Department of Industrial Engineering

# Design of a Pipeline Flexibility Bench

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In this thesis "*Design of pipeline flexibility bench"* an exertion has been made to make a conclusive study of fluid flow through pipe. Analysis has been carried out using analytical and numerical approach, where advance computer aided engineering tools such as CFD & FEM approach is used to study the flow behaviour in different pipe configurations. Results show a significant change in wall shear stress distribution with the change of flow properties and pipe configuration.

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**Rizwan Ghani**
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***DEDICATED TO***

***NATURE----***_The Volume of which Almighty ALLAH is the author._

# Abstract

In this thesis "*Design of pipeline flexibility bench"* an exertion has been made to make a conclusive study of fluid flow through pipe. This report comprise of 7 Chapters, the introductory chapters deal briefly with the concepts and needs of flexibility bench design for solving the problem of load and vibration problems in the piping industry. In chapter 3, Different materials have been suggested, discussed by their properties and by comparison, PVC (poly vinyl) is selected for the final design. In the chapter 4 head losses, which occur due to geometry of the pipe, material of the pipe and friction factors are figured out and calculated by the analytical modelling. Velocities and shearing stresses are calculated by MATLAB in chapter 4. Changing turbulent velocity profile with radial distance, shearing stresses are calculated in the analytical modelling. In chapter 5, with the help of ANSYS fluid flow has been analysed at straight pipe, one bend pipe and u-shape pipe. In CFD part different effects have been analysed by changing the velocities for different cases, shearing stresses also calculated by changing the rage of velocities. A comparison has been study of shearing stress calculated from analytical modelling and shearing stresses calculated by CFD modelling. In the end, structural analysis has been performed for different cases at different pressure to decide where to put suitable support which can control the load and vibration in the pipe. A final flexibility bench is design with proper support and these supports can be modify at different flow rate. Thermal effects for flexibility has be described for the final design.

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## List of Symbols:

| S.N | Name | Symbol | Unit |
|---|---|---|---|
| 1 | Length | l | m |
| 2 | Temperature | T | k |
| 3 | Mass | m | kg |
| 4 | Weight | W | N |
| 5 | Pressure | P | Pa |
| 6 | Density | $\rho$ | Kg/$m^3$ |
| **7** | Acceleration due to gravity | g | m/$s^2$ |
| 8 | Average velocity | Aavg | m/s |
| 9 | Area | A | $m^2$ |
| 10 | Mass flow rate | ṁ | Kg/s |
| 11 | Dynamic viscosity | µ | Pa*s |
| 12 | Volumetric flow rate | v̇ | $m^3$/s |
| 13 | Kinematic viscosity | V | $m^2$/s |
| 14 | Head loss | $h_L$ | M |
| 15 | Reynolds number | Re | |
| 16 | Prandtl number | Pr | |
| 17 | Nusselt number | Nu | |

| | | | |
|---|---|---|---|
| 18 | Internal Diameter | Di | M |
| 19 | External Diameter | Do | M |
| 20 | Turbulent Intensity | $i_T$ | |
| 21 | Heat capacity | $c_P$ | j/kg*K |
| 22 | Thermal conductivity | k | W/m*K |

## Nomenclature:

| | |
|---|---|
| **BC** | Before Christ |
| **AD** | anno Domini |
| **PVC** | polyvinyl chloride |
| **PE** | Potential Energy |
| **KE** | kinetic Energy |
| **CFD** | Computational fluid dynamics |
| **ASME** | American Society of Mechanical Engineers |
| **ANSI** | American National Standards Institute |
| **CAD** | Computer aided Design |

# CHAPTER 1

## 1 Introduction

Piping technology has been playing significant role in fluid flow applications. Circular pipes or even non-circular pipes are commonly being in used in different industries for different flow purposes. Dealing with the fluid flow is not an easy job. In any plant when piping are installed, many important factors like safety, operability, maintenance of piping and cost factor are considered. When we install pipe, fluid in the pipe and weight of pipe itself creates certain loads and due to this load, piping network can fail, heavy losses may occur.

With the passage of time piping evaluation showed that vibration in the piping was one of the most important factor that cases damages in the piping network system. The research about damages and other vibrational factors were focused in the piping in the following different areas

1. Experimental analysis
2. Investigating problems using numerical analysis
3. Investigating through simulation analysis

Using above three mentioned approaches results can be modify and enhanced and production area was able to avoided heavy losses. Fluid flow through pipes and the accurate measurements provide us information which is very useful to control system, process analysis, can measure productivity as well as energy consumption.

Dealing with structural vibration is kind of very complicated. Even following the basic principal of structural material still it is hard to solve structural vibration problems. When we deal with fluid flow, a number of important factors need to be considered and putting support on the right area to control vibration in the pipes is extremely important. Deciding where to put these supports for pipes we need to go through pipeline flow, impact of flow on pipeline walls and how shearing stresses can be controlled by providing sufficient supports.

As piping loads including piping, fluid and fittings weight, piping general arrangement drawing and thermal forces, moments & displacement of piping are considered into the basic parameters used to select supports, it is very relevant to consider that a combination of these different aspects will lead to distinct support configurations. As per clause 321.1.1 of code ASME B31.1, the objective of support design shall be directed towards preventing a series of faults as for example excessive stresses in the supporting (or restraining) elements or excessive thrust and moments on connected equipment (such as pumps and turbine) [1].

In industry several configurations must be used due to spatial restrictions of thermal restriction as can be shown in Figure 1 and 2.

*Figure 1: Different configuration of piping and piping support a) [2], b) [3]*

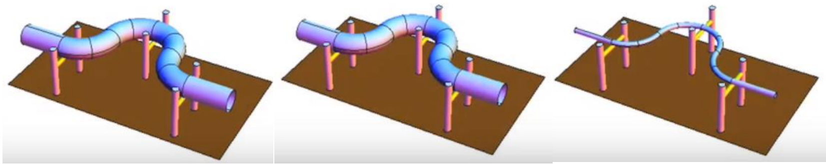


*Figure 2 Example of variations due to thermal expansion [4]*

Basic type of supports used in refinery are:

- Anchors
- Guides
- Line stops
- Rests

The anchors restricts all six degree of freedom (i.e., Rotational, Longitudinal & Axial). The guides It restricts the longitudinal movements but free to move in rotational and axial. Line stops supports are restrict the axial movements but free to move in the rotational and longitudinal. The role of rests take the vertical load generated due to pipe weight, thermal loads and occasional loads.

The aim of this thesis proposal is to measure the behaviours of flow through pipes and due to vibration try to find out where to put proper support for pipe so by putting support vibration will be minimize. Also the main idea is to reproduce a section of the pipeline that will be transporting water, connected to a close system that will contain tank/pump system to keep the flow into the system. The tank should have a heating system as well. Parallel to the pipe a "structure" having mobile supports that can be located in different position is built. Different displacement gages and torsion gages can be installed in specific

The objectives of this thesis are as follows:

1. Most commonly used methods for piping flexibility analysis
2. Accidents related to flexibility Design
3. Suggest materials for pipe flexibility bench
4. Design guidelines for pipe flexibility bench
5. Available software for flexibility analysis.

# CHAPTER 2

## 2 Literature Review

### 2.1 Role of piping in early civilizations

Piping technology has been playing an important role for advancement of human’s different area of interest since the earliest civilization. Piping technology shows us development with the passage of time. The enhancement areas of piping technology is directly proportional to the evolution of different industries. With the passage of time industrial revolution happened, optimal results also required in every aspects of piping technology. Piping technology played the key role for example in the steam power. Later on many discoveries were made by researchers like in oil and gas technology, use of plastic in the industry, chemical technology and many more. Before going to describe the concept of piping design and flexibility in the above mentioned technologies, it is important to describe the early history of the piping for the better understanding to next levels of piping flexibility bench. There are many civilizations in the history where human used piping technology for different purposes. The list of these civilizations and elaboration of piping role for different aspects are given below [5].

- Mesopotamia
- China
- Indus Valley
- Egypt
- Crete
- Greece
- Rome
- Middle Ages
- Renaissance
- The Age of Enlightenment

## 2.2 Mesopotamia

In Greek meaning Mesopotamia was the part between two rivers. It was an ancient region the eastern Mediterranean. Today's history, it is mostly part of Iraq and little bit of Syria, Iran and Turkey. It was multicultural civilization. Many discoveries were made by this civilization. For example invention of wheels, agricultural tools, wine and some weapons for wars, all these inventions credited to Mesopotamia civilization. Mesopotamia civilization was the first who used piping techniques in irrigation sector. Canals were made, water was carried out through these canals to certain level. Because of big desert water evaporation factor happened and then piping idea came in the Mesopotamia irrigation system. Pipes were in the cylindrical form and made of baked clay. The Pipes were passed through underground the land [5] [6].

### 2.2.1 China

The history literature elaborates that at the same time in the other corner of world the Chinese also used piping techniques for different purposes. There way of method was bit different as compared to Mesopotamia. The chines mostly used bamboo pipes to bring the water to their villages. They used wooden plug to control the flow of the water. To avoid the leakage of the water they used to put wax on the wooden plug [5].

### 2.2.2 Indus valley

When the term "early civilization" comes in mind, normally Egyptian or Mesopotamia civilization comes in mind. But in the early 19$^{th}$ century, a team of archaeologists made a discovery named as Indus valley civilization. This discovery was enough to tell the people that only Egypt and Mesopotamia were not the early civilization. This valley is located near the Indus river of Pakistan. Urban planning was phenomenal in the Indus civilization and somehow considered more advanced as compared to Egyptian and Mesopotamian civilization. In the Indus valley twins city were discovered named as "Mohenjo-Daro and Harappa" under the layered of Indus river sand. In these two cities, the pipes which were made from rough clay, the pipes were shaped, backed and placed back to back to pass the water [7]. Those pipes were produced in standard sized which was quite interesting. These pipes were 1 feet long in length and 4" in diameter [5] [8]. The water was passed through these pipes and in the street slabs used to cover the water.

### 2.2.3 Egypt

Earliest civilization of Egypt, not so many reports gives the clue of using pipes but few reports gives the proof of using piping techniques in the Egyptian early civilization. That time people used Nile River for the agriculture. The technique was transport waster same as Mesopotamian used to transport waster. In the urban area, the cleaned water was obtained from the local wells. The reports from the Egyptian civilization elaborate the usage of copper pipes in cylindrical form [5].

### 2.2.4 Crete

It is Greek island and considered the largest island of Greek. Crete was the centre of Minoan civilization (referred as Bronze Age civilization). The reports elaborates that it was the first European civilization. This civilization had the smartest water supply. In the civilization there were many systems which were based on local conditions and these systems involved to deal the following water resource engineering

- Groundwater exploitation and Wells
- Construction and use of fountains.
- Storage of rainwater and its usage
- Aqueducts and Piping
- Bathrooms and sanitary facilities also relating its uses of water

Figure 3 gives us the dimensions of the pipes. The pipes were designed and constructed in sections of about 600 to 750 mm each. They used pipes for water supply to the Minoan palace. They also used the rough baked clay for making pipes. The shapes of pipes were conical. One end designed narrow ended and the other one is large one. As it can be observed in the figure 1, the narrow ended side fitted in the front of large ended side so water can carry though those [5].

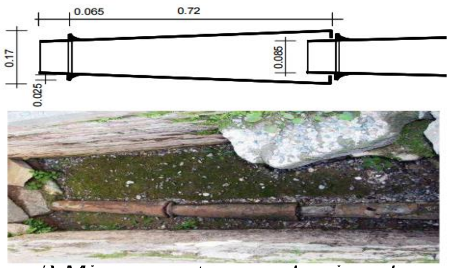



*Figure 3 Minoan water supply pipes for palace [9]*

### 2.2.5 Greece

The Grete and mainland Greece both developed many techniques for different purposes like transportation, irrigation and save the water from rain etc. the era of Greece normally considered 1600 BC to 300 BC. The Greece used rough clay made stone, bronze and lead pipes for different type of water supply techniques. In the mainland Greece the piping design was quite like Grete. One end designed large and the other end was narrow. The narrow placed to the large front area and fitted them nicely. No reports showed that the pipes were welded. Their fabrication techniques were different. From the report it has been estimated that their pipe joints must be reliable. Due to different terrain, the pipes had ability to bear the pressures [5].

### 2.2.6 Rome

The history of piping technology would not be complete until unless defining the piping engineering in the Rome. The Rome was prominent in the water in the piping engineering. Their piping techniques were different from the competitive civilization. Romans used such a system which helped to carry the clean water also the wastewater. Even the present technology of water management cannot compete the some old roman water management system.  The roman era was from 400 BC to 150 BC. The romans used to get water from different places for example initially they fetched the water from Tiber river. The drinking water obtained from wells. With the passage of time, the population increased, roman felt the upgradation of aqueducts system. In the Rome, the roman piping system used to carry water in three separate outdoor

- Public baths
- Fountains
- Private houses

### 2.2.7 Codes, Standard and Practices:

300 gallons water is provided for per person which is quite nice figure even if we compare today’s standard. Providing such amount of water, fountains and some powerful house played important role as they were used as storage tanks. Roman water management and piping system was under a commissioner. The commissioner had countless slaves which were supposed to take care of piping and water chain supply. Even in that civilization romans had quality inspectors for piping and water supply system [5].

Moreover, romans were quite advance for selecting material for the pipes. The following type of materials were used in piping

- Lead pipes
- Wood pipes but iron collars at joints
- Earthen-wear, Bronze
- In higher authority villas, silver pipes were used

### 2.2.8 Middle Ages:

In the middle ages, piping and water management system largely ignored after the fall of Roman Empire. The advancement started reverse town again become dependent on wells, spring stored water and rivers. Wastewater thrown away in the streets. Only some religious families were able to maintained metallic waster and earthenware piping system. A colour coded diagram found as a proof which is quite similar today's piping and instrumentation diagram. In Middle age civilization trees have been hollowed from centre and water transported from them.

#### 2.2.8.1 The Industrial Revolution:

Industrial revolution in piping technology came in 19th century. Researcher and scientists accelerated progress and in oil industry. Piping technology implemented in gas distribution and steam water. The usage of wood dropping as wrought iron and projecting flat rim were taking place. For the purpose of gas lighting, piping technology introduced for the first time in 1807. The pipes which were used in the gas piping are made from musket barrel. These musket barrel were achieved from the Napoleonic war. On the other hand in United States of America in 1816 Baltimore, the first gas transmission line was introduced. In the third decade of 19th century, Cornelius Whitehouse was the first man who developed fabricating pipe in one furnace pass from hot stripes.

These strips formed from bells. After five year, in Philadelphia the same technology introduced. Bessemer process geared up for making quality steel in large quantity in the half of \`9th century. Bessemer process triggered the production of pipes. E. L. Drake. Discovered oil in 1859 in Pennsylvania. Almost six years after the discovery, the oil has been transported through wagons. Eventually S. Van Syckel successfully installed pipes over six mile from the oil drilling field till the loading station. The dimension of his pipeline was 2. The pipes length was 15 feet. The pipes were wrought iron and well welded. This was a remarkable achievement of piping technology in oil industry [5]. In the end of $19^{th}$ century, seamless pipes introduced in the market. In a very short period of time seamless pipes industry covered a major part of the market. Also in that time steam was used in the transportation especially in the steam boats. Pipes were used to carry steam in boats and underground places. Welding, pumping and choice of materials made piping technology more advance in the $20^{th}$ century. A standard level material allowed in piping due to safety factor [5].

### 2.2.9 Piping history in England

If we observe the early piping history of England in term of piping technology we come to know that it stretches back to the Roman Empire. In 43 AD when romans entered and occupied England they found some engineering work about piping. Romans found series of canals and rolled led piping system. Water was transported through river, lacks were made to store water. And this water was ended in the central location of town or city and with the help of piping and served to fountains and residential areas. It was the maximum level of approach of transporting water in England. Romans were very good in piping technology, during the romans period sanitary facilities increased in England. With that improvement, people were able to get cleaner water but in the $6^{th}$ AD when barbarians and Irish again gained England from romans the sanitary care developments were no longer in England. The following eras are also important to elaborate in term of piping in England

- Medieval history of piping
- Victorian history of pipes
- Modern history of pipes

#### 2.2.9.1 Medieval history of piping

As time passed, Christianity dominated in England, the people related the romans baths as vain. Many of them considered romans baths and houses as debauchery and glowered them for using them. They did not care the sanitary facilities developed by the romans. Clean drinking water was no longer cleaner. People turned intro vine and bear as drinking choice. In the era only upper class had good conditioned baths. Even the religious houses done nothing in real piping. Only progress was in water and sanitation throughout middle ages gutters were made in the streets. Covered tunnels also made but without proper planning towards river.

In the medieval era, the situation about piping was quite dire and demanding for progress. Eventually in 1460 hull installed with pipes throughout the residential areas. In 1584, water system through piping was developed and waster transported to the town and this water stored in cistern. The Oxford city gullies the spring water and stored in 20,000 gallon tank for the public use [10].

#### 2.2.9.2 Victorian history of piping

The industrial revolution and in Victorian era, the prominent development has been seen in England. In this era central pumps were built in the populated area but limited condition. These pumps opened for certain time for everyday for water and depends on residents how much water they can save during this time. The aristocrat families which had grander houses were fitted with piping. This piping approach was only built in the first floor. People used to carry water by hand for the second floor. In the beginning of 1800 many England cities gained pipelines and aqueduct and transported clean water from almost 20 mile away.

In 1947 to 1948 public health act was passed and in the public health act if someone try to pollute the water will be fined or it will be considered offensive crime. Even till now England government invest 5 million pound for research and engineering work related to sewerage piping line.

## 2.3 PVC

With the parallel of time, many inventions happened and PVC (Polyvinyl chloride) is one of them in piping industry. It resolved many problems in piping technology if different aspects of usage. Polyvinyl chloride took many process in piping as a final product. Polyvinyl chloride was discovered in 1835, initially it was named as off white material who can bear 180 degrees C without degradation. That time polyvinyl was hard to control because of his nature and kept in laboratory for many years. His polymers were very strong. On the basis of physical and chemical properties scientists and researchers made a conclusion that it can be used where industry has durability and toughness problems while selecting materials.

Germany was the first country who initiated industrial development of PCV. From 1912 till 1920 they kept trying to find copolymer of PCV. The reason behind finding copolymer of PCV is to get such kind of materials which is easy to process. The scientists got huge success is this attempt. The next decade was quite remarkable in term of revolution of PVC piping. 1n 1932 the first pipe tubes were invented. These tubes were made of copolymers of PCV. Attempts were made and after three years using mill machine and hydraulic extruder, first PCV pipes were produced for industry usage. The production of these PVC pipes mainly contained two steps. The first step to melt the PVC powder on the mill machine and second step contained rolling the sheets till the final pipe.

However, making pipes by mill machine and extruder was used for celluloid. This process was not suitable for PVC pipe production. Due to this reason production was not up to the expectation. On the bases of inner smooth surface, resistance to chemical and lack of taste initial PCV pipes were supposed for water chain supply and also for waste water chain supply in the urban areas. The experiment was quite successful in the beginning and from 1936 till 1939 almost four hundreds houses were installed of PCV piping. Pipes were installed for both kind of supply, drinking water and waste water supply. After that within three years this techniques of supplying water spread in various cities including central Germany.

The installed pipes throughout the Germany was so good that even after damage from world war 2, the PVC installed pipes are still in use. From 1950 till 1960 many advances changes occurred both in piping and fitting technology. The results were outstanding and thus many companies started to produce PVC pipes. During the decade PVC industry scale occupied the European industry and in the parallel American companies also started the production of PVC piping. After 1960 piping industry did not look back and continued to progress. PVC piping industry started competing the traditional product in following various fields [11].

- Gas Distribution
- Sewer and Drainage
- Water Distribution
- Electrical industry
- Chemical industry
- Waste and vent Piping

## 2.4 Bernoulli' Equation:

From the law of thermodynamics when know that energy can be changed from one form to another form. In case of fluid flow there are three form of energy

1. PE = $w * \text{H}$ (2.1)

   Here is the height, w is the weight. Potential energy can also be written as

   PE = mgH (2.2)

2. When the velocity acts in the fluid flow, it creates Kinetic energy and it can be written as KE = $\frac{mv^2}{2}$, or it can be written as KE = W$*\frac{v^2}{2g}$ (2.3)

3. The third one is pressure energy

   FE = $\frac{w*\text{P}}{\gamma}$ or $\frac{w*\text{P}}{\rho g}$ as we know that $\gamma = \rho g$ (2.4)

In any point in the fluid flow in the pipe the total energy can be written as

$$\text{E = PE + KE + FE} \quad (2.5)$$

So equation 4 can be written as

$$\text{E} = \text{mgH} + \text{W}*\frac{v^2}{2g} + \frac{w*\text{P}}{\rho g} \quad (2.6)$$

From the conservation of energy principal we know that the total energy at two points remains equal if no energy is added or removed. So the above equation can be rearrange as following

$$\text{E} = \text{mgH} + \frac{mv^2}{2} + \frac{mp}{\rho} \quad (2.7)$$

From the Bernoulli equation, it explains that mechanical energy of various form in the fluid flow along the streamline is the same at all points. On that kind of streamline steady state flow the Bernoulli equation can be written as

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gH_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gH_2 + H_L \quad (2.8)$$

Here, $H_L$ is the head loss due to different factors like friction, fitting, bends and valves. $H_1$ and $H_2$ are the inlet and outlet height respectively. The frictions and pumps creates major losses and valves, fitting and bends create minor losses

$$h_{L-minor=} K * \frac{v^2 avg}{2g}$$ , here k is minor loss coefficient (2.9)

$$h_{L-major=} f \ * \frac{L}{D} * \ \frac{v^2 avg}{2g} \qquad (2.10)$$

Here f = is the friction factor

L= is the length of the pipe (m)

$Vavg$ = Average velocity (m/s)

D = is the internal diameter (m)

G = acceleration due to gravity (m/$s^2$) [12].

## 2.5 Euler Equation:

In mid-1700's, Euler equations was discovered by Leonard Euler. Euler was the student of Bernoulli and while solving of various fluid dynamics problems he discovered those equations. These equations are the simplification form of Naiver- Stokes equations of fluid dynamics. Euler equations are actually the relationship between velocity, pressure and density. Euler equations are based on following assumptions [13].

- Fluid is non viscous. That means no friction losses
- Fluid is homogenous or in compressive. In other words we keep constant the mass density of the fluid
- The flow is along the streamline and continuous
- Over the section, velocity should be uniform.
- Only pressure forces and gravity forces can affect the fluid dynamics

Now a days, in general we can say that Euler equations have time dependent continuity equation for conservation of mass and three conservation of three time–dependent of momentum equations. For the steady form and incompressible form we can the following Euler equations as .For 2-Dimensions. [13]:

Continuity: $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$ (2.11)

X-momentum: $\partial\ (\rho u^2)/\ \partial x + \partial\ (\rho u v)/\ \partial y = -\ \partial p/\ \partial x$ (2.12)

Y-momentum: $\partial(\ \rho u v)/\ \partial x + \partial(\ \rho v^2)/\ \partial y = -\ \partial p/\ \partial y$ (2.13)

For imcompressive form they can be written as:

Continuity: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (2.14)

X-momentum: $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0 = - 1/\rho\ \frac{\partial p}{\partial x}$ (2.15)

Y-momentum: $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0 = - 1/\rho\ \frac{\partial p}{\partial y}$ (2.16)

Here, two independent variable are in X and Y coordinates domain.

P is the pressure

$\rho$ is the density

U component is the x direction and v in the y direction. $\partial$ is used here for partial differential [13].

## 2.6 Navier-Stokes Equation:

These equations were derived by G. G. Stokes and M. Navier in the 1800's. That is why those equations are Navier-Stokes equations. G. G. Stokes derived in England and M. Navier in France. Those equations are kind of extensions of Euler equation. These equations address the relationship of velocity, pressure, temperature and density in the moving fluid. These equation are very complex and mathematically can be written as [14]

Continuity: $\frac{\partial\ \rho}{\partial\ t} + \frac{\partial\ \rho u}{\partial\ x} + \frac{\partial\ \rho v}{\partial\ y} + \frac{\partial\ \rho w}{\partial\ z} = 0$ (2.17)

X-momentum: $\frac{\partial\ \rho u}{\partial\ t} + \frac{\partial\ \rho u^2}{\partial\ x} + \frac{\partial\ \rho uv}{\partial\ y} + \frac{\partial\ \rho uw}{\partial\ z} = - \frac{\partial\ p}{\partial\ x} + 1/R_{er}\ [\ \partial\ \tau_{xx}/\partial\ x + \partial\ \tau_{xy}/\partial\ y + \partial\ \tau_{xz}/\partial\ z\ ]$ (2.18)

Y-momentum: $\frac{\partial\ \rho v}{\partial\ t} + \frac{\partial\ \rho uv}{\partial\ x} + \frac{\partial\ \rho v^2}{\partial\ y} + \frac{\partial\ \rho vw}{\partial\ z} = - \frac{\partial\ p}{\partial\ y} + 1/R_{er}\ [\ \partial\ \tau_{xy}/\partial\ x + \partial\ \tau_{yy}/\partial\ y + \partial\ \tau_{yz}/\partial\ z\ ]$ (2.19)

Z-momentum= $\frac{\partial\ \rho w}{\partial\ t} + \frac{\partial\ \rho uw}{\partial\ x} + \frac{\partial\ \rho vw}{\partial\ y} + \frac{\partial\ \rho w^2}{\partial\ z} = - \frac{\partial\ p}{\partial\ z} + 1/R_{er}\ [\ \partial\ \tau_{xz}/\partial\ x + \partial\ \tau_{yz}/\partial\ y + \partial\ \tau_{zz}/\partial\ z\ ]$ (2.20)

For energy: $\frac{\partial\ E_t}{\partial\ t} + \frac{\partial\ uE_t}{\partial\ x} + \frac{\partial\ vE_t}{\partial\ y} + \frac{\partial\ wE_t}{\partial\ z} = - \frac{\partial\ up}{\partial\ x} - \frac{\partial\ vp}{\partial\ y} - \frac{\partial\ wp}{\partial\ z} - 1/R_{er}P_{er}[\ \partial\ q_x/\partial\ x + \partial\ q_y/\partial\ y + \partial\ q_z/\partial\ z\ ] + 1/R_{er}[\ \partial\ /\partial\ x\ (\ u\tau_{xx} + v\tau_{xy} + w\tau_{xz}\ ) + \partial\ /\partial\ y\ (\ u\tau_{xy} + v\tau_{yy} + w\tau_{yz}\ ) + \partial\ /\partial\ z\ (\ u\tau_{xz} + v\tau_{yz} + w\tau_{zz}\ )\ ]$ (2.21)

Here,

t = is the time

q = is the heat flux

p = is the pressure

x,y,z are the coordinates

u,v,w are the velocity components

$E_t$ is the total energy

$\partial$ is the partial derivatives. This symbol was different in euler equation. This symbol indicates that all independent variable are being hold.

$\rho$ is the density and $\tau$ is the stress.

## 2.7 Types of Fluid Flow

In fluid flow, velocity, density and pressure create external forces on the pipe. To understand the creation of these forces on external wall of the pipes, it is important to know the behaviour details of the flow. There are three kind of fluid flow

1. Laminar flow
2. Turbulent flow
3. Transitional flow

### 2.7.1 Laminar Flow:

When the flow rate in the pipe becomes slow, all the molecules travels parallel to axes of the pipe. This kind of flow called laminar flow. In the laminar flow, molecules near the wall move bit slower as compared to molecules in the centre. Due to this reason, flow becomes parabolic. Pipe internal roughness also effect the flow. Therefore, we can say rougher the pipe from inside, causes the friction and increase the pressure drop. The laminar flow is kind of critical to solve any problem in fluid dynamics.

When the fluid particles flow in the straight line parallel to pipeline wall with low velocities without any distance between the layers, this kind of flow also known as streamline flow. Reynolds number helps us to recognize the type of flow. When the Reynolds number is less than 2300, the flow is consider as laminar so we can say that Reynolds number is used as parameter to determine the type of flow [12]. The velocity profile of laminar flow is shown in figure number 4.

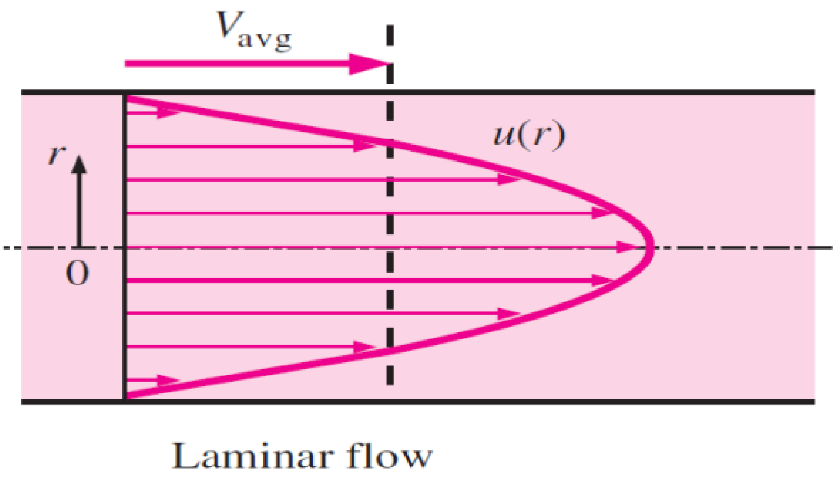



*Figure 4: velocity profile of laminar flow in pipe [12]*

#### 2.7.1.1 Pressure drop in laminar flow:

In the fluid flow anslysis, the interesting thing is that the pressure drop in the pipe for laminar flow has direct relation with power consumption by the pump. To maintain the flow this consumption happens. Pressure drop in the fluid flow for laminar can be calculated by this formula [12]

Pressure Drop $\Delta p = \frac{128\ \mu L \dot{v}}{\pi D^4}$ (2.22)

$$\Delta p = \frac{128\ \mu L Vavg}{D^2} \quad (9) \quad (2.23)$$

Here $\dot{v} = Vavg\frac{\pi D^2}{4}$

$\dot{v}$ = Volumetric flow rate ($\frac{m^2}{s}$)

$\mu$ = Dynamic viscosity (Pa*s)

Vavg = average velocity (m/s)

L = Length (m) and D = Diameter

Δp can be zero if there is no friction but in real cases it does not happen. The main formula clearly tell us that pressure drop is directly proportional to the viscosity. So in the laminar flow, in pressure drop viscous effects will be the main reason.

#### 2.7.1.2 Head loss in laminar flow:

For laminar flow head loss can be calculated by using Hagen-Poiseuille equation. For the horizontal pipe the head loss can be calculated by the following formula

$$H_L = \Delta p/\rho g \tag{2.24}$$

Here $\rho$ is the density, p is the pressure and g is the acceleration due to gravity

### 2.7.2 Turbulent Flow in Pipes

Because of his fluctuation complexity, it is very difficult to find the exact theory regarding turbulent flow in pipes. To understand the phenomena of turbulent flow is important because it creates shear stresses on the wall of the pipes. Calculating these shear stress is quite complicated. Velocity boundary layer can be observe and understand till some extent from the following figure number 5.

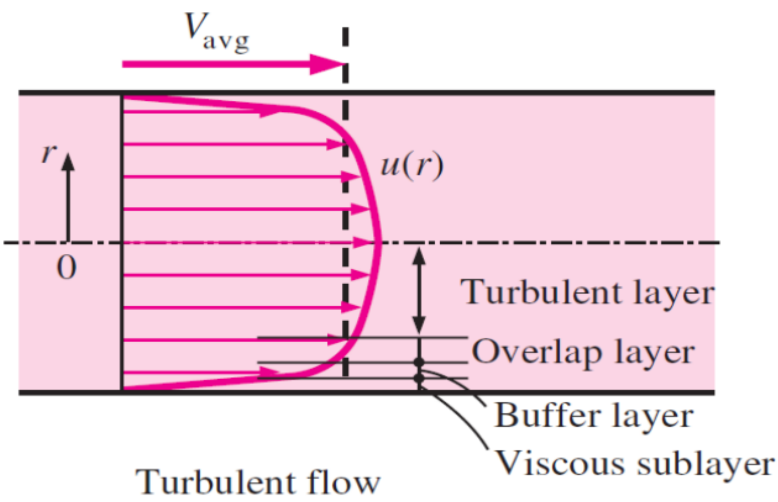



*Figure 5: velocity profile in the turbulent flow [12]*

From the above figure number 5 different layers come to in observation. These layers are:

- Viscous sublayer
- Buffer layer
- Overlap layer
- Turbulent layer

When fluid layer contact the boundary layer in the pipe, it comes to complete stop due to no-slip condition. The Viscous sublayer and overlap layer are dominated by the viscous effects but overlap layer is dominated by turbulent effect as the distance increase from the wall boundary. In the centre of the region turbulent effect dominates. The turbulent intensity provides information about turbulent level. The turbulent intensity is denoted by $i_T$. Turbulent length is denoted by$L_T$. Turbulent length gives the information about fluctuation of the fluid in the pipe

#### 2.7.2.1 Head loss for turbulent flow:

In the fluid flow, certain amount of energy is required to push the flow in the pipe. In other word pressure difference provides this in the pipe. The inner surface caused to waste some energy and also reduces the seed of the flow in the pipe. The loss due to this friction is called head loss in turbulent flow. Higher the velocity, greater the head loss until unless inner surface is quite smooth. In the turbulent flow, the liquid in the centre has the highest velocity because there friction is the minimum. Darcy’s equation is used for the calculation the head loss when the flow is fully developed [12]

$$h_{L-=} f \ * \frac{L}{D} * \ \frac{v^2 avg}{2g} \tag{2.25}$$

Here f = is the friction factor

L= is the length of the pipe (m)

$Vavg$ = Average velocity (m/s)

D = is the internal diameter (m)

G = acceleration due to gravity (m/$s^2$)

### 2.7.3 Transitional Flow:

Transitional flow is the flow which is between laminar and turbulent flow. In the transitional flow Reynolds number is greater than 2300 but less than 4000. In that type of flow viscous and Reynolds stresses are equal that is why it's called transitional flow.

## 2.8 Factors that affect Head Losses

Following are the factors that affect the head losses.

- Flow rate
- Internal diameter
- Roughness of the pipe wall
- Straightness of the pipe

As elaborated above that square of the velocity related to head loss. That means higher the flow rate will increase the velocity and when velocity will be higher viscosity will create loss in the result.

Internal diameter also affect the head loss. If the internal diameter is bigger it will decrease the flow rate. When the flow rate is low, it is obvious that it will decrease the velocity and in result due is friction head loss will be low. But when internal diameter is not bigger, and flow rate decreases, due to friction the head loss increase.

Inside rough surface also affect head losses. In that scenario in the centre velocity profile increase but at wall boundaries flow area reduces and due to friction head loss happens. Then bends disturb the flow of the fluid and head loss increases [12].

## 2.9 Entrance Region

After discussing types of fluid flow in pipes and head losses, the entrance region is important to elaborate in a pipeline flow. Entrance region is considered that region when fluid enters in the pipe at uniform velocity. When the fluid starts flowing through the pipes, the surface layers tries to stop it and in the result velocity increase at the centre of the pipes. In the entrance region flow phenomena, boundary layers can be divided into two regions

1. Boundary layer region
2. Irrotational Region Layer

In the following figure number 6, the entrance flow and behaviour of the fluid flow can be seen in details. From the figure we can see that at irrotational flow region the velocity profile is kind of uniform but at velocity boundary layer, viscous affect is dominating and flow is in the processing for developing.

The region between start of the entrance fluid in the pipe till the region where velocity profile fully developed is called hydrodynamic dynamic region. The region which starts from fully developed velocity profile is called hydrodyanamically fully developed region as shown in the figure 6.

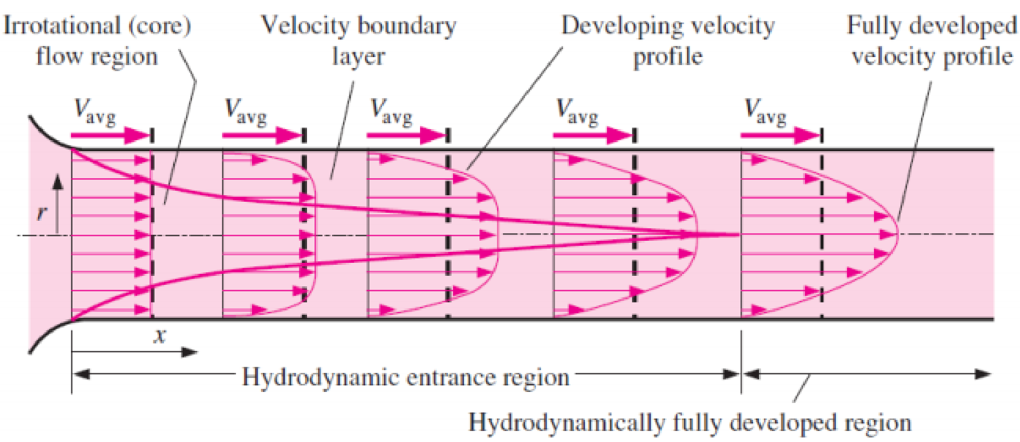



*Figure 6: Entrance region with development of fluid velocity in the pipe [12]*

### 2.9.1 Entry Length

From the following figure, the concept of entry length will be clearer.

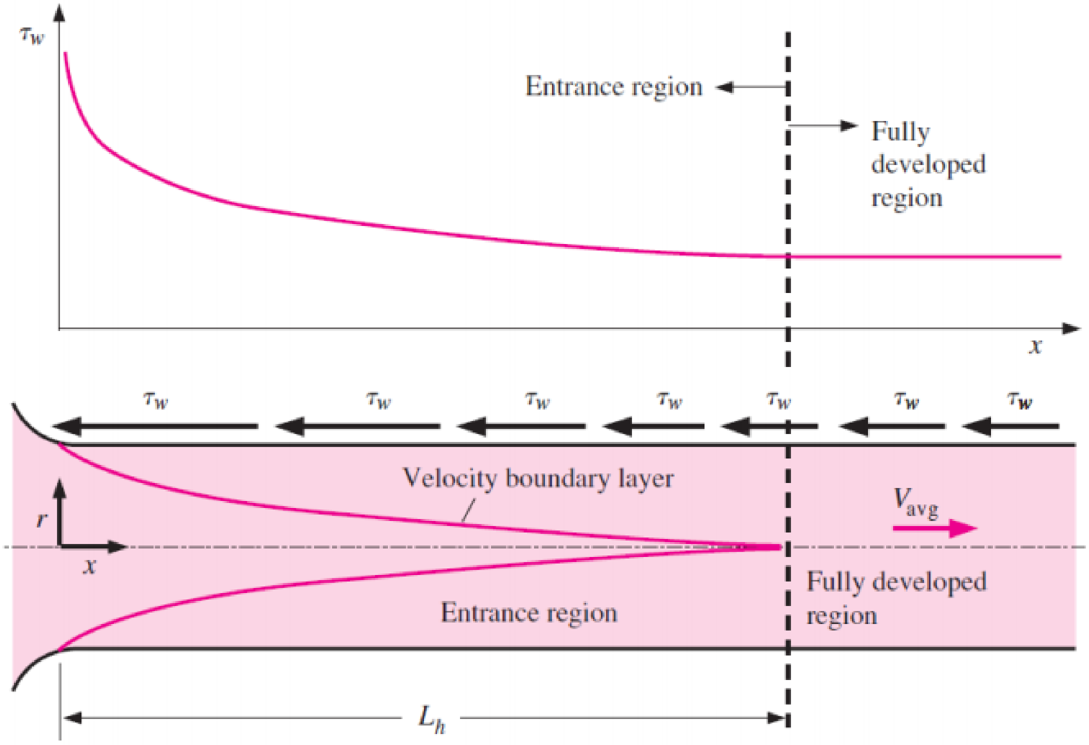



*Figure 7: Entry length and information about shear stresses in the direction of the flow [12]*

From the figure number 7, we can say that entrance region is called entry length or in other words it is the distance from the entering point of the fluid till the flow is fully developed. In the laminar flow the approximately entry length is 0.05ReD [12]

## 2.10 Reynolds number, Nusselt number and Prandtl number

When the transition happens from laminar to turbulent flow, this transition depends many factors like geometry, surface roughness, velocity even surface temperature, types of fluids and other rest factors. In 1810s, Reynolds was the first man who discovered that the flow regime depends on the ration of inertial forces and viscous forces. This ration is called Reynolds number. This is the two forces, hence it is a dimensionless quantity. Mathematically we can write

Re = Inertial forces / viscous forces

$$= Vavg * D / \nu = \rho * Vavg * D / \mu \tag{2.26}$$

Here Vavg = average velocity (m/s)

D = internal Diameter (m)

μ = Dynamic Viscosity (Pa * s)

ν = $\rho/\mu$ is the Kinematic Viscosity of the fluid ($m^2/s$)

### 2.10.1 Critical Reynolds number:

Critical Reynolds number is the number at which the fluid flow turns into turbulent flow. Critical Reynolds number calculation depends on the viscous parameters. Different constrains for Reynolds number is given as

- if $Re \leq 2300$ the flow is laminar
- If $2300 \leq Re \geq 4000$ the flow is transitional as explained earlier
- If $Re \geq 4000$ the flow is turbulent

The flow condition and structure are the good example of critical Reynolds number.

### 2.10.2 Nusselt number:

Mathematically Nusselt number can be written as

$$Nu = CD/K \tag{2.27}$$

Here, C represents conduction and k represents convection. D is the internal diameter of the pipe. Form the formula Nusselt number can be described as the ratio between conduction and convection is called Nusselt number. Fully developed flow in circular pipes can be calculated from the following formula [12]

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^{1/3} \tag{2.28}$$

### 2.10.3 Prandtl Number:

Mathematically it can be written as

$$Pr = \mu c_p / K \tag{2.29}$$

Here $c_p$ is the heat capacity

μ = 1.002 * $10^{-3}$ (Pa*s)

K = is the thermal conductivity

From the formula, prandtl number is called the ratio between fluid ability to store heat and transfer heat through conduction. Prandtl number is independent of system geometry.

## 2.11 Friction factors:

It is necessary to elaborate friction factor after elaborating the head losses, entrance region and Reynolds number. In head losses, friction caused many losses. Like Reynolds number friction factor is also a dimensionless factor as it is the ratio of two quantities. Friction factor depends on the velocity, density, diameter and viscosity. Frictions of wall roughness can also be mention as friction factor. Generally is can be written as [12]

$$f \propto (Re, \varepsilon/D) \tag{2.30}$$

### 2.11.1 Friction factor for laminar flow:

Friction factor for laminar depends on Reynolds number. Mathematically it can be written as

$$f = 64/ Re_D \tag{2.31}$$

It gives the friction factor for fully developed laminar flow. The equation is independent the ration of roughness and diameter of the pipe.

### 2.11.2 Friction factor for turbulent flow:

In the laminar flow friction factor is dependent only Reynolds number but in case of turbulent flow friction factor is calculated based on roughness of the pipe and Reynolds number also. Calculating friction factor for turbulent flow is bit complicated as compared to laminar. For the turbulent friction factor we need to elaborate Colebrook equation and Moody diagram

### 2.11.3 Colebrook Equation:

Colebrook equation for calculating friction factor for turbulent is little bit complicated. In 1939 discovered a formula for calculating friction factor by combing the data of transition phase and turbulent flow of the pipe. This combination includes both smooth and rough pipes, later on the equation is known as Colebrook equation. Mathematically it can be written as

$$1 / \sqrt{f} = -2\, log_{10} \left[ \frac{\varepsilon/D}{3.7} + \frac{2.51}{Re_{D} \sqrt{f}} \right] \tag{2.32}$$

The interesting thing about Colebrook equation is that it only works on turbulent flow condition and it calculates friction loss coefficients ducts, tubes and pipes.

### 2.11.4 Moody Diagram:

The figure number 8 of moody diagram is given as

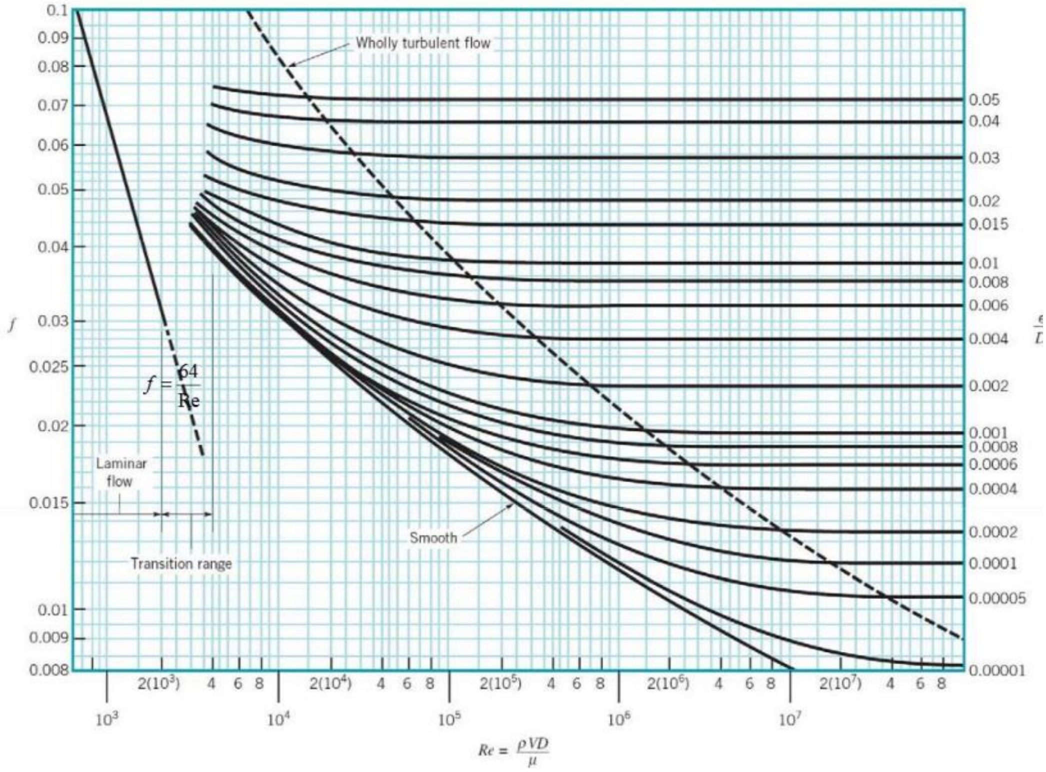



*Figure 8 moody diagram [12]*

After two year later of Colebrook equation, Lewis F. Moody's diagram comes into the form and later on known as Moody diagram. There is variation between laminar and turbulent flow when it comes to calculation friction factor. When the flow is fully developed in the turbulent flow, in that state the friction factor depends on the height of roughness and diameter of the pipe. This dependency cannot be obtained theoretically although researchers used artificially roughness but still results were not accurate. The most complicated part to determine the friction coefficient in the transit phase. Moody diagram provide guidance to calculate the friction factor in the transitional phase [12]. Moody chart was derived from the moody diagram and we can make following observation from moody chart [12]

- Moody chart tells that in the laminar flow, increasing the Reynolds number friction factor decreases. This friction factor do not depend on the surface roughness.
- Because of no-slip condition, the friction factor can be minimum but not zero although if the surface is smooth. It increases with surface roughness.
- The figure number 9 shows the shaded area in the moody chart. This shaded region is from laminar to turbulent regime. Guessing flow in this region is quite a challenge, the flow can be laminar region or turbulent region, and it depends on flow disturbances. The values of friction factor in this region can be laminar or turbulent. In this region even the small relative roughness can increase the friction factor. It is not wrong to say that in this region the data is least reliable.
- The moody chart also give information that friction factor curves correspond to relative roughness curves are horizontal. The flow in this region is called fully rough turbulent flow because increasing the Reynolds number viscous sublayer decreases and at certain stage it becomes so thin that it can be negligible.

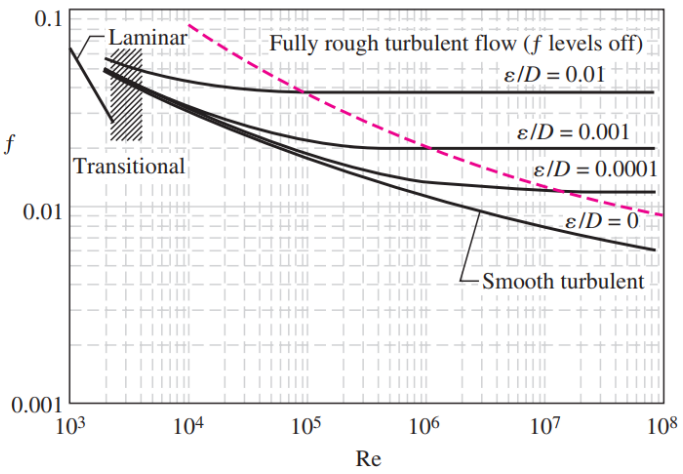



*Figure 9 transition region from laminar to turbulent regime [12]*

## 2.12 Role of Pumping and water tanks

The role of tanks is obvious to store water depends on the amount how much water is required. In the pumping system, pumps have direct relation to the pressure. The pump provide the pressure to overcome the operating system. By pumping, the required flow rate can be achieved in the water piping system. The operating pressure depends on many things like flow through the system and arrangement of the system. the arrangement of system relate to pipe length, fitting, pipe size, pressure on the liquid surface and elevation of the liquid etc. to achieve the required flow it is necessary to calculate the operating pressure of the system. The following figure number 10 will help us understand the phenomena and mathematical calculation of pumping in pumping arrangement system [15].

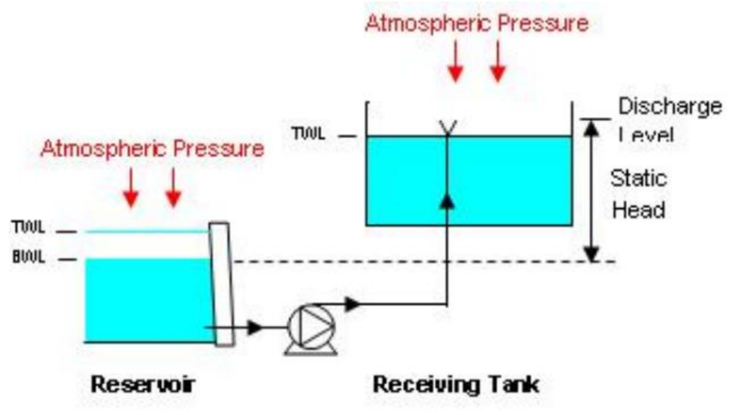



*Figure 10 Pumping arrangement for mathematical calculation of pumping water [15]*

Before going to water supply, normally water is pumped from reservoir into the tank where water is stored. The operating pressure of the pump is measured in the meter in SI unit system. Moreover, to maintain the dimensional consistency any value converted from kPa into meter and 1kpa = 0.102 m [15]. This is measured by a water filed U tube manometer. From the pumping arrangement system the total operating pressure can be defined as

$$H_{total} = H_s + H_D + ( P_{RT} - P_{res} ) \quad (2.33)$$

Here,

$H_s$ is the static head

$H_D$ is dynamic head

$P_{RT}$ is the pressure on the water surface in the tank

$P_{res}$ is the pressure where water is pumped, in other word pressure of water surface in the reservoir. As it is obvious that atmospheric pressure changes with changing the height but the change in the pressure occurs over the pump height is not so much and it can be negligible. That mean $P_{RT}$ - $P_{res}$ near to zero or can be negligible so, the above equation can be written as [15]

$$H_{total} = H_s + H_D \quad (2.34)$$

The dynamic head can be calculated from the following formula

$$H_D = \frac{Kv^2}{2g} \tag{2.35}$$

Here, k is the loss coefficient, v is the velocity in the pipe (m/s) and g the acceleration due to gravity (m/$s^2$). And we can calculate the velocity by the following formula

$$V = Q/A \tag{2.36}$$

Here, Q is the flow rate through pipe ($m^3$/s) and A is the pipe section area ($m^2$).

## 2.13 Piping Supports

This topic will elaborate the role of supports in the piping and| which will help us to put the suitable support in the design phase. The concept of piping span is important to discuss regarding piping support.

### 2.13.1 Piping Span

Span is the function of pipe weight that the support can handle. As the pipe size increases the weight of the pipe also increases, also weight of fluid in the pipe also matters. From the piping span, the location of the piping depends on 4 factors [16].

1. Pipe size, pipe size affects the weight
2. Configuration of pipe, configuration affects the location of the support
3. Location of valves and fitting
4. Structure available for the support

### 2.13.2 Piping classification

Piping supports can be classify in the following three groups [17].

1. General details
2. Construction details
3. Faction details

#### 2.13.2.1 Piping classification as general details:

Normally the foundations or structures are built on ground and pipeline need to supported from the base. Being built on ground, they react an equal and opposite reaction in the piping. In the general details classification piping supports can be classified into two parts

1. Primary support, directly connect to pipe and part of support assembly
2. Secondary support, part of support assembly, connected to foundation and support the primary support.

### 2.13.2.2 Pipe support classification construction details:

In the construction details, piping support can further classify flowing types as

1. Rigid supports
2. Elastic supports
3. Adjustable supports

Following table comparison between these supports will give better understating

*Table 1- comparison between different kinds of supports*

| Rigid supports | Elastic Supports | Adjustable supports |
|---|---|---|
| 1. This support is simple and maximum use in piping<br>2. It does not have ability to adjust erection tolerance in the piping<br>3. Directly rest on structure<br><br>Common types are:<br>1. Welded<br>2. Clamps<br>3. Trunnion<br>4. Valve holder | 1. Mostly use in piping<br>2. It can even support the pipe when it moves up or down at support point<br><br>Common types are:<br>1. Spring support<br>2. Constant type spring support | 1. More likely rigid support<br>2. Deals with nuts and bolts<br>3. It has ability to adjust erection tolerance in the piping<br>4. Mostly used at critical locations |

### 2.13.3 Pipe support classification function details:

The supports classified as functions wise can be elaborated in details as follows

#### 2.13.3.1 Loose support:

It support the pipes weight vertically. This support let the pipe move transverse and axial directions but in vertically way.

#### 2.13.3.2 Longitudinal guide:

This support mostly used with loose support. It restricts the movement of the pipe in tranverse direction.

#### 2.13.3.3 Transverse guide:

This support let the movement in the transverse direction. The other name of this support is axial stop. This type of supports is not commonly used as compare to above mentioned ones.

#### 2.13.3.4 Fixed point:

We can also call it anchor. It restricts movement all three direction as well as rotation also.

#### 2.13.3.5 Limit stop:

As name indicated that it also allow limited movements. It control the further movements when they go beyond the limits. Mostly used in selective cases.

#### 2.13.3.6 Welded and non-welded anchors:

The welded support try to stops the linear and rotational movements but non-welded only stop the linear movement in all direction. It is considered the combination of transverse and longitudinal guide.

#### 2.13.3.7 Special supports:

This support is used when construction and functional details are different to support a pipe. In that case special supports are designed and prepared.

Flexibility analysis:

## 2.14 Flexibility analysis:

The concept of flexibility analysis is very important to understand to design a flexibility bench. In the stress analysis functions two things are need to keep in mind by stress engineers while making layout design. The first things is amount of conditions in flexibility layout and the other thing is how to establish flexibility methods in the layout. There are number of criteria which at least define the minimum acceptable flexibility. The amount of conditions in the flexibility can be divided into two categories

1. Maximum limit of stress range in the pipe
2. The limiting forcing values and moments which piping is permitted to impose in the connecting equipment system.

In the second thing as mentioned in the above paragraph, flexibility analysis experts need to decide which criteria applies. The criteria choice has the following choice

1. On the base of previous experience, accepting layout
2. Using the approximate methods, analyse the layout
3. Performing comprehensive stress analysis.

### 2.14.1 Code and regulation in piping flexibility:

Piping codes and regulations are the basic necessary things in the pipeline flexibility design, the code also covers the safety which is the basic factor in any pipeline construction. Codes and regulations ensure the actions with all applicable code regulations to design a flexibility bench at national or local level. Design specifications decide which codes are required for different purposes. The piping codes are the core thing for analysis of loading in piping system as well as flexibility analysis.

Some codes and standard in the present use are American national codes for pressure piping ANSI B 31. Globally it is most widely code which is in practice for piping. The history of this code published in 1935 and it is considered the American tentative standard cod for pressure piping. After many improvements, American standard code for piping pressure ASA-B31.1 is published in 1942. The code and regulations are mandatory and recoments the thickness of pipes for internal and external pressure. These codes recommends for external expension and allow and define stresses for different materials. The following table will give us information about different codes for different categories [18].

Table 2 : Codes Standards for different industries

| | |
|---|---|
| ASME B 31.1 | For power piping, associated with power boilers |
| ASME B 31.3 | Use for petroleum refinery piping, |
| ASME B 31.4 | Deals oil transportation piping |
| ASME B 31.5 | For Refrigeration piping |
| ASME B 31.8 | Deals with gas transmission and distribution piping |
| ASME B 31.9 | For building services piping |
| ASME B 31.11 | Use for slurry transportation piping |

### 2.14.2 Flexibility and Stiffness of piping:

Mechanics engineering perspective, the concept of flexibility and stiffness in application need to be understood. Mathematically flexibility and stiffness are inverse to each other. in the practical way flexibility mentions to the piping configuration. The piping configuration means able to absorb thermal movements by using loops. The movements allows the pipe to displace itself. In the flexibility scenario, making the pipe more flexible is much better to remove the problems. The stiffness can be defined as it the amount of force required to produce unit displacement. The displacement can be translational or rotational [19].

### 2.14.3 Criteria for Flexibility analysis:

From the above table, ASME B31.3 and ASME B 31.1 and their details give us information that formal flexibility is not required for piping that duplicates or replaces withour marginal changes [2]. Moreover, power piping code and processes piping codes give criteria when formal flexibility is not required. Many software packages are good to perform flexibility analysis but they are limited, they do not make compute component design. In the flexibility the component design requires either a closed form solution or finite results. When no intermediate supports are included, formula for two anchor piping system is given as

$$\frac{\mathrm{Dy}}{(\mathrm{L}-\mathrm{U})^2} \subseteq \mathrm{k}_1 \tag{2.37}$$

Where D = external diameter of the pipe (mm)

y = is the resultant of total displacement strains (mm)

L = is length which is developed between pipes (m)

U = is the anchor distance (m)

$\mathrm{k}_1$= 30 $s_{A/E_a}$ ( in./ft)$^2$

$S_a$ = is the allowable displacement

$E_a$ = is the reference modulus of elasticity

There is no guarantee or proof available that the above formula give accurate result. It is not applicable for certain systems which are under severe cyclic conditions. The above criteria selection formula can also be used for U-bends when L/U > 2.5 [19]. Empirical flexibility criteria is also another example for computing the parameters for flexibility.

Flexibility in torsion:

Flexibility in torsion can be calculated from the following formula

$$\theta = \frac{TL}{GJ}$$

Where θ = is the angle of twist in radius

T = is the torsion moment Lb/ $inch^2$

L = length in inch

G = modulus of rigidity Lb/ $inch^2$

J = polar moment of inertia

### 2.14.4 Suggested criteria to maintain level of piping flexibility:

There are no such standard rule to choose which criteria should use for flexibility analysis. Although computer flexibility analysis helps us to save the time but one can do manual flexibility analysis. Using the following guidelines, times and expenses can be saved while using formal computing flexibility analysis [19].

1. Formal computer flexibility is performed
2. Visual inspection and short cut manual calculation is performed
3. Special consideration should be given

In the flexibility analysis, during severe operating temperature conditions, the conditions are sustained during start-up, normal or regeneration scenarios. The analysis is for the maximum temperature differentials [19].

## 2.15 Incidents Due to poop flexibility piping design:

Piping engineering suffered a lot due to different reasons. Sometime some damaged happened lack of knowledge in engineering mechanics. In the current era due to improvements of computer aided design and computer aided manufacturing, there is less misunderstanding in piping flexibility as compared to previous eras. There many incidents happened due to poor piping design, cannot cover all of them but Filxborough is of one of them. Flixborogh disaster happened in 1974. From the reports the disaster happened due to poor qualified team and poor temporary piping design. 28 people died in that incident and billions of dollar sank as industrial damage. In the figure 11, the graphic view can been seen from ground.

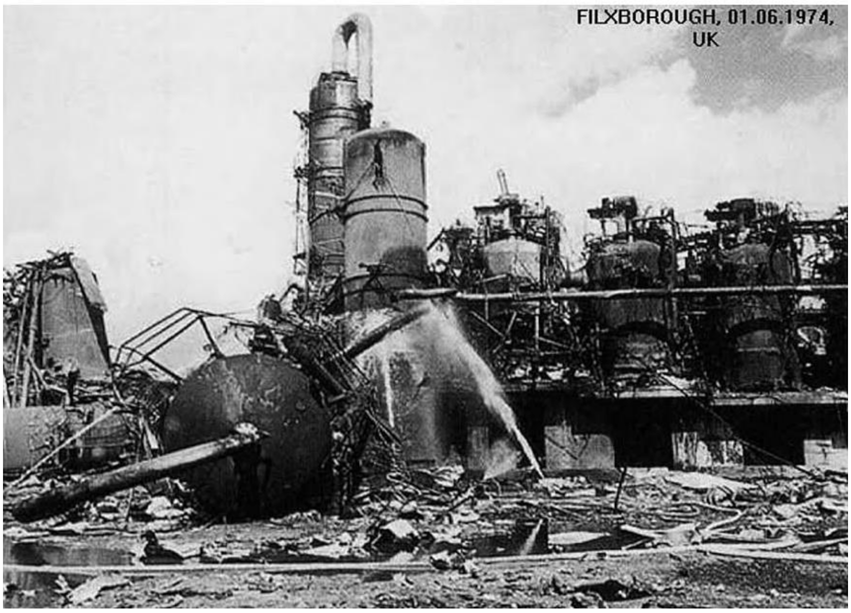



*Figure 11: flixborogh disaster due to bad piping design*

## 2.16 Stress

The idea of stress generated when researchers tried to find out the strength or failure of solid materials. In technical words, we can say that stress is actually distribution of some internal traction or forces and external forces. Due to this internal and external traction, the object remains stable. Mathematical we can write as [20]

$$T = \lim_{\Delta s \to 0} \frac{\Delta F}{\Delta s} = \frac{dF}{ds} \tag{2.38}$$

Here T is the bound vector. The flowing figure number 12 will more clear the idea about bound vector T. this the external traction.

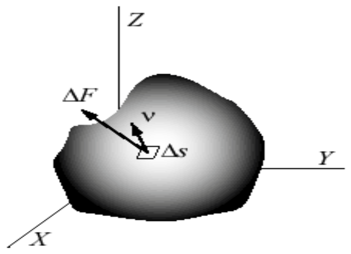



*Figure 12 Traction vector [20]*

The external traction cannot elaborated until unless external forces and internal forces, where they applied on the surface of the object is not specified. In the same manner, we can also elaborate the internal stress of surface of body. The internal traction can be understand by the following figure number 13.

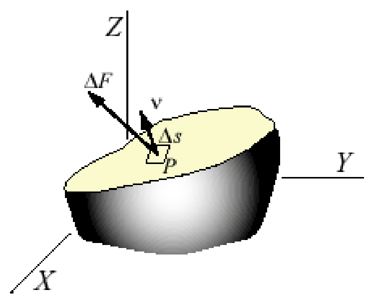



*Figure 13 internal traction within the solid [20]*

After the basic elaboration of stress the most general state of stress at a point can be describe by the following components

### 2.16.1 Normal stresses:

In mechanics, normal stress is the most common stress. The load divided by its area is called normal stress and mathematical relation can be written as

$$\sigma = P/A \qquad (2.39)$$

Here P is the pressure, A is the area of the surface as shown in the figure and $\sigma$ is denoted for normal stress. $\tau$ is used for other kind of stresses. If the same case in applied in perpendicular way, the equation can be written as

$$P = \int \sigma dA \qquad (2.40)$$

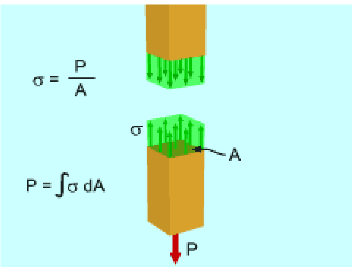



*Figure 14 Normal stress [21]*

### 2.16.2 Shearing stress:

Shearing stress can be define by the following equation

$$\tau = F/A \qquad (2.41)$$

Here $\tau$ is the sheaing stress, F is the force which is applied and A is the cross section area. If a shear force is applied on the beam, the internal shear stress can also be called beam share and it can be written as

$$\tau = VQ/It \qquad (2.42)$$

Here,

- $\tau$ = shear stress
- V= sheer force of the location
- T= is the thickness of the material which is perpendicular to the shear stress
- I = is the moment of inertia of the cross section area

The formula of shear stress in the fluids can be written as

$$\tau\,(y) = \mu \frac{\partial u}{\partial y} \tag{2.43}$$

Here, μ is the dynamic velocity of the fluid, u is the velocity along boundary surface, y represents the height of the boundary [22].

The following figure the plate stress on the cubic body can be written as

$$\sigma_x, \sigma_y, \ \tau_{xy} \text{ and } \ \sigma_z = \tau_{zx} = \tau_{zy} = 0 \tag{2.44}$$

The state of stress also occurs on free surface i.e without subjecting the external force at any point of the surface as shown in the figure 6b

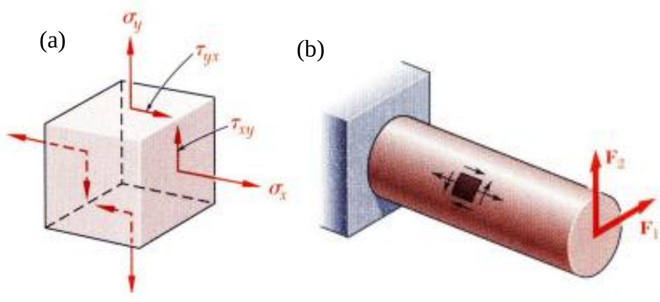



*Figure 15 (a), (b) Plane stress on cubic and structural element [23]*

### 2.16.3 Transformation of plane stress

Suppose at point Q, state of plane stress exits with $\sigma_z = \tau_{zx} = \tau_{zy} = 0$ and it is defined in previous section. Stress componets also defined there which are $\sigma_x, \sigma_y, \ \tau_{xy}$. These components associted with elements in figure 7 [23].

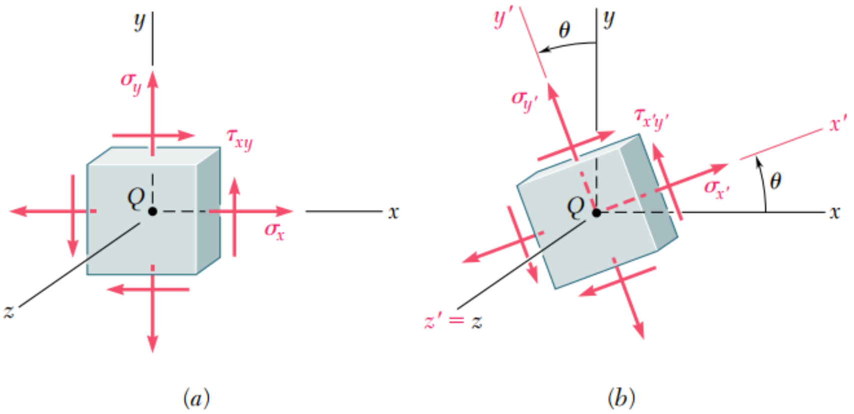



*Figure 16 Transformation of stress [23]*

To determine stress components after some rotation with θ angel about z axis in term of $\sigma_x, \sigma_y, \tau_{xy}$ and angleθ, for this consider the conditions for equlibrium of prismatic element with faces perpendicular to the x, y and x-axis. Figure 8 will help us to give the oblique face which is represented by ΔA. Then vertical and horizontal faces becomes ΔAcos θ and ΔAsinθ.

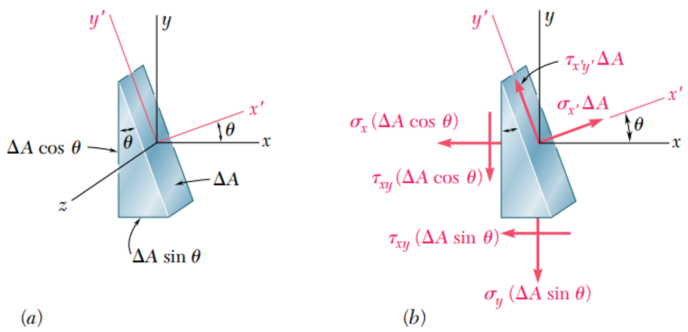



*Figure 17: Prismatic elements with faces [23]*

Using components along x′ and y′ axes and using equilibrium equations we can get the following equation for σx′, $\tau_{x'y'}$ and $\sigma_{y'}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \tag{2.45}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \tag{2.46}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \tag{2.47}$$

Since $\sigma_{z'} = \sigma_z$, so we can say that in plane stress case, some of normal stress applied on cubib materials, these stress are independent of the orientation.

## 2.17 Turbulent shearing stress and viscosity

Viscosity has direct relation to resistance of the flow. Different fluid materials have different viscosities and their flow in the pipe is different from each other. There are forces of attraction between the molecules, in case of liquid those forces are high enough to keep them together but in rigid state scenario can be different. Fluid has many surfaces during the flow in the pipe and when fluid flow over the other surface attaches each other as they have force of attraction between them. It will be not wrong to say that there is some shearing forces taking places between layers when liquid layer above the next layer keeps on moving in the pipe. To overcome the friction between layers fluid in the pipes need energy [12]

# CHAPTER 3

## 3 Design of Experiment

After the comprehensive study about literature review, this chapter elaborate research approaches, tools and methodologies which will lead us to determine the flexibility of pipeline and support of pipeline. For that purpose first a good material for piping is very necessary. In the market a long variety of pipes is available but keeping in mind the water flow rate through pipes but with some certain temperature.

### 3.1.1 Material Selection

The problem of design procedure is to find pipeline configuration and other important factor that make the design safe as safety wise and cost effective. Moreover, our fluid flow application medium is water so we need to focus the materials which are good for water flow according to following factors

- Life and durability
- Strength of the pipe
- Cost on transportation and installation
- Water carrying capacity
- Joint process
- Maintenance and repairs

In the market several type of pipes are available which can be divided into three types

1. Metallic Pipes
2. Cement pipes
3. plastic pipes

Metallic pipes includes the steel pipes, galvanised iron pipes and cast iron pipes. The cement pipes based on the category of concrete cement pipes and asbestos cement pipes. The third type of pipes which is PVC. PCV consist of plasticised polyvinyl chlorides pipes. The complete comparison of these three pipes will help us to select the pipe for our final design. First of all elaboration required for steel pipes [24].

Table 3- For Steel

| Properties | Drawbacks |
|---|---|
| 1. Strong<br>2. Durable<br>3. Can bear high temperature<br>4. Can get longer in length as compare to other pipes<br>5. Easily welded | 1. Heavy in weight-wise<br>2. Expensive |

Table 4- For Cast iron

| Properties | Drawbacks |
|---|---|
| 1. Mostly used in water supply<br>2. Good for handling pressure due to external thickness<br>3. Easy in manufacturing, layout and joining | 1. Heavy in weight<br>2. Transportation and installation is expensive due to weight<br>3. Not suitable for inaccessible places<br>4. Short in length |

Table 5- For cement Pipe

| Properties | Drawbacks |
|---|---|
| 1. Non-corrosive nature<br>2. Extreme strong<br>3. Durable | 1. Expensive<br>2. Being bulky and heavy they are more costly to handle. This reason also make them costly in installation and transportation |

Table 6- For PVC Pipes

| Properties | Drawbacks |
| --- | --- |
| 1. Non—corrosive<br>2. Extremely light<br>3. Easily handle and transport<br>4. Can get in long length<br>5. Cost effective<br>6. Low installation cost | 1. Expansion and contraction<br>2. Can deform due to temperature. |

From the above comparison, we can easily select PVC as our selected materials for design a pipeline flexibility bench. The selection of PVC as a martial is easy because our liquid flow is water, it can be at normal temperature or it can be warm, in both cases PVC can bear the conditions applied for the final design. Operating temperature for PVC with pressure is $38C^0$ and without pressure is $60C^0$. The heat distortion temperature is $54\text{-}80C^0$ and vicat softening temperature is $92C^0$ [25]

### 3.1.2 ANSYS:

ANSYS is dominant software which covers large spans of physics, it provides access to virtually any field related to engineering simulation that a design process requires. In our proposal thesis, **ANSYS** is used for fluid flow simulation and structural analysis of fluid flow piping. In the fluid part CFD (Computational Fluid Dynamics) is a tool which provides high level of flexibility and accuracy which helps to optimize design. **ANSYS** solves complex fluid problems and help us to make better and faster decision in very short time. In the structural analysis part, it help us to solve complex structural engineering problems. **ANSYS** gives us structural analysis for all experiences levels such as, reliability, high quality, automated meshing etc.

### 3.1.3 Chart Flow:

The following chart flow will give overall idea about design and analysis approach for flexibility of pipe.

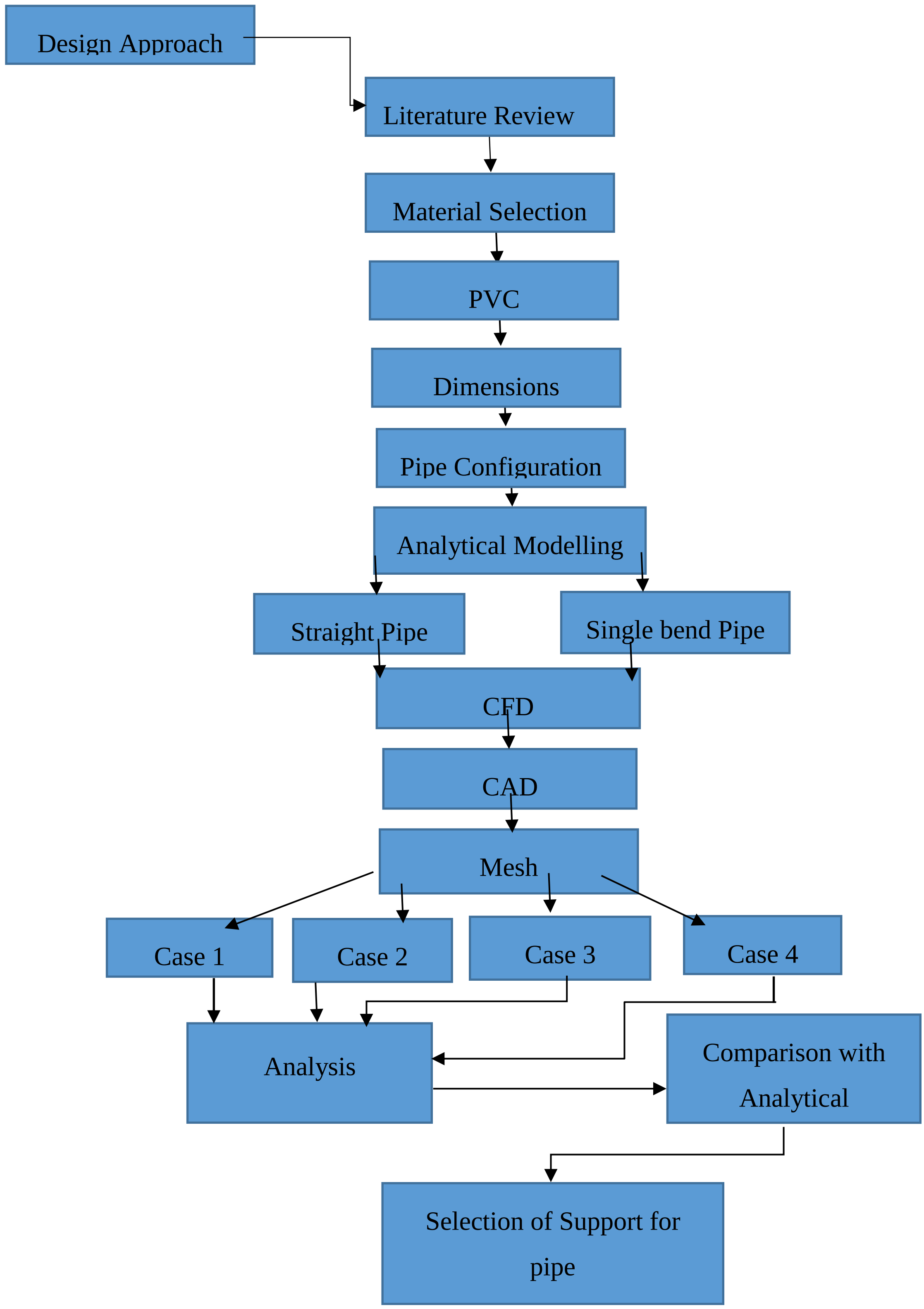
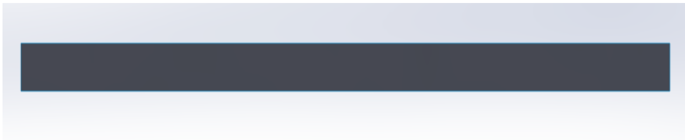
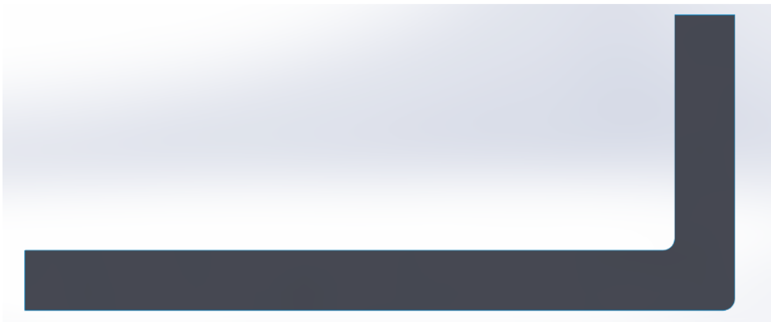
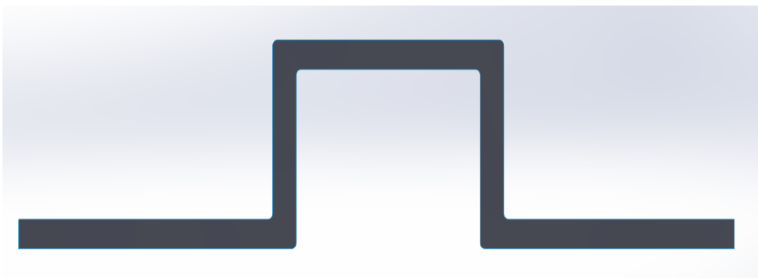


Table 7– explaining cases

| **Case No** | |
|---|---|
| **Case 1- straight pipe** |  |
| **Case 2- Single Bend (Sharp Corners)** |  |
| **Case 3- Single Bend (Round Corners)** |  |
| **Case 4- U shape bend** |  |

# CHAPTER 4

## 4 Analytical Modelling

When it comes to modelling of structural response due to fluid flow interaction, the key factor here is determining the nature of flow in question. Flows can be laminar, transitional or turbulent. The distinction between them lies in balance between the inertial and viscous forces in the fluid, which is described by non-dimensional Reynolds number, *Re*, defined for a flow in a pipe as:

$$Re = \frac{\rho V d}{\mu} \tag{4.1}$$

where ρ is fluid density (kg/m$^3$), $V$ is fluid velocity (m/s), $d$ is pipe diameter (or hydraulic diameter) (m), $\mu$ is dynamic viscosity of fluid (Pa·s). The fluid velocity $V$ can be defined as:

$$V = \frac{q}{A_c} \tag{4.2}$$

where, $q$ is the volume flow rate (m$^3$/s) and $A_c$ is the cross-sectional area of a circular pipe (m$^2$), which can be calculated as:

$$A_c = \frac{\pi d^2}{4} \tag{4.3}$$

where $d$ is pipe diameter (m). The type of flow (laminar, transitional or turbulent) depends on value of Reynolds number, *Re*, for the particular flow, and the "breakpoints" are following:

- If $Re \leq 2300$ than flow is laminar.
- If $2300 \leq Re \leq$ than flow is transitional.
- If $4000 \leq Re$ than flow is turbulent.

These "breakpoints" are not explicit, for example, in carefully controlled experimental conditions it is possible to have laminar flow for Reynolds number up to 100,000, however, in real, practical situations this may be exceedingly hard to achieve, and thus absolute majority of flows encountered in engineering applications are turbulent.

The change in type of flow changes the processes occurring in the flow, most notably, the viscous and inertial forces and interactions within fluid. This is, again, described by Reynolds number, physical meaning of which is ratio between inertial and viscous forces in the flow. At low Reynolds numbers, when the flow is laminar, viscous forces in fluid dominate, and the flow is characterized by highly ordered motion, smooth streamlines (moreover, for case of steady laminar flow the streamlines, streaklines and pathlines all coincide) and minimum or non-existent mixing, mass, momentum and energy transfer between different layers of fluids (only mechanism for those in laminar flow is molecular diffusion). Turbulent flows are sufficiently opposite of laminar ones, i.e. turbulent flows are characterized by random and rapid fluctuations of swirling regions of fluid, called eddies, throughout the flow. These fluctuations provide an additional mechanism for mass, momentum and energy transfer, in addition to supporting vigorous mixing in the turbulent fluid, thus supporting and enhancing the mechanism of molecular diffusion. The end result is significantly increased values of friction, heat transfer and mass transfer coefficient. Moreover, eddy motion, vigorous mixing and random fluctuations affect the behavior of flow even further. For example, in case of turbulent flow the streamlines, streakline and pathlines will not match, as in case of steady laminar flow and will change constantly, over time. To expand on this concept, one needs to return to the definition of fluid velocity, $V$ and flow rate, $q$ above. Using the principle of conservation of mass, the flow rate can be defined as:

$$q = VA_c = \iint_{A_c} u(r)\, dA_c \tag{4.4}$$

where $u(r)$ is the velocity profile and $dA_c$ represents the integration over area, and in case of circular pipe, the integration over circular cross-section, enclosed by pipe radius $R = d/2$, where $d$ is pipe diameter. The equation for velocity profile $u(r)$ for the region of hydrodynamically developed flow, i.e. the region where flow profile is developed and time-averaged velocity profile remains constant, meaning that:

$$\frac{\partial u(r,x)}{\partial x} = 0 \Longrightarrow u = u(r) \tag{4.5}$$

can be obtained as follows. Consider and infinitesimal element of fluid of radius $r$, differential thickness $dr$ and length $dx$ located somewhere within a flow. Laws of mechanics require that force balance on this element $\sum F = 0$. This results in following expression:

$$(2\pi drP)_x - (2\pi drP)_{x+dx} + (2\pi dx\tau)_r - (2\pi dx\tau)_{r+dr} = 0$$

where $P$ is the pressure and τ is viscous shear stress. The preceding equation shows that in hydrodynamically developed flow in a horizontal pipe viscous and pressure forces balance each other. After some simple mathematical operations the force balance equation cane written in the following form:

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_{dr}}{dr} = 0 \tag{4.6}$$

Differentiating and taking limits *dr*, *dx* → 0 results in:

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \tag{4.7}$$

By the definition, the viscous shear stress is equal to

$$\tau = -\mu\left(\frac{du}{dr}\right) \tag{4.8}$$

where μ is absolute viscosity of fluid. Using the assumption that viscosity of fluid remains unchanged during the flow, i.e. μ = constant, the force balance can be rewritten, by inserting the definition of viscous shear stress in it, as follows:

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} \tag{4.9}$$

The term $du/dr$ is negative for a flow, and the negative sign is included in expression for τ in order to obtain positive values of τ (alternatively, once can use equality $du/dr = -du/dy$ using the definition of $y = R - r$). The left hand side of previous equality is a function of $r$, while right hand side is a function of $x$. This equality must hold for any given value of $r$ and $x$, and the equality of a form $f(r) = g(x)$ can only be satisfied if both $f(r)$ and $g(x)$ are equal to the same constant, which leads to conclusion that $dP/dx$ = constant. One way to verify that is to write a force balance in term of an infinitesimal segment of a pipe with differential length $dx$ as follows:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R} \tag{4.10}$$

here $\tau_w$ is the wall shear stress, and $\tau_w$ = constant as viscosity and velocity profiles are constant in hydrodynamically developed region. Hence, two important conclusion follow:

1. The quantity $dP/dx$ is constant.

2. The quantity $du/dr = 0$ at $r = R$, as $\tau_w$ = constant at a distance $R$ throughout the pipe, meaning $\tau_w$ is independent of $x$.

The latter is an important factor in determining the solution equation of velocity profile $u(r)$. As $\tau_w$ = constant and $du/dr = 0$ on the pipe wall at radius $R$ it leads to conclusion that $u$ = constant on the boundary (since $r = R$, which is some positive, real value), or, more precisely, $u = 0$ due to no-slip condition. The no-slip condition dictates, that the fluid, directly attached to surface, should match the velocity of said surface, and in case of stationary surface, which own velocity is zero, the fluid velocity, by no-slip condition, should also be zero. This principle gives one of needed boundary conditions in order to obtain the solution to equation of velocity profile, which in general form can be obtained as:

$$V = \frac{q}{A_c} = \frac{\iint_{A_c} u(r)\, dA_c}{A_c} = \frac{\int_0^R u(r)\, 2\pi r\, dr}{\pi R^2} = \frac{2}{R^2}\int_0^R u(r)\, r\, dr \tag{4.11}$$

As the integration is done over area, meaning two integrations are required, the general expression for velocity profile has two constants of integrations, thus requiring two boundary conditions in order to be solved explicitly. The general expression of velocity profile, after necessary integrations is:

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2 \tag{4.12}$$

where $C_1$ and $C_2$ are constants of integration. Since the no-slip condition requires $\tau_w$ = constant at both sides of the pipe, and by symmetry and force balance once can conclude that at $r = -R$, $\tau = -\tau_w$, this means that shear stress distribution in the pipe is symmetrical along the centerline and therefore $du/dr = 0$ at $r = 0$. This gives secondary boundary condition, required to obtain explicit solution for the velocity profile $u(r)$ and inserting those boundary conditions into relevant expression yields:

$$u(r) = \frac{-R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) \tag{4.13}$$

As terms $-R^2/4\mu$ and $dP/dx$ are equal to some constants this means that the flow profile is governed by the term $1 - r^2/R^2$ which gives a parabolic expression and thus velocity profile for laminar flow is parabolic. Inserting the newly obtained expression for velocity profile into the expression for fluid velocity $V$ yields:

$$V = \frac{2}{R^2}\int_0^R u(r)\, r\, dr = \frac{2}{R^2}\int_0^R \frac{-R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right) r\, dr = \frac{-R^2}{8\mu}\left(\frac{dP}{dx}\right) \tag{4.14}$$

which can be rewritten as:

$$u(r) = 2V_{avg}\left(1-\frac{r^2}{R^2}\right) \tag{4.15}$$

where $V_{avg}$ is the average fluid velocity in a fluid and $V_{avg} = V$. The maximum velocity, $u_{max}$, of fluid for laminar flow is equal to:

$$u_{max} = 2V_{avg} \tag{4.16}$$

and it occurs at centerline at $r = 0$. The minimum velocity in a flow occurs at point $r = \pm R$ and $u_{min} = 0$ due to no-slip condition. The general notion is to call the region where $u \geq 0.98V_{avg}$ as core (also referred as irrotational) region, and region, where $u \leq 0.98V_{avg}$ as boundary layer region. This, along with parabolic shape of velocity profile for laminar flow can be explained by viscous effects, molecular diffusion and momentum conservation principle. As the velocity of fluid directly at surface is zero, the viscous interaction will slow down directly adjacent layers of fluid. This will result of certain amount of fluid being displaced from those layers inwards, towards the center of the pipe. Due to momentum conservation principle, this extra mass will slow down next adjacent layers (though progressively to lesser extent) as momentum should be conserved. The process is governed by molecular diffusion. This displacement and momentum conservation will result in development of shear stress in the fluid in the pipe. The displacement is zero directly at centerline, and, as a result – the velocity there is maximum and shear stress is minimum in the flow and is equal to zero.

Returning back to the term $dP/dx$ which shows the pressure change per some length. Assuming pipe of length $L$ and setting $dx = L$ yields following expression:

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} \tag{4.17}$$

where $P_1$ is initial pressure, i.e. pressure at pipe entrance and $P_2$ is pressure at some distance $L$, for example, at the pipe end. It is easy to see that value of the term $P_2 - P_1$ is negative, and thus $dP/dx$ can be viewed as pressure loss. Rewriting the expression for velocity by inserting updated expression for $dP/dx$ results in:

$$V = \frac{-R^2}{8\mu}\left(\frac{dP}{dx}\right) = \frac{-R^2}{8\mu}\left(\frac{P_2 - P_1}{L}\right) = \frac{R^2(P_1 - P_2)}{8\mu L} = \frac{R^2\Delta P}{8\mu L} \tag{4.18}$$

where $\Delta P$ is the pressure loss and it can be obtained as:

$$\Delta P = \frac{8\mu LV}{R^2} = \frac{32\mu LV}{d^2} \tag{4.19}$$

Moreover, the expression

$$\frac{dP}{dx} = -\frac{2\tau_w}{R} \tag{4.20}$$

can be modified by insertion of term $\Delta P/L$ in place of $dP/dx$ as follows:

$$\frac{\Delta P}{L} = \frac{2\tau_w}{R} \tag{4.21}$$

from which the wall shear stress $\tau_w$ can be obtained as:

$$\tau_w = \frac{\Delta PR}{2L} = \frac{\Delta Pd}{4L} = \frac{d}{4}\Delta P_L \tag{4.22}$$

where $\Delta P_L$ is the pressure drop per unit length. Alternatively, pressure loss can be calculated as:

$$\Delta P = f\frac{L}{d}\frac{\rho V^2}{2} \tag{4.23}$$

where $\rho V^2/2$ is the dynamic pressure and $f$ is the Darcy-Weisbach friction factor equal to:

$$f = \frac{8\tau_w}{\rho V^2} \tag{4.24}$$

Therefore, the wall shear stress alternatively can be calculated as:

$$\tau_w = \frac{f\rho V^2}{8} \tag{4.25}$$

where $f$ is friction factor. The friction factor is not necessarily should be Darcy-Weisbach friction factor, as it will be shown later, for turbulent flows friction factors can be different, depending on the range of parameters. In case of laminar flow and Darcy-Weisbach friction factor, the pressure loss and wall shear stress equation can be rewritten as:

$$\Delta P = f\frac{L}{d}\frac{\rho V^2}{2} = \frac{8\tau_w}{\rho V^2}\frac{L}{d}\frac{\rho V^2}{2} = \frac{4\tau_w L}{d} \Longrightarrow \tau_w = \frac{\Delta Pd}{4L} \tag{4.26}$$

which is exactly matches the equation obtained via force balance. Moreover, in case of laminar flow in circular pipe the friction factor $f$ can be obtained as follows:

$$\frac{32\mu LV}{d^2} = f\frac{L}{d}\frac{\rho V^2}{2} \Longrightarrow f = \frac{64\mu LVd}{L\rho V^2 d^2} = \frac{64\mu}{\rho Vd} = \frac{64}{Re} \quad (4.27)$$

Note that this friction factor is valid only for laminar flow within a circular pipe, as its derivation is directly dependent on definition of average velocity, and by extension – the definition of velocity profile $u(r)$, being equal to those, derived previously in this section. Finally, dividing the pressure loss per unit length $\Delta P_L$ by quantity $\rho g$ which represents specific gravity of the fluid one can obtain the head loss per unit length $h_L$ as:

$$h_L = \frac{\Delta P_L}{\rho g} \quad (4.28)$$

and therefore, the preceding calculations of wall shear stress and friction factor can be done in terms of head loss. The head loss represents the additional height that the fluid needs to be raised by a pump in order to overcome frictional losses in the pipe. The head loss is caused by viscosity, and it is directly related to wall shear stress. The equations for wall shear stress, pressure loss and head loss are also valid for case of turbulent flow, however, the expressions for friction factor in the form of $64/Re$ as well as Darcy-Weisbach friction factor itself are not valid for turbulent flow, due to differences in flow behavior between laminar and turbulent flow, as it will be shown in subsequent discussion.

First of all, the velocity profile for turbulent flow differs of that of laminar flow significantly. This difference arises due to random fluctuations in the flow, primarily due to vigorous mixing and eddy motion at high frequency (in order of $10^3$ eddies per second). Since eddies provide additional source of mass, momentum and energy transport the velocity of turbulent flow, $u$, can be expressed in two components – the time-averaged component $\bar{u}$ and fluctuating component $u'$ as follows

$$u = \bar{u} + u' \quad (4.29)$$

As have been discussed previously, the velocity $u$ and associated velocity profile $u(r)$ do affect other values of interest, such as pressure changes, shear stress etc. Therefore, one can conclude that these values will also vary in random nature as the velocity itself, and for turbulent flow they can be written as $P = \bar{P} + P'$, $\tau = \bar{\tau} + \tau'$ etc. (4.30)

Moreover, the eddy motion can introduce an upward movement of fluid particles in the flow. Consider turbulent flow in a horizontal circular pipe, with the upward eddy motion of fluid particles in a layer with lower velocity (located closer to the wall) to an adjacent layer with higher velocity (located closer to the centerline of the pipe) through some infinitesimal differential area *dA*, as a result of velocity fluctuation, for clarity, denoted as *v'*. The mass flow rate of the fluid particles passing through *dA* is equal to ρ*v'dA*, and, due to conservation of mass and momentum, the effect of this mass flow is a reduction of average velocity of the layer above *dA*. The momentum transfer causes the horizontal velocity component to increase by *u'* and therefore its momentum in the horizontal direction increases at a rate ρ*v'dAu'* which must be equal to the decrease in the momentum of adjacent upper fluid layer. As the force in a given direction is equal to the rate of change of momentum in that direction, the horizontal fore acting on the fluid element above *dA* due to passage of velocity particles through the *dA* is:

$$dF = -\rho u' v' dA \tag{4.31}$$

Thus, the shear force per unit area due to eddy motion of fluid particles is equal to:

$$\frac{dF}{dA} = -\rho u' v' \tag{4.32}$$

and it can be viewed as instantaneous turbulent shear stress. By extension, the time-averaged turbulent shear stress can be expressed as:

$$\tau_{turb} = -\rho \overline{u' v'} \tag{4.33}$$

where $\overline{u'v'}$ is the time average of product of the fluctuating velocity components *u'* and *v'*. Note that $\overline{u'v'} \neq 0$ even though individual fluctuating velocity components $\overline{u'}$, $\overline{v'}$ and $\overline{u'}\,\overline{v'}$ are equal to zero in time-averaged turbulent flow. The term $\overline{u'v'}$ is usually a negative quantity and thus a negative sign is added in the expression for turbulent shear stress to obtain positive values of it. Moreover, terms such as $-\rho\overline{u'v'}$ or $-\rho\overline{u'^2}$ are frequently called as Reynolds stresses or turbulent stresses.

The expression for turbulent shear stress can be rewritten in order to make it consistent with laminar shear stress as follows:

$$\tau_{turb} = \mu_t \frac{\partial \bar{u}}{\partial y} \tag{4.34}$$

where $\mu_t$ is the turbulent (eddy viscosity) and $\partial\bar{u}/\partial y$ is the velocity gradient. Note that unlike turbulent flow velocity profile equation where velocity gradient $du/dr$ is a full differentiation with respect to $r$, the velocity gradient for turbulent flow $\partial\bar{u}/\partial y$ is a partial derivative of $\bar{u}$ which is dependent on both horizontal and vertical velocity components, $u'$ and $v'$, respectively and differentiation is done with respect to $y = R - r$, i.e. the differentiation is done "per layer" basis, indicating the change of velocity gradient per layer. Furthermore, the total shear stress consists of laminar part, which is dominant source of shear stress in boundary layer and turbulent part, which is dominant in core region, and total stress can be obtained as:

$$\tau_{total} = \tau_{lam} + \tau_{turb} = (\mu + \mu_t)\frac{\partial\bar{u}}{\partial y} \tag{4.35}$$

where μ is the absolute viscosity of the fluid.

Unlike laminar flow, the expressions for the velocity profile in turbulent flow are based on both analysis and measurements and thus they are semi-empirical in nature with constants determined from experimental data. In addition, while laminar flow profile is parabolic and can be defined by two distinct regions – the boundary layer region and core (irrotational) region, this is not the case for turbulent flow. The turbulent flow consists of much thinner boundary layer, also referred as viscous, laminar, linear or wall layer, in which the viscous effect are dominant, and as it can be easily guessed – this layer is located near surfaces. Next to viscous layer is the buffer layer, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects. Above the buffer layer is the inertial sublayer, also called as overlap or transition layer, in which the turbulent effects are much more significant but still not dominant. Finally, above that is the turbulent (or outer) layer, which occupies the remaining part of the flow in which turbulent effect dominate over molecular diffusion (viscous) effects.

Flow characteristics are quite different in different regions, and thus it is difficult to come up with an analytic relation for the velocity profile for the entire flow, unlike the case of laminar flow. The idealized approach requires performing dimensional analysis, identifying critical values, parameters and functional forms, coupled with experimental verification and development of velocity profile for each separate layer in the flow, followed by making those parameterizations piecewise continuous in order to obtain a complete expression for turbulent velocity profile. As one can see, this approach is cumbersome, mathematically intensive, requires experimental data and verification, and thus is not suitable to vast majority of practical applications. Instead, in majority of engineering applications, semi-empirical relations and simplifications are used, in order to obtain a decent mathematical approximations (within few %) for turbulent flow, which are not that computationally expensive.

From numerous empirical velocity profiles derived over multiple studies, one of the simplest and most used ones is the power-law velocity profile, given as:

$$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{\frac{1}{n}} \tag{4.36}$$

where, for simplicity $u = \bar{u} = V = V_{avg}$. Since $y = R - r$ the preceding equation can be rewritten in terms of $r$ as:

$$\frac{u}{u_{max}} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \tag{4.37}$$

where the exponent $n$ is a constant whose value depends on the Reynolds number, and the value of $n$ increases with increase in Reynolds number value. Generally, the value of $n = 7$ is said to approximate majority of turbulent flows, encountered in practice, (known as one-seventh power law) however if more precise estimates are needed the value of $n$ can be calculated as a function of $Re$ with one of the most common expressions being:

$$n = 1.03 \ln Re - 3.6 \tag{4.38}$$

With equation for velocity profile for turbulent flow established it is possible to reuse now equations for wall shear stress $\tau_w$ and pressure loss $\Delta P$ derived in previous discussion on laminar flow. However, those equations depend on the friction factor $f$ and how it has been discussed previously, due to significantly increased effects of friction in turbulent flow, due to eddy motion, the friction factors for laminar flow, such as Darcy-Weisbach friction factors cannot be used for turbulent flow. Instead, other semi-empirical expressions for friction factor have to be used.

Those friction factors can be classified into two different ways – implicit or explicit, and for hydrodynamically rough or smooth pipes. The major difference in previous that implicit expressions require iterative procedures and solvers in order to calculate values matching experimental or simulation values with good accuracy. Explicit relations usually have some margin of error (within few %) when compared to experimental data, however no iterations are needed and solution is readily available. The latter classification scheme relies on knowledge of equivalent roughness factor, ε of the pipe. If ε = 0 it is said that the pipe is hydrodynamically smooth and equivalent roughness does not have any sort of impact on friction values. If ε ≠ 0, than the pipe is hydrodynamically rough and equivalent roughness does influence friction values, usually by increasing the value of friction factor of the flow, and as a result – the values of shear stress in the pipe. Note that there is a certain uncertainty with values of ε for most engineering pipes and flows, and these results are approximate. In any case, the best known expression for friction factor for turbulent flow is the Colebrook equation, defined as:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \tag{4.39}$$

where the term ε/*d* is the relative roughness of the pipe. The equation, by default assumes that ε ≠ 0, meaning pipe is rough, however, it can also be used for smooth pipes. Note that values of constants in Colebrook are a "best-fit" to the experimental values, and thus different authors can employ slightly different values for those constants. The Colebrook equation is an implicit equation in *f* thus requiring a few iterations to obtain a good convergence. The one of most well-known explicit equations in *f* for flow in hydrodynamically rough pipes is the Haaland equation, given as:

$$\frac{1}{\sqrt{f}} \cong -1.8\log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/d}{3.7}\right)^{1.11}\right] \tag{4.40}$$

The values from Haaland equation typically are within 2% of values obtained from Colebrook equation, and thus, if more accurate results are needed, the Haaland equation can be used as a first guess in Colebrook equation.

If equivalent roughness is zero, the friction factor will be at minimum, but still not zero, due to no-slip conditions at the wall. In this case other empirical expressions are needed to calculate the friction factor. One of the most-known equations for turbulent flow in smooth pipes is a Prandtl equation, which is a direct result of simplification of Colebrook equation for case when ε = 0, and it is given as:

$$\frac{1}{\sqrt{f}} = 2.0\log\left(Re\sqrt{f}\right) - 0.8 \tag{4.41}$$

Again, the equation is implicit in $f$. For explicit equation in $f$ for flow in hydrodynamically smooth pipes a few empirical equations exist, with Blasius equation being the most widely used one. The Blasius equation is given by following expression:

$$f = (100Re)^{-0.25} \tag{4.42}$$

The equation is valid for the range of $10^4 \leq Re \leq 10^5$. At higher values of $Re$, the Blasius equation tends to underestimate the friction factor, and thus a new equation is needed, with one of the most used ones being the Filonenko equation, given as follows:

$$f = (0.79 \ln Re - 1.64)^{-2} \tag{4.43}$$

The Filonenko equation is valid for the range of $10^4 \leq Re \leq 10^8$. Both Blasius and Filonenko equations are explicit in $f$ and are only dependent on value of Reynolds number, which makes the attractive for modelling turbulent flows in smooth pipes.

This concludes investigation into the theory of the laminar and turbulent flows. With established equations and described theoretical concepts it is possible to calculate the flow and resultant shear stresses in a few proof-of-concept cases. In order to facilitate process a *MATLAB* script has been written with all the necessary equations and inputs, needed to calculate the flow. This script is available as an appendix for this work.

## 4.1 Case for analytical modelling

For the first proof-of-concept case the simple horizontal pipe was selected with properties, summarized in Table 8.

Table 8- Simple horizontal pipe with dimensions

| Parameter | Symbol | Value |
|---|---|---|
| Pipe length (m) | $L$ | 1 |
| Pipe inner diameter (in) | $d$ | 2 |
| Pipe outer diameter (in) | $D$ | 2.375 |
| Wall thickness (in) | $t$ | 0.1875 |
| Temperature (°C) | $T$ | 15 |

| | | |
|---|---|---|
| Pipe material | - | PVC |
| Fluid | - | Water |
| Flow velocity (m/s) | $V$ | 0.5, 1, 2, 3 |

Table 1. Operating conditions for case 1.

The pipe with dimensions for this case is given in Figure 18.

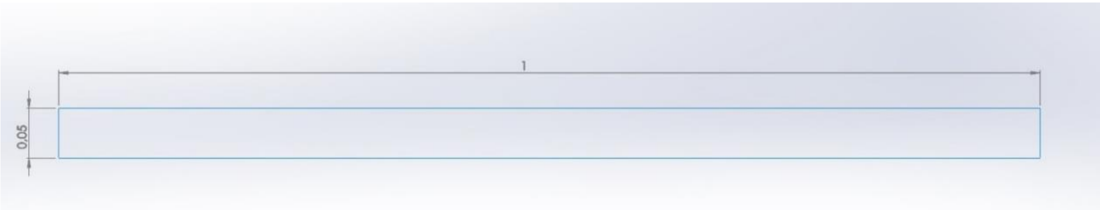



*Figure 18: Pipe dimensions in case 1.*

First of all, it is necessary to calculate such properties of water, as density and viscosity for a flow at 15 °C. The density of water can be calculated using the following expression:

$$\rho = \frac{\rho_0}{1-\beta\Delta T} \tag{4.44}$$

in which, $\rho_0$ is the density of water at 0 °C, $\beta$ is the volumetric expansion coefficient of water ($m^3/K$) and $\Delta T$ is the change in temperature, defined as

$$\Delta T = T_0 - T \tag{4.45}$$

where $T_0$ is a temperature corresponding to 0 °C. Inserting all the necessary values of constants in equation of $\rho$ gives following density of water at 15 °C:

$$\rho = \frac{\rho_0}{1-\beta\Delta T} = \frac{999.8396}{1-0.0002*(-15)} = 996.8491\ {}^{kg}/{}_{m^3} \tag{4.46}$$

The viscosity of water at 15 °C can be calculated using empiric Vogel equations, given as:

$$\mu = e^{A+\frac{B}{C+T}} \tag{4.47}$$

where $A$, $B$, and $C$ are empirical constants given as:

$A = -3.7188$ $\quad$ $B = 578.919$ $\quad$ $C = -137.546$

and the temperature $T$ in this case is in Kelvin. Substitution of relevant values yields:

$$\mu = e^{A+\frac{B}{C+T}} = e^{-3.7188+\frac{578.919}{-137.546+288.15}} = 1.13335\ mPa\ \cdot s = 1.13335 * 10^{-3}\ Pa \cdot s \quad (4.48)$$

With those values established, it is now necessary to establish type of flow this system will experience. For the case of velocity, $V$, being equal to 0.5 m/s, the value of Reynolds number $Re$ is equal to:

$$Re = \frac{\rho V d}{\mu} = \frac{996.8491*0.5*(0.0254*2)}{1.13335*10^{-3}} = 22341 \quad (4.49)$$

which indicates that the flow is clearly turbulent. The next step would be identification of correct expression for friction factor $f$. Since the pipe is made out from PVC, one can assume that the pipe is hydrodynamically smooth, as it is a common notion for calculating flows in pipes, made out from plastics and glass. Therefore, the equitant roughness, ε can be assumed to be equal to zero, and one of equations for such cases, for example Prandtl or Filonenko can be used. Since Prandtl equation is implicit in $f$, the Filonenko equation will be used instead, to avoid iterative procedure. Inserting the required quantities in Filonenko equation results in following value of friction factor $f$:

$$f = (0.79 \ln Re - 1.64)^{-2} = (0.79 \ln 22341 - 1.64)^{-2} = 0.0254 \quad (4.50)$$

With friction factor value known, it is possible to obtain the pressure drop $\Delta P$ as follows:

$$\Delta P = f \frac{L}{d} \frac{\rho V^2}{2} = 0.0254 * \frac{1}{0.0254*2} * \frac{996.8491*0.5^2}{2} = 62.37\ Pa \quad (4.51)$$

With pressure drop value obtained, it is possible to directly calculate the wall shear stress $\tau_w$ in a pipe in a following manner:

$$\tau_w = \frac{\Delta P d}{4L} = \frac{62.37*(0.0254*2)}{4*1} = 0.7921\ Pa \quad (4.52)$$

Alternatively, the wall shear stress can be calculated directly, by using the friction factor $f$ as follows:

$$\tau_w = \frac{f \rho V^2}{8} = \frac{0.0254*996.8491*0.5^2}{8} = 0.7921\ Pa \quad (4.53)$$

As it can be seen, both values are exact, therefor, in further calculations one can use any of those formulas, or a combination of both, in order to check the results. The shear stress distribution can be obtained using following relationship:

$$\tau = \frac{2 * \tau_w * r}{d} \quad (4.54)$$

Inserting the necessary values results in following expression:

$$\tau = \frac{2 * \tau_w * r}{d} = \frac{2 * 0.7921 * r}{0.0508} = 31.185 r \quad (4.55)$$

Graphically, the shear stress distribution in a pipe is given in Figure 19.

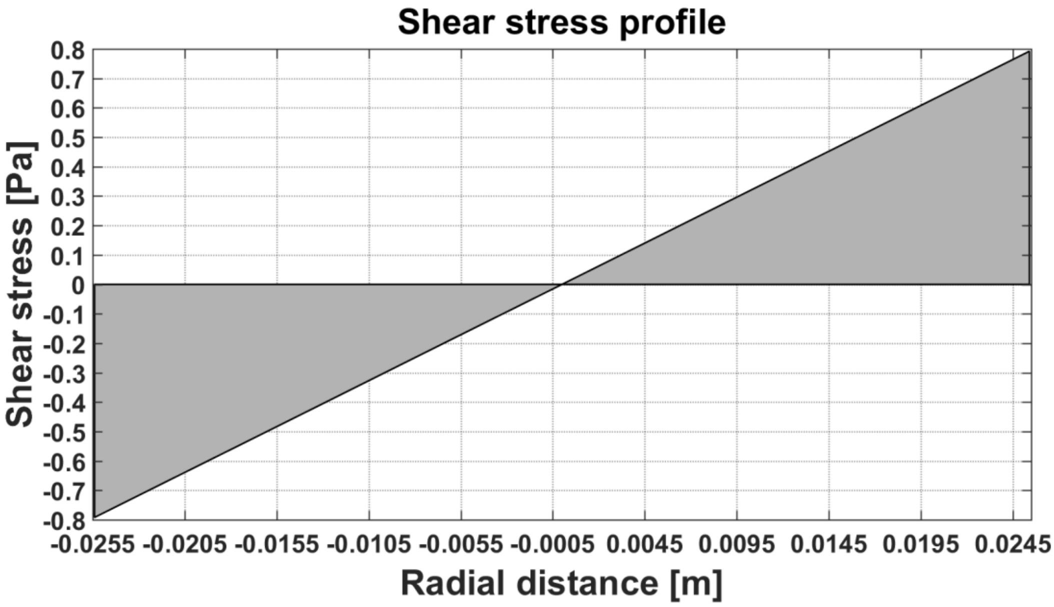



*Figure 19: Shear stress profile for 0.5 m/s flow.*

The value of exponent *n* for turbulent flow at this Reynolds number is:

$$n = 1.03 \ln 22341 - 3.6 = 6.7146 \quad (4.56)$$

which is sufficiently close to typically used value of *n* = 7. With value of exponent *n* determined the velocity profile for a flow in this case is given as:

$$\frac{u}{u_{max}} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} = \left(1 - \frac{r}{0.0254}\right)^{\frac{1}{6.7146}} \quad (4.57)$$

The maximum velocity in a flow can be obtained as follows:

$$u_{max} = \frac{(n+1) * (2n+1)}{2n^2} V \quad (4.58)$$

which can be written as:

$$u_{max} = \frac{(n+1)*(2n+1)}{2n^2}V = \frac{(6.7146+1)*(2*6.7146+1)}{2*6.7146^2} * 0.5 = 0.6172\ {}^{m}/_{s} \qquad (4.59)$$

The resultant velocity profile is given in Figure 20.

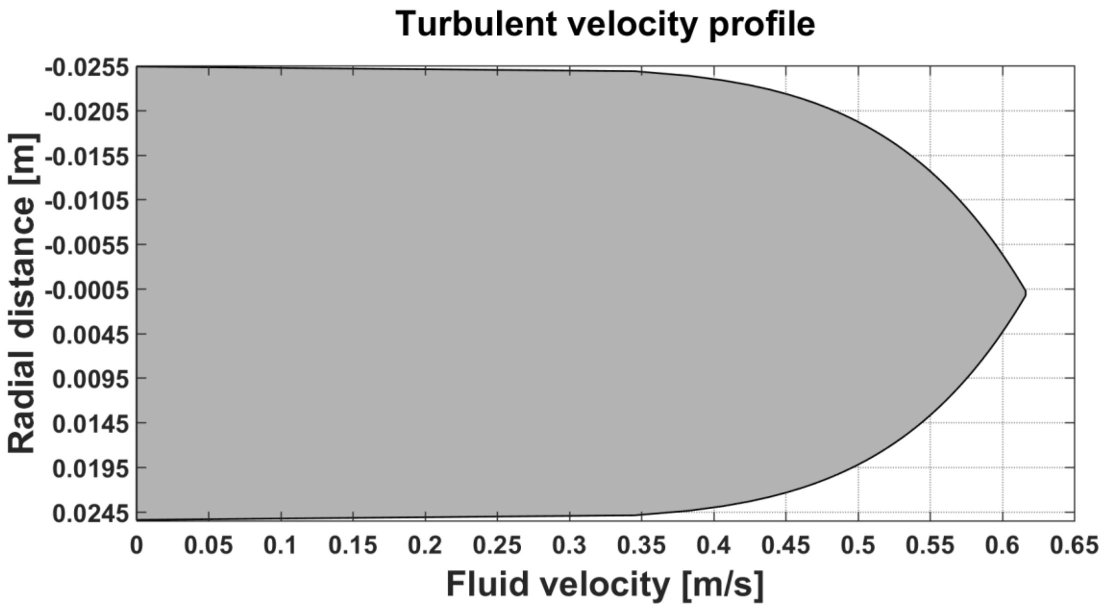



*Figure 20: Velocity profile for 0.5 m/s flow.*

This concludes the investigation for the case of 0.5 m/s flow. Proceeding further, the calculations for the case of 1 m/s are done next, using the exactly same scheme, starting with calculation of Reynolds number:

$$Re = \frac{996.8491*1*(0.0254*2)}{1.13335*10^{-3}} = 44681 \qquad (4.60)$$

The friction factor *f* for this case is equal to:

$$f = (0.79 \ln 44681 - 1.64)^{-2} = 0.0215 \qquad (4.61)$$

Note that friction factor in this case in less than for 0.5 m/s flow. This is because at higher Reynolds number the friction effects decrease, as friction becomes progressively less dependent on Reynolds number. In the extreme case of *Re* → ∞ the Colebrook equation will assume the following form:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/d}{3.7}\right) \quad (4.62)$$

which is known as von Kármán equation and it is explicit in *f*. However, for the case of $\varepsilon = 0$ the von Kármán equation breaks down as logarithm of 0 is undefined. Proceeding further, the pressure drop and wall shear stresses are calculated as:

$$\Delta P = 0.0215 * \frac{1}{0.0254*2} * \frac{996.8491*1^2}{2} = 211.0195\ Pa \quad (4.63)$$

Observe the significantly increased value of pressure drop, when compared to the case of 0.5 m/s. This is due to the fact that $\Delta P$ is increases as square of velocity, with minor contribution from change in friction factor. As wall shear stress is directly dependent on $\Delta P$ roughly the same increase can be expected in the values of shear stress. For the case of 1 m/s the wall shear stress $\tau_w$ is equal to:

$$\tau_w = \frac{\Delta P d}{4L} = \frac{211.0195*(0.0254*2)}{4*1} = 2.6799\ Pa \quad (4.64)$$

$$\tau_w = \frac{f\rho V^2}{8} = \frac{0.0215*996.8491*1^2}{8} = 2.6799\ Pa \quad (4.65)$$

Again, both equations are in agreement, and the shear stresses for this case are given as:

$$\tau = \frac{2*2.6799*r}{0.0508} = 105.5079r \quad (4.66)$$

and the shear stress profile is given graphically in Figure 21.

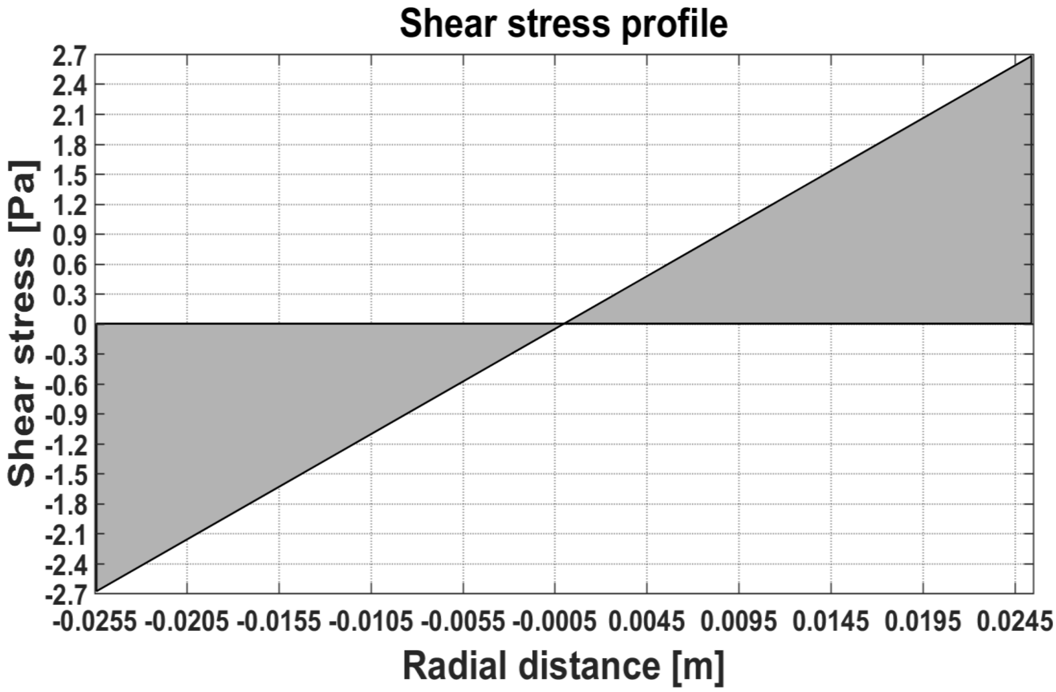



*Figure 21: Shear stress profile for 1 m/s flow.*

The value of exponent *n* for turbulent flow at this Reynolds number is:

$$n = 1.03 \ln 44681 - 3.6 = 7.4285 \tag{4.67}$$

and the velocity profile for a flow in this case is:

$$\frac{u}{u_{max}} = \left(1 - \frac{r}{0.0254}\right)^{\frac{1}{7.4285}} \tag{4.68}$$

with maximum velocity equal to:

$$u_{max} = \frac{(7.4285+1)*(2*7.4285+1)}{2*7.4285^2} * 1 = 1.211\ {}^{m}/_{s} \tag{4.69}$$

The velocity profile for this case graphically is given in Figure 22.

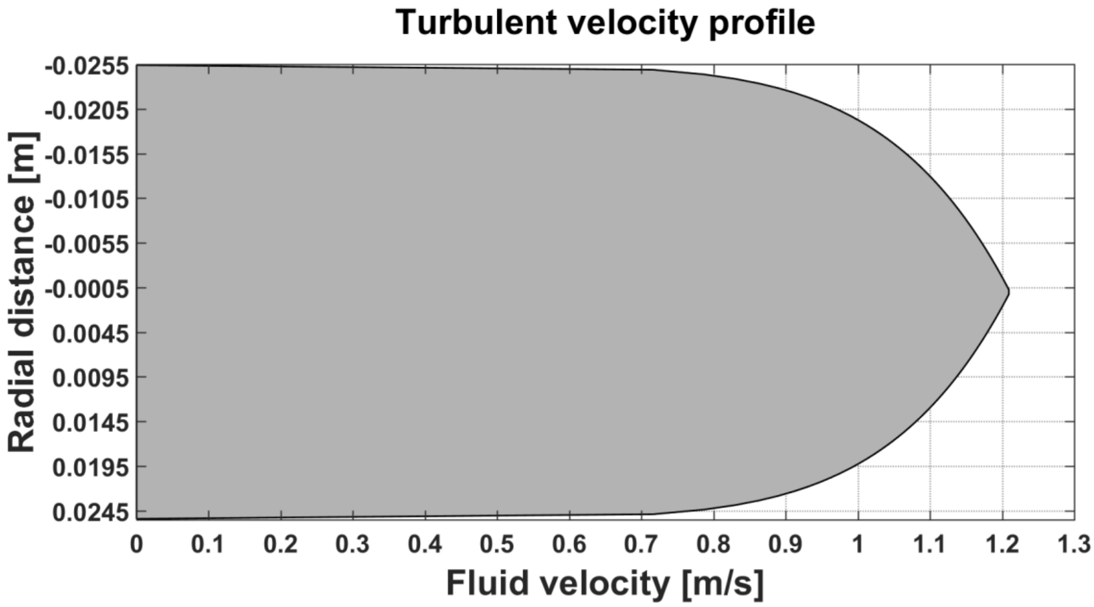



*Figure 22: Velocity profile for 1 m/s flow.*

For the case of 2 m/s flow for this pipe the results are following:

$$Re = \frac{996.8491*2*(0.0254*2)}{1.13335*10^{-3}} = 89363 \tag{4.70}$$

$$f = (0.79 \ln 89363 - 1.64)^{-2} = 0.0184 \tag{4.71}$$

$$\Delta P = 0.0184 * \frac{1}{0.0254*2} * \frac{996.8491*2^2}{2} = 723.2514\ Pa \tag{4.72}$$

$$\tau_w = \frac{\Delta P d}{4L} = \frac{211.0195*(0.0254*2)}{4*1} = 9.1853\ Pa \tag{4.73}$$

$$\tau_w = \frac{f \rho V^2}{8} = \frac{0.0184*996.8491*2^2}{8} = 9.1853\ Pa \tag{4.74}$$

$$\tau = \frac{2*9.1853*r}{0.0508} = 361.626r \tag{4.75}$$

$$n = 1.03 \ln 89363 - 3.6 = 8.1425 \tag{4.76}$$

$$\frac{u}{u_{max}} = \left(1 - \frac{r}{0.0254}\right)^{\frac{1}{8.1425}} \tag{4.77}$$

$$u_{max} = \frac{(8.1425+1)*(2*8.1425+1)}{2*8.1425^2} * 2 = 2.3835\ {}^{m}/_{s} \tag{4.78}$$

The shear stress and turbulent velocity profiles are given in Figures 23 and 24, respectively.

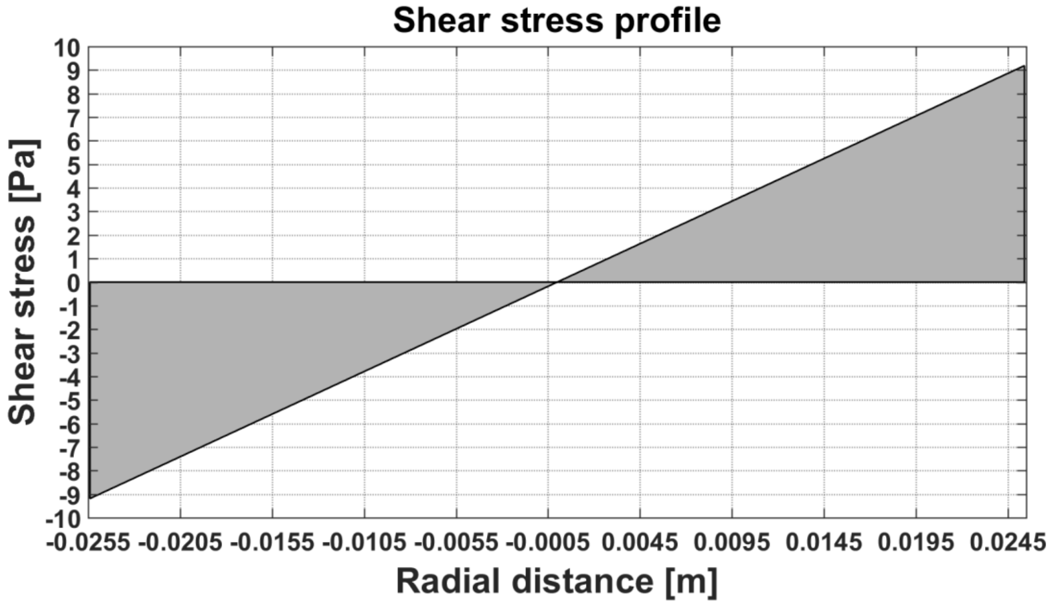



*Figure 23: Shear stress profile for 2 m/s flow.*

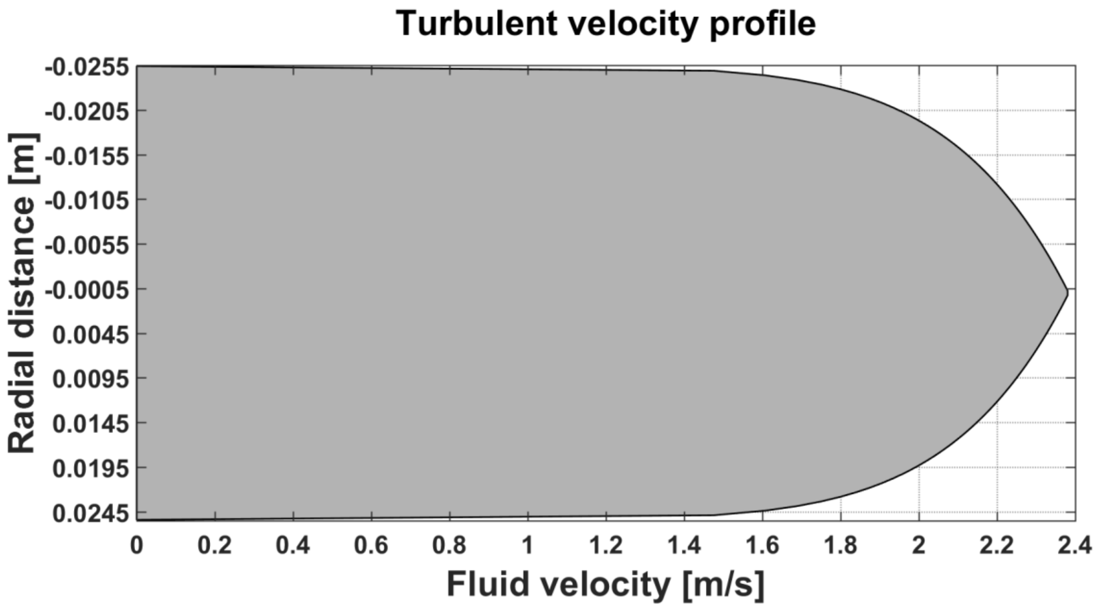



*Figure 24: Velocity profile for 2 m/s flow.*

Finally, for the case of 3 m/s flow the results are following:

$$Re = \frac{996.8491 * 3 * (0.0254 * 2)}{1.13335 * 10^{-3}} = 134044 \tag{4.79}$$

$$f = (0.79 \ln 134044 - 1.64)^{-2} = 0.0169 \tag{4.80}$$

$$\Delta P = 0.0169 * \frac{1}{0.0254*2} * \frac{996.8491*3^2}{2} = 1494.5\ Pa \quad (4.81)$$

$$\tau_w = \frac{\Delta P d}{4L} = \frac{1494.5*(0.0254*2)}{4*1} = 18.9803\ Pa \quad (4.82)$$

$$\tau_w = \frac{f\rho V^2}{8} = \frac{0.0169*996.8491*3^2}{8} = 18.9803\ Pa \quad (4.83)$$

$$\tau = \frac{2*18.9803*r}{0.0508} = 747.2559r \quad (4.84)$$

$$n = 1.03\ln 134044 - 3.6 = 8.5601 \quad (4.85)$$

$$\frac{u}{u_{max}} = \left(1 - \frac{r}{0.0254}\right)^{\frac{1}{8.5601}} \quad (4.86)$$

$$u_{max} = \frac{(8.5601+1)*(2*8.5601+1)}{2*8.5601^2} * 2 = 3.5462\ {}^{m}/_{s} \quad (4.87)$$

The shear stress and turbulent velocity profiles for 3 m/s flow are given in Figures 25 and 26, respectively.

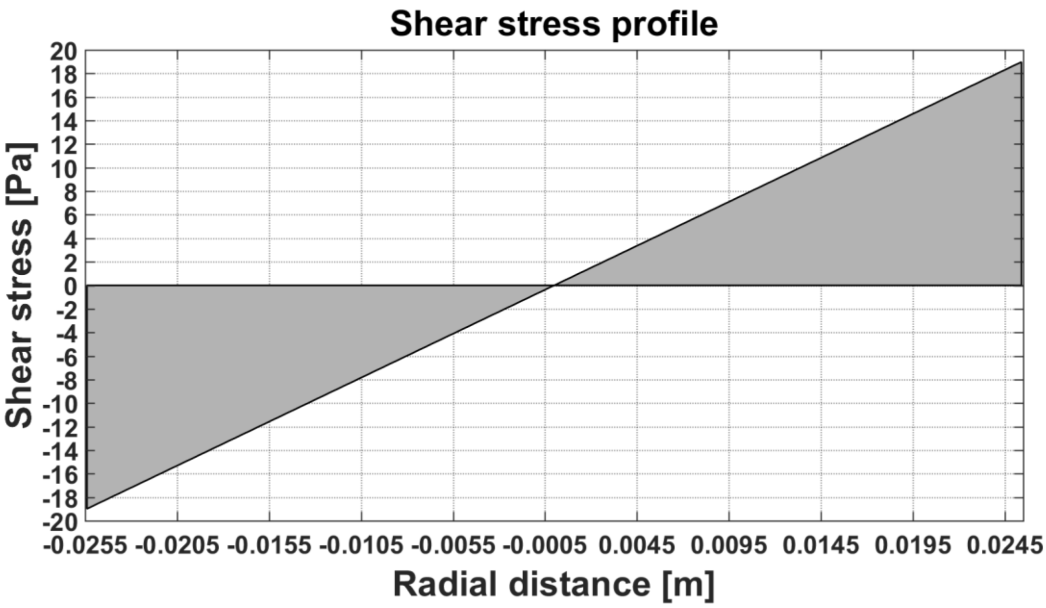



*Figure 25: Shear stress profile for 3 m/s flow.*

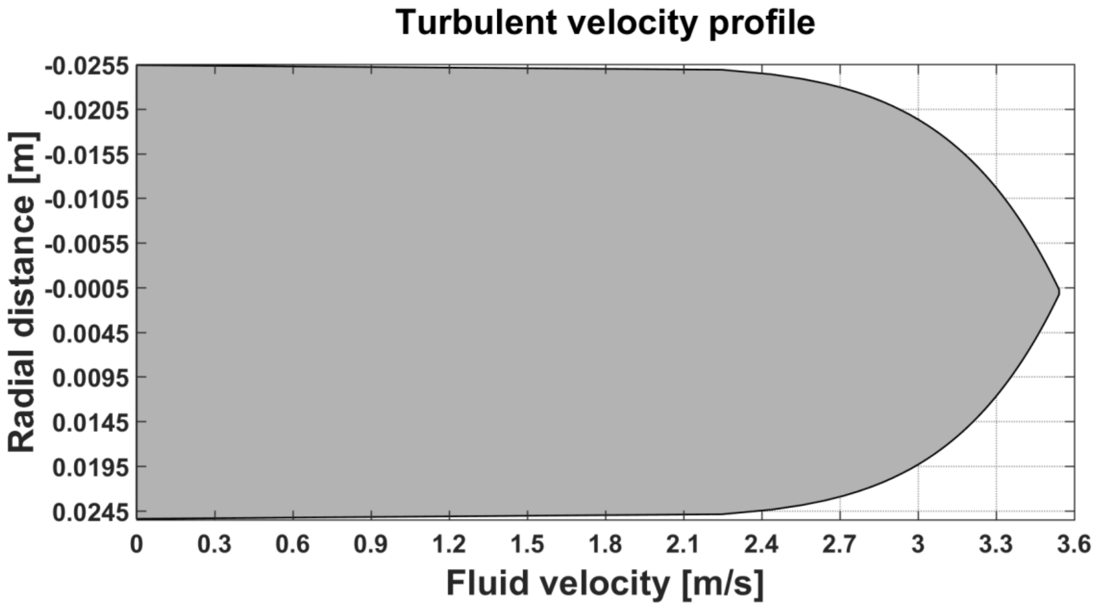



*Figure 26: Velocity profile for 3 m/s flow.*

# CHAPTER 5

## 5 CFD Modelling of flow in pipe:

After analytical modeling, this chapter will elaborate the result of CFD analysis and how much results differ with analytically modeling results. In the analytical modelling chapter, shear stress calculated for two cases, which were straight pipe and other one in bending pipe. Dimensions of the pipe already mentioned in previous chapter. In the CFD modelling, 4 cases are going to analysis.

Table 9 (Operating conditions)

| | |
|---|---|
| **Flow Velocity** | 1.49 m/s |
| **Pipe diameter** | 0.0508 m |
| **Turbulence model** | Reynolds stress |
| **Mesh Type Used** | Hexahedral |

### 5.1 Case 1 for straight pipe:

The operating conditions and dimensions in the analytical modelling are assigned in back-step geometry model of fluid flow CFD model. After inserting all properties, the mesh which is another step in simulation of the fluid flow, is made. After creating mesh the fluid flow simulated and in results, such parameters as velocity profile, pressure gradients, shear stress plots etc. will the effects of fluid flow on the pipe wall.

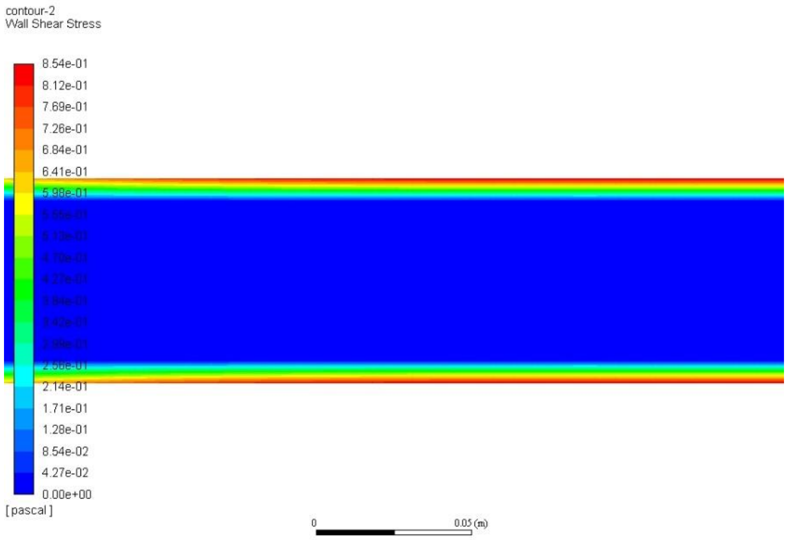



*Figure 27: Wall shear stress profile for case1*

Figure 27 shows the distribution of wall shear stress for the case 1 for 2 m/s flow. The maximum results of 8.54 Pa agrees reasonably well with predicted analytical result of 9.19 Pa. The difference can be explained that the estimation of wall stress in CFD simulation is performed at some distance close to the wall, but not directly at the wall as velocity gradient, *du/dy* at the wall is infinity, due to the way the equations for turbulent flow is structured (it is also evidenced by the fact that the highest shear stress in Figure 18 protracts to some "depth" in the pipe, with respect to the wall, while shear stress distribution in analytical model shows slight drop in stress values when moving away from the wall). Moreover, Figure 28 shows comparison of wall shear stresses in CFD simulations for case 1 at velocities of 0.5, 1, 2 and 3 m/s.

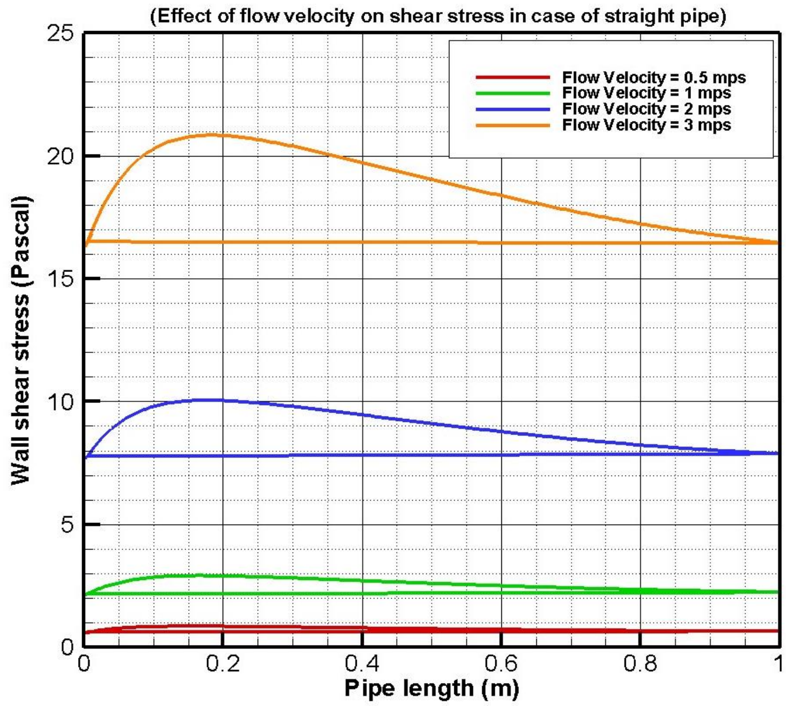



*Figure 28: shear stresses profile of case 1*

The results from Figure 28 agree well with analytical calculation performed in previous chapter. Note that the highest values of wall shear stress occur in all the flows at around 0.175 meters in the pipe, clearly within the entrance region in the pipe. This result is expected from general flow theory, as highest values of wall shear stress are expected in entrance region due to "formation" of flow regime, i.e. flow is not steady, however, analytical models normally do not take into account the stress value in entrance region, due to complexity of modelling non-steady flows and furthermore non-steady turbulent flows. Observe that even past entrance region, with entrance length for turbulent flows typically characterized as being equal to ten pipe diameters, there is continuous shear stress drop throughout the remainder of the pipe. The explanation for it is that friction in the pipe, even in hydrodynamically smooth as made from PVC, causes the pressure drop, which leads to decrease in instantaneous velocity and shear stress values, which, as shown in previous chapter, depend on the value of pressure and/or pressure drop. The main conclusion of this comparison is that CFD setup has been done correctly, the values are well in agreement with established theory and any question, pertaining to flow physics in CFD simulations can be readily answered using the established flow theory. The main advantage of this is that it is possible to rely on CFD simulations in order to solve flows in more complicated cases, which will be too cumbersome to solve analytically, for example, the case 2 which is a turbulent flow in a bended pipe, with distance from entrance to the bend being less than ten pipe diameters, thus resulting in undeveloped flow.

## 5.2 Case 2: Single bend pipe.

In this case a comparison will be made for flow behaviour in pipe with single bend for two situations: one with flow in the pipe being undeveloped with distance from entrance to the bend being 0.5 meters, which is slightly less than estimated entrance length, and one with this distance being equal to 1 meter, which is clearly larger than entrance length. The purpose of this is to study how the bend and the length of the pipe effect flow behaviour. Figure 29 shows the velocity profile for undeveloped flow.

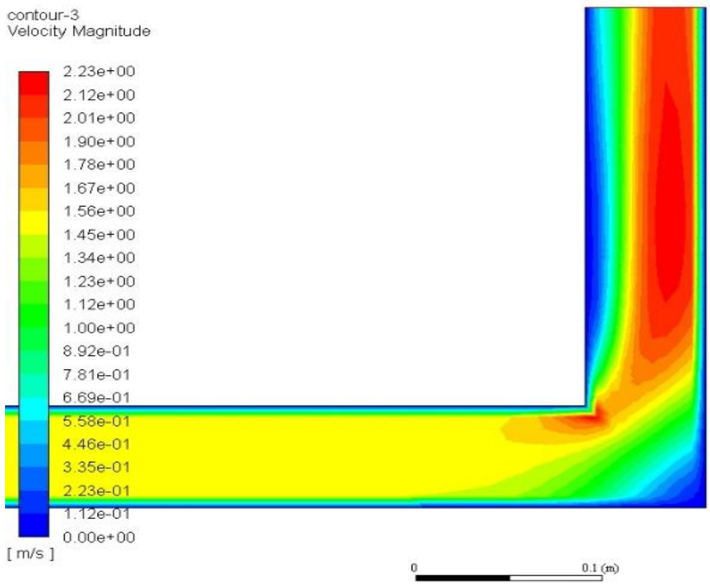



*Figure 29: velocity profile undeveloped flow of one bend pipe case 2*

As it can be seen from Figure 20, the highest velocity occurs just after passing the bend. The result can be explained by applying Bernoulli principle and keeping in mind that the bend will cause significant pressure loss, thus requiring the velocity increase past the bend in order to satisfy said principle. In addition, observe that "left-hand" side of the pipe past bend has a significant region of zero velocity. From analytical modelling, presented in Chapter 4 once can expect that the velocity profile past the bend will be equal for both sides of the pipe, with respect to centreline. This asymmetry is readily explainable by "no-slip" condition, as fluid moving past sharp obstacle, is forced to stick to said obstacle at its surface, thus excluding any sort of rapid recovery of velocity, as the rest of the fluid will be forced to move past the obstacle at some distance and fluid in adjacent sections to the obstacle cannot recover velocity fast enough after the bend as the velocity increase is bounded by energy and momentum transfer via eddy motion, meaning it will take certain time (and distance) for eddies to "equalize" the flow. Furthermore, observe the region with zero velocity adjacent to the angle of the bend. Again it is arising from no-slip condition at both sides of the angle of stationary pipe thus requiring the flow to be "locked" in circular motion, in order to satisfy both conservation of momentum and no-slip condition (this is also known as Coanda effect). Finally, Figure 30 gives the wall shear distribution for undeveloped case.

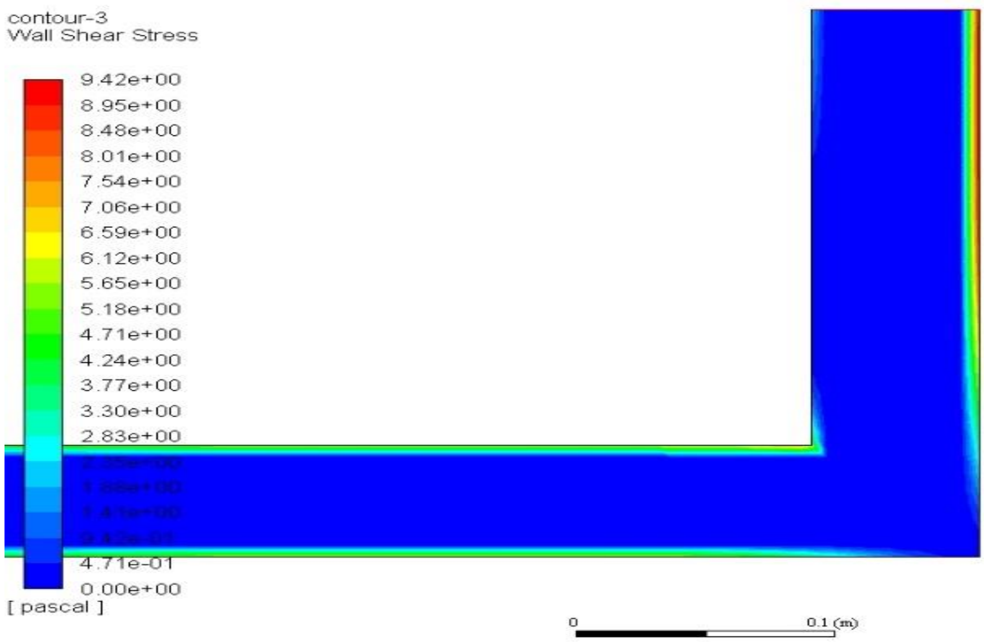



*Figure 30: Wall shear stresses of undeveloped flow for one bend pipe*

As expected the highest wall shear stress occurs at the point with maximum velocity, i.e., the flow in region, adjacent to the "right-hand" side of the pipe after the bend. Corollary, the zero values in wall shear stress occur at the point where fluid velocity is equal to zero, which can be easily seen when comparing Figures 29 and 30. Figure 31 shows the velocity profile distribution for the single bend pipe with developed flow.

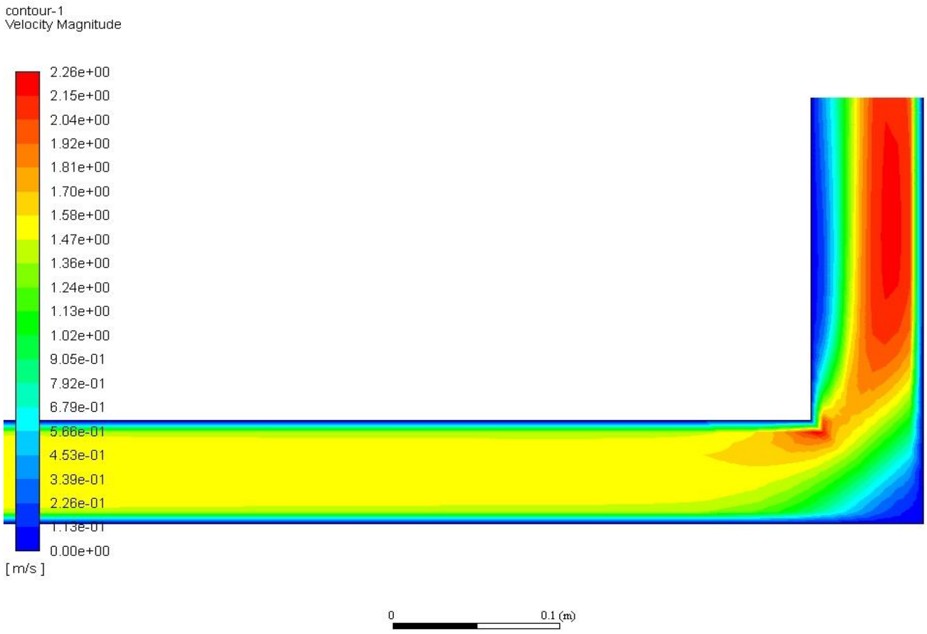



*Figure 31: velocity profile for developed flow*

From Figure 31 it can be seen that flow behavior for developed flow is almost the same as in case foe undeveloped flow, thus same considerations and conclusions apply for both cases. When comparing Figures 29 and 31 one difference is slight change of maximum velocity of the flow. For developed flow the maximum velocity is 2.26 m/s, while for undeveloped one the maximum velocity is 2.23 m/s, which suggests that in terms of wall shear stresses the undeveloped flow should have slightly higher values than developed flow. The wall shear stress for developed flow with single bend is given in Figure 32.

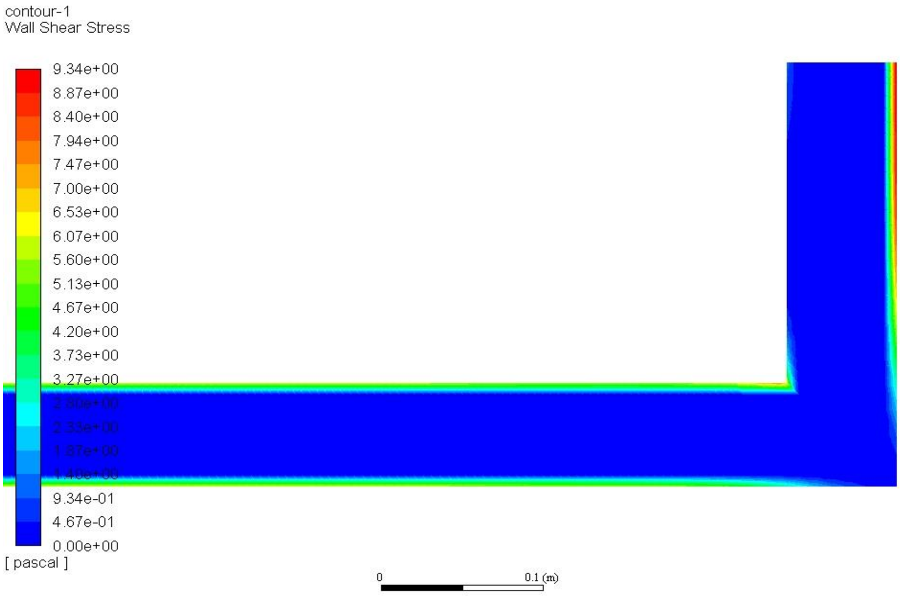



*Figure 32: shear stresses on developed flow*

Figure 32 shows that maximum shearing stress for developed flow is equal to 9.34 Pa, while for undeveloped flow it is equal to 9.42 Pa, as it can be seen from Figure 21. This reinforces two notions – that the wall shear stress is dependent on or a function of velocity, and that decreases in velocity from maximum is caused by higher pressure drop, and by extension – higher shear stresses. Finally, Figure 33 shows shear stress profiles for both undeveloped and developed flow for case 2. As it can be seen, the differences for two cases are minimal and curve behaviour is the same, meaning underling physics in both flows is the same.

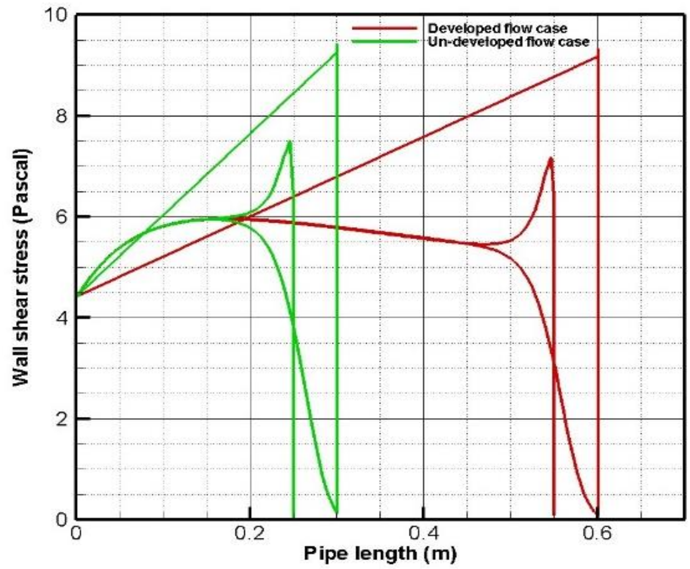



*Figure 33: shear stresses comparison between developed and undeveloped flow case*

## 5.3 Case 3: Corner shape pipe and round shape pipe:

Case 3 shows the impact of geometry on the values of shear stress. For this case the pipe with developed flow is chosen, however, in one of the scenarios the bend is made rounded, while in another the bend remains sharp as in original geometry. Figures 34 and 35 show velocity and shear stress profiles, respectively, for case with sharp bend.

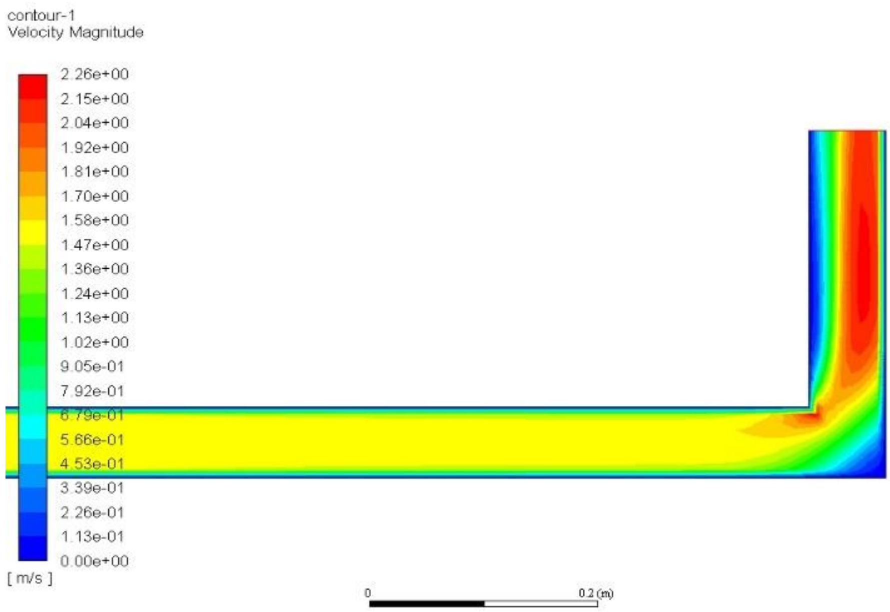



*Figure 34: velocity flow of corner type one bend pipe*

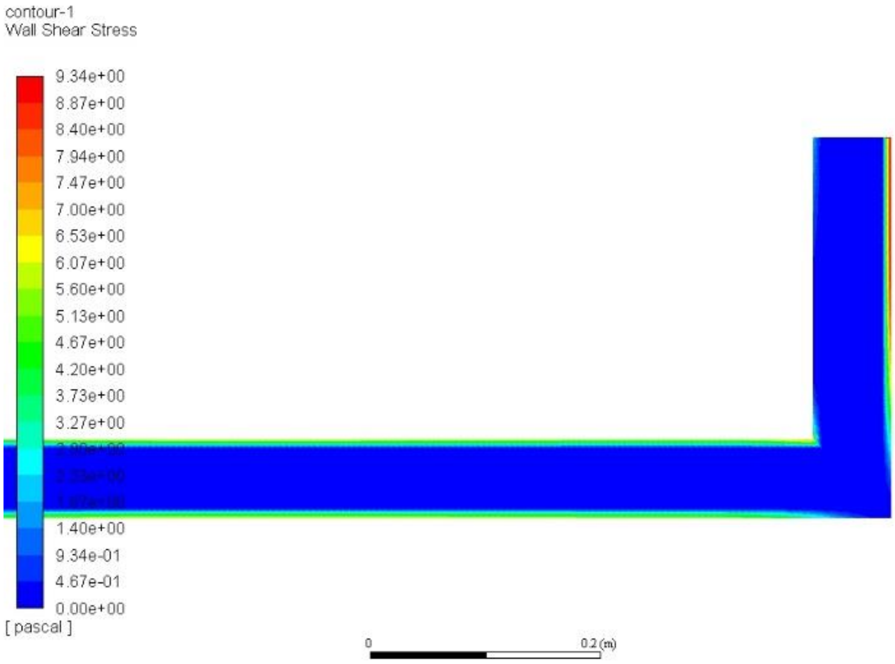



*Figure 35: shearing stress profile of one bend corner pipe.*

From Figures 34 and 35 one can see that flow behaviour is identical to that in case of developed flow form case 2, with maximum values of velocity and shearing stress being identical. Figures 36 and 37 show the velocity and shear stress profiles, respectively, for the case of rounded bend.

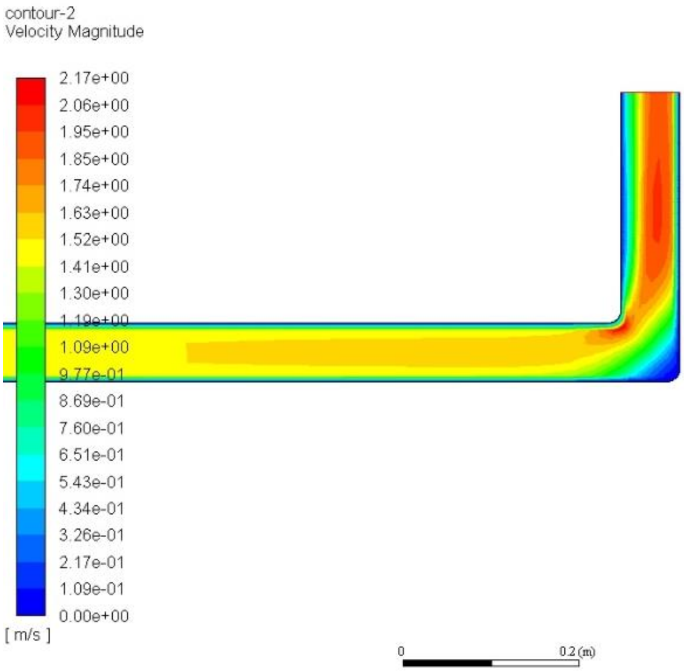



*Figure 36: velocity profile of one bend rounded pipe*

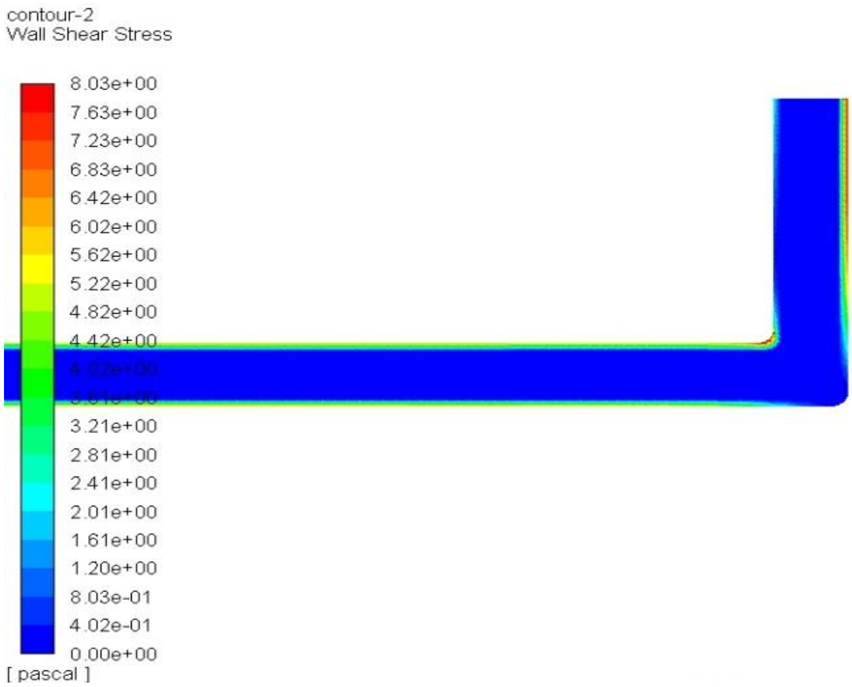



*Figure 37: shear stress of one bend rounded pipe*

Observe that for rounded pipe both velocity and shear stress values are lower than the respective values when comparing with the sharp bend. These values are 2.18 m/s and 8.03 Pa, respectively, and for sharp edge they are equal to 2.26 m/s and 9.34 Pa. This difference is dependent purely on geometry on the bend. As the round edge represents easier to pass obstacle, which means the flow does not have to deviate as much from initial trajectory in order to pass the obstacle, as it can be seen when comparing Figures 34 and 36, which in turn, means that eddy motion can transport mass and energy much efficiently, thus restoring centreline "symmetry" considerably faster. This increased transfer rate causes the observed drop in velocity and by extension – shear stress values. Again, this supports the idea that maximum shear stress is a function of both velocity and pressure drop in the CFD simulations. Moreover, Figure 38 gives the comparison of shear stress profiles for both of these cases. The impact of rounded bend on the shear stress profile is clearly seen by overall drop in shear stress values past the bend.

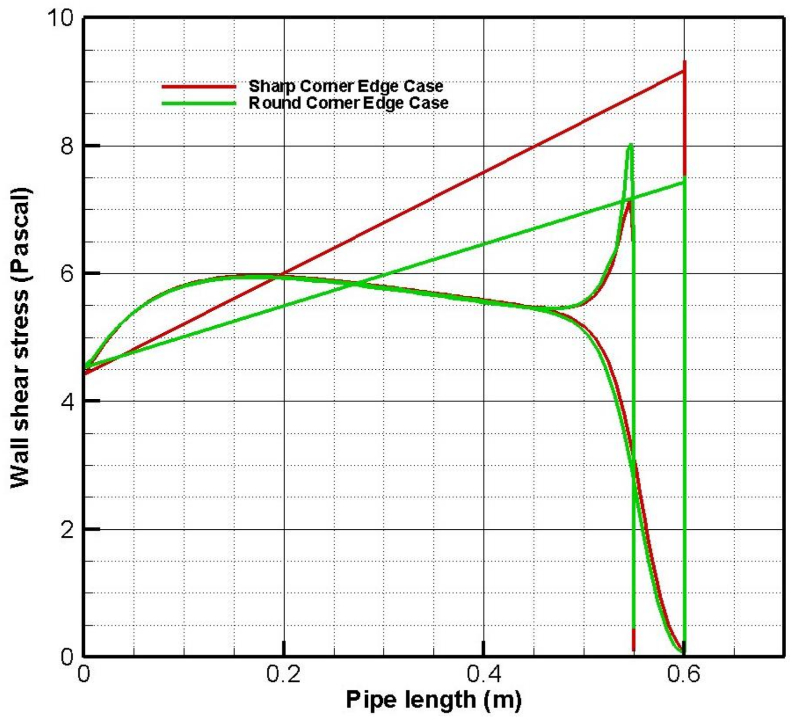



*Figure 38: shear stress comparison between corner and rounded pipe flow*

### 5.3.1 Case 4: for U-shape pipe:

The last case is the pipe with U-shaped bend. The total pressure and shear stress profiles are given in Figure 39.

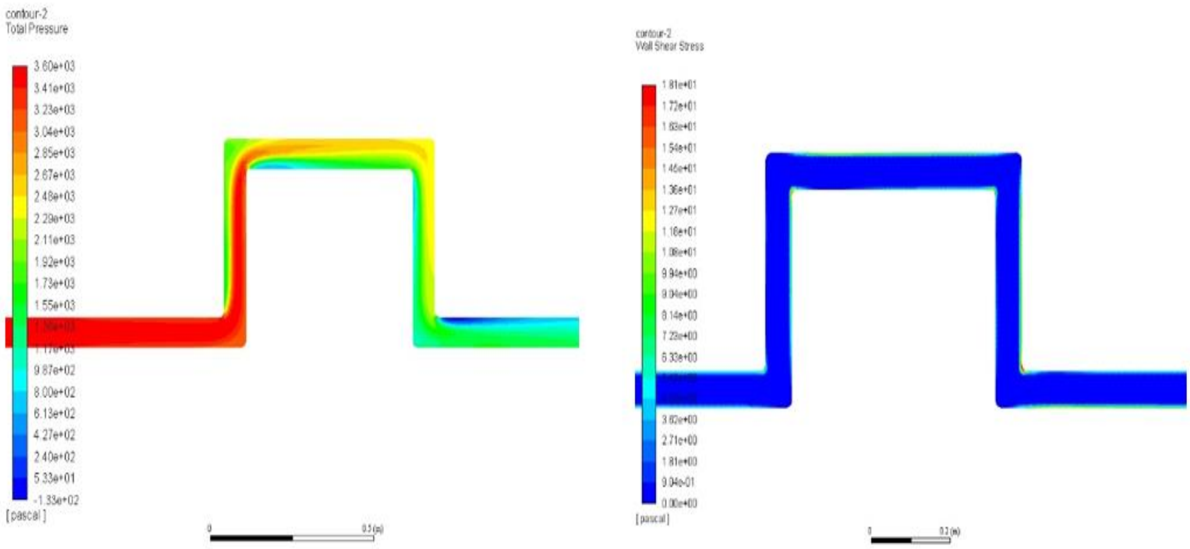



*Figure 39: pressure profile and shear stress profile of u-shape pipe*

From the pressure profile one can see impact of the bends on total pressure in the system. At the first bane, the flow with total pressure of an order of 3.5 kPa, gets "split" into two distinct parts as it is forced to move round obstacle. The part with higher total pressure is the part of the flow which will have higher flow velocities, based on principles, explained in cases 2 and 3. After second bend, the flow is "split" further again, decreasing the pressure even further. Finally, after last bend the same process happens again, decreasing the pressure to the minimum and at some points negative pressure is developed, meaning that the flow and the pipe will be subjected to suction forces in these parts. These large pressure drops are corresponding to the locations of maximum shear stresses in the flow, as seen in the shear stress profile plot in Figure 30. The shearing stress profile for pipe with U-shaped bend is given in Figure 40.

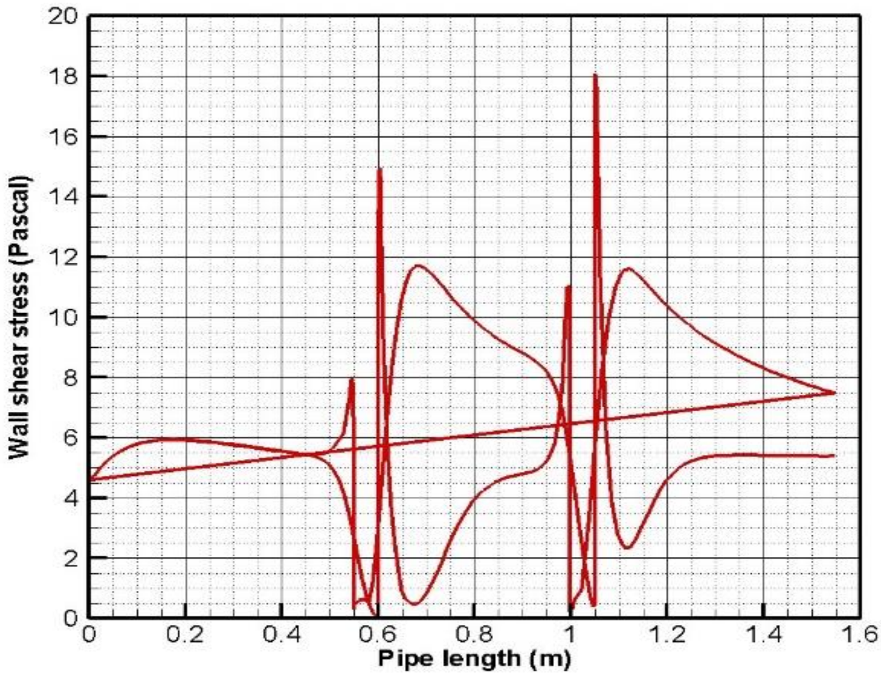



*Figure 40: shearing stress plot for u-shape pipe*

## 5.4 Full scale model

The full scale model of proposed test rig has been design with 2 U-bends and a large, long section, ending in sharp downward bend. The test rig is a closed-loop section with water tank and pipe in it, however, for CFD simulations only piping system was tested thus giving the impression of an open-loop section. The pipe diameter is 2 inches, and the volume flow rate is 50 and 100 gal/min, thus giving average flow velocity of 1.49 and 2.99 m/s [25]. Since physics in both cases will be the same only the results for 1.49 m/s case is given in graphical form in Figures 32 and 33, and shear stress profile comparison is given in Figure 34. Since the piping system consist of U-bends and a simple bend of the end, the detailed description of physical processes in it won't be given, as for every region of interest one can refer to the descriptions and explanations of cases 1–4, given previously in this chapter, as the physical processes of the flow in the full model is a combination of all preceding cases. The scope of interest in this simulation is estimation of region with highest shear stresses in order to install the supports on a pipe to damp vibrations in most efficient way. The preliminary information about possible support locations can be incurred from plots of dynamic pressure and velocity profiles, as was discussed previously, the wall shear stress is a function of pressure and velocity change. Figure 41 gives the plots of dynamic pressure and velocity profiles for the full scale model.

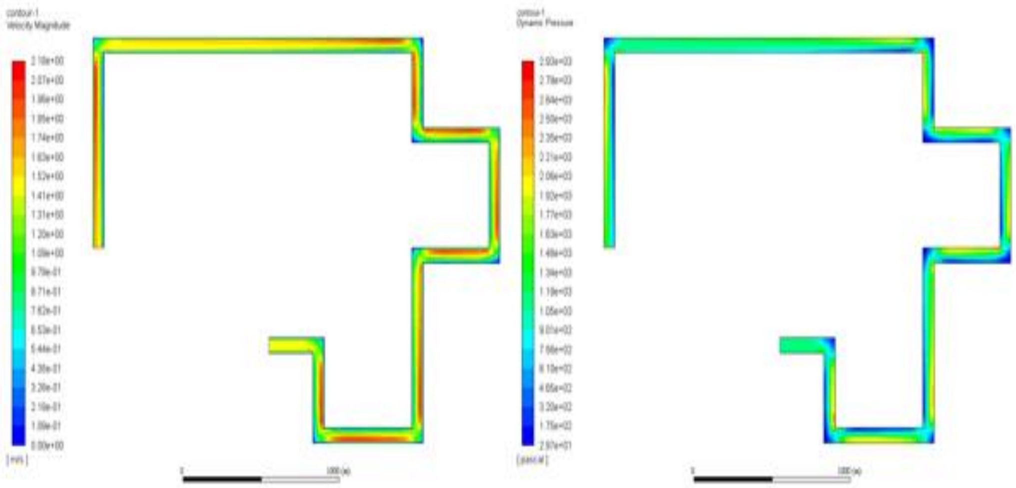


*Figure 41: Dynamic pressure (left) and velocity (right) profiles of full scale model.*

From Figure 32 one can see that maximum velocities and dynamic pressures in a system occur just after the 90° degree bends, irrespective if this is U-shaped bend or a simple bend. Therefore, for maximum efficiency, it is advantageous to put supports somewhere in-between the bends, ideally, in each section. Figure 42 gives the wall shear stress distribution for full scale model.

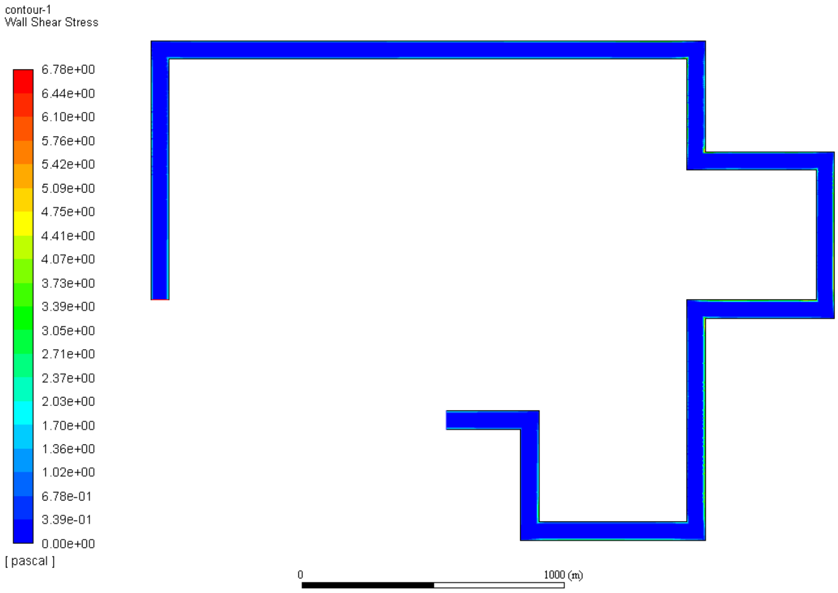



*Figure 42: Wall shear stress distribution for full scale model*

From Figure 42 it is easy to see that maximum shear stresses occur at the angles of bends, however placing supports here might be difficult, mainly due to geometric constraints. In addition, Figure 42 shows elevated levels of wall shear stress, somewhat close to maximum values in a simulation, in a straight segments of a pipe in-between the bends, which coincides with regions of maximum velocities and dynamic pressures, observed in Figure 41, therefore supporting the idea of placing supports at those locations, in-between the bends. The more detailed location of supports can be estimated after performing structural simulations of the system segments and/or full scale model. Finally, Figure 43 gives comparison between shear stress values for full scale model for cases of 1.49 m/s and 2.99 m/s velocities, corresponding to volume flow rate of 50 and 100 gal/min, respectively.

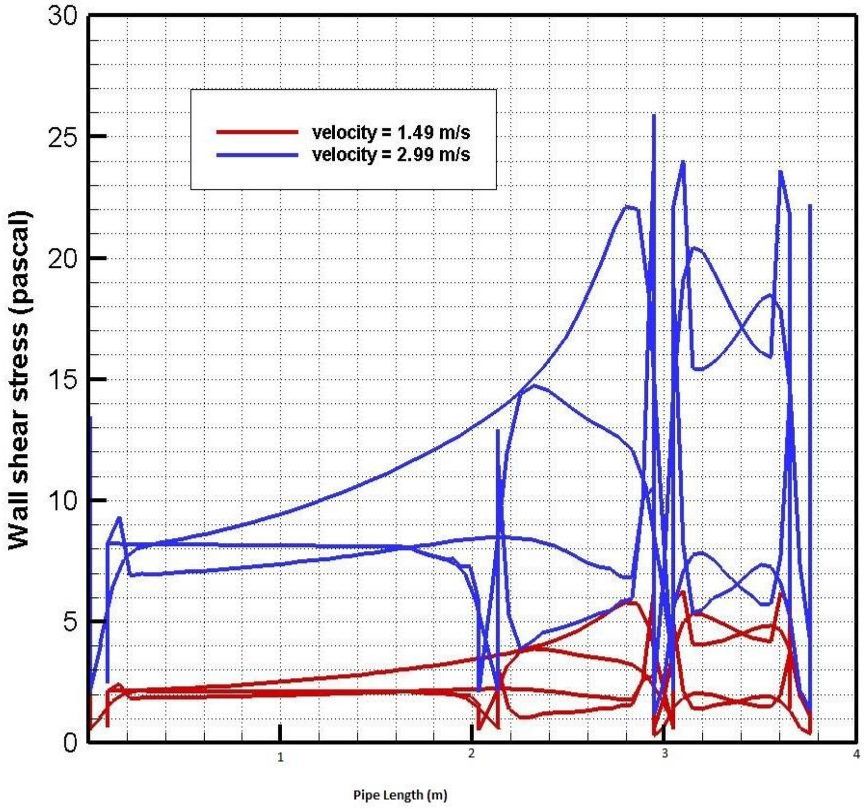



*Figure 43: Wall shear stress distribution for full scale model*

The main observation from Figure 34 is considerably higher values of wall shear stress at 2.99 m/s flow. This can be explained, by the fact that shear stress grows as approximately a square of velocity, as it can be inferred from analytical derivations in previous chapter. Indeed, the maximum value of wall shear stress for 1.49 m/s flow is 6.78 Pa, while for 2.99 m/s flow it is 26.0 Pa. The reason, why shear stress does not grow exactly as square of velocity is the decrease in friction factor at higher values of Reynolds number, on which the wall shear stress is linearly dependent, in addition to changes in pressure drop values, and their corresponding ratios, when comparing cases of 1.49 m/s and 2.99 m/s flows.

# CHAPTER 6

## 6 Structural Analysis

### 6.1 Case 1:

In this case we have a straight pipe having a 2 inch diameter. It has a length of 1m. We have made a 2D analysis using Ansys APDL. The material, which we have choose for this analysis, is Polyvinyl Chloride (PVC). By applying its material properties on the pipe we have simulate the results related to maximum shear stress and the displacement along y component. The results are being obtained by applying constraints on different positions on the pipe. Some of the values we have applied for the pipe are as follows

- Young's modulus for PVC=2.14 GPa
- Poisson's ratio = 0.4
- Density = 1580 kg/$m^3$
- Pressure = 63.27 Pa

#### 6.1.1 At node 0.175:

By applying a fixed constraint at the point, that is 0.175m out of 1m pipe. Figure 44 shows the meshing of the pipe the equivalent size of the mesh is 0.025m.The fixed constraints are applied as shown in figure 44 is at 0.175m and the pressure applied is along the x component.

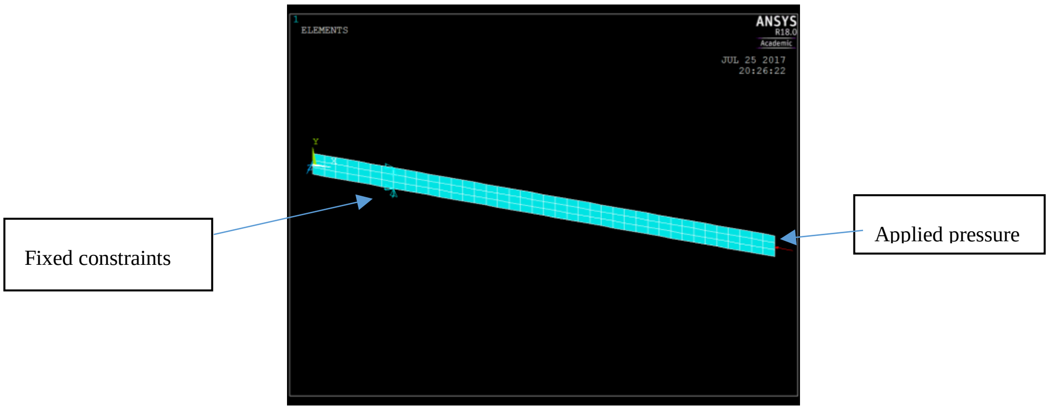



*Figure 44: Loading conditions at point 0.1*

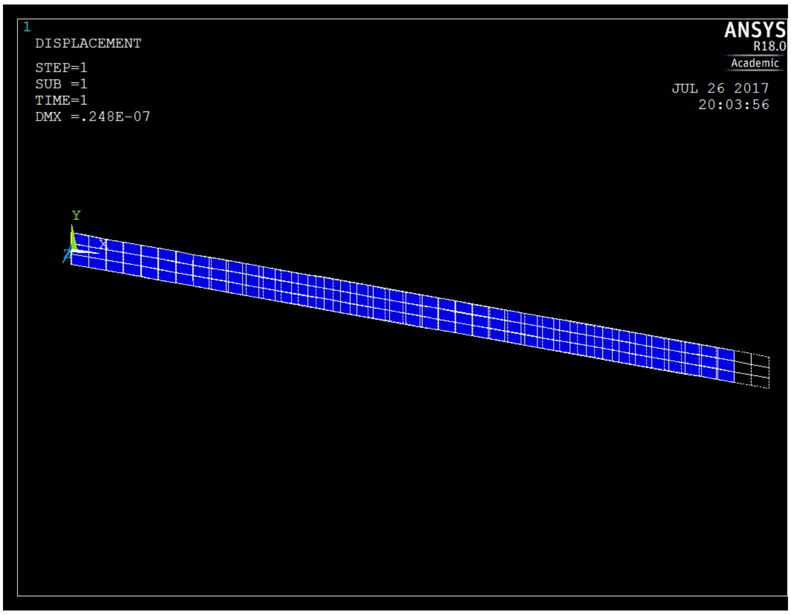



*Figure 45: Deformed and undeformed shape*

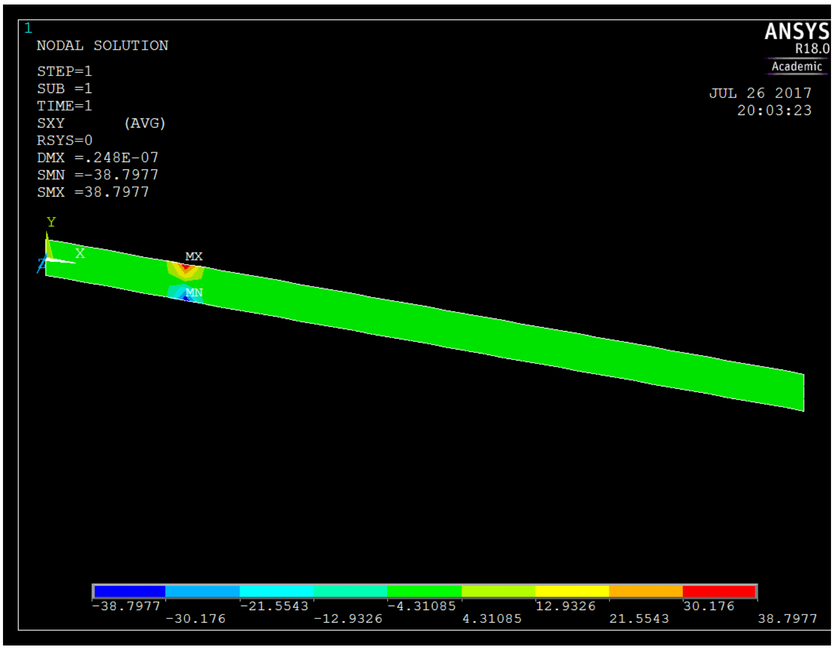



*Figure 46: Shear stress along xy-axis*

Figure 46 shows the results of maximum and minimum shear stress along the xy component if we apply the pressure of 63.27 Pa.

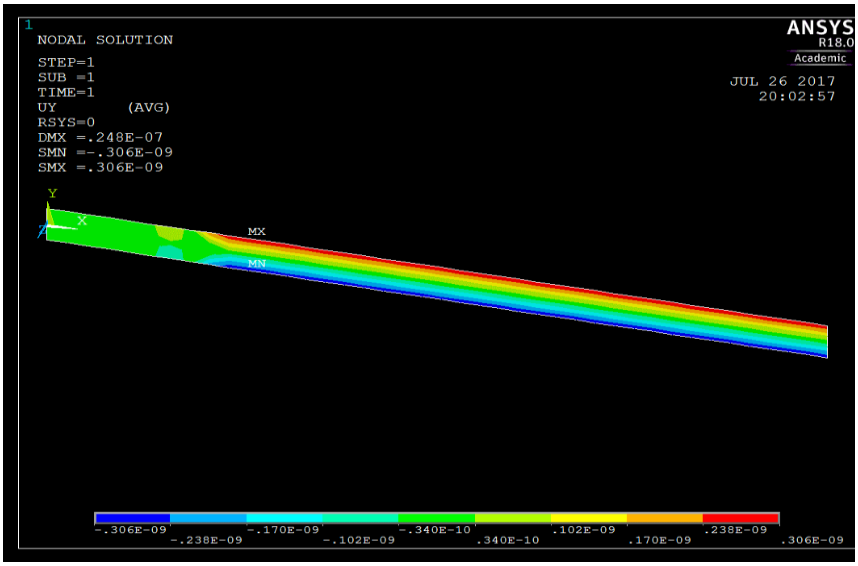



*Figure 47: Y displacement using Nodal analysis*

The results obtained from figure 47 demonstrates that there is negligible displacement along y component that is there is no deformation along the pipe if we apply pressure along the x component. The max deformation is 0.248E-07m that is considered approximately equal to 0.

### 6.1.2 At Node 0.5:

To make some comparisons and to check the condition of supports we applied constraints at different point also. As shown in figure 48 we have applied the fixed support at the middle of the pipe and the same pressure along the x component. Same properties that is the material of PVC is applied for this pipe.

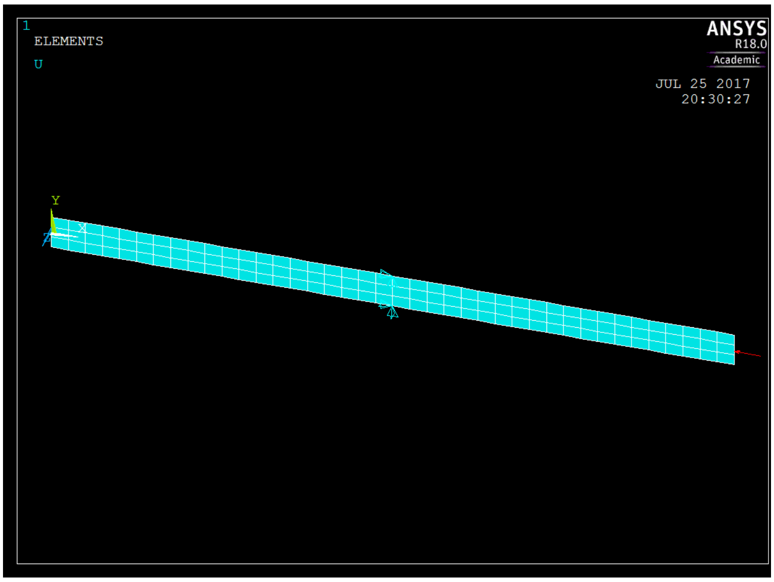



*Figure 48: Loads applied on point 0.5*

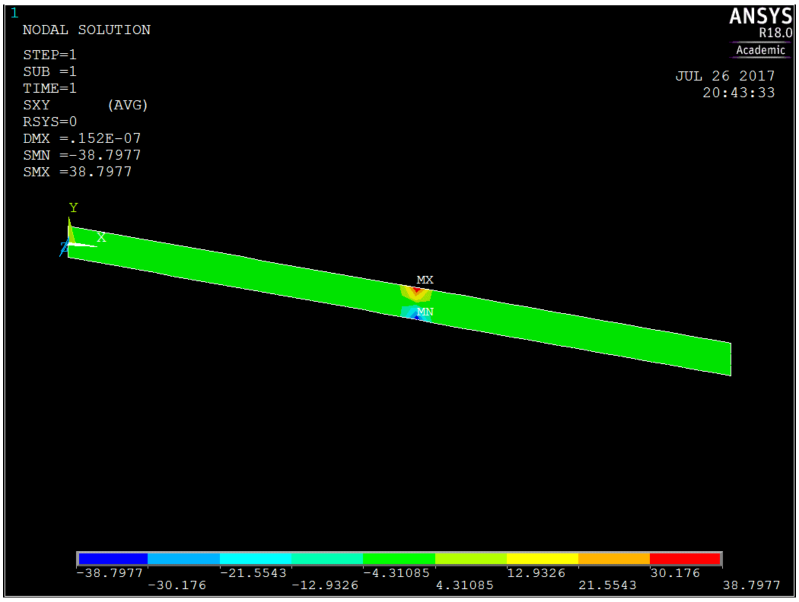



*Figure 49: xy shear stress*

Figure 49 shows the results of maximum shear stress along the pipe. By applying pressure along the x component we have the values of maximum and minimum shear stress that is similar to that when we have the fixed constraint at point 0.175m.Figure 50 shows the y-displacement component of the pipe and it is also similar to the case, as we have discussed above.

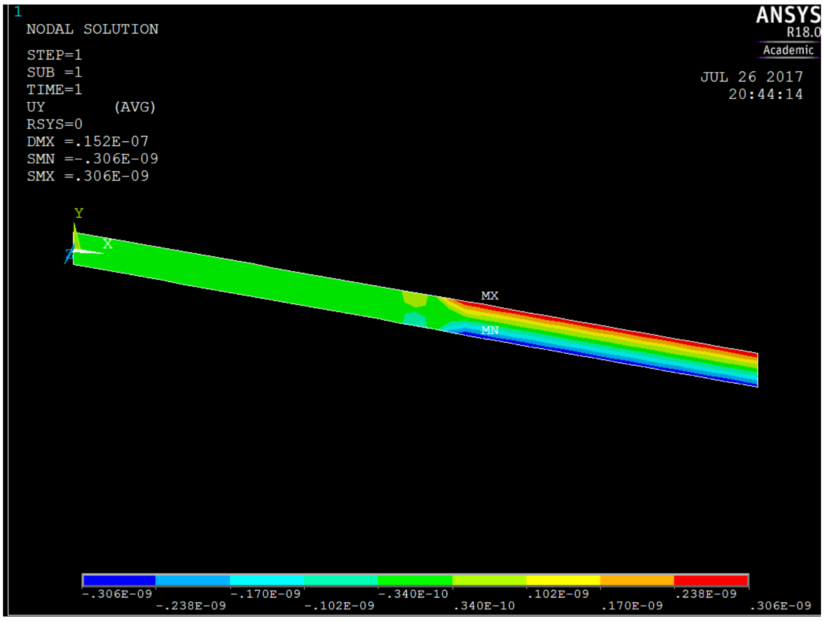



*Figure 50: Y displacement Nodal analysis*

Thus for this case 1 we have concluded that the effect of constraints whether they are attached at the middle point or at the start does not have much of impact on the bending of pipe.

## 6.2 Case 2:

For case 2 we have a same size pipe of 2 inch diameter. In this case we have a sharp bend on the pipe. Applied pressure is higher as compared to case 1.We need to check the effect of constraints on different positions. The analysis is made in Ansys Apdl. Some of the values we have applied for the pipe are as follows

- Young’s modulus for PVC=2.14 GPa
- Poisson’s ratio = 0.4
- Density = 1580 kg/m$^3$
- Pressure = 1500 Pa

As shown in figure 51 mesh equivalent size for this pipe is 0.0254m.We have applied load on two different points as show in figure 52.The pressure is along the xy component.

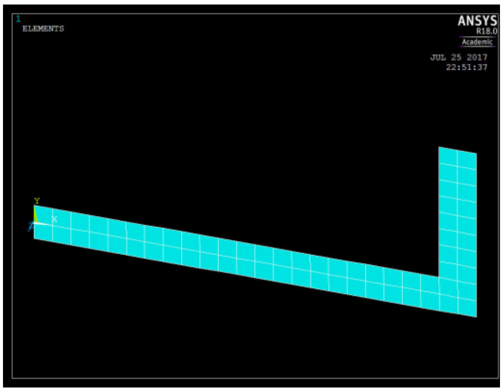



*Figure 51: Mesh for case 2*

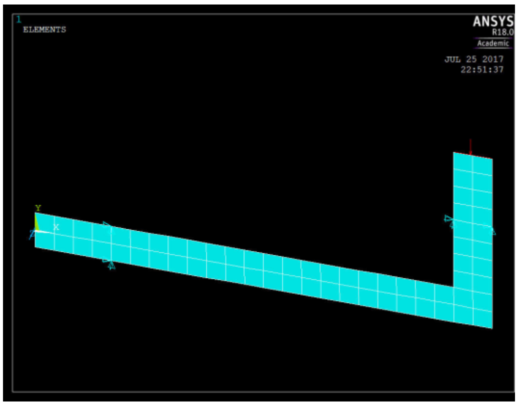



*Figure 52: Loading fixed constraints and Pressure*

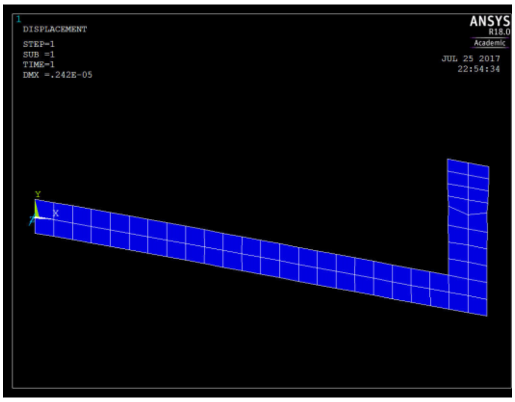



*Figure 53: Deformed shape*

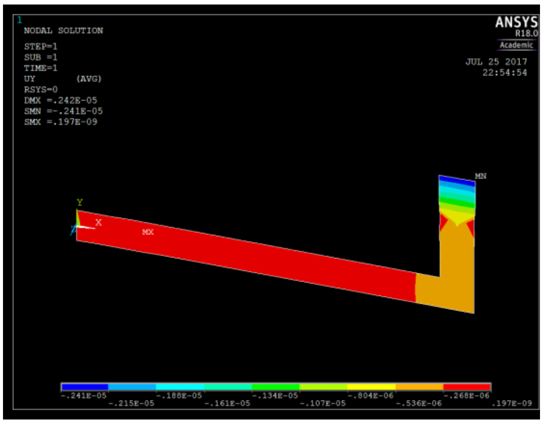



*Figure 54: Y Component displacement*

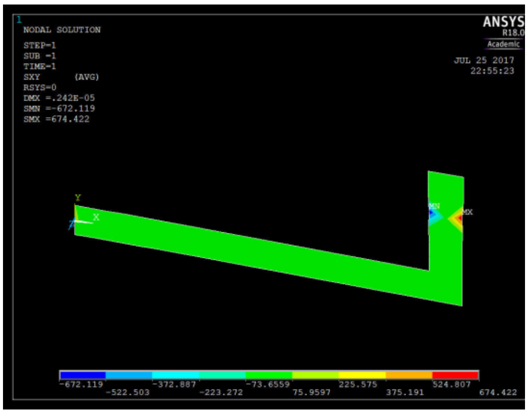



*Figure 55: xy shear stress*

The results shows that the effect of fixed constraints on the pipe allows the pipe to withstand pressure of 1500 Pascals. Maximum deflection is negligible in this case that is 0.242E-5 ~0.00000242m.Figure 55 shows the maximum and minimum shear stress along the wall of the pipe.

## 6.3 Case 3:

For case 3 we have a same size pipe of 2 inch diameter. In this case we have a curved bend on the pipe. Applied pressure is same as compared to case 2.We need to check the effect of constraints on different positions. The analysis is made in Ansys Apdl. Some of the values we have applied for the pipe are as follows

- Young's modulus for PVC=2.14 GPa
- Poisson's ratio = 0.4
- Density = 1580 kg/m$^3$

- Pressure = 1500 Pa

As shown in figure 56 mesh equivalent size for this pipe is 0.0254m.We have applied load on two different points as show in figure 56.The pressure is along the xy component.

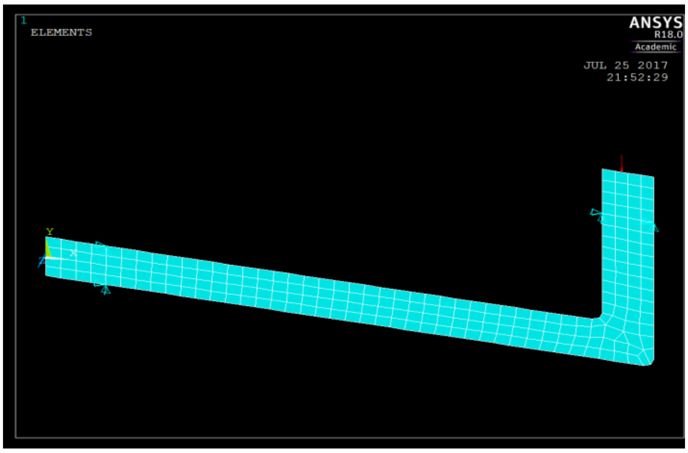



*Figure 56: Applying loads and pressure*

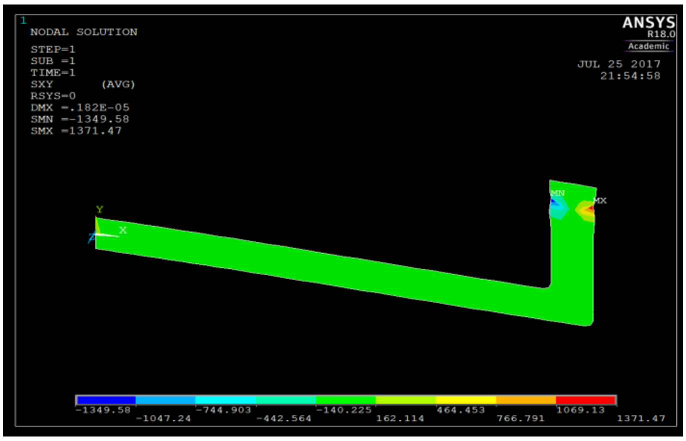



*Figure 57: xy shear stress*

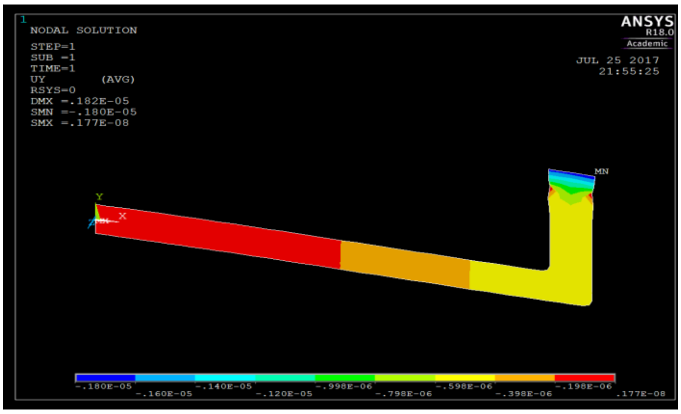



*Figure 58: y Component displacement*

For this case the curved bend gives higher shear stress along the pipe .Deflection is still negligible in this case that is similar to case-2.

## 6.4 Final Model:

After going through structural analysis cases as elaborated above, designed a main model with suitable support for the pipeline flexibility bench. In figure number 59, the dimensions can be seen as. With the help of previous chapter of CFD analysis, different support are applied. The main advantages of structural and CFD analysis is that, keeping result of those analysis one can modify theses support according to change in flow rate and pressure.

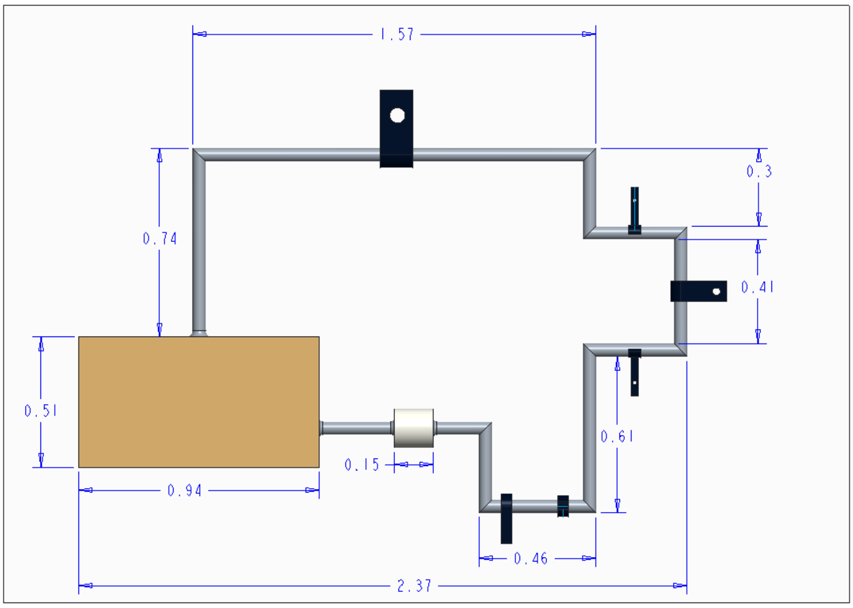



*Figure 59: Dimensions of main model*

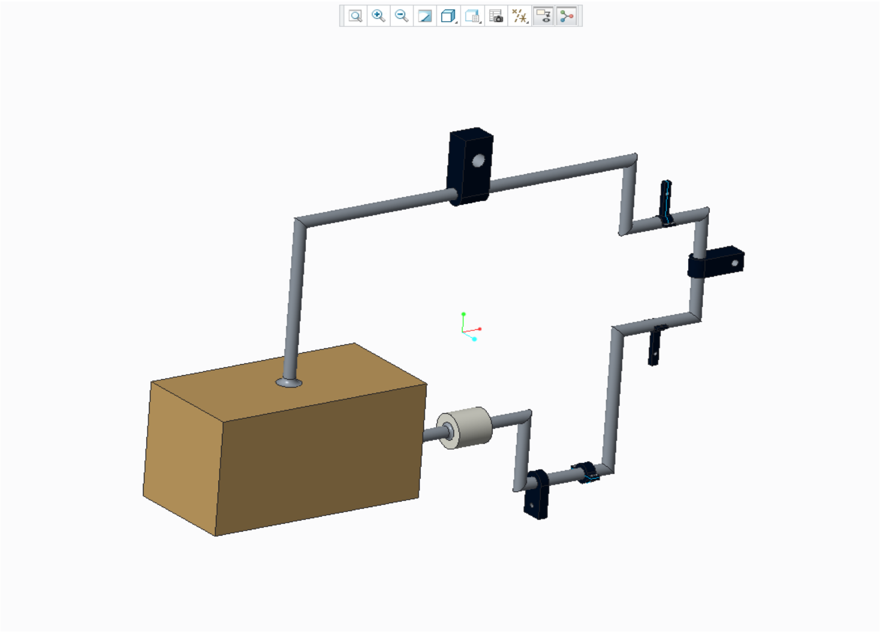


*Figure 60: the CAD main model for pipeline flexibility bench*

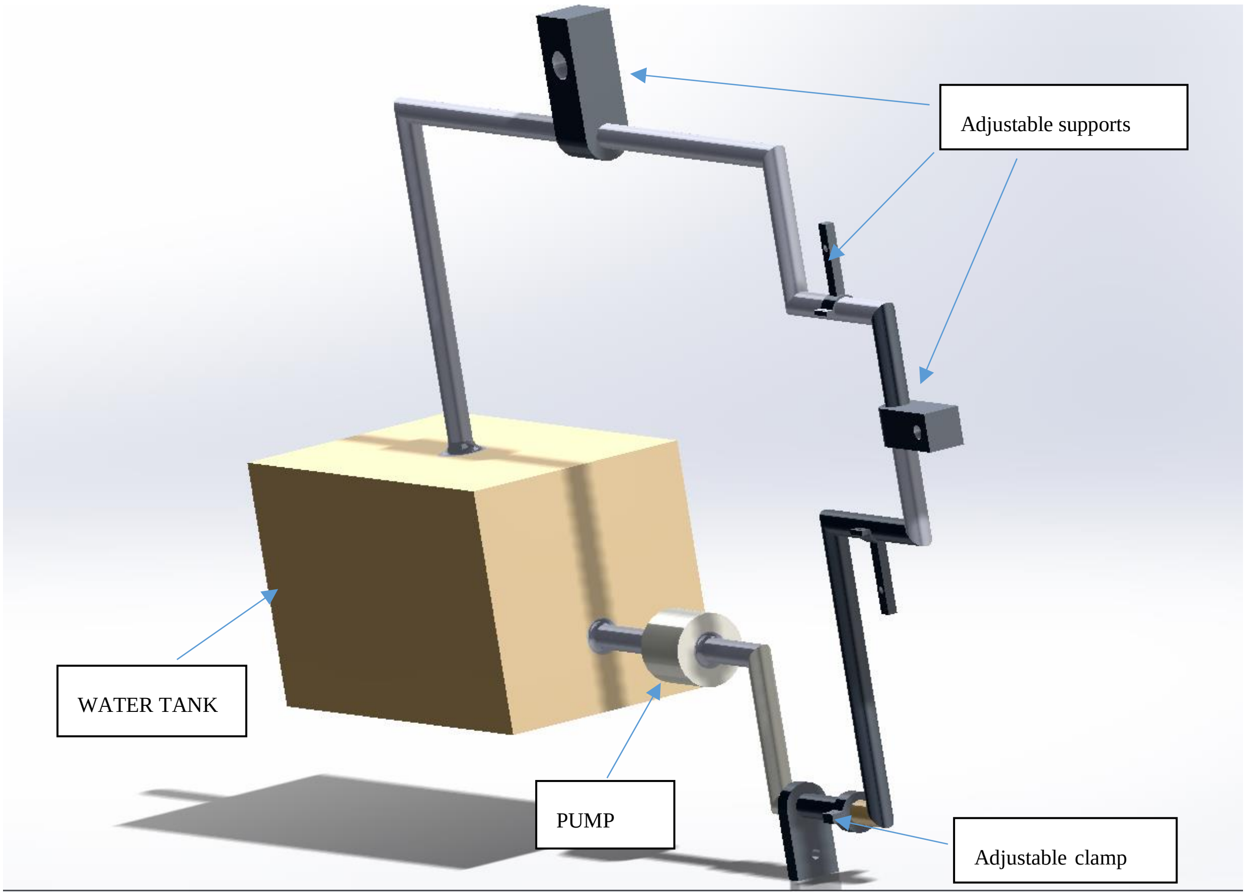



*Figure 61: 3D view of final design with adjustable supports*

Figure 59 shows the length of the overall pipe at different parts it is written over there. This length is in meters. So, it is possible to build this design practically in the lab and different analysis can be performed. In the figure 61, 3D model is shown of final design, water tank is shown, and the capacity of this tank is up to 100 gallon as flow velocity is taken 2.99 m /s in the CFD final design in figure number 42. The pump is also shown in the figure 61. The main task of the pump to circulate the water in the loop. According to CFD and Structural analysis results, the adjustable supports are placed in the final design and main objective of the proposal is achieved. The supports are adjustable, the student can modify these supports on different condition of fluid flow through the pipe.

## 6.5 Effect on temperature on flexibility:

The effect on temperature on flexibility of the pipe can be explained as follows. For PVC pipe the linear thermal expansion coefficient is equal to 54–110 m/(m·K), giving the average value of 82 m/(m·K), while for water this coefficient is equal to 51 m/(m·K). The coefficient is linear and not volumetric, however it can be used as an estimate of volumetric expansion, considering that volume is a function of cube of a linear dimension, such as length.

Consider, for example, a change of temperature from 15 to 80 °C. Since the linear expansion coefficient for water is lower than that of PVC, being approximately 0.62 of the value for PVC, the relative change in expansion between water and PVC will be 0.62 times the change in temperature (e.g. from 15 to 80 °C). Thus, PVC will expand considerably more than water. Since water will be forced to fill entire area of a pipe, and velocity is being dependent on volume flow rate, the lesser change of volume for water when compared to PVC will cause velocity to decrease, as velocity $V = q/A$, where $q$ is volume flow rate of water and $A$ is the area of the pipe will be "skewed" in favour of PVC, since it expands by large amount. Thus, the drop of velocity will occur. The drop of velocity will lead to drop in pressure and as a result – drop in shear stress values. Moreover, since the viscosity of water plays the defining role in values of shear stress, and viscosity decreases as temperature increases, a further drop in wall shear stress values will occur. Moreover, expanded PVC pipe will exert a normal stress on the support, assuming support was fitted for 15 °C flow temperature assumption, thus giving an interference fit. The end result is that the support will be more efficient in damping vibrations, as this sort of stress and interference fit will cause system to become overdamped, assuming a critical damping ratio was maintained for nominal conditions at 15 °C.

The only pronounced effect of temperature change on PVC pipe can be seen with creep behaviour of PVC under elevated temperatures. However, creep is intrinsic properties of solids and not liquids as water, and therefore, cannot be a function of flow. Moreover, creep effects are time-dependent on longer time scale (on order of month or even years) and can only be realistically detected visually if a long, inadequately supported (for example, only end supports are present in segment of a size of meter or higher) segment of PVC pipe has a constant flow at elevated temperatures for this period of time. The end result of it will be visible sag of centre section of a pipe [25].

# CHAPTER 7

# 7 Conclusion & Future Work

## 7.1 Conclusion

In the piping industry, piping flexibility design has been challenging from many decades. Human suffered many death causalities and economic losses due to poor piping modelling and piping stresses. An intelligent design of pipeline flexibility is the backbone of an accurate and safe piping system.

Due to geometry constrains, placing supports in the fluid flow pipe is bit difficult. To solve this problem steady state CFD analysis and structural modelling has been performed on different type of pipe such as straight pipe, one bend pipe and u-shaped pipe in chapter 6 and 7. The comparison of the analysis results from those cases showed us the maximum shearing stresses near the bending area and on some other part of pipes. The maximum shearing stresses help us to decide placing the suitable supports on the pipe so, these supports can control the loads and vibrations in the pipe.

The flexibility pipeline design is made in a way that if someone can modify the supports while changing the velocity and pressure and can easily analyse the maximum shear stresses on the pipeline wall and can replace the supports constrains. This design will help us to optimize in the risk factor such as vibration and loads of the pipe and safety factor will increase in the piping industry. This thesis report shows the selection of the suitable materials for pipeline flexibility design for water keeping in mind the thermal effect on the pipe in term of safety and cost.

## 7.2 Future Work

Following are the recommendations to continue this research work in future

- Transient time dependant CFD analysis of flow in pipes
- Dynamic stress analysis including the vibration distribution spectrum overtime.
- Numerical study of material failure characteristics under various operating conditions

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## Appendix -1: (*MATLAB Code*)

```
 % Fluid calculations
% in this case fluid = water

% Density calculation via volumetric expansion
 g = 9.80655; % gravitational acceleration (m/s^2)

 rho0_w = 999.8396; % density of water at 0 °C (kg/m^3)

b = 0.0002; % water volumetric expansion coefficient (m^3/°C)

T0 = 273.15 + 0; % reference water temperature in K at 0 °C
T = 273.15 + 15; %operating temperature (K)
dT = T0 - T; % temperature difference for
% volumetric expansion calculatuion (K)
 rho = rho0_w/(1-(b*dT)); % water density at 80 °C (kg/m^3)

 % Viscocity calcultion (Vogel equation)

 A = -3.7188;
 B = 578.919;
 C = -137.546;

% A, B and C are empirical constants for water in Vogel equation

 E = exp(1); % the value of Euler number e
mu = E^(A + (B/(C+T))) * 10^-3; % viscoscity of water at

 % temperature T (Pa*s)

 % Pipe parameters

 d = 0.0254 * 2; % internal diameter of a pipe (m)

 D = 0.0254 * 2.375; % outer diameter of a pipe (m)

 t = (D-d) / 2; % wall thickness of a pipe (m)
L = 1; % length of a pipe (m)
Ac = 1/4 * pi * d^2; % internal cross-section of a pipe (m^2)
 R = d/2; % pipe internal radius (m)
r = linspace(-d/2, d/2, 100); % some radial distance in a pipe (m);
% symbolic, needed for plots
 e = 0; % pipe roughness; 0 means hydrodynamically smooth (PVC)

% Flow parameters
```

```
q = 6.30902 * 10^-5 * 50; % volume flow rate (m^3/s)
 v = 3; % flow speed (m/s)

 %Flow calculations

Re = rho*d*v/mu; % Reynolds number
% If Re < 2300 flow is laminar, if Re > 4000 flow is turbulent (2 different
% sets of formulae is used here). If 2300 < Re < 4000 flow is transitional
 % (alterating between laminar at turbulent). The script will stop if flow
% is in transitional range, as the fluctuating nature makes investigation
% of proper friction factors highly complex.

 if Re <= 2300 % check to see if flow is laminar

f = 64/Re; % Darcy-Weisbach friction factor
%for CIRCULAR PIPE AND LAMINAR FLOW

dP = f * (L/d) * (rho*v^2/2); % pressure loss (Pa)

hl = dP/rho * g; % head loss (m)
tau_w = f * rho * v^2 / 8; % wall shear stress (N/m^2)

 tau_w_alt = (dP * d)/(4 * L); % alternate way to calculate

 % wall shear stress. Sanity check.

 u = 2 .* v *(1 - (r.^2/R^2)); % velocity profile for laminar flow
tau = 2 .* tau_w .* r /d; % shear stress distribution
u_max = 2 * v; % maximum velocity in a laminar flow (m)

elseif Re >= 4000 % check to see if flow is turbulent
if e == 0 % check to see if pipe is hydrodynamically smooth

f = (0.79 * log(Re) - 1.64)^-2; % friction factor for

 % hydrodynamically smooth pipe

else % if e > 0 than pipe is hydrodynamically rough.
% Both checks affect the value of friction factor used.
 % No check is done for case of e < 0, as this is unphysical
 % case.

 f = 0.11 * (e/D + 68/Re) ^ 0.25; % friction factor for
% hydrodynamically rough pipe
 end

n = 1.03 * log(Re) - 3.6; % power law for turbulent flow
```

```
 dP = f * (L/d) * (rho*v^2/2); % pressure loss (Pa)
hl = dP/rho * g; % head loss (m)
 tau_w = f * rho * v^2 / 8; % wall shear stress (N/m^2)
 tau_w_alt = (dP * d)/(4 * L); % alternate way to calculate

% wall shear stress. Sanity check.

tau = 2 * tau_w .* r /d; % shear stress distribution
u_max = (v * (n+1) * (2*n+1)) / (2*n^2); % maximum velocity in a

% turbulent flow (m/s)

du = -u_max / (n*R) .* (1 - abs(r) /R).^((1-n)/n); % derivative of
% velocity profile with respect to r

 tau_lam = -mu * du; % laminar contribution to total stress
tau_turb = (tau - tau_lam) ./ tau_lam; % ratio of turbulent

 % to laminar stress contribution

 u = (1 - abs(r)/R).^(1/n) * u_max; % velocity profile

% for turbulent flow

 elseif (2300 < Re) && (Re < 4000) % check to see if flow is
% in transitional range

disp('Flow is in transitional range.') % displays this message

end

 fig1 = area(r, u); % plot of velocity profile
set(fig1,'LineStyle', '-', 'LineWidth', 2);
fig1.FaceColor = [0.7, 0.7, 0.7];
grid on
set(gca, 'LineWidth', 1.5, 'GridLineStyle', ':', 'GridAlpha', 0.5,...
 'FontSize', 30, 'FontWeight', 'bold');
title('Turbulent velocity profile', 'FontSize', 42, 'FontWeight', 'bold');
axis([-0.0255 0.0255 0 3.6]);
xlabel('Radial distance [m]', 'FontSize',42,'FontWeight','bold');
ylabel('Fluid velocity [m/s]', 'FontSize',42,'FontWeight','bold');
ax = gca;
gcX = sym(ax.XLim(1):0.005:ax.XLim(2));
ax.XTick = double(gcX);
gcY = sym(ax.YLim(1):0.3:ax.YLim(2));
ax.YTick = double(gcY);
ylabh = get(gca,'YLabel');
```

```
fig2 = area(r, tau); % plot of shear stress profile
set(fig2,'LineStyle', '-', 'LineWidth', 2);
 fig2.FaceColor = [0.7, 0.7, 0.7];
grid on
set(gca, 'LineWidth', 1.5, 'GridLineStyle', ':', 'GridAlpha', 0.5,...
'FontSize', 30, 'FontWeight', 'bold');
 title('Shear stress profile', 'FontSize', 42, 'FontWeight', 'bold');
 axis([-0.0255 0.0255 -20 20]);
xlabel('Radial distance [m]', 'FontSize',42,'FontWeight','bold');
 ylabel('Shear stress [Pa]', 'FontSize',42,'FontWeight','bold');
ax = gca;
gcX = sym(ax.XLim(1):0.005:ax.XLim(2));
ax.XTick = double(gcX);
gcY = sym(ax.YLim(1):2:ax.YLim(2));
 ax.YTick = double(gcY);
ylabh = get(gca,'YLabel');
```