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Ola Flaaten

The Economics of Multispecies Harvesting

Theory and Application to the
Barents Sea Fisheries



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PREFACE

The aim of this study is twofold: to provide a theoretical and an applied analysis of multispecies fisheries. The theoretical part will include concepts and analysis which, hopefully, will be of interest not only to economists, but also to biologists and ecologists. The application of the theoretical model and analysis to the Barents Sea fisheries gives empirical content to the analysis, which is important for the advancement of fisheries management science. It is also my firm belief that this kind of work in the end will be beneficial to the people trying to make a living from harvesting marine resources.

For thousands of years man has been whaling, sealing and fishing in these cold and harsh surroundings. The relative importance of the different species in the ecosystem has changed throughout history. In the seventeenth century the abundant, slow-swimming Greenland right whale and the Biscayan right whale in the Barents Sea area were so valuable, especially to English and Dutch whalers, that the intensive exploitation of these common property resources probably were the main reason for the extinction of these two stocks. The two species are, however, still present in other parts of the North Atlantic Ocean. Except for these two stocks of whales there is no knowledge of other stocks of sea mammals or fish in this area being extinct in historical time.

With the immense development of harvesting technology, especially in our century, man has been increasingly aware of the possibility of depletion of some stocks under a "free for all" harvesting regime. For the same reason the knowledge of the biological interdependencies of the species has become increasingly demanded by those having an interest in better management of the living resources of the sea. The traditional object of study for economists is the use of limited resources for competing ends. In our context the limits of the resources lies within the ecosystem and because no multispecies model of the Barents Sea area was available, I had to design my own and

implement this by use of existing statistics. Being especially concerned with the main principles for optimal harvesting of the ecosystem, this study is carried out on a highly aggregated level with just three stocks involved. The work to be presented is an economist's contribution to a hopefully better understanding of how the ecosystem should be managed. Maybe it should be called an interdisciplinary study rather than an economic one, since to a great extent it has been necessary to integrate ecological, biological and economic knowledge to give an answer to optimal management.

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Tromsø, Norway
December, 1987

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1. INTRODUCTION

It is many years since interested theoretical biologists started studying ecological systems by means of mathematical models (e.g. Lotka, 1925 and Volterra, 1928), and studies of single species models are even older (e.g., Verhulst, 1838).

The breakthrough in the use of mathematical models in applied marine-biological research should be especially credited to the English biologists R.J.H. Beverton and S.J. Holt (Beverton and Holt, 1957) and the Canadian M.B. Schaefer (Schaefer, 1954 and 1957). The Beverton-Holt model is a one species model in which the relations to other species are not explicitly formulated. However, in a way these relations are implicitly taken care of: The mortality rates include mortality due to predation by other species; growth rates and recruitment will depend on availability of prey and on the presence of competitors, etc.

One species models of this kind have many advantages compared to more complex ecosystem models. They are quite simple, the parameters have specific biological meaning and can in many cases be estimated from catch and effort data, and the models have proved to be sufficiently reliable for management purposes. The latter is especially the case when the harvesting has concentrated on just one or a very few species in an ecosystem, or when the level of fishing effort directed against different species has been changing slowly with time.

The technical and economical developments have led to commercial exploitation of more and more fish stocks, and stocks which sustained fisheries for a very long time have been severely depleted. This is probably the main reason for the increased interest biologists and others have taken in the use of multi-species models in applied research. As far as we know the most comprehensive study undertaken until now is the North Sea model (Andersen and Ursin, 1977). It can be described as a multi-species extension of the single-species Beverton-Holt model. The interactions between fish stocks are taken care of by natural mortality and growth rates, based on the principle of "one man's loss is another man's gain".

Although the main report on the North Sea model (op.cit.) is primarily a theoretical study, the model has been used for simulation of the history of North Sea fisheries and also for prognostic purposes. The computer simulations tell i.a. that the increase of cod and other demersal species during the 1970s were due to a decrease of the herring and mackerel stocks at the end of the 1960s. Mackerel, in particular, preys hard upon the fry of herring, cod and other species, and when the mackerel stock was depleted from 3 to 1 million metric tons by heavy purse-seine fishing within a few years, the niches for other stocks were expanded. All-in-all, this change in exploitation pattern has increased the total fish harvest in the North Sea from 1.5 million m.t. in 1960 to more than 3 million tons in 1976 (Ursin, 1974, 1977 and 1978). To get more reliable estimates of the model's coefficients, The International Council for Exploitation of the Sea (ICES) has started a large research project based on stomach sampling (ICES, 1982).

Beyond the North Sea model there have been several biological studies of parts of fish stock interactions in larger ecosystems. Some of these will be mentioned here. T. Laevastu and other scientists at the Northwest and Alaska Fisheries center in Seattle have done several studies on multispecies interactions in the Bering Sea area by means of computer simulation models. Contrary to the North Sea model, which is based on number of fish, Laevastu et al. work on biomass level (Laevastu and Larkins, 1981). They are especially concerned with

"fisheries-oriented ecosystem simulation and its evaluation" (op.cit., p. 2),

and the models include the predatory effects from marine mammals. Laevastu and Favorite (1978) is a study of predation on herring from marine mammals and fish in the Bering Sea. They found that the predators' consumption was more than 1 million m.t. per year, while on the other hand the fishing amounted to only 40 thousand tons.

Ponomarenko et al. (1978) studied the predation effects on capelin in the Barents Sea from cod and haddock. They found that the annual consumption amounted to between 6.6 and 9.8 million

m.t. in the years 1974-76. In comparison, annual catches were 1.4 million m.t. in the same period, or only 15-20% of the consumption by cod and haddock.

Interactions between seals and fish stocks in the Atlantic ocean were studied by Sergeant (1973 and 1976). He found that the Harp seal in the Northwest Atlantic consumes food at a daily rate of 5% of its own body weight. Parrish and Shearer (1977) studied interactions between seals and fish in Scottish waters and concluded that the predation-consumption of fish by seals amounted to 200 thousand tons per year.

Theoretical bioeconomic studies have focused on the difference between open access harvesting and socially optimal harvesting. Multispecies analyses of these kind are to be found in Quirk and Smith (1970), Anderson (1975) and Clark (1976). All of them use generally formulated growth equations, whereas Hannesson (1983a) utilizes a Lotka-Volterra type of growth equations. By this specification of the model, he is able to show how the optimum stock levels are effected by changes in discount rate, relative prices and harvesting costs. The parameters used are assumed values and not estimated from empirical data. However, in Hannesson (1983c), which uses a two species Beverton-Holt model with the growth of individual cod being a function of the capelin biomass, some of the parameters are estimated from Barents Sea data. Conrad and Adu-Asamoah (1986) have partly estimated the parameters and partly assigned values to the parameters of the models in their multispecies analysis of tuna in the Eastern Tropical Atlantic.

Balchen (1979) describes a large research program, Oceanographic Biomodels (OBM), whose aim is

.... to establish the capability of producing mathematical/numerical models of a total marine ecological system of the Barents Sea,

The study would include physical and chemical oceanography, phytoplankton, zooplankton and fishes. So far a submodel of capelin is completed (Reed and Balchen, 1981) in addition to oceanographic parts of the study. Regarding the applicability of the project it was said:

An operational set of models is planned to be available around 1985. The most obvious applications of this system will be in estimating present and future states of the system, particularly those representing the quality, quantity and distribution of the most important species of commercial fish. It is expected, however, that the system will also have important applications in oceanographic and biological research and in studies of ocean pollution as a consequence of offshore oil activities. (Balchen, 1980, p. 67)

Concerning the fisheries the aims of the OBM of the Barents Sea are very much like those of the North-Sea model even though methods and the way of approaching reality are different. However, as the quotations demonstrate, the general aims of the former models are much wider than just being tools for fisheries management.¹⁾

At the Institute of Marine Research, Bergen a research project on "A multispecies model of the Barents Sea" (The MSB-model) has been initiated (Tjelmeland, 1986). The aims are to make the project an integrated part of the institute's marine biology research and to develop the multispecies model into an operational management tool for the Barents Sea fisheries.

Both the North-Sea model, the OBM- and the MSB-model are large, complex, resource consuming research projects with inputs from biologists, mathematicians, cyberneticians, statisticians and other groups of scientists. So far, no economists or social scientists have been involved.

Compared to the three models above our study is less ambitious regarding aims and quantitative extent on the applied biological level. We shall develop a three species model of the Lotka-Volterra type and use this for economic studies of multispecies fish communities. Hoping that our work can be of some help for better management, it will be applied to the Barents Sea fisheries. An operational model of the ecosystem does not yet exist. Therefore we will have to rely on several sources to get the necessary data for estimating the biological interdependencies of the relevant species. As will be clear later,

¹⁾ The OBM project seems to have come to an end without having reached the goals with respect to operational models of "the most important species of commercial fish".

this is not at all an easy task. Biological data gathered for use in the Beverton-Holt type of models obviously can be used to estimate parameters of other kinds of models, e.g. the Lotka-Volterra type we are going to use. However, this may raise some methodological problems, and even though they can be overcome to some extent, scientifically based data on biological interactions among species is often lacking. Therefore we will have to rely on "guesstimation" and simulation techniques to complete the three species Barents Sea model.

The aim of this study is twofold: methodological and quantitative analysis of multispecies fisheries, with an application to the Barents Sea fisheries. Even though some of the data are scanty and the methods to be used in the estimation of biological parameters are rough, the quantitative analysis should not be considered merely as an illustration of the method. If we were required to make decisions and take actions in connection with relationships covered by this study, we would (in the absence of more reliable results, and without doing more work) rely to some extent in our results. Thus, the quantitative analysis does not solely serve the purpose of illustrating a method, but we think they also give a description of some important biological and economic relationships of the Barents Sea fisheries.

A brief outline of the book is as follows. A two species biomass model is reviewed in Section two, and Section three presents a three species model based upon the former one. The concept of maximum sustainable yield frontier (MSF) is defined and presented in Section four, and a method for deriving this in the three species case is developed.

A brief description of the marine ecological system of the Barents Sea area is included in Section five. Since a fully developed and implemented multispecies model of the Barents Sea fisheries did not exist, an attempt has been made to estimate the nine biological parameters of the three species model. The "guesstimation" procedure is explained step by step in Section six.

The three stocks in the applied model are aggregated stocks with "capelin" consisting of capelin and herring, "cod" consisting of cod, haddock and saithe, and "sea mammals" consisting of 14 species of whales and two species of seals.

Economic aspects of multispecies fisheries studied in Section seven includes optimal harvesting as well as open access harvesting. The economic parameters of the model are derived from Norwegian data in Section eight.

In Section nine are shown the optimal solutions of the three species Barents Sea model (TSB-model), and the open access solutions are derived in Section ten. Finally, Section eleven comprises the summary and the conclusion of this study.

2. A TWO SPECIES MODEL

First we shall give a review of the two species model analysed in May et al. (1979), since our three species model will be based upon this. Suppose there is a prey, W_1 , on which the existence of a predator, W_2 , is based. W_1 and W_2 can be thought of as biomasses. A simple model describing the dynamics of such a system is

$$\dot{W}_1 = dW_1/dt = r_1 W_1 (1 - W_1/K) - aW_1 W_2 \quad (2.1)$$

$$\dot{W}_2 = dW_2/dt = r_2 W_2 (1 - W_2/\alpha W_1) , \quad (2.2)$$

where r_1 and r_2 are the intrinsic growth rates of the respective species. K is the carrying capacity of the total system, at which the prey will settle in the case of no predator and no harvest.

The per capita¹⁾ growth rate of the prey decreases from r_1 for stock levels close to zero, to zero for stock levels equal to the carrying capacity in case of no predators. If predators exist, the per capita growth rate for the prey equals zero for a stock level lower than the carrying capacity. The presence of predators reduces the per capita growth rate in proportion to the biomass of the predator. The predation coefficient, a , tells how much the per capita growth rate of the prey reduces per unit of the predator. Or to put it another way, a tells which share of the prey stock one unit of the predator is consuming per unit of time. The total rate of consumption is expressed in the term $aW_1 W_2$.

The predator's per capita growth rate decreases from r_2 when its own stock level is close to zero, to zero for a stock level equal to its own carrying capacity, which is proportional to the level of the prey stock. The proportionality coefficient is α .

Mathematical stability properties of the model (2.1)-(2.2) will not be discussed here.²⁾ (It can be found in the literature

¹⁾ The term "per capita" is used, even though we mean per unit of biomass.

²⁾ For the three species model to be studied, the stability conditions are derived in Appendix 8.

of theoretical ecology, e.g. in Beddington and Cook (1982), May (1974) and May (1981).) However, it is easy to see, by letting \dot{W}_1 and \dot{W}_2 equal zero in (2.1) and (2.2), that if an equilibrium point exists with both species being positive, the stock levels will be

$$W_1^* = \frac{K}{1+v}, \quad \lim_{K \rightarrow \infty} W_1^* = \frac{r_1}{a\alpha} \quad 1) \quad (2.3)$$

$$W_2^* = \frac{\alpha K}{1+v}, \quad \lim_{K \rightarrow \infty} W_2^* = \frac{r_1}{a} \quad 1) \quad (2.4)$$

where $v = \frac{a\alpha K}{r_1}$.

It should be noticed that the intrinsic growth rate of the predator, r_2 , does not affect the equilibrium values of either of the two species. The equilibrium values of both species increase with any increase in r_1 or K , ceteris paribus. From (2.3) and (2.4) it follows

$$W_2^*/W_1^* = \alpha. \quad (2.5)$$

In equilibrium α expresses the relative size of the predator stock to that of its prey.

Even though r_2 does not affect the equilibrium values of the stocks, it is of importance to the behaviour of the system outside equilibrium. Defining the "natural return time", T^R , of the species as

$$T_i^R = 1/r_i \quad i = 1, 2, \quad (2.6)$$

r_2 will affect the time the predator will need to reach equilibrium from a higher or lower level.

Suppose that the fish stocks are harvested independently with constant effort per unit of time, F_i , scaled such that

1) In a logistic single species model the equilibrium stock level with no harvesting always equals the carrying capacity. In the two species model, however, (2.3) and (2.4) demonstrate that an increase in the carrying capacity towards infinity, increases the stock levels towards limits fixed by other parameters than the carrying capacity.

$F_1 = 1$ corresponds to constant catchability coefficients equal to r_i . Then the catch rates will be

$$h_1 = r_1 F_1 W_1 \quad (2.7)$$

$$h_2 = r_2 F_2 W_2 \quad (2.8)$$

With harvesting introduced it will influence the growth rates in (2.1) and (2.2) which will be changed to

$$\dot{W}_1 = r_1 W_1 (1 - W_1/K) - a W_1 W_2 - r_1 F_1 W_1 \quad (2.9)$$

$$\dot{W}_2 = r_2 W_2 (1 - W_2/\alpha W_1) - r_2 F_2 W_2 \quad (2.10)$$

If positive equilibrium levels of W_1 and W_2 exist simultaneously, using W_{1F}^* and W_{2F}^* as symbols, they will be:

$$W_{1F}^* = \frac{K(1-F_1)}{1 + v(1-F_2)} \quad (2.11)$$

$$W_{2F}^* = \frac{\alpha K(1-F_1)(1-F_2)}{1 + v(1-F_2)} \quad (2.12)$$

With harvesting the relative stock size is

$$W_{2F}^*/W_{1F}^* = \alpha(1-F_2) \quad (2.5')$$

It is seen from (2.11) that only for $F_1 < 1$ will there exist a positive equilibrium value of the prey. If $F_1 \geq 1$ the prey-stock will be extinct, and so of course will be the predator, as seen from (2.12). The latter expression shows that only for $F_2 < 1$ and $F_1 < 1$ will the predator survive.

The equilibrium values of both species increase with decreasing fishing pressure on the prey, i.e. for reduced F_1 . More of the prey gives increased carrying capacity for the predator which can be kept on a higher level.

On the other hand, the effects on the prey and on the predator from decreased fishing pressure on the predator are the opposite of each other. From (2.11) it is seen that the equilibrium value of the prey will decrease, and from (2.12) that the predator will increase. The increased stock level for the predator means heavier predation on the prey, and thereby a reduced equilibrium level for the latter.

Defining $X_1 = W_1/K$ and $X_2 = W_2/\alpha K$ we can rewrite equations (2.9) and (2.10) as

$$dX_1/dt = r_1 X_1 (1 - F_1 - X_1 - \nu X_2) \quad (2.9')$$

$$dX_2/dt = r_2 X_2 (1 - F_2 - X_2/X_1) \quad (2.10')$$

Here the dimensionless parameter ν is defined as $\nu = \alpha K/r_1$.

The equilibrium properties of this ecological system depend only on the fishing efforts, F_1 and F_2 , and ν . The dynamics additionally involve r_1 and r_2 .

The phase-diagram for the system (2.9')-(2.10') is shown in Figure 2.1. The isoclines are found by setting $dX_1/dt = 0$ and $dX_2/dt = 0$ in (2.9') and (2.10'). This gives

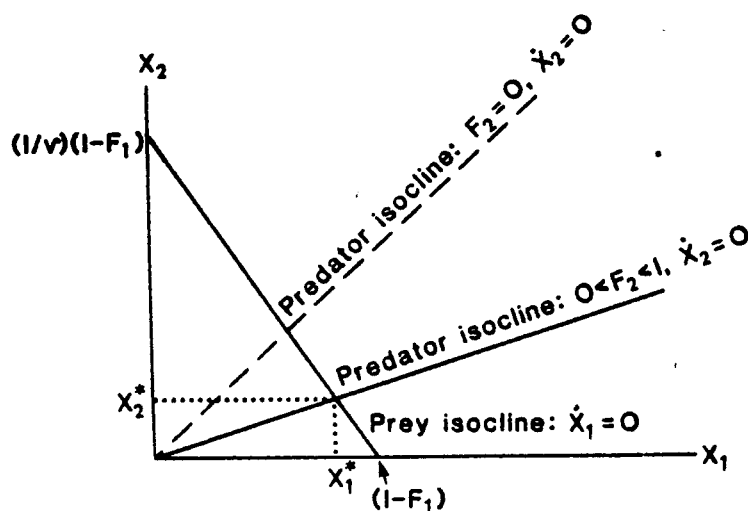


Figure 2.1. Phase diagram for a predator-prey model.

$$X_2 = (1/v)(1 - F_1 - X_1) \quad \text{for } dX_1/dt = 0 \quad (2.13)$$

$$X_2 = (1 - F_2)X_1 \quad \text{for } dX_2/dt = 0 \quad (2.14)$$

The equilibrium values of X_1 and X_2 are found where the isoclines intersect, that is for

$$X_1^* = \frac{1 - F_1}{1 + v(1 - F_2)} \quad (2.11')$$

$$X_2^* = \frac{(1 - F_1)(1 - F_2)}{1 + v(1 - F_2)} \quad (2.12')$$

X_1^* and X_2^* both equal $\frac{1}{1+v}$ in the absence of fishing, and zero in the case of $F_1 = 1$. In addition, X_2^* will equal zero if $F_2 = 1$.

The three species model to be designed and presented in the next section is an extension of the two species model shown in Equations (2.1)-(2-2) and discussed in this section.

3. A THREE SPECIES MODEL

The aim of this study is, as noted earlier, to develop a three species model of the Lotka-Volterra type, and apply it into a bioeconomic study of the Barents Sea fisheries. The model we have in mind should take care of those aspects of the Barents Sea ecology of most importance to fisheries management. Thus it should include the most important species like cod and capelin. However, we know that whales and seals also are large consumers of fish and that they compete with fishstocks over food like plankton, krill etc. If possible, therefore, one should include the marine mammals too in a multispecies model for this area. An illustration of such a model is shown in Figure 3.1.

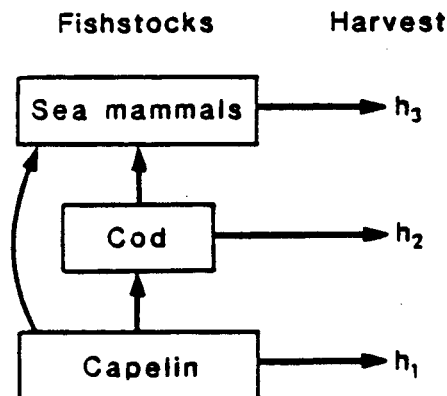


Figure 3.1. Structure of a three species model.

The marine mammals, such as whales and seals, are preying upon both demersal and pelagic species, in Figure 3.1 called cod and capelin. On the other hand cod is preying upon capelin. Limits to growth of the system are set by limiting the environment's carrying capacity for the lowest ecological level, the capelin.

The complete model on biomass form is:

$$dW_1/dt = r_1W_1(1 - W_1/K) - a_{12}W_1W_2 - a_{13}W_1W_3 - h_1 \quad (3.1)$$

$$dW_2/dt = r_2W_2(1 - W_2/(\alpha W_1)) - a_{23}W_2W_3 - h_2 \quad (3.2)$$

$$dW_3/dt = r_3 W_3 (1 - W_3 / (\beta (W_1 + W_2))) - h_3 . \quad (3.3)$$

The symbols are:

$W_i(t)$ = Biomass of species i at time t .

$dW_i(t)/dt$ = Growth rate of species i .

r_i = Intrinsic growth rate of species i .

α = Carrying capacity coefficient for species 2.

β = Carrying capacity coefficient for species 3.

a_{ij} = Preying coefficient, species j preying upon species i .

h_i = Harvest rate for species i .

We assume that the three trophic levels can be harvested independently of each other, that is, the fishing effort targeted at one species catches just that one. It is also assumed that the catch rate are functions of the target fishstock:

$$h_i = h_i(F_i, W_i) \quad (3.4)$$

where F_i is the fishing effort for species i .

The specific harvest functions are, as was the case in the two species model, assumed to be homogeneous of degree two in fishing effort and stock level. To simplify the growth equations, the scaling of fishing effort is such that $F_i = 1$ corresponds to constant catchability coefficient equal to the respective intrinsic growth rates, r_i . The harvest functions are

$$h_i = r_i F_i W_i \quad i = 1, 2, 3 . \quad (3.5)$$

By subtracting the harvest rates on the righthand side of the growth equations (3.1)-(3.3), this three species model can be written in a dimensionless form which can be of help when

analysing the behaviour of the system. Defining $X_1 = W_1/K$, $X_2 = W_2/\alpha K$, $X_3 = W_3/\alpha\beta K$ and $A_i = 1 - F_i$, the system described by equations (3.1)-(3.3) is changed to

$$dX_1/dt = r_1 X_1 (A_1 - X_1 - \gamma_{12} X_2 - \gamma_{13} X_3) \quad (3.6)$$

$$dX_2/dt = r_2 X_2 (A_2 - X_2/X_1 - \gamma_{23} X_3) \quad (3.7)$$

$$dX_3/dt = r_3 X_3 (A_3 - \alpha X_3 / (X_1 + \alpha X_2)) , \quad (3.8)$$

where $\gamma_{12} = a_{12}\alpha K/r_1$, $\gamma_{13} = a_{13}\alpha\beta K/r_1$ and $\gamma_{23} = a_{23}\alpha\beta K/r_2$.

One of the important questions raised in the ecology literature is how to predict what will happen to a disturbed ecosystem. Ideally one would like to be able to measure certain properties of such systems before they are disturbed, and then on the basis of these results to be able to predict how the system will respond to disturbance. The latter tells which properties of ecosystems tend to enhance stability against external disturbances, while the former includes concepts like global asymptotic stability, neighbourhood asymptotic stability, structural stability, resistance and resilience. Discussions on stability indicators are found in e.g. Halfon (1979) and May (1974). Often there can seemingly be a missing link on the way from pure theoretical ecology to applied studies in the field. A quotation from the ecology literature tells it this way:

A shortcoming of much of the theoretical work in ecology is that results are often not expressed or expressible as relations among readily measureable quantities. A familiar example is the often-quoted result that a necessary and sufficient condition for asymptotic stability of a system described by a community matrix is the negativity of the real parts of all the eigenvalues of that matrix. While mathematically rigorous, this result unfortunately is not very useful in situations of practical concern such as environmental impact prediction or assessment. (Harte, 1979, p. 454)

The aim of this study is to develop a rather simple ecological model, as represented by the system (3.1)-(3.3), and to apply it into a bioeconomic study of the Barents Sea fisheries. The problem is, however, that no one has yet studied

this ecological system in a way that directly can give us estimates of the biological parameters in the model. The way to handle such a problem will be dealt with later on in this study.

If the system (3.6)-(3.8) has an equilibrium solution, (X_1^*, X_2^*, X_3^*) , and $\gamma_{ij} \neq 0$ for all i, j , it is shown in Appendix 1 that the solution for X_1 is

$$X_1^* = \frac{-D_1 \pm \sqrt{D_1^2 - 4A_1D_2}}{2D_2} \quad (3.9)$$

where D_1 and D_2 are

$$D_1 = A_1A_3\gamma_{23} - 1 - \gamma_{12}A_2 - A_3(1/\alpha)\gamma_{13} - A_2A_3\gamma_{13}$$

$$D_2 = A_3(1/\alpha)\gamma_{12}\gamma_{23} - A_3\gamma_{23} .$$

X_2^* and X_3^* expressed as functions of X_1^* are

$$X_2^* = \frac{(A_2 - A_3(1/\alpha)\gamma_{23}X_1^*)X_1^*}{1 + A_3\gamma_{23}X_1^*} \quad (3.10)$$

$$X_3^* = \frac{A_3X_1^*((1/\alpha) + A_2)}{1 + A_3\gamma_{23}X_1^*} . \quad (3.11)$$

From (3.11) it can be seen that X_3^* is increasing with increased values of X_1^* , while (3.10) shows that the effect on X_2^* from increased values of X_1^* can be positive or negative. The former is due to the fact that the top-predator, X_3 , is preying on both X_1 and X_2 . Increased X_1^* directly gives a larger food base for the top-predator as well as for the intermediate species, X_2^* . However, this increase in X_3^* may cause trouble for X_2^* . If the predation effect on X_2^* from increased X_3^* is greater than the prey effect from increased X_1^* , the total result on X_2^* will be negative.

In the case of no harvesting, that is $A_i = 1$ for $i = 1, 2, 3$, it is seen from (3.10) that a necessary condition for $X_2^* > 0$ is that

$$x_1^* < \alpha/\gamma_{23} = r_2/a_{23}\beta .$$

This can be explained the following way. A large r_2 , given all other parameters being constant, gives species 2 a higher chance to survive than a small r_2 . The law of the survival of the fittest implies in this context that species 2 belongs to the fittest if, *ceteris paribus*, its intrinsic growth rate is large enough. On the other hand, species 2's chance of being positive, decreases with an increase in species 3's preying pressure and its carrying capacity as represented by a_{23} and β , respectively.

It is easy to see that the three species model described by equations (3.1)-(3.3) includes the two species model of equations (2.1)-(2.2). By setting a_{13} and a_{23} equal to zero in (3.1) and (3.2) we have the two species model. The three species model is also seen to be a general form of a three species ecological system, since it includes the cases shown in Figures 3.2 and 3.3. In the former one there is no predation from species 3, here called polar bears, on species 1, which could be fish, but only on 2, seals. Our general three species model is fitted to this system by letting $a_{13} = 0$ and by letting the carrying capacity of species 3 depend only on the biomass of species 2.

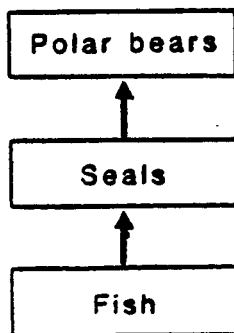


Figure 3.2. The structure of a three species predator-prey model.

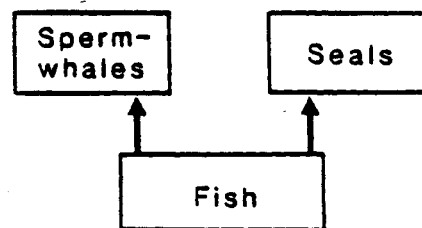


Figure 3.3. The structure of combined competition and predation in a three species model.

The model illustrated in Figure 3.3 is a case of combined competition and predation. Species 2 and 3, here called sperm whales and seals, are competing for their joint prey, the krill. Our general three species model is fitted to the competitive-predation model by letting $a_{23} = 0$ and by adjusting the carrying capacity of species 3 to depend only on the biomass of species 1.¹⁾

Equilibrium and stability conditions of the model described by Equations (3.6)-(3.8) are derived in Appendix 8. In the applied part of this study it is assumed that the main features of the ecological system can be described by the three species equilibrium model. It is not, however, obvious on a priori reasons that an ecological system should have a stable equilibrium. One could as well think of a perpetual cyclical movement of the fish stocks in preharvesting time. Such a system might be described by a limit cycle model of the kind found in Lotka (1925) and Volterra (1928).²⁾ In fact, the first attempt ever done on conducting a multispecies analysis of fishing was by means of limit cycle models. Empirical studies of the Upper Adriatic Sea's fisheries before, during and after the first world war found in D'Ancona (1926) were an important source of inspiration to the theoretical works by V. Volterra as demonstrated by this quotation:

Doctor UMBERTO D'ANCONA (D'Ancona, 1926) has many times spoken to me about the statistics which he was making in fishery in the period during the war and in periods before and after, asking me if it were possible to give a mathematical explanation of the results which he was getting in the percentages of the various species in these different periods. This request has spurred me to formulate the problem and solve it, establishing the laws which are set forth in § 7. Both D'Ancona and I working independently were equally satisfied in comparing results which were

¹⁾ The models in Figures 3.2 and 3.3 are equivalent to the ones described in notes 58 and 51, respectively, in May et al. (1979). There is only a minor difference in the numbering of the species.

²⁾ Limit cycle models also played an important role in the early development of mathematical business cycle models. (See e.g. Frisch, 1933; Frisch and Holme, 1935; Kalecki, 1935; Tinbergen, 1935; and Goodwin, 1951.)

revealed to us separately by calculus and by observation, as these results were in accord; showing for instance that man in fisheries, by disturbing the natural condition of proportion of two species, one of which feeds upon the other, causes diminution in the quantity of the species that eats the other, and an increase in the species fed upon. (Volterra, 1928, p. 4.)

Based upon his empirical studies of the fisheries of the upper Adriatic Sea, D'Ancona (1926) concluded that the predators of this sea, the sharks, ought to be decreased by increased harvest intensity. That would make it possible to increase the yields of more valuable prey stocks.

Having designed a three species model in this section, it will be of interest, from a theoretical point of view, to analyse the limits to growth and harvest of such an ecological system. This is the aim of the next section of this study.

4. THE MAXIMUM SUSTAINABLE YIELD FRONTIER

In the case of the two species model (Equations (2.9')-(2.10')) it can be of interest both from a biological and from an economical efficiency point of view to maximize the sustainable yield of one species for a specified constant level of sustainable yield of the other. This problem is equivalent to that of welfare economics: deriving the production possibility frontier by maximizing the output of one good for a specified amount of output of the other, for a fixed amount of factors of production. In the two species biological system the limited amount of factors of production are embodied in the carrying capacity and the intrinsic growth rates of the model. In the real biological world of the seas, the limited factor of production used for "production" of the two fish species usually will be the zooplankton communities.

The problem of maximizing

$$y_1 = r_1 X_1 (1 - X_1 - v X_2) \quad (4.1)$$

subject to the constraint

$$y_2 = r_2 X_2 (1 - X_2/X_1) = \text{constant} , \quad (4.2)$$

can be done by using the Lagrange-method. As shown in Beddington and May (1980), this problem gives the following quadratic equation for X_1 as a function of X_2 :

$$X_1 = (1/4)[1-(4-v)X_2] \pm (1/4)\{[1+(4-v)X_2]^2 - 8X_2[2-3vX_2]\}^{1/2} . \quad (4.3)$$

For each level of X_2 we calculate X_1 from (4.3), and the resulting yields, y_1 and y_2 , are given by (4.1) and (4.2). The locus combining the yields of the two species is shown in Figure 4.1 for $v = 2$. Such locuses shall be called the maximum sustainable yield frontier (MSF) to emphasize the connections to the concepts

used in welfare economics. MSF gives the absolute sustainable yield of either population for a specified yield of the other. All combinations of yields on or below this curve are sustainable, whereas yields to the north-east of the curve are possible for some period of time, but they are not sustainable. The star in the north-east corner corresponds to a combination of the largest possible yield of the prey and the largest possible yield of the predator, but such a combination of yields is definitely not sustainable.

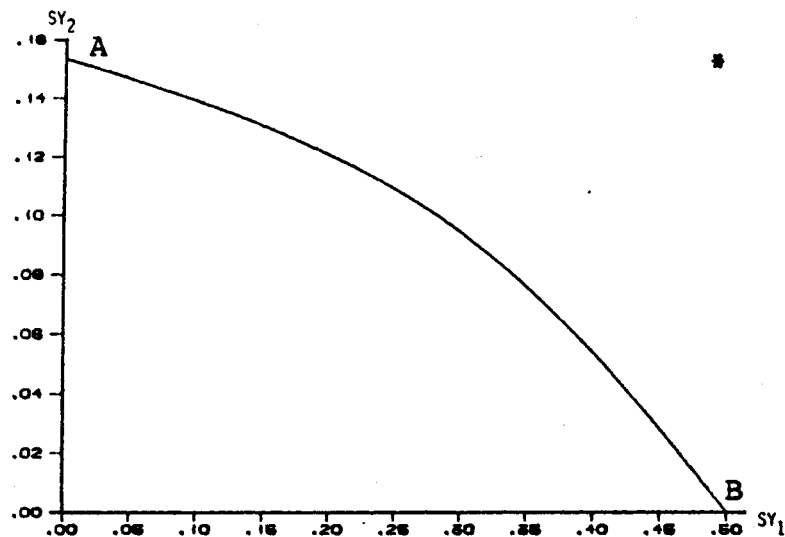


Figure 4.1. The maximum sustainable yield frontier (MSF) of a two species model shows sustainable combinations of yield of species 1 (SY_1) and species 2 (SY_2). Parameters used are $r_1 = 2.0$, $r_2 = 1.15$ and $v = 2.0$.

From the single species logistic growth model it is known that a given sustainable yield less than the maximum sustainable yield (MSY) can be harvested at two different stock levels, above or below the MSY level. These two ways of harvesting are called biological underexploitation and overexploitation, respectively. From a biological point of view the best way of harvesting is to harvest the MSY, whereas the economical optimal

yield stock level, also depend on product price, harvesting cost and discount rate in addition to biological factors.

Unit harvesting cost is usually assumed to be a decreasing function of stock level, leading to the conclusion that the resource should be biologically underexploited to reduce costs. On the other hand, a positive discount rate leads to the conclusion that the resource should be biologically overexploited since a given amount of net revenue "today" is preferred to the same amount "tomorrow". In other words, from an economic point of view, harvesting below, at or above the MSY stock level can all be optimal; it is a question of prices, costs and discount rates.

The lower branch of Equation (4.3) corresponds to a biologically inefficient harvest level, either underexploitation of the predator, or overexploitation of the prey. In the former case the predator is kept on the highest stock level of two possible ones, both giving the same sustainable yield of the predator. A higher predator stock means more consumption of the prey, thereby removing a potential prey yield. To get the highest possible sustainable yield of the prey for a given predator yield it is therefore obviously best to overexploit the predator. For similar reasons it is efficient to underexploit the prey to give more food to the predator. MSF harvesting thus means that neither shall the predator be underexploited, nor shall the prey be overexploited.

The terminal points of the MSF locus in Figure 4.1, A and B, are related to specific stock levels of the predator and the prey. At point A the predator is extinct and the prey is at its single species biological optimum level:

$$X_1 \Big|_{X_2 = y_2 = 0} = 1/2 . \quad (4.4)$$

At point B the corresponding X_1 and X_2 can be found by using Equations (4.5) and (4.6), corresponding to (3.8) and (3.9) in Beddington and May (1980):

$$Y_2 = r_2 \frac{1 + v - R}{v} X_2 \quad (4.5)$$

$$F_1 = 1 - (1 + v + R)X_2 \quad (4.6)$$

where $R = \sqrt{1+v}$.

At point B we have $F_1 = 0$, thus Equation (4.6) gives

$$X_2 \Big|_{F_1 = y_1 = 0} = \frac{1}{1 + v + R} \quad (4.7)$$

Substituting Equation (4.7) into Equation (4.5) gives the corresponding absolute maximum sustainable yield of the predator:

$$MSY_2 \Big|_{F_1 = y_1 = 0} = r_2 [(R - 1)/v]^2 \quad (4.8)$$

When $X_2 \Big|_{F_1 = y_1 = 0}$ is known, the corresponding value of X_1 is found by setting Equation (4.1) equal to zero, after having substituted for X_2 from (4.7). This gives

$$X_1 \Big|_{F_1 = y_1 = 0} = \frac{1 + R}{1 + v + R} \quad (4.9)$$

It should be noted that the following relation between X_1 and v holds:

$$X_1 \Big|_{F_1 = y_1 = 0} \begin{cases} > \\ < \end{cases} 1/2 \quad \text{if} \quad v \begin{cases} \leq \\ > \end{cases} 3 \quad (4.10)$$

This states that the absolute maximum sustainable yield of the predator occurs for an unharvested prey stock above, at or below its single species biological optimum depending on the size of the dimensionless combination of parameters, v . The smaller v is, the higher will be the prey stock level.

For the three species model the MSF locus (Figure 4.1) might be substituted by a maximum sustainable yield plane. In principal this can be done by using the Lagrange-method on the problem of maximizing y_1 subject to the constraints $y_2 = \text{constant}$ and $y_3 = \text{constant}$. Since much of the discussion on sea mammal

harvesting and/or preservation is focused on stock size, we choose to perform the maximization subject to the constraint $X_3 = \text{constant}$ rather than $y_3 = \text{constant}$. For each level of the sea mammal stock one MSF can be drawn for the two other species. With the three species system (Equations (3.6)-(3.8)) in equilibrium with harvesting we have

$$\dot{X}_1 = r_1 X_1 (1 - F_1 - \gamma_{12} X_2 - \gamma_{13} X_3) = 0 \quad (4.11)$$

$$\dot{X}_2 = r_2 X_2 (1 - F_2 - X_2/X_1 - \gamma_{23} X_3) = 0 \quad (4.12)$$

$$\dot{X}_3 = r_3 X_3 (1 - F_3 - \alpha X_3 / (X_1 + \alpha X_2)) = 0 \quad (4.13)$$

By keeping species 3 constant, $X_3 = \bar{X}_3$, it is seen from Equations (4.11) and (4.12) that this has a similar effect as the levels of fishing effort, F_1 and F_2 , have. Substituting

$$\bar{F}_1 = F_1 + \gamma_{13} \bar{X}_3 \quad (4.14)$$

and

$$\bar{F}_2 = F_2 + \gamma_{23} \bar{X}_3 \quad (4.15)$$

into (4.11) and (4.12), we get

$$\dot{X}_1 = r_1 X_1 (1 - \bar{F}_1 - X_1 - \gamma_{12} X_2) \quad (4.16)$$

$$\dot{X}_2 = r_2 X_2 (1 - \bar{F}_2 - X_2/X_1) \quad (4.17)$$

Equations (4.16) and (4.17) are the same as Equations (2.9') and (2.10') except for the notation of \bar{F}_1 . Therefore, exactly the same procedure as used for arriving at the MSF locus in the two species model can be used now, remembering the condition that X_3 is kept constant at \bar{X}_3 . (This is done by varying the harvest rate, y_3 .) The problem now is to maximize the gross yield of one species

$$\bar{Y}_1 = r_1 \bar{F}_1 X_1 = r_1 X_1 (1 - X_1 - \gamma_{12} X_2) \quad (4.18)$$

subject to a constant gross yield of the other

$$\bar{y}_2 = r_2 \bar{F}_2 X_2 = r_2 X_2 (1 - X_2/X_1) = \text{constant} . \quad (4.19)$$

This is exactly the same problem as that in Equations (4.1) and (4.2), and the former results are valid here too.

Substituting γ_{12} for v in Equation (4.3) gives the same quadratic equation of X_1 as a function of X_2 :

$$X_1 = (1/4) [1 + (4 - \gamma_{12}) X_2] \pm (1/4) \{ [1 + (4 - \gamma_{12}) X_2]^2 - (8 X_2 [2 - 3 \gamma_{12} X_2])^{1/2} \} . \quad (4.20)$$

For each level of $X_2 \in [0, \frac{1}{1 + \gamma_{12} + R}]$, X_1 is calculated from (4.20) and the resulting gross yields, \bar{y}_1 and \bar{y}_2 , are given by (4.18) and (4.19). The yields harvested by man are given by:

$$y_1 = \bar{y}_1 - r_1 \gamma_{12} X_1 \bar{X}_3 \quad (4.21)$$

$$y_2 = \bar{y}_2 - r_2 \gamma_{23} X_2 \bar{X}_3 . \quad (4.22)$$

The yield of species 3 follows from

$$y_3 = r_3 \bar{X}_3 (1 - \alpha \bar{X}_3 / (X_1 + \alpha X_2)) , \quad (4.23)$$

and will vary with the X_1, X_2 combinations given by (4.20).

The procedure for finding MSF for a specified stock level of species 3, \bar{X}_3 , can be summarized as follows

1. Choose a fixed value of $\bar{X}_3 \geq 0$.
2. For $X_2 \in [0, \frac{1}{1 + \gamma_{12} + R}]$, compute X_1 from (4.20).
3. Substitute for X_1, X_2 into (4.18) and (4.19) and further into (4.21) and (4.22) to derive the yields, y_1 and y_2 .
4. Compute y_3 from (4.23).

A computer program designed for doing these calculations is shown in Appendix 2.

In Figure 4.2 are shown MSF locuses for three different levels of X_3 .¹⁾

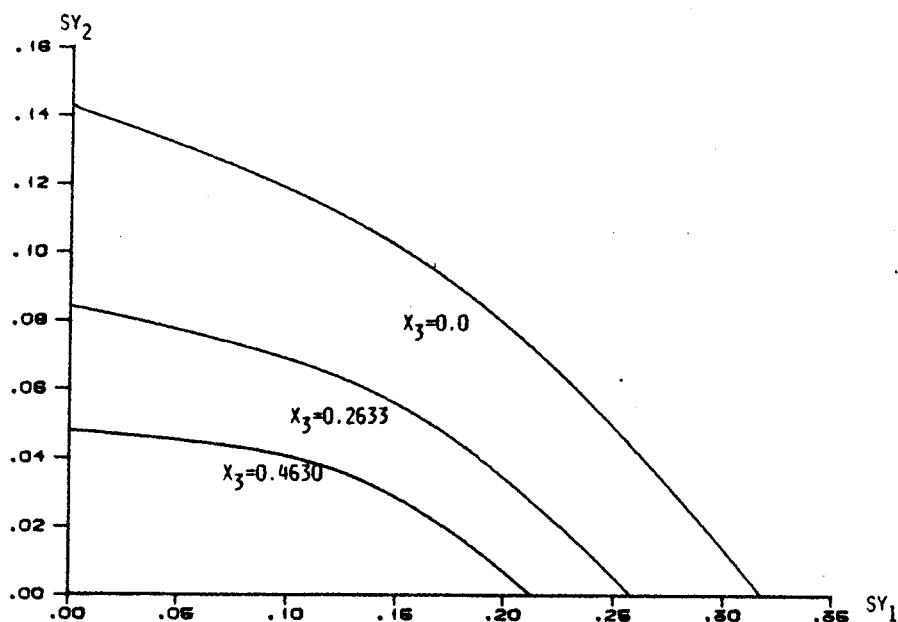


Figure 4.2. The map of maximum sustainable yield frontiers of a three species model shows sustainable combinations of yield of species 1 (SY_1) and species 2 (SY_2) for given stock levels of species 3 (X_3).

The question of economic optimal harvesting of the three species is left to be answered in Section 7. The next section, Section 5, gives a brief description of the ecological system of the Barents Sea, to which the three species model shall be applied.

¹⁾ Seen from the origin X_3 is equivalent to $W_3 = 1500, 853$ and 0 thousand tons, respectively. The parameters used are: $r_1 = 1.2704$, $r_2 = 1.1617$, $r_3 = 0.0614$, $\alpha = 0.9$, $\gamma_{12} = 2.6566$, $\gamma_{13} = 0.3571$ and $\gamma_{23} = 0.8646$. (For estimation of the parameters, see Section 6).

5. THE ECOLOGICAL SYSTEM OF THE BARENTS SEA

The Barents Sea occupies 1.405 million square km between the coast of North Norway, Svalbard, Novaya Zemlja and the Murman Coast. The greater portion of the sea occupies depths ranging from 200 to 400 m, with a mean depth of 229 m. Due to the warm Atlantic waters continually flowing into the sea from southwest, the southwestern part of the Barents Sea never freezes to ice, and the northern and eastern parts are ice-free for part of the year.

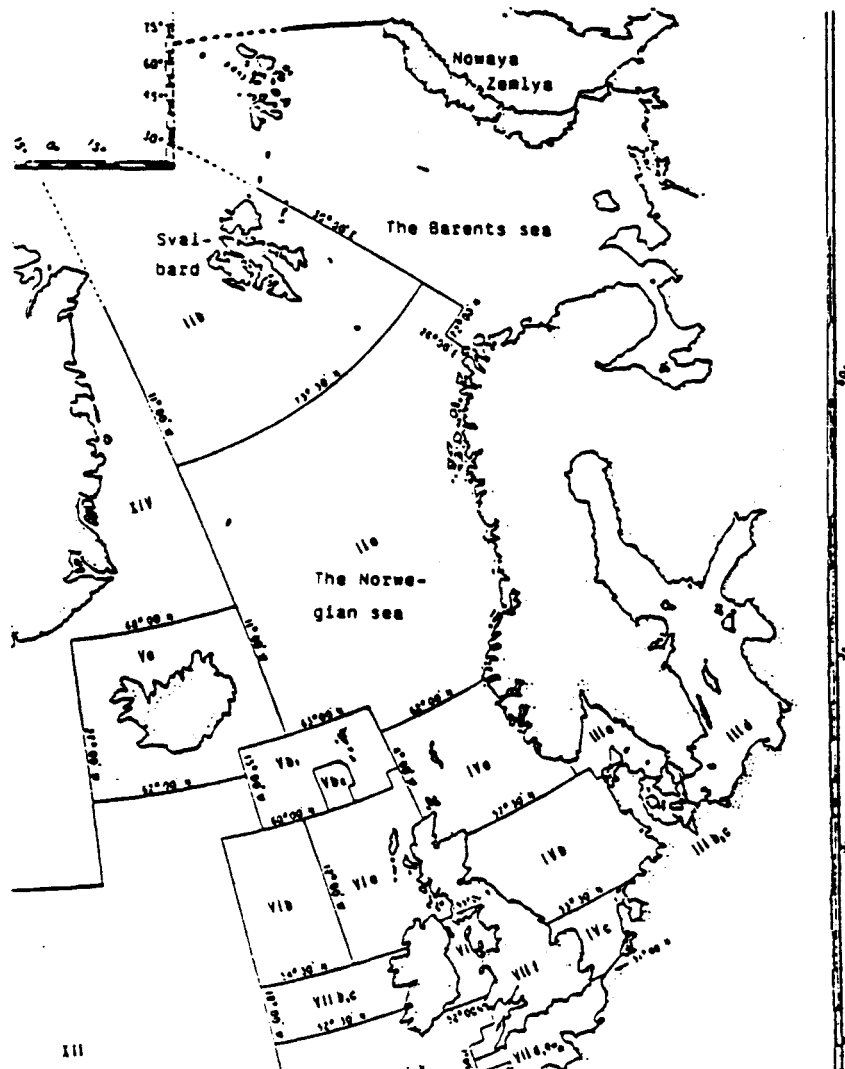


Figure 5.1. The Northeast Atlantic Ocean, including the Barents Sea.

The rich biological productivity of this area is determined by the phenomenon of vertical circulation. With cooling, the saline Atlantic water being denser sinks, bringing oxygen from the uppermost water masses bottomward. This creates an upward countercurrent bringing nutrient salts needed for the phytoplankton organisms. The production of phytoplankton is especially strong in spring in the border areas of ice-free and ice-covered sea. When the ice melts and the ice edge gradually moves to the north and northeast, the primary production is at its peak (Zenkevitch, 1956, and Gjørseter et al., 1983).

The total number of zooplankton species of the Barents Sea is relatively few, consisting of 145 species (Zenkevitch, 1956). The copepoda make up the main zooplankton biomass: 90% in the southwest part of the sea. The total biomass of zooplankton is very high:

We can assume that the total quantity of zooplankton in the Barents Sea makes up in summer approximately 100 million tons of raw material. Basically, the quantity, as we know, is produced by Calanus finmarchicus, yielding one generation a year. Thus the total zooplankton production of the Barents Sea can be assumed to equal the above-mentioned magnitude. Just for the feeding of the zooplankton mass there is needed not less than 1000 million tons of phytoplankton which compensates for the colossal consumption by its exclusive capacity to propagate at a high rate. (Zenkevitch, 1956; p. 247)

The rate of production of phytoplankton is very high, but with a comparatively small standing biomass:

Indeed, when taking into consideration the overall biomass which hardly exceeds 50 million tons, the annual production of the Barents Sea phytoplankton must be of a huge order of several thousand million tons, i.e. the ratio of the annual production to the biomass of the Barents Sea plankton in summer cannot be smaller than 50. (Zenkevitch, 1956; p. 247)

As in other seas of the world, the copepods are the basic food components for fish and other inhabitants.

In addition to the zooplankton, there is another important main source of food for the fish: the benthos inhabiting the

sea bottom. The biomass density of these varies with the type of bottom as well as with depth.

The total biomass of benthos in the entire Barents Sea is expressed by a magnitude of the order of 130 to 140 million tons. (Zenkevitch, 1956; p. 256)

In Table 5.1 are shown variations with depth in density of benthos.

Table 5.1. Variation with depth in the density of bottom population of the Barents Sea (in g/m^2).

Depth in m	The mean biomass in g/m^2	Depth in m	The mean biomass in g/m^2
0-100	310	400- 600	20
100-200	170	600-1000	2-10
200-300	90	1000-2000	1-2
300-400	50	(in the Greenland Sea)	

Source: Zenkevitch, 1956; Table 49.

114 species of fish are known to inhabit the Barents Sea, of which the most important families are: the cod family (12 species), flounder (11 species), viviparous blenny (13 species), goby (10 species) and white fish (7 species). The greater part of the families are, however, represented by one or two species in the commercial fish catches.

In the Barents Sea there are especially two species of zoo-plankton preying species of fish which have been of great commercial value: capelin (Mallotus villosus) and herring (Clupea harengus). In addition to these the pelagic species Polar cod (Boreogadus saida) has been commercially harvested since 1970. The pelagic fish species are important food sources for larger fish species, sea mammals and birds. So also is krill (Euphausiacea), feeding primarily at the second trophic

level. In the transference of energy from phytoplankton through the food web to fish, krill is an important link (Mauchline and Fisher, 1969).

Larger fish of high commercial value are cod (Gadus morhua), haddock (Melanogrammus aeglefinus) and saithe (Pollachius virens). On average for the years 1950-1980, these three species yielded more than 90% of the total annual catch of demersal and semi-pelagic species (excluding pelagic species like herring, capelin and polar cod). Other demersal species of commercial value are: red fish (Sebastes viviparus), Norway haddock (Sebastes marinus), halibut (Hippoglossus hippoglossus), Greenland halibut (Reinhardtius hippoglossoides), plaice (Pleuronectes platessa). Of high-valued species, deep water prawn (Pandalus borealis) and salmon (Salmo salar) are the most important ones.

The main food sources of cod are to some extent known:

The main food of cod consists of small pelagic fishes: herring, capelin, young cod, haddock and Polar cod. The food of cod consists of 60% of fishes. Then follow large planktonic crustaceans such as amphipods, euphausiide and decapods. In the eastern part of the sea, a considerable portion of food is made up of bottom animals, such as crabs, hermit-crabs and various other sizable representatives of amphipods, isopods and cumaceans, to a lesser degree worms and mollusks. (Zenkevitch, 1956; p. 259)

Regarding the preys of haddock, the same source says:

"Haddock, in contrast to cod, feeds on benthos, such as mollusks, worms, crustaceans and echinoderms."

The seals of the Barents Sea and adjacent areas are great consumers of fish, krill, zooplankton, benthos, etc. Like other living resources of the sea, each species of seal occupies its own niche in the ecosystem with respect to prey selection, geographical distribution, etc. The most important species for commercial purposes has been harp seal (Pagophilus groenlandicus). Its opportunistic feeding behaviour is probably a very important explanation of the large stock to be found in the Barents Sea,

¹⁾ Aggregated catch statistics for the period 1868-1980 are shown in Appendix 4.

including the White Sea. Dorofeev (1956) estimated the pre-harvesting stock level at 3-3½ million animals. Cod, capelin, herring and Polar cod are the most important preys among the commercially valuable fish species. Among other food sources reported for the harp seal are krill, deep sea prawn, flatfish, redfish and molluscs (see e.g. Bjørge et al., 1981; Kapel, 1973; Myers, 1959; Sergeant, 1973).

The hooded seal (Crystophora cristata) in the Barents Sea area is especially found in the thick, drifting ice around Svalbard. Compared to other seals in the North Atlantic the hooded seal dive deeper and for that reason probably have a higher share of deep water species on its menu (Reeves and Ling, 1981; Sergeant, 1976). Due to its preference for deep water and thick, drifting ice, the segment of the North Atlantic stock found in the Barents Sea area is not very great. The number of animals in the mid 1950s in the total stock has been estimated to be 1/2 million animals (Reeves and Ling, 1981). Walrus (Odobenus rosmarus) is the largest of the seals in these areas, but the number of individuals are small (references in Fay, 1981). So are also bearded seal (Erignathus barbatus), grey seal (Hali-choerus grypus) and common seal (Phoca vitulina). The smallest, and probably the most abundant, seal in the northeast Atlantic and the Arctic Ocean is the ringed seal (Phoca hispida). Estimates of stock size vary widely, from 2.5 million to 6-7 million animals (Frost and Lowry, 1981; Stirling and Calvert, 1979). It is not known how many of these are present in the Barents Sea. About half of the ringed seal's food consists of fish of which polar cod (Boreogadus saida) is definitely the most important species (Gjertz and Lydersen, 1986).

Contrary to fish and seals which occupy the Barents Sea area all the year round, most of the whales utilize the area through intensive feeding in summer. In fall they migrate south to warmer parts of the Atlantic Ocean to breed and have their calves. These migratory patterns also are most efficient from a physiological point of view. The intensive summer feeding in the Arctic gives the animals a surplus of energy which is stored mainly as blubber. The storage of energy is

gradually decreased during winter time when food resources are scarce. The warmer climate of the temperature zone helps to save energy for purely life processes.

Despite earlier years intensive hunting of the big baleen whales, all species are still found in the Barents Sea area: Blue whale (Balaenoptera musculus), fin-whale (Balaenoptera physalus), humpback whale (Megaptera novaeangliae) and sei-whale (Balaenoptera borealis). In addition to these rare big baleen whales the smaller minke whale (Balaenoptera acutorostrata) still is plentiful with stock estimates for the early 1980s ranging between 50 000 and 100 000 individuals, with an average weight of 4 tons (Rørvik, 1981).

The blue whale and the sei-whale mostly feed on krill and other crustaceans animals, but they have also been observed preying on small pelagic fish like herring and capelin.

Small pelagic fish seem to form a higher share of the diet of the fin and humpback whales than for the other two species of big baleen whales. The humpback also preys on other kinds of fish like cod. The total number of big baleen whales in the Barents Sea is uncertain, but guesstimates say 2-3000 with a biomass of 80-120 thousand tons (references in Holm, 1983).

In addition to the five baleen whales described here, there used to be two other species in this part of the Arctic. The black right whale and the Greenland right whale were two plentiful, slow swimming species in the Barents Sea, especially around the islands of Svalbard. These stocks have, however, disappeared, probably because of heavy harvesting of them in the 17th century by English and Dutch whalers.

The toothed whales in the Barents Sea consist of 12 species of which 3 are rare. The group is very heterogeneous with respect to individual size, with the sperm whale (Physeter catodon) as the biggest (30-40 tons) and the porpoise (Phocaena phocaena) as the smallest (less than 100 kg). Common species include: pilot whale (Globicephala melaena), white whale (Delphiapeterus leucas), white-sided dolphin (Lagenorhynchus acutus), bottlenose whale (Hyperoodon ampullatus) and killer whale (Orcinus orca). The total biomass of sperm whales in the

Barents Sea area - in summer time - has been guesstimated to vary between 150 and 175 thousand tons in the 1950s, 1960s and 1970s. The biomass of smaller toothed whales has been guesstimated to be 50 thousand tons in the same time period. (See sources in Holm, 1983.)

The sperm whale's diet consists primarily of squid and deep water fish, but also a certain amount of cod fish. Food sources for the bottlenose, the most plentiful of the small toothed whales in this area, are much like those of the sperm whale, whereas the killer whale and the others mainly feed on fishes, such as cod and other demersal species and on pelagic species like herring and capelin.

A brief description of the ecological system of the Barents Sea area ought to include the polar bear (Ursus maritimus). The population biology of this species is well documented (Larsen, 1986a). The total population in the area from east Greenland to Franz Josef Land and Novaya Zemlya is now (mid 1980s) more than 5000 animals, of which 2500 are in the Svalbard area (Larsen, 1986b). Ninety percent of the polar bear's food is ringed seal, 5% other seals and the rest is fish. An important chain in the ecological system in this part of the Arctic seems to be polar cod - ringed seal - polar bear. Nevertheless it will not be included in this study since these species' commercial value has been of minor importance compared to many other living resources of the Barents Sea.

One of the main objectives of this study is to quantify to which extent each of the trophic levels of the marine ecology system should be harvested. Despite the complexity of such a system we shall consider just some of the species on the following three trophic levels: (1) plankton preying pelagic fish species, (2) carnivore fish species and (3) fish consuming sea mammals. The species considered are, however, among the most important with respect to biomass in the ecological system and with respect to value for the fishing industry.

6. ESTIMATING BIOLOGICAL PARAMETERS OF THE THREE SPECIES
BARENTS SEA MODEL

Before presenting the parameter estimates of the three species Barents Sea (TSB) model, we shall briefly describe some of the problems of parameter estimation in single species models. Almost all population analysis of North Atlantic fish stocks have been carried out on the basis of Beverton-Holt types of models (Beverton and Holt, 1957). The use of aggregated biomass models to assess North Atlantic stocks is almost non-existent in the literature. However, it is possible to estimate parameters in the latter types of models from parameters in the former type of models.

Applied fish population analysis of demersal species is usually based upon assumptions like: the existence of a stock-recruitment relation, age specific growth functions and constant natural mortality. Estimation of parameters is done on the basis of catch and effort data by means of e.g. Virtual Population Analysis (VPA). For cod in the North Atlantic Ocean estimates of biological parameters for several stocks are given in Garrod (1977). Some of the estimates for the three largest stocks are shown in Table 6.1. (The complete table is shown in Appendix 5.)

Table 6.1. Some characteristics of the largest Atlantic cod stocks.

	1	2	3
	Recruits 2 year old (in millions)	Maximum sustain- able yield per recruit (kg)	W_{MSY} ¹⁾ (in million metric tons)
Iceland	300	1.56	2.28
Arcto-Norwegian	1250	0.57	4.13
East New Found- land/Labrador	2000	0.40	3.16

Source: Garrod (1977) (also see Appendix 5).

¹⁾ W_{MSY} = Necessary stock level to produce maximum sustainable yield.

The single species logistic growth equation is the one used in the Schaefer model (Schaefer, 1954 and 1957):

$$\dot{W} = rW(1 - W/K) . \quad (6.1)$$

r is the intrinsic growth rate, and K is the carrying capacity of the ecological niche of the stock. The maximum sustainable yield (MSY) in this model is:

$$MSY = rK/4 \quad \text{for} \quad W_{MSY} = K/2 . \quad (6.2)$$

The growth curve, and therefore the sustainable yield curve, is symmetric around $W = K/2$. "Skewed" growth models, where the yield curve is asymmetrical and has its maximum at $W \neq K/2$, also have been used in fish population analysis (Pella and Tomlinson, 1969). However, we shall stick to the symmetrical growth model since this is the simplest one to handle and since the biological studies in this field do not contradict such a presumption regarding cod (Garrod, 1977).¹⁾ Using the data on maximum yield per recruit (Y/R), number of recruits (R) and MSY-biomass (W_{MSY}) in Table 6.1, the MSY figures can be calculated from the formula:

$$\hat{MSY} = (Y/R)R . \quad (6.3)$$

With the additional assumption that the yield pattern of the stock may be described by the logistic growth equation, r and K can be found from the formulas:

$$\hat{K} = 2W_{MSY} \quad (6.4)$$

$$\hat{r} = \frac{2\hat{MSY}}{W_{MSY}} , \quad (6.5)$$

where \hat{MSY} is from (6.3).

¹⁾ In a Beverton-Holt model with constant recruitment and age specific growth, yield per recruit and total yield are determined by the age of capture and the effort level. For a given age of capture and effort level a corresponding stock level exists. Therefore, it is possible to have a yield-stock relationship in a biomass model without assuming density dependent recruitment and growth.

Using data from Table 6.1 and the Equations (6.3)-(6.5), the calculated MSY, r and K are shown in Table 6.2.

Table 6.2. Calculated logistic growth equation parameters for three cod stocks.

	\hat{MSY} (million tons)	\hat{K} (million tons)	\hat{r}
Iceland	0.468	4.46	0.41
Arcto-Norwegian	0.713	8.26	0.35
East Newfoundland/Labrador	0.800	6.32	0.51

The parameters in Table 6.2 can not be directly used in two or three species models since they are derived on the assumption that sustainable yield is a "net sustainable yield" of which the total can be harvested by man. In contrast the prey-related parameters in a multispecies model must take care of the prey's production of food for the predator in addition to the net sustainable yield which can be harvested by man. Nonetheless the single species model and the calculated parameters in Table 6.2 can be of interest in the case of a stable environment for the cod stocks, i.e. for stable stocks of preys and predators and moderate variations in harvesting. In the case of increased harvesting of the preys or the predators of the cod stock we would expect a change in single species model parameters such that the sustainable yield estimates (included MSY) are also changed. For these reasons the calculated parameters of the single species model cannot be directly used in the three species model. We shall therefore have to rely on other methods where some of the parameters of the three stocks are "guesstimated" simultaneously, or where the interrelationships between parameters are taken care of in other ways.

By "guesstimation" is meant, in this connection, that relevant biological literature is studied and information essential to our problem is extracted. When two or more sources

give somewhat different figures for the same variable, parameter etc., a choice is made as to which to rely on or the average of them is used. All the steps and assumptions made are explained in detail so that the reader can critically appraise and modify them if that is felt to be desirable.

In the TSB-model there are 9 biological parameters:

The predation coefficients	:	a_{12} , a_{13} and a_{23}
Intrinsic growth rates	:	r_1 , r_2 and r_3
Carrying capacity related coefficients:		K , α and β

In the introductory part of this study references are given to some works by fisheries biologists on predators' consumption rates of preys. The methods used vary from very detailed and comprehensive stomach sampling surveys, e.g. ICES (1982) and Ponomarenko et al. (1978), via indirect methods using the received knowledge of the species' physiology, metabolism, etc., e.g. Sergeant (1969), to guesstimation and simulation technique, e.g. Christensen (1982), Laevastu and Favorite (1978) and Reed and Balchen (1981).

Based on several biological studies the predation coefficients have been guesstimated by the author elsewhere (Flaaten, 1984b; and Flaaten, 1984c). When stocks are measured in thousand tons the guesstimates are

$$\begin{aligned}\hat{a}_{12} &= 0.25 \cdot 10^{-6} \\ \hat{a}_{13} &= 0.14 \cdot 10^{-6} \\ \hat{a}_{23} &= 0.31 \cdot 10^{-6}\end{aligned}\tag{6.6}$$

The consumption functions used, $C_{ij} = a_{ij}W_iW_j$, are based on the assumption of opportunistic feeding. That is, each unit of predator always eats a constant portion of the prey stock. The "opposite" would be when one unit of the predator eats a constant quantity of the prey, independent of the size of the prey stock (provided that the prey stock is big enough to meet this demand). Such feeding can be termed specific feeding.

After having calculated the predation coefficients, there still remain six parameters to be estimated. Looking at the complete model in Equations (3.1)-(3.3) it should be noticed that the parameters r_1 , α , β , and K , occur in pairs in the three equations: r_1 and K in the first, r_2 and α in the second and r_3 and β in the third. In other words there are only two unknown parameters in each of the three equations (with the a_{ij} now known). This is an important observation which will be used to simplify the estimation procedure. In equilibrium, that is when $dW_i/dt = 0$ in Equations (3.1)-(3.3), all stocks and harvest rates are constant. In this case Equation (3.1) gives the following relation between r_1 and K , with equilibrium stocks and harvest rates denoted \bar{W}_1 and \bar{h}_1 , respectively.

$$r_1 = \frac{S_1}{1 - \bar{W}_1/K} \quad (6.7)$$

where S_1 is a constant equal to

$$S_1 = a_{12}\bar{W}_2 + a_{13}\bar{W}_3 + \bar{h}_3/\bar{W}_3 . \quad (6.8)$$

The equilibrium relation between r_2 and α follows from (3.2):

$$r_2 = \frac{S_2}{1 - \bar{W}_2/\alpha\bar{W}_1} \quad (6.9)$$

with S_2 equal to

$$S_2 = a_{23}\bar{W}_3 + \bar{h}_2/\bar{W}_2 . \quad (6.10)$$

Finally, the relation between r_3 and β is:

$$r_3 = \frac{S_3}{1 - \bar{W}_3/\beta(\bar{W}_1 + \bar{W}_2)} \quad (6.11)$$

with S_3 equal to

$$S_3 = \bar{h}_3/\bar{W}_3 . \quad (6.12)$$

With Equations (6.7), (6.9) and (6.11) the original problem of estimating six independent parameters have been reduced to that of estimating just three with the other three being given by the said equations. This result will be used later on in the guesstimation process, which will proceed in several steps.

First approach

As a first approach to guesstimation of the biological parameters, a couple of assumptions are made with respect to the equilibrium situation of the TSB-model:

1. The three species model has an equilibrium with catches and stock sizes equal to the averages for the years 1951-80, except for the stock of sea mammals where 1950-54 figures are used since more recent data are lacking for some of the whale stocks included in the sea mammal stock.
2. The values of the predation coefficients are as shown in (6.6).

Assumption 1 does not say that the ecological system of the Barents Sea has been in equilibrium in the period 1951-80, but it puts a restriction on the size of the system and its production capacity.

The equilibrium values of catches and fish stocks are shown in Table 6.3. In the last column of the table are shown the equilibrium consumption rates calculated from the other data in the table.

By using the data in Table 6.3 in Equations (6.7), (6.9) and (6.11) the following specific relationships between pairs of parameters are found:

$$r_1 = \frac{1.5557}{1 - 9.465/K} \quad (6.13)$$

with K measured in million metric tons.

$$r_2 = \frac{0.4771}{1 - 0.5496/\alpha} \quad (6.14)$$

$$r_3 = \frac{0.0216}{1 - 0.0582/\beta} \quad (6.15)$$

Table 6.3. Assumed equilibrium stocks and catch rates and derived consumption rates for the TSB-model. First approach.

i	\bar{w}_i ¹⁾ ('000 metric tons)	\bar{h}_2 ²⁾ ('000 metric tons per year)	\hat{a}_{ij} ³⁾	\bar{c}_{ij} ('000 metric tons per year)
1	9465	1285.6	$a_{12} = 0.25 \cdot 10^{-6}$	$\bar{c}_{12} = 12\ 309$
2	5202	1105.7	$a_{13} = 0.14 \cdot 10^{-6}$	$\bar{c}_{13} = 1\ 130$
3	853	18.4	$a_{23} = 0.31 \cdot 10^{-6}$	$\bar{c}_{23} = 1\ 376$

1) Average stock sizes 1951-80 for $i = 1, 2$ and 1950-54 for $i = 3$. Source: Flaaten (1984a and b).

2) Average catches 1951-80 for $i = 1, 2, 3$. Source: Flaaten and Holm (1984).

3) From (6.6).

Given the assumptions made, we know that all combinations of parameters, with (6.13)-(6.15) fulfilled, will give the same equilibrium values of the fish stocks, \bar{w}_i . However, it is not known for which combinations there will be a stable equilibrium and for which there will be an unstable equilibrium. Neither do we know for which combinations the dynamic behaviour of the system will be best. Intuitively we will prefer the ones where the simulated stocks behave as similarly as the actual fish stocks as possible (we shall return to this point later on).

It is reasonable to demand that the TSB-model with a specific set of parameters should fulfil the following two requirements:

1. With the annual harvest rates for 1868-1980 put into the simulation model the simulated stock levels for 1980 should be positive for all three species.

2. Without harvesting, i.e. for $A_i = 1$ in (3.6)-(3.8), there should be a locally stable equilibrium (or two or more equilibria) with positive stock levels for all three species.

The first requirement is reasonable since none of the three stocks have been extinct during the actual time period. The second one is reasonable since there probably was a pristine equilibrium¹⁾ in the ecosystem before man started his harvesting. It also can imply that if all harvesting came to an end, the system again would return to its former equilibrium.

A simulation model

The three species model described by the differential Equations (3.1)-(3.3), or even by the reduced form in (3.6)-(3.8) is too complex for an analytical solution to the system to be found. Such a solution would imply that for known initial level of the three stocks and for the known biological parameters of the model, the stocks at any point in time could be expressed as functions of initial stock levels, parameters and harvest rates. Even though an analytical solution can not be found, it is quite easy to find approximate solutions by use of computer simulation models.

By use of the computer program DYNAMO²⁾ (Pugh III, 1980) a program for simulating the system (3.1)-(3.3) (see Appendix 7) has been designed. The model has been implemented on a CYBER 171MP computer at the University of Tromsø. For an initial set of stock levels the model computes, in discrete time, the change in the stock levels during a short period of time and adds this to the initial levels. By repeating this computation many times the model can describe the development of the stocks for any period of time. By making the steps small enough the model simulations

¹⁾ However, see p. 17 for a brief discussion on limit cycle models

²⁾ DYNAMO is a compiler for translating and running continuous models (models described by a set of differential equations). It was developed by the industrial dynamics group at M.I.T. for simulating dynamic feedback models of business, economic, and social systems, but there is nothing in its design that precludes its use for any continuous system.

can be as accurate as desired. However, accuracy will be at the expense of using more computer resources.

For running the computer model it is necessary to know the initial stock levels, parameter values and harvest rates for the simulation period. It has been possible to find annual harvest rates of the Barents Sea area back to 1868 (Flaaten and Holm, 1984). Since the stock levels of capelin, cod and sea mammals are unknown for the initial year, 1868, they had to be chosen somewhat arbitrary to start with. In the final simulations the initial stocks are put equal to the equilibrium stock levels shown in Table 6.9.

For a specific set of values of the righthand side parameters of (6.13)-(6.15), K , α and β , the lefthand side parameters, r_1 , r_2 and r_3 , can be calculated. It should be noted that the r_i 's are decreasing functions of the r.h.s. parameters. Based on some knowledge of the ecosystem of the Barents Sea and information provided by studies based on one-species models, we have chosen to start with the set of parameters shown in Table 6.4.

Table 6.4. Parameter values based on (6.13)-(6.15).
First approach.

Exogeneous chosen:	$K = 30 \cdot 10^6$	$\alpha = 0.90$	$\beta = 0.08$
Calculated:	$r_1 = 2.2728$	$r_2 = 1.2254$	$r_3 = 0.0793$

The parameters in Table 6.4 are put into the simulation model. The problem of evaluating the results still has to be solved. One possibility is to use the sum of squares of differences between the simulated stocks and actual stocks. However, only the capelin and the cod stocks are known for a long enough period of time (1951-80), and even those data might include some errors (Flaaten, 1984b). For the sea mammals complete time series data are lacking, even though there are estimates for some stocks at some points in time (Flaaten, 1984b and c).

To explain the sum of squares method the following symbols are used:

W_{it}° = Known stock level of species i , at time t
 W_{it} = Simulated stock level of species i , at time t ,

The sum of squares of the differences of relative stock sizes is defined as

$$Q^2 = \sum_i \sum_t (W_{it} - W_{it}^{\circ}) / W_{it}^{\circ})^2 . \quad (6.16)$$

The aim of the simulation procedure should be to find the combination of parameters, which, via W_{it} , minimizes Q^2 . By using relative stock sizes instead of absolute sizes, we give each of the three species the same weight in the objective functional. Otherwise the species on the lowest trophic level would in practice be given a higher weight since it usually maintains a higher stock level than species on lower trophic levels do.

As an experiment the stocks of capelin and cod shown in Appendix 3 have been taken as given, while the stock of sea mammals has been excluded from (6.16). With the parameter-formulae (6.7), (6.9) and (6.11) put into the simulation model, the W_{it} 's (for capelin and cod) can be found for all specified combinations of the exogeneous parameters K , α and r_3 .¹⁾ A system of computer programs has been written to make these simulations and computations automatically. The chart flow in Appendix 6 shows the programs involved when the model is run.

As examples of the kind of results we get, Tables 6.5 and 6.6 show computed Q^2 -values for different combinations of K and α , for $\beta = 0.08$ and $\beta = 0.10$, respectively.

Experimenting with the simulation models has given us some insight to the problem of finding the best combination of biological parameters in the TSB-model. However, new questions and problems have also arisen as a result of these experiments. First of all, it does not seem possible to get Q^2 to reach a

¹⁾ For historical reasons (in the evolution of this work) r_3 , instead of β in (6.11) is exogeneous in the simulation model. This of course makes no change in the results.

Table 6.5. Computed Q^2 -values for $\beta = 0.08$ ($r_3 = 0.0893$).
First approach.

α $K \cdot 10^{-6}$	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.0
24	7.75	7.77	7.80	7.82	7.85	7.86	7.90	7.92	7.95	7.98	8.0
26	7.72	7.73	7.76	7.80	7.82	7.85	7.88	7.92	7.95	7.99	8.0
28	7.68	7.71	7.74	7.77	7.80	7.83	7.87	7.91	7.96	8.00	8.0
30	7.66	7.69	7.71	7.75	7.78	7.83	7.86	7.92	7.97	8.02	8.0
32	7.63	7.67	7.70	7.73	7.77	7.82	7.87	7.93	7.98	8.04	8.1
34	7.61	7.64	6.68	7.72	7.77	7.82	7.88	7.93	8.00	8.08	8.1
36	7.59	7.63	7.67	7.71	7.75	7.81	7.88	7.95	8.02	8.11	8.1

Note: Using formula (6.16), with W_{it}^* 's in Appendix 3 for capeli and cod for the years 1951-80. W_{it} 's are computed by mean of the DYNAMO simulation program shown in Appendix 7.

Table 6.6. Computed Q^2 -values for $\beta = 0.10$ ($r_3 = 0.0517$).
First approach.

α $K \cdot 10^{-6}$	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.0
24	7.73	7.75	7.76	7.79	7.80	7.82	7.85	7.86	7.89	7.91	7.9
26	7.69	7.71	7.73	7.74	7.77	7.80	7.83	7.85	7.88	7.91	7.9
28	7.66	7.68	7.70	7.72	7.74	7.78	7.80	7.84	7.88	7.93	7.9
30	7.63	7.64	7.67	7.70	7.73	7.76	7.81	7.85	7.88	7.93	7.9
32	7.60	7.62	7.65	7.67	7.71	7.75	7.79	7.84	7.90	7.95	8.0
34	7.57	7.60	7.62	7.66	7.70	7.74	7.79	7.85	7.91	7.97	8.0
36	7.56	7.58	7.61	7.65	7.69	7.74	7.79	7.86	7.92	8.00	8.0

Note: See note to Table 6.5.

minimum value for an interior set of parameter combinations.¹⁾ Secondly, it might be that the use of just two species in the objective functional, while the model also has a third species, produces some unsolved and unfortunate problems. Thirdly, the model framework we are working with might be too general to expect good empirical results. Fourthly, the stock estimates of cod and capelin are based on single species model. As noted earlier, such estimates could be biased because of changes in the harvest mix of species. Finally, the predation coefficients in (6.6) could be wrong. In a simulation framework this could probably be checked for by also varying these coefficients. If the restrictions put upon the parameters by (6.7), (6.9) and (6.11) were abolished, a larger simulation program could be made where all nine biological parameters in the TSB-model could be varied independently. Q^2 could then be calculated for a very large number of parameter combinations. This, however, is reckoned as being too time consuming and resource demanding to be included in this study. Leaving the simulation framework here we shall now return to the maximum sustainable yield frontiers (MSF) derived in Section 4.

The parameters in (6.6) and Table 6.4, which are the first approach values, can be used for computing the terminal points of the MSF. The procedure for this is described in Equations (4.18)-(4.23), and the computer program designed to do the proper calculations is shown in Appendix 2. The results for capelin and cod of course depend on the stock level of the top predator, the sea mammals. Table 6.7 shows the results for the absolute maximum sustainable yields of capelin and cod for three different stock levels of sea mammals.

¹⁾ This might be because of computer programming difficulties. For unknown reasons the program would not run more than a limited number of simulations. Several attempts have been made to solve these problems, included inquiries to the supplier of DYNAMO, but all in vain so far.

Table 6.7. Computed absolute maximum sustainable yields of capelin and cod. Million metric tons. First approach.

Sea mammals stock level \ Species	Extinct: 0	1950-54 level: 0.853	High level: 1.200
Capelin	17.05 ¹⁾	15.25 ¹⁾	14.53 ¹⁾
Cod	4.98 ²⁾	3.07 ²⁾	2.35 ²⁾

¹⁾ Provided that cod is extinct.

²⁾ Provided that capelin is unharvested by man.

Even though they are terminal points on the MSF locus, the results in Table 6.7 seem to be high compared with actual harvest rates. From the stock data in Appendix 3 it is clear that there has been a decrease in both capelin and cod stocks in the period 1951-80. In other words, the stock levels of capelin and cod have been reduced during this period by the fishermen's harvest and the sea mammals' consumption.

Let us pick an arbitrary point on the MSF locus for $W_3 = 0.853$ million m.t., say $SY_1 = 10.18$ million m.t. and $SY_2 = 1.98$ million m.t. (at this point $SY_3 = 0.02$ million m.t.). This is obviously far beyond the estimates given in the literature.¹⁾ Even a point like $SY_1 = 9.29$ million m.t. and $SY_2 = 1.69$ million m.t. on the MSF locus for $W_3 = 1.200$ million m.t. ($SY_3 = 0.01$ million tons) is significantly more than would be expected according to the biology literature. So far we seem to be trapped in the guesstimation procedure since we do not know if all nine parameters in first approach should be corrected, or just some of them. However, recent research on the feeding habits of cod in a North Norwegian fjord (Eliassen and Grotnes, 1985) gives some ideas as to which direction the guesstimation procedure should proceed.

¹⁾ See Garrod (1977); Hamre and Tjelmeland (1982); Dragesund et al. (1980); Sætersdal (1984); and references given in Flaaten (1984b).

Second approach

Balsfjorden in Northern Norway is a sheltered subarctic fjord which constitutes a miniature Barents Sea with respect to fish communities. Feeding habits of fish species in the fjord can therefore be expected to have much in common with the equivalent species in the larger ecosystem of the Barents Sea. Results in Eliassen and Grotnes (1985) and from work quoted therein (e.g. Klemetsen, 1982) might indicate that the first approach guesstimate of the coefficient a_{12} for cod's predation on capelin¹⁾ is too high. The guesstimate, $\hat{a}_{12} = 0.25 \cdot 10^{-6}$, in (6.6) is based on data from a period (1974-76) when the herring were practically extinct in the Barents Sea and adjacent areas. Because of the lack of stock estimates in Eliassen and Grotnes (1985)²⁾ and Klemetsen (1982), their data cannot be used for the guesstimates given in (6.6) in the same way as predation results in Ponomarenko et al. (1978) were used in Flaaten (1984c). We shall therefore choose to reduce a_{12} somewhat arbitrarily and see what the implications are for the TSB-model.

As noted earlier there are several reasons to believe that a_{12} in (6.6) is too high. However, we have not come across information that would lead to changes in a_{13} and a_{23} .

In this second approach of the guesstimation procedure we shall make the following two changes in the assumptions compared to the first approach:

- 1) $a_{12} = 0.125 \cdot 10^{-6}$
- 2) With average stock sizes and catch rates shown in Table 6.3 the annual relative growth rates of the three species are:

¹⁾ Remember that capelin in this report means capelin plus herring.

²⁾ Stock estimates will be published later (Eliassen, personal communication).

i	1	2	3
$\frac{dW_i/dt}{W_i}$	-0.0359	-0.0247	0.01

Assumption 2 is based on the observation of a decrease in the capelin and the cod stocks, as shown in Appendix 3. For the period 1950-80 the capelin stock decreased on average 3.59% p.a. and the cod stock 2.47% p.a.¹⁾ As noted before, we do not know the changes in the sea mammal stock as well as those of the two others. An annual increase of one per cent in the period may be right, but it could just as well have been an average of zero (for references to relevant literature, see Flaaten (1984c)).

With the assumptions made and by using the same method as described in the text concluding with (6.13)-(6.15), the specific relations between pairs of parameters now become:

$$r_1 = \frac{0.8696}{1 - 9.465/K} \quad (6.17)$$

$$r_2 = \frac{0.4523}{1 - 0.5496/\alpha} \quad (6.18)$$

$$r_3 = \frac{0.0316}{1 - 0.0582/\beta} \quad (6.19)$$

By a somewhat arbitrary choice of values of the r.h.s. parameters, (6.17)-(6.19) may be used to calculate the values of the l.h.s. parameters. The results are shown in Table 6.8.

Using Equations (3.9)-(3.11), the normalized stock levels equilibrium without harvesting (i.e. when $A_i = 1$ for $i = 1, 2, 3$) can be computed. With the second approach set of parameters from Table 6.8 the stock levels derived are shown in Table 6.9, for ordinary stocks as well as for normalized stocks.

¹⁾ The figures are found by linear regression on the formulas $W_i(t) = W_i^{1950} e^{a_i t}$ for $i = 1, 2$: $a_1 = -0.0359$ and $a_2 = -0.0247$

Table 6.8. Parameter values based on (6.17)-(6.19).
Second approach.

Exogeneous chosen	$K = 30 \cdot 10^6$	$\alpha = 0.90$	$\beta = 0.12$
Calculated	1.2704	1.1617	0.0614

Table 6.9. Equilibrium stocks without harvesting.
Second approach.

\bar{w}_1	\bar{x}_1	\bar{w}_2	\bar{x}_2	\bar{w}_3	\bar{x}_3
10.069 ¹⁾	0.336	4.759 ¹⁾	0.176	1.779 ¹⁾	0.549

¹⁾ Million metric tons.

If the Second approach parameters are correct, the equilibrium stocks shown in Table 6.9 are the pristine levels of Barents Sea area. For obvious reasons there are no stock assessment figures from pre-harvesting time to check our results against.

We are also interested in the stability of the pre-harvesting equilibrium. According to the criteria given in Appendix 8, there exists a unique non-harvesting equilibrium with all stocks positive if

$$\gamma_{23} \leq \alpha + \gamma_{13} . \quad (6.20)$$

Using the second approach parameters the following values of the γ 's are derived:

$$\begin{aligned} \gamma_{12} &= 2.6566 \\ \gamma_{13} &= 0.3571 \\ \gamma_{23} &= 0.8646 , \end{aligned} \quad (6.21)$$

and the inequality (6.20) is satisfied since $\alpha = 0.9$. According to Appendix 8 this equilibrium is locally stable if

$$x = \frac{4(\gamma_{12} - \alpha)\gamma_{23}}{B_1^2} \leq 1 \quad (6.22)$$

where

$$B_1 = -\alpha(1 + \gamma_{12}) - \gamma_{13}(1 + \alpha) + \alpha \gamma_{23} \quad (6.23)$$

in the non-harvesting case.

Inserting the values from (6.21) into (6.22) and (6.23) give

$$x = 0.5965$$

and inequality (6.22) is satisfied. The non-harvesting equilibrium therefore is a locally stable equilibrium.¹⁾

Having used the sum of squares method (see (6.16)) with the second approach parameters, the computed Q^2 values are shown in Tables 6.10 and 6.11.

Comparing the results in Tables 6.10 and 6.11 to those in Tables 6.5 and 6.6 makes it clear that according to the sum of squares criteria the second approach parameters perform better than the first approach parameters. This, however, is definitely not to say that the second approach is the best of all thinkable sets of parameters. Since time series data for the sea mammal stock are lacking, we should not expect to find the biological optimal set of parameters by use of the sum of squares method. As seen from Tables 6.10 and 6.11 the Q^2 -values are lower the smaller α is and the greater K is. Going down and to the left of $\alpha = 0.9$, $K = 30$ in the tables would, however, imply that $r_1 < r_2$, according to Equations (6.17) and (6.18). This would contradict received knowledge saying that the intrinsic growth rate of a species

¹⁾ In the same way it can be shown that with the first approach set of parameters the model has a locally stable equilibrium (without harvesting) with stocks:

$$\bar{W}_1 = 9.018, \bar{W}_2 = 5.699 \text{ and } \bar{W}_3 = 1.177 \text{ (all in million tons).}$$

Table 6.10. Computed Q^2 values for $\beta = 0.10$ ($r_3 = 0.0756$).
Second approach.

α $K \cdot 10^{-6}$	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00
24	6.26	6.40	6.55	6.70	6.87	7.05	7.25	7.48	7.72	8.00	8.30
26	6.21	6.36	6.52	6.70	6.88	7.09	7.31	7.57	7.85	8.16	8.51
28	6.16	6.32	6.50	6.69	6.90	7.13	7.38	7.65	7.96	8.31	8.71
30	6.12	6.30	6.49	6.69	6.92	7.16	7.43	7.73	8.07	8.46	8.91
32	6.09	6.28	6.48	6.70	6.93	7.19	7.48	7.81	8.18	8.60	9.09
34	6.06	6.26	6.48	6.70	6.95	7.22	7.54	7.88	8.29	8.74	9.28
36	6.04	6.25	6.46	6.71	6.97	7.26	7.59	7.96	8.38	8.87	9.46

Table 6.11. Computed Q^2 values for $\beta = 0.12$ ($r_3 = 0.0614$).
Second approach.

α $K \cdot 10^{-6}$	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00
24	6.24	6.39	6.54	6.72	6.90	7.11	7.33	7.58	7.86	8.16	8.52
26	6.19	6.36	6.53	6.72	6.92	7.15	7.41	7.68	8.00	8.35	8.75
28	6.15	6.33	6.52	6.74	6.96	7.20	7.48	7.78	8.14	8.53	8.98
30	6.12	6.31	6.51	6.73	6.97	7.24	7.54	7.88	8.25	8.69	9.20
32	6.09	6.29	6.51	6.74	7.00	7.28	7.61	7.97	8.38	8.87	9.42
34	6.06	6.28	6.50	6.76	7.03	7.33	7.67	8.05	8.51	9.02	9.63
36	6.04	6.27	6.51	6.76	7.05	7.36	7.73	8.14	8.63	9.17	9.84

usually is higher the lower the species is on the trophical ladder. Instead of spending more resources on refining the set of parameters, we shall therefore stick to that in Table 6.8 and call this the basic case. A summary of the basic case biological parameters is shown in Table 6.12.

Table 6.12. The basic case parameters of the TSB-model.

a_{12}	a_{13}	a_{23}	r_1	r_2	r_3	K	α	β
$0.125 \cdot 10^{-6}$	$0.14 \cdot 10^{-6}$	$0.31 \cdot 10^{-6}$	1.2704	1.1617	0.0614	$30 \cdot 10^6$	0.9	0.12

Sources: See the text.

With the basic case parameters the model's maximum sustainable yield frontier is shown in Figure 6.1 for four levels of the sea mammal stock.

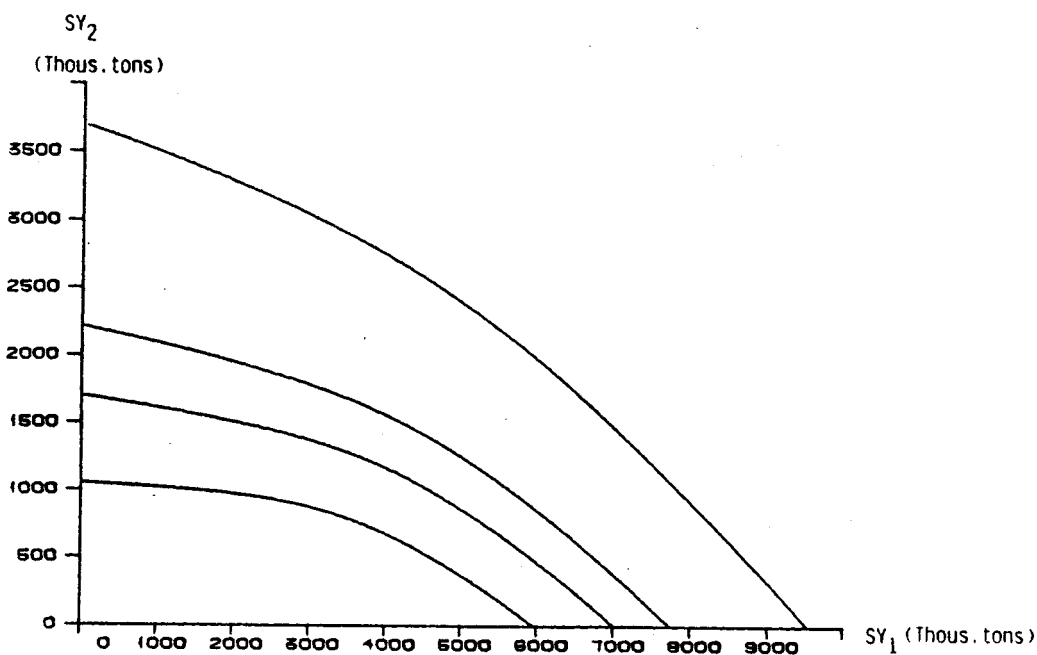


Figure 6.1. Maximum sustainable yield frontiers (MSF) for the TSB-model with basic case parameters.

The outermost MSF for cod and capelin is for sea mammal extinct ($W_3 = 0$), while the next one is for sea mammal at the 1950-54 level ($W_3 = 0.853$ million tons). The two other MSFs are for $W_3 = 1.200$ and $W_3 = 1.700$ million m.t., respectively.

In the case of no harvesting of the two predators, cod and sea mammals, the traditional sustainable yield locus for capelin is shown in Figure 6.2.

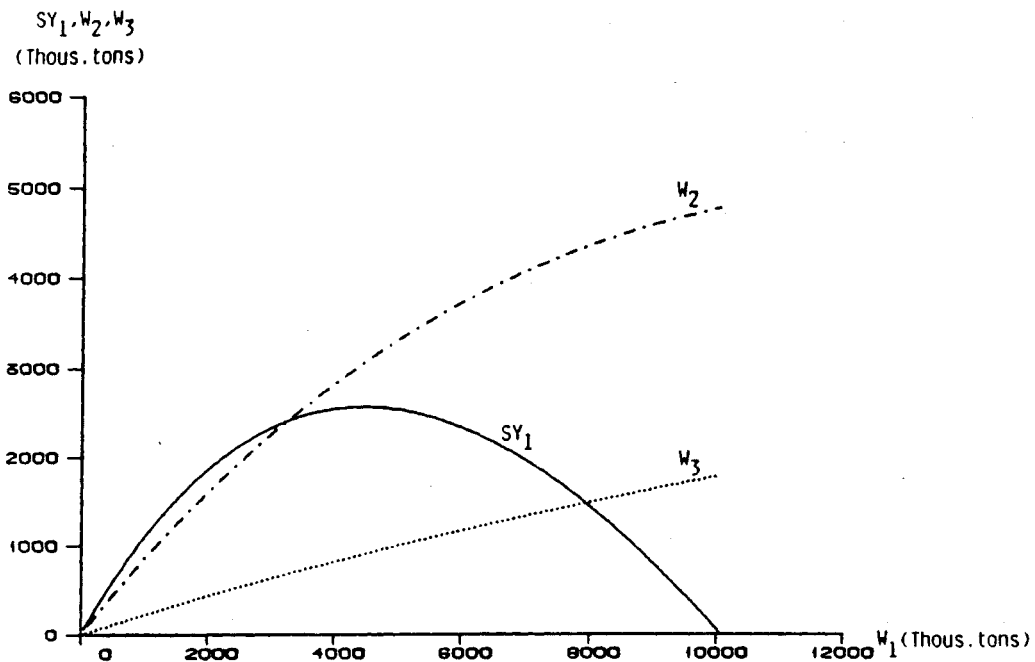


Figure 6.2. The sustainable yield curve (SY_1) for capelin in case of no harvesting of the predators in the TSB-model, with Basic case parameters. W_2 and W_3 are the corresponding unharvested stocks of cod and sea mammals respectively.

In this case the MSY for capelin is approximately 2.6 million m.t. for a stock level of 4.4 million m.t. To each level of the capelin stock the two corresponding stock levels of cod and sea mammals are also shown in Figure 6.2. Comparing Figures 6.2 and 6.1 makes it clear that a harvesting strategy leaving cod and sea mammals unharvested and only harvesting the basic prey, the

capelin, is not a good strategy. Such a strategy implies not only loss of cod and sea mammal yields, but also less capelin yield than is possible from the ecological system if more than just the lowest level of the system were harvested. The question of finding the best way of harvesting the three species is a question of economics, and the answer will depend on relative harvesting costs and product prices, as well as the discount rate.

7. ECONOMIC ASPECTS OF MULTISPECIES FISHERIES

The harvest rates introduced to the biological model represent a kind of predation effect from mankind. Since the beginning of time mankind has been harvesting natural resources both on land and offshore. In some cases the land resources, like grazing fields, hunting areas etc., have been common property resources. However, in most "civilized" cultures such resources have been private property, including governmental or collective forms of ownership where laws or social rules limit the individual's access to the resource. The living resources of the high seas on the contrary have a very long history of being common property, meaning that any member of the society who wanted to utilize the resource were free to do so. The only factors that influenced the number of participators were private benefits and costs.

In particular two kinds of questions regarding the use of common property resources have been raised by economists. Firstly, what will be the results of free access to the resource? Secondly, what would be the optimal utilization of the resource? The former involves questions of extinction, stock level, harvest rate, fishing effort, costs, benefits etc., and so also does the latter, but in addition other questions are raised: How to regulate the use of the resource, what will the rent be and, perhaps also, how to distribute the rent?

Studies of the economic use of fisheries resources have become numerous after the seminal article by the Canadian economist H. Scott Gordon (Gordon, 1954). Even long before that the Danish economist J. Warming wrote an article, unfortunately in Danish, on the same subject (Warming, 1911). An excellent review of the fisheries economics theory and literature is the article by Munro and Scott, 1985. The main result from the theory is that fishing effort should be limited by fiscal or other means, in order to gain a rent from the resource to the society.

Optimal harvesting

We are now returning to our multispecies model for a thorough study of the economic optimal harvesting of such an ecological system. In addition to the symbols already defined, the following ones will be needed

δ = The social rate of discount.

p_i = Price per unit standardized harvest of species i .

c_i = Cost per unit standardized harvest of species i .

b_i = Net profit per unit standardized harvest of species i .

π_i = Total profit from harvesting species i .

Let the growth of the species of a general n -species model be described by the following system of differential equations

$$\dot{X}_i = G_i(X_1, \dots, X_n) - y_i, \quad i = 1, \dots, n. \quad (7.1)$$

The following properties of the price, cost and profit functions will be assumed:

$$p_i = \text{constant}, \quad i = 1, \dots, n \quad (7.2)$$

$$c_i = c_i(X_i), \quad i = 1, \dots, n \quad (7.3)$$

$$b_i(X_i) = p_i - c_i(X_i), \quad i = 1, \dots, n \quad (7.4)$$

$$\pi_i = \pi_i(y_i, X_i) = b_i(X_i)y_i, \quad i = 1, \dots, n. \quad (7.5)$$

The optimal harvesting of the ecosystem is assumed to be equivalent to maximizing the objective functional

$$J = \int_0^{\infty} e^{-\delta t} (\sum_i b_i(X_i)y_i) dt \quad (7.6)$$

This is done subject to the state equations (7.1), as well as the usual constraints, including the control variables

$$0 \leq y_i \leq y_i^{\max}, \quad i = 1, \dots, n \quad (7.7)$$

The following procedure for solving this optimization problem is

the same as the one used by Clark (1976, ch. 9), for a two-species model.

Neglecting the abnormal case, the Hamiltonian is

$$H = e^{-\delta t} \left(\sum_i b_i(X_i) y_i \right) + \sum_i \lambda_i (G_i(X_1, \dots, X_n) - y_i) . \quad (7.8)$$

The λ_i are the adjoint variables. First we consider the case of "multi-singular" control, where the coefficients of the control variables y_i vanish identically

$$\frac{\partial H}{\partial y_i} = 0 \quad i = 1, \dots, n \quad (7.9)$$

$$\lambda_i = e^{-\delta t} b_i(X_i) \quad i = 1, \dots, n \quad (7.10)$$

$$\dot{\lambda}_i = -\delta e^{-\delta t} b_i(X_i) . \quad i = 1, \dots, n . \quad (7.11)$$

The adjoint equations are

$$\dot{\lambda}_i = -\frac{\partial H}{\partial X_i} = -e^{-\delta t} (b_i'(X_i) y_i) - \sum_j \lambda_j G_{ij} \quad (7.12)$$

where

$$G_{ij} = \frac{\partial G_j(X_1, \dots, X_n)}{\partial X_i} \quad \text{and} \quad b_i'(X_i) = \frac{db_i(X_i)}{dX_i} ,$$

$$i = 1, \dots, n .$$

Substituting for λ_i from (7.10) into (7.12) and using $y_i = G_i(X_1, \dots, X_n)$ in equilibrium (i.e. for $\dot{X}_i = 0$) we derive

$$\dot{\lambda}_i = -e^{-\delta t} (b_i'(X_i) G_i(X_1, \dots, X_n) + \sum_j b_j(X_j) G_{ij}) , \quad (7.13)$$

$$i = 1, \dots, n .$$

Equating $\dot{\lambda}_i$ from (7.11) and (7.13) gives

$$\sum_j b_j(X_j) G_{ij} + b_i'(X_i) G_i(X_1, \dots, X_n) = \delta b_i(X_i) , \quad (7.14)$$

$$i = 1, \dots, n .$$

Thus the case of "multi-singular" control in the n-species model corresponds to the case of singular control in the one-species model, known from Clark and Munro (1975):

$$b(X)G'(X) + b'(X)G(X) = b(X) \quad (7.15)$$

where $G'(X) = \frac{dG(X)}{dX}$. In particular equations (7.14) yield an optimal equilibrium $X_i = X_i^*$, $i = 1, \dots, n$. Essential to the multispecies model are the cross-dependencies, G_{ij} , which of course do not appear in the single species model. The economic interpretation of the result of the multispecies analysis is quite similar to that of the single species model. Dividing through (7.14) with $b_i(X_i)$ gives

$$G_{ii} + \sum_{j, j \neq i} \frac{b_j(X_j)G_{ij}}{b_i(X_i)} + \frac{b'_i(X_i)}{b_i(X_i)} G_i(X_1, \dots, X_n) = \delta, \quad (7.16)$$

$$i = 1, \dots, n.$$

The lefthand side is species i 's own rate of interest, which should equal the social rate of discount on the righthand side. The first two terms on the l.h.s. together form the instantaneous marginal product of the species. It consists of two parts, where the direct one (G_{ii}) is equivalent to the one in a single-species model. The second part is the indirect part of the instantaneous marginal product via other species. The last term on the l.h.s. is the marginal stock effect; that is, the cost-reducing effect an increase in the level of one species has on its own harvesting.

A common economic interpretation of the singular path of single species models can be generalized to cover the multi-species case. Let

$$\pi(X_1, \dots, X_n) = \sum_i b_i(X_i)G_i(X_1, \dots, X_n). \quad (7.17)$$

π is the total sustainable rent associated with sustainable harvesting at given stock levels. Then

$$\frac{\partial \pi}{\partial X_i} = b_i'(X_i)G_i(X_1, \dots, X_n) + \sum_j b_j(X_j)G_{ij} \quad (7.18)$$

$$i = 1, \dots, n,$$

where $G_{ij} = \frac{\partial G_j(X_1, \dots, X_n)}{\partial X_i}$. Now (7.16) can be written as

$$\frac{1}{\delta} \frac{\partial \pi}{\partial X_i} = b_i(X_i), \quad i = 1, \dots, n. \quad (7.19)$$

The lefthand side is often referred to as the marginal user cost which is the loss in present value of sustainable harvesting when the capital asset, the fish stock, is reduced by one marginal unit. The righthand side is the net current value of harvesting the stock at the margin. In other words, optimal harvesting of the ecological system requires that, for each of the stocks, which can be thought of as the assets of the social manager's resource portfolio, the present value of future losses from reducing the stock through harvesting should equal the current net benefits from that harvesting.

It is well-known from the analysis of single species models that an increase in the own price of fish has a negative effect on the optimal stock, whilst the effect of a cost increase is positive. These and other results of the single species analysis are shown in Table 7.1.

Table 7.1. Expected effects from parameter increases on optimal stock (W) and net present value (NPV) in the Schaefer model.

Parameter	Effect on	
	W	NPV
r	+	+
K	+	+
p	-	+
c	+	-
δ	-	-

The question is now - what will happen when there are two or more biologically interacting species in the ecological system? Since a n-species model usually is too complex for an analytical solution to be derived, we shall answer the question within the framework of a two species model.

To simplify the notation, let

$$\dot{X}_1 = f(X_1, X_2) \quad (7.20)$$

$$\dot{X}_2 = g(X_1, X_2) . \quad (7.21)$$

From (7.3) and (7.4) we have that the net profit per unit of harvest is a function of own price, effort cost and stock size:

$$b_i = b_i(p_i, c_i, X_i), \quad i = 1, 2 . \quad (7.22)$$

The partial derivatives of the unit profit functions have the following signs

$$b_{ip} = \frac{\partial b_i(\cdot)}{\partial p_i} > 0, \quad b_{ic} = \frac{\partial b_i(\cdot)}{\partial c_i} < 0, \quad b_{ix} = \frac{\partial b_i(\cdot)}{\partial x_i} > 0, \\ i = 1, 2 . \quad (7.23)$$

(7.23) tells that the two species are harvested cost-independent of each other and sold in separate markets.

Equations (7.16) and (7.17) implicitly give the state variables, the X_i 's, as functions of the biological and the economic parameters. Differentiating (7.19) with respect to p_1 , when $n = 2$, and rearranging somewhat gives:

$$\left(\frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x} \right) \frac{\partial X_1}{\partial p_1} + \frac{\partial^2 \pi}{\partial X_1 \partial X_2} \frac{\partial X_2}{\partial p_1} = \delta b_{1p} \quad (7.24)$$

$$\frac{\partial^2 \pi}{\partial X_2 \partial X_1} \frac{\partial X_1}{\partial p_1} + \left(\frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x} \right) \frac{\partial X_2}{\partial p_1} = 0 . \quad (7.25)$$

Using Cramer's rule we find from (7.24) and (7.25):

$$\frac{\partial X_1}{\partial p_1} = \frac{\delta b_{1p} \left(\frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x} \right)}{|D|} \quad (7.26)$$

$$\frac{\partial X_2}{\partial p_1} = \frac{-\delta b_{1p} \frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|} \quad (7.27)$$

$$\text{where } D = \begin{vmatrix} \frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x} & \frac{\partial^2 \pi}{\partial X_1 \partial X_2} \\ \frac{\partial^2 \pi}{\partial X_2 \partial X_1} & \frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x} \end{vmatrix}.$$

The second order conditions for the existence of an interior solution to the maximization problem in (7.6) are, for $n = 2$

$$\begin{vmatrix} \frac{\partial^2 \pi}{\partial X_1^2} & \frac{\partial^2 \pi}{\partial X_1 \partial X_2} \\ \frac{\partial^2 \pi}{\partial X_2 \partial X_1} & \frac{\partial^2 \pi}{\partial X_2^2} \end{vmatrix} > 0 \quad \text{and} \quad \frac{\partial^2 \pi}{\partial X_1^2} < 0,$$

and from this it follows that $\partial^2 \pi / \partial X_2^2 < 0$. The second order conditions imply $|D| > 0$ since $b_{ix} > 0$. Assuming the second order conditions are satisfied, it is seen from (7.26) that we have $\partial X_1 / \partial p_1 < 0$, since $b_{1p} > 0$ and $b_{2x} > 0$. Thus, the effect on the optimal prey stock from an own price increase is negative. This result is the same as for the single species model. To determine the effect from the increased prey price on the predator stock Equation (7.27) shows that it is necessary to know the sign of $\partial^2 \pi / \partial X_2 \partial X_1$. Using the growth functions (7.20) and (7.21) in (7.17) and differentiating twice gives

$$\frac{\partial^2 \pi}{\partial X_1 \partial X_2} = b_{1x} f_2 + b_1 f_{21} + b_2 g_{21} + b_{2x} g_1 \quad (7.28)$$

$$\frac{\partial^2 \pi}{\partial X_2 \partial X_1} = b_{1x} f_2 + b_1 f_{12} + b_2 g_{12} + b_{2x} g_1, \quad (7.29)$$

$$\text{where } f_i = \frac{\partial f(X_1, X_2)}{\partial X_i}, \quad g_i = \frac{\partial g(X_1, X_2)}{\partial X_i}, \quad f_{ij} = \frac{\partial f(X_1, X_2)}{\partial X_i \partial X_j} \quad \text{and}$$

$$g_{ij} = \frac{\partial g(X_1, X_2)}{\partial X_i \partial X_j}.$$

According to Young's theorem we have

$$\frac{\partial^2 \pi}{\partial X_1 \partial X_2} = \frac{\partial^2 \pi}{\partial X_2 \partial X_1}. \quad (7.30)$$

Using the explicitly formulated growth functions from the two species model, Equations (2.9') and (2.10'), the following partial derivatives of first and second order are derived:

$$\begin{aligned} f_1 &= r_1(1-2X_1-vX_2) & f_2 &= -vr_1X_1 \\ g_1 &= r_2X_2^2/X_1^2 & g_2 &= r_2(1-2X_2/X_1) \\ f_{11} &= -2r_1 & f_{12} &= -vr_1 \\ f_{21} &= -vr_1 & f_{22} &= 0 \\ g_{11} &= -2r_2X_2^2/X_1^3 & g_{12} &= 2r_2X_2/X_1^2 \\ g_{21} &= 2r_2X_2/X_1^2 & g_{22} &= -2r_2/X_1. \end{aligned} \quad (7.31)$$

Using (7.23) and (7.31) it is seen that the first two terms of (7.28) are both negative. The sign of the third one, b_2g_{21} , depends on whether the predator is valuable enough to be harvested at a positive net profit or if the harvesting costs exceed the proceeds. In the latter case, i.e. when $b_2 < 0$, the optimal solution requires payment of a bounty to the harvester to compensate his loss from harvesting at a low stock level. When $b_2 \ll 0$ at the optimum, the predator shall be called a "nuisance". Even though the last term, b_2Xg_1 , is positive, it is likely that it is outweighed by the three others, so that $\partial^2 \pi / \partial X_1 \partial X_2 < 0$, hence $\partial X_2 / \partial p_1 > 0$. In other words, when the predator is a nuisance, an increase in the price of the prey will lead to a larger optimal stock of the predator. Such a price change initially augments

the value of the resource capital of the prey proportionally to the price increase. However, it pays to transform some of the increased wealth into capital in general, rewarding the social manager with the interest expressed by the discount rate. This transformation may be controlled directly through harvesting of the prey, or indirectly by letting the predator harvest the prey. Hence, the effect of an increased price of the prey is an increased optimal stock of the predator. The investment in the predator stock is rewarded by increased revenues and reduced harvesting cost of this species.

The possibility of $\partial^2 \pi / \partial X_1 \partial X_2$ being positive does exist. To see under which conditions this is likely, we shall assume that the unit profit function is of the Schaefer type

$$b_i = p_i - \frac{c_i}{X_i}, \quad i = 1, 2 \quad (7.32)$$

where p_i is the price per unit of standardized harvest and c_i is the cost per unit of "fishing effort".¹⁾ Using (7.23), (7.31) and (7.32) and inserting into (7.28) gives the following result:

$$\begin{aligned} \frac{\partial^2 \pi}{\partial X_1 \partial X_2} &= -\frac{c_1}{X_2^2} \nu r_1 X_1 - \left(p_1 - \frac{c_1}{X_1}\right) \nu r_1 + \left(p_2 - \frac{c_2}{X_2}\right) \frac{2r_2 X_2}{X_1^2} \\ &+ \frac{c_2}{X_2^2} \frac{r_2 X_2^2}{X_1^2} = \frac{r_2 X_2 (2p_2 - c_2/X_2)}{X_1^2} - p_1 \nu r_1. \end{aligned} \quad (7.33)$$

When $b_2 \gg 0$ at the optimum, the predator will be called a "valuable". The likelihood of $\partial^2 \pi / \partial X_1 \partial X_2$ being positive is greater the more of a valuable the predator is, and the lower the price of the prey, p_1 , is. The more inefficient the predator is as transmuter of the prey, that is the lower ν is, the more likely this result will occur. To summarize, if p_2 and r_2 are large enough and/or p_1 , r_1 , ν and c_2 small enough, it is possible to have $\partial X_2 / \partial p_1 < 0$.

Having derived the partial derivatives of the optimal stocks with respect to the price of the prey, it is now easy

¹⁾ Precise definitions will be given below in Equations (8.1), (8.7) and (8.8).

to verify that the effects of a change in the effort cost of harvesting the prey are:

$$\frac{\partial X_1}{\partial c_1} = \frac{\delta b_{1c} \left(\frac{\partial^2 \pi}{X_2^2} - \delta b_{2x} \right)}{|D|} \quad (7.34)$$

$$\frac{\partial X_2}{\partial c_1} = \frac{-\delta b_{1c} \frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|} \quad (7.35)$$

Since b_{1c} and b_{1p} have opposite signs, the sign of (7.34) and (7.35) must be opposite to those of (7.26) and (7.27). In other words, the effect on the optimal prey stock of an increase in own effort cost is positive, whilst the effect on the predator stock depends on whether the predator is a nuisance or a valuable.

The effect on the optimal stocks from a change in the price of the predator is found by differentiating (7.19) with respect to p_2 , for $n = 2$, and solving the equations for $\partial X_1 / \partial p_2$ and $\partial X_2 / \partial p_2$:

$$\frac{\partial X_1}{\partial p_2} = \frac{-\delta b_{2p} \frac{\partial^2 \pi}{\partial X_1 \partial X_2}}{|D|} \quad (7.36)$$

$$\frac{\partial X_2}{\partial p_2} = \frac{\delta b_{2p} \left(\frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x} \right)}{|D|} \quad (7.37)$$

Since $b_{2p} > 0$, $b_{1x} > 0$ and $\partial^2 \pi / X_1^2 < 0$ we always have $\partial X_2 / \partial p_2 < 0$. The optimal predator stock is a decreasing function of its own price. Comparing $\partial X_1 / \partial p_2$ in (7.36) to $\partial X_2 / \partial p_1$ in (7.27) it is seen that they will always have the same sign, positive or negative, depending on whether $\partial^2 \pi / \partial X_1 \partial X_2$ ¹⁾ is negative or positive, respectively. According to the previous discussion on the sign

1) Recall $\frac{\partial^2 \pi}{\partial X_1 \partial X_2} = \frac{\partial^2 \pi}{\partial X_2 \partial X_1}$.

of $\partial^2\pi/\partial X_1\partial X_2$, it is most likely that if the predator is a nuisance a rise in the predator price causes a rise in the optimal prey stock ($\partial X_1/\partial p_2 > 0$). In this case the reduction in the optimal predator stock leaves more of the prey to be harvested by man at a lower unit harvesting cost caused by the increased prey stock.

On the other hand, if the predator is a valuable and the main value of the prey is as feed for the predator, we have seen that $\partial X_2/\partial p_1 < 0$, hence $\partial X_1/\partial p_2 < 0$. When the predator is the valuable resource, the optimal prey stock decreases as a result of an increase in the predator price. In this case the optimal prey and predator stocks move in the same direction.

The effects on optimal stocks from changes in the predator's effort cost are:

$$\frac{\partial X_1}{\partial c_2} = \frac{-\delta b_{2c} \frac{\partial^2 \pi}{\partial X_1 \partial X_2}}{|D|} \quad (7.38)$$

$$\frac{\partial X_2}{\partial c_2} = \frac{\delta b_{2c} \left(\frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x} \right)}{|D|} \quad (7.39)$$

Comparing (7.36) and (7.38) it is seen that $\partial X_1/\partial p_2$ and $\partial X_1/\partial c_2$ have opposite signs since b_{2p} and b_{2c} are opposite in sign. This is also the case for $\partial X_2/\partial p_2$ and $\partial X_2/\partial c_2$ as seen from (7.37) and (7.39).

To see how changes in the discount rate affect the optimal stock levels, we may use the same method as used for studying price changes. Differentiating (7.19) with respect to δ , for $n = 2$, and solving the equations for $\partial X_1/\partial \delta$ and $\partial X_2/\partial \delta$ gives these results:

$$\frac{\partial X_1}{\partial \delta} = \frac{b_1 \left(\frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x} \right) - b_2 \frac{\partial^2 \pi}{\partial X_1 \partial X_2}}{|D|} \quad (7.40)$$

$$\frac{\partial X_2}{\partial \delta} = \frac{b_2 \left(\frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1X} \right) - b_1 \frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|} \quad (7.41)$$

Since $\partial^2 \pi / \partial X_2^2 < 0$ and $b_{2X} > 0$, it is seen from (7.40) that when $\partial^2 \pi / \partial X_1 \partial X_1 < 0$ and $b_2 < 0$, $\partial X_1 / \partial \delta$ is unambiguously negative. This is a likely result when the predator is a nuisance. The optimal prey stock is reduced by an increase in the discount rate, and this result is clearly a parallel to the single species case. The increased discount rate makes it more costly to keep a large stock, therefore a part of it is transmuted into capital in general, yielding rent as expressed by the discount rate.

From (7.41) it is seen that since, by assumption, $\partial^2 \pi / \partial X_1^2 < 0$ and $b_{1X} > 0$, $\partial X_2 / \partial \delta$ is positive if $\partial^2 \pi / \partial X_1 \partial X_2 < 0$ and $b_2 < 0$.¹⁾ In the case where the predator is a nuisance, Equation (7.41) states the optimal management strategy is to increase this stock when the discount rate is increased. The larger predator stock helps reduce the prey to its lower optimal stock level caused by the increased discount rate. As noted above, a rise in the discount rate makes it more costly to keep a large prey stock, therefore a part of it is transmuted into capital in general. Another part is transmuted into predator resource capital, thus the losses from harvesting the predator are reduced because of the lowered unit harvesting cost and/or increased revenues.

When the predator is a valuable, that is if $b_2 \gg 0$ at the optimum, the possibility of $\partial X_2 / \partial \delta$ being negative emerges. In this case both types of resource capital, the predator and the prey stocks, are reduced by partial transformation into capital in general.

The possibility of simultaneously having $\partial^2 \pi / \partial X_1 \partial X_2 > 0$ and $b_2 < 0$, or vice versa, cannot be ruled out. Therefore, it may be possible to have $\partial X_1 / \partial \delta > 0$. According to (7.33), the case with negative unit profit of the predator, combined with a

¹⁾ The possibility of having $\partial X / \partial \delta > 0$ for one species have been asserted in Hannesson, 1983a.

positive, but low unit profit of the prey and poorly reproducing prey might produce this result.

The joint harvesting of a predator-prey ecological system can give the traditional result of a rise in the discount rate, decreased optimal stocks, or the untraditional result of an increase in one of the stocks. The latter result is most likely in cases where the predator clearly is a nuisance to be harvested not (only) for the sake of its own value, but to increase the availability and the profitability of the prey harvest.

Having derived the effects on the optimal stocks from partial changes in prices and discount rate, it can now be shown that there are close relationships between these effects. From (7.26), (7.27) and (7.40) we find

$$\begin{aligned} \frac{\partial X_1}{\partial \delta} &= \frac{\frac{b_1}{\delta b_{1p}} \left(\frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x} \right) \delta b_{1p} - \frac{b_2}{\delta b_{1p}} \frac{\partial^2 \pi}{\partial X_2 \partial X_1} \delta b_{1p}}{|D|} \\ &= \frac{1}{\delta b_{1p}} \left(b_1 \frac{\partial X_1}{\partial p_1} + b_2 \frac{\partial X_2}{\partial p_1} \right), \end{aligned} \quad (7.42)$$

since $\frac{\partial^2 \pi}{\partial X_1 \partial X_2} = \frac{\partial^2 \pi}{\partial X_2 \partial X_1}$.

From (7.36), (7.37) and (7.41) we find

$$\begin{aligned} \frac{\partial X_2}{\partial \delta} &= \frac{\frac{b_2}{\delta b_{2p}} \delta b_{2p} \left(\frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x} \right) - \frac{b_1}{\delta b_{2p}} \delta b_{2p} \frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|} \\ &= \frac{1}{\delta b_{2p}} \left(b_1 \frac{\partial X_1}{\partial p_2} + b_2 \frac{\partial X_2}{\partial p_2} \right). \end{aligned} \quad (7.43)$$

As shown in connection with (7.26) and (7.27), $\partial X_1 / \partial p_1$ and $\partial X_2 / \partial p_1$ are negative and positive, respectively, when the predator is a

nuisance. In this case (7.42) shows that $\partial X_1 / \partial \delta$ unambiguously is negative. In connection with (7.36) and (7.37) it has been shown that $\partial X_1 / \partial p_2$ is positive and $\partial X_2 / \partial p_2$ is negative when the predator is a nuisance. From (7.43) it now follows that $\partial X_2 / \partial \delta$ unambiguously is positive in this case. This result, that the optimal resource stock may increase with increased discount rate, is contrary to received wisdom from single species models.

Having derived some analytical results of the two species model, we now proceed with the three species model which will be applied to the Barents Sea fisheries in the next section. Rewriting Equation (7.16) in the three species case gives:

$$G_{11} + \frac{b_2(X_2)}{b_1(X_1)} G_{12} + \frac{b_3(X_2)}{b_1(X_1)} G_{13} + \frac{b_1'(X_1)}{b_1(X_1)} G_1(X_1) = \delta \quad (7.44)$$

$$\frac{b_1(X_1)}{b_2(X_2)} G_{21} + G_{22} + \frac{b_3(X_3)}{b_2(X_2)} G_{23} + \frac{b_2'(X_2)}{b_2(X_2)} G_2(X_2) = \delta \quad (7.45)$$

$$\frac{b_1(X_1)}{b_3(X_3)} G_{31} + \frac{b_2(X_2)}{b_3(X_3)} G_{32} + G_{33} + \frac{b_3'(X_3)}{b_3(X_3)} G_3(X_3) = \delta \quad (7.46)$$

Equations (7.44)-(7.46) implicitly determine the three state variables $X_1 = X_1^*$, $X_2 = X_2^*$ and $X_3 = X_3^*$ as functions of the biological parameters in the ecological model (3.6)-(3.8) and of the economic parameters in the price and cost functions (7.2)-(7.3). In this respect the three species model gives the same results as does the single species model: the optimal solutions of the state variables are independent of time. Once reached, the optimal combination of stock levels should be kept constant and the harvesting accordingly take place on a sustainable yield basis. The optimal sustainable harvest rates are implicitly given by the growth Equations (3.6)-(3.8) since the net growth of the stocks identically equals zero in equilibrium.

Because of the relatively complex dynamic properties of the model it has not been possible to derive any explicit solution for the optimal stocks from Equations (7.44)-(7.46). Therefore it has been necessary to design a computer program to find the

optimal solution for a given set of biological and economic parameters. The program OPT is shown in Appendix 11.

If the maximization problem does not have an interior solution, for example because one of the constraints on the harvest rates in (7.7) becomes binding, Equations (7.44)-(7.46) do not give the solution to this constrained maximization problem.¹⁾ In the long run there are few reasons to believe that the upper constraints should be binding in our case since the harvesting capacity can be expanded by building more vessels.²⁾ Since negative harvest rates have no biological meaning, the lower constraints in (7.7) might well become binding. If, for example, one of the three species has a sufficiently low price or high enough harvesting cost, it may be that the optimal solution implies no harvesting of that particular species.

Corner solutions can be checked for by use of the current value Hamiltonian and the necessary conditions for maximum. The current value Hamiltonian is defined by

$$H^0 = e^{\delta t} H = \sum_i b_i(X_i) y_i + e^{\delta t} \sum_i \lambda_i (G_i(X_1, X_2, X_3) - y_i), \quad (7.47)$$

and the current value adjoint variables by

$$\mu_i = e^{\delta t} \lambda_i, \quad i = 1, 2, 3. \quad (7.48)$$

If $y_1 = 0$ becomes binding, Equation (7.44) has to be replaced by

$$G_1(X_1, X_2, X_3) = 0. \quad (7.49)$$

The optimal stock levels can now be found from equations (7.43), (7.46) and (7.49). To test whether $y_1 = 0$ belongs to the optimal solution we have to check if

¹⁾ Constrained optimal control theory can be found in Kamien and Schwartz (1981).

²⁾ For a discussion on problems involved when the upper constraint becomes binding, see Clark and Munro (1975).

$$\mu_1 \geq b_1(X_1^*) \quad (7.50)$$

where X_1^* , together with X_2^* and X_3^* , are found from (7.45), (7.46) and (7.49). It can be shown (Seierstad and Sydsæter, 1987) that $\mu_i(t)$ satisfies

$$\dot{\mu}_i = - \frac{\partial H^0(\cdot)}{\partial X_i} + \delta \mu_i \quad (7.51)$$

In equilibrium, that is when $\dot{\mu}_i = 0$, this implies for $i = 1$

$$\mu_1 = (1/\delta) \sum_j \mu_j G_{1j}(X_1, X_2, X_3) \quad (7.52)$$

since $y_1 = 0$.

We can now solve for μ_1 and perform the test in (7.50).

So far we have been concerned about the optimal equilibrium solution (X_1^*, X_2^*, X_3^*) . The problem of determining the best way of moving from an initial point (X_1^0, X_2^0, X_3^0) to the aforementioned optimal one still remains to be solved. However, this seems to be a formidable piece of work to be left to the mathematicians. Even the case of two species is very difficult to handle in a general way (cf. Clark, 1976, Ch. 9.3). His conclusion on "practical approach paths" for a two species system is

Even the simplistic rule

$$h_1 = \begin{cases} h_{\max} & \text{if } X > X^* \\ 0 & \text{if } X < X^* \end{cases}$$

similar for h_2 , although suboptimal is surely a practically acceptable approach. The conclusion: if you know where you want to be and if many feasible approach paths are available, do not be concerned if the ideal path is not apparent. (Clark, 1976, p. 323).

Open access harvesting

In the case of open access harvesting it is well known from the fisheries economic literature that the resource rent will be dissipated. When all species in an ecosystem are common property resources, the equilibrium harvesting will take place such that none of the fisheries are rent yielding.¹⁾ In addition to lack of incentives to invest in the stock the individual fisherman also does not have incentives to consider the interdependencies of species. Necessary conditions for bioeconomic equilibrium are therefore found by using Equations (7.4) and (7.5):

$$\pi_i = (p_i - c_i(X_i))y_i = 0, \quad (i = 1, \dots, n). \quad (7.53)$$

Unless $y_i = 0$, which will be the case when $p_i < c_i(X_i)$, we must have

$$p_i = c_i(X_i), \quad (i = 1, \dots, n), \quad (7.54)$$

in equilibrium.

With the simple price and cost functions we have assumed, Equation (7.54) implicitly gives the open access equilibrium stocks as functions of own price and cost:

$$X_i^\infty = X_i(p_i, c_i), \quad i = 1, \dots, n. \quad (7.55)$$

This is to say that each of the stock levels only depends on own price and harvesting costs. Substituting from (7.55) into (7.1) gives the harvest rates in open access:

$$y_i = G_i(X_1(p_1, c_1), \dots, X_n(p_n, c_n)), \quad i = 1, \dots, n. \quad (7.56)$$

The open access harvest rates depend on all prices and effort costs, whilst the stock levels only depend on own price and effort cost. This is in case of an interior solution with all three species being harvested.

¹⁾ However, the possibility of having a consumer's surplus and/or a producer's surplus does exist. See Copes, 1972.

Whether the equilibrium point in (7.53) exists or not and whether it is stable or not are questions that in principle can be answered.

With stocks and harvest rates known we can use the tests in Appendix 8 to check if equilibrium and stability conditions are met.

In the case where

$$p_i < c_i(X_i^\infty) \quad (7.57)$$

for one of the three species, the open access fishery implies no harvesting of this species. Equilibrium stocks and harvest rates can still be found, but the procedure will be somewhat different from the case where the conditions in (7.54) are met for all $i = 1, \dots, n$. Let us have a closer look at the three species model and assume that harvest price and costs for species 2, cod, are such that (7.57) is fulfilled in equilibrium. To find the equilibrium stocks and harvest rates it is necessary to specify the cost functions $c_i(X_i)$. Assuming pure cod fish technology¹⁾ for all three species the cost functions are

$$c_i(X_i) = c_i/X_i, \quad (7.58)$$

where c_i is the unit harvesting cost for catching the last fish of the stock and is assumed to be constant. Using (7.58) and (7.54) the equilibrium stocks of capelin and sea mammals are found:

$$X_i^\infty = c_i/p_i \quad (i = 1,3) . \quad (7.59)$$

In addition to X_1^∞ and X_3^∞ we also know that

$$A_2 = 1 - F_2 = 1 \quad (7.60)$$

in equilibrium with no harvest of cod. By simple arithmetic the

¹⁾We shall return to the concept of cod fish technology in Section 8.

three species model in (3.6)-(3.8) gives the following equilibrium solutions for X_2 , F_1 and F_3 , with X_1 , X_3 and F_2 now known:

$$X_2^\infty = (A_1 - \gamma_{23}X_3^\infty)X_1^\infty \quad (7.61)$$

$$A_1 = X_1^\infty + \gamma_{12}(A_2 - \gamma_{23}X_3^\infty)X_1^\infty + \gamma_{13}X_3^\infty \quad (7.62)$$

$$A_3 = \alpha X_3^\infty / (X_1^\infty + \alpha(A_2 - \gamma_{23}X_3^\infty)X_1^\infty) \quad (7.63)$$

This is an example where the open-access stock size of one of the species, cod, does not depend on own price and effort cost, but on stock sizes of the two other species, capelin and sea mammals, as shown in Equation (7.61). Substituting A_i and X_i ($i = 1, 2, 3$) from (7.59)-(7.63) into the harvest functions (3.5), and remembering that $A_i = 1 - F_i$, the open-access equilibrium harvest rates are found for this special case.

In the next section are derived the economic parameters necessary for application of the three species bioeconomic model to the Barents Sea fisheries. The optimal solutions will be presented and discussed in Section 9, and the open-access solution in Section 10.

8. ESTIMATING ECONOMIC PARAMETERS OF THE THREE SPECIES BARENTS SEA MODEL

The cod fisheries

Since 1968 the Directorate of Fisheries has conducted cost and income surveys for the Norwegian fleet of fishing vessels above 40 feet operating year round. In 1980 they also started undertaking such surveys for the smaller inshore boats.

In this study we use the average cod-fish vessel above 40 feet as the numeraire for fishing effort in the cod sector of the three species model. The average cod-fish vessel is calculated from a very heterogeneous group which consists of inshore and near-shore vessels as well as large sea-going freezing trawlers. The latter are, according to usual profitability measures, the most profitable vessels in the Norwegian fishing fleet for demersal species. Therefore calculations will also be performed with the average freezing factory trawler as the numeraire.

The data needed for each class of vessels are

- i) quantity and price of catch
- ii) total costs (capital and operating costs, including labour).

In the short run analysis hull, engine, electronic equipment, gear etc. are fixed factors of production, while fuel, bait etc. are variable. A bioeconomic study is primarily a long run analysis; therefore the average vessel fully equipped and with all factors of production necessary for one year of operation will be used as the unit of fishing effort. In other words, the total fishing effort targeted at the stock can be varied by varying the number of standardized (average) vessel years.

Table 8.1 shows quantity, value and price figures for the years 1978-80 for the average vessel in the cod fisheries and for the average Norwegian factory vessel. Prices in the second column include the price support from the Government to vessels fishing for demersal species, except the factory vessels which

Table 8.1. Catch, price and cost data.¹⁾ Cod fisheries²⁾ 1978-80.

Vessel group	Year	Catch				Cost ⁵⁾	
		1	2	3	4	5	6
		Quantity ⁴⁾ (metric tons)	Price (1000 Nkr/ m.t.)	Net price (1000 Nkr/ m.t.)	Value (1000 Nkr)	Alternative A ⁶⁾ (1000 Nkr)	Alternative B ⁶⁾ (1000 Nkr)
Average Cod- vessel ³⁾	1978	464.5	2.4816	2.1915	1152.7	1337.9	1207.2
	1979	470.0	2.7023	2.2883	1270.1	1468.6	1350.3
	1980	420.7	3.1648	2.5039	1331.4	1533.5	1385.1
	average	451.7	2.7829	2.3279	1251.4	1446.7	1314.2
Factory vessel ⁷⁾	1978	2796.8	4.6743		13073.4	13874.4	13498.2
	1979	2917.8	5.2813		15409.7	14637.2	13329.0
	1980	2407.3	4.5306		10906.3	13717.3	12923.0
	average	2707.3	4.8287		13129.9	14076.3	13250.1

1) Nominal prices.

2) Distant water and coastal fisheries primarily for cod, haddock, saithe and redfish (wet fish trawlers and factory vessels included).

3) The following sources have been used: Value and cost figures: Lønnsomhetsundersøkelser (1978), (1979) and (1980). Price: Calculated from Fiskeri Statistikk (1981), Table 10. Net price: Price support from the Government have been deducted from price (Kristoffersen and Flaaten, 1984). Quantity: Calculated by dividing value by price.

4) Live weight.

5) Including remuneration of labour (wage, share, skipper's share etc.).

6) Alternative A: Includes replacement cost depreciation on vessel and calculated interest on owner's net capital. Alternative B: Includes depreciation on vessel based on historic cost, interest on owner's net capital is not included.

7) Source for quantity and value: Årsberetning vedk. Norges fiskerier (1978) and (1980). Prices have been calculated ($P = V/Q$). Source for cost: Depreciation and interest on owner's capital are from Lønnsomhetsundersøkelser (1978), (1979) and (1980), and the rest are from Årsberetning vedk. Norges fiskerier (1978) and (1980).

are not eligible for such subsidies. The distribution of subsidies between "producers" and "consumers", which in this case means the fish harvesting and the processing industry, respectively, as usual depends on elasticity relations on the two sides. In this case we have assumed infinitely elastic demand for raw fish, so that all the subsidies go to the harvesting sector. 80-90% of the value of the Norwegian harvest of demersal species are exported to the world market where Norwegian producers are price takers for the majority of their products. This is the reason for using the assumption of infinitely elastic demand for raw fish. Prices net of subsidies from the Government are shown in the third column in Table 8.1.

The cost of fishing effort can be calculated in many ways. There might be deviations between social and private costs, and each can be defined in different ways. For our purpose, a long run bioeconomic analysis, we would like to use social costs, including the opportunity cost of capital and labour. For labour costs we have chosen to use the actual remuneration of fishermen, including lay income, wage, skipper's lay, etc. Alternative A in Table 8.1, column 5 assumes replacement-cost depreciation of the vessel, and includes calculated interest on owner's net capital. Alternative B in column 6 is based on the historic-cost of the vessel, and interest on owner's net capital is not included. For all years, the costs according to alternative A exceed the value of the catch for the average cod vessel as well as for the average factory vessel. Except for factory vessels in 1979, this is also the case for alternative B, which is the lower cost alternative. In open access fisheries with homogeneous vessels one would expect average cost of fishing effort to equal average revenues in bioeconomic equilibrium. In the case of Norwegian cod fisheries the fleet is definitely not homogeneous and this should increase the possibility of the existence of "producers surplus" or "intramarginal rent" in the fishery (Copes, 1972). However, in this case there is what we can call a small negative producers surplus, and we can think of several possible explanations for this:

- i) There can be something wrong with the accounting surveys, either in the methods used or in the data the fishermen have reported.
- ii) Fishing vessel owners also have revenues from other than the sale of catches; for example, interest on bank deposits and other financial incomes, skipper's lay, freight revenues, etc. In 1980, as a compensation for increased fuel prices, they received some Government subsidies which was not directly connected to actual cost or income.
- iii) The fishery need not have been in bioeconomic equilibrium in the years 1978-1980. Overinvestment in the fleet in earlier years could have resulted in depreciation exceeding the difference between total proceeds and operating costs. After some years of negative investment in the fleet the open access fishery might reach bioeconomic equilibrium (Clark, Clarke and Munro, 1979).

Explanation i) is probably of minor importance in this case since the surveys have been conducted every year since 1968 and the methods used seem to be adequate. The results have been extensively used by Government branches, the fishing industry and researchers and no serious objections have been raised against the surveys. Regarding ii), the accounting surveys for the years 1978-80 reveal income other than from sale of catches equivalent to 2-6% of total revenues.

We shall now leave these problems and stick to the cost figures in Table 8.1.

The capelin fisheries

In Table 8.2, column 1, are shown average catches for all vessels in the capelin fleet and for large purse seiners with a licensed cargo capacity of 8,000 hectoliters or more. The figures are for the years 1978, 1979 and 1980 and for the average of all three years. Average prices paid to fishermen are shown in the second column, and the value of the catches in the third.

Table 8.2. Catch, price and cost data.¹⁾ Capelin fisheries²⁾ 1978-80.

Vessel group	Year	1			2		3	4		5
		Quantity ⁴⁾ (metric tons)	Price (1000 Nkr/ m.t.)	Value (1000 Nkr)	Catch		Value (1000 Nkr)	Alternative A ⁶⁾ (1000 Nkr)	Alternative B ⁶⁾ (1000 Nkr)	Cost ⁵⁾
Average Capelin-vessel	1978	4378.8	0.5011	2194.2		2194.2	2941.5	2513.9		
	1979	4927.0	0.4907	2417.9		2417.9	3292.4	2789.3		
	1980	4578.1	0.5935	2727.1		2727.1	3494.8	2898.1		
	average	4628.0	0.5284	2443.1		2443.1	3242.9	2733.8		
Large purse seiner ⁷⁾	1978	9863.1	0.5011	4942.4		4942.4	6524.3	5500.6		
	1979	9753.0	0.4907	4785.8		4785.8	6765.2	5897.9		
	1980	8942.9	0.5935	5307.6		5307.6	6993.9	5754.2		
	average	9519.7	0.5284	5011.9		5011.9	6761.1	5717.6		

1) Nominal prices.

2) Fishing with seine and trawl for herring, mackerel, capelin, sprat, Norway pout, sandeel, blue whiting and polar cod.

3) The following sources have been used: Value and cost figures: Lønnsomhetsundersøkelser (1978), (1979) and (1980). Price: Calculated from Fiskeri Statistikk (1981), Table 10. Quantity: Calculated by dividing value by price.

4) Live weight.

5) Including remuneration of labour (wage, share, skipper's share etc.).

6) Alternative A: Includes replacement-cost depreciation on vessel and calculated interest on owner's net capital. Alternative B: Includes depreciation on vessel based on historic-cost; interest on owner's net capital is not included.

7) Value and cost figures are from: Lønnsomhetsundersøkelser (1978), (1979) and (1980). Price has been calculated from Fiskeri Statistikk (1981), Table 10. Quantity has been calculated (Q = V/P).

Governmental price support have been practically non existing in this part of the fishing industry. Average total cost per vessel have been calculated the same way as for the cod vessels. The results are shown in columns 4 and 5.

The average capelin vessel includes purse seiners between 100 and 220 feet long, with cargo capacity ranging from 150 to 1,500 metric tons, as well as trawlers between 70 and 130 feet long. Formally both purse seine and trawl fishing for capelin have been under licensing restrictions for many years; purse seining since 1973 and trawl fishing even longer (Fiskeridepartementet, 1983). In the former case, the licensing system has been an effective barrier to entry, whereas the licenses for trawl fishing for capelin, sandeel, Norway pout, etc. were more like a registration system until 1979 when the Government effectively stopped issuing new licenses.

In the years 1978-1980 limited entry by licensing has been combined with individual vessel quotas in the most valuable fishery: the capelin fishery in the Barents Sea and on the coast of Finnmark. In these years the capelin counted for 75-80% of the total value of the purse seiners' catch. The other 20-25% was mainly mackerel, herring and blue whiting. The use of limited entry schemes in the capelin fishery has given the participating vessels and crews a share in the resource rent. Without the introduction of licenses and quotas in the early 1970s, the capelin stock would probably have been heavily depleted due to entry of many vessels made redundant after the depletion of the rich herring and mackerel stocks (Norwegian Spring Spawning Herring, North Sea herring and North Sea mackerel) in the 1960s and early 1970s.¹⁾

Norwegian limited entry schemes were mainly introduced in the capelin fishery to avoid collapse of the stock and not primarily to manage the fishery in an economically optimal manner. That would have meant far fewer, and bigger, vessels than those actually participating in the fishery (Flåm, 1980). It is easy

¹⁾ The restrictions on entry combined with quotas obviously were not enough to avoid the depletion of the stock throughout the 1980s. A nearly collapse of the stock in 1985 therefore necessitated practically nil-quotas for 1986 and 1987.

to understand this from the fact that "whole year operated" purse seiners in 1978 and 1980 operated 34.5 and 29.5 weeks respectively, whereas the average for all "whole year operated" vessels were 39.0 and 38.5 weeks. Fewer purse seiners with higher average operating time per year would have reduced the capital costs for the fleet. The cost data for the average capelin vessel presented in Table 8.2 do not take such things into consideration, but only presents actual reported costs for the years 1978-1980.

It can be seen from Table 8.2 that for the capelin fleet as well the total average cost exceeds average revenues not only in the case of alternative A but also for the lower cost alternative B. The reasons for this are much the same as mentioned for the cod fleet. Due to institutional arrangements in the management of the Norwegian capelin fishery the vessels are not allowed to utilize their full catching potential, and the redundancy rates in many cases are higher for the larger than for the smaller vessels. This result is mainly created through the allocation system for vessel quotas, and also by the regulated transportation of capelin from the fishing grounds to the reduction plants. The vessels get an individual quota which is an increasing, concave function of licensed cargo capacity, and the larger vessels are told by the Regulatory board to transport their catches to plants further away from fishing grounds than the smaller ones. These are probably the main reason why the large purse seiners have higher costs per ton caught than the average for all seiners.

Whaling and sealing

Whaling and sealing in Norway are typical seasonal activities conducted by multipurpose vessels. In this study, however, revenues and cost figures for vessel year equivalents (VYE) are needed, and recalculation from seasonal data will therefore be necessary.

Since the mid 1970s the only kind of whaling in Norway has been that of small whales in the North East Atlantic by small vessels (usually between 50 and 90 feet), while the harvesting

Table 8.3. Data on Norwegian whaling in the North Atlantic, 1978-80. (Nominal prices).

1 Year	2 No. of vessels ¹⁾	3 Average length of vessels ¹⁾ (feet)	4 Average catch per vessel		5 Whaling in % of total annual operation ³⁾			6 Calculated VYE		7 Total cost per vessel ⁴⁾			
			Quantity ²⁾ (m.t., Live weight)	Revenues ¹⁾ (1000 Nkr)	Price (1000 Nkr/ton)	(i)	(ii)	(iii)	(iv)	Quantity (m.t., Live weight)	Revenues (1000 Nkr)	Alternative A (1000 Nkr)	Alternative B (1000 Nkr)
1978	87	67.0	64.0	249.8 ¹⁾ (557.6)	3.903	44.8	35	34	38	168.5	657.4	670.3	574.8
1979	84	66.4	83.7	371.5 ¹⁾ (740.4)	4.438	50.2	--	--	45	186.0	825.6	881.0	723.4
1980	89	68.6	79.5	400.2 ¹⁾ (1042.9)	5.034	38.4	36	35	37	215.0	1081.6	1265.1	1076.4
Weighted average	86.7	67.3	75.7	340.6	4.499					199.0	854.8	942.0	794.5

1) Source: Småhvalfangsten, 1978; 1979; 1980. Revenue data in parenthesis are from Lønnsomhetsundersøkelsen, 1978; 1979; 1980 (Vesselgroup no. 29/1978 and no. 28 for 1979 and 1980: "Smallwhaling") and include all fishing activities.

2) Calculated from "live weight" data in Flaaten (1984b).

3) (i) Revenues from whaling in % of total revenues, according to sources in note 1).

(ii)-(iii) According to sources in note 1): Time spent on harvest by "use of gun" and "whaling-operations", respectively, as % of total time spent on fishing and whaling.

(iv) "Average" of the figures in (i)-(iii); to be used in the calculation of catch of whale and revenues from whaling per vessel year equivalent (VYE) in column 6.

4) See note 6 in Table 8.1.

Table 8.4. Data on Norwegian sealing. *) 1978-80. (Nominal prices.)

1 Year	2 No. of vessels ¹⁾	3 Average vessel size ¹⁾ (feet)	4 Average catch per vessel ²⁾			5 Days at sealing in % of no. of days in operation	6 Calculated VYE		7 Cost per vessel ⁴⁾	
			Quantity (m.t., live weight)	Value (1000 Nkr)	Price (1000 Nkr/m.t.)		Quantity (m.t., live weight)	Value (1000 Nkr)	Alternative A (1000 Nkr)	Alternative B (1000 Nkr)
1978	19	118.8	304.8	501.5	1.645	18.5	1647.3	2710.3	4211.9	3546.9
1979	18	121.1	417.2	711.6	1.706	18.1	2304.0	3929.8	4558.6	3849.4
1980	15	125.7	405.0	686.1	1.694	24.7	1642.4	2783.2	4607.5	3883.8
Weighted average	17.3	121.6	372.6	627.5	1.684	20.2	1844.6	3106.4	4446.0	3748.8

*) Average figures for sealing on three main catching areas referred to in the text.

1) Source: Selfangsten, 1978; 1979; 1980.

2) Calculated from sources in note 1.

3) Sources: No. of days at sealing are from Selfangsten, 1978; 1979; 1980, and total no. of days in operation are from Lønnsomhetsundersøkelsene (Vesselgroup no. 21/1978 and no. 22/1979 and 1980: Purse seiners 4000-5999 hectoliter licensed cargo capacity).

4) See note 6 in Table 8.1.

of the larger baleen whales (fin, sei, humpback etc.) came to an end in the early 1970s due to low profitability in the industry.

In Table 8.3 are shown data for Norwegian whaling in the North Atlantic for the years 1978-80. The quantity figures shown are live weight, and prices are calculated per unit of live weight. The VYE data in column 6 tell what quantity, and the corresponding value, a vessel operating year round would have caught if the availability of whales and other factors affecting the productivity had been the same for the whole year as for the actual harvesting season.¹⁾

In Table 8.4 are shown similar data for Norwegian sealing in the North Atlantic areas. Sealing has usually taken place on three different catching grounds: Newfoundland, the West Ice (Jan Mayen - Greenland area) and the East Ice (The White Sea and the Barents Sea area). The average size of sealing vessels has been largest on the Newfoundland grounds, while the East-Ice vessels have been the smallest. Due to lack of specific accounting surveys for the sealing industry, we have chosen to let the cost side be represented by purse seiners of 4000-5999 hecto-liter cargo capacity. Discussions with industry representatives revealed that this vessel group can be used as a proxy for the average sealer regarding size, catching capacity and total costs.

As seen before for the capelin and the codfish industries, revenues were usually exceeded somewhat by total costs. This is also the case for the sealing industry where the costs (alternatives A and B) exceed calculated VYE revenues in seven out of eight possible comparisons between columns 6 and 7 in Table 8.4. In addition to the possible explanations for this as discussed in the preceding sections, it is possible that we have made a mistake in choice of proxy for the cost side of the sealing industry. However, having discussed this matter with the industry representatives, it seems most likely that the cost data in Table 8.4 are not far from the truth. The economic situation in the

¹⁾ VYE are used in an attempt to keep the applied part of the analysis as simple as possible and in accordance with the theoretical part. Seasonal variations in economic, biological or technical factors, e.g. the catchability coefficient, of course could have been introduced and this would have made the analysis more realistic, but on the cost of simplicity (see Flaaten, 1983; and Flaaten 1987b).

Table 8.5. Data for a designed general purpose sea mammal harvesting vessel. 1978-80.

1	2		3		4	5	
	Catch per vessel		Total cost per vessel ²⁾				
	Quantity ¹⁾ (m.t., Live weight)	Value (1000 Nkr)	Alternative A (1000 Nkr)	Alternative B (1000 Nkr)			
Average 1978-80	322.4	1032.6	3.203	1222.3	1030.8	9111	No. of SM vessels (VYE) 30.7

1) 1 ton of SM consists of 54% whale and 46% seal.

2) See note 6 in Table 8.1.

3) Includes all catches of sea mammals in the Barents Sea and adjacent areas.

sealing industry has been gradually worsening during the 1970s, due to reduced quotas and declining prices (in real terms) for industry products. Unlike most parts of the Norwegian fishing industry, the sealing industry rarely received subsidies from the Government until 1982 when economic conditions were so severe that the whole harvesting was threatened to come to an end. For this reason the discrepancies between revenue and cost data in Table 8.4 might be reasonable, and not unexpected.

Vessels in the Norwegian whaling fleet are usually only 50-90 feet long, whereas the sealers are 100-160 feet. Nevertheless we shall design a standardized vessel for sea mammal harvesting to suit the three species model. The ratio between the average total catch of whales and the average total catch of seal biomass in the period 1978-80 is 1.17. Requiring one unit of the sea mammal (SM) vessel to catch whales and seals in the same ratio, and by using the VYE figures from Tables 8.3 and 8.4, we find that a SM vessel should consist of 92% of a VYE whaler and 8% of a VYE sealer. The data for such a multipurpose SM harvesting vessel are shown in Table 8.5. Dividing the average annual total catch of whales and seals in column 4 by the catching capacity in column 2, we get the number of constructed SM vessels (measured in VYE) in column 5 which would have been necessary to harvest the actual catch of whale and seal biomass. It should also be noticed that using cost alternative B the vessel's harvest revenues would just have exceeded the costs.

Standardized prices and costs

Normalized prices and costs in Equations (7.2)-(7.5) can be found by calculations from ordinary revenue and cost survey generated data. The following price symbols will be used:

p_i^0 = Price per unit harvest (i.e., per unit of h_i)

p_i = Price per unit standardized harvest (i.e., per unit of y_i).

The prices are related in the following way (when substituting for $y_i = h_i/K_i$):

$$p_i = \frac{p_i^0 h_i}{Y_i} = p_i^0 K_i \quad (8.1)$$

where the K_i are defined as: $K_i = \begin{cases} K & i = 1 \\ \alpha K & i = 2 \\ \alpha \beta K & i = 3 \end{cases}$.

Having defined the standardized prices, we now proceed with the concept of standardized unit harvesting cost. For this purpose, however, it will be necessary to specify the production function of the fishery.

The production function in equation (3.5) has been used in both theoretical and empirical fisheries economic studies. Recent empirical research has proved that the function is suitable for use in studies of demersal fisheries (Hannesson, 1983b and Shrank et al., 1984). On the other hand, studies of pelagic, schooling species have revealed that such a function overstates the effect on harvest rate from the stock size (Bjørndal, 1985 and Ulltang, 1976). A more general formulation of the production function could be the Cobb-Douglas type:

$$h_i = r_i F_i^{\sigma} W_i^{\gamma} \quad (i = 1, 2, 3) \quad (8.2)$$

Let us refer to the case of $\sigma = \gamma = 1$ as the pure cod-fish technology, whereas the case of $\sigma = 1$ and $\gamma = 0$ is the pure pelagic fish technology. For pelagic species (herring), the studies referred to found estimates on stock output elasticities, γ , varying from close to zero (Ulltang, 1976) to 0.13 and 0.34¹⁾ (Bjørndal, 1985). The effort output elasticities, σ , are usually not significantly different from unity, except in the study by Bjørndal where the estimates are 0.68 and 0.69²⁾

¹⁾ Bjørndal (1985) used four different specifications of the production function. The results quoted here are from the models which seem to have been most successful in interpreting the data. The additional two models, A1 and B1 in his study, gave stock output elasticities of 0.62 and 0.19, respectively.

²⁾ The concept of fishing effort is more complex in Bjørndal (1985) than here. He uses "boat-days", "number of boats" and "size of boats" as three independent factors of production. The elasticities quoted here are those with respect to number of boats.

for the two most successful formulations of the production functions. A priori it would be reasonable to expect that there are few crowding externalities taking place in the Barents Sea fisheries; that is, we should not expect $\sigma < 1$. On the other hand, if the vessels in one fishery were jointly operated by one owner, or if they otherwise cooperated in fish searching etc., it could be that the effort output elasticity is greater than one. However, the Norwegian vessels are usually owned by independent fishermen or small companies, and the sharing of fishing information only takes place occasionally. For these reasons we shall simplify the study by assuming that $\sigma = 1$ for fishing on all three species. On the other hand we shall distinguish between the cases of pure cod fish technology ($\gamma = \sigma = 1$) and that of pure pelagic fish technology ($\gamma = 0$ & $\sigma = 1$).

The symbols of costs and related parameters for species i are:

- c_i^0 = Cost per vessel year equivalent (VYE)
- \bar{c}_i = Cost per unit of fishing effort, F_i
- TC_i = Total costs per year
- N_i = Number of VYE
- k_i = Catch per VYE
- $c_i(W_i)$ = Cost per unit of harvest
- $c_i(X_i)$ = Cost per unit of standardized harvest.

The definitions of and relations between different cost concepts are:

$$TC_i = c_i^0 N_i = \bar{c}_i F_i . \quad (8.3)$$

The catch per VYE is:

$$k_i = h_i / N_i . \quad (8.4)$$

For the unit harvesting cost we shall distinguish between the case of pure cod fish technology and that of pure pelagic fish technology.

Pure cod fish technology

In this case $\gamma = \sigma = 1$ in the harvest function (8.2). By using vessel year equivalent (VYE) as a measure of fishing effort, we substitute (8.5) into (8.2).

$$q_i N_i = r_i F_i \quad (8.5)$$

q_i is the catchability coefficient. This gives the following production function:

$$h_i = q_i N_i W_i \quad (8.6)$$

(8.3) and (8.6) give the unit harvesting cost

$$c_i(W_i) = TC_i/h_i = c_i/W_i \quad (8.7)$$

where

$$c_i = c_i^0 W_i^0 / k_i^0 \quad (8.8)$$

W_i^0 and k_i^0 are stock size and catch per VYE, respectively, both related to a specific point in time (actually, a year or another short period, during which the stock can be treated as if it were constant). c_i is the unit harvesting cost for $W = 1$; in other words, the unit harvesting cost for catching the last fish in the stock. The unit harvesting cost for standardized harvest is

$$c_i(X_i) = TC_i/y_i = c_i/X_i \quad (8.9)$$

where

$$y_i = h_i/K_i \quad (8.10)$$

Pure pelagic fish technology

In this case we have $\sigma = 1$ and $\gamma = 0$ in the production function (8.2). Instead of (8.6) we now get

$$h_i = k_i N_i. \quad (8.11)$$

The unit harvesting cost in this case is constant:

$$c_i(W_i) = TC_i/h_i = c_i^0/k_i^0. \quad (8.12)$$

The unit harvesting cost for a standardized harvest is

$$c_i(X_i) = TC_i/y_i = c_i^0 K_i/k_i^0. \quad (8.13)$$

From the economic data in Tables 8.1-8.5 we can now derive the economic parameters of the TSB-model, and the optimal stationary stock levels can be computed from Equations (7.42)-(7.44). Putting the economic data into the price and cost functions of the model we get the results shown in Table 8.6. Using the price of cod as numeraire gives price and cost data shown in Table 8.7.

Table 8.6. Basic price and cost parameters of the TSB-model.
(The average of 1978-1980 data.)

i	p_1^0 ⁴⁾ (1000 Nkr/ m.t.)	c_i^0 (1000 Nkr/VYE)		k_i^0 (M.t./VYE)	w_i^0 (Million m.t.)	c_i (million Nkr)	
		Alterna- tive A	Alterna- tive B			Alterna- tive A	Alterna- tive B
1 ¹⁾	0.5284	3242.9	2733.8	4628.0	6.320	4428.6	3733.2
2 ²⁾	2.3279	1446.7	1314.2	451.7	2.893	9265.7	8417.2
3 ³⁾	3.2029	1222.3	1030.8	322.4	0.853	3233.9	2727.3

1) Data for the average capelin fishing vessel.

2) Data for the average cod fishing vessel.

3) Data for a general purpose sea mammal vessel.

4) Prices net of government subsidies.

Table 8.7. Basic price and cost parameters of the TSB-model with the price of cod as numeraire. (The average of 1978-1980 data.)

i	p_i ⁴⁾	c_i^0 (m.t./VYE)		c_i (1000 m.t.)	
		Alternative A	Alternative B	Alternative A	Alternative B
1 ¹⁾	0.2270	1393.1	1174.4	1902.4	1603.7
2 ²⁾	1.0000	621.5	564.5	3980.3	3615.8
3 ³⁾	1.3759	525.1	442.8	1389.2	1171.6

1)-4) See footnotes to Table 8.6.

The estimates of the biological parameters in Section 6 and the economic parameters in this section will be used in the next section to derive the optimal solutions of the TSB-model. The open access solutions will be presented and discussed in Section 10.

9. OPTIMAL SOLUTIONS OF THE THREE SPECIES BARENTS SEA MODEL

Equations (7.42)-(7.44) give the necessary conditions for an interior solution to the maximization problem in Equation (7.6). If such a solution exists, it can be found by solving the three equations for the three state variables, the fish stocks. Unfortunately the non-linearities of Equations (7.42)-(7.44) are too complex for an analytical solution for X_i^* ($i = 1, 2, 3$) to be found. A simulation program, OPT (see Appendix 11), has been created to find numerical solutions for X_i^* ($i = 1, 2, 3$) from the three golden rule equations. The partial derivatives of the growth functions with respect to stocks are needed in this computation, and they are shown in Appendix 10.

With substitution of the basic case biological parameters in Table 6.12, the basic economic parameters in Table 8.6 and a discount rate of five percent into Equations (7.42)-(7.44), the computer program OPT (Appendix 11) generated the following optimal normalized stock levels:

$$\begin{aligned} X_1^* &= 0.5022 \\ X_2^* &= 0.2648 \\ X_3^* &= 0.0138 . \end{aligned} \tag{9.1}$$

The corresponding equilibrium catch levels are found by substituting (9.1) into (7.1). Doing this reveals a negative catch rate for the capelin stock. According to the discussion in Section 7, the solution in (9.1) can not be an optimal solution since the lower bound on the capelin catch rate ($y_1 = 0$) becomes binding.

At this stage we avoid the constrained maximization problem by making a change in one of the economic parameters, hoping that this will give an interior solution to the original problem. Intuitively an increase in the price of capelin, ceteris paribus, can make the capelin fishery profitable enough to give such a solution. The average catch of capelin in the years 1978-80 was 1.86 million m.t. (see Appendix 4) and the

average price was 0.5284 Nkr/kg (Table 8.6).¹⁾ Between 90 and 99% of the capelin landings have been used for reduction to fish meal and oil during the 1970s and 1980s. The rest has been mostly used for fresh and frozen products and for roe production. Under the current system Norwegian fishermen's organizations have been granted the legal right to fix minimum prices on raw fish. Thus, they are in a position where, via price discrimination, they can charge a monopoly price in some markets (Hannesson, 1985). This is probably the main reason why capelin used for other purposes fetches significantly higher prices to the fishermen than capelin used for reduction into meal and oil. For 1978-80 quantities and prices of capelin for other purposes are shown in Appendix 12. The average price of capelin used for fresh, frozen and roe products was 1.503 Nkr/kg or three times higher than the average ex-vessel price of capelin for all uses.

Increasing the price of capelin by 50, 100 and 150% to 0.7926, 1.0568 and 1.3210 Nkr/kg respectively, and keeping all other biological and economic parameters equal to that in Tables 6.12 and 8.6 gives the optimal normalized stock levels and catch rates shown in Table 9.1. The corresponding figures in million tons are shown in Table 9.2.

Table 9.1. Optimal stocks and harvest rates in normalized units.¹⁾

P_1^0 (1000 Nkr/m.t.)	X_1^*	X_2^*	X_3^*	Y_1^*	Y_2^*	Y_3^*
²⁾ 0.7926 (0.3405)	0.404155	0.212338	0.018729	0.012866	0.113080	0.001117
²⁾ 1.0568 (0.4540)	0.361002	0.183105	0.021832	0.066389	0.100807	0.001290
²⁾ 1.3210 (0.5675)	0.341609	0.162232	0.023262	0.095082	0.095179	0.001367

¹⁾ Based on biological parameters in Table 6.12 and economic parameters in Table 8.6.

²⁾ Prices with price of cod as the numeraire are given in parenthesis.

¹⁾ Remember that quantity refers to the total harvest from the Barents Sea area, while price refers to that of Norwegian landings.

$$\begin{aligned}
\text{NPV} &= (1/\delta) \sum_{i=1}^3 (p_i - c_i/X_i^*) y_i^* \\
&= (1/\delta) \sum_{i=1}^3 (p_i^0 - c_i/W_i^*) h_i^* = (1/\delta) \sum_{i=1}^3 \pi_i^* . \quad (9.4)
\end{aligned}$$

The definitions of p_i and c_i are found in (8.1) and (8.8), respectively. The relationships between the two types of harvest rates are

$$h_i^* = y_i^* K_i \quad \text{with} \quad K_i = \begin{cases} K & i = 1 \\ \alpha K & \text{for } i = 2 \\ \alpha \beta K & i = 3 \end{cases} . \quad (9.5)$$

The computational results for the sustainable rent, π_i^* , and NPV are shown in Table 9.4.¹⁾ The price of cod is used as numeraire. The table includes the extreme values of p_1^0 to give simultaneous positive rent from capelin and cod.

Table 9.4. Sustainable rent, π_i^* , and net present value, NPV, of optimal harvesting in the case of interior solution. Thousand of cod units.¹⁾

p_1^0 (in cod units)	π_1^*	π_2^*	π_3^*	NPV
0.3234	0	1208.368	-65.192	22865.8
0.3405	80.341	1127.580	-64.916	22860.1
0.4540	609.241	731.140	-63.494	25537.7
0.5675	1172.307	448.469	-62.752	31160.5
0.7429	2243.833	0	-62.601	43624.0

¹⁾ One unit of cod is 1.0 metric ton.

So far in this section the results are based on the assumption that the solution to the optimization problem in (7.6) is an interior one, implicitly found from Equations (7.42)-(7.44).

¹⁾ The computer program OPV designed to compute optimal sustainable rents and net present value is an extended version of the program OPT which compute the optimal stock levels. They are both shown in Appendix 11.

To verify that this solution is the optimal one, we need to perform the test described in Section 7. A closer look at the results in Table 9.4 reveals a curious result. The net present value of optimal harvesting is lower for $p_1^0 = .3405$ than for $p_1^0 = .3234$. A priori one would rather expect the opposite result, and therefore this makes us suspicious about the reliability of the findings in the low range of p_1^0 . The alternative to simultaneous harvest of all three species would be to leave the cheap species, the capelin, in the sea as feed for the more valuable one, the cod.

The aforementioned test will reveal whether the interior or the corner solution is the optimal one. To illustrate this we shall compare the net present values of sustainable harvesting for the two alternatives. The net present value of the interior solution as a function of p_1^0 is shown as NPV_1 in Figure 9.1. The corner solution, i.e. when $y_1 = 0$, is found from Equations

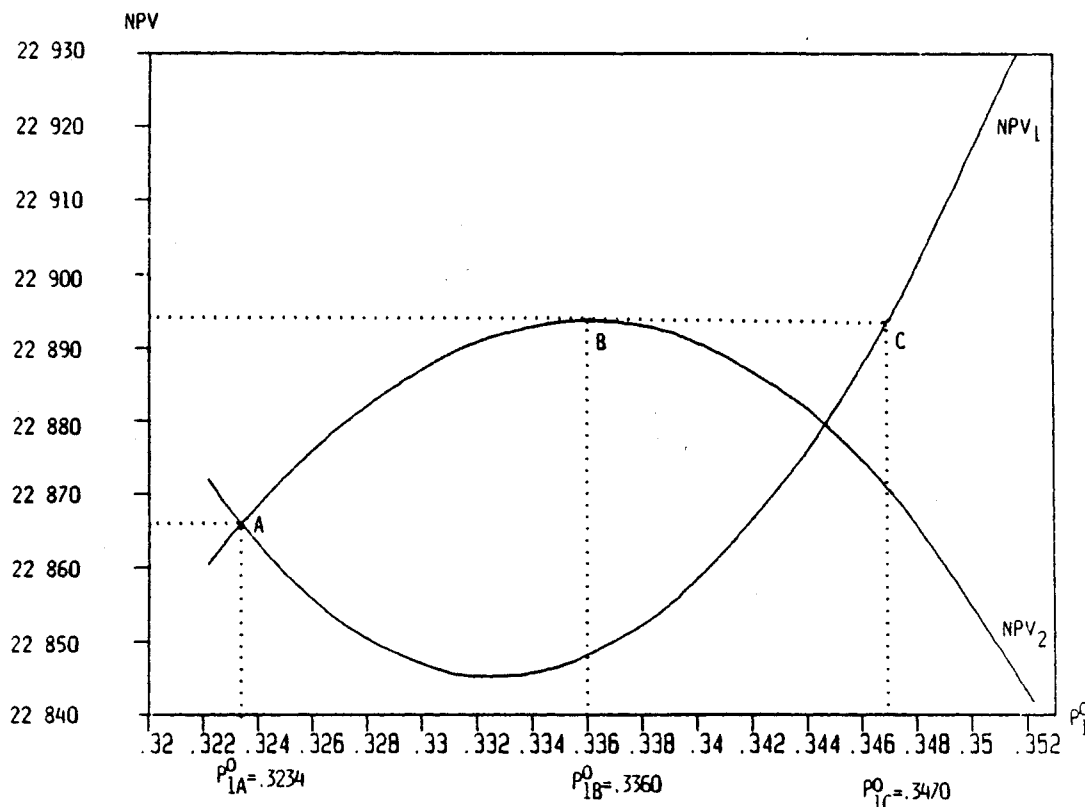


Figure 9.1. Net present value, in thousand units of cod, of interior solution (NPV_1) and the corner solution (NPV_2).

(7.43), (7.44) and (7.47). The net present value of this harvesting strategy as a function of p_1^0 is shown as NPV_2 in Figure 9.1.

From Figure 9.1 we see that $NPV_1 = NPV_2$ at two points: when $p_1^0 = 0.3447$ and when $p_1^0 = 0.3234$. The latter one is the lower limit for the interior solution which will give positive harvest rates for all three species. For lower capelin prices the golden rule equations (7.42)-(7.44) give negative harvest rates of capelin, as is seen for $p_1^0 = 0.2270$; and this has no biological meaning. Moving along the NPV_1 curve from A to the right it first decreases, then increases as a function of p_1^0 . This is what the findings in Table 9.4 indicated. The NPV_2 curve has the opposite shape with a maximum at point B.

The complete solution to our maximization problem will be as follows. For capelin prices to the right of C the interior solution will be the optimal one, and the NPV_1 curve shows the level of the objective functional for the infinite horizon equilibrium harvests. For lower prices of capelin; that is, to the left of C in Figure 9.1, the optimal result will be at point B.¹⁾ Among all feasible combinations of harvest rates consistent with Equations (7.43), (7.44) and (7.47), there is one, from an economic point of view, which is better than the others and this corresponds to p_{1B}^0 . Regardless of what the price of capelin is, as long as it is lower than p_{1C}^0 , the optimal solution will be found as if the price equals p_{1B}^0 .

Implementation of the optimal solution require use of indirect means like fees/subsidies on harvest or fishing effort, or direct means like quota and effort restrictions. Using a simple biomass model with a one dimensional fishing effort concept it can be shown that from an efficiency point of view it does not matter which of the means are used (Clark, 1980). However, the more complex the model is made, e.g. by extending to several year classes, geographical distribution, seasonal

¹⁾ The corresponding stock levels are $X_1^* = 0.420009$, $X_2^* = 0.215913$ and $X_3^* = 0.017887$, and the harvest rates are $y_1 = 0$, $y_2 = 0.118006$ and $y_3 = 0.001069$. Stability tests of this equilibrium points give $x = 0.0341$ and $R = 9054$, which imply a locally stable equilibrium.

growth and migration, and multipurpose vessels and gear, the larger the number of means necessary in order to achieve the optimal result (Clark, 1980; Flaaten, 1983; Flaaten 1987a). For the TSB-model it is clear that the optimal stationary harvesting regime can be kept by use of just three regulatory means. Whether harvest fees/subsidies or effort fees/subsidies are used to deprive the fishermen for the resource rent, the results are the same: the harvesting sector gets paid its alternative cost and the government collects the rent.

The following symbols will be used in deriving the formulas for calculation of the fees/subsidies:

\tilde{p}_i^0 = Net price of species i received by the fishermen

\tilde{c}_i^0 = Cost of fishing effort paid by the fishermen

t_{pi} = Landing fee/subsidy on harvest

t_{ei} = Effort fee/subsidy.

Price and cost definitions are

$$\tilde{p}_i^0 = p_i^0(1-t_{pi}) \quad (9.6)$$

$$\tilde{c}_i^0 = \tilde{c}_i^0(1+t_{ei}) . \quad (9.7)$$

Simple arithmetic shows that the tax rates are found from the formulas:

$$t_{pi} = b_i^*/p_i \quad (9.8)$$

$$t_{ei} = b_i^*/(c_i/X_i^*) , \quad (9.9)$$

where $p_i = p_i^0 K_i$ with K_i defined in connection with (8.1). b_i^* is the rent per unit of standardized harvest, X_i^* is the optimal stationary stock level and c_i , the cost of harvesting the last unit of the stock, is defined in (8.8).

In Table 9.5 are shown the optimal stationary harvest and effort fees/subsidies as a share of harvest price and effort cost, respectively.

Table 9.5. Optimal stationary fees and subsidies (-) as a share of harvest price (t_{pi}) and effort cost (t_{ei}) in the case of the interior solution.

p_1^o (in cod units)	t_{p1}	t_{p2}	t_{p3}	t_{e1}	t_{e2}	t_{e3}
0.3405 ¹⁾	0.6115	0.3693	-13.0323	1.5742	0.5856	-0.9287
0.4540	0.6738	0.2686	-11.0323	2.0658	0.3673	-0.9171
0.5675	0.7242	0.1746	-10.3169	2.6264	0.2115	-0.9116

¹⁾ Based on the interior solution. As explained in the text the corner solution at B in Figure 9.1 will be the optimal solution for this price of capelin.

Table 9.5 clearly shows that capelin and cod should be taxed, the former more heavily than the latter, and that the tax on capelin should be higher the higher the price of capelin is. For cod the result is the opposite with respect to capelin price.

The necessity of heavily subsidization of the harvesting of sea mammals is evident from Table 9.5. Using price support would mean that the harvesters should receive ten to thirteen times as much subsidies as they get for their products in the market. With a low price of capelin, the optimal solution is to leave much of the capelin in the sea as feed for the cod and harvest the cod instead. Since the sea mammals' predation effect on cod is more significant than on capelin (see (6.21)), it is optimal to exploit the sea mammals most heavily when cod is expensive relative to capelin. That is why the optimal subsidies on sea mammals in Table 9.5 is highest for the lowest price on capelin.

Given that there exists an interior solution to our maximization problem it is seen that the optimal stocks are derived from Equations (7.42)-(7.44). The stocks are implicit functions

of the biological and economic parameters. To find how sensitive the solution is with respect to changes in the parameters we have computed the elasticities¹⁾ of the optimal stocks with respect to biological and economic parameters, defined as:

$$\check{W}_{ij}^* = \frac{\partial W_i^* / W_i^*}{\partial P(j) / P(j)} \quad , \quad (9.10)$$

where $P(j)$ is parameter j . The results are shown in Table 9.6 for standardized prices of capelin set equal to 0.3405 and 0.5675. In the table are also shown the corresponding elasticities of net present value of the sustainable rent:

$$\check{NPV}_j = \frac{\partial NPV / NPV}{\partial P(j) / P(j)} \quad . \quad (9.11)$$

The results from the TSB-model in Table 9.6 show a positive effect on the stocks from increases in the own intrinsic growth rate. This is equivalent to that of the Schaefer model. Both for the low and the high capelin price will there be a negative impact on the sea mammal stock when r_1 increases. The effect from r_2 , however, is dependent on the price of capelin. When capelin is expensive relative to cod, an increase in r_2 slightly increases the optimal stock of sea mammals. This is because of the sea mammals' relative stronger predation on cod than on capelin (see (6.21)) in the TSB-model. A larger stock of sea mammals means a net contribution to capelin harvest in this case.

Increased biological productivity of capelin and cod, via r_1 and r_2 , has a significant positive impact on the net present value. The magnitude is, of course, dependent on the harvest prices. Due to the sea mammals being a top predator of relatively low value an increase in its intrinsic growth rate, r_3 , has a negative, but small, impact on NPV. K and α have, like r_1 and r_2 , a significant positive impact on NPV, whereas the

¹⁾ Elasticities were approximated by computing the effects of one percent increase in the parameters.

Table 9.6. Elasticities of optimal stocks and net present value of sustainable harvesting with respect to biological and economic parameters, for $p_1^0 = 0.3405$. (For $p_1^0 = 0.5675$ in brackets. ¹⁾) Based on the interior solutions.

P(j) i	\dot{W}_{ij}^*			\dot{NPV}_j
	1	2	3	
r_1	0.359 { 0.492 }	0.348 { 0.439 }	-0.854 { -1.275 }	1.738 { 2.329 }
r_2	0.214 { 0.070 }	0.097 { 0.030 }	-0.310 { -1.127 }	1.058 { 0.311 }
r_3	-0.001 { -0.002 }	-0.002 { -0.004 }	0.123 { 0.159 }	-0.063 { -0.046 }
K	0.546 { 0.739 }	0.174 { 0.013 }	-0.618 { -0.769 }	1.214 { 1.298 }
α	0.001 { -0.096 }	0.690 { 0.591 }	-1.226 { -0.894 }	1.165 { 0.232 }
β	0.0(+){ 0.0(+)} -0.540 { -0.522 }	0.001 { 0.001 }	-0.020 { -0.034 }	0.0(-){ 0.0(-)}
a_{12}	-0.540 { -0.522 }	-0.401 { -0.437 }	1.084 { 1.334 }	-1.666 { -1.543 }
a_{13}	-0.002 { -0.003 }	0.005 { 0.016 }	-0.374 { -0.772 }	-0.001 { -0.004 }
a_{23}	0.002 { 0.003 }	-0.004 { -0.015 }	-0.667 { -0.278 }	-0.002 { 0.0(+)}
p_1^0	-0.454 { -0.163 }	-0.516 { -0.579 }	0.641 { 0.148 }	0.067 { 1.078 }
p_2^0	0.540 { 0.424 }	0.093 { 0.029 }	-2.285 { -2.186 }	2.846 { 1.783 }
p_3^0	0.0(-){ 0.0(-)}	0.0(-){ 0.0(-)}	0.011 { 0.010 }	0.004 { 0.004 }
c_1^0	0.187 { 0.121 }	0.205 { 0.161 }	-0.262 { -0.123 }	-0.019 { -0.281 }
c_2^0	-0.261 { -0.373 }	0.243 { 0.415 }	0.876 { 1.115 }	-1.760 { -1.487 }
c_3^0	-0.011 { -0.012 }	-0.019 { -0.023 }	1.073 { 1.094 }	-0.113 { -0.081 }
δ	-0.034 { -0.044 }	-0.040 { -0.028 }	1.020 { 0.999 }	-1.040 { -1.027 }

¹⁾ + or - sign in parenthesis after 0.0 indicates that the elasticity is of positive or negative magnitude, respectively, but close to zero.

effect from β is negligible. Changes in the sea mammals' related coefficients, r_3 and β , have minor effects on NPV because of the heavily depleted stock and low sea mammal yield at the optimum. That is also why changes in a_{13} and a_{23} have minor impacts on NPV. However, these preying coefficients' negative impact on the optimal sea mammal stock is significant. The more food the sea mammals need, measured by a_{13} and a_{23} , the lower the stock should be. Since both capelin and cod, directly or indirectly, are valuable species, an increase in the latter's consumption of the former has a great negative impact on the net present value. As a conclusion we can say that the NPV of optimal harvesting is rather sensitive to changes in biological parameters related to capelin and cod, but not very sensitive to changes in sea mammal related biological parameters due to this species' low optimal stock level.

With respect to an own price increase, capelin is seen, from Table 9.6, to be affected the same way as in a single species model. The optimal stock decreases and the net present value increases. This is also what we should expect from the analysis of the two species predator-prey model in Section 7. According to the theoretical analysis the optimal predator stock should increase with a rise in the price of the prey when the predator is a nuisance. Table 9.6 shows that in the TSB-model this is the case for the sea mammal stock when the capelin price rises, whereas the optimal stocks of cod and capelin decrease in this case.

A rise in the capelin price initially augments the value of this species proportionally to the price increase. However, it pays to transform some of the increased wealth into capital in general, and some into resource capital of the top predator. This increase in the sea mammal stock helps reduce the intermediate stock, the cod, thus reducing the total consumption of the capelin which has now become more valuable. Increased capital in general rewards the social manager with the interest expressed by the discount rate, and the top predator capital rewards him with increased revenues and reduced harvesting costs of this species. In addition there is an increase in the net benefits of capelin harvest

due to the reduced predation pressure from the cod stock.

Table 9.6 shows that a rise in the price of cod, p_2 , has a much stronger impact on the optimal capelin stock than on the cod stock; both, however, increase. In the economic analysis of the two species model in Section 7 we showed that, both for the prey and for the predator, the optimal stock decreases with an own price increase. In the TSB-model the cod is a prey compared to the sea mammals, but a predator compared to the capelin. From the two species analysis we therefore would think that the optimal cod stock in the TSB-model would decrease with an own price increase. However, as Table 9.6 shows, the effect on the optimal cod stock from an own price increase is positive, even though it is small. The reason for this is probably that the capelin, as well as the cod, is prey for the sea mammals. A rise in the price of cod initially augments the value of the resource capital of this species proportionally to the price increase. However, since cod is the most important prey for the sea mammals, it now pays to reduce the use of it as an expensive feed for the sea mammals. This is done by the reduction of the sea mammal stock which enables a minor increase of the cod stock. On the other hand, the capelin stock should be increased significantly to supply the cod stock with more, and relatively cheaper, food than before the rise of the price of cod.

The effects on the optimal stocks from an increase in the price of sea mammals are, as Table 9.6 show, hardly noticeable. Note, however, that the effects are of opposite sign to the results derived in the theoretical two species analysis in Section 7. In the TSB-model the optimal sea mammal stock increases a little with an own price increase, whereas in the two species model the predator is always negatively affected by an own price increase. The explanation for this might be that the optimal sea mammal stock is so small that a price increase for this species contributes very little to the value of the resource stock. Therefore it does not pay to reduce this capital stock and transform it into capital in general. It rather should be increased slightly to reduce the negative net profit from the harvest of the sea mammals.

In the two species analysis in Section 7 we found symmetry between the effects of prices and costs on the optimal stocks. When the effect of a price increase on one of the stocks is negative, the effect of an increase in the corresponding cost is positive. The results in Table 9.6 show such symmetric effects on the optimal stocks in the case of changes in capelin price and cost. For changes in prices and costs of the two other species this, however, is not the case. Note, for example, that the optimal sea mammal stock increases both with an increase in the price and in the cost of the sea mammals. The effect of a cost increase, is, however, very strong, whereas the effect of a price increase is very weak as shown above. When the effort cost of the sea mammal harvest increases with one percent, the optimal sea mammal stock increases slightly more than one percent. The increased stock reduces the unit harvest cost, thus almost restoring harvest profit to its original level. The harvest profit of sea mammals is anyway negative.

An increase in one of the prices has a positive effect on NPV in the TSB-model, whereas a cost increase has a negative impact just as in the single species model. With respect to capelin and cod the effect on NPV from a price increase is more important the larger the previous harvest of the price increased resource. For a relatively low price of capelin the optimal harvest of that particular species is low; therefore, a price increase contributes little to the total economic result measured by NPV.

For capelin and cod the effect from an increased discount rate, δ , in the TSB-model is similar to that of the single species model. The optimal stock levels decrease with a partial increase in the discount rate. For the sea mammal stock, however, the result is the opposite. An increased discount rate, *ceteris paribus*, makes the optimal stock level significantly higher. The larger predator stock helps reduce the prey to its lower optimal stock level caused by the increased discount rate. As noted in the discussion in Section 7, a rise in the discount rate makes it more costly to keep a large prey stock, therefore a part of it is transmuted into capital in general. In the case of the

TSB-model, cod and capelin are both preys relative to the sea mammals, so a part, although a small one, of each of them is transmuted into capital in general. Another part is transmuted into the predator resource capital, the sea mammals. Thus, the losses from harvesting the predator are reduced, probably because of the lowered unit harvesting cost, but also because of increased revenues from this species.

With respect to the net present value, it is seen from Table 9.6 that a discount rate increase has a negative effect as is the case with the single species model. The NPV might be compared to the price of a bond which decreases when the interest rate increases. In our case the NPV expresses the potential market value of the resource capital represented by the stocks of the three species, provided the stocks are at the optimum.

Having presented and discussed the optimal solutions of the TSB-model in the section, we shall proceed with the open access solutions in the next section, and compare them with the optimal solutions.

10. OPEN ACCESS SOLUTIONS

Until 1977 the living resources of the Barents Sea and adjacent areas were common property resources exploited by fishermen from most European fishing nations. The implementation of the Extended Fisheries Jurisdiction (EFJ), made possible by the United Nations' third law of the sea conference, in principle transformed the fishing resources of the area from international common property resources to the property of the coastal states of Norway and the Soviet Union. However, there are still disputes regarding the jurisdictional right for Norway to manage the fisheries resources of the Svalbard zone (Churchill, 1985); and the management problems with respect to the whale stocks in the area still prevail (Hoel, 1985).

To give an impression of the magnitude of the fisheries of the Barents Sea and adjacent areas, annual catch rates of sea mammals are depicted in Figure 10.1 and catches of capelin and cod in Figure 10.2 for the period 1868-1980. The harvest of sea mammals has fluctuated widely with significant peaks in the mid 1880s, in the 1890s, in the early 20th century, from the mid 1920s to the outbreak of the Second World War and in the early 1950s. A detailed empirical analysis of the causes of the fluctuating sea mammal harvest is beyond the scope of this study. According to the theory of common property resources, the causes should be sought in biological, technical, market or cost relations.

For capelin and cod it should be noticed that for practically all years until 1960 the catches of capelin were well below that of cod. The "capelin" peak after the mid 1960s is mainly herring and it coincides with the commencement of the almost total depletion of the Atlanto Scandian herring (Sætersdal, 1980). The peak ten years later consists almost entirely of capelin, caught by Norway and the Soviet Union. The collapse of the capelin stock, however, did not occur before the mid 1980s (Anon., 1986).

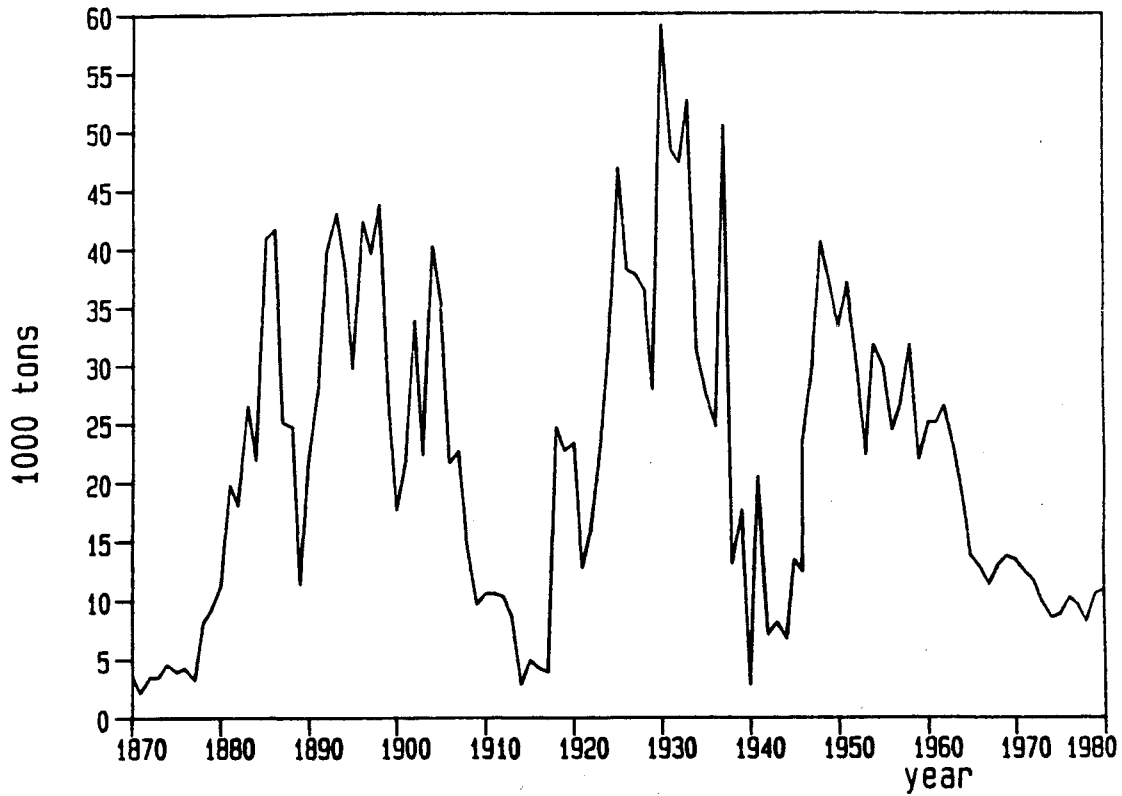


Figure 10.1. Annual harvest of sea mammals in the Barents Sea area, 1868-1980, in thousand metric tons.

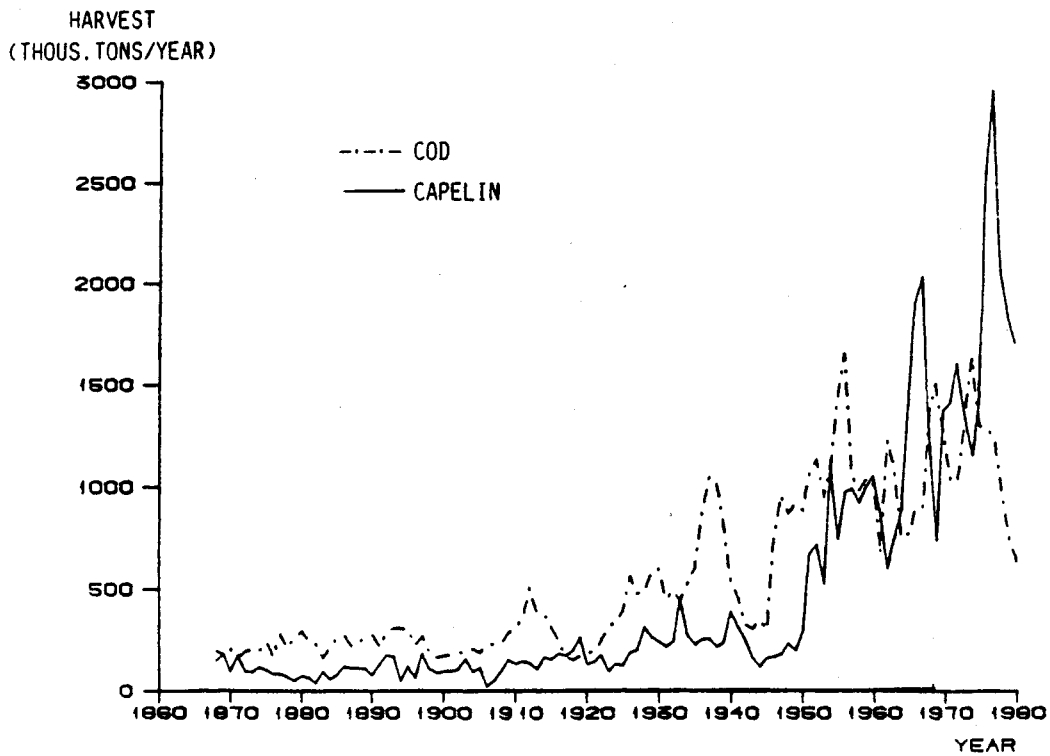


Figure 10.2. Annual harvests of capelin and cod in the Barents Sea area, 1868-1980, in thousand metric tons.

Using the economic and biological data from previous sections of this study we can analyse the open access solution of the Barents Sea fisheries. This is done within the framework of the TSB-model and is restricted to comparisons of equilibrium situations. With the basic case biological parameters in Table 6.12 and the basic economic parameters in Table 8.6, the open access equilibrium of the TSB-model is shown in the first lines of Tables 10.1 and 10.2 in normalized and metric units, respectively. For comparisons the optimal solution from Section 9 are included in the tables.

Table 10.1. Stock levels and harvest rates under open access and optimal management. Normalized units.

P_1^0 ('000 Nkr/ m.t.)	Harvesting regime	X_1	X_2	X_3	Y_1	Y_2	Y_3
1) 0.5284 (0.2270)	Open access	0.235503	0.133918	0.262812	0.094208	0.031757	0.005416
	Optimal	0.420009	0.215913	0.017887	0	0.118006	0.001069
1) 0.7926 (0.3405)	Open access	0.157002	0.121327	0.262812	0.085135	0	0.001799
	Optimal ²⁾	0.404155	0.212338	0.018729	0.012866	0.113080	0.001117

1) Standardized price with price of cod as the numeraire is given in parenthesis.

2) Interior solution, cf. Figure 9.1. The optimum is equivalent to the corner solution shown in this table for $p_1^0 = 0.5284$ Nkr/kg.

The open access fishery were analysed in the last part of Section 7. There it was shown that if relative prices and costs imply simultaneous harvesting of all three species, the open access stock level of each species will depend only on own price and effort cost. The stock levels are, in general, found from Equation (7.55), but in the particular case of cod fish technology Equation (7.59) gives the open access solution for all three species.

Table 10.2. Stock levels and harvest rates under open access and optimal management. Million metric tons.

p_1^0 ('000 Nkr/ m.t.)	Harvesting regime	W_1	W_2	W_3	h_1	h_2	h_3
1) 0.5284 (0.2270)	Open access	7.065	3.616	0.852	2.826	0.857	0.018
	Optimal	12.600	5.830	0.058	0	3.186	0.003
1) 0.7926 (0.3405)	Open access	4.710	3.276	0.852	2.554	0	0.006
	Optimal ²⁾	12.125	5.733	0.061	0.386	3.053	0.004

1) and 2): See notes to Table 10.1.

The open access solution for basic prices and costs, that is for $p_1^0 = 0.5284$ Nkr/kg, is derived using the method leading to Equations (7.55) and (7.56). This solution implies simultaneous harvesting of all three species, and it proves to be locally stable. The harvest rates seem to be reasonable compared to actual harvests shown in Figures 10.1 and 10.2, except for the harvest of capelin which seems to be somewhat high. It is, however, important to stress that the solution is based on the assumption of pure cod fish technology for all three species. If we, on the contrary, had assumed pure pelagic fish technology for the capelin fishery, it is easy to demonstrate the extinction of all three species would be the result. As long as the profit from harvesting one ton of capelin is positive, it is positive for any stock level including the last unit of the stock. Being a common property means that no one has the incentive to leave the last unit of the resource unharvested. With the basic prey removed from the ecosystem, the predators, cod and sea mammals, are also extinct in the TSB-model.

The question of what kind of harvest technology prevails in the industries may be essential to the question of extinction of common property fish resources. To some extent we have

discussed this subject in Section 8. Nevertheless, here we shall stick to the assumption of pure cod fish technology in the harvest of all three species in the TSB-model.

Comparing the optimal solution with the open access solution for $p_1^0 = 0.5284$ Nkr/kg, it is seen from Tables 10.1 and 10.2 that the stocks of capelin and cod should be significantly increased and that of sea mammals should be dramatically depleted. This drastic removal of the top predator, the sea mammals, makes it possible almost to quadruple the harvest of cod. A capelin price of 0.5284 Nkr/kg is so low that it is optimal not to harvest this resource. It should rather remain in the sea as feed for cod under optimal management of the interdependent species, while in the open access case the capelin will be heavily exploited.

The open access fishery can affect the stocks to the extent that one of them, for example the cod stock, is reduced to such a low equilibrium level that fishing for this particular species is unprofitable. The analysis in section 7, and in particular Equations (7.59) and (7.61), show that the open access stock level of cod in this case is a function of prices and costs of the two other species, capelin and sea mammals.

With a 50% increase in the price of capelin, *ceteris paribus*, from 0.5285 Nkr/kg to 0.7926 Nkr/kg, the open access harvesting of capelin will be more intensive reducing the stock from 7 million tons to 4.7 million tons. Such a reduction in the availability of prey reduces the cod stock so much that it makes the harvesting of this resource unprofitable. To find the complete open access solution in this case it is therefore necessary use Equations (7.59) and (7.61). The results are presented in Tables 10.1 and 10.2 for $p_1^0 = 0.7926$ Nkr/kg. Under open access a 50% partial price increase of capelin renders the cod stock unharvested, reduces the harvest of capelin somewhat and reduces the harvest rate of sea mammals by as much as two-thirds. Altogether, the gross value of the total open access harvests is nearly halved, especially due to the unprofitability of the cod fishery.

In the case of the pure cod fish technology, Equation (7.59) shows that the open access stock level of a harvested species depends only on the own price and effort cost. For sea mammals this is illustrated in Tables 10.1 and 10.2, where the stock level of this species is unchanged from line one to line three, because price and cost are unchanged. For capelin, however, the stock level is reduced because of increased own price. For two different levels of p_1 , p_1^1 and p_1^2 , Equation (7.59) implies the following relationship between the two levels of the capelin stock, X_1^1 and X_1^2 , when cost is unchanged:

$$\frac{X_1^1}{X_1^2} = \frac{p_1^2}{p_1^1} . \quad (10.1)$$

For capelin we have from Table 10.1:

$$\frac{p_1^2}{p_1^1} = \frac{.3405}{.2270} = 1.5 . \quad (10.2)$$

That is why the ratio of the stock levels of capelin on line one and three is

$$\frac{X_1^1}{X_1^2} = \frac{.235503}{.157002} = 1.5 . \quad (10.3)$$

In this respect the result is similar to the result of the single species analysis of open access fisheries. Under open access harvesting no one has the incentive to invest in the capelin stock today in order to be able to harvest more of it tomorrow. Now, in the multispecies context we might say that no one has the incentive to invest in the capelin stock today so as to harvest more cod tomorrow.

Under open access harvesting it is interesting to notice the strong effect on the harvest rate of sea mammals from an increase in the price of capelin. This happens despite the constant equilibrium level of sea mammals. The cause is to be found in the reduction of the prey stocks, especially that of the capelin. The sustainable yield of sea mammals is therefore sensitive to changes in the availability of preys within the range of stock levels considered here.

11. SUMMARY AND CONCLUSION

The aim of this study has been twofold: to provide a theoretical and an applied analysis of multispecies fisheries. A two species biomass model is reviewed in Section Two, and Section Three presents a three species model based upon the former one. The concept of maximum sustainable yield frontier (MSF) is defined and presented in Section Four, and a method for deriving this in the three species case is developed. The biological MSF concept is closely related to the concept of production possibility frontier in economic theory.

The applied part of this study is devoted to the fisheries of the Barents Sea area in the North East Atlantic Ocean. Therefore, a brief description of the marine ecological system of the area is included in Section Five. Since a fully developed and implemented multispecies model of the Barents Sea fisheries did not exist, an attempt has been made to estimate the nine biological parameters of the model. The "guesstimation" procedure is explained step by step and the assumptions made are explicitly put forward in Section Six, such that the reader can critically appraise and modify them if that is felt to be desirable.

The three stocks in the applied model are aggregated stocks with "capelin" consisting of capelin and herring, "cod" consisting of cod, haddock and saithe, and "sea mammals" consisting of 14 species of whales and two species of seals.

Economic aspects of multispecies fisheries studied in Section Seven includes optimal harvesting levels as well as open access harvesting levels. In addition to the traditional study of interior solutions, we also discuss the possibility of corner solutions and analyse in some detail a couple of cases. With respect to the effects on the optimal stocks from an increase in the discount rate, this was studied theoretically within the framework of a two species predator-prey model. We found that the untraditional result of an increase in the predator stock, is most likely in cases where the predator clearly is a nuisance

to be harvested not (only) for the sake of its own value, but to increase the availability and the profitability of the prey harvest.

The economic parameters of the model are derived from Norwegian data in Section Eight and together with the biological data in Section Six, they are used in Section Nine to derive the optimal solutions of the three species Barents Sea model (TSB-model). With the basic set of biological and economic parameters the optimal solution proves to be a corner solution with no harvesting of capelin. However, the argument is that this result is a consequence of the assumed constant price of capelin. Introduction of elastic demand would probably alter this finding as indicated by the computed results from increased price of capelin. The optimal solution requires taxation of the capelin and the cod fisheries and heavily subsidization of the sea mammal harvesting due to the latter's imposed costs on the two former through predation. Sensitivity analysis of the optimal results in Section Nine indicates that this finding is very robust against changes in biological parameters, and, within reasonable limits, this main finding is also robust against changes in most economic parameters.

Open access solutions of the TSB-model are presented in Section Ten, and it is concluded that all three stocks are substantially reduced compared to the pristine levels. However, the optimal stocks of capelin and cod are not only larger than open access stocks, but even larger than the pristine levels. This paradoxical result is made possible by the heavily reduced stock of sea mammals which expands the ecological niche for capelin and cod. The relative importance of capelin and cod in the optimum very much depends on the relative prices and harvest costs of these two species. For example, when the price of capelin is one-third of the price of cod it is optimal only to harvest cod, whereas if the capelin price rises to three-quarters of the price of cod, *ceteris paribus*, it becomes optimal only to harvest the capelin. However, in either case, the sea mammals should always be kept at a relative low stock level.

One of the main policy recommodations of this study is that the sea mammals should be heavily depleted to increase the surplus production of fish resources for man. Controversial it might be, our findings are nevertheless rational from an economic point of view. It should, however, be stressed that this result may be somewhat modified if the resource is assigned an optional value from people's willingness to pay for keeping the stock at higher level. A biological argument that also may weaken our result is the eventual existence of critical depensation for lower stock levels. The TSB-model is, as most stock assessment models of whales and seals also are, based on the assumption of pure compensation. Together with the assumption of pure cod fish technology in the harvesting sector this is a safeguard against extinction. In the TSB-model the optimal stock level of sea mammals increases somewhat with an increase in the price of capelin, *ceteris paribus*. This is also the case for increased harvest costs of cod or capelin, and for an increased discount rate. Since the sea mammals' predation pressure on cod is relatively higher than that on capelin, a reduction in the price of cod increases the optimal stock level of sea mammals. A reduced price of cod allows a larger consumption of cod by sea mammals, hence, an increase in the optimal stock of the sea mammals.

As the quotation from D'Ancona (1926) in Section Three shows, the recognition of the necessity of harvesting the predator to increase the yield of the prey is not entirely new. Even long before that, in 1859, this was asserted by Charles Darwin in his famous book "The Origin of Species" as the following quotation demonstrate:

The amount of food for each species of course gives the extreme limit to which each can increase; but very frequently it is not the obtaining food, but the serving as prey to other animals which determines the average number of a species. Thus, there seems to be little doubt that the stock of partridges, grouse and hares on any large estate depends chiefly on the destruction of vermin. If not one head of game were shot during the next twenty years in England, and, at the same time, if no vermin were destroyed, there would, in all probability, be less game than at present, although hundreds of thousands of game animals are now annually shot. (Darwin, 1882, pp. 53-54; quoted from Volterra, 1928, pp. 21-22.)

In the context of the Barents Sea fisheries the "vermin" consist of several species of fish-consuming whales and seals, and this is also the case for the fisheries in other areas of the North Atlantic.

Several extensions of this work can be thought of in both the theoretical and in the applied field. The former includes multispecies studies of resources where, e.g., one of the stocks is "transboundary"¹⁾, i.e. the fish stock migrate across the boundary of the Extended Economic Zones of two countries. If one country has the main jurisdiction of the prey and another the main jurisdiction of the predator the complexity of the model clearly increases. Another extension of the theoretical analysis could be to introduce seasonal variations in the growth rates of the stocks, the catchability coefficients or in the prices of the harvests.²⁾ This clearly enhances the complexity of the bio-economic system and of the economic solution to the optimization problem.

In the applied field more research needs to be devoted to other living resources of the Barents Sea by adding more stocks to the TSB-model. Deep water prawn is a species whose harvest immensely increased, both in quantitative and value terms, from the mid 1970s to the mid 1980s. Some scientists believe this quantitatively increase has been possible mainly because of the depletion of the cod stock. If this is true the optimal solution will probably tell us to reduce the cod stock more than in the case of the TSB-model with capelin as the only prey.

The most important fish consumers included in the stock of sea mammals in this study are the Minke whale and the Harp seal; and one could think of dividing the sea mammals into two or more proper stocks to study how different combinations of whaling and sealing effect the optimal result. This could also include economic studies of transboundary resources since the migratory

¹⁾ Munro (1979) is a theoretical study in the economics of a transboundary resource within the framework of a one species model.

²⁾ Optimal harvesting of one renewable resource with seasonal growth is analysed in Flaaten (1983).

whales are combined Norwegian property and internationally common property resources, while the Harp seal stock of the Barents Sea mainly is the property of the USSR.

In the last couple of decades the harvest of whales and seals has been an increasingly touchy question for all nations with traditional interests in utilizing the marine resources. Studies in political science of the behaviour of governments, environmental organizations, the International Whaling Commission, etc. will also be of interest for the political implementation of the economic optimal solutions to multispecies harvesting problems (e.g. see Hoel, 1985).

In scientific disciplines other than social sciences we can think of detailed biological studies of predation and of interspecies relations with respect to carrying capacities and growth rates. Cost-benefit analysis linked to this study might be of some help in the selection of biological projects.

The economic history of Arctic societies is closely related to natural resources. Multispecies studies of the living resources will probably improve historians' understanding of the development of such societies, since technology, relative prices and costs change with time and alter the relative importance of the species in the ecological system. For example, how did the early intensive hunting of whales and seals in the Barents Sea area affect the development of coastal fisheries and communities in North Norway? And how severe will the effect on this fishery dependent region be as a result of a complete ban on whaling and sealing? The quantitative analysis of this study does not solely serve the purpose of illustrating a method, but the author thinks they also give a description of some important biological and economic relationships of the Barents Sea fisheries. The sincere answer to the last question therefore is that such an experiment in the long run may prove disastrous for this fragile, resource-based region; and the same conclusion will probably also apply to similar societies in other parts of the Arctic.

APPENDICES

		page
Appendix 1.	Equilibrium solutions of the three species model.	118
Appendix 2.	Program for computation of the MSF-data.	120
Appendix 3.	Aggregated stock levels of capelin and cod 1950-1980.	123
Appendix 4.	Aggregated catches of capelin, cod and sea mammals, 1868-1980.	124
Appendix 5.	Characteristics of North Atlantic cod stocks.	126
Appendix 6.	Chart flow. Programs for computing the sum of squares of the differences of relative stock sizes.	127
Appendix 7.	DYNAMO simulation program for the TSB-model, with harvest rates 1868-1980 included.	128
Appendix 8.	Equilibrium and stability conditions of the three species model.	134
Appendix 9.	A SIMULA program for computation of equilibrium stocks and harvest rates.	146
Appendix 10.	The partial derivatives of the growth functions.	148
Appendix 11.	Two FORTRAN programs for computation of the TSB-model's optimal stocks, sustainable rents and net present value.	149
Appendix 12.	Norwegian capelin and herring catches used for fresh and frozen products 1978-80.	153

EQUILIBRIUM SOLUTIONS OF THE THREE SPECIES MODEL

If the system (3.6)-(3.8) have an equilibrium solution different from $X_i = 0$ for all i , and $\gamma_{ij} \neq 0$ for all i, j , it is found easily from

$$A_1 - X_1 - \gamma_{12}X_2 - \gamma_{13}X_3 = 0 \quad (\text{A1.1})$$

$$A_2 - X_2/X_1 - \gamma_{23}X_3 = 0 \quad (\text{A1.2})$$

$$A_3 - \alpha X_3 / (X_1 + \alpha X_2) = 0 \quad (\text{A1.3})$$

Reformulating (A1.2) gives

$$X_2 = A_2 X_1 - \gamma_{23} X_1 X_3 \quad (\text{A1.4})$$

Substituting for X_2 in (A1.3) and reformulating gives

$$X_3 = \frac{A_3 X_1 \left(\frac{1}{\alpha} + A_2 \right)}{1 + A_3 \gamma_{23} X_1} \quad (\text{A1.5})$$

Substituting for X_3 in (A1.4) gives

$$X_2 = \frac{(A_2 - \gamma_{23} A_3 \frac{1}{\alpha} X_1) X_1}{1 + A_3 \gamma_{23} X_1} \quad (\text{A1.6})$$

We have now expressed X_2 and X_3 as functions of X_1 . Substituting for X_2 and X_3 into (A1.1) and reformulating gives

$$\frac{A_1 + D_1 X_1 + D_2 X_1^2}{1 + A_3 \gamma_{23} X_1} = 0 \quad (\text{A1.7})$$

$$A_1 + D_1 X_1 + D_2 X_2^2 = 0 \quad (\text{A1.8})$$

where

$$D_1 = A_1 A_3 \gamma_{23} - 1 - \gamma_{12} A_2 - A_3 (1/\alpha) \gamma_{13} - A_2 A_3 \gamma_{13}$$

$$D_2 = A_3 (1/\alpha) \gamma_{12} \gamma_{23} - A_3 \gamma_{23} .$$

The solution of X_1 from (A1.8) is

$$X_1 = \frac{-D_1 \pm \sqrt{D_1^2 - 4A_1 D_2}}{2D_2} . \quad (A1.9)$$

PROGRAM FOR COMPUTATION OF THE MSF-DATA

BEGIN

COMMENT

```

*****
*
*   FILNAVN PROGRAM      : SY12
*   FILNAVN KOMPILERT    : SY12B
*   FILNAVN INNDATA      : INPUT
*   FILNAVN UTDATA       : LESES INN
*   PROGRAMMERINGSSPRÅK : SIMULA 1.2-353
*   TYPE MASKIN          : CYBER 171MP
*   OPERATIVSYSTEM       : NOS 2.0-531/528
*   PROGRAMMERER         : OLA FLÅTEN
*   INSTITUSJON          : IFF, UNIV. I TROMSØ
*   SEKSJON              : AVDELING FOR ØKONOMI
*   DATO                 : 05/11/1985
*   VERSJON              : 1
*
*****;

```

```

COMMENT PROGRAMMET LESER INN HØYESTE/LAVESTE VERDI FOR EN
PARAMETER OG VERDIER FOR 9 ANDRE. DATAENE LEGGES UT
PÅ EN FIL TIL GRAFISK FREMSTILLING;

```

```

REAL ARRAY PARAM(1:10);
REAL A12,A13,A23,A1,A2,A3,G12,G13,G23,K1,K2,X1,X2,X3,W1,W2,W3,
Y1,Y2,Y3,H1,H2,H3,SQ,X,XMIN,XMAX,XSTEP;
TEXT ARRAY TPARAM(1:10);
INTEGER I,PARAMNR;
TEXT FILNAVN;
REF(OUTFILE)UT;

```

```

COMMENT *****
*           H O V E D P R O G R A M           *
*****;

```

```

FILNAVN:-BLANKS(7);
OUTTEXT("LES INN NAVN PÅ DATAFIL");OUTIMAGE;INIMAGE;
FILNAVN:=-INTEXT(7);

```

BEGIN

```

UT:-NEW OUTFILE(FILNAVN.STRIP);
UT.OPEN(BLANKS(80),0);
FOR I:=1 STEP 1 UNTIL 10 DO
  TPARAM(I):-BLANKS(2);

```

```

TPARAM(1):="U1";
TPARAM(2):="U2";
TPARAM(3):="U3";
TPARAM(4):="R1";
TPARAM(5):="R2";
TPARAM(6):="R3";
TPARAM(7):="A ";
TPARAM(8):="B ";
TPARAM(9):="K ";
TPARAM(10):="X2";

```

```
PARAM(1):=0.0000;
PARAM(2):=0.0000;
PARAM(3):=0.0000;
PARAM(4):=1.2704;
PARAM(5):=1.1617;
PARAM(6):=0.0614;
PARAM(7):=0.90;
PARAM(8):=0.12;
PARAM(9):=30*10**6;
PARAM(10):=0.05;
```

```
FOR I:=1 STEP 1 UNTIL 10 DO
```

```
  BEGIN
```

```
    SETPOS(20);OUTINT(I,2);OUTTEXT(" ");
```

```
    OUTTEXT(TPARAM(I));OUTTEXT(" = ");
```

```
    OUTFIX(PARAM(I),4,14);OUTIMAGE;
```

```
  END FOR I;
```

```
  OUTIMAGE;OUTTEXT("NR. PÅ PARAMETER SOM SKAL FORANDRES");
```

```
  OUTIMAGE;OUTIMAGE;
```

```
  PARAMNR:=ININT;
```

```
  OUTTEXT("LES INN MIN OG MAX FOR ");
```

```
  OUTTEXT(TPARAM(PARAMNR));OUTIMAGE;OUTIMAGE;
```

```
  XMIN:=INREAL; XMAX:=INREAL;
```

```
  XSTEP:=(XMAX-XMIN)/50;
```

```
FOR X:=XMIN STEP XSTEP UNTIL XMAX DO
```

```
  BEGIN
```

```
    PARAM(PARAMNR):=X;
```

```
    A12:=0.125/10**6;
```

```
    A13:=0.140/10**6;
```

```
    A23:=0.310/10**6;
```

```
    W3:=1500;
```

```
    X3:=1000*W3/(PARAM(7)*PARAM(8)*PARAM(9));
```

```
    X2:=PARAM(10);
```

```
    G12:=A12*PARAM(7)*PARAM(9)/PARAM(4);
```

```
    G13:=A13*PARAM(7)*PARAM(8)*PARAM(9)/PARAM(4);
```

```
    G23:=A23*PARAM(7)*PARAM(8)*PARAM(9)/PARAM(5);
```

```
    K1:=1+(4-G12)*X2;
```

```
    K2:=8*X2*(2-3*G12*X2);
```

```
    SQ:=K1**2-K2;
```

```
    IF SQ<0 THEN
```

```
      BEGIN
```

```
        OUTTEXT("VERDI I KVADRATROT ER NEGATIV, SETTES TIL 0");
```

```
        OUTIMAGE;
```

```
        SQ:=0;
```

```
        UT.OUTCHAR('*');
```

```
      END ELSE
```

```
        UT.OUTCHAR(' ');
```

```

X1:=K1/4+(SQRT(SQ))/4;

Y1:=PARAM(4)*X1*(1-X1-G12*X2)-PARAM(4)*X1*G13*X3;
Y2:=PARAM(5)*X2*(1-X2/X1)-PARAM(5)*X2*G23*X3;
Y3:=PARAM(6)*X3*(1-X3/(X1/PARAM(7)+X2));

W1:=X1*PARAM(9)/1000;
W2:=X2*PARAM(7)*PARAM(9)/1000;

H1:=PARAM(9)*Y1/1000;
H2:=PARAM(7)*PARAM(9)*Y2/1000;
H3:=PARAM(7)*PARAM(8)*PARAM(9)*Y3/1000;

UT.OUTFIX(Y1,6,9);UT.OUTFIX(Y2,6,9);
UT.OUTFIX(Y3,6,9);UT.OUTFIX(X1,6,9);UT.OUTFIX(X2,6,9);
UT.OUTFIX(X3,6,9);UT.OUTIMAGE;
END FOR X;

OUTIMAGE;OUTTEXT("DATA PÅ FILEN ");OUTTEXT(FILNAVN);
OUTIMAGE;OUTIMAGE;
END FILNAVN;
END PROGRAM;

```

AGGREGATED STOCK LEVELS OF CAPELIN AND COD,¹⁾ 1950-1980

1950	11.690	6.652
1951	13.210	7.031
1952	14.800	7.699
1953	15.620	7.770
1954	15.930	7.971
1955	16.190	7.982
1956	16.130	6.835
1957	14.050	5.311
1958	13.230	4.972
1959	13.170	5.425
1960	11.330	4.825
1961	9.760	4.942
1962	8.020	4.770
1963	6.950	3.917
1964	7.950	3.996
1965	9.410	4.687
1966	7.920	5.326
1967	6.270	5.686
1968	5.900	5.746
1969	6.680	5.301
1970	6.990	4.637
1971	5.587	4.594
1972	7.000	5.049
1973	5.210	5.362
1974	6.230	5.226
1975	8.585	4.560
1976	6.926	4.099
1977	5.942	3.666
1978	6.309	3.134
1979	5.665	2.810
1980	6.985	2.734

¹⁾ "Capelin" includes capelin (Mallotus villosus) and herring (Clupea harengus). "Cod" includes cod (Gadus morhua), haddock (Melanogrammus aeglefinus) and saithe (Pollachius virens).
Source: Flaaten (1984a).

AGGREGATED CATCHES OF CAPELIN, COD AND SEA MAMMALS,¹⁾ 1868-1980

1868	149.2	198.2	3.0
1869	182.6	176.0	2.0
1870	95.4	207.2	3.4
1871	177.5	171.3	2.2
1872	95.5	197.4	3.7
1873	92.5	209.9	3.4
1874	117.9	200.3	4.6
1875	102.9	242.7	3.8
1876	84.4	168.4	4.1
1877	83.6	280.6	3.2
1878	69.1	213.5	8.0
1879	51.6	266.7	9.1
1880	75.8	294.5	11.2
1881	66.9	246.6	19.7
1882	38.8	233.6	17.9
1883	95.5	162.5	26.3
1884	58.8	222.1	22.0
1885	80.3	257.8	41.1
1886	121.5	277.1	41.5
1887	114.6	226.2	25.1
1888	112.1	246.2	24.6
1889	112.9	258.6	11.2
1890	78.5	276.8	22.4
1891	130.7	219.4	28.3
1892	177.1	289.3	39.8
1893	170.2	311.1	43.2
1894	51.2	311.4	38.0
1895	119.8	304.9	29.7
1896	66.3	229.0	42.4
1897	184.2	272.0	39.6
1898	109.3	185.3	43.7
1899	89.7	167.6	26.8
1900	100.8	176.2	17.6
1901	100.3	180.7	22.0
1902	107.4	191.9	33.8
1903	161.1	193.3	22.3
1904	95.2	210.5	40.1
1905	114.6	192.3	35.2
1906	27.4	228.6	21.5
1907	54.3	238.5	22.5
1908	102.0	243.0	14.4
1909	155.4	289.7	9.6
1910	135.5	317.7	10.7
1911	147.0	360.2	10.6
1912	139.0	505.9	10.3
1913	107.7	389.2	8.3
1914	166.2	380.6	2.8
1915	158.7	313.5	5.0
1916	185.1	248.4	4.2
1917	174.7	169.0	4.0
1918	199.5	154.1	24.7
1919	266.2	173.9	22.6

¹⁾ For definitions of "capelin" and "cod", see note in Appendix 3. "Sea mammals" includes harp seal (Pagophilus groenlandicus), hooded seal (Crystophora cristata), blue whale (Balaenoptera musculus), fin whale (Balaenoptera physalus), humpback whale (Megaptera novae-canalia), sei whale (Balaenoptera borealis), minke whale (Balaenoptera acutorostrata), sperm whale (Physeter catodon), porpoise (Phocaena phocaena), pilot whale (Globicephala melaena), white whale (Delphiapeterus leucas), white-sided dolphin (Lagenorhynchus acutus), bottlenose whale (Hyperoodon ampullatus) and killer whale (Orcinus orca).

1920	134.5	181.7	23.3
1921	145.0	202.7	12.8
1922	179.9	250.0	16.0
1923	98.6	317.4	22.8
1924	132.3	340.9	31.0
1925	125.9	396.8	46.8
1926	192.4	565.3	38.0
1927	204.6	478.5	37.7
1928	315.3	489.2	36.5
1929	264.9	589.0	27.8
1930	243.3	602.0	58.8
1931	220.0	452.5	48.3
1932	246.8	490.0	47.3
1933	455.0	445.7	52.5
1934	271.4	529.7	31.0
1935	229.6	611.9	27.4
1936	256.9	878.3	24.6
1937	257.3	1052.2	50.6
1938	218.5	1020.8	13.0
1939	239.5	821.4	17.8
1940	390.9	547.7	3.0
1941	314.2	457.8	20.3
1942	253.2	325.1	6.9
1943	162.7	307.7	8.1
1944	119.4	350.8	6.6
1945	163.5	305.3	13.3
1946	169.9	748.6	12.5
1947	184.9	963.4	23.4
1948	235.7	872.6	29.9
1949	199.4	917.1	40.6
1950	297.9	890.6	33.1
1951	676.5	1065.5	36.9
1952	719.5	1138.5	29.5
1953	527.1	952.9	22.4
1954	1097.9	1104.7	31.7
1955	749.3	1435.6	29.7
1956	977.4	1654.1	24.4
1957	996.4	1062.5	26.6
1958	927.0	987.6	31.9
1959	1005.2	1043.1	22.0
1960	1053.9	1027.8	25.0
1961	869.1	674.5	24.9
1962	604.9	1231.5	26.6
1963	750.9	1098.0	22.2
1964	889.2	752.6	18.5
1965	1393.2	773.2	13.9
1966	1908.8	890.1	12.7
1967	2035.7	895.4	11.3
1968	1237.8	1368.5	13.0
1969	742.1	1504.0	13.8

1970	1376.5	1255.4	13.5
1971	1414.4	1028.1	12.4
1972	1605.9	1036.0	11.8
1973	1342.9	1337.7	9.8
1974	1154.9	1637.9	8.6
1975	1421.4	1301.2	9.0
1976	2546.4	1289.6	10.3
1977	2958.0	1240.5	9.7
1978	2048.2	984.5	8.1
1979	1832.5	746.8	10.6
1980	1704.5	652.5	11.0

CHARACTERISTICS OF NORTH ATLANTIC COD STOCKS

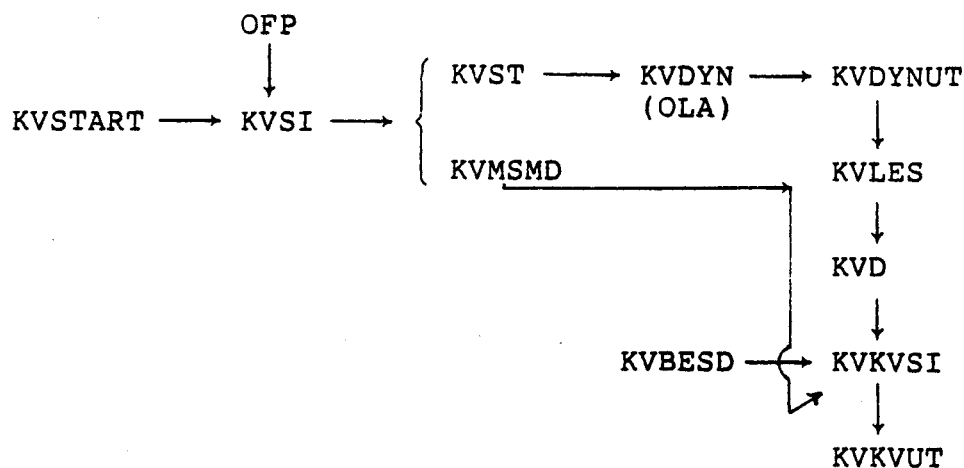
Table A5. Characteristics of North Atlantic cod stocks (natural mortality 0.2-0.3; fecundity = 500 eggs/gram female).

	Recruits			Growth			Mature			Exploitation				Average catch, 1961-70 (000t)
	2 year olds x 10 ⁶	K	L _∞ (cm)	W _∞ (kg)	W (kg) (5 year)	50% age recruitment (t ₂ years)	Y _∞ /R ₂ (kg) at F _{max}	Biomass ^a (10 ⁶ t)	F _{max}	F _{recent}	F _{max}	F _{recent}		
North Sea, south	75	0.33	111	17	6.1	3	1.93	0.70	0.25	0.5	0.25	0.5	111	
north/central	50	0.27	119	15	5.2	3	1.57	0.99	0.25	0.3	0.25	0.3	54	
West Scotland	(14)	0.24	108	14	6.6	3	1.94	(0.13)	0.3	0.6	0.3	0.6	18	
Irish Sea/Bristol Channel	4	0.33	101	12	6.5	2.3	1.25	0.04	0.3	0.85	0.3	0.85	6	
Faroe, Plateau/Bank	16	0.19	115	14	4.5	3/4	1.87	0.10	0.3	0.55	0.3	0.55	28	
Iceland	300	0.16	118	15	3.4	7/8	1.56	2.28	0.8	0.8	0.8	0.8	395	
Baltic	250	0.20	90	6	1.5	4	0.50	0.39	<0.9	0.9	<0.9	1.0	139	
Arcto-Norwegian	1250	0.12	130	17	1.5	7/8	0.57	4.13	0.5	0.6	0.5	0.6	750	
East and southwest Greenland	80		'SLOW'			8/9	0.78	0.24	0.65	0.65	0.65	0.65	102	
West Greenland	180	0.2	100	8	1.9	7/8	0.99	0.74	0.50	0.50	0.50	0.50	264	
Labrador	2000	{0.15	81	5	0.8	7/8	0.40	3.16	0.40	0.60	0.40	0.60	639	
East Newfoundland		{0.11	114	12		4	0.65	NK	0.50	>0.5	0.50	>0.5	30	
Flemish Cap	50	NK	NK	NK	NK	4/5	0.87	1.08	0.2	0.6	0.2	0.6	103	
Grand Bank	200	0.10	152	25	1.4	4	0.63	0.27	0.3	0.5	0.3	0.5	61	
St. Pierre Bank	80	0.10	137	18	1.1	4/5	0.56	0.28	0.4	0.3	0.4	0.3	83	
West Newfoundland	NK	0.05	143	20		4/5	0.86	0.19	0.45	0.5	0.45	0.5	65	
South Gulf of St. Lawrence	125	?	?	?	1.0	4	1.40	0.08	0.35	0.6	0.35	0.6	25	
Banquereau	75	0.14	105	11	2.1	4/5	2.30	NK	0.3	>0.3	0.3	>0.3	37	
Brown's/Lahave Bank	16	0.20	117	16		3/4							110	
Georges Bank	NK	0.12	142	20	3.0	3							3087	
Miscellaneous coastal														
Total														

^a Biomass of 2 year old + based on exploitation of average recruitment at F_{max} given.
^b Icelandic and ICES authorities give estimates which differ according to age groups used as reference of full recruitment. Estimates of fishing mortality/age group agree.
 () Tentative.
 NK Not known.
 ? Varied widely in last decade

Source: Garrod (1977), Table 18.

CHART FLOW. PROGRAMS FOR COMPUTING THE SUM OF SQUARES OF THE DIFFERENCES OF RELATIVE STOCK SIZES



The programs:

KVSTART	:	Control Commands
KVSI (KVSIB)	:	The main program (SIMULA)
OFP	:	Password
KVST	:	Control Commands
KVMSMD	:	Data (min step max)
KVDYN	:	DYNAMO-program
KVDYNUT	:	DYNAMO out
KVLES (KVLEB)	:	SIMULA-program
KVD	:	Data (results)
KVKVSI (KVKVSI B)	:	SIMULA (min. Q^2)-program
KVBESD	:	Data (stocks 1951-80)
KVKVUT	:	Tables

A DYNAMO SIMULATION PROGRAM FOR THE TSB-MODEL, WITH HARVEST
RATES 1868-1980 INCLUDED

TIME 1868-1980

OPT SVALL, R

* L O D D E *

L LODDE. K=LODDE. J+DT*RLODDE. JK
N LODDE=NLODDE
C NLODDE=10.069E6
R RLODDE. KL=R1. K*LODDE. K*(1-LODDE. K/K)-A12*LODDE. K*TORSK. K-A13*LODDE. K
X *SEL. K-LFANGST. K
A LFANGST. K=TABHL (LFAN, TIME. K, 1868, 1980, 1)
A R1. K=0.8696/(1-9.465E6/K)
C A12=0.1250E-6
C ALFA=0.90
C K=30E6
C A13=0.1400E-6
A BETA. K=0.0582*R3/(R3-0.0316)

* T O R S K *

L TORSK. K=TORSK. J+DT*RTORSK. JK
N TORSK=NTORSK
C NTORSK=4.759E6
R RTORSK. KL=R2. K*TORSK. K*(1-TORSK. K/(ALFA*LODDE. K))-A23*TORSK. K*SEL. K
X -TFANGST. K
A TFANGST. K=TABHL (TFAN, TIME. K, 1868, 1980, 1)
A R2. K=0.4523/(1-0.5496/ALFA)
C A23=0.3100E-6

* S E L *

L SEL. K=SEL. J+DT*RSEL. JK
N SEL=NSEL
C NSEL=1.779E6
R RSEL. KL=R3*SEL. K*(1-SEL. K/(BETA. K*(LODDE. K+TORSK. K)))-SFANGST. K
A SFANGST. K=TABHL (SFAN, TIME. K, 1868, 1980, 1)
C R3=0.0614
C Y3=0.035

FANGST AV LODDE, TORSK OG SEL (51 LINJER)

T LFAN=149.2E3, 182.6E3, 95.4E3, 177.5E3, 95.5E3, 92.5E3, 117.9E3,
X 102.9E3, 84.4E3, 83.6E3, 69.1E3, 51.6E3, 75.8E3, 66.9E3,
X 38.8E3, 95.5E3, 58.8E3, 80.3E3, 121.5E3, 114.6E3, 112.1E3,
X 112.9E3, 78.5E3, 130.7E3, 177.1E3, 170.2E3, 51.2E3, 119.8E3,
X 66.3E3, 184.2E3, 109.3E3, 89.7E3, 100.8E3, 100.3E3, 107.4E3,
X 161.1E3, 95.2E3, 114.6E3, 27.4E3, 54.3E3, 102.0E3, 155.4E3,
X 135.5E3, 147.0E3, 139.0E3, 107.7E3, 166.2E3, 158.7E3, 185.1E3,
X 174.7E3, 199.5E3, 266.2E3, 134.5E3, 145.0E3, 179.9E3, 98.6E3,
X 132.3E3, 125.9E3, 192.4E3, 204.6E3, 315.3E3, 264.9E3, 243.3E3,
X 220.0E3, 246.8E3, 455.0E3, 271.4E3, 229.6E3, 256.9E3, 257.3E3,
X 218.5E3, 239.5E3, 390.9E3, 314.2E3, 253.2E3, 162.7E3, 119.4E3,
X 163.5E3, 169.9E3, 184.9E3, 235.7E3, 199.4E3, 297.9E3, 676.5E3,
X 719.5E3, 527.1E3, 1097.9E3, 749.3E3, 977.4E3, 996.4E3, 927.0E3,
X 1005.2E3, 1053.9E3, 869.1E3, 604.9E3, 750.9E3, 889.2E3, 1393.2E3,
X 1908.8E3, 2035.7E3, 1237.8E3, 742.1E3, 1376.5E3, 1414.4E3, 1605.9E3,
X 1342.9E3, 1154.9E3, 1421.4E3, 2546.4E3, 2958.0E3, 2048.2E3, 1832.5E3,
X 1704.5E3

T TFAN=198. 2E3, 176. 0E3, 207. 2E3, 171. 3E3, 197. 4E3, 209. 9E3, 200. 3E3,
X 242. 7E3, 168. 4E3, 280. 6E3, 213. 5E3, 266. 7E3, 294. 5E3, 246. 6E3,
X 233. 6E3, 162. 5E3, 222. 1E3, 257. 8E3, 277. 1E3, 226. 2E3, 246. 2E3,
X 258. 6E3, 276. 8E3, 219. 4E3, 289. 3E3, 311. 1E3, 311. 4E3, 304. 9E3,
X 229. 0E3, 272. 0E3, 185. 3E3, 167. 6E3, 176. 2E3, 180. 7E3, 191. 9E3,
X 193. 3E3, 210. 5E3, 192. 3E3, 228. 6E3, 238. 5E3, 243. 0E3, 289. 7E3,
X 317. 7E3, 360. 2E3, 505. 9E3, 389. 2E3, 380. 6E3, 313. 5E3, 248. 4E3,
X 169. 0E3, 154. 1E3, 173. 9E3, 181. 7E3, 202. 7E3, 250. 0E3, 317. 4E3,
X 340. 9E3, 396. 8E3, 565. 3E3, 478. 5E3, 489. 2E3, 589. 0E3, 602. 0E3,
X 452. 5E3, 490. 0E3, 445. 7E3, 529. 7E3, 611. 9E3, 878. 3E3, 1052. 2E3,
X 1020. 8E3, 821. 4E3, 547. 7E3, 457. 8E3, 325. 1E3, 307. 7E3, 350. 8E3,
X 305. 3E3, 748. 6E3, 963. 4E3, 872. 6E3, 917. 1E3, 890. 6E3, 1065. 5E3,
X 1138. 5E3, 952. 9E3, 1104. 7E3, 1435. 6E3, 1654. 1E3, 1062. 5E3, 987. 6E3,
X 1043. 1E3, 1027. 8E3, 674. 5E3, 1231. 5E3, 1098. 0E3, 752. 6E3, 773. 2E3,
X 890. 1E3, 895. 4E3, 1368. 5E3, 1504. 0E3, 1255. 4E3, 1028. 1E3, 1036. 0E3,
X 1337. 7E3, 1637. 9E3, 1301. 2E3, 1289. 6E3, 1240. 5E3, 984. 5E3, 746. 8E3,
X 652. 5E3
T SFAN=3. 0E3, 2. 0E3, 3. 4E3, 2. 2E3, 3. 7E3, 3. 4E3, 4. 6E3,
X 3. 8E3, 4. 1E3, 3. 2E3, 8. 0E3, 9. 1E3, 11. 2E3, 19. 7E3,
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X 4. 0E3, 24. 7E3, 22. 6E3, 23. 3E3, 12. 8E3, 16. 0E3, 22. 8E3,
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X 13. 3E3, 12. 5E3, 23. 4E3, 29. 9E3, 40. 6E3, 33. 1E3, 36. 9E3,
X 29. 5E3, 22. 4E3, 31. 7E3, 29. 7E3, 24. 4E3, 26. 6E3, 31. 9E3,
X 22. 0E3, 25. 0E3, 24. 9E3, 26. 6E3, 22. 2E3, 18. 5E3, 13. 9E3,
X 12. 7E3, 11. 3E3, 13. 0E3, 13. 8E3, 13. 5E3, 12. 4E3, 11. 8E3,
X 9. 8E3, 8. 6E3, 9. 0E3, 10. 3E3, 9. 7E3, 8. 1E3, 10. 6E3,
X 11. 0E3
* TREARTSMODELL MED TIDSAVH. PARAMETRE 1868-1980/2080
N TIME=NTIME
C NTIME=1868
SPEC LENGTH=1980, DT=0. 05, PRTPER=1, PLTPER=1
PLOT LODDE=L, TORSK=T, SEL=S

Simulation results with the basic case parameters from Table 6.12.

PAGE 2 TREARTSMODELL MED TIDSAVH. PARAMETRE 1868-1980

TIME E+00	LODDE E+03	TORSK E+03	SEL E+03	LODDE=CAPELIN
1868.0	10069.	4759.0	1779.0	TORSK=COD
1869.0	10022.	4611.0	1775.7	SEL =SEA MAMMALS
1870.0	10128.	4535.5	1771.8	
1871.0	10251.	4527.0	1768.7	
1872.0	10323.	4544.2	1766.3	
1873.0	10393.	4557.2	1764.1	
1874.0	10415.	4575.3	1762.1	
1875.0	10414.	4575.6	1760.1	
1876.0	10429.	4593.8	1758.5	
1877.0	10425.	4589.2	1757.5	
1878.0	10458.	4575.7	1754.6	
1879.0	10489.	4583.1	1749.3	
1880.0	10521.	4568.7	1742.9	
1881.0	10550.	4584.1	1731.9	
1882.0	10558.	4635.8	1718.5	
1883.0	10501.	4712.7	1703.1	
1884.0	10400.	4761.2	1686.1	
1885.0	10346.	4757.5	1662.8	
1886.0	10323.	4760.7	1630.6	
1887.0	10307.	4805.7	1607.4	
1888.0	10261.	4855.2	1594.1	
1889.0	10211.	4872.3	1587.8	
1890.0	10193.	4865.5	1583.1	
1891.0	10182.	4888.1	1570.2	
1892.0	10123.	4903.9	1549.4	
1893.0	10100.	4896.7	1521.8	
1894.0	10163.	4921.2	1496.1	
1895.0	10202.	4979.9	1478.6	
1896.0	10170.	5063.6	1460.3	
1897.0	10057.	5130.8	1437.6	
1898.0	9945.	5179.8	1414.8	
1899.0	9868.	5246.8	1398.5	

PAGE 3 TREARTSMODELL MED TIDSAVH. PARAMETRE 1868-1980

1900.0	9781.	5272.2	1395.7
1901.0	9717.	5259.3	1395.2
1902.0	9695.	5240.3	1386.8
1903.0	9679.	5235.3	1377.8
1904.0	9687.	5231.1	1366.5
1905.0	9720.	5259.1	1349.0
1906.0	9751.	5288.3	1341.2
1907.0	9781.	5301.1	1340.4
1908.0	9761.	5302.8	1343.0
1909.0	9714.	5265.4	1351.8
1910.0	9715.	5198.7	1361.9
1911.0	9774.	5136.1	1371.0
1912.0	9878.	5039.6	1380.1
1913.0	10042.	5005.2	1390.1
1914.0	10102.	5049.0	1404.1
1915.0	10065.	5085.1	1419.6
1916.0	9976.	5122.0	1433.7
1917.0	9861.	5152.3	1447.5
1918.0	9745.	5159.6	1450.7
1919.0	9643.	5136.6	1443.7
1920.0	9642.	5105.7	1437.1
1921.0	9714.	5099.3	1435.6
1922.0	9750.	5082.0	1438.2
1923.0	9826.	5040.8	1435.9
1924.0	9939.	5019.5	1426.7
1925.0	10028.	5018.3	1406.6
1926.0	10103.	4978.1	1383.5
1927.0	10189.	4970.3	1366.0
1928.0	10174.	5025.6	1350.1
1929.0	10126.	5017.0	1339.3
1930.0	10173.	4985.3	1318.6
1931.0	10237.	5075.6	1287.6
1932.0	10169.	5202.7	1263.9
1933.0	9958.	5272.8	1238.6

PAGE 4 TREARTSMODELL MED TIDSAVH. PARAMETRE 1868-1980

1934.0	9800.	5262.4	1220.7
1935.0	9847.	5195.3	1215.7
1936.0	9987.	5050.4	1214.0
1937.0	10273.	4851.5	1201.5
1938.0	10667.	4769.6	1194.2
1939.0	10915.	4881.1	1205.4
1940.0	10763.	5152.7	1221.7
1941.0	10329.	5363.6	1237.1
1942.0	9953.	5446.1	1248.6
1943.0	9715.	5435.3	1255.4
1944.0	9646.	5356.4	1281.1
1945.0	9661.	5297.5	1293.7
1946.0	9734.	5104.1	1302.6
1947.0	10059.	4792.7	1306.2
1948.0	10514.	4654.9	1301.7
1949.0	10866.	4678.5	1290.2
1950.0	11029.	4759.1	1278.2
1951.0	10876.	4765.5	1268.7
1952.0	10703.	4634.5	1260.1
1953.0	10782.	4587.7	1258.3
1954.0	10668.	4577.7	1256.0
1955.0	10645.	4334.9	1248.9
1956.0	11059.	3955.1	1245.2
1957.0	11592.	3876.9	1243.9
1958.0	11774.	4190.5	1240.8
1959.0	11514.	4468.6	1240.6
1960.0	11053.	4602.2	1243.6
1961.0	10663.	4774.2	1244.0
1962.0	10445.	4733.2	1243.0
1963.0	10560.	4527.9	1242.4
1964.0	10593.	4607.5	1246.0
1965.0	10163.	4753.6	1253.6
1966.0	9392.	4634.2	1261.7
1967.0	8762.	4325.1	1266.9

PAGE 5 TREARTSMODELL MED TIDSAVH. PARAMETRE 1868-1980

1968.0	8921.	3857.5	1268.6
1969.0	10181.	3404.9	1270.0
1970.0	11517.	3255.5	1276.0
1971.0	12172.	3448.8	1286.4
1972.0	12141.	3786.3	1299.6
1973.0	11805.	3918.9	1313.8
1974.0	11807.	3709.1	1328.5
1975.0	12067.	3521.5	1342.9
1976.0	11719.	3508.3	1356.0
1977.0	10852.	3434.6	1365.4
1978.0	10489.	3413.8	1372.0
1979.0	10597.	3617.8	1378.2
1980.0	10495.	3944.5	1384.1

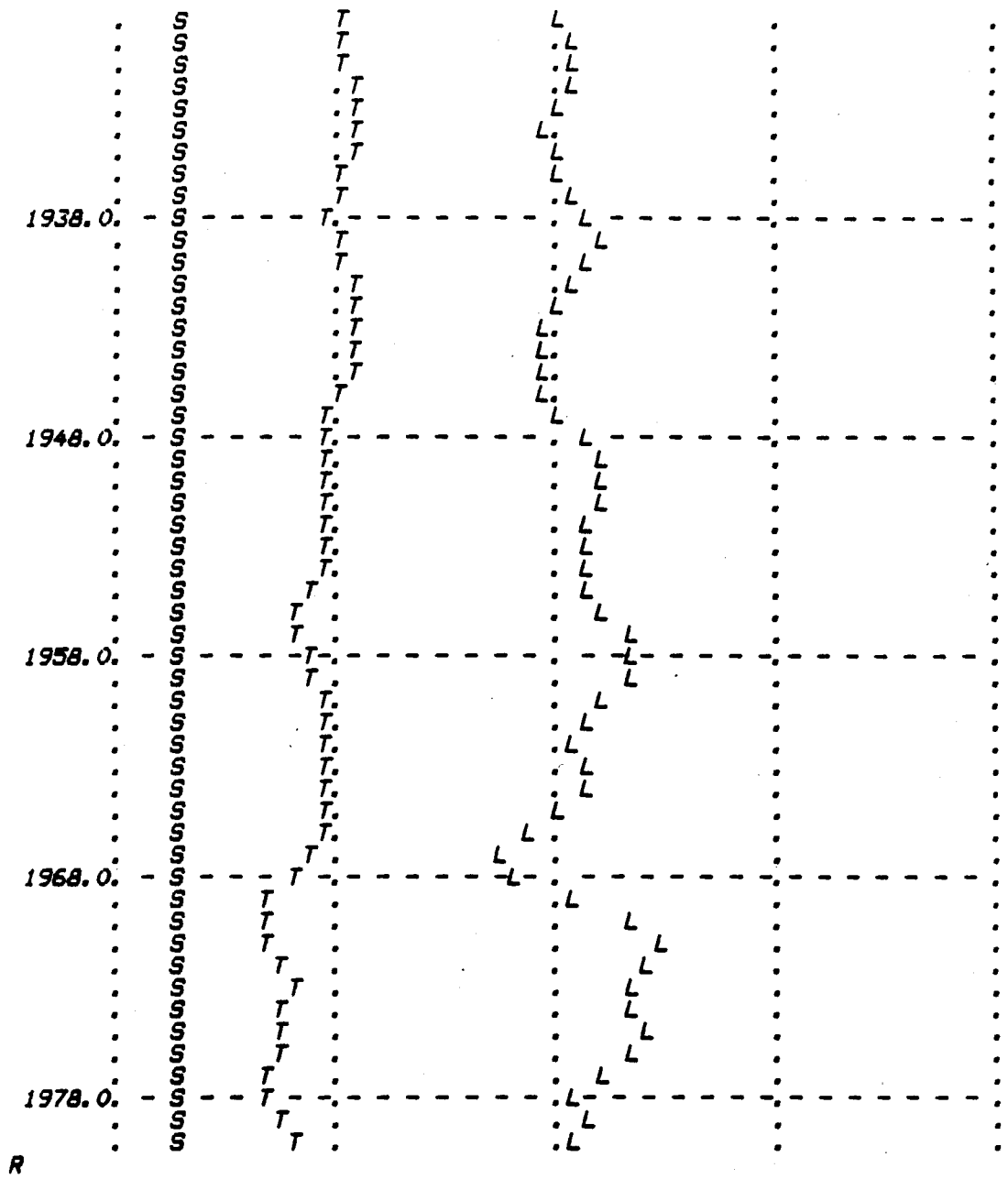
Q
T

Plotted results:

PAGE 6 TREARTSMODELL MED TIDSAVH. PARAMETRE 1868-1980/20

LODDE=L TORSK=T SEL=S

	0.000M	5.000M	10.000M	15.000M	20.000M LTS
1868.0.	S	T	L		
.	S	T	L		
.	S	T	L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
1878.0.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
1888.0.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
1898.0.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
1908.0.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
1918.0.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
.	S	T	.L		
1928.0.	S	T	.L		



PAGE 7 TREARTSMODELL MED TIDSAVH. PARAMETRE 1868-1980/20

EQUILIBRIUM AND STABILITY CONDITIONS OF THE THREE SPECIES MODEL

Conditions for Existence of Equilibrium

The three equations that must be satisfied if an equilibrium with all 3 stocks present exists are from (3.6)-(3.8)

$$A_1 = X_1 + \gamma_{12}X_2 + \gamma_{13}X_3$$

$$A_2 = X_2/X_1 + \gamma_{23}X_3$$

$$A_3(X_1 + \alpha X_2) = \alpha X_3 .$$

The third equation can be used to eliminate X_3 from the first two equations giving

$$X_2 = \frac{\alpha A_1 - (\alpha + \gamma_{13}A_3)X_1}{\alpha(\gamma_{12} + \gamma_{13}A_3)} \quad (\text{A9.1})$$

and

$$X_2 = \frac{(\alpha A_2 - \gamma_{23}A_3X_1)X_1}{\alpha(1 + A_3\gamma_{23}X_1)} . \quad (\text{A9.2})$$

Note that $X_3 > 0$ if $X_1 > 0$ and $X_2 > 0$ by the third equation above.

Eliminating X_2 from (A9.1) and (A9.2) and simplifying gives

$$(\gamma_{12} - \alpha)\gamma_{23}A_3X_1^2 + B_1X_1 + \alpha A_1 = 0 , \quad (\text{A9.3})$$

where

$$-B_1 = \alpha(1 + \gamma_{12}A_2) + \gamma_{13}A_3(1 + \alpha A_2) - \alpha\gamma_{23}A_1A_3 .$$

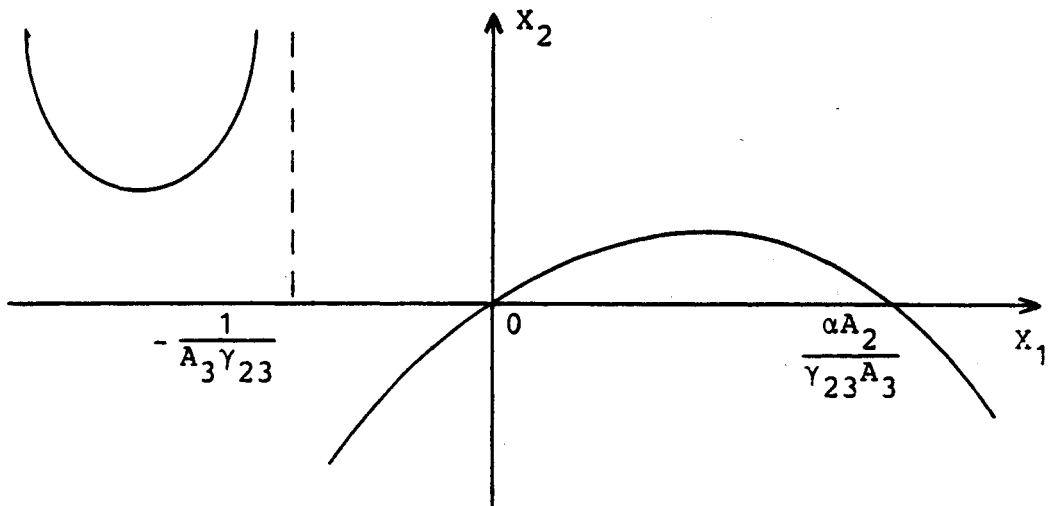
By the quadratic formula the formal solution is

$$X_1 = \frac{-B_1 \pm \sqrt{B_1^2 - 4\alpha\gamma_{23}(\gamma_{12} - \alpha)A_1A_3}}{2(\gamma_{12} - \alpha)\gamma_{23}A_3} \quad (A9.4)$$

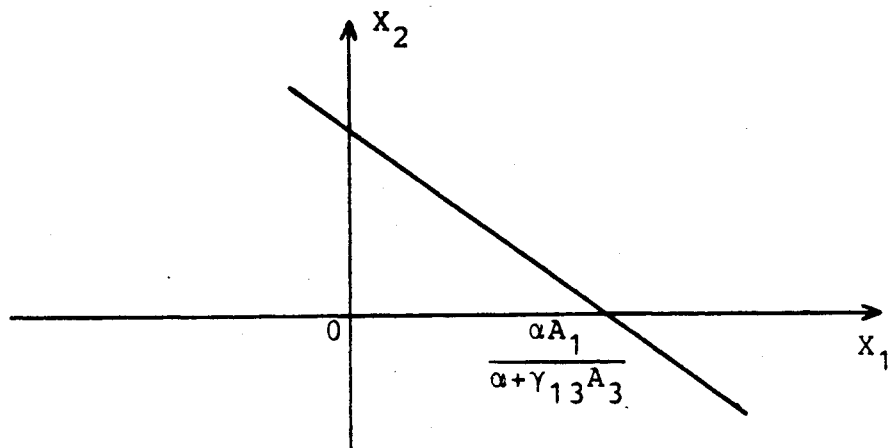
Note that by (2) $X_2 > 0$ if and only if

$$0 < X_1 < \frac{\alpha A_2}{\gamma_{23}A_3} \quad (A9.5)$$

In fact, the graph of (A9.2) has the form



The graph of (A9.1) has the form



Stability of Equilibrium

One looks at the matrix A of the linearization of the system (3.5)-(3.8) at the equilibrium point. Stability (local) require that all eigenvalues have nonpositive real parts which is true if

$$\text{tr } A < 0, \quad \Delta^2 A > 0, \quad |A| < 0 \quad (\Delta^2 A = \text{sum of the 3 principal } 2 \times 2 \text{ minors})$$

(i.e. all coefficients in characteristic equation are positive).

From (3.6)-(3.8) in the text

$$A = \begin{bmatrix} r_1(A_1 - 2X_1 - \gamma_{12}X_2 - \gamma_{13}X_3) & -r_1\gamma_{12}X_1 & -r_1\gamma_{13}X_1 \\ r_2 \frac{X_2^2}{X_1^2} & r_2(A_2 - 2X_2/X_1 - \gamma_{23}X_3) & -r_2\gamma_{23}X_2 \\ \frac{r_3\alpha X_2^3}{(X_1 + \alpha X_2)^2} & \frac{r_3\alpha^2 X_3^2}{(X_1 + \alpha X_2)^2} & r_3(A_3 - 2\alpha X_3/(X_1 + \alpha X_2)) \end{bmatrix}.$$

Note that $X_1 + \gamma_{12}X_2 + \gamma_{13}X_3 = A_1$ from (3.6), $X_2/X_1 + \gamma_{23}X_3 = A_2$ from (3.7) and $\alpha X_3/(X_1 + \alpha X_2) = A_3$ from (3.8) so

$$A = \begin{bmatrix} -r_1X_1 & -r_1\gamma_{12}X_1 & -r_1\gamma_{13}X_1 \\ r_2 \frac{X_2^2}{X_1^2} & -r_2 \frac{X_2}{X_1} & -r_2\gamma_{23}X_2 \\ r_3 \frac{A_3^2}{\alpha} & r_3A_3^2 & -r_3A_3 \end{bmatrix}.$$

There are then the following possibilities:

1. The roots of (A9.4) have the opposite signs (or (A9.3) is linear in which case there is a positive root). This happens when $\alpha - \gamma_{12} \geq 0$. In this case the line in the second graph intersects the curve in the first graph with a positive X_2 if and only if (since there is at most one intersection)

$$\frac{\alpha A_1}{\alpha + \gamma_{13} A_3} \leq \frac{\alpha A_2}{\gamma_{23} A_3}$$

or

$$\gamma_{23} A_1 A_3 \leq \alpha A_2 + \gamma_{13} A_2 A_3 .$$

2. If $\alpha - \gamma_{12} < 0$, and both roots of (A9.4) are real then they both have the same sign and this sign is positive providing $B_1 < 0$. In case

$$\frac{\alpha A_1}{\alpha + \gamma_{13} A_3} \leq \frac{\alpha A_2}{\gamma_{23} A_3} \quad \text{again}$$

one sees from the graphs that only the lesser root

$$X_1 = \frac{-B_1 - \sqrt{B_1^2 - 4\alpha\gamma_{23}(\gamma_{12} - \alpha)A_1A_3}}{2(\gamma_{12} - \alpha)\gamma_{23}A_3}$$

has $X_2 > 0$. Indeed this condition guarantees the existence of an equilibrium with $X_2 > 0$ so it implies that $B_1 < 0$ and the discriminant is positive.

3. If $\frac{\alpha A_1}{\alpha + \gamma_{13} A_3} > \frac{\alpha A_2}{\gamma_{23} A_3}$ and $\alpha - \gamma_{12} < 0$ then from the graphs either both roots are complex or both have $X_2 > 0$. The second case occurs when discriminant is nonnegative.

In summary, there is a unique equilibrium with all stocks positive if

$$\gamma_{23} A_1 A_3 \leq \alpha A_2 + \gamma_{13} A_2 A_3 .$$

If $\gamma_{23} A_1 A_3 > \alpha A_2 + \gamma_{13} A_2 A_3$ there is a double equilibrium if

$$\alpha < \gamma_{12} \quad \text{and} \quad B_1^2 - 4\alpha\gamma_{23}(\gamma_{12} - \alpha)A_1 A_3 > 0 ,$$

and a single equilibrium if

$$\alpha < \gamma_{12} \quad \text{and} \quad B_1^2 - 4\alpha\gamma_{23}(\gamma_{12} - \alpha)A_1 A_3 = 0 .$$

Otherwise there is no equilibrium with all stocks positive.

Then

$$\text{tr } A = -r_1 X_1 - r_2 \frac{X_2}{X_1} - r_3 A_3 < 0$$

and

$$\begin{aligned} \Delta^2 A &= \begin{vmatrix} -r_1 X_1 & -r_1 \gamma_{12} X_1 \\ r_2 \frac{X_2^2}{X_1^2} & -r_2 \frac{X_2}{X_1} \end{vmatrix} + \begin{vmatrix} -r_1 X_1 & -r_1 \gamma_{13} X_1 \\ r_3 \frac{A_3^2}{\alpha} & -r_3 A_3 \end{vmatrix} + \begin{vmatrix} -r_2 \frac{X_2}{X_1} & -r_2 \gamma_{23} X_2 \\ r_3 A_3^2 & -r_3 A_3 \end{vmatrix} \\ &= r_1 r_2 \left(X_2 + \gamma_{12} \frac{X_2^2}{X_1} \right) + r_1 r_3 \left(A_3 X_1 + \frac{\gamma_{13}}{\alpha} A_3^2 X_1 \right) \\ &\quad + r_2 r_3 \left(A_3 \frac{X_2}{X_1} + \gamma_{23} A_3^2 X_2 \right) > 0. \end{aligned}$$

Moreover

$$\begin{aligned} |A| &= (-r_1 X_1) (r_2 X_2) \left(r_3 \frac{A_3}{\alpha} \right) \begin{vmatrix} -1 & -\gamma_{12} & -\gamma_{13} \\ \frac{X_2}{X_1^2} & -\frac{1}{X_1} & -\gamma_{23} \\ A_3 & \alpha A_3 & -\alpha \end{vmatrix} \\ &= \frac{-r_1 r_2 r_3}{\alpha} X_1 X_2 A_3 \left\{ -\frac{\alpha}{X_1} + \gamma_{12} \gamma_{23} A_3 - \alpha \gamma_{13} A_3 \frac{X_2}{X_1} \right. \\ &\quad \left. - \gamma_{13} A_3 \frac{1}{X_1} - \alpha \gamma_{23} A_3 - \alpha \gamma_{12} \frac{X_2}{X_1^2} \right\} \\ &= \frac{r_1 r_2 r_3}{\alpha} A_3 \frac{X_2}{X_1} \left\{ (\gamma_{12} - \alpha) \gamma_{23} A_3 X_1^2 - (\alpha + \gamma_{13} A_3) X_1 \right. \\ &\quad \left. - \alpha (\gamma_{12} + \gamma_{13} A_3) X_2 \right\}. \end{aligned}$$

Now

$$\alpha(\gamma_{12} + \gamma_{13}A_3)X_2 = \alpha A_2 - (\alpha + \gamma_{13}A_3)X_1 \quad (\text{equivalent to (3.10) in text}).$$

Thus

$$|A| = \frac{r_1 r_2 r_3}{\alpha} A_3 \frac{X_2}{X_1} \left\{ (\gamma_{12} - \alpha) \gamma_{23} A_3 X_1^2 - \alpha A_2 \right\}, \quad (\text{A9.6})$$

or using the quadratic equation defining X_1

$$|A| = \frac{r_1 r_2 r_3}{\alpha} A_3 \frac{X_2}{X_1} \left\{ -B_1 X_1 - \alpha(A_1 + A_2) \right\}. \quad (\text{A9.7})$$

In sum, an equilibrium is locally stable only if

$$(\gamma_{12} - \alpha) \gamma_{23} A_3 X_1^2 - \alpha A_2 < 0, \quad (\text{A9.8})$$

or equivalently,

$$-B_1 X_1 - \alpha(A_1 + A_2) < 0. \quad (\text{A9.9})$$

Some general observations about stability can then be made.

In particular if $\gamma_{12} - \alpha \leq 0$, then by (A9.3) the (unique) equilibrium is locally stable.

If, on the other hand, $\gamma_{12} - \alpha > 0$, then stability requires

$$X_1^2 < \frac{\alpha A_2}{(\gamma_{12} - \alpha) \gamma_{23} A_3} \quad \text{by (A9.3)}.$$

Using the formula for X_1 , this is equivalent to

$$[-B_1 \pm \sqrt{B_1^2 - 4(\gamma_{12} - \alpha)\gamma_{23}A_1A_3}]^2 < 4(\gamma_{12} - \alpha)\alpha\gamma_{23}A_2A_3,$$

which can be rewritten as

$$[1 \pm \sqrt{1-x}]^2 < \frac{A_2}{A_1} x$$

where

$$x = \frac{4(\gamma_{12} - \alpha)\gamma_{23}A_1A_3}{B_1^2} \leq 1$$

(note that when $\gamma_{12} - \alpha > 0$, and an equilibrium exists then $-B_1 > 0$).

Suppose now there is a double equilibrium (so $x < 1$). The equilibrium with the larger value of X_1 is stable if and only if

$$2 - x + 2\sqrt{1-x} < \frac{A_2}{A_1} x$$

$$2\sqrt{1-x} < (1 + \frac{A_2}{A_1})x - 2.$$

Therefore stability requires

$$x \geq \frac{2A_1}{A_1 + A_2} \quad (\text{i.e. RHS} \geq 0)$$

and

$$4 - 4x < 4 - 4(1 + \frac{A_2}{A_1})x + (1 + \frac{A_2}{A_1})^2 x^2,$$

which simplifies to

$$0 < -4 \frac{A_2}{A_1} + \left(\frac{A_1 + A_2}{A_1} \right)^2 x$$

or

$$x > \frac{4A_1 A_2}{(A_1 + A_2)^2} .$$

Now suppose $A_1 \geq A_2$. Then

$$\frac{2A_2}{A_1 + A_2} \leq 1 \leq \frac{2A_1}{A_1 + A_2}$$

and $x \geq \frac{2A_1}{A_1 + A_2}$ is false.

Therefore this equilibrium is unstable.

On the other hand if $A_1 < A_2$, then

$$\frac{2A_1}{A_1 + A_2} < 1 < \frac{2A_2}{A_1 + A_2}$$

and hence

$$\frac{2A_1}{A_1 + A_2} < \frac{4A_1 A_2}{(A_1 + A_2)^2} ,$$

so the equilibrium is stable if $x > \frac{4A_1 A_2}{(A_1 + A_2)^2}$.

The equilibrium corresponding to the smaller value of X_1 can be analyzed in a similar manner.

Stability requires

$$2 - x - 2\sqrt{1-x} < \frac{A_2}{A_1} x$$

or

$$2 - \left(1 + \frac{A_2}{A_1}\right)x < 2\sqrt{1-x} .$$

Thus the equilibrium is stable if

$$x > \frac{2A_1}{A_1 + A_2} \quad (\text{i.e. LHS} < 0)$$

or if

$$4 - 4\left(1 + \frac{A_2}{A_1}\right)x + \left(1 + \frac{A_2}{A_1}\right)^2 x^2 < 4 - 4x ,$$

which simplifies to

$$-4 \frac{A_2}{A_1} + \left(\frac{A_1 + A_2}{A_1}\right)^2 x < 0$$

or

$$x < \frac{4A_1 A_2}{(A_1 + A_2)^2} .$$

Then in the case $A_1 > A_2$

$$x > \frac{2A_1}{A_1 + A_2} \quad \text{is not possible}$$

so stability requires

$$x < \frac{4A_1 A_2}{(A_1 + A_2)^2} .$$

In the case $A_1 \leq A_2$

$$\frac{2A_1}{A_1+A_2} \leq \frac{4A_1A_2}{(A_1+A_2)^2}$$

so one of the two inequalities must be true at least and the equilibrium is stable.

Finally, in the case $x = 1$, the equilibrium is stable if

$$1 < \frac{A_2}{A_1} .$$

These results can be summarized as follows:

1. If $\alpha \geq \gamma_{12}$, the unique equilibrium is stable.
2. If $\alpha < \gamma_{12}$ and $x = 1$, the unique equilibrium is stable if $1 < A_2/A_1$, unstable if $1 > A_2/A_1$.
3. If $\alpha < \gamma_{12}$ and $x < 1$, then the equation defining X_1 may have two solutions but the larger root may have $X_2 < 0$.
In any case

(a) if $A_2 = A_1$, the smaller root corresponds to a stable equilibrium and the larger root (if relevant) to an unstable equilibrium. The case includes the case of no fishing.

(b) if $A_1 > A_2$, the larger root (if relevant) is unstable and the smaller root is stable only if $x < 4A_1A_2/(A_1+A_2)^2$.

(c) if $A_1 < A_2$, the smaller root is stable and the larger root (if relevant) is stable only if $x > 4A_1A_2/(A_1+A_2)^2$.

A SIMULA PROGRAM FOR COMPUTATION OF EQUILIBRIUM STOCKS AND
HARVEST RATES

```

BEGIN
COMMENT
*****
*
*   FILNAVN PROGRAM      : KVFYSI
*   FILNAVN KOMPILERT    : KVFYSB
*   FILNAVN INNDATA      : INPUT
*   FILNAVN UTDATA       : LESES INN
*   PROGRAMMERINGSsprak : SIMULA 1.2-353
*   TYPE MASKIN          : CYBER 171MP
*   OPERATIVSYSTEM       : NOS 2.0-531/528
*   PROGRAMMERER         : SIGFUS KRISTMANSSON/
*                       OLA FLATEN
*   INSTITUSJON          : POLLINN/IFF
*   SEKSJON              : MARKED/ØKONOMI
*   DATO                  : 11/04/1985
*   VERSJON              : 2
*
*****;

COMMENT PROGRAMMET LESER INN HØYESTE/LAVESTE VERDI FOR EN PARAMETER
OG VERDIER FOR 11 ANDRE. DATAENE LEGGES UT PÅ EN FIL TIL GRAFISK
FREMSTILLING;

REAL ARRAY PARAM(1:12);
REAL A12, A13, A23, A1, A2, A3, G12, G13, G23, K1, K2, X1, X2, X3, W1, W2, W3,
Y1, Y2, Y3, H1, H2, SQ, X, XMIN, XMAX, XSTEP;
TEXT ARRAY TPARAM(1:12);
INTEGER I, PARAMNR;
TEXT FILNAVN;
REF(OUTFILE)UT;

COMMENT *****
*   HOVEDPROGRAM
*****;

FILNAVN:=-BLANKS(7);
OUTTEXT("LES INN NAVN PÅ DATAFIL");OUTIMAGE;INIMAGE;
FILNAVN:=INTEXT(7);
BEGIN
UT:=-NEW OUTFILE(FILNAVN.STRIP);
UT.OPEN(BLANKS(80),0);
FOR I:=1 STEP 1 UNTIL 12 DO
  TPARAM(I):=-BLANKS(3);

TPARAM(1):="U1";
TPARAM(2):="U2";
TPARAM(3):="U3";
TPARAM(4):="R1";
TPARAM(5):="R2";
TPARAM(6):="R3";
TPARAM(7):="A ";
TPARAM(8):="B ";
TPARAM(9):="K ";
TPARAM(10):="A12";
TPARAM(11):="A13";
TPARAM(12):="A23";

PARAM(1):=0.0000;
PARAM(2):=0.0000;
PARAM(3):=0.0000;
PARAM(4):=1.2704;
PARAM(5):=1.1617;
PARAM(6):=0.0614;
PARAM(7):=0.90;
PARAM(8):=0.12;
PARAM(9):=30*10**6;
PARAM(10):=0.125/10**6;
PARAM(11):=0.140/10**6;
PARAM(12):=0.310/10**6;

```

```

FOR I:=1 STEP 1 UNTIL 12 DO
BEGIN
  SETPOS(20);OUTINT(I,3);OUTTEXT(" ");
  OUTTEXT(TPARAM(I));OUTTEXT(" = ");
  OUTFIX(PARAM(I),12,22);OUTIMAGE;
END FOR I;
OUTIMAGE;OUTTEXT("NR. PA PARAMETER SOM SKAL FORANDRES");
OUTIMAGE;OUTIMAGE;
PARAMNR:=ININT;
OUTTEXT("LES INN MIN OG MAX FOR ");
OUTTEXT(TPARAM(PARAMNR));OUTIMAGE;OUTIMAGE;
XMIN:=INREAL; XMAX:=INREAL;
XSTEP:=(XMAX-XMIN)/50;

FOR X:=XMIN STEP XSTEP UNTIL XMAX DO
BEGIN
  PARAM(PARAMNR):=X;

  A1:=1-PARAM(1)/PARAM(4);
  A2:=1-PARAM(2)/PARAM(5);
  A3:=1-PARAM(3)/PARAM(6);

  G12:=PARAM(10)*PARAM(7)*PARAM(9)/PARAM(4);
  G13:=PARAM(11)*PARAM(7)*PARAM(8)*PARAM(9)/PARAM(4);
  G23:=PARAM(12)*PARAM(7)*PARAM(8)*PARAM(9)/PARAM(5);

  K1:=A1*A3*623-1-G12*A2-A3*G13/PARAM(7)-A2*A3*G13;
  K2:=A3*623*(G12/PARAM(7)-1);

  SQ:=K1**2-4*A1*K2;
  IF SQ<0 THEN
  BEGIN
    OUTTEXT("VERDI I KVADRATROT ER NEGATIV, SETTES TIL 0");OUTIMAGE;
    SQ:=0;
    UT.OUTCHAR('*');
  END ELSE
    UT.OUTCHAR(' ');

  X1:=(-K1-SQRT(SQ))/(2*K2);

  H1:=1/PARAM(7)+A2;
  H2:=1+A3*623*X1;

  X2:=((A2-A3*623*X1/PARAM(7))*X1)/H2;
  X3:=(A3*H1*X1)/H2;

  W1:=X1*PARAM(9)/1000;
  W2:=X2*PARAM(7)*PARAM(9)/1000;
  W3:=X3*PARAM(7)*PARAM(8)*PARAM(9)/1000;

  Y1:=PARAM(1)*W1;
  Y2:=PARAM(2)*W2;
  Y3:=PARAM(3)*W3;

  UT.OUTFIX(X,12,22);
  UT.OUTFIX(W1,3,12);UT.OUTFIX(W2,3,12);
  UT.OUTFIX(W3,3,12);UT.OUTIMAGE;
END FOR X;

OUTIMAGE;OUTTEXT("DATA PA FILEN ");OUTTEXT(FILNAVN);
OUTIMAGE;OUTIMAGE;
END FILNAVN;
END PROGRAM;

```

THE PARTIAL DERIVATIVES OF THE GROWTH FUNCTIONS

$$G_{ij} = \frac{\partial F_j(X_1, X_2, X_3)}{\partial X_i} \quad (i, j = 1, 2, 3)$$

$$G_{11} = r_1(1 - 2X_1 - \gamma_{12}X_2 - \gamma_{13}X_3)$$

$$G_{21} = -\gamma_{12}r_1X_1$$

$$G_{31} = -\gamma_{13}r_1X_1$$

$$G_{12} = r_2X_2^2/X_1^2$$

$$G_{22} = r_2(1 - 2X_2/X_1 - \gamma_{23}X_3)$$

$$G_{32} = -\gamma_{23}r_2X_2$$

$$G_{13} = r_3X_3^2/(\alpha(X_1/\alpha + X_2)^2)$$

$$G_{23} = r_3X_3^2/(X_1/\alpha + X_2)^2$$

$$G_{33} = r_3 - 2r_3X_3/(X_1/\alpha + X_2)$$

TWO FORTRAN PROGRAMS FOR COMPUTATION OF THE TSB-MODEL'S OPTIMAL STOCKS, SUSTAINABLE RENTS AND NET PRESENT VALUE

The OPT-program

```
PROGRAM TEST2(INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT)
REAL X(3),FVEC(3),XTOL,WA(51),R(14),A(6),Y(3),V(4)
REAL B(6),C(4)
COMMON R
EXTERNAL FCN
```

C
C
C

```
A INNEHOLDER DE BIOLOGISKE PARAMETRENE I SAMME
REKKEFØLGE SOM PÅ ARKET.
```

```
A(1)=1.2704
A(2)=1.1617
A(3)=0.0614
A(4)=0.90
A(5)=0.12
A(6)=3E7
```

C
C
C

```
B INNEHOLDER DE ØKONOMISKE PARAMETRENE I SAMME
REKKEFØLGE SOM PÅ ARKET.
```

```
B(1)=0.7926
B(2)=2.3279
B(3)=3.2029
B(4)=3.7332E6
B(5)=8.4173E6
B(6)=2.7273E6
```

C
C
C

```
C INNEHOLDER DE FASTE PARAMETRENE I SAMME REKKEFØLGE
SOM PÅ ARKET.
```

```
C(1)=0.125E-6
C(2)=0.140E-6
C(3)=0.310E-6
C(4)=0.05
```

C

```

R(1)=A(1)
R(2)=A(2)
R(3)=A(3)
R(4)=A(4)
R(5)=B(4)
R(6)=B(5)
R(7)=B(6)
R(8)=B(1)*A(6)
R(9)=B(2)*R(4)*A(6)
R(10)=B(3)*R(4)*A(5)*A(6)
R(11)=C(1)*R(4)*A(6)/R(1)
R(12)=C(2)*R(4)*A(5)*A(6)/R(1)
R(13)=C(3)*R(4)*A(5)*A(6)/R(2)
R(14)=C(4)
WRITE(2,1)
FORMAT("GI VERDIER FOR X1,X2 OG X3")
READ(2,*) (X(I),I=1,3)
TOL=SQRT(X02AAF(0.0))
IFAIL=0
N=3
CALL COSNBF(FCN,N,X,FVEC,TOL,WA,51,IFAIL)
WRITE(2,*) IFAIL
Y(1)=R(1)*X(1)*(1-X(1)-R(11)*X(2)-R(12)*X(3))
Y(2)=R(2)*X(2)*(1-X(2)/X(1)-R(13)*X(3))
Y(3)=R(3)*X(3)*(1-X(3)/((X(1)/R(4))+X(2)))
V(1)=(R(8)-R(5)/X(1))*Y(1)
V(2)=(R(9)-R(6)/X(2))*Y(2)
V(3)=(R(10)-R(7)/X(3))*Y(3)
V(4)=(V(1)+V(2)+V(3))/R(14)
WRITE(2,*) (X(I),I=1,3)
WRITE(2,*) (Y(I),I=1,3)
WRITE(2,*) (V(I),I=1,4)
FORMAT(F12.8)
END
SUBROUTINE FCN(N,X,FVEC,IFLAG)
INTEGER N,IFLAG
REAL X(N),FVEC(N),R(14)
COMMON R
FVEC(1)=R(1)*(X(1)**2*R(8)-X(1)*R(5))*(1-2*X(1)
* -R(11)*X(2)-R(12)*X(3))+R(2)*(R(9)*X(2)**2-R(6)*X(2))
* +(R(10)*X(3)**2-R(7)*X(3))*R(3)*R(4)*X(1)**2/
* (X(1)+R(4)*X(2))**2
* +R(5)*R(1)*X(1)*(1-X(1)-R(11)*X(2)-R(12)*X(3))
* -R(14)*(R(8)*X(1)**2-R(5)*X(1))
FVEC(2)=-R(2)*R(11)*X(2)*(R(8)*X(1)**2-R(5)*X(1))
* +R(2)*(R(9)*X(2)-R(6))*(X(1)-2*X(2)-R(13)*X(1)*X(3))
* +(R(10)*X(3)**2-R(7)*X(3))
* *R(3)*R(4)**2*X(1)*X(2)/(X(1)+R(4)*X(2))**2
* +R(6)*R(2)*(X(1)-X(2)-R(13)*X(1)*X(3))
* -R(14)*X(1)*(R(9)*X(2)-R(6))
FVEC(3)=-R(1)*R(12)*X(3)*(X(1)*R(8)-R(5))
* -R(2)*R(13)*X(3)*(R(9)*X(2)-R(6))
* +R(3)*(X(3)*R(10)-R(7))*(1-2*R(4)*X(3)/(X(1)+R(4)*X(2)))
* +R(7)*R(3)*(1-R(4)*X(3)/(X(1)+R(4)*X(2)))
* -(R(10)*X(3)-R(7))*R(14)
RETURN
END

```

The OPV-program

```
PROGRAM TEST2(INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT)
REAL (3),FVEC(3),XTOL,WA(51),R(14),A(6)
REAL B(6),C(4)
COMMON R
EXTERNAL FCN

C
C   A INNEHOLDER DE BIOLOGISKE PARAMETRENE I SAMME
C   REKKEFØLGE SOM PÅ ARKET.
A(1)=1.2704
A(2)=1.1617
A(3)=0.0614
A(4)=0.90
A(5)=0.12
A(6)=3.0E7

C
C   B INNEHOLDER DE ØKONOMISKE PARAMETRENE I SAMME
C   REKKEFØLGE SOM PÅ ARKET.
B(1)=0.5284
B(2)=2.3279
B(3)=3.2029
B(4)=3.7332E6
B(5)=8.4172E6
B(6)=2.7273E6

C
C   C INNEHOLDER DE FASTE PARAMETRENE I SAMME REKKEFØLGE
C   SOM PÅ ARKET.
C(1)=0.125E-6
C(2)=0.140E-6
C(3)=0.310E-6
C(4)=0.05

C
C
R(1)=A(1)
R(2)=A(2)
R(3)=A(3)
R(4)=A(4)
R(5)=B(4)
R(6)=B(5)
R(7)=B(6)
R(8)=B(1)*A(6)
R(9)=B(2)*R(4)*A(6)
R(10)=B(3)*R(4)*A(5)*A(6)
R(11)=C(1)*R(4)*A(6)/R(1)
R(12)=C(2)*R(4)*A(5)*A(6)/R(1)
R(13)=C(3)*R(4)*A(5)*A(6)/R(2)
R(14)=C(4)
WRITE(2,1)
1  FORMAT("GI VERDIER FOR X1,X2 OG X3")
READ(2,*) (X(I),I=1,3)
TOL=SQRT(X02AAF(0.0))
IFAIL=0
N=3
CALL C05NBF(FCN,N,X,FVEC,TOL,WA,51,IFAIL)
WRITE(2,*) IFAIL
```

```

WRITE(2,*) (X(I),I=1,3)
END
SUBROUTINE FCN(N,X,FVEC,IFLAG)
INTEGER N,IFLAG
REAL X(N),FVEC(N),R(14)

COMMON R
FVEC(1)=R(1)*(X(1)**2*R(8)-X(1)*R(5))*(1-2*X(1)
*-R(11)*X(2)-R(12)*X(3))+R(2)*(R(9)*X(2)**2-R(6)*X(2))
**+(R(10)*X(3)**2-R(7)*X(3))*R(3)*R(4)*X(1)**2/
*(X(1)+R(4)*X(2))**2
**+R(5)*R(1)*X(1)*(1-X(1)-R(11)*X(2)-R(12)*X(3))
*-R(14)*(R(8)*X(1)**2-R(5)*X(1))
FVEC(2)=-R(2)*R(11)*X(2)*(R(8)*X(1)**2-R(5)*X(1))
**+R(2)*(R(9)*X(2)-R(6))*(X(1)-2*X(2)-R(13)*X(1)*X(3))
**+(R(10)*X(3)**2-R(7)*X(3))
**R(3)*R(4)**2*X(1)*X(2)/(X(1)+R(4)*X(2))**2
**+R(6)*R(2)*(X(1)-X(2)-R(13)*X(1)*X(3))
*-R(14)*X(1)*(R(9)*X(2)-R(6))
FVEC(3)=-R(1)*R(12)*X(3)*(X(1)*R(8)-R(5))
*-R(2)*R(13)*X(3)*(R(9)*X(2)-R(6))
**+R(3)*(X(3)*R(10)-R(7))*(1-2*R(4)*X(3)/(X(1)+R(4)*X(2)))
**+R(7)*R(3)*(1-R(4)*X(3)/(X(1)+R(4)*X(2)))
*-(R(10)*X(3)-R(7))*R(14)
RETURN
END

```

Table A12. Norwegian capelin and herring catches used for fresh and frozen products 1978-80.

Year	Capelin		Herring ¹⁾		Total		
	Quantity (metric ton)	Value ('000 Nkr)	Quantity (metric ton)	Value ('000 Nkr)	Quantity (metric ton)	Value ('000 Nkr)	Price ²⁾ ('000 Nkr/m.t.)
1978	6924	6431	3412	13303	10336	19734	1.9092
1979	24754	25952	1670	8545	26424	34497	1.3055
1980	28314	32967	2261	6682	30575	39649	1.2968
Average price	-	-	-	-	-	-	1.503

1) Includes winter herring and fat herring.

2) Price equals Value divided by Quantity in the Total column.

Source: Fisheries Statistics, 1978, 1979 and 1980.

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