that. If the subject is all about mathematics I believe there will be some impatience, because you do not get the natural flow that we had on that particular day. I really appreciate the balance we got that day, to get the mathematics in while they performed activities …and talked about it …and related it and associated it to mathematics, yes … that is more natural.

According to Therese the students' basic knowledge the second day differed from their basic knowledge the first day; angles seemed to concern them in a way. She mentioned one girl who had some trouble with the concept of right angle. During the second day she believed this girl understood what was meant by a 'right angle'.

Furthermore she pointed out that the first day's focus was on climbing while the second day's focus was on mathematics and she appreciated *"the natural progression to get more focus on the subject."* Therese's writings can be interpreted as belonging to field C in

Figure 2; to be that she has experienced that mathematising of climbing with respect to angles, can function as a tool to avoid misconceptions in mathematics. Frode's writings do not indicate a similar focus.

According to Frode the students' attitude towards mathematics was very negative when he started teaching them this autumn. He was not sure if he has managed to change any of this, but many of the students had begun to claim that they enjoyed mathematics.

Frode was asked if he believed that the students would be relating angles to climbing in the future. He was sure about that: *"We have worked with angles related to climbing. I believe that in future talk about angles the word climbing will show up, and thus they will think about what we did."* This way of connecting angles to climbing is interpreted as use of climbing bodies as model for angles. Because Therese made a similar claim about models, their statements are interpreted as belonging to field A in Figure 2.

The Trainees' Presentation

When Therese and the other trainee had their presentation to their fellow trainees they could have focused on the climbing approach to angles. But they chose to demonstrate the meso space perpendicular bisection. There could be several reasons for their choice. Maybe the inductive meso space demonstration of some given mathematics was more obvious to grasp than the point of mathematising. Or maybe these teachers were searching for something new and thus would approve of anything.

Maybe the researcher's enthusiasm for mathematising of climbing had influenced Therese and the other trainee and overwhelmed them; as Socrates questions the rhetorician: *"…he need not know the truth about things; he has only to discover some way of persuading the ignorant that he has more knowledge than those who know?"* (Plato, 1968, p. 547). Maybe these two trainees were just as convinced about the climbing approach to angles as the audience who was able to see "The Emperor's new clothes".

Some months later the researcher asked if Therese could explain why she chose to let the other trainees do just the perpendicular bisection. Without hesitating she answered that this was a simple and practical task that was easy to perform in the actual room, and that they had had some discussion before they decided what to do. She continued, *"By activating the other students we could describe how to do mathematics in a practical way and we illustrated how you wanted to work with mathematics."* This claim can be interpreted to be that she thinks that the researcher's focus is practical work just like what the curriculum points out. However, maybe she just recognised and appreciated the inductive approach without being able to label it. The awareness of her strong antipathy against traditional deductive teaching supports an interpretation which includes both possibilities.

Frode's Teaching Practice

During the interview Frode suggested that it could be an idea to try to integrate some mathematics into his physical education lessons. A couple of weeks after day two, he was e-mailed and asked how this had worked out. Frode answered that throughout two physical education lessons with the class he tried to have mathematics in the back of his mind. He explained that he had used the words 'line', 'velocity', 'angle' and 'direction' in his instructions. *"Many of the concepts are mathematical, but many were everyday concepts which we regularly use in everyday language."* Regarding physical education as additional subject he wrote:

> We do many coordinating exercises where especially the angle concepts are used; usually the students do somersaults in many different ways. I see that we can use the concept of rotation here. I give instructions to the students like that *"in your next jump you shall rotate horizontally 360°"*

Frode's descriptions of these lessons show that he is mathematising his physical education lessons to a large extent, and that his mathematising of the students' activity leads to deductive use of mathematics as part of his physical education teaching. Mathematising does not seem to be part of the students' activity. Even though his work here is quite close to field A in Figure 2, it is interpreted just to belong to the field 'What Frode thinks the researcher's intentions are'.

Frode writes: *"he he, you have opened my eyes a bit here"*. This is a strong statement and it indicates that Frode has attained some of the intentions of the TCE; mathematising has started to become part of his own teaching practice. Frode is becoming acquainted with mathematising students' physical experiences: *"It is all about possibilities, not limitations"*.

Frode's attitude towards mathematising as element of the physical education lessons has changed and he has started changing his approach to physical education teaching. But according to Goldin (2002) attitudes are just moderately stable predispositions that involve a balance of affect and cognition. If Frode does not experience success in his attempts to integrate mathematics into the physical education lessons, there is a great risk that he does not attain the TCE's intentions.

The Third Day

The first day none of the teachers wrote anything about the climbing approach to angles. Maybe one more climbing day would help the researcher to guide the teachers to field A in Figure 1. However, a broader insight into the intentions of the TCE would give a better fundament for possible abstaining from the actual intentions as well.

In the morning Therese and the students started out writing about their memories from the first two days and their expectations to this day. Unfortunately Frode was prevented from taking part in this writing session. This lack of data made Therese's writings less useful because they could not be compared to what Frode wrote.

The previous time Therese thoroughly learned that demonstrations are useful simply to create an image of something to copy; to learn something the students need to have something in their own hands and try it themselves afterwards. In addition she had learned new ways of thinking about angles. Her text is interpreted to be that she has attained the curriculum's claim that the teaching shall be based on the students' practical experiences and she explains why she disapproves of the deductive 'teacher showing and explaining' that 1987 curriculum (KUD, 1986) recommended.

Therese had no expectations concerning mathematics this day. She was curious about the day and expected a nice day with possibilities for her to give some climbing advices to the students.

Afterwards Therese enjoyed watching the students making routes and discussing what holds that were natural to use related to their movements. And *"It seems as if the students' conceptions of angle are more profound now than last time."* Her writing can be interpreted to concern mathematising but because of a lack of clear data this statement does not undergo more profound analyses.

Furthermore Therese pointed out a misconception caused by language; the Norwegian word *rett* means both straight and right. *"A straight leg with a 180° angle can quite easily be called a right angle in Norwegian. And that is not unnatural because that is what we connect with the word right."* This writing can be interpreted as a description of how she experienced that the students' mathematising of climbing helped them to get over an expected misconception; this writing belongs to field C in Figure 2 because Frode does not make any claim about how to prevent possible misconceptions.

Most of Frode's writing concerned mathematics and mathematising. He even described the students' mathematising of climbing:

> At the end of the day, during the presentations, I observed that the students had learned to put angles down in words. They managed to ascend the climbing wall and from different positions they named angles in their bodies. For instance our elbow can shape a right angle.

Furthermore he appreciated that the students had found out how to use a table as a tool for organising and structuring their information; the students were able to analyse their information after having put it into a table. This activity is the students own mathematising of climbing but not with respect to angles this time. What Frode describes here is a further development of the students' mathematising of climbing and this is interpreted to belong into field D in Figure 2 because Therese did not describe this.

A Couple of Months later

The data can be interpreted to indicate that the teachers to a large extent have attained the intentions of the TCE. However, maybe the teachers just wrote what they expected that the researcher wanted them to write. Maybe the teachers did not want to disappoint the researcher for some reason. Thus the data needed careful validating and the teachers were e-mailed some months later and asked to reply in two to ten lines: *"Can climbing be used as basis for teaching about angles?"*

Therese's reply arrived less than two hours later. She starts out claiming that there are lots of angles both in the climbing bodies and in the climbing gear for belaying. Further she argues that she finds the adjusting of angles in arms and legs to be an element in the climbing moves. Her writing is interpreted to be that the students can mathematise their climbing; that she has attained the TCE's intentions:

> The climber who is conscious about this can feel it in her own climbing and make active use of it as an element in the climbing technique. Good climbing technique is based on the least possible use of force. This is active thinking about angles.

Furthermore Therese describes how the rope's angles related to the belay plate change when you belay someone. She describes angles between the rope and the wall related to the wall's steepness and she adds that the wall includes lots of angles with separate names. She finishes by pointing out that the angles in climbing are really visible.

Therese's text is interpreted to be that mathematising of climbing with respect to angles is easy because the climbing context is pervaded with static and dynamic angles both in the ropes, in the wall and in the climbers' bodily joints. Maybe she just claims this; she can not be interpreted to really have attained the intentions before she has tried to implement it in her own teaching. However, she has got as far as possible at that given moment. In addition she points to the students' positive attitude toward this activity: Most people experience climbing as exciting and fun.

Frode's reply arrived three days later. He wrote 15 lines concerning his opinion about physical activity in school in general, "… *Children enjoy physical activities and so do adults. Physically active children are happy children!"* followed by 11 lines where he focuses on the question. 'Experience' is the only reason he gives for his opinion. His answer to the question about angles is a clear yes and he continues to argue that children need physical activity.

There was a great risk in interpreting Frode's e-mail as that he was not convinced about anything related to climbing and angles. The TCE is a comparative case study with only two informants and thus it was natural to make one more inquiry to investigate if this really was Frode's answer to the question.

According to Lerman (2002) it is a methodological weakness to assume that interviews and questionnaires can reveal beliefs which is the main determinant of a teacher's action in the classroom. There was a risk that Therese and Frode's e-mails did not reflect what they really meant about climbing as fundament for teaching mathematics. Maybe their e-mails just revealed what they thought the researcher expected them to write or what they felt persuaded to write. They were asked to reply on a question which concerned the intentions of the TCE, but there was no guarantee that their answers would be interpreted as belonging to field A in Figure 2.

A final Visit

In the end of June Therese verified most of the writings about her while Frode spontaneously explained that his last e-mail was meant as a start of some longer writing, but that this longer writing never was continued. So his last e-mail really did not reflect what he meant.

Frode explained that when his students worked with time and velocity during this spring's mathematics lessons, they started with performing a 'running experiment' and finished with making a written report that explained what they had done and how they could find the average velocity. What he says here is interpreted as the students' mathematising of their own meso space activity.

It is not quite clear whether Frode's approach was inductive or deductive, but he has guided the students to build a bridge between their embodied meso space experiences and school mathematics. Frode immediately made a new version of his reply to the question. At first he wrote that climbing was a great fundament for teaching about angles,

> Children use their bodies to shape different angles. This gives them a closer relationship to angles. The students in my class enjoy climbing and after the climbing days some of the students said: *'Angles are fun!'* I believe the students will remember *'angles'* in their future climbing.

Both he and Therese were interpreted to have attained the intentions of the TCE and thus their writings are interpreted to belong to field A in Figure 2.

Frode claims that he believes in students' mathematising as a part of mathematics teaching and that he himself is aware of it. In addition he shows that his own teaching practice is changing. His students had mathematised their own running this spring and they had even made a written report about this. This is what Lerman (2002) claims, a change in teaching practice is related to a change in belief, and thus it indicates that Frode has attained the TCE's intentions.

DISCUSSION

Leatham (2006) warns researchers against assuming that teachers easy can articulate their beliefs. He also warns against the idea that there is a one-to one correspondence between what teachers state and what researchers think those statements mean. Particularly the second warning matches the TCE's analysis; as shown in Frode's first e-mail reply to the question about climbing as basis for teaching about angles. The main data source of the TCE is the teachers' written statements. These data's limitations have to be discussed in the analyses. Thus the analyses of the teachers' written statements were presented to the teachers in order to have the analyses as close as possible to what the teachers' really meant.

According to Brekke, Kobberstad, Lie and Turmo (1998) it has been problematic for Norwegian students to grasp that 180° is an angle. A strengthened rope represents a 180° angle where both of the sides are visible. At the end of day two one of the teachers wrote *"Many of the students thought that the angle disappeared when the rope was straightened….. But I believe they absorbed that the straight rope represents a 180° angle."* This writing indicates that the angles shaped by climbing ropes can represent a useful contribution to the teaching about angles; that students' mathematising of the belaying of climbers had proved to be useful to extend the students conceptions of angles.

According to Gravemeijer and Cobb (in press) the Dutch RME (Realistic Mathematics Education) has emerged

> in resistance to instructional and design approaches that treated mathematics as a ready-made product… A process of guided reinvention then…requires the instructional starting points to be experimentally real for the students, which means that one has to present the students problem situations in which they can reason and act in a personally meaningful manner.(ibid., p.15)

In the TCE the students' conceptions of angle are treated as something the students create as an integrated part of the development of their climbing talk. One intention of the TCE is that the instruction's starting point is the students' own climbing. Except for one boy, all of the students took part in the climbing and none of the climbers asked why they had to climb or what they needed these experiences for; this is interpreted to be that the students found the activity to be meaningful to them.

According to van den Heuvel-Panhuizen (2003, p. 13)

> Models are attributed the role of bridging the gap between the informal understanding connected to the 'real' and imagined reality on the one side, and the understanding of formal systems on the other.

This corresponds to the intention of the TCE; to guide the students to build a bridge between their (embodied meso space) experiences and school mathematics.

Gravemeijer and Cobb (in press) claim that different researchers would not necessarily develop identical theoretical constructs when analysing the same set of design experiment data. Therefore, what is generalised is a way of interpreting and understanding specific cases, and one goal of analysing innovations is the development of theory that can feed forward to guide future research and instructional design (ibid.). Frode's approach to teaching about distance and velocity can be interpreted as a generalisation of how he has attained the intentions of the TCE.

FINDINGS AND CONCLUSIONS

The TCE is a comparative case study with two different informants. The possibilities of generality are to be found in regularities in different cases, here represented by fields A and B in Figure 2. Andersen (2003) claims that if you find regularities in the most different cases, it will indicate generality and robustness.

Both of the teachers were fully aware that about one week before the first day more than half of the students failed on a task about comparing the size of five different angles. Despite that none of them mentioned the climbing approach to angles in their texts at the end of day one and thus their writings were interpreted to belong to field B in Figure 2. In the end of day three their writings are interpreted as belonging to field A; they wrote several lines about the students' mathematising of climbing.

The above findings indicate that these two different teachers undergo a similar development towards attaining the TCE's intentions. First, appreciating the inductive meso space approach and second, discovering the mathematising approach. The teachers here met three new aspects in teaching and seemed to need time to grasp the TCE's intentions. Time is a delimited element in daily teaching, and thus a short instructional DVD is made in order to teach teachers about the TCE's approach to angles.

The TCE aimed to inquire how the teachers' acquaintance with mathematising of climbing developed. Thus it is important to point out that they entered the TCE from different starting points and that 'mathematising' most likely still do not mean the same to both of them.

The teachers have experienced the TCE as participating in developmental work because the final analyses of the students' work were not worked out before afterwards. Maybe the teachers' respond to the results of these analyses would have influenced their claims and attainments of the TCE's intentions.

The students' mathematising of climbing gives opportunities for the students later on to revert to a lower level of understanding, like for instance the basic recognition of a bent bodily joint and it also gives opportunities for further work with vectors.

Six months after the last climbing day Therese is working with outdoor education at a non-degree granting college and there she pays no attention to mathematics. Frode is approaching the subject of geometry in his teaching schedule and he claims that his way of teaching differs from the researcher's. This can be interpreted as what Leatham (2006) claims; that a prevalent pitfall of research on mathematics education is to assume that teachers easy can articulate their beliefs. Maybe Frode felt persuaded to attain what he thought was the researcher's intentions with the TCE. But he claims: *"You must let the students perform activities that they enjoy. The challenge is to find the mathematics in these activities."*

The teachers needed to get acquainted with inductive enactive mathematics teaching, meso space mathematics teaching and mathematising of climbing. The analyses of the TCE indicate that the teachers needed to get acquainted with inductive enactive meso space teaching before they were able to grasp the students' mathematising of climbing. This gives rise to future research: *"Is inductive enactive teaching experience necessary for teachers to grasp students' mathematising?"*

NOTES

[1] Therese is a member of IMGA–United International Mountain Guide Association, IFMGA–International Federation Montagne Guide Association

[2] To belay means to secure the climber with a breaking device connected to the rope in case the climber will fall. The climber then will be hanging in the rope.

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