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Solution of Pure Scattering Radiation Transport Equation (RTE) using Finite Difference Method (FDM)

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Abstract. Radiative transfer is the physical phenomenon of energy transfer in the form of electromagnetic radiation. The propagation of radiation through a medium is affected by absorption, emission, and scattering. Radiative Transfer Equation (RTE) have been applied in a many subjects including optics, astrophysics, atmospheric science, remote sensing, etc. Analytic solutions for RTE exist for simple cases, but, for more realistic media with complex multiple scattering effects, numerical methods are required. In the RTE, six different independent variables define the radiance at any spatial and temporal point. By making appropriate assumptions about the behavior of photons in a scattering medium, the number of independent variables can be reduced. These assumptions lead to the diffusion theory (or diffusion equation) for photon transport. In this work, the diffusive form of RTE is discretized, using a Forward-Time Central-Space (FTCS) Finite Difference Method (FDM). The results reveal the radiance penetration according to Beer-Lambert law.

Keywords: Radiation Transport Modelling (RTM), Radiation Transport Equation (RTE) Finite Difference Method (FDM), Forward-Time Central-Space (FTCS)

Radiation Transport Equation.

$$\frac{\partial I_v(\boldsymbol{r},\widehat{\boldsymbol{n}},t)}{c\partial t}+\widehat{\Omega}.\nabla I_v(\boldsymbol{r},\widehat{\boldsymbol{n}},t)+\left(k_{v,s}+k_{v,a}\right)I_v(\boldsymbol{r},\widehat{\boldsymbol{n}},t)=j_v(\boldsymbol{r},t)+\frac{1}{4\pi}k_{v,s}\int_{\Omega}I_v(\boldsymbol{r},\widehat{\boldsymbol{n}},t)d\Omega$$

I_v is spectral radiance of electromagnetic waves
c is speed of light
$\widehat{\Omega}$ is the vectorial position of a solid angle
$k_{v,s}$ is the scattering opacity of the medium
$k_{v,a}$ is the absorption opacity of the medium
j_v is the emission coefficient of the medium
t is time variable

Two Dimensional Pure Scattering Radiation Transport Equation.

$$\frac{\partial\phi_v(\boldsymbol{r},t)}{c\ \partial t}=D\nabla^2\phi_v(\boldsymbol{r},t)=D\left(\frac{\partial^2\phi_v(\boldsymbol{r},t)}{\partial x^2}+\frac{\partial^2\phi_v(\boldsymbol{r},t)}{\partial y^2}\right)$$

$\phi_v(\boldsymbol{r},t)$ is radiance intensity
c is speed of light
x, y are the space dimensions
D is diffusion coefficient
t is time variable

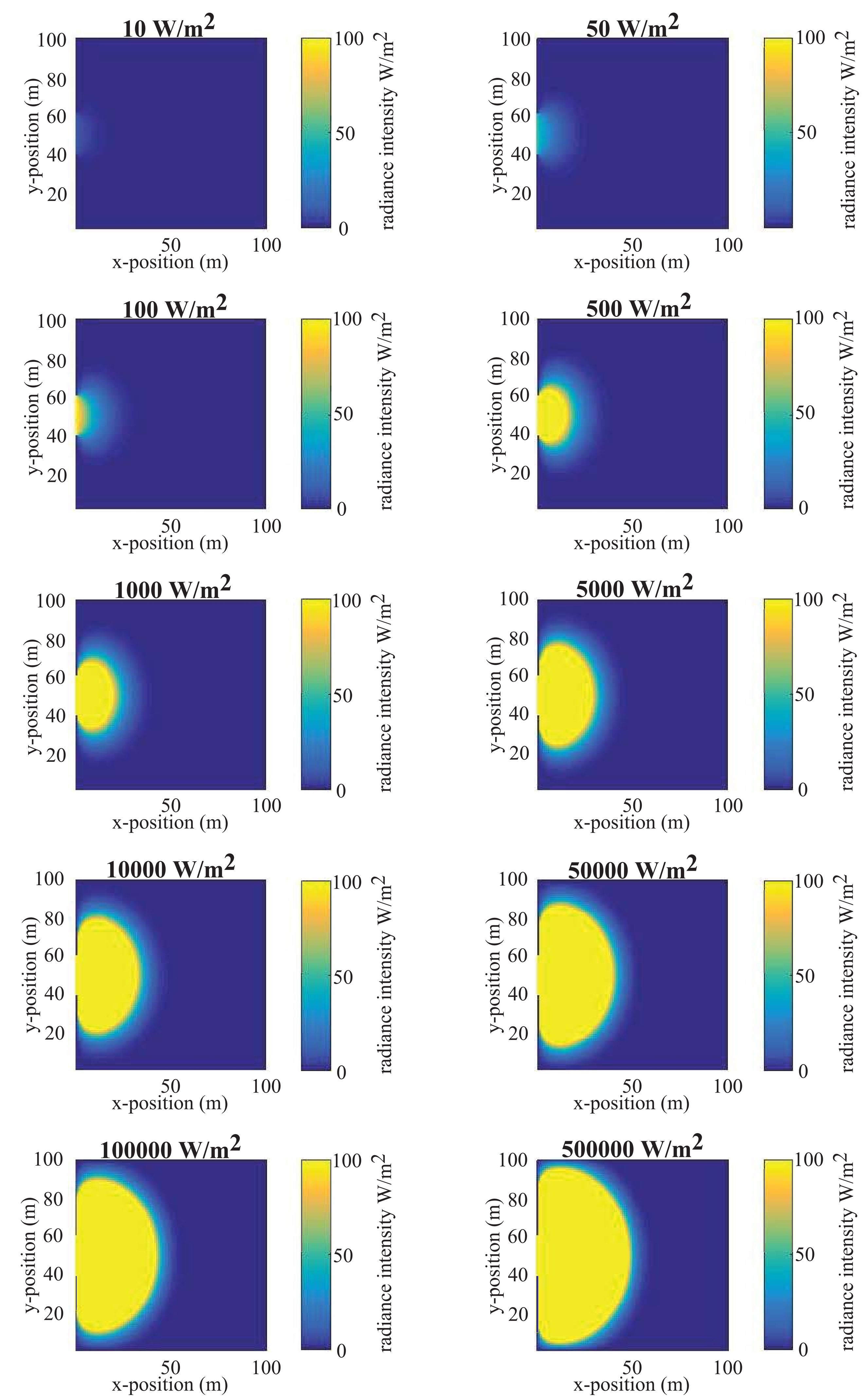

Figure. Contour plots with various radiance intensities.

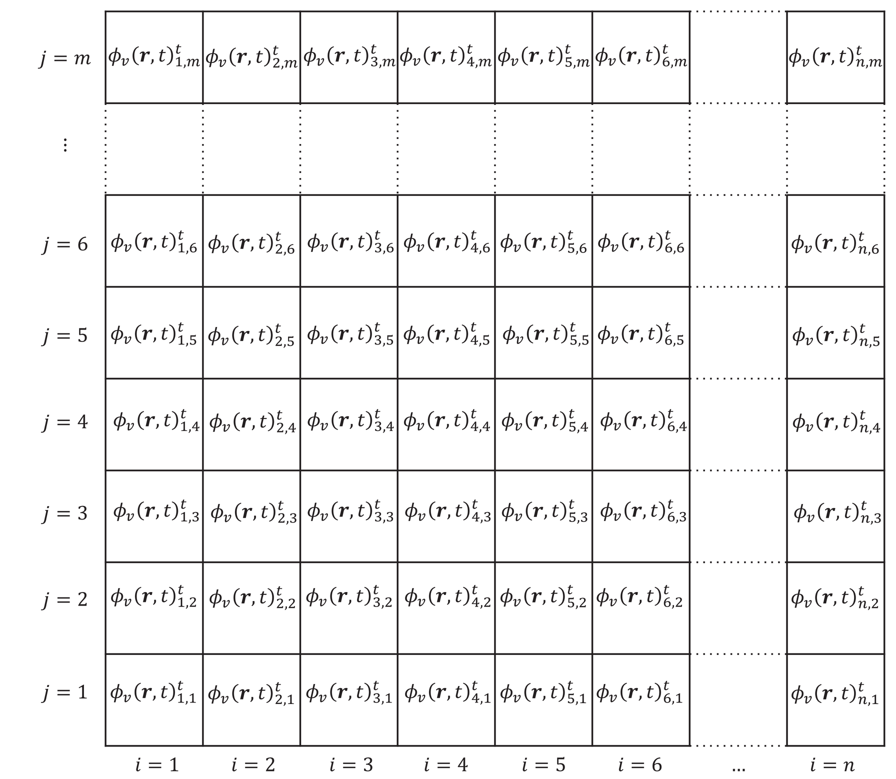

Figure. Radiance intensities in the two-dimensional discretized domain. Indices i and j refer to the nodal position of radiance intensity.

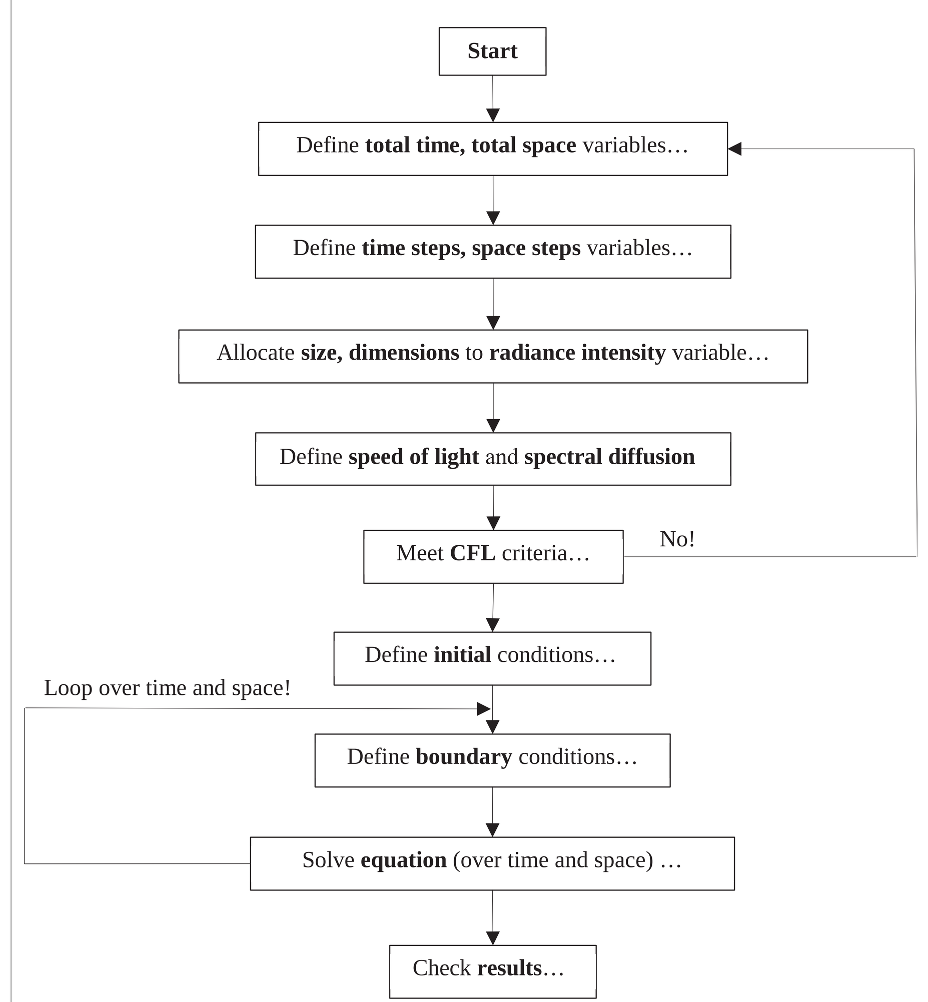

Figure. Flow chart of the method of solution.

Forward-Time Central-Space (FTCS) Finite Difference Method.

$$\partial\phi_v(\boldsymbol{r},t)_{i,j}^{t+1}=\partial\phi_v(\boldsymbol{r},t)_{i,j}^{t}+D\frac{\left(\partial\phi_v(\boldsymbol{r},t)_{i+1,j}^{t}-2\partial\phi_v(\boldsymbol{r},t)_{i,j}^{t}+\partial\phi_v(\boldsymbol{r},t)_{i-1,j}^{t}\right)}{(\Delta x)^2}\Delta t\ +\ D\frac{\left(\partial\phi_v(\boldsymbol{r},t)_{i,j+1}^{t}-2\partial\phi_v(\boldsymbol{r},t)_{i,j}^{t}+\partial\phi_v(\boldsymbol{r},t)_{i,j-1}^{t}\right)}{(\Delta y)^2}\Delta t$$

subscripts i and j represent the computational points in the space domain
superscript t represents the transient state

Conclusion. The Radiation Transport Equation (RTE) can be solved using a Forward-Time Central-Space (FTCS) Finite Difference Method (FDM) in special cases such as a highly diffusive mediums (as discussed above). The results show the radiance intensity in space, which reflects on the radiance penetration.

Future Work. This work can be further extended by varying the diffusion coefficient and observing its impact on the radiance penetration. It will help to find the validity limits of Radiation Transport Equation (RTE) solution using Forward-Time Central-Space (FTCS) Finite Difference Method (FDM).