A contribution to the economics of multispecies harvesting with special attention to the Barents Sea fisheries

by

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CONTENTS

SUMMARY

1. Introduction

This is a thesis on the utilization of renewable resources with species interactions, within the field of natural resource economics. It comprises theoretical analysis as well as empirical application of the theory to the fisheries of the Barents Sea and adjacent areas, and consists of the following monograph and two papers:

1. The Economics of Multispecies Harvesting – Theory and Application to the Barents Sea Fisheries. Springer-Verlag. Berlin–Tokyo (1988).
2. The Economics of Predator–Prey Harvesting, in Rights Based Fishing (eds. P.A. Neher, R. Arnason and N. Mollett) 485–503, Kluwer Academic Publ. Dordrecht–London (1989).
3. Bioeconomics of Sustainable Harvest of Competing Species. *J. of Environm. Econ. and Managem.* Forthcoming.

In this text we shall refer to these as Papers 1, 2 and 3, respectively.

Within the field of fisheries economics especially two kinds of questions are raised. First, what does an open access harvesting regime imply for the levels of the resource stock, fishing effort and harvest rate? Second, what is the economic optimal resource stock, fishing effort and harvest rate, and furthermore, how can this economic solution be implemented by means of harvest fees, effort taxes or quantitative control of the fishing activities? To answer such questions it is common to assume that the Pareto-criterions are fulfilled for the rest of the economy. That is, the only externality to be considered is the one in the fishing industry, therefore the problem is of a "first-best" nature.

It is well-known from the theory of "second-best" that if one of the conditions for Pareto-optimum for one or another reason is not fulfilled, neither should the others be in general (Lipsey and Lancaster, 1956). As noted above it is quite common in the field of fisheries economics, as well as in natural resource economics in general, to assume that the problem considered is one of "first-best" nature. This, however, does not mean that the real economy is in such a state, but the assumption should rather be looked upon as a simplifying one to keep the analysis within a reasonable limit. When interpreting the results of the analysis, one obviously must take account of the limits the assumptions impose, in particular if the implications for management are going to be applied to the industry.

The assumption that the resource management problem is of "first-best" nature may be compared to that of the normative theory of taxation where the state of comparison very often is a "first-best" economy with lump sum transfers as the system of taxation. Nevertheless, every economist knows that such a system hardly exists in the real world. The advantage of this approach is, however, that problems of efficiency can be handled in a stringent way. The literature on economics of fisheries nearly always consider problems of "first-best" nature. This thesis follows that tradition. The next section classifies the literature on fisheries economics and places this thesis in the proper group. Each of the three parts of the thesis are summarized and discussed in the subsequent sections.

2. Classification of fisheries economics models and analysis

Fisheries economics models and analysis may be classified in several ways, for example in static and dynamic, or, theoretical and empirical, or, deterministic and stochastic models. Figure 1 illustrates a classification suitable for this particular thesis. The main distinction is between singlespecies and multispecies models. Singlespecies models are further divided into autonomous and non-autonomous models, where the former are characterized by their time invariant parameters. The multispecies models may have biological or harvest technological interactions. In the front of figure 1 the deterministic models are classified as dynamic or static depending on whether they discount future benefits and costs or not. Stochastic models explicitly take care of the uncertainty prevalent in most biological and economic systems.

	Singlespecies		Multispecies	
	Autonomous	Non-autonomous	Biological interaction	Harvest interaction
Static	1	4	7	10
Dynamic	2	5	8	11
Stochastic	3	6	9	12

Figure 1. Classification of fisheries economic models and analyses.

Let us now go through the 12 groups of fisheries economics models and briefly discuss some of the main works in the field.

Models in the first group, static, autonomous singlespecies models, are the classics in fisheries economics theory. Gordon (1954) and Schaefer (1957) are the most famous ones. A Scandinavian thesis ought to mention the work of Warming (1911) as well, which includes the major elements of the theory of open access harvesting, later known under the heading "the tragedy of the commons".

Group 2, dynamic, autonomous singlespecies models, has its forerunner in non-technical analysis in Scott (1955) and Gordon (1956). Crutchfield and Zellner (1962) is the first attempt on mathematical formulation and solution of the dynamic harvesting problem by means of classical calculus of variation. This was a part of an empirical analysis of the Pacific-halibut fishery in the northeast Pacific. In the late 1960s and early 1970s optimal control theory became a standard tool in economics (Dorfman, 1969), especially in capital theory. Also in fisheries economics theory this tool was applied at an early stage (Plourde 1970, 1971; Quirk and Smith, 1970). Several papers applying control theory followed, with Clark and Munro (1975) and Clark (1976) among the most well-known. The former treats the resource stock as capital, and the interpretations of the solutions are similar to that of capital theory.

In the 1980s there has been an increasing interest in stochastic, autonomous singlespecies models of Group 3. Every fish resource shows smaller or greater variations in recruitment, growth and/or natural mortality. Also economic parameters such as market prices, ex-vessel prices, input prices etc. are important. Therefore, it is not a surprise that such variations more and more are explicitly included in fisheries economic models and analysis (see e.g. Lewis, 1981; Andersen, 1982; Charles, 1983 a and b; Andersen and Sutinen, 1984; Clark, Munro and Charles, 1985; Spulber, 1985; and Hannesson, 1989).

Static, singlespecies models with non-autonomous parameters in Group 4 are, in a way, contradictory and have hardly any place in the literature. However, such models might be of pedagogical value to show how the optimal, or open access stock level, vary with the time variant parameters.

Dynamic, non-autonomous singlespecies models in Group 5, are of interest for the management of natural resources with intra- or inter-annual variations in biological or economic parameters. Papers on theoretical analyses of such phenomena includes Hannesson (1974) ch. 5.4; Clark and Munro (1975), Clark (1976) and Flaaten (1983). Henriksen (1986) is an application of this theory to the Barents Sea capelin fishery where non-autonomous biological as well as economic parameters are prevailing.

A model with stochastic recruitment and intra-annual, or seasonal, growth of a fish species would be an example of a singlespecies, stochastic, non-autonomous model in Group 6. Little has been published on this type of models yet, but elements in e.g. Flaaten (1983) and Hannesson (1989) may be integrated to form such models.

Group 7 comprises multispecies models with biological interaction between the species. Such interactions may be of predator–prey type, competitive or a combination of predator–prey and competition. Bioeconomic analysis of multispecies fisheries is fairly new, however, Larkin (1963) includes some economic elements. Maximizing combined yield of two species, having given constant weights to each of them, is equivalent to maximizing gross revenue from the combined fisheries. Hannesson (1974) ch. 3 and Anderson (1975) include fish prices as well as harvesting costs and maximizes the net economic yield from the combined harvest. Pikitch (1988) gives a thorough review of papers on fisheries with biological and technical interactions, mainly in the field of natural sciences.

Dynamic multispecies models with biological interactions, Group 8, explicitly take notice of the timelag between the investment/disinvestment in a natural resource and the altered harvest possibilities this gives. To make comparisons of revenues and costs at different points in time it is necessary to discount to the same point in time. Quirk and Smith (1970) is an early paper on this subject, but the somewhat complex model used does not make the interpretations of the results simple.

The two species model and the bioeconomic analysis in Clark (1976) ch. 9 is complex enough to include the biological interesting points, but still he manages to make the economic interpretations easy to compare with these of the singlespecies models. Other papers of interest in this group includes Silvert and Smith (1977), Hannesson (1983) and Conrad and Adu-Asamoah (1986). In the field of mathematical ecology there are several papers extending the analysis in the former. The latter includes, i.a. empirical analysis of the tuna fisheries in the eastern, tropical Atlantic, using a model of two competing species.

Group 9 comprises stochastic multispecis models with biological interactions. Mendelssohn (1980) and Yeung (1986) are examples of papers on this subject.

Static multispecies models with harvest interactions, Group 10, are valid for analysing fisheries where the gear simultaneously catches more than one species. Anderson (1975) is one of the very few bioeconomic papers in this group. European Communities (1987) gives a thorough review of fisheries where such externalities are prevalent.

The models of Group 11, dynamic multispecies models with harvest interactions, extend the analyses undertaken by Group 10 models by including the discounting of future revenues and costs. Clark (1976) ch. 9 includes dynamic analysis of technological interdependent fisheries.

Stochastic multispecies models with harvest technical interactions, Group 12, are nonexisting in the bioeconomic literature. Elements from the models in Group 9 may be useful in the analysis of Group 12 models.

All three parts of this thesis belong to Group 8, dynamic multispecies models with biological interactions. The subsequent sections of this summary are brief reviews of the three parts of the thesis with special emphasis on the question raised, the results found, the connections between the papers and how they are related to the literature in this field.

According to the introduction in Paper 1 the aim of that monograph is to give a

> ... methodological and quantitative analysis of multispecies fisheries, with an application to the Barents Sea fisheries.
> (Paper 1, p. 5.)

This is essentially the purpose of this thesis as well. To be more specific, the questions raised include: What are the open access equilibrium solutions for the fish stocks, fishing effort and harvest rates, and what are the corresponding optimal solutions? How are these solutions affected by changes in the economic parameters such as the ex-vessel prices of fish, harvesting costs and the discount rate? When are the implications for management of the two species models equivalent to that of singlespecies analysis? What is the adequate concept of multispecies models to be compared to the concept of maximum sustainable yield (MSY) known from the singlespecies models? What are the implications for management of different biological and economic objectives for the fisheries? Does a three species model add something to the analysis compared to that of a two species model?

In Paper 1 some of these questions are answered within a theoretical as well as an empirical context. The theory is applied to an investigation of the fisheries of the Barents Sea and adjacent areas. The three species Barents Sea model (the TSB-model) includes species at three different ecological levels: plankton preying fish, fish preying fish and fish preying sea mammals. At each ecological level two or more species are aggregated into one. In the following section is given a brief review of the 11 sections of Paper 1 and comparisons are made to the relevant literature on dynamic multispecies models with biological interactions. The subsequent two sections of this summary discuss Papers 2 and 3, respectively, which are purely theoretical, partly supporting the analysis of Paper 1 and partly extending the analysis of the latter by raising other related questions.

3. The economics of multispecies harvesting – theory and application to the Barents Sea fisheries

This monograph consists of 11 chapters and 12 appendices. The introduction gives a review of relevant biological and economic literature (published before 1986/87) and the purpose of the work is put forward. To understand the relative great emphasis put on the applied part of the work, including Flaaten (1984 a–c), it is of importance to stress the lack of an empirical based biological multispecies model for the Barents Sea when this work commenced. Biologists and other natural scientists are, however, currently working on such a model (see Tjelmeland and Bogstad, 1989).

Chapter 2 presents a fundamental predator–prey model from Leslie (1948) and May et al. (1979). In Ch. 3 the two species model is extended to include a top predator preying on the two other species, and whose carrying capacity depends on the total biomass of its preys. Selective harvest technology for each of the three species is introduced, and equilibrium stock levels are derived to be functions of biological and harvest technological parameters.

Hannesson (1983) is a theoretical analysis of a predator–prey system, and it is easy to show that the model he is using has the same isoclines as the model in ch. 2 of Paper 1. However, the dynamics of the two models are different. The questions raised in Paper 1 include the ones asked in Hannesson (1983), but the ways they are answered are different.

The three species model designed in ch. 3 is an extension of the three species models in May et al. (1979). This has been done to include the top predators of the Barents Sea, seals and whales, in the TSB-model. The sea mammals prey on the plankton feeders, capelin and herring, as well as on their main predators, cod, haddock and saithe. The three aggregated stocks are called sea mammals, capelin and cod. Another extension of the analysis compared to May et al. (1979) is the deduction of the equilibrium stocks as well as the conditions for equilibrium and stability, shown in appendices 1 and 8.

Based on Beddington and May (1980) it is shown in ch. 4 how to derive the limit to sustainable harvesting in a two species model. This limit proves to be a concave curve in the yield plane of the two stocks, and it is named the maximum sustainable yield frontier (MSF). This compares to the production possibility frontier, known from the economic welfare theory, and is the two species model's correspondent to the MSY concept of singlespecies models. It is also shown in ch. 4 how to derive the MSF for the two stocks at the lowest ecological levels for a given stock level of the top predator. The alternative would be a possibility plane in the room of yields for the three species. The MSF curves, and the combination of stocks giving MSF, proves to be useful in the analysis of different economic and biological harvesting regimes in Paper 3.

Ch. 5 of Paper 1 gives a short review of the ecosystem of the Barents Sea, from phytoplankton and zooplankton to fish and sea mammals. In a global context this ecosystem is considered to be a simpel one, despite nearly 150 species of zooplankton and 115 species of fish. Of the approximately 25 species of sea mammals most of the whale species utilizes the Barents Sea, the coastal areas of Norway and the Norwegian Sea as feeding grounds in the summertime. Herring, capelin and cod make the larger part of the total biomass of fish, and these species have been the most valuable ones from a commercial point of view.

Ch. 6 explains the process of "guesstimating" the biological parameters of the TSB-model. The available set of data was too poor to use a standard estimation procedure to find the nine biological parameters. For some species reliable time series data were lacking, as well as precise estimates of the stock sizes. The main types of biological interactions among the stocks of the TSB-model have to some extent been known, in the meaning of which species eat which and how much, but not to such an extent that we could defend using traditional statistical methods. Instead we used what we called a "guesstimation" procedure:

> By "guesstimation" is meant, in this connection, that relevant biological literature is studied and information essential to our problem is extracted. When two or more sources give somewhat different figures for the same variable, parameter etc., a choice is made as to which to rely on or the average of

> them is used. All the steps and assumptions made are explained in detail so that the reader can critically appraise and modify them if that is felt to be desirable.
> (Paper 1, pp. 35–36.)

To limit the attainable set of biological parameters of the TSB-model we demanded there should be some constraints on the yield and stock sizes of the model. The biological limits of the ecological system are the limits to sustainable economic development of the fisheries based on these resources. The set of biological parameters finally arrived at imply sustainable yields that do not contradict received biological knowledge for the species in the Barents Sea and adjacent areas.

Among the cited papers in Group 8 above, only Conrad and Adu-Asamoah (1986) include an empirical analysis of multispecies harvesting. It has been done partly by exogenous assigning of parameters and partly by using parameters estimated in single species contexts. Unless adjusted for, such use of parameters from single species models may cause distortions in the multispecies model. In Paper 1 the interactions between the three stocks have been taken notice of in the "guesstimation" procedure. Possible sustainable yield of one stock is therefore in tune with the possible sustainable yields of the other two stocks. Increasing the reliability of the biological part of the TSB-model, or other simple multispecies models, requires more methodological as well as empirical work. However, this research effort should rather be left to biologists, statisticians and other natural scientists. Nevertheless, recent literature in these fields seem to indicate that a partial approach to empirical multispecies modelling will still prevail for some time, see e.g. Magnusson and Pálsson (1989).

Ch. 7 is a theoretical bioeconomic analysis of multispecies harvesting. For a two species predator–prey model it is shown how the optimal equilibrium stocks are affected by changes in harvest prices, harvest costs and the social rate of discount. It is found that the optimal predator stock may increase by an increase in the discount rate. The effect on any of the stocks from an increased harvest price of the other species may be positive or negative depending on the biological interactions and the net unit value of the harvest rates at the optimum. The effects on the optimal stock levels from changes in the economic parameters are analysed also in Hannesson (1983), but the model and the method used are different from what we have used in ch. 7. It is also shown in ch. 7 what to do when the optimal solution is not in the interior of the sustainable yield area.

The economic parameters of the TSB-model are calculated in ch. 8 by using data from i.a. the Norwegian accounting surveys for fishing vessels. These parameters are used in the bioeconomic analyses in the subsequent chapters of Paper 1.

The TSB-model's optimal equilibrium stock levels, harvest rates and economic rent are derived in ch. 9. Optimal harvest and effort taxes are also found. The basic case biological and economic parameters from the preceding chapters, the TSB-model implies a long run optimal equilibrium solution with the cod fishery as the dominant fishery. Capelin should not be harvested, but rather be left in the sea as feed for the cod. The sea mammals' consumption of cod and capelin is so significant that it pays to harvest them heavily. The optimal equilibrium stock level of the sea mammals is below the open access stock level, and therefore the harvest of these must be subsidized. Losses in this branch of the fishery is more than offset by he gains to be reaped in the cod fishery.

The conclusion that capelin should not be harvested at the optimum changes when, ceteris paribus, the price of capelin increases somewhat. *If* the total harvest could be sold for prices paid for roe-capelin and capelin for other consumption purposes, the optimal solution would be to keep the cod stock at a loss giving level and let the capelin fishery be the rent yielding one.

In scientific papers using control theory it is very often assumed that the optimum is an interior solution, i.e. that the parameter set is such that the optimum is not at the boundary of the attainable solutions. As shown in ch. 9, and noted above, the TSB-model implies an optimal solution at the boundary of the attainable set of solutions for the "basic case parameters". The test used to reveal such cases is described and explained, and the desired optimum is found.

The optimal equilibrium stock levels, harvest rates and the net present value of rent

(NPV) from the combined fisheries are functions of biological and economic parameters. To investigate how sensitive these solutions are to changes in the parameters, we take the elasticities of these endogenous variables with respect to each of the parameters. We found that NPV is sensitive to changes in most of the biological and economic parameters related to the capelin and cod stocks, but rather insensitive to changes in the parameters related to the sea mammals. The optimal equilibrium stock of sea mammals increases with an increase in the social rate of discount.

As noted above boundary solutions are of interest in models such as the TSB-model. Other kinds of boundary, or corner solutions may arise in single species models, for example when there are several cohorts to be fished wholly or partly selectively, or when there are two or more types of vessels or gear fishing in a biomass model. The former type of corner solution arises in Hannesson (1978), while an example of the latter is found in Clark and Kirkwood (1979). The lack of examples in the fisheries economics literature on the kind of boundary solutions analysed in section 9 is probably because of the lack of applied multispecies models.

The optimal stock level for capelin derived in ch. 9 is, for the basic case parameters, greater than the pristine level. Such a result clearly is not possible to have in single species models. Henriksen (1984) found the optimal spawning biomass of capelin (*Mallotus villosus*) in the Barents Sea to be approximately 30% of the pristine stock level, using a 7% social rate of discount. Kristmannsson (1980) found the optimal spawning stock level of herring (*Clupea harrengus*) off Iceland to be 25–30% of the pristine level, using a 7% social rate of discount. Bjørndal (1987 and 1988) arrived at an optimal level of spawning biomass of North Sea herring equal to 40% of the pristine level in case of costless harvesting and 60% in case of stock dependent harvesting costs. In both cases the discount rate was 6%. All of these applied analyses of plankton preying fish stocks conclude with optimal relative equilibrium stock levels far below what we found for the stocks of the TSB-model. This should not come as a surprise knowing the important role of herring and capelin as transformers of plankton to fish in the north Atlantic.

In a bioeconomic analysis of cod (*Gadus morhua*) in the Barents Sea, Hannesson

(1978) does not explicitly state the optimal and the pristine stock levels, but implicitly they can be found. The former is somewhat larger than the MSY stock level, when using 10% social rate of discount. There are, to our knowledge, no published papers on bioeconomic singlespecies analysis to the Barents Sea area's stocks of seals and whales, but for other areas there are some. Clark (1985) pp. 25–27 analyses an aggregated stock of whales in the Antarctic and find the optimal equilibrium stock level to be approximately 35% of the pristine level, given a 5% social rate of discount. In a bioeconomic analysis of the harp seals (*Pagophilus groenlandicus*) in the northwest Atlantic Conrad and Bjørndal (1989) calculated the optimal stock level to 35% of the pristine level, given 6% discount rate and no stock dependent harvesting costs. Stock dependent harvesting costs would have increased the optimal stock level. In these two bioeconomic singlespecies analysis of sea mammals the optimal stock levels are significantly higher than found in the TSB-model, but still lower than the MSY stock level. The TSB-model includes the costs of predation which the sea mammals impose the capelin and cod fisheries, therefore, our results are reasonable compared to the results of the singlespecies analysis.

In ch. 10 of Paper 1 is shown the open access solution for the stock levels and harvest rates, provided that the Schaefer harvest function is valid in all three fisheries. Compared to the optimal solutions discussed in the preceding chapter there are especially two results which ought to be mentioned. First, the capelin stock is too heavily fished under an open access regime. Second, the sea mammals are harvested too lightly. Through history the relative rates of harvesting of the three ecological levels have changed dramatically, towards greater emphasis on the plankton preying species. This is probably a result of changes in relative prices and harvest costs of the three stocks.

Given the assumption of Schaefer harvest technologies in all three fisheries, none of the stocks of the TSB-model will be extinct under open access harvesting. This is in accordance with historical facts, with a couple of exceptions. However, with the current fish finding and gear technology, relying on historical facts and open access harvesting in the future may prove disastrous to many real fish stocks. An empirical analysis of the North Sea herring by Bjørndal and Conrad (1987) indicates that this stock probably would have been extinct unless the fishery had been closed after the 1977-season. This despite their calculation of the long run open access equilibrium spawning stock level to approximately 10% of the pristine level. The reasons for this being partly the difference of the vessels adaptation in the short and in the long run, and the time delay between spawning and recruitment.

In the TSB-model each of the stocks is an aggregate of two or more real species close to each other in the ecological system. The question of extinction or not under open access harvesting should rather be answered for each of the real species since there are significant differences among some of them related to animal behaviour and gear technology. For example, the minke whale (*Balaenoptera acutorostrata*) is probably less vulnerable to extinction than the harp seals of the Barents Sea. The reason being that the former is distributed over a greater part of the sea, individuals are relatively small, and furthermore, they are difficult to spot when they come to the surface to breath for a very short time. The harp seals, however, congregate on the ice to give birth to pups and are easy to harvest during this period.

For a "high" price, or "low" harvesting cost of capelin it is shown in ch. 10 that open access harvesting will reduce this stock to such a low level that it can not sustain the cod stock at its open access level, implying that there will be no cod fishery in this case. Such cases were analysed theoretically in ch. 7. Obviously, it is not possible to have such results in deterministic singlespecies models, except for the trivial case when even the pristine stock level is below the minimum rent yielding level.

Ch. 11 summarizes Paper 1 and points out possible extensions of this work. Papers 2 and 3 extend the theoretical analysis of Paper 1. They will be briefly reviewed in the two following sections.

4. The economics of predator–prey harvesting

This paper, Paper 2, extends the predator–prey analysis of Paper 1 ch. 7, especially by investigating solutions at the boundary of the sustainable yield area. By using comparative statistics, the analysis, including the graphics and the calculus, could be kept very simple.

In section 3 is shown for which conditions the open access harvesting implies equilibria at the boundary of the sustainable yield area. Even if the pristine stock level is too low to economically sustain open access harvesting, this may change if the stock level of the predator is reduced by harvesting. Decreased stock level of the predator increases the stock size of the prey. On the other hand, profitable harvesting of the predator may become unprofitable if increased harvesting of the prey reduces this stock as well as that of the predator. Cases like this may occur because of exogenous changes in market prices and harvest costs.

In section 4 is shown that the optimal stock level of the prey can not be less than the open access level, whereas the optimal stock level of the predator may be below its own open access level. The latter case arises when the predator is a "trash" and the prey is an "inexpensive-to-catch valuable". It is also shown that increased discount rate implies increased optimal stock level of the predator when it is below its open access level. Increased discount rate always implies decreased optimal stock level of the prey. As noted above it is quite common in bioeconomic theory to assume that the optimal solution is an interior solution. In the case of predator–prey harvesting this would imply a solution within the sustainable yield area with positive harvest rates of both species. If the stock levels derived from the golden-rule equations do not satisfy these constraints, the optimal solution has to be found the way reviewed at the end of section 4.

Clark (1985) ch. 5.3 analyses predator–prey harvesting in general and shows that the optimal stock level of the predator may be below its open access level, whereas the prey species have to be above its open access level. As we have shown, the results may be different when solutions at the boundary of the sustainable yield area are allowed.

In Paper 1 ch. 7 we claimed to have found that the optimal stock level of the prey might increase with increased discount rate, whereas in Paper 2, section 4 it is found that the prey species always decrease with increased discount rate. The reason for these seemingly contradictory statements is that in the former case we did not distinguish between interior and boundary solutions. When the optimal solution is at the boundary of the sustainable yield area with the predator as the only harvested species, increased discount rate implies decreased stock level of the predator and increased stock level of the prey. It is only in this case with no harvesting of the prey at the optimum that this species may increase with an increased discount rate. This also implies that even if a prey is left unexploited by man at a low rate of discount, it may be optimal to harvest this species at a higher rate of discount. The optimal solution in this case changes from a boundary solution to a solution in the interior of the sustainable yield area. Except for the cases of extinction of one species, boundary solutions have received little attention in the bioeconomic literature (see e.g. Hannesson (1983) and Clark (1985) ch. 5).

5. Bioeconomics of sustainable harvest of competing species

This paper, Paper 3, extends the two species analysis of Paper 1 ch. 7 to the case of two competing species. The main question asked is what are the implications for management of different biological and economic optimization criteria? The solutions found are compared both between them and with solutions from singlespecies analysis.

In section 3 is shown that the maximum sustainable yield frontier (MSF) is of the same importance in this Gause model of two competing species as the MSY is in singlespecies models. The combinations of stocks giving MSF form a hyperbola branch through the area of sustainable yields of the phase plane of the two species, whereas the MSY-stock level in singlespecies models is a single point at the biomass axis of the yield-biomass plane.

In section 4 is shown that economic rent may be earned for some combinations of stocks in the sustainable yield area. Given the Schaefer harvest functions the isoprofit lines prove to be ellipses. In case of costless harvesting it is shown that the graph through the points of maximum economic yield from the combined harvest of the two species is a part of a hyperbola branch lying outside the MSF hyperbola. In section 5 is shown that for various price ratios of the two species the graph through the points of maximum present value (MPV) of equilibrium rent is part of a hyperbola inside the MSF-hyperbola.

It is also shown that optimal harvesting may imply harvesting any of the two competing species at a loss at the equilibrium. The optimal equilibrium stock levels depend on the biological and economic parameters, and it is shown how the optimum is affected by changes in each of the economic parameters.

Compared to Paper 1, Paper 3 is different not only because of the analysis of two competing species instead of the predator–prey interactions analysed in the former. More important is the investigation of implications for management of various optimization criteria. Ecological conditions put restrictions on the sustainable economic development of the resource industries. In such a connection the ecological MSF criterion is merely a special case of the MPV criterion.

Paper 3 does not, like Paper 2, put emphasis on boundary solutions, except for the terminal points of the MSF, MEY and MPV graphs. The isoprofit ellipses and the hyperbolas of the MSF, MEY and MPV stocks have not been shown in the cited literature of Group 8. Neither has the result of the two hyperbolas making upper and lower bound of sustainable economic harvest of the two competing resources.

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Ola Flaaten

The Economics of Multispecies Harvesting

Theory and Application to the Barents Sea Fisheries

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PREFACE

The aim of this study is twofold: to provide a theoretical and an applied analysis of multispecies fisheries. The theoretical part will include concepts and analysis which, hopefully, will be of interest not only to economists, but also to biologists and ecologists. The application of the theoretical model and analysis to the Barents Sea fisheries gives empirical content to the analysis, which is important for the advancement of fisheries management science. It is also my firm belief that this kind of work in the end will be beneficial to the people trying to make a living from harvesting marine resources.

For thousands of years man has been whaling, sealing and fishing in these cold and harsh surroundings. The relative importance of the different species in the ecosystem has changed throughout history. In the seventeenth century the abundant, slow-swimming Greenland right whale and the Biscayan right whale in the Barents Sea area were so valuable, especially to English and Dutch whalers, that the intensive exploitation of these common property resources probably were the main reason for the extinction of these two stocks. The two species are, however, still present in other parts of the North Atlantic Ocean. Except for these two stocks of whales there is no knowledge of other stocks of sea mammals or fish in this area being extinct in historical time.

With the immense development of harvesting technology, especially in our century, man has been increasingly aware of the possibility of depletion of some stocks under a "free for all" harvesting regime. For the same reason the knowledge of the biological interdependencies of the species has become increasingly demanded by those having an interest in better management of the living resources of the sea. The traditional object of study for economists is the use of limited resources for competing ends. In our context the limits of the resources lies within the ecosystem and because no multispecies model of the Barents Sea area was available, I had to design my own and implement this by use of existing statistics. Being especially concerned with the main principles for optimal harvesting of the ecosystem, this study is carried out on a highly aggregated level with just three stocks involved. The work to be presented is an economist's contribution to a hopefully better understanding of how the ecosystem should be managed. Maybe it should be called an interdisiplinary study rather than an economic one, since to a great extent it has been necessary to integrate ecological, biological and economic knowledge to give an answer to optimal management.

OLA FLAATEN

Tromsø, Norway
December, 1987

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1. INTRODUCTION

It is many years since interested theoretical biologists started studying ecological systems by means of mathematical models (e.g. Lotka, 1925 and Volterra, 1928), and studies of single species models are even older (e.g., Verhulst, 1838).

The breakthrough in the use of mathematical models in applied marine-biological research should be especially credited to the English biologists R.J.H. Beverton and S.J. Holt (Beverton and Holt, 1957) and the Canadian M.B. Schaefer (Schaefer, 1954 and 1957). The Beverton-Holt model is a one species model in which the relations to other species are not explicitly formulated. However, in a way these relations are implicitly taken care of: The mortality rates include mortality due to predation by other species; growth rates and recruitment will depend on availability of prey and on the presence of competitors, etc.

One species models of this kind have many advantages compared to more complex ecosystem models. They are quite simple, the parameters have specific biological meaning and can in many cases be estimated from catch and effort data, and the models have proved to be sufficiently reliable for management purposes. The latter is especially the case when the harvesting has concentrated on just one or a very few species in an ecosystem, or when the level of fishing effort directed against different species has been changing slowly with time.

The technical and economical developments have led to commercial exploitation of more and more fish stocks, and stocks which sustained fisheries for a very long time have been severely depleted. This is probably the main reason for the increased interest biologists and others have taken in the use of multi-species models in applied research. As far as we know the most comprehensive study undertaken until now is the North Sea model (Andersen and Ursin, 1977). It can be described as a multi-species extension of the single-species Beverton-Holt model. The interactions between fish stocks are taken care of by natural mortality and growth rates, based on the principle of "one man's loss is another man's gain".

Although the main report on the North Sea model (op.cit.) is primarily a theoretical study, the model has been used for simulation of the history of North Sea fisheries and also for prognostic purposes. The computer simulations tell i.a. that the increase of cod and other demersal species during the 1970s were due to a decrease of the herring and mackerel stocks at the end of the 1960s. Mackerel, in particular, preys hard upon the fry of herring, cod and other species, and when the mackerel stock was depleted from 3 to 1 million metric tons by heavy purse-seine fishing within a few years, the niches for other stocks were expanded. All-in-all, this change in exploitation pattern has increased the total fish harvest in the North Sea from 1.5 million m.t. in 1960 to more than 3 million tons in 1976 (Ursin, 1974, 1977 and 1978). To get more reliable estimates of the model's coefficients, The International Council for Exploitation of the Sea (ICES) has started a large research project based on stomach sampling (ICES, 1982).

Beyond the North Sea model there have been several biological studies of parts of fish stock interactions in larger ecosystems. Some of these will be mentioned here. T. Laevastu and other scientists at the Northwest and Alaska Fisheries center in Seattle have done several studies on multispecies interactions in the Bering Sea area by means of computer simulation models. Contrary to the North Sea model, which is based on number of fish, Laevastu et al. work on biomass level (Laevastu and Larkins, 1981). They are especially concerned with

> "fisheries-oriented ecosystem simulation and its evaluation" (op.cit., p. 2),

and the models include the predatory effects from marine mammals. Laevastu and Favorite (1978) is a study of predation on herring from marine mammals and fish in the Bering Sea. They found that the predators' comsumption was more than 1 million m.t. per year, while on the other hand the fishing amounted to only 40 thousand tons.

Ponomarenko et al. (1978) studied the predation effects on capelin in the Barents Sea from cod and haddock. They found that the annual consumption amounted to between 6.6 and 9.8 million m.t. in the years 1974-76. In comparison, annual catches were 1.4 million m.t. in the same period, or only 15-20% of the consumption by cod and haddock.

Interactions between seals and fish stocks in the Atlantic ocean were studied by Sergeant (1973 and 1976). He found that the Harp seal in the Northwest Atlantic consumes food at a daily rate of 5% of its own body weight. Parrish and Shearer (1977) studied interactions between seals and fish in Scottish waters and concluded that the predation-consumption of fish by seals amounted to 200 thousand tons per year.

Theoretical bioeconomic studies have focused on the difference between open access harvesting and socially optimal harvesting. Multispecies analyses of these kind are to be found in Quirk and Smith (1970), Anderson (1975) and Clark (1976). All of them use generally formulated growth equations, whereas Hannesson (1983a) utilizes a Lotka-Volterra type of growth equations. By this specification of the model, he is able to show how the optimum stock levels are effected by changes in discount rate, relative prices and harvesting costs. The parameters used are assumed values and not estimated from empirical data. However, in Hannesson (1983c), which uses a two species Beverton-Holt model with the growth of individual cod being a function of the capelin biomass, some of the parameters are estimated from Barents Sea data. Conrad and Adu-Asamoah (1986) have partly estimated the parameters and partly assigned values to the parameters of the models in their multispecies analysis of tuna in the Eastern Tropical Atlantic.

Balchen (1979) describes a large research program, Oceanographic Biomodels (OBM), whose aim is

> .... to establish the capability of producing mathematical/ numerical models of a total marine ecological system of the Barents Sea, ...

The study would include physical and chemical oceanography, phytoplankton, zooplankton and fishes. So far a submodel of capelin is completed (Reed and Balchen, 1981) in addition to oceanographic parts of the study. Regarding the applicability of the project it was said:

> An operational set of models is planned to be available around 1985. The most obvious applications of this system will be in estimating present and future states of the system, particularly those representing the quality, quantity and distribution of the most important species of commercial fish. It is expected, however, that the system will also have important applications in oceanographic and biological research and in studies of ocean pollution as a consequence of offshore oil activities. (Balchen, 1980, p. 67)

Concerning the fisheries the aims of the OBM of the Barents Sea are very much like those of the North-Sea model even though methods and the way of approaching reality are different. However, as the quotations demonstrate, the general aims of the former models are much wider than just being tools for fisheries management.1)

At the Institute of Marine Research, Bergen a research project on "A multispecies model of the Barents Sea" (The MSB-model) has been initiated (Tjelmeland, 1986). The aims are to make the project an integrated part of the institute's marine biology research and to develop the multispecies model into an operational management tool for the Barents Sea fisheries.

Both the North-Sea model, the OBM- and the MSB-model are large, complex, resource consuming research projects with inputs from biologists, mathematicians, cyberneticians, statisticians and other groups of scientists. So far, no economists or social scientists have been involved.

Compared to the three models above our study is less ambitious regarding aims and quantitative extent on the applied biological level. We shall develop a three species model of the Lotka-Volterra type and use this for economic studies of multispecies fish communities. Hoping that our work can be of some help for better management, it will be applied to the Barents Sea fisheries. An operational model of the ecosystem does not yet exist. Therefore we will have to rely on several sources to get the necessary data for estimating the biological interdependencies of the relevant species. As will be clear later,

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1) The OBM project seems to have come to an end without having reached the goals with respect to operational models of "the most important species of commercial fish".

this is not at all an easy task. Biological data gathered for use in the Beverton-Holt type of models obviously can be used to estimate parameters of other kinds of models, e.g. the Lotka-Volterra type we are going to use. However, this may raise some methodological problems, and even though they can be overcome to some extent, scientifically based data on biological interactions among species is often lacking. Therefore we will have to rely on "guesstimation" and simulation techniques to complete the three species Barents Sea model.

The aim of this study is twofold: methodological and quantitative analysis of multispecies fisheries, with an application to the Barents Sea fisheries. Even though some of the data are scanty and the methods to be used in the estimation of biological parameters are rough, the quantitative analysis should not be considered merely as an illustration of the method. If we were required to make decisions and take actions in connection with relationships covered by this study, we would (in the absence of more reliable results, and without doing more work) rely to some extent in our results. Thus, the quantitative analysis does not solely serve the purpose of illustrating a method, but we think they also give a description of some important biological and economic relationships of the Barents Sea fisheries.

A brief outline of the book is as follows. A two species biomass model is reviewed in Section two, and Section three presents a three species model based upon the former one. The concept of maximum sustainable yield frontier (MSF) is defined and presented in Section four, and a method for deriving this in the three species case is developed.

A brief description of the marine ecological system of the Barents Sea area is included in Section five. Since a fully developed and implemented multispecies model of the Barents Sea fisheries did not exist, an attempt has been made to estimate the nine biological parameters of the three species model. The "guesstimation" procedure is explained step by step in Section six.

The three stocks in the applied model are aggregated stocks with "capelin" consisting of capelin and herring, "cod" consisting of cod, haddock and saithe, and "sea mammals" consisting of 14 species of whales and two species of seals.

Economic aspects of multispecies fisheries studied in Section seven includes optimal harvesting as well as open access harvesting. The economic parameters of the model are derived from Norwegian data in Section eight.

In Section nine are shown the optimal solutions of the three species Barents Sea model (TSB-model), and the open access solutions are derived in Section ten. Finally, Section eleven comprises the summary and the conclusion of this study.

2. A TWO SPECIES MODEL

First we shall give a review of the two species model analysed in May et al. (1979), since our three species model will be based upon this. Suppose there is a prey, W_1, on which the existence of a predator, W_2, is based. W_1 and W_2 can be thought of as biomasses. A simple model describing the dynamics of such a system is

$$\dot{W}_1 = dW_1/dt = r_1W_1(1 - W_1/K) - aW_1W_2 \tag{2.1}$$

$$\dot{W}_2 = dW_2/dt = r_2W_2(1 - W_2/\alpha W_1) , \tag{2.2}$$

where r_1 and r_2 are the intrinsic growth rates of the respective species. K is the carrying capacity of the total system, at which the prey will settle in the case of no predator and no harvest.

The per capita[1)] growth rate of the prey decreases from r_1 for stock levels close to zero, to zero for stock levels equal to the carrying capacity in case of no predators. If predators exist, the per capita growth rate for the prey equals zero for a stock level lower than the carrying capacity. The presence of predators reduces the per capita growth rate in proportion to the biomass of the predator. The predation coefficient, a, tells how much the per capita growth rate of the prey reduces per unit of the predator. Or to put it another way, a tells which share of the prey stock one unit of the predator is consuming per unit of time. The total rate of consumption is expressed in the term aW_1W_2.

The predator's per capita growth rate decreases from r_2 when its own stock level is close to zero, to zero for a stock level equal to its own carrying capacity, which is proportional to the level of the prey stock. The proportionality coefficient is α.

Mathematical stability properties of the model (2.1)-(2.2) will not be discussed here.[2)] (It can be found in the literature

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[1)] The term "per capita" is used, even though we mean per unit of biomass.

[2)] For the three species model to be studied, the stability conditions are derived in Appendix 8.

of theoretical ecology, e.g. in Beddington and Cook (1982), May (1974) and May (1981).) However, it is easy to see, by letting $\dot{W}_1$ and $\dot{W}_2$ equal zero in (2.1) and (2.2), that if an equilibrium point exists with both species being positive, the stock levels will be

$$W_1^* = \frac{K}{1+\nu}\ , \qquad \lim_{K\to\infty} W_1^* = \frac{r_1}{a\alpha}\ ^{1)} \tag{2.3}$$

$$W_2^* = \frac{\alpha K}{1+\nu}\ , \qquad \lim_{K\to\infty} W_2^* = \frac{r_1}{a}\ ^{1)} \tag{2.4}$$

where $\nu = \frac{a\alpha K}{r_1}$.

It should be noticed that the intrinsic growth rate of the predator, r_2, does not affect the equilibrium values of either of the two species. The equilibrium values of both species increase with any increase in r_1 or K, ceteris paribus. From (2.3) and (2.4) it follows

$$W_2^*/W_1^* = \alpha\ . \tag{2.5}$$

In equilibrium α expresses the relative size of the predator stock to that of its prey.

Even though r_2 does not affect the equilibrium values of the stocks, it is of importance to the behaviour of the system outside equilibrium. Defining the "natural return time", T^R, of the species as

$$T_i^R = 1/r_i \qquad i = 1,2\ , \tag{2.6}$$

r_2 will affect the time the predator will need to reach equilibrium from a higher or lower level.

Suppose that the fish stocks are harvested independently with constant effort per unit of time, F_i, scaled such that

---

1) In a logistic single species model the equilibrium stock level with no harvesting always equals the carrying capacity. In the two species model, however, (2.3) and (2.4) demonstrate that an increase in the carrying capacity towards infinity, increases the stock levels towards limits fixed by other parameters than the carrying capacity.

$F_1 = 1$ corresponds to constant catchability coefficients equal to r_i. Then the catch rates will be

$$h_1 = r_1F_1W_1 \tag{2.7}$$

$$h_2 = r_2F_2W_2 \ . \tag{2.8}$$

With harvesting introduced it will influence the growth rates in (2.1) and (2.2) which will be changed to

$$\dot{W}_1 = r_1W_1(1 - W_1/K) - aW_1W_2 - r_1F_1W_1 \tag{2.9}$$

$$\dot{W}_2 = r_2W_2(1 - W_2/\alpha W_1) - r_2F_2W_2 \ . \tag{2.10}$$

If positive equilibrium levels of W_1 and W_2 exist simultaneously, using W^*_{1F} and W^*_{2F} as symbols, they will be:

$$W^*_{1F} = \frac{K(1-F_1)}{1 + \nu(1-F_2)} \tag{2.11}$$

$$W^*_{2F} = \frac{\alpha K(1-F_1)(1-F_2)}{1 + \nu(1-F_2)} \ . \tag{2.12}$$

With harvesting the relative stock size is

$$W^*_{2F}/W^*_{1F} = \alpha(1-F_2) \ . \tag{2.5'}$$

It is seen from (2.11) that only for $F_1 < 1$ will there exist a positive equilibrium value of the prey. If $F_1 \geq 1$ the prey-stock will be extinct, and so of course will be the predator, as seen from (2.12). The latter expression shows that only for $F_2 < 1$ and $F_1 < 1$ will the predator survive.

The equilibrium values of both species increase with decreasing fishing pressure on the prey, i.e. for reduced F_1. More of the prey gives increased carrying capacity for the predator which can be kept on a higher level.

On the other hand, the effects on the prey and on the predator from decreased fishing pressure on the predator are the opposite of each other. From (2.11) it is seen that the equilibrium value of the prey will decrease, and from (2.12) that the predator will increase. The increased stock level for the predator means heavier predation on the prey, and thereby a reduced equilibrium level for the latter.

Defining $X_1 = W_1/K$ and $X_2 = W_2/\alpha K$ we can rewrite equations (2.9) and (2.10) as

$$dX_1/dt = r_1 X_1 (1 - F_1 - X_1 - \nu X_2) \qquad (2.9')$$

$$dX_2/dt = r_2 X_2 (1 - F_2 - X_2/X_1) \; . \qquad (2.10')$$

Here the dimensionless parameter ν is defined as $\nu = a\alpha K/r_1$.

The equilibrium properties of this ecological system depend only on the fishing efforts, F_1 and F_2, and ν. The dynamics additionally involve r_1 and r_2.

The phase-diagram for the system (2.9')-(2.10') is shown in Figure 2.1. The isoclines are found by setting $dX_1/dt = 0$ and $dX_2/dt = 0$ in (2.9') and (2.10'). This gives

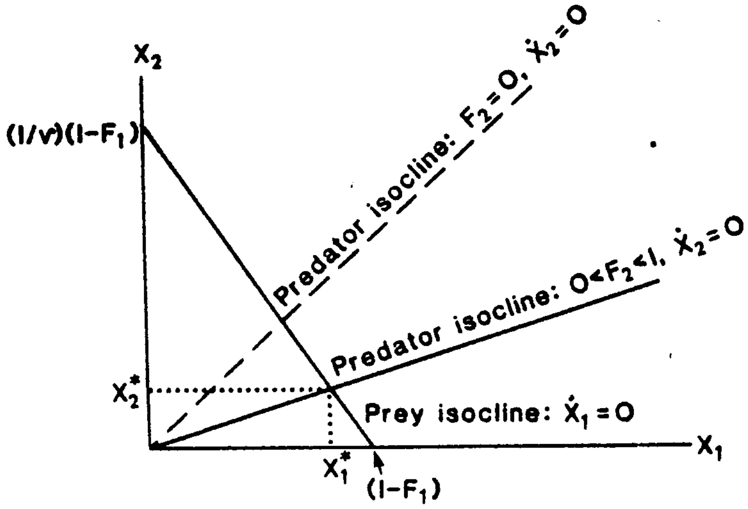

Figure 2.1. Phase diagram for a predator-prey model.

$$X_2 = (1/\nu)(1 - F_1 - X_1) \qquad \text{for } dX_1/dt = 0 \tag{2.13}$$

$$X_2 = (1 - F_2)X_1 \qquad \text{for } dX_2/dt = 0 \ . \tag{2.14}$$

The equilibrium values of X_1 and X_2 are found where the isoclines intersect, that is for

$$X_1^* = \frac{1 - F_1}{1 + \nu(1-F_2)} \tag{2.11'}$$

$$X_2^* = \frac{(1-F_1)(1-F_2)}{1 + \nu(1-F_2)} \tag{2.12'}$$

X_1^* and X_2^* both equal $\frac{1}{1+\nu}$ in the absence of fishing, and zero in the case of $F_1 = 1$. In addition, X_2^* will equal zero if $F_2 = 1$.

The three species model to be designed and presented in the next section is an extension of the two species model shown in Equations (2.1)-(2-2) and discussed in this section.

3. A THREE SPECIES MODEL

The aim of this study is, as noted earlier, to develop a three species model of the Lotka-Volterra type, and apply it into a bioeconomic study of the Barents Sea fisheries. The model we have in mind should take care of those aspects of the Barents Sea ecology of most importance to fisheries management. Thus it should include the most important species like cod and capelin. However, we know that whales and seals also are large consumers of fish and that they compete with fishstocks over food like plankton, krill etc. If possible, therefore, one should include the marine mammals too in a multispecies model for this area. An illustration of such a model is shown in Figure 3.1.

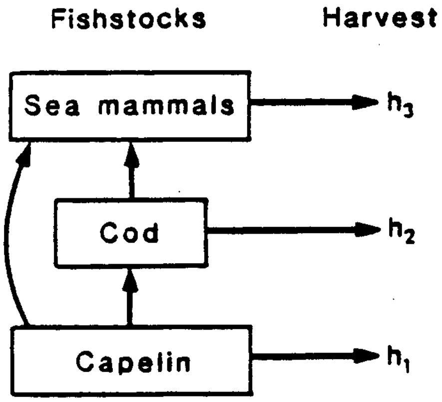

Figure 3.1. Structure of a three species model.

The marine mammals, such as whales and seals, are preying upon both demersal and pelagic species, in Figure 3.1 called cod and capelin. On the other hand cod is preying upon capelin. Limits to growth of the system are set by limiting the environment's carrying capacity for the lowest ecological level, the capelin.

The complete model on biomass form is:

$$dW_1/dt = r_1W_1(1 - W_1/K) - a_{12}W_1W_2 - a_{13}W_1W_3 - h_1 \quad (3.1)$$

$$dW_2/dt = r_2W_2(1 - W_2/(\alpha W_1)) - a_{23}W_2W_3 - h_2 \quad (3.2)$$

$$dW_3/dt = r_3W_3(1 - W_3/(\beta(W_1 + W_2))) - h_3 \ . \qquad (3.3)$$

The symbols are:

$W_i(t)$ = Biomass of species i at time t.

$dW_i(t)/dt$ = Growth rate of species i.

r_i = Intrinsic growth rate of species i.

α = Carrying capacity coefficient for species 2.

β = Carrying capacity coefficient for species 3.

a_{ij} = Preying coefficient, species j preying upon species i.

h_i = Harvest rate for species i.

We assume that the three trophic levels can be harvested independently of each other, that is, the fishing effort targeted at one species catches just that one. It is also assumed that the catch rate are functions of the target fishstock:

$$h_i = h_i(F_i,W_i) \qquad (3.4)$$

where F_i is the fishing effort for species i.

The specific harvest functions are, as was the case in the two species model, assumed to be homogeneous of degree two in fishing effort and stock level. To simplify the growth equations, the scaling of fishing effort is such that $F_i = 1$ corresponds to constant catchability coefficient equal to the respective intrinsic growth rates, r_i. The harvest functions are

$$h_i = r_iF_iW_i \qquad i = 1,2,3 \ . \qquad (3.5)$$

By subtracting the harvest rates on the righthand side of the growth equations (3.1)-(3.3), this three species model can be written in a dimensionless form which can be of help when analysing the behaviour of the system. Defining $X_1 = W_1/K$, $X_2 = W_2/\alpha K$, $X_3 = W_3/\alpha\beta K$ and $A_i = 1 - F_i$, the system described by equations (3.1)-(3.3) is changed to

$$dX_1/dt = r_1X_1(A_1 - X_1 - \gamma_{12}X_2 - \gamma_{13}X_3) \qquad (3.6)$$

$$dX_2/dt = r_2X_2(A_2 - X_2/X_1 - \gamma_{23}X_3) \qquad (3.7)$$

$$dX_3/dt = r_3X_3(A_3 - \alpha X_3/(X_1 + \alpha X_2)) , \qquad (3.8)$$

where $\gamma_{12} = a_{12}\alpha K/r_1$, $\gamma_{13} = a_{13}\alpha\beta K/r_1$ and $\gamma_{23} = a_{23}\alpha\beta K/r_2$.

One of the important questions raised in the ecology literature is how to predict what will happen to a disturbed ecosystem. Ideally one would like to be able to measure certain properties of such systems before they are disturbed, and then on the basis of these results to be able to predict how the system will respond to disturbance. The latter tells which properties of ecosystems tend to enhance stability against external disturbances, while the former includes concepts like global asymptotic stability, neighbourhood asymptotic stability, structural stability, resistance and resilience. Discussions on stability indicators are found in e.g. Halfon (1979) and May (1974). Often there can seemingly be a missing link on the way from pure theoretical ecology to applied studies in the field. A quotation from the ecology literature tells it this way:

> A shortcoming of much of the theoretical work in ecology is that results are often not expressed or expressible as relations among readily measureable quantities. A familiar example is the often-quoted result that a necessary and sufficient condition for asymptotic stability of a system described by a community matrix is the negativity of the real parts of all the eigenvalues of that matrix. While mathematically rigorous, this result unfortunately is not very useful in situations of practical concern such as environmental impact prediction or assessment. (Harte, 1979, p. 454)

The aim of this study is to develop a rather simple ecological model, as represented by the system (3.1)-(3.3), and to apply it into a bioeconomic study of the Barents Sea fisheries. The problem is, however, that no one has yet studied this ecological system in a way that directly can give us estimates of the biological parameters in the model. The way to handle such a problem will be dealt with later on in this study.

If the system (3.6)-(3.8) has an equilibrium solution, (X_1^*, X_2^*, X_3^*), and $\gamma_{ij} \neq 0$ for all i,j, it is shown in Appendix 1 that the solution for X_1 is

$$X_1^* = \frac{-D_1 \pm \sqrt{D_1^2 - 4A_1D_2}}{2D_2} \qquad (3.9)$$

where D_1 and D_2 are

$$D_1 = A_1A_3\gamma_{23} - 1 - \gamma_{12}A_2 - A_3(1/\alpha)\gamma_{13} - A_2A_3\gamma_{13}$$

$$D_2 = A_3(1/\alpha)\gamma_{12}\gamma_{23} - A_3\gamma_{23} .$$

X_2^* and X_3^* expressed as functions of X_1^* are

$$X_2^* = \frac{(A_2 - A_3(1/\alpha)\gamma_{23}X_1^*)X_1^*}{1 + A_3\gamma_{23}X_1^*} \qquad (3.10)$$

$$X_3^* = \frac{A_3X_1^*((1/\alpha) + A_2)}{1 + A_3\gamma_{23}X_1^*} . \qquad (3.11)$$

From (3.11) it can be seen that X_3^* is increasing with increased values of X_1^*, while (3.10) shows that the effect on X_2^* from increased values of X_1^* can be positive or negative. The former is due to the fact that the top-predator, X_3, is preying on both X_1 and X_2. Increased X_1^* directly gives a larger food base for the top-predator as well as for the intermediate species, X_2^*. However, this increase in X_3^* may cause trouble for X_2^*. If the predation effect on X_2^* from increased X_3^* is greater than the prey effect from increased X_1^*, the total result on X_2^* will be negative.

In the case of no harvesting, that is $A_i = 1$ for i = 1,2,3, it is seen from (3.10) that a necessary condition for $X_2^* > 0$ is that

$$X_1^* < \alpha/\gamma_{23} = r_2/a_{23}\beta \; .$$

This can be explained the following way. A large r_2, given all other parameters being constant, gives species 2 a higher chance to survive than a small r_2. The law of the survival of the fittest implies in this context that species 2 belongs to the fittest if, ceteris paribus, its intrinsic growth rate is large enough. On the other hand, species 2's chance of being positive, decreases with an increase in species 3's preying pressure and its carrying capacity as represented by a_{23} and β, respectively.

It is easy to see that the three species model described by equations (3.1)-(3.3) includes the two species model of equations (2.1)-(2.2). By setting a_{13} and a_{23} equal to zero in (3.1) and (3.2) we have the two species model. The three species model is also seen to be a general form of a three species ecological system, since it includes the cases shown in Figures 3.2 and 3.3. In the former one there is no predation from species 3, here called polar bears, on species 1, which could be fish, but only on 2, seals. Our general three species model is fitted to this system by letting a_{13} = 0 and by letting the carrying capacity of species 3 depend only on the biomass of species 2.

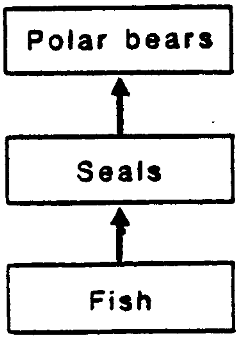

Figure 3.2. The structure of a three species predator-prey model.

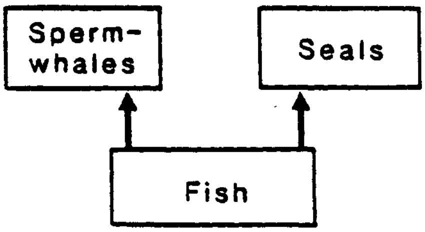

Figure 3.3. The structure of combined competition and predation in a three species model.

The model illustrated in Figure 3.3 is a case of combined competition and predation. Species 2 and 3, here called sperm whales and seals, are competing for their joint prey, the krill. Our general three species model is fitted to the competitive-predation model by letting $a_{23} = 0$ and by adjusting the carrying capacity of species 3 to depend only on the biomass of species 1.[1)]

Equilibrium and stability conditions of the model described by Equations (3.6)-(3.8) are derived in Appendix 8. In the applied part of this study it is assumed that the main features of the ecological system can be described by the three species equilibrium model. It is not, however, obvious on apriori reasons that an ecological system should have a stable equilibrium. One could as well think of a perpetual cyclical movement of the fish stocks in preharvesting time. Such a system might be described by a limit cycle model of the kind found in Lotka (1925) and Volterra (1928).[2)] In fact, the first attempt ever done on conducting a multispecies analysis of fishing was by means of limit cycle models. Empirical studies of the Upper Adriatic Sea's fisheries before, during and after the first world war found in D'Ancona (1926) were an important source of inspiration to the theoretical works by V. Volterra as demonstrated by this quotation:

> Doctor UMBERTO D'ANCONA (D'Ancona, 1926) has many times spoken to me about the statistics which he was making in fishery in the period during the war and in periods before and after, asking me if it were possible to give a mathematical explanation of the results which he was getting in the percentages of the various species in these different periods. This request has spurred me to formulate the problem and solve it, establishing the laws which are set forth in § 7. Both D'Ancona and I working independently were equally satisfied in comparing results which were

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[1)] The models in Figures 3.2 and 3.3 are equivalent to the ones described in notes 58 and 51, respectively, in May et al. (1979). There is only a minor difference in the numbering of the species.

[2)] Limit cycle models also played an important role in the early development of mathematical business cycle models. (See e.g. Frisch, 1933; Frisch and Holme, 1935; Kalecki, 1935; Tinbergen, 1935; and Goodwin, 1951.)

> revealed to us separately by calculus and by observation, as these results were in accord; showing for instance that man in fisheries, by disturbing the natural condition of proportion of two species, one of which feeds upon the other, causes diminuition in the quantity of the species that eats the other, and an increase in the species fed upon. (Volterra, 1928, p. 4.)

Based upon his empirical studies of the fisheries of the upper Adriatic Sea, D'Ancona (1926) concluded that the predators of this sea, the sharks, ought to be decreased by increased harvest intensity. That would make it possible to increase the yields of more valuable prey stocks.

Having designed a three species model in this section, it will be of interest, from a theoretical point of view, to analyse the limits to growth and harvest of such an ecological system. This is the aim of the next section of this study.

4. THE MAXIMUM SUSTAINABLE YIELD FRONTIER

In the case of the two species model (Equations (2.9')-(2.10')) it can be of interest both from a biological and from an economical efficiency point of view to maximize the sustainable yield of one species for a specified constant level of sustainable yield of the other. This problem is equivalent to that of welfare economics: deriving the production possibility frontier by maximizing the output of one good for a specified amount of output of the other, for a fixed amount of factors of production. In the two species biological system the limited amount of factors of production are embodied in the carrying capacity and the intrinsic growth rates of the model. In the real biological world of the seas, the limited factor of production used for "production" of the two fish species usually will be the zooplankton communities.

The problem of maximizing

$$y_1 = r_1X_1(1 - X_1 - \nu X_2) \tag{4.1}$$

subject to the constraint

$$y_2 = r_2X_2(1 - X_2/X_1) = \text{constant} , \tag{4.2}$$

can be done by using the Lagrange-method. As shown in Beddington and May (1980), this problem gives the following quadratic equation for X_1 as a function of X_2:

$$X_1 = (1/4)[1-(4-\nu)X_2] \pm (1/4)\{[1+(4-\nu)X_2]^2 - 8X_2[2-3\nu X_2]\}^{\frac{1}{2}} . \tag{4.3}$$

For each level of X_2 we calculate X_1 from (4.3), and the resulting yields, y_1 and y_2, are given by (4.1) and (4.2). The locus combining the yields of the two species is shown in Figure 4.1 for $\nu = 2$. Such locuses shall be called the maximum sustainable yield frontier (MSF) to emphasize the connections to the concepts used in welfare economics. MSF gives the absolute sustainable yield of either population for a specified yield of the other. All combinations of yields on or below this curve are sustainable, whereas yields to the north-east of the curve are possible for some period of time, but they are not sustainable. The star in the north-east corner corresponds to a combination of the largest possible yield of the prey and the largest possible yield of the predator, but such a combination of yields is definitely not sustainable.

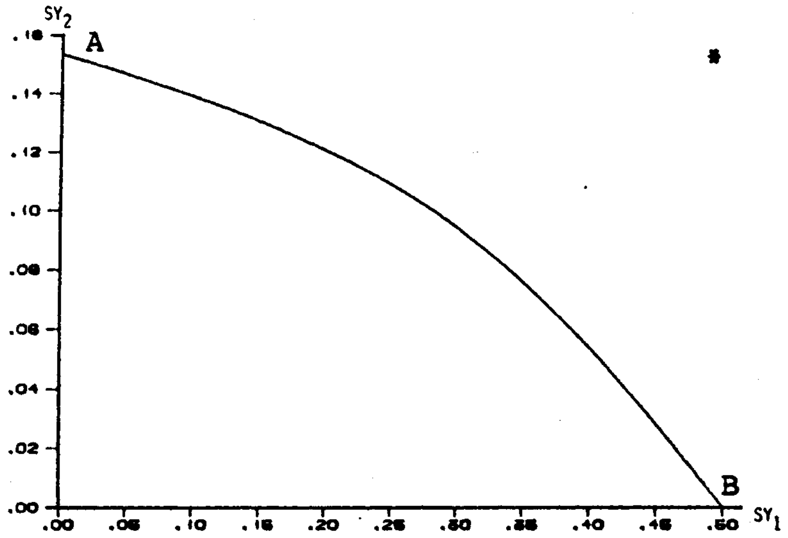

Figure 4.1. The maximum sustainable yield frontier (MSF) of a two species model shows sustainable combinations of yield of species 1 (SY_1) and species 2 (SY_2). Parameters used are $r_1 = 2.0$, $r_2 = 1.15$ and $\nu = 2.0$.

From the single species logistic growth model it is known that a given sustainable yield less than the maximum sustainable yield (MSY) can be harvested at two different stock levels, above or below the MSY level. These two ways of harvesting are called biological underexploitation and overexploitation, respectively. From a biological point of view the best way of harvesting is to harvest the MSY, whereas the economical optimal yield stock level, also depend on product price, harvesting cost and discount rate in addition to biological factors.

Unit harvesting cost is usually assumed to be a decreasing function of stock level, leading to the conclusion that the resource should be biologically underexploited to reduce costs. On the other hand, a positive discount rate leads to the conclusion that the resource should be biologically overexploited since a given amount of net revenue "today" is preferred to the same amount "tomorrow". In other words, from an economic point of view, harvesting below, at or above the MSY stock level can all be optimal; it is a question of prices, costs and discount rates.

The lower branch of Equation (4.3) corresponds to a biologically inefficient harvest level, either underexploitation of the predator, or overexploitation of the prey. In the former case the predator is kept on the highest stock level of two possible ones, both giving the same sustainable yield of the predator. A higher predator stock means more consumption of the prey, thereby removing a potential prey yield. To get the highest possible sustainable yield of the prey for a given predator yield it is therefore obviously best to overexploit the predator. For similar reasons it is efficient to underexploit the prey to give more food to the predator. MSF harvesting thus means that neither shall the predator be underexploited, nor shall the prey be overexploited.

The terminal points of the MSF locus in Figure 4.1, A and B, are related to specific stock levels of the predator and the prey. At point A the predator is extinct and the prey is at its single species biological optimum level:

$$X_1 \Big|_{X_2 = Y_2 = 0} = 1/2 \; . \tag{4.4}$$

At point B the corresponding X_1 and X_2 can be found by using Equations (4.5) and (4.6), corresponding to (3.8) and (3.9) in Beddington and May (1980):

$$Y_2 = r_2 \frac{1 + \nu - R}{\nu} X_2 \tag{4.5}$$

$$F_1 = 1 - (1 + \nu + R)X_2 \tag{4.6}$$

where $R = \sqrt{1+\nu}$.

At point B we have $F_1 = 0$, thus Equation (4.6) gives

$$X_2\Big|_{F_1 = y_1 = 0} = \frac{1}{1 + \nu + R} \; . \tag{4.7}$$

Substituting Equation (4.7) into Equation (4.5) gives the corresponding absolute maximum sustainable yield of the predator:

$$MSY_2\Big|_{F_1 = y_1 = 0} = r_2[(R - 1)/\nu]^2 \; . \tag{4.8}$$

When $X_2\Big|_{F_1 = y_1 = 0}$ is known, the corresponding value of X_1 is found by setting Equation (4.1) equal to zero, after having substituted for X_2 from (4.7). This gives

$$X_1\Big|_{F_1 = y_1 = 0} = \frac{1 + R}{1 + \nu + R} \; . \tag{4.9}$$

It should be noted that the following relation between X_1 and ν holds:

$$X_1\Big|_{F_1 = y_1 = 0} \gtreqless 1/2 \qquad \text{if} \qquad \nu \lesseqgtr 3 \; . \tag{4.10}$$

This states that the absolute maximum sustainable yield of the predator occurs for an unharvested prey stock above, at or below its single species biological optimum depending on the size of the dimensionless combination of parameters, ν. The smaller ν is, the higher will be the prey stock level.

For the three species model the MSF locus (Figure 4.1) might be substituted by a maximum sustainable yield plane. In principal this can be done by using the Lagrange-method on the problem of maximizing y_1 subject to the constraints y_2 = constant and y_3 = constant. Since much of the discussion on sea mammal harvesting and/or preservation is focused on stock size, we choose to perform the maximization subject to the constraint X_3 = constant rather than y_3 = constant. For each level of the sea mammal stock one MSF can be drawn for the two other species. With the three species system (Equations (3.6)-(3.8)) in equilibrium with harvesting we have

$$\dot{X}_1 = r_1X_1(1 - F_1 - \gamma_{12}X_2 - \gamma_{13}X_3) = 0 \tag{4.11}$$

$$\dot{X}_2 = r_2X_2(1 - F_2 - X_2/X_1 - \gamma_{23}X_3) = 0 \tag{4.12}$$

$$\dot{X}_3 = r_3X_3(1 - F_3 - \alpha X_3/(X_1 + \alpha X_2)) = 0\ . \tag{4.13}$$

By keeping species 3 constant, $X_3 = \overline{X}_3$, it is seen from Equations (4.11) and (4.12) that this has a similar effect as the levels of fishing effort, F_1 and F_2, have. Substituting

$$\overline{F}_1 = F_1 + \gamma_{13}\overline{X}_3 \tag{4.14}$$

and

$$\overline{F}_2 = F_2 + \gamma_{23}\overline{X}_3 \tag{4.15}$$

into (4.11) and (4.12), we get

$$\dot{X}_1 = r_1X_1(1 - \overline{F}_1 - X_1 - \gamma_{12}X_2) \tag{4.16}$$

$$\dot{X}_2 = r_2X_2(1 - \overline{F}_2 - X_2/X_1)\ . \tag{4.17}$$

Equations (4.16) and (4.17) are the same as Equations (2.9') and (2.10') except for the notation of $\overline{F}_i$. Therefore, exactly the same procedure as used for arriving at the MSF locus in the two species model can be used now, remembering the condition that X_3 is kept constant at $\overline{X}_3$. (This is done by varying the harvest rate, y_3.) The problem now is to maximize the gross yield of one species

$$\overline{y}_1 = r_1\overline{F}_1X_1 = r_1X_1(1 - X_1 - \gamma_{12}X_2) \tag{4.18}$$

subject to a constant gross yield of the other

$$\bar{y}_2 = r_2\bar{F}_2X_2 = r_2X_2(1 - X_2/X_1) = \text{constant} . \qquad (4.19)$$

This is exactly the same problem as that in Equations (4.1) and (4.2), and the former results are valid here too.

Substituting γ_{12} for ν in Equation (4.3) gives the same quadratic equation of X_1 as a function of X_2:

$$X_1 = (1/4)[1+(4-\gamma_{12})X_2] \pm (1/4)\{[1+(4-\gamma_{12})X_2]^2 - (8X_2[2-3\gamma_{12}X_2]\}^{1/2} . \qquad (4.20)$$

For each level of $X_2 \in \left[0, \frac{1}{1+\gamma_{12}+R}\right]$, X_1 is calculated from (4.20) and the resulting gross yields, $\bar{y}_1$ and $\bar{y}_2$, are given by (4.18) and (4.19). The yields harvested by man are given by:

$$y_1 = \bar{y}_1 - r_1\gamma_{12}X_1\bar{X}_3 \qquad (4.21)$$

$$y_2 = \bar{y} - r_2\gamma_{23}X_2\bar{X}_3 . \qquad (4.22)$$

The yield of species 3 follows from

$$y_3 = r_3\bar{X}_3(1 - \alpha\bar{X}_3/(X_1 + \alpha X_2)) , \qquad (4.23)$$

and will vary with the X_1, X_2 combinations given by (4.20).

The procedure for finding MSF for a specified stock level of species 3, $\bar{X}_3$, can be summarized as follows

1. Choose a fixed value of $\bar{X}_3 \geq 0$.
2. For $X_2 \in \left[0, \frac{1}{1+\gamma_{12}+R}\right]$, compute X_1 from (4.20).
3. Substitute for X_1, X_2 into (4.18) and (4.19) and further into (4.21) and (4.22) to derive the yields, y_1 and y_2.
4. Compute y_3 from (4.23).

A computer program designed for doing these calculations is shown in Appendix 2.

In Figure 4.2 are shown MSF locuses for three different levels of X_3.[1]

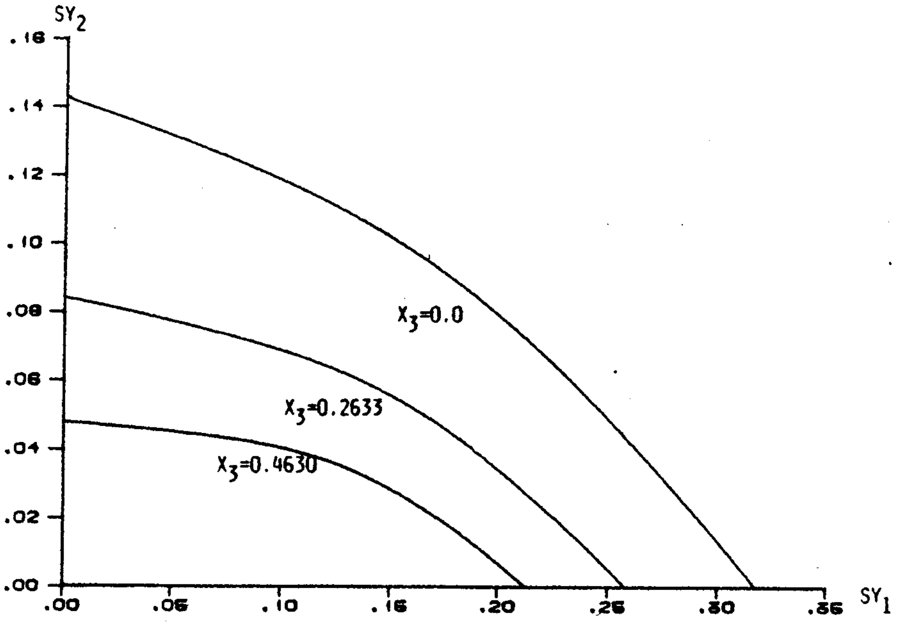

Figure 4.2. The map of maximum sustainable yield frontiers of a three species model shows sustainable combinations of yield of species 1 (SY_1) and species 2 (SY_2) for given stock levels of species 3 (X_3).

The question of economic optimal harvesting of the three species is left to be answered in Section 7. The next section, Section 5, gives a brief description of the ecological system of the Barents Sea, to which the three species model shall be applied.

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1) Seen from the origin X_3 is equivalent to W_3 = 1500, 853 and 0 thousand tons, respectively. The parameters used are: r_1 = 1.2704, r_2 = 1.1617, r_3 = 0.0614, α = 0.9, γ_{12} = 2.6566, γ_{13} = 0.3571 and γ_{23} = 0.8646. (For estimation of the parameters, see Section 6).

5. THE ECOLOGICAL SYSTEM OF THE BARENTS SEA

The Barents Sea occupies 1.405 million square km between the coast of North Norway, Svalbard, Novaja Semlja and the Murman Coast. The greater portion of the sea occupies depths ranging from 200 to 400 m, with a mean depth of 229 m. Due to the warm Atlantic waters continually flowing into the sea from southwest, the southwestern part of the Barents Sea never freezes to ice, and the northern and eastern parts are ice-free for part of the year.

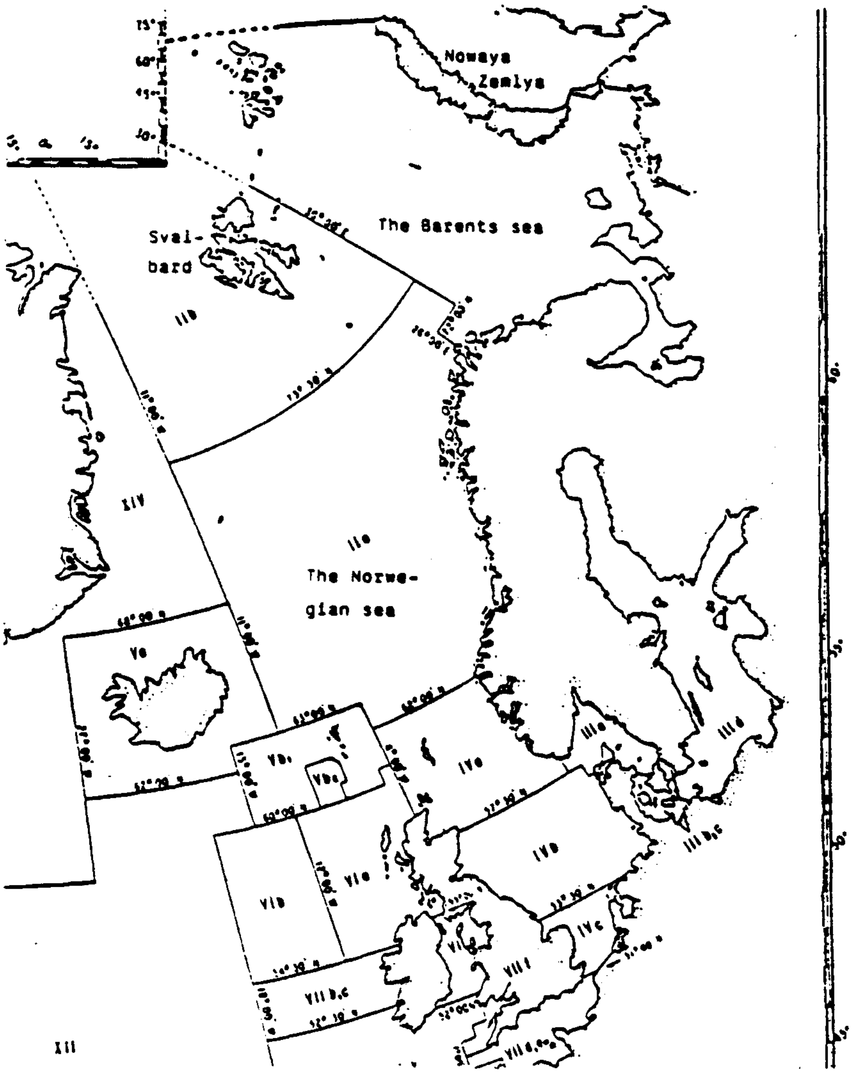

Figure 5.1. The Northeast Atlantic Ocean, including the Barents Sea.

The rich biological productivity of this area is determined by the phenomenon of vertical circulation. With cooling, the saline Atlantic water being denser sinks, bringing oxygen from the uppermost water masses bottomward. This creates an upward countercurrent bringing nutrient salts needed for the phytoplankton organisms. The production of phytoplankton is especially strong in spring in the border areas of ice-free and ice-covered sea. When the ice melts and the ice edge gradually moves to the north and northeast, the primary production is at its peak (Zenkevitch, 1956, and Gjøsæter et al., 1983).

The total number of zooplankton species of the Barents Sea is relatively few, consisting of 145 species (Zenkevitch, 1956). The copepoda make up the main zooplankton biomass: 90% in the southwest part of the sea. The total biomass of zooplankton is very high:

> We can assume that the total quantity of zooplankton in the Barents Sea makes up in summer approximately 100 million tons of raw material. Basically, the quantity, as we know, is produced by Calanus finmarchicus, yielding one generation a year. Thus the total zooplankton production of the Barents Sea can be assumed to equal the abovementioned magnitude. Just for the feeding of the zooplankton mass there is needed not less than 1000 million tons of phytoplankton which compensates for the colossal consumption by its exclusive capacity to propagate at a high rate. (Zenkevitch, 1956; p. 247)

The rate of production of phytoplankton is very high, but with a comparatively small standing biomass:

> Indeed, when taking into consideration the overall biomass which hardly exceeds 50 million tons, the annual production of the Barents Sea phytoplankton must be of a huge order of several thousand million tons, i.e. the ratio of the annual production to the biomass of the Barents Sea plankton in summer cannot be smaller than 50. (Zenkevitch, 1956; p. 247)

As in other seas of the world, the copepods are the basic food components for fish and other inhabitants.

In addition to the zooplankton, there is another important main source of food for the fish: the benthos inhabiting the sea bottom. The biomass density of these varies with the type of bottom as well as with depth.

> The total biomass of benthos in the entire Barents Sea is expressed by a magnitude of the order of 130 to 140 million tons. (Zenkevitch, 1956; p. 256)

In Table 5.1 are shown variations with depth in density of benthos.

Table 5.1. Variation with depth in the density of bottom population of the Barents Sea (in g/m^2).

Depth in m	The mean biomass in g/m^2	Depth in m	The mean biomass in g/m^2
0-100	310	400- 600	20
100-200	170	600-1000	2-10
200-300	90	1000-2000	1-2
300-400	50	(in the Greenland Sea)	

Source: Zenkevitch, 1956; Table 49.

114 species of fish are known to inhabit the Barents Sea, of which the most important families are: the cod family (12 species), flounder (11 species), viviparous blenny (13 species), goby (10 species) and white fish (7 species). The greater part of the families are, however, represented by one or two species in the commercial fish catches.

In the Barents Sea there are especially two species of zoo-plankton preying species of fish which have been of great commercial value: capelin (*Mallotus villosus*) and herring (*Clupea harengus*). In addition to these the pelagic species Polar cod (*Boreogadus saida*) has been commercially harvested since 1970. The pelagic fish species are important food sources for larger fish species, sea mammals and birds. So also is krill (*Euphauciacea*), feeding primarily at the second trophic level. In the transference of energy from phytoplankton through the food web to fish, krill is an important link (Mauchline and Fisher, 1969).

Larger fish of high commercial value are cod (*Gadus morhua*), haddock (*Melanogrammus aeglefinus*) and saithe (*Pollachius virens*). On average for the years 1950-1980, these three species yielded more than 90% of the total annual catch of demersal and semi-pelagic species (excluding pelagic species like herring, capelin and polar cod). Other demersal species of commercial value are: red fish (*Sebastes viviparus*), Norway haddock (*Sebastes marinus*), halibut (*Hippoglossus hippoglossus*), Greenland halibut (*Reinhardtius hippoglossoides*), plaice (*Pleuronectes platessa*). Of high-valued species, deep water prawn (*Pandalus borealis*) and salmon (*Salmo salar*) are the most important ones.

The main food sources of cod are to some extent known:

> The main food of cod consists of small pelagic fishes: herring, capelin, yound cod, haddock and Polar cod. The food of cod consists of 60% of fishes. Then follow large planktonic crustaceans such as amphipods, euphausiide and decapods. In the eastern part of the sea, a considerable portion of food is made up of bottom animals, such as crabs, hermit-crabs and various other sizable representatives of amphipods, isopods and cumaceans, to a lesser degree worms and mollusks. (Zenkevitch, 1956; p. 259)

Regarding the preys of haddock, the same source says:

> "Haddock, in contrast to cod, feeds on benthos, such as mollusks, worms, crustaceans and echinoderms."

The seals of the Barents Sea and adjacent areas are great consumers of fish, krill, zooplankton, benthos, etc. Like other living resources of the sea, each species of seal occupies its own niche in the ecosystem with respect to prey selection, geographical distribution, etc. The most important species for commercial purposes has been harp seal (*Pagophilus groenlandicus*
Its opportunistic feeding behaviour is probably a very important explanation of the large stock to be found in the Barents Sea,

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[1] Aggregated catch statistics for the period 1868-1980 are shown in Appendix 4.

including the White Sea. Dorofeev (1956) estimated the pre-harvesting stock level at 3-3½ million animals. Cod, capelin, herring and Polar cod are the most important preys among the commercially valuable fish species. Among other food sources reported for the harp seal are krill, deep sea prawn, flatfish, redfish and molluscs (see e.g. Bjørge et al., 1981; Kapel, 1973; Myers, 1959; Sergeant, 1973).

The hooded seal (*Crystophora cristata*) in the Barents Sea area is especially found in the thick, drifting ice around Svalbard. Compared to other seals in the North Atlantic the hooded seal dive deeper and for that reason probably have a higher share of deep water species on its menu (Reeves and Ling, 1981; Sergeant, 1976). Due to its preference for deep water and thick, drifting ice, the segment of the North Atlantic stock found in the Barents Sea area is not very great. The number of animals in the mid 1950s in the total stock has been estimated to be 1/2 million animals (Reeves and Ling, 1981). Walrus (*Odobenus rosmarus*) is the largest of the seals in these areas, but the number of individuals are small (references in Fay, 1981). So are also bearded seal (*Erignathus barbatus*), grey seal (*Halichoerus grypus*) and common seal (*Phoca vitulina*). The smallest, and probably the most abundant, seal in the northeast Atlantic and the Arctic Ocean is the ringed seal (*Phoca hispida*). Estimates of stock size vary widely, from 2.5 million to 6-7 million animals (Frost and Lowry, 1981; Stirling and Calvert, 1979). It is not known how many of these are present in the Barents Sea. About half of the ringed seal's food consists of fish of which polar cod (*Boreogadus saida*) is definitely the most important species (Gjertz and Lydersen, 1986).

Contrary to fish and seals which occupy the Barents Sea area all the year round, most of the whales utilize the area through intensive feeding in summer. In fall they migrate south to warmer parts of the Atlantic Ocean to breed and have their calves. These migratory patterns also are most efficient from a physiological point of view. The intensive summer feeding in the Arctic gives the animals a surplus of energy which is stored mainly as blubber. The storage of energy is gradually decreased during winter time when food resources are scarce. The warmer climate of the temperature zone helps to save energy for purely life processes.

Despite earlier years intensive hunting of the big baleen whales, all species are still found in the Barents Sea area: Blu whale (Balaenoptera musculus), fin-whale (Balaenoptera physalu humpback whale (Megaptera novacanglia) and sei-whale (Balaenopt borealis). In addition to these rare big baleen whales the smaller minke whale (Balaenoptera acutorostrata) still is plentiful with stock estimates for the early 1980s ranging between 50 000 and 100 000 individuals, with an average weight of 4 tons (Rørvik, 1981).

The blue whale and the sei-whale mostly feed on krill and other crustaceas animals, but they have also been observed preying on small pelagic fish like herring and capelin.

Small pelagic fish seem to form a higher share of the die of the fin and humpback whales than for the other two species of big baleen whales. The humpback also preys on other kinds of fish like cod. The total number of big baleen whales in th Barents Sea is uncertain, but guesstimates say 2-3000 with a biomass of 80-120 thousand tons (references in Holm, 1983).

In addition to the five baleen whales described here, there used to be two other species in this part of the Arctic. The black right whale and the Greenland right whale were two plentiful, slow swimming species in the Barents Sea, especiall around the islands of Svalbard. These stocks have, however, disappeared, probably because of heavy harvesting of them in the 17th century by English and Dutch whalers.

The toothed whales in the Barents Sea consist of 12 speci of which 3 are rare. The group is very heterogeneous with respect to individual size, with the sperm whale (Physeter catodon) as the biggest (30-40 tons) and the porpoise (Phocaen phocaene) as the smallest (less than 100 kg). Common species include: pilot whale (Globicephala melaena), white whale (Delphiapeterus leucas), white-sided dolphin (Lagenorhynchus acutus), bottlenose whale (Hyperoodon ampullatus) and killer whale (Orcinus orca). The total biomass of sperm whales in t

Barents Sea area - in summer time - has been guesstimated to vary between 150 and 175 thousand tons in the 1950s, 1960s and 1970s. The biomass of smaller toothed whales has been guesstimated to be 50 thousand tons in the same time period. (See sources in Holm, 1983.)

The sperm whale's diet consists primarily of squid and deep water fish, but also a certain amount of cod fish. Food sources for the bottlenose, the most plentiful of the small toothed whales in this area, are much like those of the sperm whale, whereas the killer whale and the others mainly feed on fishes, such as cod and other demersal species and on pelagic species like herring and capelin.

A brief description of the ecological system of the Barents Sea area ought to include the polar bear (Ursus maritimus). The population biology of this species is well documented (Larsen, 1986a). The total population in the area from east Greenland to Franz Josef Land and Novaya Zemlya is now (mid 1980s) more than 5000 animals, of which 2500 are in the Svalbard area (Larsen, 1986b). Ninety percent of the polar bear's food is ringed seal, 5% other seals and the rest is fish. An important chain in the ecological system in this part of the Arctic seems to be polar cod - ringed seal - polar bear. Nevertheless it will not be included in this study since these species' commercial value has been of minor importance compared to many other living resources of the Barents Sea.

One of the main objectives of this study is to quantify to which extent each of the trophic levels of the marine ecology system should be harvested. Despite the complexity of such a system we shall consider just some of the species on the following three trophic levels: (1) plankton preying pelagic fish species, (2) carnivore fish species and (3) fish consuming sea mammals. The species considered are, however, among the most important with respect to biomass in the ecological system and with respect to value for the fishing industry.

6. ESTIMATING BIOLOGICAL PARAMETERS OF THE THREE SPECIES BARENTS SEA MODEL

Before presenting the parameter estimates of the three species Barents Sea (TSB) model, we shall briefly describe some of the problems of parameter estimation in single species models. Almost all population analysis of North Atlantic fish stocks have been carried out on the basis of Beverton-Holt types of models (Beverton and Holt, 1957). The use of aggregated biomas models to assess North Atlantic stocks is almost non-existent i the literature. However, it is possible to estimate parameters in the latter types of models from parameters in the former typ of models.

Applied fish population analysis of demersal species is usually based upon assumptions like: the existence of a stock-recruitment relation, age specific growth functions and constan natural mortality. Estimation of parameters is done on the basis of catch and effort data by means of e.g. Virtual Population Analysis (VPA). For cod in the North Atlantic Ocean estimates of biological parameters for several stocks are given in Garrod (1977). Some of the estimates for the three largest stocks are shown in Table 6.1. (The complete table is shown in Appendix 5.)

Table 6.1. Some characteristics of the largest Atlantic cod stocks.

	1	2	3
	Recruits 2 year old (in millions)	Maximum sustainable yield per recruit (kg)	W_{MSY}[1] (in million metric tons)
Iceland	300	1.56	2.28
Arcto-Norwegian	1250	0.57	4.13
East New Foundland/Labrador	2000	0.40	3.16

Source: Garrod (1977) (also see Appendix 5).

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[1] W_{MSY} = Necessary stock level to produce maximum sustainable yield.

The single species logistic growth equation is the one used in the Schaefer model (Schaefer, 1954 and 1957):

$$\dot{W} = rW(1 - W/K) \ . \qquad (6.1)$$

r is the intrinsic growth rate, and K is the carrying capacity of the ecological niche of the stock. The maximum sustainable yield (MSY) in this model is:

$$MSY = rK/4 \qquad \text{for} \qquad W_{MSY} = K/2 \ . \qquad (6.2)$$

The growth curve, and therefore the sustainable yield curve, is symmetric around $W = K/2$. "Skewed" growth models, where the yield curve is asymmetrical and has its maximum at $W \neq K/2$, also have been used in fish population analysis (Pella and Tomlinson, 1969). However, we shall stick to the symmetrical growth model since this is the simplest one to handle and since the biological studies in this field do not contradict such a presumption regarding cod (Garrod, 1977).1) Using the data on maximum yield per recruit (Y/R), number of recruits (R) and MSY-biomass (W_{MSY}) in Table 6.1, the MSY figures can be calculated from the formula:

$$\widehat{MSY} = (Y/R)R \ . \qquad (6.3)$$

With the additional assumption that the yield pattern of the stock may be described by the logistic growth equation, r and K can be found from the formulas:

$$\hat{K} = 2W_{MSY} \qquad (6.4)$$

$$\hat{r} = \frac{2\widehat{MSY}}{W_{MSY}} \ , \qquad (6.5)$$

where $\widehat{MSY}$ is from (6.3).

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1) In a Beverton-Holt model with constant recruitment and age specific growth, yield per recruit and total yield are determined by the age of capture and the effort level. For a given age of capture and effort level a corresponding stock level exists. Therefore, it is possible to have a yield-stock relationship in a biomass model without assuming density dependent recruitment and growth.

Using data from Table 6.1 and the Equations (6.3)-(6.5), the calculated MSY, r and K are shown in Table 6.2.

Table 6.2. Calculated logistic growth equation parameters for three cod stocks.

	$\hat{\text{MSY}}$ (million tons)	$\hat{K}$ (million tons)	$\hat{r}$
Iceland	0.468	4.46	0.41
Arcto-Norwegian	0.713	8.26	0.35
East Newfoundland/Labrador	0.800	6.32	0.51

The parameters in Table 6.2 can not be directly used in two or three species models since they are derived on the assumption that sustainable yield is a "net sustainable yield" of which the total can be harvested by man. In contrast the prey-related parameters in a multispecies model must take care of the prey's production of food for the predator in addition to the net sustainable yield which can be harvested by man. Nonetheless the single species model and the calculated parameters in Table 6.2 can be of interest in the case of a stable environment for the cod stocks, i.e. for stable stocks of preys and predators and moderate variations in harvesting. In the case of increased harvesting of the preys or the predators of the cod stock we would expect a change in single species model parameters such that the sustainable yield estimates (included MSY) are also changed. For these reasons the calculated parameters of the single species model cannot be directly used in the three species model. We shall therefore have to rely on other methods where some of the parameters of the three stocks are "guesstimated" simultaneously, or where the interrelationships between parameters are taken care of in other ways.

By "guesstimation" is meant, in this connection, that relevant biological literature is studied and information essential to our problem is extracted. When two or more sources give somewhat different figures for the same variable, parameter etc., a choice is made as to which to rely on or the average of them is used. All the steps and assumptions made are explained in detail so that the reader can critically appraise and modify them if that is felt to be desirable.

In the TSB-model there are 9 biological parameters:

The predation coefficients : a_{12}, a_{13} and a_{23}
Intrinsic growth rates : r_1, r_2 and r_3
Carrying capacity related coefficients: K, α and β

In the introductory part of this study references are given to some works by fisheries biologists on predators' consumption rates of preys. The methods used vary from very detailed and comprehensive stomach sampling surveys, e.g. ICES (1982) and Ponomarenko et al. (1978), via indirect methods using the received knowledge of the species' physiology, metabolism, etc., e.g. Sergeant (1969), to guesstimation and simulation technique, e.g. Christensen (1982), Laevastu and Favorite (1978) and Reed and Balchen (1981).

Based on several biological studies the predation coefficients have been guesstimated by the author elsewhere (Flaaten, 1984b; and Flaaten, 1984c). When stocks are measured in thousand tons the guesstimates are

$$\begin{aligned} \hat{a}_{12} &= 0.25 \cdot 10^{-6} \\ \hat{a}_{13} &= 0.14 \cdot 10^{-6} \\ \hat{a}_{23} &= 0.31 \cdot 10^{-6} \end{aligned} \tag{6.6}$$

The consumption functions used, $C_{ij} = a_{ij}W_iW_j$, are based on the assumption of opportunistic feeding. That is, each unit of predator always eats a constant portion of the prey stock. The "opposite" would be when one unit of the predator eats a constant quantity of the prey, independent of the size of the prey stock (provided that the prey stock is big enough to meet this demand). Such feeding can be termed specific feeding.

After having calculated the predation coefficients, there still remain six parameters to be estimated. Looking at the complete model in Equations (3.1)-(3.3) it should be noticed that the parameters r_1, α, β, and K, occur in pairs in the three equations: r_1 and K in the first, r_2 and α in the second and r_3 and β in the third. In other words there are only two unknown parameters in each of the three equations (with the a_{ij} now known). This is an important observation which will be used to simplify the estimation procedure. In equilibrium, that is when $dW_i/dt = 0$ in Equations (3.1)-(3.3), all stocks and harvest rates are constant. In this case Equation (3.1) gives the following relation between r_1 and K, with equilibrium stocks and harvest rates denoted $\bar{W}_1$ and $\bar{h}_1$, respectively.

$$r_1 = \frac{S_1}{1 - \bar{W}_1/K} \tag{6.7}$$

where S_1 is a constant equal to

$$S_1 = a_{12}\bar{W}_2 + a_{13}\bar{W}_3 + \bar{h}_3/\bar{W}_3 \ . \tag{6.8}$$

The equilibrium relation between r_2 and α follows from (3.2):

$$r_2 = \frac{S_2}{1 - \bar{W}_2/\alpha\bar{W}_1} \tag{6.9}$$

with S_2 equal to

$$S_2 = a_{23}\bar{W}_3 + \bar{h}_2/\bar{W}_2 \ . \tag{6.10}$$

Finally, the relation between r_3 and β is:

$$r_3 = \frac{S_3}{1 - \bar{W}_3/\beta\ (\bar{W}_1 + \bar{W}_2)} \tag{6.11}$$

with S_3 equal to

$$S_3 = \bar{h}_3/\bar{W}_3 \ . \qquad (6.12$$

With Equations (6.7), (6.9) and (6.11) the original problem of estimating six independent parameters have been reduced to that of estimating just three with the other three being given by the said equations. This result will be used later on in the guesstimation process, which will proceed in several steps.

First approach

As a first approach to guesstimation of the biological parameters, a couple of assumptions are made with respect to the equilibrium situation of the TSB-model:

1. The three species model has an equilibrium with catches and stock sizes equal to the averages for the years 1951-80, except for the stock of sea mammals where 1950-54 figures are used since more recent data are lacking for some of the whale stocks included in the sea mammal stock.

2. The values of the predation coefficients are as shown in (6.6).

Assumption 1 does not say that the ecological system of the Barents Sea has been in equilibrium in the period 1951-80, but it puts a restriction on the size of the system and its production capacity.

The equilibrium values of catches and fish stocks are shown in Table 6.3. In the last column of the table are shown the equilibrium consumption rates calculated from the other data in the table.

By using the data in Table 6.3 in Equations (6.7), (6.9) and (6.11) the following specific relationships between pairs of parameters are found:

$$r_1 = \frac{1.5557}{1 - 9.465/K} \tag{6.13}$$

with K measured in million metric tons.

$$r_2 = \frac{0.4771}{1 - 0.5496/\alpha} \tag{6.14}$$

$$r_3 = \frac{0.0216}{1 - 0.0582/\beta} \cdot \tag{6.15}$$

Table 6.3. Assumed equilibrium stocks and catch rates and derived consumption rates for the TSB-model. First approach.

i	$\bar{W}_i$ 1) ('000 metric tons)	$\bar{h}_2$ 2) ('000 metric tons per year)	$\hat{a}_{ij}$ 3)	$\bar{C}_{ij}$ ('000 metric tons per year)
1	9465	1285.6	$a_{12} = 0.25\ 10^{-6}$	$\bar{C}_{12} = 12\ 309$
2	5202	1105.7	$a_{13} = 0.14\ 10^{-6}$	$\bar{C}_{13} = 1\ 130$
3	853	18.4	$a_{23} = 0.31\ 10^{-6}$	$\bar{C}_{23} = 1\ 376$

1) Average stock sizes 1951-80 for i = 1,2 and 1950-54 for i =
Source: Flaaten (1984a and b).

2) Average catches 1951-80 for i = 1,2,3. Source: Flaaten and Holm (1984).

3) From (6.6).

Given the assumptions made, we know that all combinations of parameters, with (6.13)-(6.15) fulfilled, will give the same equilibrium values of the fish stocks, $\bar{W}_i$. However, it is not known for which combinations there will be a stable equilibrium and for which there will be an unstable equilibrium. Neither d we know for which combinations the dynamic behaviour of the system will be best. Intuitively we will prefer the ones wher the simulated stocks behave as similarly as the actual fish stocks as possible (we shall return to this point later on).

It is reasonable to demand that the TSB-model with a spec fic set of parameters should fulfil the following two requirements:

1. With the annual harvest rates for 1868-1980 put into the simulation model the simulated stock levels for 1980 shoul be positive for all three species.

2. Without harvesting, i.e. for $A_i = 1$ in (3.6)-(3.8), there should be a locally stable equilibrium (or two or more equilibria) with positive stock levels for all three species.

The first requirement is reasonable since none of the three stocks have been extinct during the actual time period. The second one is reasonable since there probably was a pristine equilibrium[1] in the ecosystem before man started his harvestin It also can imply that if all harvesting came to an end, the system again would return to its former equilibrium.

A simulation model

The three species model described by the differential Equations (3.1)-(3.3), or even by the reduced form in (3.6)-(3.8) is too complex for an analytical solution to the system to be found. Such a solution would imply that for known initial leve of the three stocks and for the known biological parameters of t model, the stocks at any point in time could be expressed as functions of initial stock levels, parameters and harvest rates Even though an analytical solution can not be found, it is quit easy to find approximate solutions by use of computer simulatio models.

By use of the computer program DYNAMO[2] (Pugh III, 1980) a program for simulating the system (3.1)-(3.3) (see Appendix 7) has been designed. The model has been implemented on a CYBER 171MP computer at the University of Tromsø. For an initial set of stock levels the model computes, in discrete time, the change in the stock levels during a short period of time and adds this to the initial levels. By repeating this computation many times the model can describe the development of the stocks for any peri of time. By making the steps small enough the model simulations

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[1] However, see p. 17 for a brief discussion on limit cycle models

[2] DYNAMO is a compiler for translating and running continuous models (models described by a set of differential equations). It was developed by the industrial dynamics group at M.I.T. for simulating dynamic feedback models of business, economic, and social systems, but there is nothing in its design that precludes its use for any continuous system.

can be as accurate as desired. However, accuracy will be at the expense of using more computer resources.

For running the computer model it is necessary to know the initial stock levels, parameter values and harvest rates for the simulation period. It has been possible to find annual harvest rates of the Barents Sea area back to 1868 (Flaaten and Holm, 1984). Since the stock levels of capelin, cod and sea mammals are unknown for the initial year, 1868, they had to be chosen somewhat arbitrary to start with. In the final simulations the initial stocks are put equal to the equilibrium stock levels shown in Table 6.9.

For a specific set of values of the righthand side parameters of (6.13)-(6.15), K, α and β, the lefthand side parameters, r_1, r_2 and r_3, can be calculated. It should be noted that the r_i's are decreasing functions of the r.h.s. parameters. Based on some knowledge of the ecosystem of the Barents Sea and information provided by studies based on one-species models, we have chosen to start with the set of parameters shown in Table 6.4.

Table 6.4. Parameter values based on (6.13)-(6.15).
First approach.

Exogeneous chosen:	$K = 30 \cdot 10^6$	$\alpha = 0.90$	$\beta = 0.08$
Calculated:	$r_1 = 2.2728$	$r_2 = 1.2254$	$r_3 = 0.0793$

The parameters in Table 6.4 are put into the simulation model. The problem of evaluating the results still has to be solved. One possibility is to use the sum of squares of differences between the simulated stocks and actual stocks. However, only the capelin and the cod stocks are known for a long enough period of time (1951-80), and even those data might include some errors (Flaaten, 1984b). For the sea mammals complete time series data are lacking, even though there are estimates for some stocks at some points in time (Flaaten, 1984b and c).

To explain the sum of squares method the following symbols are used:

W^o_{it} = Known stock level of species i, at time t
W_{it} = Simulated stock level of species i, at time t,

The sum of squares of the differences of relative stock sizes is defined as

$$Q^2 = \sum_i \sum_t (W_{it} - W^o_{it})/W^o_{it})^2 \; . \tag{6.16}$$

The aim of the simulation procedure should be to find the combination of parameters, which, via W_{it}, minimizes Q^2. By using relative stock sizes instead of absolute sizes, we give each of the three species the same weight in the objective functional. Otherwise the species on the lowest trophic level would in practice be given a higher weight since it usually maintains a higher stock level than species on lower trophic levels do.

As an experiment the stocks of capelin and cod shown in Appendix 3 have been taken as given, while the stock of sea mammals has been excluded from (6.16). With the parameter-formulae (6.7), (6.9) and (6.11) put into the simulation model, the W_{it}'s (for capelin and cod) can be found for all specified combinations of the exogeneous parameters K, α and r_3.[1] A system of computer programs has been written to make these simulations and computations automatically. The chart flow in Appendix 6 shows the programs involved when the model is run.

As examples of the kind of results we get, Tables 6.5 and 6.6 show computed Q^2-values for different combinations of K and α, for $\beta = 0.08$ and $\beta = 0.10$, respectively.

Experimenting with the simulation models has given us some insight to the problem of finding the best combination of biological parameters in the TSB-model. However, new questions and problems have also arisen as a result of these experiments. First of all, it does not seem possible to get Q^2 to reach a

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[1] For historical reasons (in the evolution of this work) r_3, instead of β in (6.11) is exogeneous in the simulation model. This of course makes no change in the results.

Table 6.5. Computed Q^2-values for $\beta = 0.08$ ($r_3 = 0.0893$). First approach.

$K \cdot 10^{-6}$ \ α	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.0
24	7.75	7.77	7.80	7.82	7.85	7.86	7.90	7.92	7.95	7.98	8.0
26	7.72	7.73	7.76	7.80	7.82	7.85	7.88	7.92	7.95	7.99	8.0
28	7.68	7.71	7.74	7.77	7.80	7.83	7.87	7.91	7.96	8.00	8.0
30	7.66	7.69	7.71	7.75	7.78	7.83	7.86	7.92	7.97	8.02	8.0
32	7.63	7.67	7.70	7.73	7.77	7.82	7.87	7.93	7.98	8.04	8.1
34	7.61	7.64	6.68	7.72	7.77	7.82	7.88	7.93	8.00	8.08	8.1
36	7.59	7.63	7.67	7.71	7.75	7.81	7.88	7.95	8.02	8.11	8.1

Note: Using formula (6.16), with W_{it}^*'s in Appendix 3 for capeli and cod for the years 1951-80. W_{it}'s are computed by mean of the DYNAMO simulation program shown in Appendix 7.

Table 6.6. Computed Q^2-values for $\beta = 0.10$ ($r_3 = 0.0517$). First approach.

$K \cdot 10^{-6}$ \ α	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.0
24	7.73	7.75	7.76	7.79	7.80	7.82	7.85	7.86	7.89	7.91	7.9
26	7.69	7.71	7.73	7.74	7.77	7.80	7.83	7.85	7.88	7.91	7.9
28	7.66	7.68	7.70	7.72	7.74	7.78	7.80	7.84	7.88	7.93	7.9
30	7.63	7.64	7.67	7.70	7.73	7.76	7.81	7.85	7.88	7.93	7.9
32	7.60	7.62	7.65	7.67	7.71	7.75	7.79	7.84	7.90	7.95	8.0
34	7.57	7.60	7.62	7.66	7.70	7.74	7.79	7.85	7.91	7.97	8.0
36	7.56	7.58	7.61	7.65	7.69	7.74	7.79	7.86	7.92	8.00	8.

Note: See note to Table 6.5.

minimum value for an interior set of parameter combinations.[1] Secondly, it might be that the use of just two species in the objective functional, while the model also has a third species, produces some unsolved and unfortunate problems. Thirdly, the model framework we are working with might be too general to expect good empirical results. Fourthly, the stock estimates of cod and capelin are based on single species model. As noted earlier, such estimates could be biased because of changes in the harvest mix of species. Finally, the predation coefficients in (6.6) could be wrong. In a simulation framework this could probably be checked for by also varying these coefficients. If the restrictions put upon the parameters by (6.7), (6.9) and (6.11) were abolished, a larger simulation program could be made where all nine biological parameters in the TSB-model could be varied independently. Q^2 could then be calculated for a very large number of parameter combinations. This, however, is reckoned as being too time consuming and resource demanding to be included in this study. Leaving the simulation framework here we shall now return to the maximum sustainable yield frontiers (MSF) derived in Section 4.

The parameters in (6.6) and Table 6.4, which are the first approach values, can be used for computing the terminal points of the MSF. The procedure for this is described in Equations (4.18)-(4.23), and the computer program designed to do the proper calculations is shown in Appendix 2. The results for capelin and cod of course depend on the stock level of the top predator, the sea mammals. Table 6.7 shows the results for the absolute maximum sustainable yields of capelin and cod for three different stock levels of sea mammals.

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[1] This might be because of computer programming difficulties. For unknown reasons the program would not run more than a limited number of simulations. Several attempts have been made to solve these problems, included inquiries to the supplier of DYNAMO, but all in vain so far.

Table 6.7. Computed absolute maximum sustainable yields of capelin and cod. Million metric tons. First approach.

Sea mammals stock level Species	Extinct: 0	1950-54 level: 0.853	High level: 1.200
Capelin	17.05[1]	15.25[1]	14.53[1]
Cod	4.98[2]	3.07[2]	2.35[2]

[1] Provided that cod is extinct.

[2] Provided that capelin is unharvested by man.

Even though they are terminal points on the MSF locus, the results in Table 6.7 seem to be high compared with actual harvest rates. From the stock data in Appendix 3 it is clear that there has been a decrease in both capelin and cod stocks in the period 1951-80. In other words, the stock levels of capelin and cod have been reduced during this period by the fishermen's harvest and the sea mammals' consumption.

Let us pick an arbitrary point on the MSF locus for W_3 = 0.853 million m.t., say SY_1 = 10.18 million m.t. and SY_2 = 1.98 million m.t. (at this point SY_3 = 0.02 million m.t.). This is obviously far beyond the estimates given in the literature.[1] Even a point like SY_1 = 9.29 million m.t. and SY_2 = 1.69 million m.t. on the MSF locus for W_3 = 1.200 million m.t. (SY_3 = 0.01 million tons) is significantly more than would be expected according to the biology literature. So far we seem to be trapped in the guesstimation procedure since we do not know if all nine parameters in first approach should be corrected, or just some of them. However, recent research on the feeding habits of cod in a North Norwegian fjord (Eliassen and Grotnes, 1985) gives some ideas as to which direction the guesstimation procedure should proceed.

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[1] See Garrod (1977); Hamre and Tjelmeland (1982); Dragesund et al. (1980); Sætersdal (1984); and references given in Flaaten (1984b).

Second approach

Balsfjorden in Northern Norway is a sheltered subarctic fjord which constitutes a miniature Barents Sea with respect to fish communities. Feeding habits of fish species in the fjord can therefore be expected to have much in common with the equivalent species in the larger ecosystem of the Barents Sea. Results in Eliassen and Grotnes (1985) and from work quoted therein (e.g. Klemetsen, 1982) might indicate that the first approach guesstimate of the coefficient a_{12} for cod's predation on capelin[1)] is too high. The guesstimate, $\hat{a}_{12} = 0.25 \cdot 10^{-6}$, in (6.6) is based on data from a period (1974-76) when the herring were practically extinct in the Barents Sea and adjacent areas. Because of the lack of stock estimates in Eliassen and Grotnes (1985)[2)] and Klemetsen (1982), their data cannot be used for the guesstimates given in (6.6) in the same way as predation results in Ponomarenko et al. (1978) were used in Flaaten (1984c). We shall therefore choose to reduce a_{12} somewhat arbitrarily and see what the implications are for the TSB-model.

As noted earlier there are several reasons to believe that a_{12} in (6.6) is too high. However, we have not come across information that would lead to changes in a_{13} and a_{23}.

In this second approach of the guesstimation procedure we shall make the following two changes in the assumptions compared to the first approach:

1) $a_{12} = 0.125 \cdot 10^{-6}$

2) With average stock sizes and catch rates shown in Table 6.3 the annual relative growth rates of the three species are:

[1)] Remember that capelin in this report means capelin plus herring.

[2)] Stock estimates will be published later (Eliassen, personal communication).

i	1	2	3
$\frac{dW_i/dt}{W_i}$	-0.0359	-0.0247	0.01

Assumption 2 is based on the observation of a decrease in the capelin and the cod stocks, as shown in Appendix 3. For the period 1950-80 the capelin stock decreased on average 3.59% p.a. and the cod stock 2.47% p.a.[1] As noted before, we do not know the changes in the sea mammal stock as well as those of the two others. An annual increase of one per cent in the period may be right, but it could just as well have been an average of zero (for references to relevant literature, see Flaaten (1984c)).

With the assumptions made and by using the same method as described in the text concluding with (6.13)-(6.15), the specif relations between pairs of parameters now become:

$$r_1 = \frac{0.8696}{1 - 9.465/K} \qquad (6.17)$$

$$r_2 = \frac{0.4523}{1 - 0.5496/\alpha} \qquad (6.18)$$

$$r_3 = \frac{0.0316}{1 - 0.0582/\beta} \, . \qquad (6.19)$$

By a somewhat arbitrary choice of values of the r.h.s. parameters, (6.17)-(6.19) may be used to calculate the values of th l.h.s. parameters. The results are shown in Table 6.8.

Using Equations (3.9)-(3.11), the normalized stock levels equilibrium without harvesting (i.e. when $A_i = 1$ for i = 1,2,3) can be computed. With the second approach set of parameters fr Table 6.8 the stock levels derived are shown in Table 6.9, for ordinaty stocks as well as for normalized stocks.

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[1] The figures are found by linear regression on the formulas $W_i(t) = W_i^{1950} e^{a_i t}$ for i = 1,2: $a_1 = -0.0359$ and $a_2 = -0.0247$

Table 6.8. Parameter values based on (6.17)-(6.19). Second approach.

Exogeneous chosen	$K = 30 \cdot 10^6$	$\alpha = 0.90$	$\beta = 0.12$
Calculated	1.2704	1.1617	0.0614

Table 6.9. Equilibrium stocks without harvesting. Second approach.

$\bar{W}_1$	$\bar{X}_1$	$\bar{W}_2$	$\bar{X}_2$	$\bar{W}_3$	$\bar{X}_3$
10.069[1)]	0.336	4.759[1)]	0.176	1.779[1)]	0.549

[1)] Million metric tons.

If the Second approach parameters are correct, the equilibrium stocks shown in Table 6.9 are the pristine levels of Barents Sea area. For obvious reasons there are no stock assessment figures from pre-harvesting time to check our results against.

We are also interested in the stability of the pre-harvestinc equilibrium. According to the criteria given in Appendix 8, therε exists a unique non-harvesting equilibrium with all stocks positive if

$$\gamma_{23} \leq \alpha + \gamma_{13} \; . \qquad (6.20)$$

Using the second approach parameters the following values of the γ's are derived:

$$\begin{aligned} \gamma_{12} &= 2.6566 \\ \gamma_{13} &= 0.3571 \qquad (6.21) \\ \gamma_{23} &= 0.8646 \; , \end{aligned}$$

and the inequality (6.20) is satisfied since $\alpha = 0.9$. According to Appendix 8 this equilibrium is locally stable if

$$x = \frac{4(\gamma_{12} - \alpha)\gamma_{23}}{B_1^{\,2}} \leq 1 \qquad (6.22)$$

where

$$B_1 = -\alpha(1 + \gamma_{12}) - \gamma_{13}(1 + \alpha) + \alpha\,\gamma_{23} \qquad (6.23)$$

in the non-harvesting case.

Inserting the values from (6.21) into (6.22) and (6.23) give

$$x = 0.5965$$

and inequality (6.22) is satisfied. The non-harvesting equilibrium therefore is a locally stable equilibrium.[1)]

Having used the sum of squares method (see (6.16)) with the second approach parameters, the computed Q^2 values are shown in Tables 6.10 and 6.11.

Comparing the results in Tables 6.10 and 6.11 to those in Tables 6.5 and 6.6 makes it clear that according to the sum of squares criteria the second approach parameters perform better than the first approach parameters. This, however, is definitely not to say that the second approach is the best of all thinkable sets of parameters. Since time series data for the sea mammal stock are lacking, we should not expect to find <u>the</u> biological optimal set of parameters by use of the sum of squares method. As seen from Tables 6.10 and 6.11 the Q^2-values are lower the smalle α is and the greater K is. Going down and to the left of $\alpha = 0.9$, $K = 30$ in the tables would, however, imply that $r_1 < r_2$, accordin to Equations (6.17) and (6.18). This would contradict received knowledge saying that the intrinsic growth rate of a species

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[1)] In the same way it can be shown that with the first approach set of parameters the model has a locally stable equilibrium (without harvesting) with stocks:
$\overline{W}_1 = 9.018$, $\overline{W}_2 = 5.699$ and $\overline{W}_3 = 1.177$ (all in million tons).

Table 6.10. Computed Q^2 values for $\beta = 0.10$ ($r_3 = 0.0756$). Second approach.

α / $K \cdot 10^{-6}$	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00
24	6.26	6.40	6.55	6.70	6.87	7.05	7.25	7.48	7.72	8.00	8.30
26	6.21	6.36	6.52	6.70	6.88	7.09	7.31	7.57	7.85	8.16	8.51
28	6.16	6.32	6.50	6.69	6.90	7.13	7.38	7.65	7.96	8.31	8.71
30	6.12	6.30	6.49	6.69	6.92	7.16	7.43	7.73	8.07	8.46	8.91
32	6.09	6.28	6.48	6.70	6.93	7.19	7.48	7.81	8.18	8.60	9.09
34	6.06	6.26	6.48	6.70	6.95	7.22	7.54	7.88	8.29	8.74	9.28
36	6.04	6.25	6.46	6.71	6.97	7.26	7.59	7.96	8.38	8.87	9.46

Table 6.11. Computed Q^2 values for $\beta = 0.12$ ($r_3 = 0.0614$). Second approach.

α / $K \cdot 10^{-6}$	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00
24	6.24	6.39	6.54	6.72	6.90	7.11	7.33	7.58	7.86	8.16	8.52
26	6.19	6.36	6.53	6.72	6.92	7.15	7.41	7.68	8.00	8.35	8.75
28	6.15	6.33	6.52	6.74	6.96	7.20	7.48	7.78	8.14	8.53	8.98
30	6.12	6.31	6.51	6.73	6.97	7.24	7.54	7.88	8.25	8.69	9.20
32	6.09	6.29	6.51	6.74	7.00	7.28	7.61	7.97	8.38	8.87	9.42
34	6.06	6.28	6.50	6.76	7.03	7.33	7.67	8.05	8.51	9.02	9.63
36	6.04	6.27	6.51	6.76	7.05	7.36	7.73	8.14	8.63	9.17	9.84

usually is higher the lower the species is on the trophical ladder. Instead of spending more resources on refining the set of parameters, we shall therefore stick to that in Table 6.8 and call this the basic case. A summary of the basic case biological parameters is shown in Table 6.12.

Table 6.12. The basic case parameters of the TSB-model.

a_{12}	a_{13}	a_{23}	r_1	r_2	r_3	K	α	β
$0.125 \cdot 10^{-6}$	$0.14 \cdot 10^{-6}$	$0.31 \cdot 10^{-6}$	1.2704	1.1617	0.0614	$30 \cdot 10^{6}$	0.9	0.12

Sources: See the text.

With the basic case parameters the model's maximum sustainable yield frontier is shown in Figure 6.1 for four levels of the sea mammal stock.

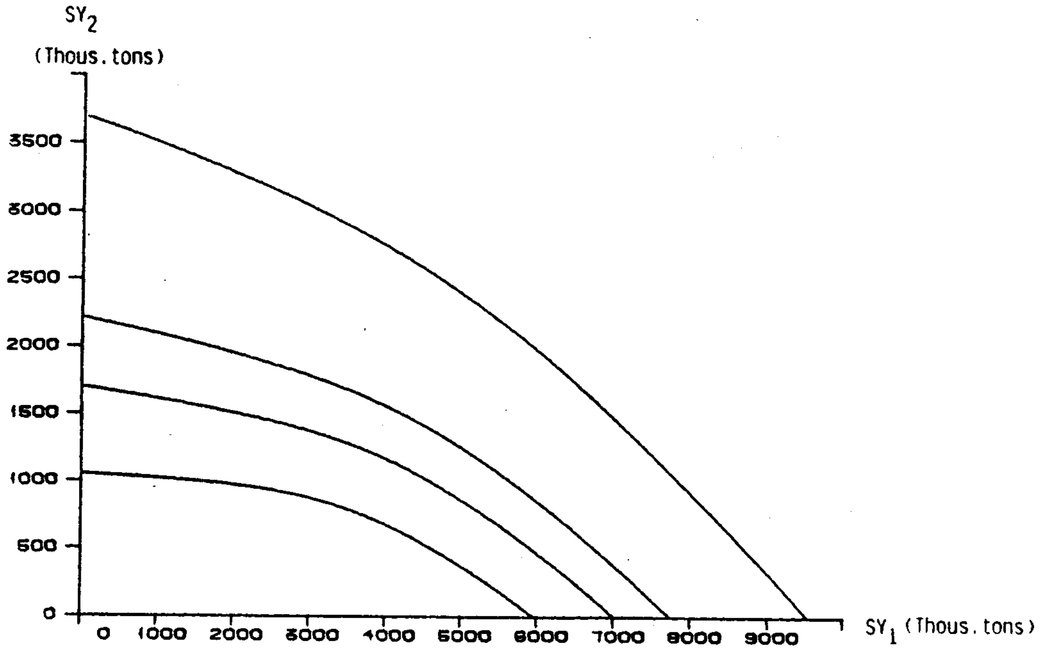

Figure 6.1. Maximum sustainable yield frontiers (MSF) for the TSB-model with basic case parameters.

The outermost MSF for cod and capelin is for sea mammal extinct ($W_3 = 0$), while the next one is for sea mammal at the 1950-54 level ($W_3 = 0.853$ million tons). The two other MSFs are for $W_3 = 1.200$ and $W_3 = 1.700$ million m.t., respectively.

In the case of no harvesting of the two predators, cod and sea mammals, the traditional sustainable yield locus for capelin is shown in Figure 6.2.

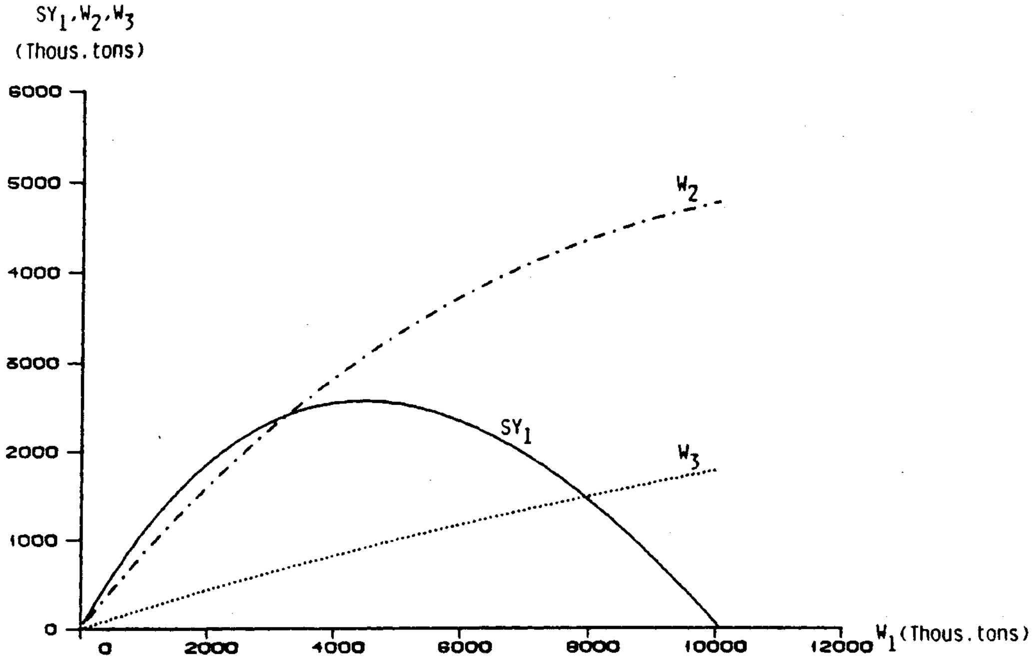

Figure 6.2. The sustainable yield curve (SY_1) for capelin in case of no harvesting of the predators in the TSB-model, with Basic case parameters. W_2 and W_3 are the corresponding unharvested stocks of cod and sea mammals respectively.

In this case the MSY for capelin is approximately 2.6 million m.t. for a stock level of 4.4 million m.t. To each level of the capelin stock the two corresponding stock levels of cod and sea mammals are also shown in Figure 6.2. Comparing Figures 6.2 and 6.1 makes it clear that a harvesting strategy leaving cod and sea mammals unharvested and only harvesting the basic prey, the capelin, is not a good strategy. Such a strategy implies not only loss of cod and sea mammal yields, but also less capelin yield than is possible from the ecological system if more than just the lowest level of the system were harvested. The questic of finding the best way of harvesting the three species is a question of economics, and the answer will depend on relative harvesting costs and product prices, as well as the discount rate.

7. ECONOMIC ASPECTS OF MULTISPECIES FISHERIES

The harvest rates introduced to the biological model represent a kind of predation effect from mankind. Since the beginning of time mankind has been harvesting natural resources both on land and offshore. In some cases the land resources, like grazing fields, hunting areas etc., have been common property resources. However, in most "civilized" cultures such resources have been private property, including governmental or collective forms of ownership where laws or social rules limit the individual's access to the resource. The living resources of the high seas on the contrary have a very long history of being common property, meaning that any member of the society who wanted to utilize the resource were free to do so. The only factors that influenced the number of participators were private benefits and costs.

In particular two kinds of questions regarding the use of common property resources have been raised by economists. Firstly, what will be the results of free access to the resource? Secondly, what would be the optimal utilization of the resource? The former involves questions of extinction, stock level, harvest rate, fishing effort, costs, benefits etc., and so also does the latter, but in addition other questions are raised: How to regulate the use of the resource, what will the rent be and, perhaps also, how to distribute the rent?

Studies of the economic use of fisheries resources have become numerous after the seminal article by the Canadian economist H. Scott Gordon (Gordon, 1954). Even long before that the Danish economist J. Warming wrote an article, unfortunately in Danish, on the same subject (Warming, 1911). An excellent review of the fisheries economics theory and literature is the article by Munro and Scott, 1985. The main result from the theory is that fishing effort should be limited by fiscal or other means, in order to gain a rent from the resource to the society.

Optimal harvesting

We are now returning to our multispecies model for a thorou study of the economic optimal harvesting of such an ecological system. In addition to the symbols already defined, the followi ones will be needed

δ = The social rate of discount.
p_i = Price per unit standardized harvest of species i.
c_i = Cost per unit standardized harvest of species i.
b_i = Net profit per unit standardized harvest of species i.
π_i = Total profit from harvesting species i.

Let the growth of the species of a general n-species model be described by the following system of differential equations

$$\dot{X}_i = G_i(X_1,\ldots\ldots,X_n) - y_i\ , \qquad i = 1,\ldots,n. \qquad (7.1)$$

The following properties of the price, cost and profit functions will be assumed:

$$p_i = \text{constant}\ , \qquad i = 1,\ldots,n \qquad (7.2)$$

$$c_i = c_i(X_i)\ , \qquad i = 1,\ldots,n \qquad (7.3)$$

$$b_i(X_i) = p_i - c_i(X_i)\ , \qquad i = 1,\ldots,n \qquad (7.4)$$

$$\pi_i = \pi_i(y_i,X_i) = b_i(X_i)y_i\ , \qquad i = 1,\ldots,n. \qquad (7.5)$$

The optimal harvesting of the ecosystem is assumed to be equivalent to maximizing the objective functional

$$J = \int_0^\infty e^{-\delta t}(\sum_i b_i(X_i)y_i)dt \qquad (7.6)$$

This is done subject to the state equations (7.1), as well as the usual constraints, including the control variables

$$0 \le y_i \le y_i^{max}\ , \qquad i = 1,\ldots,n \qquad (7.7$$

The following procedure for solving this optimization problem i the same as the one used by Clark (1976, ch. 9), for a two-species model.

Neglecting the abnormal case, the Hamiltonian is

$$H = e^{-\delta t}\left(\sum_i b_i(X_i)y_i\right) + \sum_i \lambda_i(G_i(X_1,\ldots,X_n) - y_i) . \qquad (7.8)$$

The λ_i are the adjoint variables. First we consider the case of "multi-singular" control, where the coefficients of the control variables y_i vanish identically

$$\frac{\partial H}{\partial y_i} = 0 \qquad i = 1,\ldots,n \qquad (7.9)$$

$$\lambda_i = e^{-\delta t}\, b_i(X_i) \qquad i = 1,\ldots,n \qquad (7.10)$$

$$\dot{\lambda}_i = -\,\delta e^{-\delta t}\, b_i(X_i) . \qquad i = 1,\ldots,n . \qquad (7.11)$$

The adjoint equations are

$$\dot{\lambda}_i = -\frac{\partial H}{\partial X_i} = -e^{-\delta t}(b_i'(X_i)y_i) - \sum_j \lambda_j G_{ij} \qquad (7.12)$$

where

$$G_{ij} = \frac{\partial G_j(X_1,\ldots,X_n)}{\partial X_i} \quad \text{and} \quad b_i'(X_i) = \frac{db_i(X_i)}{dX_i} ,$$

$$i = 1,\ldots,n .$$

Substituting for λ_i from (7.10) into (7.12) and using $y_i = G_i(X_1,\ldots,X_n)$ in equilibrium (i.e. for $\dot{X}_i = 0$) we derive

$$\dot{\lambda}_i = -\,e^{-\delta t}\left(b_i'(X_i)G_i(X_1,\ldots,X_n) + \sum_j b_j(X_j)G_{ij}\right) , \qquad (7.13)$$

$$i = 1,\ldots,n .$$

Equating $\dot{\lambda}_i$ from (7.11) and (7.13) gives

$$\sum_j b_j(X_j)G_{ij} + b_i'(X_i)G_i(X_1,\ldots,X_n) = \delta b_i(X_i) , \qquad (7.14)$$

$$i = 1,\ldots,n .$$

Thus the case of "multi-singular" control in the n-species model corresponds to the case of singular control in the one-species model, known from Clark and Munro (1975):

$$b(X)G'(X) + b'(X)G(X) = b(X) \tag{7.15}$$

where $G'(X) = \frac{dG(X)}{dX}$. In particular equations (7.14) yield an optimal equilibrium $X_i = X_i^*$, $i = 1,\dots,n$. Essential to the multispecies model are the cross-dependencies, G_{ij}, which of course do not appear in the single species model. The economic interpretation of the result of the multispecies analysis is quite similar to that of the single species model. Dividing through (7.14) with $b_i(X_i)$ gives

$$G_{ii} + \sum_{j,j\neq i} \frac{b_j(X_j)G_{ij}}{b_i(X_i)} + \frac{b_i'(X_i)}{b_i(X_i)} G_i(X_1,\dots,X_n) = \delta , \qquad i = 1,\dots,n . \tag{7.16}$$

The lefthand side is species i's own rate of interest, which should equal the social rate of discount on the righthand side. The first two terms on the l.h.s. together form the instantaneous marginal product of the species. It consists of two parts, where the direct one (G_{ii}) is equivalent to the one in a single-species model. The second part is the indirect part of the instantaneous marginal product via other species. The last term on the l.h.s. is the marginal stock effect; that is, the cost-reducing effect an increase in the level of one species has on its own harvesting.

A common economic interpretation of the singular path of single species models can be generalized to cover the multispecies case. Let

$$\pi(X_1,\dots,X_n) = \sum_i b_i(X_i)G_i(X_1,\dots,X_n) . \tag{7.17}$$

π is the total sustainable rent associated with sustainable harvesting at given stock levels. Then

$$\frac{\partial\pi}{\partial X_i} = b_i'(X_i)G_i(X_1,\ldots,X_n) + \sum_j b_j(X_j)G_{ij} \qquad (7.18)$$

$$i = 1,\ldots,n ,$$

where $G_{ij} = \frac{\partial G_j(X_1,\ldots,X_n)}{\partial X_i}$. Now (7.16) can be written as

$$\frac{1}{\delta}\frac{\partial\pi}{\partial X_i} = b_i(X_i), \qquad i = 1,\ldots,n . \qquad (7.19)$$

The lefthand side is often referred to as the marginal user cost which is the loss in present value of sustainable harvesting when the capital asset, the fish stock, is reduced by one marginal unit. The righthand side is the net current value of harvesting the stock at the margin. In other words, optimal harvesting of the ecological system requires that, for each of the stocks, which can be thought of as the assets of the social manager's resouce portfolio, the present value of future losses from reducing the stock through harvesting should equal the current net benefits from that harvesting.

It is well-known from the analysis of single species models that an increase in the own price of fish has a negative effect on the otimal stock, whilst the effect of a cost increase is positive. These and other results of the single species analysis are shown in Table 7.1.

Table 7.1. Expected effects from parameter increases on optimal stock (W) and net present value (NPV) in the Schaefer model.

Parameter	Effect on	
	W	NPV
r	+	+
K	+	+
p	-	+
c	+	-
δ	-	-

The question is now - what will happen when there are two or more biologically interacting species in the ecological system? Since a n-species model usually is too complex for an analytical solution to be derived, we shall answer the question within the framework of a two species model.

To simplify the notation, let

$$\dot{X}_1 = f(X_1, X_2) \qquad (7.20)$$

$$\dot{X}_2 = g(X_1, X_2)\ . \qquad (7.21)$$

From (7.3) and (7.4) we have that the net profit per unit of harvest is a function of own price, effort cost and stock size:

$$b_i = b_i(p_i, c_i, X_i), \qquad i = 1,2\ . \qquad (7.22)$$

The partial derivatives of the unit profit functions have the following signs

$$b_{ip} = \frac{\partial b_i(\cdot)}{\partial p_i} > 0, \quad b_{ic} = \frac{\partial b_i(\cdot)}{\partial c_i} < 0, \quad b_{ix} = \frac{\partial b_i(\cdot)}{\partial x_i} > 0\ , \qquad i = 1,2\ . \qquad (7.23)$$

(7.23) tells that the two species are harvested cost-independent of each other and sold in separate markets.

Equations (7.16) and (7.17) implicitly give the state variables, the X_i's, as functions of the biological and the economic parameters. Differentiating (7.19) with respect to p_1, when $n = 2$, and rearranging somewhat gives:

$$\left(\frac{\partial^2\pi}{X_1{}^2} - \delta b_{1x}\right)\frac{\partial X_1}{\partial p_1} + \frac{\partial^2\pi}{\partial X_1 \partial X_2}\frac{\partial X_2}{\partial p_1} = \delta b_{1p} \qquad (7.24)$$

$$\frac{\partial^2\pi}{\partial X_2 \partial X_1}\frac{\partial X_1}{\partial p_1} + \left(\frac{\partial^2\pi}{\partial X_2^2} - \delta b_{2x}\right)\frac{\partial X_2}{\partial p_1} = 0\ . \qquad (7.25$$

Using Cramer's rule we find from (7.24) and (7.25):

$$\frac{\partial X_1}{\partial p_1} = \frac{\delta b_{1p}\left(\frac{\partial^2\pi}{\partial X_2^2} - \delta b_{2x}\right)}{|D|} \tag{7.26}$$

$$\frac{\partial X_2}{\partial p_1} = \frac{-\delta b_{1p} \frac{\partial^2\pi}{\partial X_2 \partial X_1}}{|D|} \tag{7.27}$$

$$\text{where } D = \begin{vmatrix} \frac{\partial^2\pi}{\partial X_1^2} - \delta b_{1x} & \frac{\partial^2\pi}{\partial X_1 \partial X_2} \\ \frac{\partial^2\pi}{\partial X_2 \partial X_1} & \frac{\partial^2\pi}{\partial X_2^2} - \delta b_{2x} \end{vmatrix} .$$

The second order conditions for the existence of an interior solution to the maximization problem in (7.6) are, for n = 2

$$\begin{vmatrix} \frac{\partial^2\pi}{\partial X_1^2} & \frac{\partial^2\pi}{\partial X_1 \partial X_2} \\ \frac{\partial^2\pi}{\partial X_2 \partial X_1} & \frac{\partial^2\pi}{\partial X_2^2} \end{vmatrix} > 0 \quad \text{and} \quad \frac{\partial^2\pi}{\partial X_1^2} < 0 ,$$

and from this it follows that $\partial^2\pi/\partial X_2^2 < 0$. The second order conditions imply $|D| > 0$ since $b_{ix} > 0$. Assuming the second order conditions are satisfied, it is seen from (7.26) that we have $\partial X_1/\partial p_1 < 0$, since $b_{1p} > 0$ and $b_{2x} > 0$. Thus, the effect on the optimal prey stock from an own price increase is negative. This result is the same as for the single species model. To determine the effect from the increased prey price on the predator stock Equation (7.27) shows that it is necessary to know the sign of $\partial^2\pi/\partial X_2 \partial X_1$. Using the growth functions (7.20) and (7.21) in (7.17) and differentiating twice gives

$$\frac{\partial^2\pi}{\partial X_1 \partial X_2} = b_{1x}f_2 + b_1 f_{21} + b_2 g_{21} + b_{2x}g_1 \tag{7.28}$$

$$\frac{\partial^2\pi}{\partial X_2 \partial X_1} = b_{1x}f_2 + b_1 f_{12} + b_2 g_{12} + b_{2x}g_1 , \tag{7.29}$$

where $f_i = \dfrac{\partial f(X_1,X_2)}{\partial X_i}$, $g_i = \dfrac{\partial g(X_1,X_2)}{\partial X_i}$, $f_{ij} = \dfrac{\partial f(X_1,X_2)}{\partial X_i \partial X_j}$ and $g_{ij} = \dfrac{\partial g(X_1,X_2)}{\partial X_i \partial X_j}$.

According to Young's theorem we have

$$\frac{\partial^2 \pi}{\partial X_1 \partial X_2} = \frac{\partial^2 \pi}{\partial X_2 \partial X_1} \; . \tag{7.30}$$

Using the explicitly formulated growth functions from the two species model, Equations (2.9') and (2.10'), the following partial derivatives of first and second order are derived:

$$\begin{array}{llll} f_1 & = r_1(1-2X_1-\nu X_2) & f_2 & = -\nu r_1 X_1 \\ g_1 & = r_2 X_2^2/X_1^2 & g_2 & = r_2(1-2X_2/X_1) \\ f_{11} & = -2r_1 & f_{12} & = -\nu r_1 \\ f_{21} & = -\nu r_1 & f_{22} & = 0 \\ g_{11} & = -2r_2 X_2^2/X_1^3 & g_{12} & = 2r_2 X_2/X_1^2 \\ g_{21} & = 2r_2 X_2/X_1^2 & g_{22} & = -2r_2/X_1 \; . \end{array} \tag{7.31}$$

Using (7.23) and (7.31) it is seen that the first two terms of (7.28) are both negative. The sign of the third one, $b_2 g_{21}$, depends on whether the predator is valuable enough to be harvested at a positive net profit or if the harvesting costs exceed the proceeds. In the latter case, i.e. when $b_2 < 0$, the optimal solution requires payment of a bounty to the harvester to compensate his loss from harvesting at a low stock level. When $b_2 \ll 0$ at the optimum, the predator shall be called a "nuisance". Even though the last term, $b_{2X} g_1$, is positive, it is likely that it is outweighted by the three others, so that $\partial^2\pi/\partial X_1 \partial X_2 < 0$, hence $\partial X_2/\partial p_1 > 0$. In other words, when the predator is a nuisance, an increase in the price of the prey will lead to a larger optimal stock of the predator. Such a price change initially augments the value of the resource capital of the prey proportionally to the price increase. However, it pays to transform some of the increased wealth into capital in general, rewarding the social manager with the interest expressed by the discount rate. This transformation may be controlled directly through harvesting of the prey, or indirectly by letting the predator harvest the prey. Hence, the effect of an increased price of the prey is an increased optimal stock of the predator. The investment in the predator stock is rewarded by increased revenues and reduced harvesting cost of this species.

The possibility of $\partial^2\pi/\partial X_1\partial X_2$ being positive does exist. To see under which conditions this is likely, we shall assume that the unit profit function is of the Schaefer type

$$b_i = p_i - \frac{c_i}{X_i}, \qquad i = 1,2 \tag{7.32}$$

where p_i is the price per unit of standardized harvest and c_i is the cost per unit of "fishing effort".[1] Using (7.23), (7.31) and (7.32) and inserting into (7.28) gives the following result:

$$\frac{\partial^2\pi}{\partial X_1\partial X_2} = -\frac{c_1}{X_2^2}\nu r_1 X_1 - \left(p_1 - \frac{c_1}{X_1}\right)\nu r_1 + \left(p_2 - \frac{c_2}{X_2}\right)\frac{2r_2X_2}{X_1^2}$$

$$+ \frac{c_2}{X_2^2}\frac{r_2X_2^2}{X_1^2} = \frac{r_2X_2(2p_2 - c_2/X_2)}{X_1^2} - p_1\nu r_1 . \tag{7.33}$$

When $b_2 >> 0$ at the optimum, the predator will be called a "valuable". The likelihood of $\partial^2\pi/\partial X_1\partial X_2$ being positive is greater the more of a valuable the predator is, and the lower the price of the prey, p_1, is. The more inefficient the predator is as transmuter of the prey, that is the lower ν is, the more likely this result will occur. To summarize, if p_2 and r_2 are large enough and/or p_1, r_1, ν and c_2 small enough, it is possible to have $\partial X_2/\partial p_1 < 0$.

Having derived the partial derivatives of the optimal stocks with respect to the price of the prey, it is now easy

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[1] Precise definitions will be given below in Equations (8.1), (8.7) and (8.8).

to verify that the effects of a change in the effort cost of harvesting the prey are:

$$\frac{\partial X_1}{\partial c_1} = \frac{\delta b_{1c}\left(\frac{\partial^2 \pi}{X_2^2} - \delta b_{2x}\right)}{|D|} \tag{7.34}$$

$$\frac{\partial X_2}{\partial c_1} = \frac{-\delta b_{1c} \frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|} \quad . \tag{7.35}$$

Since b_{1c} and b_{1p} have opposite signs, the sign of (7.34) and (7.35) must be opposite to those of (7.26) and (7.27). In other words, the effect on the optimal prey stock of an increase in own effort cost is positive, whilst the effect on the predator stock depends on whether the predator is a nuisance or a valuable.

The effect on the optimal stocks from a change in the price of the predator is found by differentiating (7.19) with respect to p_2, for $n = 2$, and solving the equations for $\partial X_1/\partial p_2$ and $\partial X_2/\partial p_2$:

$$\frac{\partial X_1}{\partial p_2} = \frac{-\delta b_{2p} \frac{\partial^2 \pi}{\partial X_1 \partial X_2}}{|D|} \tag{7.36}$$

$$\frac{\partial X_2}{\partial p_2} = \frac{\delta b_{2p}\left(\frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x}\right)}{|D|} \quad . \tag{7.37}$$

Since $b_{2p} > 0$, $b_{1x} > 0$ and $\partial^2\pi/X_1^2 < 0$ we always have $\partial X_2/\partial p_2 < 0$. The optimal predator stock is a decreasing function of its own price. Comparing $\partial X_1/\partial p_2$ in (7.36) to $\partial X_2/\partial p_1$ in (7.27) it is seen that they will always have the same sign, positive or negative, depending on whether $\partial^2\pi/\partial X_1 \partial X_2$ 1) is negative or positive, respectively. According to the previous discussion on the sign

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1) Recall $\frac{\partial^2 \pi}{\partial X_1 \partial X_2} = \frac{\partial^2 \pi}{\partial X_2 \partial X_1}$.

of $\partial^2\pi/\partial X_1\partial X_2$, it is most likely that if the predator is a nuisance a rise in the predator price causes a rise in the optimal prey stock ($\partial X_1/\partial p_2 > 0$). In this case the reduction in the optimal predator stock leaves more of the prey to be harvested by man at a lower unit harvesting cost caused by the increased prey stock.

On the other hand, if the predator is a valuable and the main value of the prey is as feed for the predator, we have seen that $\partial X_2/\partial p_1 < 0$, hence $\partial X_1/\partial p_2 < 0$. When the predator is the valuable resource, the optimal prey stock decreases as a result of an increase in the predator price. In this case the optimal prey and predator stocks move in the same direction.

The effects on optimal stocks from changes in the predator's effort cost are:

$$\frac{\partial X_1}{\partial c_2} = \frac{-\delta b_{2c} \dfrac{\partial^2\pi}{\partial X_1 \partial X_2}}{|D|} \tag{7.38}$$

$$\frac{\partial X_2}{\partial c_2} = \frac{\delta b_{2c} \left(\dfrac{\partial^2\pi}{\partial X_1^2} - \delta b_{1x}\right)}{|D|} \quad . \tag{7.39}$$

Comparing (7.36) and (7.38) it is seen that $\partial X_1/\partial p_2$ and $\partial X_1/\partial c_2$ have opposite signs since b_{2p} and b_{2c} are opposite in sign. This is also the case for $\partial X_2/\partial p_2$ and $\partial X_2/\partial c_2$ as seen from (7.37) and (7.39).

To see how changes in the discount rate affect the optimal stock levels, we may use the same method as used for studying price changes. Differentiating (7.19) with respect to δ, for $n = 2$, and solving the equations for $\partial X_1/\partial\delta$ and $\partial X_2/\partial\delta$ gives these results:

$$\frac{\partial X_1}{\partial \delta} = \frac{b_1 \left(\dfrac{\partial^2\pi}{\partial X_2^2} - \delta b_{2x}\right) - b_2 \dfrac{\partial^2\pi}{\partial X_1 \partial X_2}}{|D|} \tag{7.40}$$

$$\frac{\partial X_2}{\partial} = \frac{b_2 \left(\frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x}\right) - b_1 \frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|} \quad . \tag{7.41}$$

Since $\partial^2\pi/\partial X_2^2 < 0$ and $b_{2x} > 0$, it is seen from (7.40) that when $\partial^2\pi/\partial X_1\partial X_1 < 0$ and $b_2 < 0$, $\partial X_1/\partial\delta$ is unambiguously negative. This is a likely result when the predator is a nuisance. The optimal prey stock is reduced by an increase in the discount rate, and this result is clearly a parallel to the single species case. The increased discount rate makes it more costly to keep a large stock, therefore a part of it is transmuted into capital in general, yielding rent as expressed by the discount rate.

From (7.41) it is seen that since, by assumption, $\partial^2\pi/\partial X_1^2 < 0$ and $b_{1x} > 0$, $\partial X_2/\partial\delta$ is positive if $\partial^2\pi/\partial X_1\partial X_2 < 0$ and $b_2 < 0$.[1] In the case where the predator is a nuisance, Equation (7.41) states the optimal management strategy is to increase this stock when the discount rate is increased. The larger predator stock helps reduce the prey to its lower optimal stock level caused by the increased discount rate. As noted above, a rise in the discount rate makes it more costly to keep a large prey stock, therefore a part of it is transmuted into capital in general. Another part is transmuted into predator resource capital, thus the losses from harvesting the predator are reduced because of the lowered unit harvesting cost and/or increased revenues.

When the predator is a valuable, that is if $b_2 \gg 0$ at the optimum, the possibility of $\partial X_2/\partial\delta$ being negative emerges. In this case both types of resource capital, the predator and the prey stocks, are reduced by partial transformation into capital in general.

The possibility of simultaneously having $\partial^2\pi/\partial X_1\partial X_2 > 0$ and $b_2 < 0$, or vice versa, cannot be ruled out. Therefore, it may be possible to have $\partial X_1/\partial\delta > 0$. According to (7.33), the case with negative unit profit of the predator, combined with a

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[1] The possibility of having $\partial X/\partial\delta > 0$ for one species have been asserted in Hannesson, 1983a.

positive, but low unit profit of the prey and poorly reproducing prey might produce this result.

The joint harvesting of a predator-prey ecological system can give the traditional result of a rise in the discount rate, decreased optimal stocks, or the untraditional result of an increase in one of the stocks. The latter result is most likely in cases where the predator clearly is a nuisance to be harvested not (only) for the sake of its own value, but to increase the availability and the profitability of the prey harvest.

Having derived the effects on the optimal stocks from partial changes in prices and discount rate, it can now be shown that there are close relationships between these effects. From (7.26), (7.27) and (7.40) we find

$$\frac{\partial X_1}{\partial \delta} = \frac{\frac{b_1}{\delta b_{1p}} \left(\frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x}\right) \delta b_{1p} - \frac{b_2}{\delta b_{1p}} \frac{\partial^2 \pi}{\partial X_2 \partial X_1} \delta b_{1p}}{|D|}$$

$$= \frac{1}{\delta b_{1p}} \left(b_1 \frac{\partial X_1}{\partial p_1} + b_2 \frac{\partial X_2}{\partial p_1}\right) , \qquad (7.42)$$

since $\frac{\partial^2 \pi}{\partial X_1 \partial X_2} = \frac{\partial^2 \pi}{\partial X_2 \partial X_1}$.

From (7.36), (7.37) and (7.41) we find

$$\frac{\partial X_2}{\partial \delta} = \frac{\frac{b_2}{\delta b_{2p}} \delta b_{2p} \left(\frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x}\right) - \frac{b_1}{\delta b_{2p}} \delta b_{2p} \frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|}$$

$$= \frac{1}{\partial b_{2p}} \left(b_1 \frac{\partial X_1}{\partial p_2} + b_2 \frac{\partial X_2}{\partial p_2}\right) . \qquad (7.43)$$

As shown in connection with (7.26) and (7.27), $\partial X_1/\partial p_1$ and $\partial X_2/\partial p_1$ are negative and positive, respectively, when the predator is a nuisance. In this case (7.42) shows that $\partial X_1/\partial\delta$ unambiguously is negative. In connection with (7.36) and (7.37) it has been shown that $\partial X_1/\partial p_2$ is positive and $\partial X_2/\partial p_2$ is negative when the predator is a nuisance. From (7.43) it now follows that $\partial X_2/\partial\delta$ unambiguously is positive in this case. This result, that the optimal resource stock may increase with increased discount rate, is contrary to received wisdom from single species models.

Having derived some analytical results of the two species model, we now proceed with the three species model which will be applied to the Barents Sea fisheries in the next section. Rewriting Equation (7.16) in the three species case gives:

$$G_{11} + \frac{b_2(X_2)}{b_1(X_1)} G_{12} + \frac{b_3(X_2)}{b_1(X_1)} G_{13} + \frac{b_1'(X_1)}{b_1(X_1)} G_1(X_1) = \delta \qquad (7.44)$$

$$\frac{b_1(X_1)}{b_2(X_2)} G_{21} + G_{22} + \frac{b_3(X_3)}{b_2(X_2)} G_{23} + \frac{b_2'(X_2)}{b_2(X_2)} G_2(X_2) = \delta \qquad (7.45)$$

$$\frac{b_1(X_1)}{b_3(X_3)} G_{31} + \frac{b_2(X_2)}{b_3(X_3)} G_{32} + G_{33} + \frac{b_3'(X_3)}{b_3(X_3)} G_3(X_3) = \delta\,. \qquad (7.46)$$

Equations (7.44)-(7.46) implicitly determine the three state variables $X_1 = X_1^*$, $X_2 = X_2^*$ and $X_3 = X_3^*$ as functions of the biological parameters in the ecological model (3.6)-(3.8) and of the economic parameters in the price and cost functions (7.2)-(7.3). In this respect the three species model gives the same results as does the single species model: the optimal solutions of the state variables are independent of time. Once reached, the optimal combination of stock levels should be kept constant and the harvesting accordingly take place on a sustainable yield basis. The optimal sustainable harvest rates are implicitly given by the growth Equations (3.6)-(3.8) since the net growth of the stocks identically equals zero in equilibrium.

Because of the relatively complex dynamic properties of the model it has not been possible to derive any explicit solution for the optimal stocks from Equations (7.44)-(7.46). Therefore it has been necessary to design a computer program to find the optimal solution for a given set of biological and economic parameters. The program OPT is shown in Appendix 11.

If the maximization problem does not have an interior solution, for example because one of the constraints on the harvest rates in (7.7) becomes binding, Equations (7.44)-(7.46) do not give the solution to this constrained maximization problem.[1] In the long run there are few reasons to believe that the upper constraints should be binding in our case since the harvesting capacity can be expanded by building more vessels.[2] Since negative harvest rates have no biological meaning, the lower constraints in (7.7) might well become binding. If, for example, one of the three species has a sufficiently low price or high enough harvesting cost, it may be that the optimal solution implies no harvesting of that particular species.

Corner solutions can be checked for by use of the current value Hamiltonian and the necessary conditions for maximum. The current value Hamiltonian is defined by

$$H^o = e^{\delta t}H = \sum_i b_i(X_i)y_i + e^{\delta t} \sum_i \lambda_i(G_i(X_1,X_2,X_3) - y_i) , \quad (7.47)$$

and the current value adjoint variables by

$$\mu_i = e^{\delta t} \lambda_i , \qquad i = 1,2,3 . \quad (7.48)$$

If $y_1 = 0$ becomes binding, Equation (7.44) has to be replaced by

$$G_1(X_1,X_2,X_3) = 0 . \quad (7.49)$$

The optimal stock levels can now be found from equations (7.43), (7.46) and (7.49). To test whether $y_1 = 0$ belongs to the optimal solution we have to check if

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[1] Constrained optimal control theory can be found in Kamien and Schwartz (1981).

[2] For a discussion on problems involved when the upper constraint becomes binding, see Clark and Munro (1975).

$$\mu_1 \geq b_1(X_1^*) \tag{7.50}$$

where X_1^*, together with X_2^* and X_3^*, are found from (7.45), (7.46) and (7.49). It can be shown (Seierstad and Sydsæter, 1987) that $\mu_i(t)$ satisfies

$$\dot{\mu}_i = -\frac{\partial H^o(\cdot)}{\partial X_i} + \delta\, \mu_i\ . \tag{7.51}$$

In equilibrium, that is when $\dot{\mu}_i = 0$, this implies for $i = 1$

$$\mu_1 = (1/\delta) \sum_j \mu_j G_{1j}(X_1, X_2, X_3) \tag{7.52}$$

since $y_1 = 0$.

We can now solve for μ_1 and perform the test in (7.50).

So far we have been concerned about the optimal equilibrium solution (X_1^*, X_2^*, X_3^*). The problem of determining the best way of moving from an initial point (X_1^o, X_2^o, X_3^o) to the aforementioned optimal one still remains to be solved. However, this seems to be a formidable piece of work to be left to the mathematicians. Even the case of two species is very difficult to handle in a general way (cf. Clark, 1976, Ch. 9.3). His conclusion on "practical approach paths" for a two species system is

> Even the simplistic rule
>
> $$h_1 = \begin{cases} h_{max} & \text{if } X > X^* \\ 0 & \text{if } X < X^* \end{cases}$$
>
> similar for h_2, although suboptimal is surely a practically acceptable approach. The conclusion: if you know where you want to be and if many feasible approach paths are available, do not be concerned if the ideal path is not apparent. (Clark, 1976, p. 323).

Open access harvesting

In the case of open access harvesting it is well known from the fisheries economic literature that the resource rent will be dissipated. When all species in an ecosystem are common property resources, the equilibrium harvesting will take place such that none of the fisheries are rent yielding.[1] In addition to lack of incentives to invest in the stock the individual fisherman also does not have incentives to consider the interdependencies of species. Necessary conditions for bioeconomic equilibrium are therefore found by using Equations (7.4) and (7.5):

$$\pi_i = (p_i - c_i(X_i))y_i = 0 \ , \qquad (i = 1,\ldots,n) \ . \qquad (7.53)$$

Unless $y_i = 0$, which will be the case when $p_i < c_i(X_i)$, we must have

$$p_i = c_i(X_i) \ , \qquad (i = 1,\ldots,n) \ , \qquad (7.54)$$

in equilibrium.

With the simple price and cost functions we have assumed, Equation (7.54) implicitly gives the open access equilibrium stocks as functions of own price and cost:

$$X_i^{\infty} = X_i(p_i,c_i) \ , \qquad i = 1,\ldots,n \ . \qquad (7.55)$$

This is to say that each of the stock levels only depends on own price and harvesting costs. Substituting from (7.55) into (7.1) gives the harvest rates in open access:

$$y_i = G_i(X_1(p_1,c_1),\ldots,X_n(p_n,c_n)) \ , \qquad i = 1,\ldots,n. \qquad (7.56)$$

The open access harvest rates depend on all prices and effort costs, whilst the stock levels only depend on own price and effort cost. This is in case of an interior solution with all three species being harvested.

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[1] However, the possibility of having a consumer's surplus and/or a producer's surplus does exist. See Copes, 1972.

Whether the equilibrium point in (7.53) exists or not and whether it is stable or not are questions that in principle can be answered.

With stocks and harvest rates known we can use the tests in Appendix 8 to check if equilibrium and stability conditions are met.

In the case where

$$p_i < c_i(X_i^{\infty}) \tag{7.57}$$

for one of the three species, the open access fishery implies no harvesting of this species. Equilibrium stocks and harvest rates can still be found, but the procedure will be somewhat different from the case where the conditions in (7.54) are met for all i = 1,...,n. Let us have a closer look at the three species model and assume that harvest price and costs for species 2, cod, are such that (7.57) is fulfilled in equilibrium. To find the equilibrium stocks and harvest rates it is necessary to specify the cost functions $c_i(X_i)$. Assuming pure cod fish technology[1] for all three species the cost functions are

$$c_i(X_i) = c_i/X_i \ , \tag{7.58}$$

where c_i is the unit harvesting cost for catching the last fish of the stock and is assumed to be constant. Using (7.58) and (7.54) the equilibrium stocks of capelin and sea mammals are found:

$$X_i^{\infty} = c_i/p_i \qquad\qquad (i = 1,3) \ . \tag{7.59}$$

In addition to X_1^{∞} and X_3^{∞} we also know that

$$A_2 = 1 - F_2 = 1 \qquad . \tag{7.60}$$

in equilibrium with no harvest of cod. By simple arithmetic the

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[1] We shall return to the concept of cod fish technology in Section 8.

three species model in (3.6)-(3.8) gives the following equilibrium solutions for X_2, F_1 and F_3, with X_1, X_3 and F_2 now known:

$$X_2^\infty = (A_1 - \gamma_{23}X_3^\infty)X_1^\infty \tag{7.61}$$

$$A_1 = X_1^\infty + \gamma_{12}(A_2 - \gamma_{23}X_3^\infty)X_1^\infty + \gamma_{13}X_3^\infty \tag{7.62}$$

$$A_3 = \alpha X_3^\infty/(X_1^\infty + \alpha(A_2 - \gamma_{23}X_3^\infty)X_1^\infty \;. \tag{7.63}$$

This is an example where the open-access stock size of one of the species, cod, does not depend on own price and effort cost, but on stock sizes of the two other species, capelin and sea mammals, as shown in Equation (7.61). Substituting A_i and X_i $(i = 1,2,3)$ from (7.59)-(7.63) into the harvest functions (3.5), and remembering that $A_i = 1-F_i$, the open-access equilibrium harvest rates are found for this special case.

In the next section are derived the economic parameters necessary for application of the three species bioeconomic model to the Barents Sea fisheries. The optimal solutions will be presented and discussed in Section 9, and the open-access solution in Section 10.

8. ESTIMATING ECONOMIC PARAMETERS OF THE THREE SPECIES BARENTS SEA MODEL

The cod fisheries

Since 1968 the Directorate of Fisheries has conducted cost and income surveys for the Norwegian fleet of fishing vessels above 40 feet operating year round. In 1980 they also started undertaking such surveys for the smaller inshore boats.

In this study we use the average cod-fish vessel above 40 feet as the numeraire for fishing effort in the cod sector of the three species model. The average cod-fish vessel is calculated from a very heterogeneous group which consists of inshore and near-shore vessels as well as large sea-going freezing trawlers. The latter are, according to usual profitability measures, the most profitable vessels in the Norwegian fishing fleet for demersal species. Therefore calculations will also be performed with the average freezing factory trawler as the numeraire.

The data needed for each class of vessels are

i) quantity and price of catch
ii) total costs (capital and operating costs, including labour).

In the short run analysis hull, engine, electronic equipment, gear etc. are fixed factors of production, while fuel, bait etc. are variable. A bioeconomic study is primarily a long run analysis; therefore the average vessel fully equipped and with all factors of production necessary for one year of operation will be used as the unit of fishing effort. In other words, the total fishing effort targeted at the stock can be varied by varying the number of standardized (average) vessel years.

Table 8.1 shows quantity, value and price figures for the years 1978-80 for the average vessel in the cod fisheries and for the average Norwegian factory vessel. Prices in the second column include the price support from the Government to vessels fishing for demersal species, except the factory vessels which

Table 8.1. Catch, price and cost data.[1] Cod fisheries[2] 1978-80.

Vessel group	Year	1	2	3	4	5	6
		Catch				Cost[5]	
		Quantity[4] (metric tons)	Price (1000 Nkr/ m.t.)	Net price (1000 Nkr/ m.t.)	Value (1000 Nkr)	Alternative A[6] (1000 Nkr)	Alternative B[6] (1000 Nkr)
Average Cod-vessel[3]	1978	464.5	2.4816	2.1915	1152.7	1337.9	1207.2
	1979	470.0	2.7023	2.2883	1270.1	1468.6	1350.3
	1980	420.7	3.1648	2.5039	1331.4	1533.5	1385.1
	average	451.7	2.7829	2.3279	1251.4	1446.7	1314.2
Factory vessel[7]	1978	2796.8	4.6743		13073.4	13874.4	13498.2
	1979	2917.8	5.2813		15409.7	14637.2	13329.0
	1980	2407.3	4.5306		10906.3	13717.3	12923.0
	average	2707.3	4.8287		13129.9	14076.3	13250.1

[1] Nominal prices.

[2] Distant water and coastal fisheries primarily for cod, haddock, saithe and redfish (wet fish trawlers and factory vessels included).

[3] The following sources have been used: Value and cost figures: Lønnsomhetsundersøkelser (1978), (1979) and (1980). Price: Calculated from Fiskeri Statistikk (1981), Table 10. Net price: Price support from the Government have been deducted from price (Kristoffersen and Flaaten, 1984). Quantity: Calculated by dividing value by price.

[4] Live weight.

[5] Including remuneration of labour (wage, share, skipper's share etc.).

[6] Alternative A: Includes replacement cost depreciation on vessel and calculated interest on owner's net capital. Alternative B: Includes depreciation on vessel based on historic cost, interest on owner's net capital is not included.

[7] Source for quantity and value: Årsberetning vedk. Norges fiskerier (1978) and (1980). Prices have been calculated (P = V/Q). Source for cost: Depreciation and interest on owner's capital are from Lønnsomhetsundersøkelser (1978), (1979) and (1980), and the rest are from Årsberetning vedk. Norges fiskerier (1978) and (1980).

are not eligible for such subsidies. The distribution of subsidies between "producers" and "consumers", which in this case means the fish harvesting and the processing industry, respectively, as usual depends on elasticity relations on the two sides. In this case we have assumed infinitely elastic demand for raw fish, so that all the subsidies go to the harvesting sector. 80-90% of the value of the Norwegian harvest of demersal species are exported to the world market where Norwegian producers are price takers for the majority of their products. This is the reason for using the assumption of inifinitely elastic demand for raw fish. Prices net of subsidies from the Government are shown in the third column in Table 8.1.

The cost of fishing effort can be calculated in many ways. There might be deviations between social and private costs, and each can be defined in different ways. For our purpose, a long run bioeconomic analysis, we would like to use social costs, including the opportunity cost of capital and labour. For labour costs we have chosen to use the actual remuneration of fishermen, including lay income, wage, skipper's lay, etc. Alternative A in Table 8.1, column 5 assumes replacement-cost depreciation of the vessel, and includes calculated interest on owner's net capital. Alternative B in column 6 is based on the historic-cost of the vessel, and interest on owner's net capital is not included. For all years, the costs according to alternative A exceed the value of the catch for the average cod vessel as well as for the average factory vessel. Except for factory vessels in 1979, this is also the case for alternative B, which is the lower cost alternative. In open access fisheries with homogeneous vessels one would expect average cost of fishing effort to equal average revenues in bioeconomic equilibrium. In the case of Norwegian cod fisheries the fleet is definitely not homogeneous and this should increase the possibility of the existence of "producers surplus" or "intramarginal rent" in the fishery (Copes, 1972). However, in this case there is what we can call a small negative producers surplus, and we can think of several possible explanations for this:

i) There can be something wrong with the accounting surveys, either in the methods used or in the data the fishermen have reported.

ii) Fishing vessel owners also have revenues from other than the sale of catches; for example, interest on bank deposits and other financial incomes, skipper's lay, freight revenues, etc. In 1980, as a compensation for increased fuel prices, they received some Government subsidies which was not directly connected to actual cost or income.

iii) The fishery need not have been in bioeconomic equilibrium in the years 1978-1980. Overinvestment in the fleet in earlier years could have resulted in depreciation exceeding the difference between total proceeds and operating costs. After some years of negative investment in the fleet the open access fishery might reach bioeconomic equilibrium (Clark, Clarke and Munro, 1979).

Explanation i) is probably of minor importance in this case since the surveys have been conducted every year since 1968 and the methods used seem to be adequate. The results have been extensively used by Government branches, the fishing industry and researchers and no serious objections have been raised against the surveys. Regarding ii), the accounting surveys for the years 1978-80 reveal income other than from sale of catches equivalent to 2-6% of total revenues.

We shall now leave these problems and stick to the cost figures in Table 8.1.

The capelin fisheries

In Table 8.2, column 1, are shown average catches for all vessels in the capelin fleet and for large purse seiners with a licensed cargo capacity of 8,000 hectoliters or more. The figures are for the years 1978, 1979 and 1980 and for the average of all three years. Average prices paid to fishermen are shown in the second column, and the value of the catches in the third.

Table 8.2. Catch, price and cost data.[1] Capelin fisheries[2] 1978-80.

Vessel group	Year	1	2	3	4	5
		Catch			Cost[5]	
		Quantity[4] (metric tons)	Price (1000 Nkr/ m.t.)	Value (1000 Nkr)	Alternative A[6] (1000 Nkr)	Alternative B[6] (1000 Nkr)
Average Capelin-vessel	1978	4378.8	0.5011	2194.2	2941.5	2513.9
	1979	4927.0	0.4907	2417.9	3292.4	2789.3
	1980	4578.1	0.5935	2727.1	3494.8	2898.1
	average	4628.0	0.5284	2443.1	3242.9	2733.8
Large purse seiner[7]	1978	9863.1	0.5011	4942.4	6524.3	5500.6
	1979	9753.0	0.4907	4785.8	6765.2	5897.9
	1980	8942.9	0.5935	5307.6	6993.9	5754.2
	average	9519.7	0.5284	5011.9	6761.1	5717.6

[1] Nominal prices.

[2] Fishing with seine and trawl for herring, mackerel, capelin, sprat, Norway pout, sandeel, blue whiting and polar cod.

[3] The following sources have been used: Value and cost figures: Lønnsomhetsundersøkelser (1978), (1979) and (1980). Price: Calculated from Fiskeri Statistikk (1981), Table 10. Quantity: Calculated by dividing value by price.

[4] Live weight.

[5] Including remuneration of labour (wage, share, skipper's share etc.).

[6] Alternative A: Includes replacement-cost depreciation on vessel and calculated interest on owner's net capital. Alternative B: Includes depreciation on vessel based on historic-cost; interest on owner's net capital is not included.

[7] Value and cost figures are from: Lønnsomhetsundersøkelser (1978), (1979) and (1980). Price has been calculated from Fiskeri Statistikk (1981), Table 10. Quantity has been calculated (Q = V/P).

Governmental price support have been practically non existing in this part of the fishing industry. Average total cost per vessel have been calculated the same way as for the cod vessels. The results are shown in columns 4 and 5.

The average capelin vessel includes purse seiners between 100 and 220 feet long, with cargo capacity ranging from 150 to 1,500 metric tons, as well as trawlers between 70 and 130 feet long. Formally both purse seine and trawl fishing for capelin have been under licensing restrictions for many years; purse seining since 1973 and trawl fishing even longer (Fiskeridepartementet, 1983). In the former case, the licensing system has been an effective barrier to entry, whereas the licenses for trawl fishing for capelin, sandeel, Norway pout, etc. were more like a registration system until 1979 when the Government effectively stopped issuing new licenses.

In the years 1978-1980 limited entry by licensing has been combined with individual vessel quotas in the most valuable fishery: the capelin fishery in the Barents Sea and on the coast of Finnmark. In these years the capelin counted for 75-80% of the total value of the purse seiners' catch. The other 20-25% was mainly mackerel, herring and blue whiting. The use of limited entry schemes in the capelin fishery has given the participating vessels and crews a share in the resource rent. Without the introduction of licenses and quotas in the early 1970s, the capelin stock would probably have been heavily depleted due to entry of many vessels made redundant after the depletion of the rich herring and mackerel stocks (Norwegian Spring Spawning Herring, North Sea herring and North Sea mackerel) in the 1960s and early 1970s.1)

Norwegian limited entry schemes were mainly introduced in the capelin fishery to avoid collapse of the stock and not primarily to manage the fishery in an economically optimal manner. That would have meant far fewer, and bigger, vessels than those actually participating in the fishery (Flåm, 1980). It is easy

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1) The restrictions on entry combined with quotas obviously were not enough to avoid the depletion of the stock throughout the 1980s. A nearly collapse of the stock in 1985 therefore necessitated practically nil-quotas for 1986 and 1987.

to understand this from the fact that "whole year operated" purse seiners in 1978 and 1980 operated 34.5 and 29.5 weeks repsectively, whereas the average for all "whole year operated" vessels were 39.0 and 38.5 weeks. Fewer purse seiners with higher average operating time per year would have reduced the capital costs for the fleet. The cost data for the average capelin vessel presented in Table 8.2 do not take such things into consideration, but only presents actual reported costs for the years 1978-1980.

It can be seen from Table 8.2 that for the capelin fleet as well the total average cost exceeds average revenues not only in the case of alternative A but also for the lower cost alternative B. The reasons for this are much the same as mentioned for the cod fleet. Due to institutional arrangements in the management of the Norwegian capelin fishery the vessels are not allowed to utilize their full catching potential, and the redundancy rates in many cases are higher for the larger than for the smaller vessels. This result is mainly created through the allocation system for vessel quotas, and also by the regulated transportation of capelin from the fishing grounds to the reduction plants. The vessels get an individual quota which is an increasing, concave function of licensed cargo capacity, and the larger vessels are told by the Regulatory board to transport their catches to plants further away from fishing grounds than the smaller ones. These are probably the main reason why the large purse seiners have higher costs per ton caught than the average for all seiners.

Whaling and sealing

Whaling and sealing in Norway are typical seasonal activities conducted by multipurpose vessels. In this study, however, revenues and cost figures for vessel year equivalents (VYE) are needed, and recalculation from seasonal data will therefore be necessary.

Since the mid 1970s the only kind of whaling in Norway has been that of small whales in the North East Atlantic by small vessels (usually between 50 and 90 feet), while the harvesting

Table 8.3. Data on Norwegian whaling in the North Atlantic, 1978-80. (Nominal prices).

1	2	3	4			5				6		7	
Year	No. of vessels1)	Average length of vessels1) (feet)	Average catch per vessel			Whaling in % of total annual operation3)				Calculated VYE		Total cost per vessel4)	
			Quantity2) (m.t., Live weight)	Revenues1) (1000 Nkr)	Price (1000 Nkr/ton)	(i)	(ii)	(iii)	(iv)	Quantity (m.t. Live weight)	Revenues (1000 Nkr)	Alternative A (1000 Nkr)	Alternative B (1000 Nkr)
1978	87	67.0	64.0	249.81) (557.6)	3.903	44.8	35	34	38	168.5	657.4	670.3	574.8
1979	84	66.4	83.7	371.51) (740.4)	4.438	50.2	--	--	45	186.0	825.6	881.0	723.4
1980	89	68.6	79.5	400.21) (1042.9)	5.034	38.4	36	35	37	215.0	1081.6	1265.1	1076.4
Weighted average	86.7	67.3	75.7	340.6	4.499					199.0	854.8	942.0	794.5

1)Source: Småhvalfangsten, 1978; 1979; 1980. Revenue data in parenthesis are from Lønnsomhetsundersøkelser, 1978; 1979; 1980 (Vesselgroup no. 29/1978 and no. 28 for 1979 and 1980: "Smallwhaling") and include all fishing activities.

2)Calculated from "live weight" data in Flaaten (1984b).

3)(i) Revenues from whaling in % of total revenues, according to sources in note 1).

(ii)-(iii) According to sources in note 1): Time spent on harvest by "use of gun" and "whaling-operations", respectively, as % of total time spent on fishing and whaling.

(iv) "Average" of the figures in (i)-(iii); to be used in the calculation of catch of whale and revenues from whaling per vessel year equivalent (VYE) in column 6.

4)See note 6 in Table 8.1.

Table 8.4. Data on Norwegian sealing.*) 1978-80. (Nominal prices.)

1	2	3	4			5	6		7	
Year	No. of vessels[1]	Average vessel size[1] (feet)	Average catch per vessel[2]			Days at sealing in % of no. of days in operation	Calculated VYE		Cost per vessel[4]	
			Quantity[1] (m.t., live weight)	Value (1000 Nkr)	Price (1000 Nkr/ m.t.)		Quantity (m.t., live weight)	Value (1000 Nkr)	Alternative A (1000 Nkr)	Alternative B (1000 Nkr)
1978	19	118.8	304.8	501.5	1.645	18.5	1647.3	2710.3	4211.9	3546.9
1979	18	121.1	417.2	711.6	1.706	18.1	2304.0	3929.8	4558.6	3849.4
1980	15	125.7	405.0	686.1	1.694	24.7	1642.4	2783.2	4607.5	3883.8
Weighted average	17.3	121.6	372.6	627.5	1.684	20.2	1844.6	3106.4	4446.0	3748.8

*) Average figures for sealing on three main catching areas referred to in the text.

[1] Source: Selfangsten, 1978; 1979; 1980.

[2] Calculated from sources in note 1.

[3] Sources: No. of days at sealing are from Selfangsten, 1978; 1979; 1980, and total no. of days in operation are from Lønnsomhetsundersøkelsene (Vesselgroup no. 21/1978 and no. 22/1979 and 1980: Purse seiners 4000-5999 hectoliter licensed cargo capacity).

[4] See note 6 in Table 8.1.

of the larger baleen whales (fin, sei, humpback etc.) came to an end in the early 1970s due to low profitability in the industry.

In Table 8.3 are shown data for Norwegian whaling in the North Atlantic for the years 1978-80. The quantity figures shown are live weight, and prices are calculated per unit of live weight. The VYE data in column 6 tell what quantity, and the corresponding value, a vessel operating year round would have caught _if_ the availability of whales and other factors affecting the productivity had been the same for the whole year as for the actual harvesting season.[1)]

In Table 8.4 are shown similar data for Norwegian sealing in the North Atlantic areas. Sealing has usually taken place on three different catching grounds: Newfoundland, the West Ice (Jan Mayen - Greenland area) and the East Ice (The White Sea and the Barents Sea area). The average size of sealing vessels has been largest on the Newfoundland grounds, while the East-Ice vessels have been the smallest. Due to lack of specific accounting surveys for the sealing industry, we have chosen to let the cost side be represented by purse seiners of 4000-5999 hectoliter cargo capacity. Discussions with industry representatives revealed that this vessel group can be used as a proxy for the average sealer regarding size, catching capacity and total costs.

As seen before for the capelin and the codfish industries, revenues were usually exceeded somewhat by total costs. This is also the case for the sealing industry where the costs (alternatives A and B) exceed calculated VYE revenues in seven out of eight possible comparisons between columns 6 and 7 in Table 8.4. In addition to the possible explanations for this as discussed in the preceeding sections, it is possible that we have made a mistake in choice of proxy for the cost side of the sealing industry. However, having discussed this matter with the industry representatives, it seems most likely that the cost data in Table 8.4 are not far from the truth. The economic situation in the

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[1)] VYE are used in an attempt to keep the applied part of the analysis as simple as possible and in accordance with the theoretical part. Seasonal variations in economic, biological or technical factors, e.g. the catchability coefficient, of course could have been introduced and this would have made the analysis more realistic, but on the cost of simplicity (see Flaaten, 1983; and Flaaten 1987b).

Table 8.5. Data for a designed general purpose sea mammal harvesting vessel. 1978-80.

1	2			3		4	5
	Catch per vessel			Total cost per vessel[2]		Total annual catch of sea mammals[3] (m.t., Live weight)	No. of SM vessels (VYE)
	Quantity[1] (m.t., Live weight)	Value (1000 Nkr)	Price (1000 Nkr/ton)	Alternative A (1000 Nkr)	Alternative B (1000 Nkr)		
Average 1978-80	322.4	1032.6	3.203	1222.3	1030.8	9111	30.7

[1] 1 ton of SM consists of 54% whale and 46% seal.

[2] See note 6 in Table 8.1.

[3] Includes all catches of sea mammals in the Barents Sea and adjacent areas.

sealing industry has been gradually worsening during the 1970s, due to reduced quotas and declining prices (in real terms) for industry products. Unlike most parts of the Norwegian fishing industry, the sealing industry rarely received subsidies from the Government until 1982 when economic conditions were so severe that the whole harvesting was threatened to come to an end. For this reason the discrepancies between revenue and cost data in Table 8.4 might be reasonable, and not unexpected.

Vessels in the Norwegian whaling fleet are usually only 50-90 feet long, whereas the sealers are 100-160 feet. Nevertheless we shall design a standardized vessel for sea mammal harvesting to suit the three species model. The ratio between the average total catch of whales and the average total catch of seal biomass in the period 1978-80 is 1.17. Requiring one unit of the sea mammal (SM) vessel to catch whales and seals in the same ratio, and by using the VYE figures from Tables 8.3 and 8.4, we find that a SM vessel should consist of 92% of a VYE whaler and 8% of a VYE sealer. The data for such a multipurpose SM harvesting vessel are shown in Table 8.5. Dividing the average annual total catch of whales and seals in column 4 by the catching capacity in column 2, we get the number of constructed SM vessels (measured in VYE) in column 5 which would have been necessary to harvest the actual catch of whale and seal biomass. It should also be noticed that using cost alternative B the vessel's harvest revenues would just have exceeded the costs.

Standardized prices and costs

Normalized prices and costs in Equations (7.2)-(7.5) can be found by calculations from ordinary revenue and cost survey generated data. The following price symbols will be used:

p_i^o = Price per unit harvest (i.e., per unit of h_i)

p_i = Price per unit standardized harvest (i.e., per unit of y_i).

The prices are related in the following way (when substituting for $y_i = h_i/K_i$):

$$p_i = \frac{p_i^o h_i}{y_i} = p_i^o K_i \tag{8.1}$$

where the K_i are defined as: $K_i = \begin{cases} K & i = 1 \\ \alpha K & i = 2 \\ \alpha\beta K & i = 3 \end{cases}$.

Having defined the standardized prices, we now proceed with the concept of standardized unit harvesting cost. For this purpose, however, it will be necessary to specify the production function of the fishery.

The production function in equation (3.5) has been used in both theoretical and empirical fisheries economic studies. Recent empirical research has proved that the function is suitable for use in studies of demersal fisheries (Hannesson, 1983b and Shrank et al., 1984). On the other hand, studies of pelagic, schooling species have revealed that such a function overstates the effect on harvest rate from the stock size (Bjørndal, 1985 and Ulltang, 1976). A more general formulation of the production function could be the Cobb-Douglas type:

$$h_i = r_i F_i^{\sigma_i} W_i^{\gamma_i}, \qquad (i = 1,2,3) . \tag{8.2}$$

Let us refer to the case of $\sigma = \gamma = 1$ as the pure cod-fish technology, whereas the case of $\sigma = 1$ and $\gamma = 0$ is the pure pelagic fish technology. For pelagic species (herring), the studies referred to found estimates on stock output elasticities, γ, varying from close to zero (Ulltang, 1976) to 0.13 and 0.34[1] (Bjørndal, 1985). The effect output elasticities, σ, are usually not significantly different from unity, except in the study by Bjørndal where the estimates are 0.68 and 0.69[2]

---

[1] Bjørndal (1985) used four different specifications of the production function. The results quoted here are from the models which seem to have been most successful in interpreting the data. The additional two models, A1 and B1 in his study, gave stock output elasticities of 0.62 and 0.19, respectively.

[2] The concept of fishing effort is more complex in Bjørndal (1985) than here. He uses "boat-days", "number of boats" and "size of boats" as three independent factors of production. The elasticities quoted here are those with respect to number of boats.

for the two most successful formulations of the production functions. A priori it would be reasonable to expect that there are few crowding externalities taking place in the Barents Sea fisheries; that is, we should not expect $\sigma < 1$. On the other hand, if the vessels in one fishery were jointly operated by one owner, or if they otherwise cooperated in fish searching etc., it could be that the effort output elasticity is greater than one. However, the Norwegian vessels are usually owned by independent fishermen or small companies, and the sharing of fishing information only takes place occasionally. For these reasons we shall simplify the study by assuming that $\sigma = 1$ for fishing on all three species. On the other hand we shall distinguish between the cases of pure cod fish technology ($\gamma = \sigma = 1$) and that of pure pelagic fish technology ($\gamma = 0$ & $\sigma = 1$).

The symbols of costs and related parameters for species i are:

c_i^o = Cost per vessel year equivalent (VYE)
$\bar{c}_i$ = Cost per unit of fishing effort, F_i
TC_i = Total costs per year
N_i = Number of VYE
k_i = Catch per VYE
$c_i(W_i)$ = Cost per unit of harvest
$c_i(X_i)$ = Cost per unit of standardized harvest.

The definitions of and relations between different cost concepts are:

$$TC_i = c_i^o N_i = \bar{c}_i F_i \quad . \tag{8.3}$$

The catch per VYE is:

$$k_i = h_i/N_i \quad . \tag{8.4}$$

For the unit harvesting cost we shall distinguish between the case of pure cod fish technology and that of pure pelagic fish technology.

Pure cod fish technology

In this case $\gamma = \sigma = 1$ in the harvest function (8.2). By using vessel year equivalent (VYE) as a measure of fishing effort, we substitute (8.5) into (8.2).

$$q_i N_i = r_i F_i \tag{8.5}$$

q_i is the catchability coefficient. This gives the following production function:

$$h_i = q_i N_i W_i \ . \tag{8.6}$$

(8.3) and (8.6) give the unit harvesting cost

$$c_i(W_i) = TC_i/h_i = c_i/W_i \ , \tag{8.7}$$

where

$$c_i = c_i^o W_i^o / k_i^o \ . \tag{8.8}$$

W_i^o and k_i^o are stock size and catch per VYE, respectively, both related to a specific point in time (actually, a year or another short period, during which the stock can be treated as if it were constant). c_i is the unit harvesting cost for $W = 1$; in other words, the unit harvesting cost for catching the last fish in the stock. The unit harvesting cost for standardized harvest is

$$c_i(X_i) = TC_i/y_i = c_i/X_i \tag{8.9}$$

where

$$y_i = h_i/K_i \ . \tag{8.10}$$

Pure pelagic fish technology

In this case we have $\sigma = 1$ and $\gamma = 0$ in the production function (8.2). Instead of (8.6) we now get

$$h_i = k_i N_i \, . \tag{8.11}$$

The unit harvesting cost in this case is constant:

$$c_i(W_i) = TC_i/h_i = c_i^o/k_i^o \; . \tag{8.12}$$

The unit harvesting cost for a standardized harvest is

$$c_i(X_i) = TC_i/y_i = c_i^o K_i/k_i^o \; . \tag{8.13}$$

From the economic data in Tables 8.1-8.5 we can now derive the economic parameters of the TSB-model, and the optimal stationary stock levels can be computed from Equations (7.42)-(7.44). Putting the economic data into the price and cost functions of the model we get the results shown in Table 8.6. Using the price of cod as numeraire gives price and cost data shown in Table 8.7.

Table 8.6. Basic price and cost parameters of the TSB-model. (The average of 1978-1980 data.)

i	p_i^o 4) (1000 Nkr/ m.t.)	c_i^o (1000 Nkr/VYE)		k_i^o (M.t./VYE)	W_i^o (Million m.t.)	c_i (million Nkr)	
		Alternative A	Alternative B			Alternative A	Alternative B
1 1)	0.5284	3242.9	2733.8	4628.0	6.320	4428.6	3733.2
2 2)	2.3279	1446.7	1314.2	451.7	2.893	9265.7	8417.2
3 3)	3.2029	1222.3	1030.8	322.4	0.853	3233.9	2727.3

1) Data for the average capelin fishing vessel.
2) Data for the average cod fishing vessel.
3) Data for a general purpose sea mammal vessel.
4) Prices net of government subsidies.

Table 8.7. Basic price and cost parameters of the TSB-model with the price of cod as numeraire. (The average of 1978-1980 data.)

i	p_i^o 4)	c_i^o (m.t./VYE)		c_i (1000 m.t.)	
		Alternative A	Alternative B	Alternative A	Alternative B
1 1)	0.2270	1393.1	1174.4	1902.4	1603.7
2 2)	1.0000	621.5	564.5	3980.3	3615.8
3 3)	1.3759	525.1	442.8	1389.2	1171.6

1)-4) See footnotes to Table 8.6.

The estimates of the biological parameters in Section 6 and the economic parameters in this section will be used in the next section to derive the optimal solutions of the TSB-model. The open access solutions will be presented and discussed in Section 10.

9. OPTIMAL SOLUTIONS OF THE THREE SPECIES BARENTS SEA MODEL

Equations (7.42)-(7.44) give the necessary conditions for an interior solution to the maximization problem in Equation (7.6). _If_ such a solution exists, it can be found by solving the three equations for the three state variables, the fish stocks. Unfortunately the non-linearities of Equations (7.42)-(7.44) are too complex for an analytical solution for $X_i{*}$ (i = 1,2,3) to be found. A simulation program, OPT (see Appendix 11), has been created to find numerical solutions for $X_i{*}$ (i = 1,2,3) from the three golden rule equations. The partial derivatives of the growth functions with respect to stocks are needed in this computation, and they are shown in Appendix 10.

With substitution of the basic case biological parameters in Table 6.12, the basic economic parameters in Table 8.6 and a discount rate of five percent into Equations (7.42)-(7.44), the computer program OPT (Appendix 11) generated the following optimal normalized stock levels:

$$
\begin{aligned}
X_1^* &= 0.5022 \\
X_2^* &= 0.2648 \qquad (9.1) \\
X_3^* &= 0.0138 \,.
\end{aligned}
$$

The corresponding equilibrium catch levels are found by substituting (9.1) into (7.1). Doing this reveals a negative catch rate for the capelin stock. According to the discussion in Section 7, the solution in (9.1) can not be an optimal solution since the lower bound on the capelin catch rate ($y_1 = 0$) becomes binding.

At this stage we avoid the constrained maximization problem by making a change in one of the economic parameters, hoping that this will give an interior solution to the original problem. Intuitively an increase in the price of capelin, ceteris paribus, can make the capelin fishery profitable enough to give such a solution. The average catch of capelin in the years 1978-80 was 1.86 million m.t. (see Appendix 4) and the average price was 0.5284 Nkr/kg (Table 8.6).[1] Between 90 and 99% of the capelin landings have been used for reduction to fish meal and oil during the 1970s and 1980s. The rest has been mostly used for fresh and frozen products and for roe production. Under the current system Norwegian fishermen's organizations have been granted the legal right to fix minimum prices on raw fish. Thus, they are in a position where, via price discrimination, they can charge a monopoly price in some markets (Hannesson, 1985). This is probably the main reason why capelin used for other purposes fetches significantly higher prices to the fishermen than capelin used for reduction into meal and oil. For 1978-80 quantities and prices of capelin for other purposes are shown in Appendix 12. The average price of capelin used for fresh, frozen and roe products was 1.503 Nkr/kg or three times higher than the average ex-vessel price of capelin for all uses.

Increasing the price of capelin by 50, 100 and 150% to 0.7926, 1.0568 and 1.3210 Nkr/kg respectively, and keeping all other biological and economic parameters equal to that in Tables 6.12 and 8.6 gives the optimal normalized stock levels and catch rates shown in Table 9.1. The corresponding figures in million tons are shown in Table 9.2.

Table 9.1. Optimal stocks and harvest rates in normalized units.[1]

p_1^o (1000 Nkr/m.t.)	x_1^*	x_2^*	x_3^*	y_1^*	y_2^*	y_3^*
[2]0.7926 (0.3405)	0.404155	0.212338	0.018729	0.012866	0.113080	0.001117
[2]1.0568 (0.4540)	0.361002	0.183105	0.021832	0.066389	0.100807	0.001290
[2]1.3210 (0.5675)	0.341609	0.162232	0.023262	0.095082	0.095179	0.001367

[1] Based on biological parameters in Table 6.12 and economic parameters in Table 8.6.

[2] Prices with price of cod as the numeraire are given in parenthesis.

---

[1] Remember that quantity refers to the total harvest from the Barents Sea area, while price refers to that of Norwegian landings.

$$NPV = (1/\delta) \sum_{i=1}^{3} (p_i - c_i/X_i^*)y_i^*$$

$$= (1/\delta) \sum_{i=1}^{3} (p_i^o - c_i/W_i^*)h_i^* = (1/\delta) \sum_{i=1}^{3} \pi_i^* . \qquad (9.4)$$

The definitions of p_i and c_i are found in (8.1) and (8,8), respectively. The relationships between the two types of harvest rates are

$$h_i^* = y_i^*K_i \qquad \text{with} \qquad K_i = \begin{cases} K & \text{for } i = 1 \\ \alpha K & \text{for } i = 2 \\ \alpha\beta K & \text{for } i = 3 \end{cases} . \qquad (9.5)$$

The computational results for the sustainable rent, π_i^*, and NPV are shown in Table 9.4.[1] The price of cod is used as numeraire. The table includes the extreme values of p_1^o to give simultaneous positive rent from capelin and cod.

Table 9.4. Sustainable rent, π_i^*, and net present value, NPV, of optimal harvesting in the case of interior solution. Thousand of cod units.[1]

p_1^o (in cod units)	π_1^*	π_2^*	π_3^*	NPV
0.3234	0	1208.368	-65.192	22865.8
0.3405	80.341	1127.580	-64.916	22860.1
0.4540	609.241	731.140	-63.494	25537.7
0.5675	1172.307	448.469	-62.752	31160.5
0.7429	2243.833	0	-62.601	43624.0

[1] One unit of cod is 1.0 metric ton.

So far in this section the results are based on the assumption that the solution to the optimization problem in (7.6) is an interior one, implicitly found from Equations (7.42)-(7.44).

---

[1] The computer program OPV designed to compute optimal sustainable rents and net present value is an extended version of the program OPT which compute the optimal stock levels. They are both shown in Appendix 11.

To verify that this solution is the optimal one, we need to perform the test described in Section 7. A closer look at the results in Table 9.4 reveals a curious result. The net present value of optimal harvesting is lower for $p_1^o = .3405$ than for $p_1^o = .3234$. A priori one would rather expect the opposite result, and therefore this makes us suspicious about the reliability of the findings in the low range of p_1^o. The alternative to simultaneous harvest of all three species would be to leave the cheap species, the capelin, in the sea as feed for the more valuable one, the cod.

The aforementioned test will reveal whether the interior or the corner solution is the optimal one. To illustrate this we shall compare the net present values of sustainable harvesting for the two alternatives. The net present value of the interior solution as a function of p_1^o is shown as NPV_1 in Figure 9.1. The corner solution, i.e. when $y_1 = 0$, is found from Equations

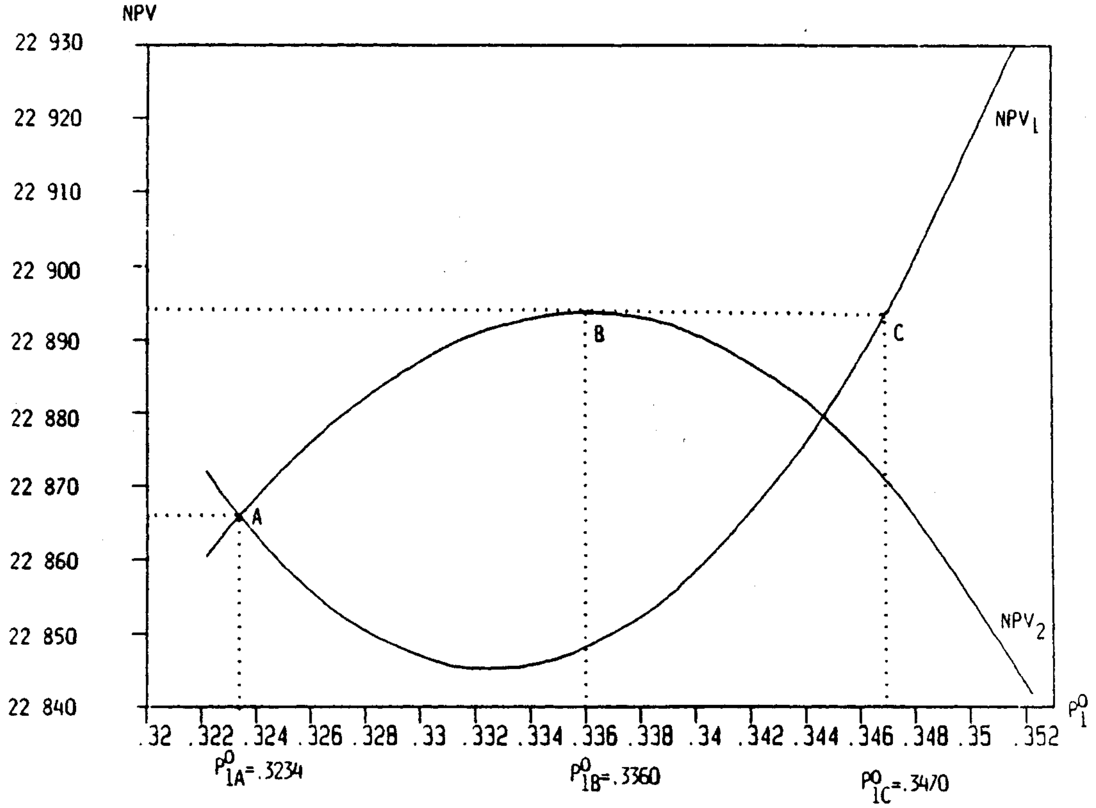

Figure 9.1. Net present value, in thousand units of cod, of interior solution (NPV_1) and the corner solution (NPV_2).

(7.43), (7.44) and (7.47). The net present value of this harvesting strategy as a function of p_1^o is shown as NPV_2 in Figure 9.1.

From Figure 9.1 we see that $NPV_1 = NPV_2$ at two points: when $p_1^o = 0.3447$ and when $p_1^o = 0.3234$. The latter one is the lower limit for the interior solution which will give positive harvest rates for all three species. For lower capelin prices the golden rule equations (7.42)-(7.44) give negative harvest rates of capelin, as is seen for $p_1^o = 0.2270$; and this has no biological meaning. Moving along the NPV_1 curve from A to the right it first decreases, then increases as a function of p_1^o. This is what the findings in Table 9.4 indicated. The NPV_2 curve has the opposite shape with a maximum at point B.

The complete solution to our maximization problem will be as follows. For capelin prices to the right of C the interior solution will be the optimal one, and the NPV_1 curve shows the level of the objective functional for the infinite horizon equilibrium harvests. For lower prices of capelin; that is, to the left of C in Figure 9.1, the optimal result will be at point B.1) Among all feasible combinations of harvest rates consistent with Equations (7.43), (7.44) and (7.47), there is one, from an economic point of view, which is better than the others and this corresponds to p_{1B}^o. Regardless of what the price of capelin is, as long as it is lower than p_{1C}^o, the optimal solution will be found as if the price equals p_{1B}^o.

Implementation of the optimal solution require use of indirect means like fees/subsidies on harvest or fishing effort, or direct means like quota and effort restrictions. Using a simple biomass model with a one dimensional fishing effort concept it can be shown that from an efficiency point of view it does not matter which of the means are used (Clark, 1980). However, the more complex the model is made, e.g. by extending to several year classes, geographical distribution, seasonal

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1) The corresponding stock levels are $X_1^* = 0.420009$, $X_2^* = 0.215913$ and $X_3^* = 0.017887$, and the harvest rates are $y_1 = 0$, $y_2 = 0.118006$ and $y_3 = 0.001069$. Stability tests of this equilibrium points give $x = 0.0341$ and $R = 9054$, which imply a locally stable equilibrium.

growth and migration, and multipurpose vessels and gear, the larger the number of means necessary in order to achieve the optimal result (Clark, 1980; Flaaten, 1983; Flaaten 1987a). For the TSB-model it is clear that the optimal stationary harvesting regime can be kept by use of just three regulatory means. Whether harvest fees/subsidies or effort fees/subsidies are used to deprive the fishermen for the resource rent, the results are the same: the harvesting sector gets paid its alternative cost and the government collects the rent.

The following symbols will be used in deriving the formulas for calculation of the fees/subsidies:

$\tilde{p}_i^o$ = Net price of species i received by the fishermen

$\tilde{c}_i^o$ = Cost of fishing effort paid by the fishermen

t_{pi} = Landing fee/subsidy on harvest

t_{ei} = Effort fee/subsidy.

Price and cost definitions are

$$\tilde{p}_i^o = p_i^o(1-t_{pi}) \tag{9.6}$$

$$\tilde{c}_i^o = \tilde{c}_i^o(1+t_{ei}) \ . \tag{9.7}$$

Simple arithmetic shows that the tax rates are found from the formulas:

$$t_{pi} = b_i^*/p_i \tag{9.8}$$

$$t_{ei} = b_i^*/(c_i/X_i^*) \ , \tag{9.9}$$

where $p_i = p_i^o K_i$ with K_i defined in connection with (8.1). b_i^* is the rent per unit of standardized harvest, X_i^* is the optimal stationary stock level and c_i, the cost of harvesting the last unit of the stock, is defined in (8.8).

In Table 9.5 are shown the optimal stationary harvest and effort fees/subsidies as a share of harvest price and effort cost, respectively.

Table 9.5. Optimal stationary fees and subsidies (-) as a share of harvest price (t_{pi}) and effort cost (t_{ei}) in the case of the interior solution.

p_1^o (in cod units)	t_{p1}	t_{p2}	t_{p3}	t_{e1}	t_{e2}	t_{e3}
0.3405[1]	0.6115	0.3693	-13.0323	1.5742	0.5856	-0.9287
0.4540	0.6738	0.2686	-11.0323	2.0658	0.3673	-0.9171
0.5675	0.7242	0.1746	-10.3169	2.6264	0.2115	-0.9116

[1] Based on the interior solution. As explained in the text the corner solution at B in Figure 9.1 will be the optimal solution for this price of capelin.

Table 9.5 clearly shows that capelin and cod should be taxed, the former more heavily than the latter, and that the tax on capelin should be higher the higher the price of capelin is. For cod the result is the opposite with respect to capelin price.

The necessity of heavily subsidization of the harvesting of sea mammals is evident from Table 9.5. Using price support would mean that the harvesters should receive ten to thirteen times as much subsidies as they get for their products in the market. With a low price of capelin, the optimal solution is to leave much of the capelin in the sea as feed for the cod and harvest the cod instead. Since the sea mammals' predation effect on cod is more significant than on capelin (see (6.21)), it is optimal to exploit the sea mammals most heavily when cod is expensive relative to capelin. That is why the optimal subsidies on sea mammals in Table 9.5 is highest for the lowest price on capelin.

Given that there exists an interior solution to our maximization problem it is seen that the optimal stocks are derived from Equations (7.42)-(7.44). The stocks are implicit functions of the biological and economic parameters. To find how sensitive the solution is with respect to changes in the parameters we have computed the elasticities[1] of the optimal stocks with respect to biological and economic parameters, defined as:

$$\check{W}_{ij}^{*} = \frac{\partial W_i^{*}/W_i^{*}}{\partial P(j)/P(j)} \quad , \tag{9.10}$$

where P(j) is parameter j. The results are shown in Table 9.6 for standardized prices of capelin set equal to 0.3405 and 0.5675. In the table are also shown the corresponding elasticities of net present value of the sustainable rent:

$$N\check{P}V_j = \frac{\partial NPV/NPV}{\partial P(j)/P(j)} \quad . \tag{9.11}$$

The results from the TSB-model in Table 9.6 show a positive effect on the stocks from increases in the own intrinsic growth rate. This is equivalent to that of the Schaefer model. Both for the low and the high capelin price will there be a negative impact on the sea mammal stock when r_1 increases. The effect from r_2, however, is dependent on the price of capelin. When capelin is expensive relative to cod, an increase in r_2 slightly increases the optimal stock of sea mammals. This is because of the sea mammals' relative stronger predation on cod than on capelin (see (6.21)) in the TSB-model. A larger stock of sea mammals means a net contribution to capelin harvest in this case.

Increased biological productivity of capelin and cod, via r_1 and r_2, has a significant positive impact on the net present value. The magnitude is, of course, dependent on the harvest prices. Due to the sea mammals being a top predator of relatively low value an increase in its intrinsic growth rate, r_3, has a negative, but small, impact on NPV. K and α have, like r_1 and r_2, a significant positive impact on NPV, whereas the

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[1] Elasticities were approximated by computing the effects of one percent increase in the parameters.

Table 9.6. Elasticities of optimal stocks and net present value of sustainable harvesting with respect to biological and economic parameters, for $p_1^o = 0.3405$. (For $p_1^o = 0.5675$ in brackets.[1]) Based on the interior solutions.

P(j) \ i	$\check{W}_{ij}^*$ 1	2	3	$\check{NPV}_j$
r_1	0.359 { 0.492 }	0.348 { 0.439}	-0.854 {-1.275}	1.738 { 2.329 }
r_2	0.214 { 0.070 }	0.097 { 0.030}	-0.310 {-1.127}	1.058 { 0.311 }
r_3	-0.001 {-0.002 }	-0.002 {-0.004}	0.123 { 0.159}	-0.063 {-0.046 }
K	0.546 { 0.739 }	0.174 { 0.013}	-0.618 {-0.769}	1.214 { 1.298 }
α	0.001 {-0.096 }	0.690 { 0.591}	-1.226 {-0.894}	1.165 { 0.232 }
β	0.0(+) { 0.0(+)}	0.001 { 0.001}	-0.020 {-0.034}	0.0(-) { 0.0(-)}
a_{12}	-0.540 {-0.522 }	-0.401 {-0.437}	1.084 { 1.334}	-1.666 {-1.543 }
a_{13}	-0.002 {-0.003 }	0.005 { 0.016}	-0.374 {-0.772}	-0.001 {-0.004 }
a_{23}	0.002 { 0.003 }	-0.004 {-0.015}	-0.667 {-0.278}	-0.002 { 0.0(+)}
p_1^o	-0.454 {-0.163 }	-0.516 {-0.579}	0.641 { 0.148}	0.067 { 1.078 }
p_2^o	0.540 { 0.424 }	0.093 { 0.029}	-2.285 {-2.186}	2.846 { 1.783 }
p_3^o	0.0(-) { 0.0(-)}	0.0(-) { 0.0(-)}	0.011 { 0.010}	0.004 { 0.004 }
c_1^o	0.187 { 0.121 }	0.205 { 0.161}	-0.262 {-0.123}	-0.019 {-0.281 }
c_2^o	-0.261 {-0.373 }	0.243 { 0.415}	0.876 { 1.115}	-1.760 {-1.487 }
c_3^o	-0.011 {-0.012 }	-0.019 {-0.023}	1.073 { 1.094}	-0.113 {-0.081 }
δ	-0.034 {-0.044 }	-0.040 {-0.028}	1.020 { 0.999}	-1.040 {-1.027 }

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[1] + or - sign in parenthesis after 0.0 indicates that the elasticity is of positive or negative magnitude, respectively, but close to zero.

effect from β is negligible. Changes in the sea mammals' related coefficients, r_3 and β, have minor effects on NPV because of the heavily depleted stock and low sea mammal yield at the optimum. That is also why changes in a_{13} and a_{23} have minor impacts on NPV. However, these preying coefficients' negative impact on the optimal sea mammal stock is significant. The more food the sea mammals need, measured by a_{13} and a_{23}, the lower the stock should be. Since both capelin and cod, directly or indirectly, are valuable species, an increase in the latter's consumption of the former has a great negative impact on the net present value. As a conclusion we can say that the NPV of optimal harvesting is rather sensitive to changes in biological parameters related to capelin and cod, but not very sensitive to changes in sea mammal related biological parameters due to this species' low optimal stock level.

With respect to an own price increase, capelin is seen, from Table 9.6, to be affected the same way as in a single species model. The optimal stock decreases and the net present value increases. This is also what we should expect from the analysis of the two species predator-prey model in Section 7. According to the theoretical analysis the optimal predator stock should increase with a rise in the price of the prey when the predator is a nuisance. Table 9.6 shows that in the TSB-model this is the case for the sea mammal stock when the capelin price rises, whereas the optimal stocks of cod and capelin decrease in this case.

A rise in the capelin price initially augments the value of this species proportionally to the price increase. However, it pays to transform some of the increased wealth into capital in general, and some into resource capital of the top predator. This increase in the sea mammal stock helps reduce the intermediate stock, the cod, thus reducing the total consumption of the capelin which has now become more valuable. Increased capital in general rewards the social manager with the interest expressed by the discount rate, and the top predator capital rewards him with increased revenues and reduced harvesting costs of this species. In addition there is an increase in the net benefits of capelin harvest due to the reduced predation pressure from the cod stock.

Table 9.6 shows that a rise in the price of cod, p_2, has a much stronger impact on the optimal capelin stock than on the cod stock; both, however, increase. In the economic analysis of the two species model in Section 7 we showed that, both for the prey and for the predator, the optimal stock decreases with an own price increase. In the TSB-model the cod is a prey compared to the sea mammals, but a predator compared to the capelin. From the two species analysis we therefore would think that the optimal cod stock in the TSB-model would decrease with an own price increase. However, as Table 9.6 shows, the effect on the optimal cod stock from an own price increase is positive, even though it is small. The reason for this is probably that the capelin, as well as the cod, is prey for the sea mammals. A rise in the price of cod initially augments the value of the resource capital of this species proportionally to the price increase. However, since cod is the most important prey for the sea mammals, it now pays to reduce the use of it as an expensive feed for the sea mammals. This is done by the reduction of the sea mammal stock which enables a minor increase of the cod stock. On the other hand, the capelin stock should be increased significantly to supply the cod stock with more, and relatively cheaper, food than before the rise of the price of cod.

The effects on the optimal stocks from an increase in the price of sea mammals are, as Table 9.6 show, hardly noticeable. Note, however, that the effects are of opposite sign to the results derived in the theoretical two species analysis in Section 7. In the TSB-model the optimal sea mammal stock increases a little with an own price increase, whereas in the two species model the predator is always negatively affected by an own price increase. The explanation for this might be that the optimal sea mammal stock is so small that a price increase for this species contributes very little to the value of the resource stock. Therefore it does not pay to reduce this capital stock and transform it into capital in general. It rather should be increased slightly to reduce the negative net profit from the harvest of the sea mammals.

In the two species analysis in Section 7 we found symmetry between the effects of prices and costs on the optimal stocks. When the effect of a price increase on one of the stocks is negative, the effect of an increase in the corresponding cost is positive. The results in Table 9.6 show such symmetric effects on the optimal stocks in the case of changes in capelin price and cost. For changes in prices and costs of the two other species this, however, is not the case. Note, for example, that the optimal sea mammal stock increases both with an increase in the price and in the cost of the sea mammals. The effect of a cost increase, is, however, very strong, whereas the effect of a price increase is very weak as shown above. When the effort cost of the sea mammal harvest increases with one percent, the optimal sea mammal stock increases slightly more than one percent. The increased stock reduces the unit harvest cost, thus almost restoring harvest profit to its original level. The harvest profit of sea mammals is anyway negative.

An increase in one of the prices has a positive effect on NPV in the TSB-model, whereas a cost increase has a negative impact just as in the single species model. With respect to capelin and cod the effect on NPV from a price increase is more important the larger the previous harvest of the price increased resource. For a relatively low price of capelin the optimal harvest of that particular species is low; therefore, a price increase contributes little to the total economic result measured by NPV.

For capelin and cod the effect from an increased discount rate, δ, in the TSB-model is similar to that of the single species model. The optimal stock levels decrease with a partial increase in the discount rate. For the sea mammal stock, however, the result is the opposite. An increased discount rate, ceteris paribus, makes the optimal stock level significantly higher. The larger predator stock helps reduce the prey to its lower optimal stock level caused by the increased discount rate. As noted in the discussion in Section 7, a rise in the discount rate makes it more costly to keep a large prey stock, therefore a part of it is transmuted into capital in general. In the case of the

TSB-model, cod and capelin are both preys relative to the sea mammals, so a part, although a small one, of each of them is transmuted into capital in general. Another part is transmuted into the predator resource capital, the sea mammals. Thus, the losses from harvesting the predator are reduced, probably because of the lowered unit harvesting cost, but also because of increased revenues from this species.

With respect to the net present value, it is seen from Table 9.6 that a discount rate increase has a negative effect as is the case with the single species model. The NPV might be compared to the price of a bond which decreases when the interest rate increases. In our case the NPV expresses the potential market value of the resource capital represented by the stocks of the three species, provided the stocks are at the optimum.

Having presented and discussed the optimal solutions of the TSB-model in the section, we shall proceed with the open access solutions in the next section, and compare them with the optimal solutions.

10. OPEN ACCESS SOLUTIONS

Until 1977 the living resources of the Barents Sea and adjacent areas were common property resources exploited by fishermen from most European fishing nations. The implementation of the Extended Fisheries Jurisdiction (EFJ), made possible by the United Nations' third law of the sea conference, in principle transformed the fishing resources of the area from international common property resources to the property of the coastal states of Norway and the Soviet Union. However, there are still disputes regarding the jurisdictional right for Norway to manage the fisheries resources of the Svalbard zone (Churchill, 1985); and the management problems with respect to the whale stocks in the area still prevail (Hoel, 1985).

To give an impression of the magnitude of the fisheries of the Barents Sea and adjacent areas, annual catch rates of sea mammals are depicted in Figure 10.1 and catches of capelin and cod in Figure 10.2 for the period 1868-1980. The harvest of sea mammals has fluctuated widely with significant peaks in the mid 1880s, in the 1890s, in the early 20th century, from the mid 1920s to the outbreak of the Second World War and in the early 1950s. A detailed empirical analysis of the causes of the fluctuating sea mammal harvest is beyond the scope of this study. According to the theory of common property resources, the causes should be sought in biological, technical, market or cost relations.

For capelin and cod it should be noticed that for practically all years until 1960 the catches of capelin were well below that of cod. The "capelin" peak after the mid 1960s is mainly herring and it coincides with the commencement of the almost total depletion of the Atlanto Scandian herring (Sætersdal, 1980). The peak ten years later consists almost entirely of capelin, caught by Norway and the Soviet Union. The collapse of the capelin stock, however, did not occur before the mid 1980s (Anon., 1986).

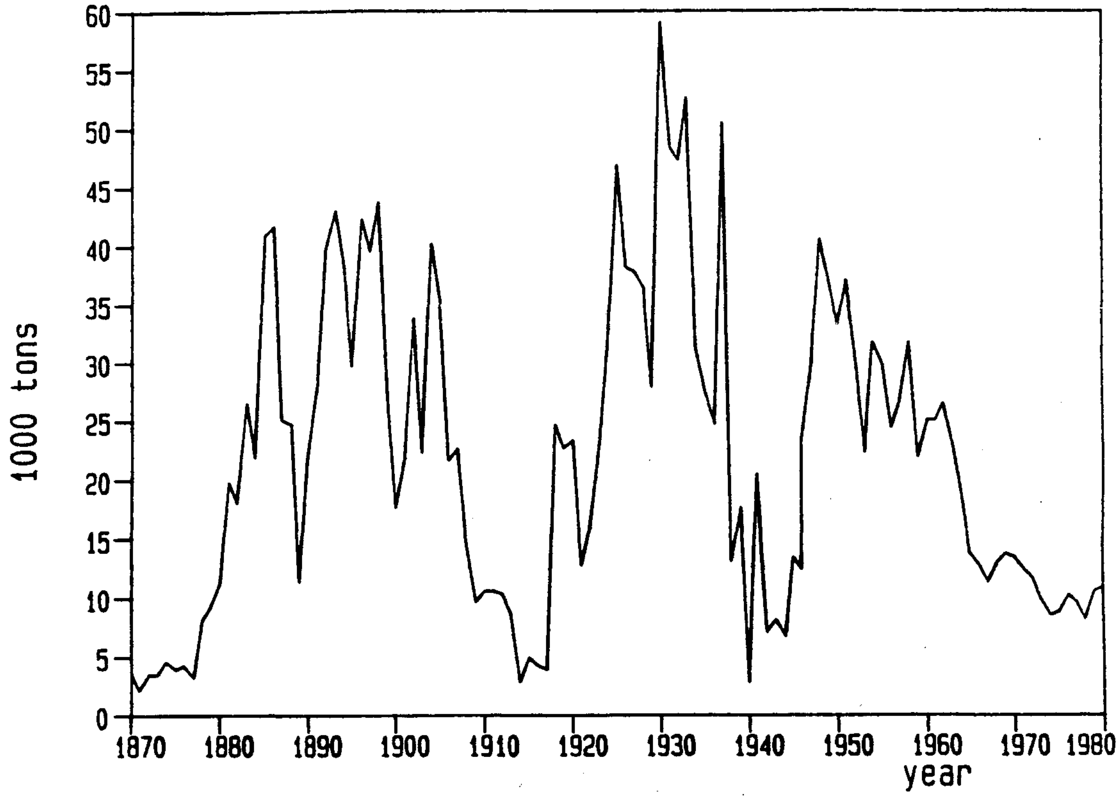


Figure 10.1. Annual harvest of sea mammals in the Barents Sea area, 1868-1980, in thousand metric tons.

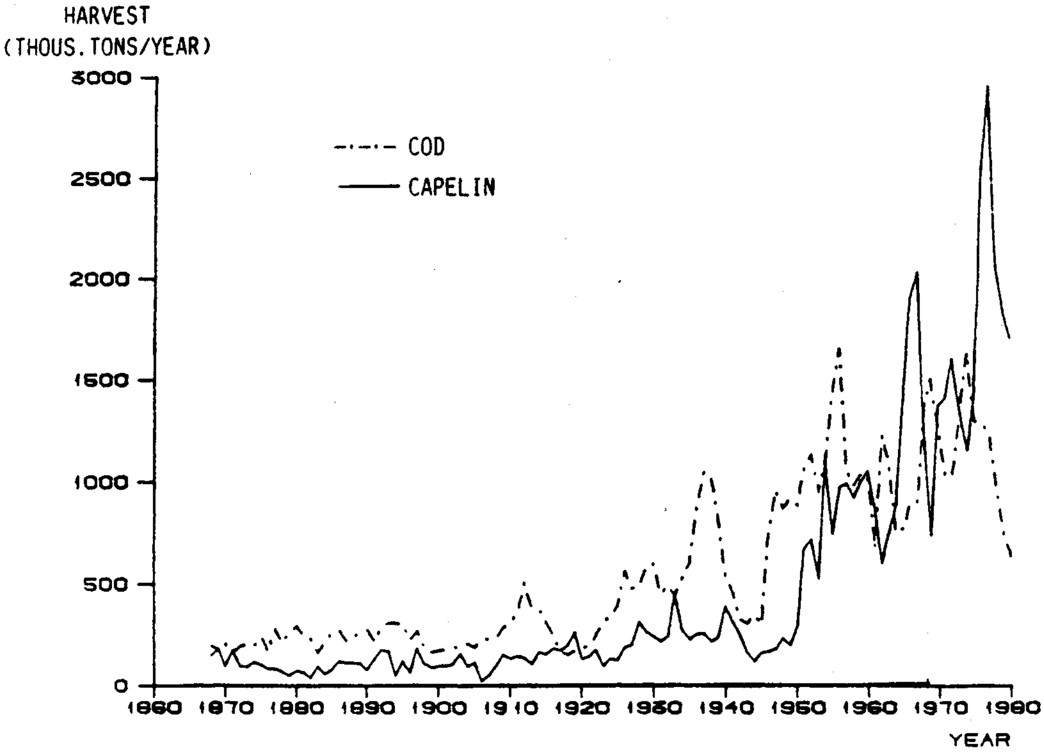


Figure 10.2. Annual harvests of capelin and cod in the Barents Sea area, 1868-1980, in thousand metric tons.

Using the economic and biological data from previous sections of this study we can analyse the open access solution of the Barents Sea fisheries. This is done within the framework of the TSB-model and is restricted to comparisons of equilibrium situations. With the basic case biological parameters in Table 6.12 and the basic economic parameters in Table 8.6, the open access equilibrium of the TSB-model is shown in the first lines of Tables 10.1 and 10.2 in normalized and metric units, respectively. For comparisons the optimal solution from Section 9 are included in the tables.

Table 10.1. Stock levels and harvest rates under open access and optimal management. Normalized units.

p_1^o ('000 Nkr/ m.t.)	Harvesting regime	x_1	x_2	x_3	y_1	y_2	y_3
1) 0.5284 (0.2270)	Open access	0.235503	0.133918	0.262812	0.094208	0.031757	0.005416
	Optimal	0.420009	0.215913	0.017887	0	0.118006	0.001069
1) 0.7926 (0.3405)	Open access	0.157002	0.121327	0.262812	0.085135	0	0.001799
	Optimal 2)	0.404155	0.212338	0.018729	0.012866	0.113080	0.001117

1) Standardized price with price of cod as the numeraire is given in parenthesis.

2) Interior solution, cf. Figure 9.1. The optimum is equivalent to the corner solution shown in this table for p_1^o = 0.5284 Nkr/kg.

The open access fishery were analysed in the last part of Section 7. There it was shown that if relative prices and costs imply simultaneous harvesting of all three species, the open access stock level of each species will depend only on own price and effort cost. The stock levels are, in general, found from Equation (7.55), but in the particular case of cod fish technology Equation (7.59) gives the open access solution for all three species.

Table 10.2. Stock levels and harvest rates under open access and optimal management. Million metric tons.

p_1^o ('000 Nkr/ m.t.)	Harvesting regime	w_1	w_2	w_3	h_1	h_2	h_3
1) 0.5284 (0.2270)	Open access	7.065	3.616	0.852	2.826	0.857	0.018
	Optimal	12.600	5.830	0.058	0	3.186	0.003
1) 0.7926 (0.3405)	Open access	4.710	3.276	0.852	2.554	0	0.006
	Optimal 2)	12.125	5.733	0.061	0.386	3.053	0.004

1) and 2): See notes to Table 10.1.

The open access solution for basic prices and costs, that is for p_1^o = 0.5284 Nkr/kg, is derived using the method leading to Equations (7.55) and (7.56). This solution implies simultaneous harvesting of all three species, and it proves to be locally stable. The harvest rates seem to be reasonable compared to actual harvests shown in Figures 10.1 and 10.2, except for the harvest of capelin which seems to be somewhat high. It is, however, important to stress that the solution is based on the assumption of pure cod fish technology for all three species. If we, on the contrary, had assumed pure pelagic fish technology for the capelin fishery, it is easy to demonstrate the extinction of all three species would be the result. As long as the profit from harvesting one ton of capelin is positive, it is positive for any stock level including the last unit of the stock. Being a common property means that no one has the incentive to leave the last unit of the resource unharvested. With the basic prey removed from the ecosystem, the predators, cod and sea mammals, are also extinct in the TSB-model.

The question of what kind of harvest technology prevails in the industries may be essential to the question of extinction of common property fish resources. To some extent we have discussed this subject in Section 8. Nevertheless, here we shall stick to the assumption of pure cod fish technology in the harvest of all three species in the TSB-model.

Comparing the optimal solution with the open access solution for $p_1^o = 0.5284$ Nkr/kg, it is seen from Tables 10.1 and 10.2 that the stocks of capelin and cod should be significantly increased and that of sea mammals should be dramatically depleted. This drastic removal of the top predator, the sea mammals, makes it possible almost to quadruple the harvest of cod. A capelin price of 0.5284 Nkr/kg is so low that it is optimal not to harvest this resource. It should rather remain in the sea as feed for cod under optimal management of the interdependent species, while in the open access case the capelin will be heavily exploited.

The open access fishery can affect the stocks to the extent that one of them, for example the cod stock, is reduced to such a low equilibrium level that fishing for this particular species is unprofitable. The analysis in section 7, and in particular Equations (7.59) and (7.61), show that the open access stock level of cod in this case is a function of prices and costs of the two other species, capelin and sea mammals.

With a 50% increase in the price of capelin, ceteris paribus, from 0.5285 Nkr/kg to 0.7926 Nkr/kg, the open access harvesting of capelin will be more intensive reducing the stock from 7 million tons to 4.7 million tons. Such a reduction in the availability of prey reduces the cod stock so much that it makes the harvesting of this resource unprofitable. To find the complete open access solution in this case it is therefore necessary use Equations (7.59) and (7.61). The results are presented in Tables 10.1 and 10.2 for $p_1^o = 0.7926$ Nkr/kg. Under open access a 50% partial price increase of capelin renders the cod stock unharvested, reduces the harvest of capelin somewhat and reduces the harvest rate of sea mammals by as much as two-thirds. Altogether, the gross value of the total open access harvests is nearly halved, especially due to the unprofitability of the cod fishery.

In the case of the pure cod fish technology, Equation (7.59) shows that the open access stock level of a harvested species depends only on the own price and effort cost. For sea mammals this is illustrated in Tables 10.1 and 10.2, where the stock level of this species is unchanged from line one to line three, because price and cost are unchanged. For capelin, however, the stock level is reduced because of increased own price. For two different levels of p_1, p_1^1 and p_1^2, Equation (7.59) implies the following relationship between the two levels of the capelin stock, X_1^1 and X_1^2, when cost is unchanged:

$$\frac{X_1^1}{X_1^2} = \frac{p_1^2}{p_1^1} \quad . \tag{10.1}$$

For capelin we have from Table 10.1:

$$\frac{p_1^2}{p_1^1} = \frac{.3405}{.2270} = 1.5 \ . \tag{10.2}$$

That is why the ratio of the stock levels of capelin on line one and three is

$$\frac{X_1^1}{X_1^2} = \frac{.235503}{.157002} = 1.5 \ . \tag{10.3}$$

In this respect the result is similar to the result of the single species analysis of open access fisheries. Under open access harvesting no one has the incentive to invest in the capelin stock today in order to be able to harvest more of it tomorrow. Now, in the multispecies context we might say that no one has the incentive to invest in the capelin stock today so as to harvest more cod tomorrow.

Under open access harvesting it is interesting to notice the strong effect on the harvest rate of sea mammals from an increase in the price of capelin. This happens despite the constant equilibrium level of sea mammals. The cause is to be found in the reduction of the prey stocks, especially that of the capelin. The sustainable yield of sea mammals is therefore sensitive to changes in the availability of preys within the range of stock levels considered here.

11. SUMMARY AND CONCLUSION

The aim of this study has been twofold: to provide a theoretical and an applied analysis of multispecies fisheries. A two species biomass model is reviewed in Section Two, and Section Three presents a three species model based upon the former one. The concept of maximum sustainable yield frontier (MSF) is defined and presented in Section Four, and a method for deriving this in the three species case is developed. The biological MSF concept is closely related to the concept of production possibility frontier in economic theory.

The applied part of this study is devoted to the fisheries of the Barents Sea area in the North East Atlantic Ocean. Therefore, a brief description of the marine ecological system of the area is included in Section Five. Since a fully developed and implemented multispecies model of the Barents Sea fisheries did not exist, an attempt has been made to estimate the nine biological parameters of the model. The "guesstimation" procedure is explained step by step and the assumptions made are explicitly put forward in Section Six, such that the reader can critically appraise and modify them if that is felt to be desirable.

The three stocks in the applied model are aggregated stocks with "capelin" consisting of capelin and herring, "cod" consisting of cod, haddock and saithe, and "sea mammals" consisting of 14 species of whales and two species of seals.

Economic aspects of multispecies fisheries studied in Section Seven includes optimal harvesting levels as well as open access harvesting levels. In addition to the traditional study of interior solutions, we also discuss the possibility of corner solutions and analyse in some detail a couple of cases. With respect to the effects on the optimal stocks from an increase in the discount rate, this was studied theoretically within the framework of a two species predator-prey model. We found that the untraditional result of an increase in the predator stock, is most likely in cases where the predator clearly is a nuisance to be harvested not (only) for the sake of its own value, but to increase the availability and the profitability of the prey harvest.

The economic parameters of the model are derived from Norwegian data in Section Eight and together with the biological data in Section Six, they are used in Section Nine to derive the optimal solutions of the three species Barents Sea model (TSB-model). With the basic set of biological and economic parameters the optimal solutions proves to be a corner solution with no harvesting of capelin. However, the argument is that this result is a consequence of the assumed constant price of capelin. Introduction of elastic demand would probably alter this finding as indicated by the computed results from increased price of capelin. The optimal solution requires taxation of the capelin and the cod fisheries and heavily subsidization of the sea mammal harvesting due to the latter's imposed costs on the two former through predation. Sensitivity analysis of the optimal results in Section Nine indicates that this finding is very robust against changes in biological parameters, and, within reasonable limits, this main finding is also robust against changes in most economic parameters.

Open access solutions of the TSB-model are presented in Section Ten, and it is concluded that all three stocks are substantially reduced compared to the pristine levels. However, the optimal stocks of capelin and cod are not only larger than open access stocks, but even larger than the pristine levels. This paradoxical result is made possible by the heavily reduced stock of sea mammals which expands the ecological niche for capelin and cod. The relative importance of capelin and cod in the optimum very much depends on the relative prices and harvest costs of these two species. For example, when the price of capelin is one-third of the price of cod it is optimal only to harvest cod, whereas if the capelin price rises to three-quarters of the price of cod, ceteris paribus, it becomes optimal only to harvest the capelin. However, in either case, the sea mammals should always be kept at a relative low stock level.

One of the main policy recommodations of this study is that the sea mammals should be heavily depleted to increase the surplus production of fish resources for man. Controversial it might be, our findings are nevertheless rational from an economic point of view. It should, however, be stressed that this result may be somewhat modified if the resource is assigned an optional value from people's willingness to pay for keeping the stock at higher level. A biological argument that also may weaken our result is the eventual existence of critical depensation for lower stock levels. The TSB-model is, as most stock assessment models of whales and seals also are, based on the assumption of pure compensation. Together with the assumption of pure cod fish technology in the harvesting sector this is a safeguard against extinction. In the TSB-model the optimal stock level of sea mammals increases somewhat with an increase in the price of capelin, ceteris paribus. This is also the case for increased harvest costs of cod or capelin, and for an increased discount rate. Since the sea mammals' predation pressure on cod is relatively higher than that on capelin, a reduction in the price of cod increases the optimal stock level of sea mammals. A reduced price of cod allows a larger consumption of cod by sea mammals, hence, an increase in the optimal stock of the sea mammals.

As the quotation from D'Ancona (1926) in Section Three shows, the recognition of the necessity of harvesting the predator to increase the yield of the prey is not entirely new. Even long before that, in 1859, this was asserted by Charles Darwin in his famous book "The Origin of Species" as the following quotation demonstrate:

> The amount of food for each species of course gives the extreme limit to which each can increase; but very frequently it is not the obtaining food, but the serving as prey to other animals which determines the average number of a species. Thus, there seems to be little doubt that the stock of partridges, grouse and hares on any large estate depends chiefly on the destruction of vermin. If not one head of game were shot during the next twenty years in England, and, at the same time, if no vermin were destroyed, there would, in all probability, be less game than at present, although hundreds of thousands of game animals are now annually shot. (Darwin, 1882, pp. 53-54; quoted from Volterra, 1928, pp. 21-22.)

In the context of the Barents Sea fisheries the "vermin" consist of several species of fish-consuming whales and seals, and this is also the case for the fisheries in other areas of the North Atlantic.

Several extensions of this work can be thought of in both the theoretical and in the applied field. The former includes multispecies studies of resources where, e.g., one of the stocks is "transboundary"[1], i.e. the fish stock migrate across the boundary of the Extended Economic Zones of two countries. If one country has the main jurisdiction of the prey and another the main jurisdiction of the predator the complexity of the model clearly increases. Another extension of the theoretical analysis could be to introduce seasonal variations in the growth rates of the stocks, the catchability coefficients or in the prices of the harvests.[2] This clearly enhances the complexity of the bioeconomic system and of the economic solution to the optimization problem.

In the applied field more research needs to be devoted to other living resources of the Barents Sea by adding more stocks to the TSB-model. Deep water prawn is a species whose harvest immensely increased, both in quantitative and value terms, from the mid 1970s to the mid 1980s. Some scientists believe this quantitatively increase has been possible mainly because of the depletion of the cod stock. If this is true the optimal solution will probably tell us to reduce the cod stock more than in the case of the TSB-model with capelin as the only prey.

The most important fish consumers included in the stock of sea mammals in this study are the Minke whale and the Harp seal; and one could think of dividing the sea mammals into two or more proper stocks to study how different combinations of whaling and sealing effect the optimal result. This could also include economic studies of transboundary resources since the migratory

---

[1] Munro (1979) is a theoretical study in the economics of a transboundary resource within the framework of a one species model.

[2] Optimal harvesting of one renewable resource with seasonal growth is analysed in Flaaten (1983).

whales are combined Norwegian property and internationally common property resources, while the Harp seal stock of the Barents Sea mainly is the property of the USSR.

In the last couple of decades the harvest of whales and seals has been an increasingly touchy question for all nations with traditional interests in utilizing the marine resources. Studies in political science of the behaviour of governments, environmental organizations, the International Whaling Commission, etc. will also be of interest for the political implementation of the economic optimal solutions to multispecies harvesting problems (e.g. see Hoel, 1985).

In scientific disiplines other than social sciences we can think of detailed biological studies of predation and of inter-species relations with respect to carrying capacities and growth rates. Cost-benefit analysis linked to this study might be of some help in the selection of biological projects.

The economic history of Arctic societies is closely related to natural resources. Multispecies studies of the living resources will probably improve historians' understanding of the development of such societies, since technology, relative prices and costs change with time and alter the relative importance of the species in the ecological system. For example, how did the early intensive hunting of whales and seals in the Barents Sea area affect the development of coastal fisheries and communities in North Norway? And how severe will the effect on this fishery dependent region be as a result of a complete ban on whaling and sealing? The quantitative analysis of this study does not solely serve the purpose of illustrating a method, but the author thinks they also give a description of some important biological and economic relationships of the Barents Sea fisheries. The sincere answer to the last question therefore is that such an experiment in the long run may prove disasterous for this fragile, resource-based region; and the same conclusion will probably also apply to similar societies in other parts of the Arctic.

APPENDICES

EQUILIBRIUM SOLUTIONS OF THE THREE SPECIES MODEL

If the system (3.6)-(3.8) have an equilibrium solution different from $X_i = 0$ for all i, and $\gamma_{ij} \neq 0$ for all i,j, it is found easily from

$$A_1 - X_1 - \gamma_{12}X_2 - \gamma_{13}X_3 = 0 \tag{A1.1}$$

$$A_2 - X_2/X_1 - \gamma_{23}X_3 = 0 \tag{A1.2}$$

$$A_3 - \alpha X_3/(X_1 + \alpha X_2) = 0 \;. \tag{A1.3}$$

Reformulating (A1.2) gives

$$X_2 = A_2X_1 - \gamma_{23}X_1X_3 \;. \tag{A1.4}$$

Substituting for X_2 in (A1.3) and reformulating gives

$$X_3 = \frac{A_3X_1(\frac{1}{\alpha} + A_2)}{1 + A_3\gamma_{23}X_1} \;. \tag{A1.5}$$

Substituting for X_3 in (A1.4) gives

$$X_2 = \frac{(A_2 - \gamma_{23}A_3\frac{1}{\alpha}X_1)X_1}{1 + A_3\gamma_{23}X_1} \;. \tag{A1.6}$$

We have now expressed X_2 and X_3 as functions of X_1. Substituting for X_2 and X_3 into (A1.1) and reformulating gives

$$\frac{A_1 + D_1X_1 + D_2X_1^2}{1 + A_3\gamma_{23}X_1} = 0 \tag{A1.7}$$

$$A_1 + D_1X_1 + D_2X_2^2 = 0 \;, \tag{A1.8}$$

where

$$D_1 = A_1A_3\gamma_{23} - 1 - \gamma_{12}A_2 - A_3(1/\alpha)\gamma_{13} - A_2A_3\gamma_{13}$$

$$D_2 = A_3(1/\alpha)\gamma_{12}\gamma_{23} - A_3\gamma_{23} \cdot$$

The solution of X_1 from (A1.8) is

$$X_1 = \frac{- D_1 \pm \sqrt{D_1{}^2 - 4A_1D_2}}{2D_2} \quad . \tag{A1.9}$$

APPENDIX 2

PROGRAM FOR COMPUTATION OF THE MSF-DATA

```
BEGIN
  COMMENT
    ****************************************************
    *                                                  *
    *    FILNAVN PROGRAM       : SY12                  *
    *    FILNAVN KOMPILERT     : SY12B                 *
    *    FILNAVN INNDATA       : INPUT                 *
    *    FILNAVN UTDATA        : LESES INN             *
    *    PROGRAMMERINGSSPRÅK : SIMULA  1.2-353         *
    *    TYPE MASKIN           : CYBER 171MP           *
    *    OPERATIVSYSTEM        : NOS 2.0-531/528       *
    *    PROGRAMMERER          : OLA FLÅTEN            *
    *    INSTITUSJON           : IFF, UNIV. I TROMSØ   *
    *    SEKSJON               : AVDELING FOR ØKONOMI  *
    *    DATO                  : 05/11/1985            *
    *    VERSJON               : 1                     *
    *                                                  *
    ****************************************************;

  COMMENT PROGRAMMET LESER INN HØYESTE/LAVESTE VERDI FOR EN
          PARAMETER OG VERDIER FOR 9 ANDRE. DATAENE LEGGES UT
          PÅ EN FIL TIL GRAFISK FREMSTILLING;


  REAL ARRAY PARAM(1:10);
  REAL A12,A13,A23,A1,A2,A3,G12,G13,G23,K1,K2,X1,X2,X3,W1,W2,W3,
       Y1,Y2,Y3,H1,H2,H3,SQ,X,XMIN,XMAX,XSTEP;
  TEXT ARRAY TPARAM(1:10);
  INTEGER I,PARAMNR;
  TEXT FILNAVN;
  REF(OUTFILE)UT;

  COMMENT ******************************************
          *          H O V E D P R O G R A M       *
          ******************************************;

  FILNAVN:-BLANKS(7);
  OUTTEXT("LES INN NAVN PÅ DATAFIL");OUTIMAGE;INIMAGE;
  FILNAVN:=INTEXT(7);
  BEGIN
    UT:-NEW OUTFILE(FILNAVN.STRIP);
    UT.OPEN(BLANKS(80),0);
    FOR I:=1 STEP 1 UNTIL 10 DO
      TPARAM(I):-BLANKS(2);

    TPARAM(1):="U1";
    TPARAM(2):="U2";
    TPARAM(3):="U3";
    TPARAM(4):="R1";
    TPARAM(5):="R2";
    TPARAM(6):="R3";
    TPARAM(7):="A ";
    TPARAM(8):="B ";
    TPARAM(9):="K ";
    TPARAM(10):="X2";
```

```
PARAM(1):=0.0000;
PARAM(2):=0.0000;
PARAM(3):=0.0000;
PARAM(4):=1.2704;
PARAM(5):=1.1617;
PARAM(6):=0.0614;
PARAM(7):=0.90;
PARAM(8):=0.12;
PARAM(9):=30*10**6;
PARAM(10):=0.05;

FOR I:=1 STEP 1 UNTIL 10 DO
BEGIN
  SETPOS(20);OUTINT(I,2);OUTTEXT(" ");
  OUTTEXT(TPARAM(I));OUTTEXT(" = ");
  OUTFIX(PARAM(I),4,14);OUTIMAGE;
END FOR I;
OUTIMAGE;OUTTEXT("NR. PÅ PARAMETER SOM SKAL FORANDRES");
OUTIMAGE;OUTIMAGE;
PARAMNR:=ININT;
OUTTEXT("LES INN MIN OG MAX FOR ");
OUTTEXT(TPARAM(PARAMNR));OUTIMAGE;OUTIMAGE;
XMIN:=INREAL; XMAX:=INREAL;
XSTEP:=(XMAX-XMIN)/50;

FOR X:=XMIN STEP XSTEP UNTIL XMAX DO
BEGIN
  PARAM(PARAMNR):=X;

  A12:=0.125/10**6;
  A13:=0.140/10**6;
  A23:=0.310/10**6;


  W3:=1500;
  X3:=1000*W3/(PARAM(7)*PARAM(8)*PARAM(9));
  X2:=PARAM(10);

  G12:=A12*PARAM(7)*PARAM(9)/PARAM(4);
  G13:=A13*PARAM(7)*PARAM(8)*PARAM(9)/PARAM(4);
  G23:=A23*PARAM(7)*PARAM(8)*PARAM(9)/PARAM(5);

  K1:=1+(4-G12)*X2;
  K2:=8*X2*(2-3*G12*X2);


  SQ:=K1**2-K2;
  IF SQ<0 THEN
  BEGIN
    OUTTEXT("VERDI I KVADRATROT ER NEGATIV, SETTES TIL 0");
    OUTIMAGE;
    SQ:=0;
    UT.OUTCHAR('*');
  END ELSE
    UT.OUTCHAR(' ');
```

```
      X1:=K1/4+(SQRT(SQ))/4;

      Y1:=PARAM(4)*X1*(1-X1-G12*X2)-PARAM(4)*X1*G13*X3;
      Y2:=PARAM(5)*X2*(1-X2/X1)-PARAM(5)*X2*G23*X3;
      Y3:=PARAM(6)*X3*(1-X3/(X1/PARAM(7)+X2));

      W1:=X1*PARAM(9)/1000;
      W2:=X2*PARAM(7)*PARAM(9)/1000;

      H1:=PARAM(9)*Y1/1000;
      H2:=PARAM(7)*PARAM(9)*Y2/1000;
      H3:=PARAM(7)*PARAM(8)*PARAM(9)*Y3/1000;


      UT.OUTFIX(Y1,6,9);UT.OUTFIX(Y2,6,9);
      UT.OUTFIX(Y3,6,9);UT.OUTFIX(X1,6,9);UT.OUTFIX(X2,6,9);
      UT.OUTFIX(X3,6,9);UT.OUTIMAGE;
    END FOR X;

    OUTIMAGE;OUTTEXT("DATA PÅ FILEN ");OUTTEXT(FILNAVN);
    OUTIMAGE;OUTIMAGE;
  END FILNAVN;
END PROGRAM;
```

AGGREGATED STOCK LEVELS OF CAPELIN AND COD,[1] 1950-1980

1950	11.690	6.652
1951	13.210	7.031
1952	14.800	7.699
1953	15.620	7.770
1954	15.930	7.971
1955	16.190	7.982
1956	16.130	6.835
1957	14.050	5.311
1958	13.230	4.972
1959	13.170	5.425
1960	11.330	4.825
1961	9.760	4.942
1962	8.020	4.770
1963	6.950	3.917
1964	7.950	3.996
1965	9.410	4.687
1966	7.920	5.326
1967	6.270	5.686
1968	5.900	5.746
1969	6.680	5.301
1970	6.990	4.637
1971	5.587	4.594
1972	7.000	5.049
1973	5.210	5.362
1974	6.230	5.226
1975	8.585	4.560
1976	6.926	4.099
1977	5.942	3.666
1978	6.309	3.134
1979	5.665	2.810
1980	6.985	2.734

---

[1] "Capelin" includes capelin (Mallotus villosus) and herring (Clupea harengus). "Cod" includes cod (Gadus morhua), haddock (Melanogrammus aeglefinus) and saithe (Pollachius virens). Source: Flaaten (1984a).

AGGREGATED CATCHES OF CAPELIN, COD AND SEA MAMMALS,[1] 1868-1980

1868	149.2	198.2	3.0
1869	182.6	176.0	2.0
1870	95.4	207.2	3.4
1871	177.5	171.3	2.2
1872	95.5	197.4	3.7
1873	92.5	209.9	3.4
1874	117.9	200.3	4.6
1875	102.9	242.7	3.8
1876	84.4	168.4	4.1
1877	83.6	280.6	3.2
1878	69.1	213.5	8.0
1879	51.6	266.7	9.1
1880	75.8	294.5	11.2
1881	66.9	246.6	19.7
1882	38.8	233.6	17.9
1883	95.5	162.5	26.3
1884	58.8	222.1	22.0
1885	80.3	257.8	41.1
1886	121.5	277.1	41.5
1887	114.6	226.2	25.1
1888	112.1	246.2	24.6
1889	112.9	258.6	11.2
1890	78.5	276.8	22.4
1891	130.7	219.4	28.3
1892	177.1	289.3	39.8
1893	170.2	311.1	43.2
1894	51.2	311.4	38.0
1895	119.8	304.9	29.7
1896	66.3	229.0	42.4
1897	184.2	272.0	39.6
1898	109.3	185.3	43.7
1899	89.7	167.6	26.8
1900	100.8	176.2	17.6
1901	100.3	180.7	22.0
1902	107.4	191.9	33.8
1903	161.1	193.3	22.3
1904	95.2	210.5	40.1
1905	114.6	192.3	35.2
1906	27.4	228.6	21.5
1907	54.3	238.5	22.5
1908	102.0	243.0	14.4
1909	155.4	289.7	9.6
1910	135.5	317.7	10.7
1911	147.0	360.2	10.6
1912	139.0	505.9	10.3
1913	107.7	389.2	8.3
1914	166.2	380.6	2.8
1915	158.7	313.5	5.0
1916	185.1	248.4	4.2
1917	174.7	169.0	4.0
1918	199.5	154.1	24.7
1919	266.2	173.9	22.6

---

[1] For definitions of "capelin" and "cod", see note in Appendix 3. "Sea mammals" includes harp seal (<u>Pagophilus groenlandicus</u>), hooded seal (<u>Crystophora cristata</u>), blue whale (<u>Balaenoptera musculus</u>), fin whale (<u>Balaenoptera physalus</u>), humpback whale (<u>Megaptera novacanglia</u>), sei whale (<u>Balaenoptera borealis</u>), minke whale (<u>Balaenoptera acutorostrata</u>), sperm whale (<u>Physeter catodon</u>), porpoise (<u>Phocaena phocaene</u>), pilot whale (<u>Globicephala melaena</u>), white whale (<u>Delphiapeterus leucas</u>), white-sided dolphin (<u>Lagenorhynchus acutus</u>), bottlenose whale (<u>Hyperoodon ampullatus</u>) and killer whale (<u>Orcinus orca</u>).

1920	134.5	181.7	23.3
1921	145.0	202.7	12.8
1922	179.9	250.0	16.0
1923	98.6	317.4	22.8
1924	132.3	340.9	31.0
1925	125.9	396.8	46.8
1926	192.4	565.3	38.0
1927	204.6	478.5	37.7
1928	315.3	489.2	36.5
1929	264.9	589.0	27.8
1930	243.3	602.0	58.8
1931	220.0	452.5	48.3
1932	246.8	490.0	47.3
1933	455.0	445.7	52.5
1934	271.4	529.7	31.0
1935	229.6	611.9	27.4
1936	256.9	878.3	24.6
1937	257.3	1052.2	50.6
1938	218.5	1020.8	13.0
1939	239.5	821.4	17.8
1940	390.9	547.7	3.0
1941	314.2	457.8	20.3
1942	253.2	325.1	6.9
1943	162.7	307.7	8.1
1944	119.4	350.8	6.6
1945	163.5	305.3	13.3
1946	169.9	748.6	12.5
1947	184.9	963.4	23.4
1948	235.7	872.6	29.9
1949	199.4	917.1	40.6
1950	297.9	890.6	33.1
1951	676.5	1065.5	36.9
1952	719.5	1138.5	29.5
1953	527.1	952.9	22.4
1954	1097.9	1104.7	31.7
1955	749.3	1435.6	29.7
1956	977.4	1654.1	24.4
1957	996.4	1062.5	26.6
1958	927.0	987.6	31.9
1959	1005.2	1043.1	22.0
1960	1053.9	1027.8	25.0
1961	869.1	674.5	24.9
1962	604.9	1231.5	26.6
1963	750.9	1098.0	22.2
1964	889.2	752.6	18.5
1965	1393.2	773.2	13.9
1966	1908.8	890.1	12.7
1967	2035.7	895.4	11.3
1968	1237.8	1368.5	13.0
1969	742.1	1504.0	13.8
1970	1376.5	1255.4	13.5
1971	1414.4	1028.1	12.4
1972	1605.9	1036.0	11.8
1973	1342.9	1337.7	9.8
1974	1154.9	1637.9	8.6
1975	1421.4	1301.2	9.0
1976	2546.4	1289.6	10.3
1977	2958.0	1240.5	9.7
1978	2048.2	984.5	8.1
1979	1832.5	746.8	10.6
1980	1704.5	652.5	11.0

APPENDIX 5

CHARACTERISTICS OF NORTH ATLANTIC COD STOCKS

Table A5. Characteristics of North Atlantic cod stocks (natural mortality 0.2-0.3; fecundity = 500 eggs/gram female).

	Recruits	Growth					Exploitation					
	2 year olds × 10^6	K	L_∞ (cm)	W_∞ (kg)	$\bar{W}$ (kg) (5 year)	Mature 50% age	50% recruitment (t_c years)	Y_w/R_2 (kg) at F_{max}	Biomass[a] 2 + at F_{max} (10^6 t)	F_{max}	F_{recent}	Average catch, 1961–70 (000 t)
North Sea, south	75	0·33	111	17	6·1	3	2	1·93	0·70	0·25	0·5	111
north/central	50	0·27	119	15	5·2	3	2	1·57	0·99	0·25	0·3	54
West Scotland	(14)	0·24	108	14	6·6	3	2	1·94	(0·13)	0·3	0·6	18
Irish Sea/Bristol Channel	4	0·33	101	12	6·5	2/3	1/2	1·25	0·04	0·3	0·85	6
Faroe, Plateau/Bank	16	0·19	115	14	4·5	3/4	3	1·87	0·10	0·3	0·55	28
Iceland	300	0·16	118	15	3·4	7/8	5/6	1·56	2·28	0·8 1·4	0·8[b] 1·0	395
Baltic	250	0·20	90	6	1·5	4	3/4	0·50	0·39	< 0·9	0·9	139
Arcto-Norwegian	1250	0·12	130	17	1·5	7/8	5/6	0·57	4·13	0·5	0·6	750
East and southwest Greenland	80		'SLOW'		1·3	8/9	5	0·78	0·24	0·65	0·65	102
West Greenland	180	0·2	100	8	1·9	7/8	5	0·99	0·74	0·50	0·50	264
Labrador East Newfoundland	2000	0·15 0·11	81 114	5 12	0·8	7/8	4 4	0·40	3·16	0·40	0·60	639
Flemish Cap	50	NK	NK	NK	NK			0·65	NK	0·50	> 0·5	30
Grand Bank	200	0·10	152	25	1·4	4/5	4	0·87	1·08	0·2	0·6	103
St. Pierre Bank	80	0·10	137	18	1·1	4/5	5	0·63	0·27	0·3	0·5	61
West Newfoundland	NK	0·05	143	20		4/5	4					83
South Gulf of St. Lawrence	125	?	?	?	1·0		4	0·56	0·28	0·4	0·3	67
Banquereau	75	0·14	105	11	1·4	4/5	4	0·86	0·19	0·45	0·5	65
Brown's/Lahave Bank	16	0·20	117	16	2·1	4/5	4	1·40	0·08	0·35	0·6	25
Georges Bank	NK	0·12	142	20	3·0	3/4	3	2·30	NK	0·3	> 0·3	37
Miscellaneous coastal												110
Total												3087

[a] Biomass of 2 year old + based on exploitation of average recruitment at F_{max} given.

[b] Icelandic and ICES authorities give estimates which differ according to age groups used as reference of full recruitment. Estimates of fishing mortality/age group agree.

( ) Tentative.

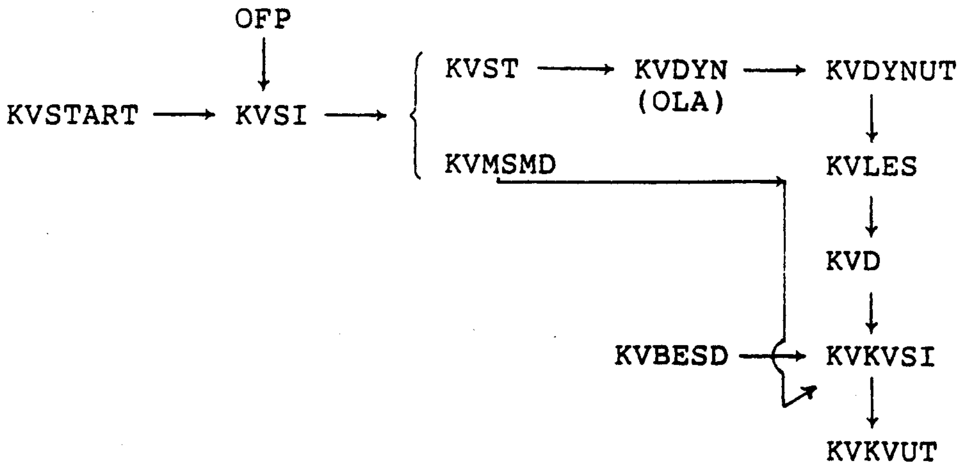
NK Not known.

? Varied widely in last decade

Source: Garrod (1977), Table 18.

APPENDIX 6

CHART FLOW. PROGRAMS FOR COMPUTING THE SUM OF SQUARES OF THE DIFFERENCES OF RELATIVE STOCK SIZES



The programs:

KVSTART	:	Control Commands
KVSI (KVSIB)	:	The main program (SIMULA)
OFP	:	Password
KVST	:	Control Commands
KVMSMD	:	Data (min step max)
KVDYN	:	DYNAMO-program
KVDYNUT	:	DYNAMO out
KVLES (KVLEB)	:	SIMULA-program
KVD	:	Data (results)
KVKVSI (KVKVSIB)	:	SIMULA (min. Q^2)-program
KVBESD	:	Data (stocks 1951-80)
KVKVUT	:	Tables

APPENDIX 7

A DYNAMO SIMULATION PROGRAM FOR THE TSB-MODEL, WITH HARVEST RATES 1868-1980 INCLUDED

```
  TIME 1868-)1980

OPT SVALL,R

 ******************
 *   L O D D E   *
 ******************

L LODDE. K=LODDE. J+DT*RLODDE. JK
N LODDE=NLODDE
C NLODDE=10. 069E6
R RLODDE. KL=R1. K*LODDE. K*(1-LODDE. K/K)-A12*LODDE. K*TORSK. K-A13*LODDE. K
X *SEL. K-LFANGST. K
A LFANGST. K=TABHL(LFAN,TIME. K,1868,1980,1)
A R1. K=0. 8696/(1-9. 465E6/K)
C A12=0. 1250E-6
C ALFA=0. 90
C K=30E6
C A13=0. 1400E-6
A BETA. K=0. 0582*R3/(R3-0. 0316)

 ******************
 *   T O R S K   *
 ******************


L TORSK. K=TORSK. J+DT*RTORSK. JK
N TORSK=NTORSK
C NTORSK=4. 759E6
R RTORSK. KL=R2. K*TORSK. K*(1-TORSK. K/(ALFA*LODDE. K))-A23*TORSK. K*SEL. K
X -TFANGST. K
A TFANGST. K=TABHL(TFAN,TIME. K,1868,1980,1)
A R2. K=0. 4523/(1-0. 5496/ALFA)
C A23=0. 3100E-6

 ******************
 *     S E L      *
 ******************

L SEL. K=SEL. J+DT*RSEL. JK
N SEL=NSEL
C NSEL=1. 779E6
R RSEL. KL=R3*SEL. K*(1-SEL. K/(BETA. K*(LODDE. K+TORSK. K)))-SFANGST. K
A SFANGST. K=TABHL(SFAN,TIME. K,1868,1980,1)
C R3=0. 0614
C Y3=0. 035

  FANGST AV LODDE, TORSK OG SEL (51 LINJER)

T LFAN=149. 2E3,182. 6E3,95. 4E3,177. 5E3,95. 5E3,92. 5E3,117. 9E3,
X 102. 9E3,84. 4E3,83. 6E3,69. 1E3,51. 6E3,75. 8E3,66. 9E3,
X 38. 8E3,95. 5E3,58. 8E3,80. 3E3,121. 5E3,114. 6E3,112. 1E3,
X 112. 9E3,78. 5E3,130. 7E3,177. 1E3,170. 2E3,51. 2E3,119. 8E3,
X 66. 3E3,184. 2E3,109. 3E3,89. 7E3,100. 8E3,100. 3E3,107. 4E3,
X 161. 1E3,95. 2E3,114. 6E3,27. 4E3,54. 3E3,102. 0E3,155. 4E3,
X 135. 5E3,147. 0E3,139. 0E3,107. 7E3,166. 2E3,158. 7E3,185. 1E3,
X 174. 7E3,199. 5E3,266. 2E3,134. 5E3,145. 0E3,179. 9E3,98. 6E3,
X 132. 3E3,125. 9E3,192. 4E3,204. 6E3,315. 3E3,264. 9E3,243. 3E3,
X 220. 0E3,246. 8E3,455. 0E3,271. 4E3,229. 6E3,256. 9E3,257. 3E3,
X 218. 5E3,239. 5E3,390. 9E3,314. 2E3,253. 2E3,162. 7E3,119. 4E3,
X 163. 5E3,169. 9E3,184. 9E3,235. 7E3,199. 4E3,297. 9E3,676. 5E3,
X 719. 5E3,527. 1E3,1097. 9E3,749. 3E3,977. 4E3,996. 4E3,927. 0E3,
X 1005. 2E3,1053. 9E3,869. 1E3,604. 9E3,750. 9E3,889. 2E3,1393. 2E3,
X 1908. 8E3,2035. 7E3,1237. 8E3,742. 1E3,1376. 5E3,1414. 4E3,1605. 9E3,
X 1342. 9E3,1154. 9E3,1421. 4E3,2546. 4E3,2958. 0E3,2048. 2E3,1832. 5E3,
X 1704. 5E3
```

```
T TFAN=198.2E3,176.0E3,207.2E3,171.3E3,197.4E3,209.9E3,200.3E3,
X 242.7E3,168.4E3,280.6E3,213.5E3,266.7E3,294.5E3,246.6E3,
X 233.6E3,162.5E3,222.1E3,257.8E3,277.1E3,226.2E3,246.2E3,
X 258.6E3,276.8E3,219.4E3,289.3E3,311.1E3,311.4E3,304.9E3,
X 229.0E3,272.0E3,185.3E3,167.6E3,176.2E3,180.7E3,191.9E3,
X 193.3E3,210.5E3,192.3E3,228.6E3,238.5E3,243.0E3,289.7E3,
X 317.7E3,360.2E3,505.9E3,389.2E3,380.6E3,313.5E3,248.4E3,
X 169.0E3,154.1E3,173.9E3,181.7E3,202.7E3,250.0E3,317.4E3,
X 340.9E3,396.8E3,565.3E3,478.5E3,489.2E3,589.0E3,602.0E3,
X 452.5E3,490.0E3,445.7E3,529.7E3,611.9E3,878.3E3,1052.2E3,
X 1020.8E3,821.4E3,547.7E3,457.8E3,325.1E3,307.7E3,350.8E3,
X 305.3E3,748.6E3,963.4E3,872.6E3,917.1E3,890.6E3,1065.5E3,
X 1138.5E3,952.9E3,1104.7E3,1435.6E3,1654.1E3,1062.5E3,987.6E3,
X 1043.1E3,1027.8E3,674.5E3,1231.5E3,1098.0E3,752.6E3,773.2E3,
X 890.1E3,895.4E3,1368.5E3,1504.0E3,1255.4E3,1028.1E3,1036.0E3,
X 1337.7E3,1637.9E3,1301.2E3,1289.6E3,1240.5E3,984.5E3,746.8E3,
X 652.5E3
T SFAN=3.0E3,2.0E3,3.4E3,2.2E3,3.7E3,3.4E3,4.6E3,
X 3.8E3,4.1E3,3.2E3,8.0E3,9.1E3,11.2E3,19.7E3,
X 17.9E3,26.3E3,22.0E3,41.1E3,41.5E3,25.1E3,24.6E3,
X 11.2E3,22.4E3,28.3E3,39.8E3,43.2E3,38.0E3,29.7E3,
X 42.4E3,39.6E3,43.7E3,26.8E3,17.6E3,22.0E3,33.8E3,
X 22.3E3,40.1E3,35.2E3,21.5E3,22.5E3,14.4E3,9.6E3,
X 10.7E3,10.6E3,10.3E3,8.3E3,2.8E3,5.0E3,4.2E3,
X 4.0E3,24.7E3,22.6E3,23.3E3,12.8E3,16.0E3,22.8E3,
X 31.0E3,46.8E3,38.0E3,37.7E3,36.5E3,27.8E3,58.8E3,
X 48.3E3,47.3E3,52.5E3,31.0E3,27.4E3,24.6E3,50.6E3,
X 13.0E3,17.8E3,3.0E3,20.3E3,6.9E3,8.1E3,6.6E3,
X 13.3E3,12.5E3,23.4E3,29.9E3,40.6E3,33.1E3,36.9E3,
X 29.5E3,22.4E3,31.7E3,29.7E3,24.4E3,26.6E3,31.9E3,
X 22.0E3,25.0E3,24.9E3,26.6E3,22.2E3,18.5E3,13.9E3,
X 12.7E3,11.3E3,13.0E3,13.8E3,13.5E3,12.4E3,11.8E3,
X 9.8E3,8.6E3,9.0E3,10.3E3,9.7E3,8.1E3,10.6E3,
X 11.0E3
* TREARTSMODELL MED TIDSAVH. PARAMETRE 1868->1980/2080
N TIME=NTIME
C NTIME=1868
SPEC LENGTH=1980,DT=0.05,PRTPER=1,PLTPER=1
PLOT LODDE=L,TORSK=T,SEL=S
```

Simulation results with the basic case parameters from Table 6.12.

LODDE=CAPELIN

TORSK=COD

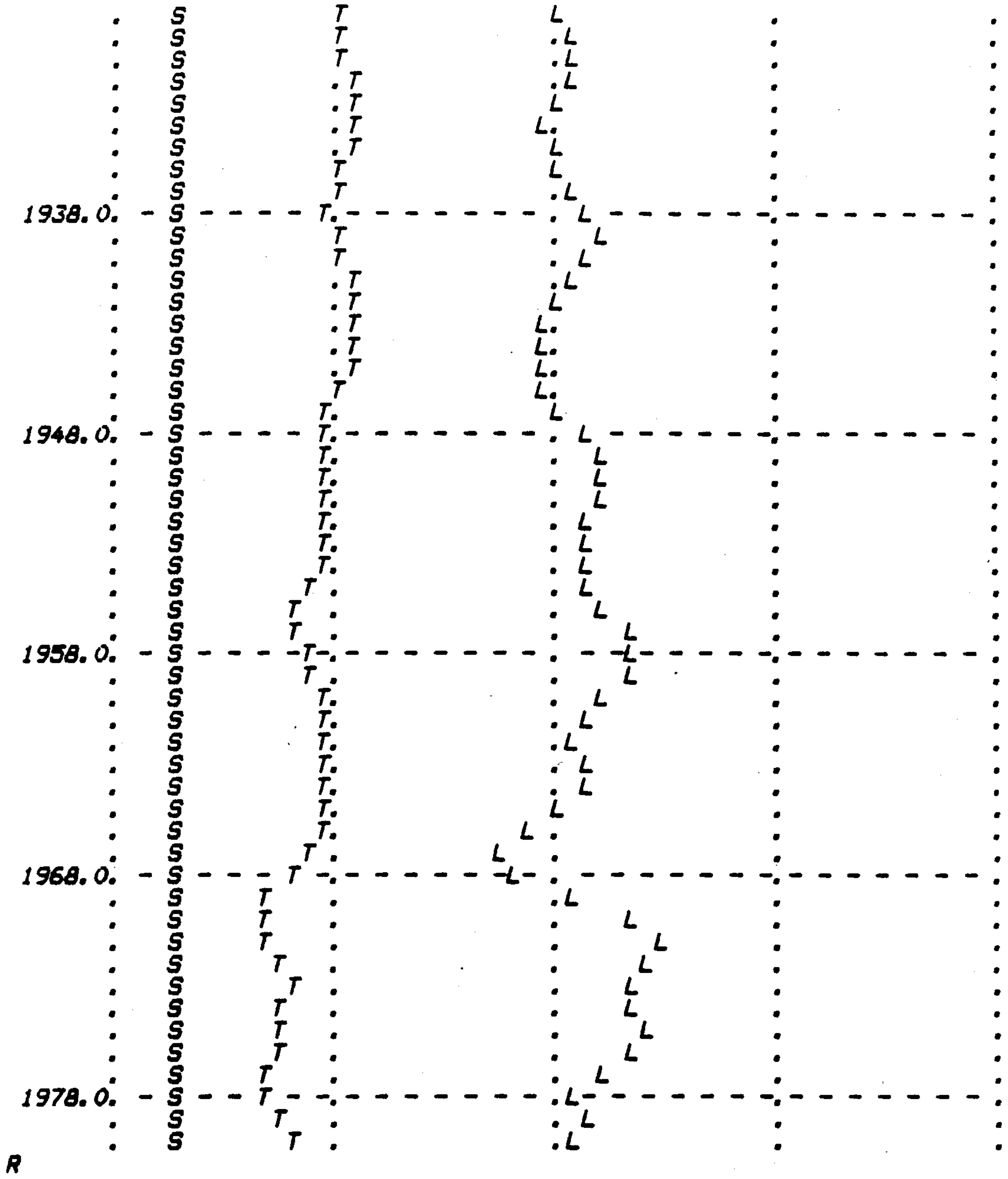
SEL =SEA MAMMALS

```
PAGE   2  TREARTSMODELL MED TIDSAVH. PARAMETRE 1868->1980

   TIME     LODDE    TORSK      SEL
   E+00      E+03     E+03     E+03
 1868.0   10069.   4759.0   1779.0
 1869.0   10022.   4611.0   1775.7
 1870.0   10128.   4535.5   1771.8
 1871.0   10251.   4527.0   1768.7
 1872.0   10323.   4544.2   1766.3
 1873.0   10393.   4557.2   1764.1
 1874.0   10415.   4575.3   1762.1
 1875.0   10414.   4575.6   1760.1
 1876.0   10429.   4593.8   1758.5
 1877.0   10425.   4589.2   1757.5
 1878.0   10458.   4575.7   1754.6
 1879.0   10489.   4583.1   1749.3
 1880.0   10521.   4568.7   1742.9
 1881.0   10550.   4584.1   1731.9
 1882.0   10558.   4635.8   1718.5
 1883.0   10501.   4712.7   1703.1
 1884.0   10400.   4761.2   1686.1
 1885.0   10346.   4757.5   1662.8
 1886.0   10323.   4760.7   1630.6
 1887.0   10307.   4805.7   1607.4
 1888.0   10261.   4855.2   1594.1
 1889.0   10211.   4872.3   1587.8
 1890.0   10193.   4865.5   1583.1
 1891.0   10182.   4888.1   1570.2
 1892.0   10123.   4903.9   1549.4
 1893.0   10100.   4896.7   1521.8
 1894.0   10163.   4921.2   1496.1
 1895.0   10202.   4979.9   1478.6
 1896.0   10170.   5063.6   1460.3
 1897.0   10057.   5130.8   1437.6
 1898.0    9945.   5179.8   1414.8
 1899.0    9868.   5246.8   1398.5


PAGE   3  TREARTSMODELL MED TIDSAVH. PARAMETRE 1868->1980

 1900.0    9781.   5272.2   1395.7
 1901.0    9717.   5259.3   1395.2
 1902.0    9695.   5240.3   1386.8
 1903.0    9679.   5235.3   1377.8
 1904.0    9687.   5231.1   1366.5
 1905.0    9720.   5259.1   1349.0
 1906.0    9751.   5288.3   1341.2
 1907.0    9781.   5301.1   1340.4
 1908.0    9761.   5302.8   1343.0
 1909.0    9714.   5265.4   1351.8
 1910.0    9715.   5198.7   1361.9
 1911.0    9774.   5136.1   1371.0
 1912.0    9878.   5039.6   1380.1
 1913.0   10042.   5005.2   1390.1
 1914.0   10102.   5049.0   1404.1
 1915.0   10065.   5085.1   1419.6
 1916.0    9976.   5122.0   1433.7
 1917.0    9861.   5152.3   1447.5
 1918.0    9745.   5159.6   1450.7
 1919.0    9643.   5136.6   1443.7
 1920.0    9642.   5105.7   1437.1
 1921.0    9714.   5099.3   1435.6
 1922.0    9750.   5082.0   1438.2
 1923.0    9826.   5040.8   1435.9
 1924.0    9939.   5019.5   1426.7
 1925.0   10028.   5018.3   1406.6
 1926.0   10103.   4978.1   1383.5
 1927.0   10189.   4970.3   1366.0
 1928.0   10174.   5025.6   1350.1
 1929.0   10126.   5017.0   1339.3
 1930.0   10173.   4985.3   1318.6
 1931.0   10237.   5075.6   1287.6
 1932.0   10169.   5202.7   1263.9
 1933.0    9958.   5272.8   1238.6
```

```
PAGE    4  TREARTSMODELL MED TIDSAVH. PARAMETRE 1868->1980

  1934.0    9800.   5262.4  1220.7
  1935.0    9847.   5195.3  1215.7
  1936.0    9987.   5050.4  1214.0
  1937.0   10273.   4851.5  1201.5
  1938.0   10667.   4769.6  1194.2
  1939.0   10915.   4881.1  1205.4
  1940.0   10763.   5152.7  1221.7
  1941.0   10329.   5363.6  1237.1
  1942.0    9953.   5446.1  1248.6
  1943.0    9715.   5435.3  1265.4
  1944.0    9646.   5356.4  1281.1
  1945.0    9661.   5297.5  1293.7
  1946.0    9734.   5104.1  1302.6
  1947.0   10059.   4792.7  1306.2
  1948.0   10514.   4654.9  1301.7
  1949.0   10866.   4678.5  1290.2
  1950.0   11029.   4759.1  1278.2
  1951.0   10876.   4765.5  1268.7
  1952.0   10703.   4634.5  1260.1
  1953.0   10782.   4587.7  1258.3
  1954.0   10668.   4577.7  1256.0
  1955.0   10645.   4334.9  1248.9
  1956.0   11059.   3955.1  1245.2
  1957.0   11592.   3876.9  1243.9
  1958.0   11774.   4190.5  1240.8
  1959.0   11514.   4468.6  1240.6
  1960.0   11053.   4602.2  1243.6
  1961.0   10663.   4774.2  1244.0
  1962.0   10445.   4733.2  1243.0
  1963.0   10560.   4527.9  1242.4
  1964.0   10593.   4607.5  1246.0
  1965.0   10163.   4753.6  1253.6
  1966.0    9392.   4634.2  1261.7
  1967.0    8762.   4325.1  1266.9


PAGE    5  TREARTSMODELL MED TIDSAVH. PARAMETRE 1868->1980

  1968.0    8921.   3857.5  1268.6
  1969.0   10181.   3404.9  1270.0
  1970.0   11517.   3255.5  1276.0
  1971.0   12172.   3448.8  1286.4
  1972.0   12141.   3786.3  1299.6
  1973.0   11805.   3918.9  1313.8
  1974.0   11807.   3709.1  1328.5
  1975.0   12067.   3521.5  1342.9
  1976.0   11719.   3508.3  1356.0
  1977.0   10852.   3434.6  1365.4
  1978.0   10489.   3413.8  1372.0
  1979.0   10597.   3617.8  1378.2
  1980.0   10495.   3944.5  1384.1
Q
T
```

Plotted results:

```
PAGE   6  TREARTSMODELL MED TIDSAVH. PARAMETRE 1868->1980/20
  LODDE=L  TORSK=T  SEL=S

   0.000M             5.000M             10.000M             15.000M            20.000M LTS
1868.0. - - -S- - - - - -T. - - - - - - - - - L - - - - - - - - - . - - - - - - - - - .
      .      S           T.                   L                   .                   .
      .      S           T.                   L                   .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
1878.0. - - -S- - - - - -T. - - - - - - - - - .L- - - - - - - - - . - - - - - - - - - .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .  L                .                   .
      .      S           T.                   .  L                .                   .
      .      S           T.                   .  L                .                   .
      .      S           T.                   .  L                .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
      .      S           T.                   .L                  .                   .
1888.0. - - -S- - - - - - T - - - - - - - - - .L- - - - - - - - - . - - - - - - - - - .
      .      S            T                   .L                  .                   .
      .      S            T                   .L                  .                   .
      .      S            T                   .L                  .                   .
      .      S            T                   L                   .                   .
      .      S            T                   L                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   .L                  .                   .
      .    S              T                   .L                  .                   .
      .    S              T                   L                   .                   .
1898.0. - -S- - - - - - - .T- - - - - - - - - L - - - - - - - - - . - - - - - - - - - .
      .    S              .T                  L                   .                   .
      .    S              .T                 L.                   .                   .
      .    S              .T                 L.                   .                   .
      .    S              .T                 L.                   .                   .
      .    S              .T                 L.                   .                   .
      .    S              .T                 L.                   .                   .
      .    S              .T                 L.                   .                   .
      .    S              .T                 L.                   .                   .
      .    S              .T                 L.                   .                   .
1908.0. - -S- - - - - - - .T- - - - - - - - -L. - - - - - - - - - . - - - - - - - - - .
      .    S              .T                 L.                   .                   .
      .    S              .T                 L.                   .                   .
      .    S              T                  L.                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   L                   .                   .
1918.0. - -S- - - - - - - T - - - - - - - - -L. - - - - - - - - - . - - - - - - - - - .
      .    S              T                  L.                   .                   .
      .    S              T                  L.                   .                   .
      .    S              T                  L.                   .                   .
      .    S              T                  L.                   .                   .
      .    S              T                  L.                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   L                   .                   .
      .    S              T                   .L                  .                   .
1928.0. - -S- - - - - - - T - - - - - - - - - .L- - - - - - - - - . - - - - - - - - - .
```



PAGE 7 TREARTSMODELL MED TIDSAVH. PARAMETRE 1868->1980/20

EQUILIBRIUM AND STABILITY CONDITIONS OF THE THREE SPECIES MODEL

Conditions for Existence of Equilibrium

The three equations that must be satisfied if an equilibrium with all 3 stocks present exists are from (3.6)-(3.8)

$$A_1 = X_1 + \gamma_{12}X_2 + \gamma_{13}X_3$$

$$A_2 = X_2/X_1 + \gamma_{23}X_3$$

$$A_3(X_1+\alpha X_2) = \alpha X_3 .$$

The third equation can be used to eliminate X_3 from the first two equations giving

$$X_2 = \frac{\alpha A_1 - (\alpha+\gamma_{13}A_3)X_1}{\alpha(\gamma_{12}+\gamma_{13}A_3)} \tag{A9.1}$$

and

$$X_2 = \frac{(\alpha A_2-\gamma_{23}A_3X_1)X_1}{\alpha(1+A_3\gamma_{23}X_1)} . \tag{A9.2}$$

Note that $X_3 > 0$ if $X_1 > 0$ and $X_2 > 0$ by the third equation above.

Eliminating X_2 from (A9.1) and (A9.2) and simplifying gives

$$(\gamma_{12}-\alpha)\gamma_{23}A_3X_1{}^2 + B_1X_1 + \alpha A_1 = 0 , \tag{A9.3}$$

where

$$-B_1 = \alpha(1+\gamma_{12}A_2) + \gamma_{13}A_3(1+\alpha A_2) - \alpha\gamma_{23}A_1A_3 .$$

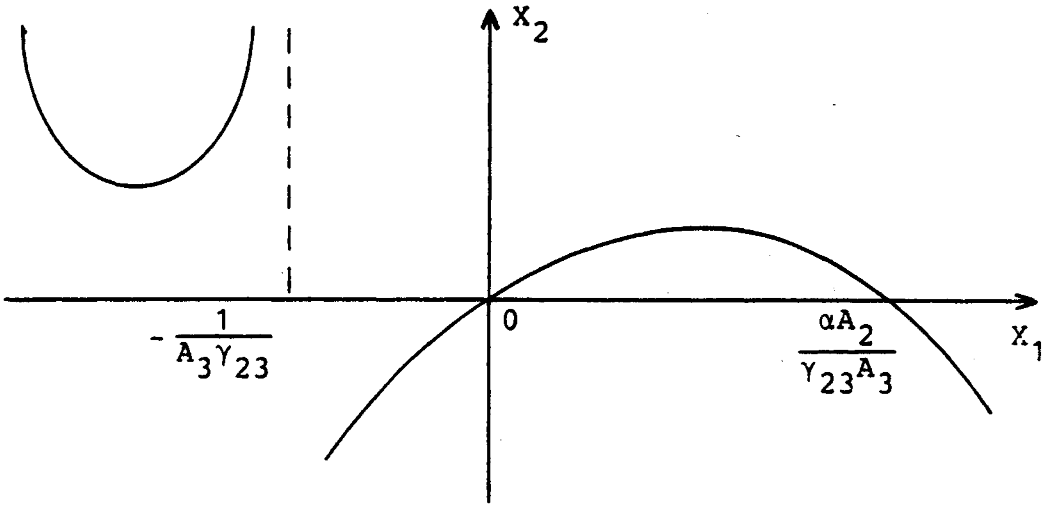
By the quadratic formula the formal solution is

$$X_1 = \frac{-B_1 \pm \sqrt{B_1^2 - 4\alpha\gamma_{23}(\gamma_{12}-\alpha)A_1A_3}}{2(\gamma_{12}-\alpha)\gamma_{23}A_3} \quad . \qquad (A9.4)$$

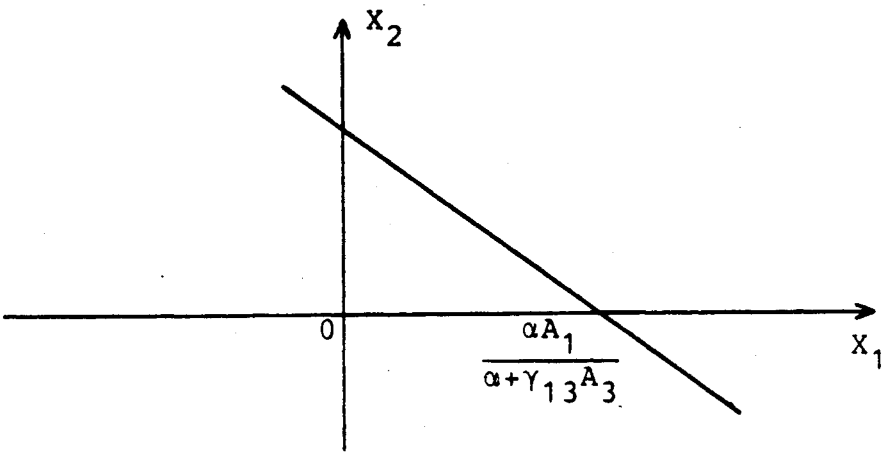
Note that by (2) $X_2 > 0$ if and only if

$$0 < X_1 < \frac{\alpha A_2}{\gamma_{23}A_3} \quad . \qquad (A9.5)$$

In fact, the graph of (A9.2) has the form



The graph of (A9.1) has the form



Stability of Equilibrium

One looks at the matrix A of the linearization of the system (3.6)-(3.8) at the equilibrium point. Stability (local) require that all eigenvalues have nonpositive real parts which is true if

$$\mathrm{tr}\,A < 0\ , \quad \Lambda^2 A > 0\ , \quad |A| < 0 \qquad (\Lambda^2 A = \text{sum of the 3 principal } 2\times 2 \text{ minors})$$

(i.e. all coefficients in characteristic equation are positive). From (3.6)-(3.8) in the text

$$A = \begin{bmatrix} r_1(A_1-2X_1-\gamma_{12}X_2-\gamma_{13}X_3) & -r_1\gamma_{12}X_1 & -r_1\gamma_{13}X_1 \\ r_2\dfrac{X_2^2}{X_1^2} & r_2(A_2-2X_2/X_1-\gamma_{23}X_3) & -r_2\gamma_{23}X_2 \\ \dfrac{r_3\alpha X_2^3}{(X_1+\alpha X_2)^2} & \dfrac{r_3\alpha^2X_3^2}{(X_1+\alpha X_2)^2} & r_3(A_3-2\alpha X_3/(X_1+\alpha X_2) \end{bmatrix} .$$

Note that $X_1+\gamma_{12}X_2+\gamma_{13}X_3 = A_1$ from (3.6), $X_2/X_1 + \gamma_{23}X_3 = A_2$ from (3.7) and $\alpha X_3/(X_1+\alpha X_2) = A_3$ from (3.8) so

$$A = \begin{bmatrix} -r_1X_1 & -r_1\gamma_{12}X_1 & -r_1\gamma_{13}X_1 \\ r_2\dfrac{X_2^2}{X_1^2} & -r_2\dfrac{X_2}{X_1} & -r_2\gamma_{23}X_2 \\ r_3\dfrac{A_3^2}{\alpha} & r_3A_3^2 & -r_3A_3 \end{bmatrix} .$$

There are then the following possibilities:

1. The roots of (A9.4) have the opposite signs (or (A9.3) is linear in which case there is a positive root). This happens when $\alpha-\gamma_{12} \geq 0$. In this case the line in the second graph intersects the curve in the first graph with a positive X_2 if and only if (since there is at most one intersection)

$$\frac{\alpha A_1}{\alpha+\gamma_{13}A_3} \leq \frac{\alpha A_2}{\gamma_{23}A_3}$$

or

$$\gamma_{23}A_1A_3 \leq \alpha A_2 + \gamma_{13}A_2A_3 \ .$$

2. If $\alpha-\gamma_{12} < 0$,and both roots of (A9.4) are real then they both have the same sign and this sign is positive providing $B_1 < 0$. In case

$$\frac{\alpha A_1}{\alpha+\gamma_{13}A_3} \leq \frac{\alpha A_2}{\gamma_{23}A_3} \quad \text{again}$$

one sees from the graphs that only the lesser root

$$X_1 = \frac{-B_1 - \sqrt{B_1{}^2-4\alpha\gamma_{23}(\gamma_{12}-\alpha)A_1A_3}}{2(\gamma_{12}-\alpha)\gamma_{23}A_3}$$

has $X_2 > 0$. Indeed this condition guarantees the existence of an equilibrium with $X_2 > 0$ so it implies that $B_1 < 0$ and the discriminant is positive.

3. If $\frac{\alpha A_1}{\alpha+\gamma_{13}A_3} > \frac{\alpha A_2}{\gamma_{23}A_3}$ and $\alpha-\gamma_{12} < 0$ then from the graphs either both roots are complex or both have $X_2 > 0$. The second case occurs when discriminant is nonnegative.

In summary, there is a unique equilibrium with all stocks positive if

$$\gamma_{23}A_1A_3 \leq \alpha A_2 + \gamma_{13}A_2A_3 \ .$$

If $\gamma_{23}A_1A_3 > \alpha A_2 + \gamma_{13}A_2A_3$ there is a double equilibrium if

$$\alpha < \gamma_{12} \quad \text{and} \quad B_1^2 - 4\alpha\gamma_{23}(\gamma_{12}-\alpha)A_1A_3 > 0 \ ,$$

and a single equilibrium if

$$\alpha < \gamma_{12} \quad \text{and} \quad B_1^2 - 4\alpha\gamma_{23}(\gamma_{12}-\alpha)A_1A_3 = 0 \ .$$

Otherwise there is no equilibrium with all stocks positive.

Then

$$\operatorname{tr} A = -r_1X_1 - r_2\,\frac{X_2}{X_1} - r_3A_3 < 0$$

and

$$\Lambda^2 A = \begin{vmatrix} -r_1X_1 & -r_1\gamma_{12}X_1 \\ r_2\,\dfrac{X_2{}^2}{X_1{}^2} & -r_2\,\dfrac{X_2}{X_1} \end{vmatrix} + \begin{vmatrix} -r_1X_1 & -r_1\gamma_{13}X_1 \\ r_3\,\dfrac{A_3{}^2}{\alpha} & -r_3A_3 \end{vmatrix} + \begin{vmatrix} -r_2\,\dfrac{X_2}{X_1} & -r_2\gamma_{23}X_2 \\ r_3A_3{}^2 & -r_3A_3 \end{vmatrix}$$

$$= r_1r_2\Big(X_2 + \gamma_{12}\,\frac{X_2{}^2}{X_1}\Big) + r_1r_3\Big(A_3X_1 + \frac{\gamma_{13}}{\alpha}\,A_3{}^2X_1\Big)$$

$$+\, r_2r_3\Big(A_3\,\frac{X_2}{X_1} + \gamma_{23}A_3{}^2X_2\Big) > 0\,.$$

Moreover

$$|A| = (-r_1X_1)(r_2X_2)\Big(r_3\,\frac{A_3}{\alpha}\Big)\begin{vmatrix} -1 & -\gamma_{12} & -\gamma_{13} \\ \dfrac{X_2}{X_1{}^2} & -\dfrac{1}{X_1} & -\gamma_{23} \\ A_3 & \alpha A_3 & -\alpha \end{vmatrix}$$

$$= \frac{-r_1r_2r_3}{\alpha}\,X_1X_2A_3\Big\{-\frac{\alpha}{X_1} + \gamma_{12}\gamma_{23}A_3 - \alpha\gamma_{13}A_3\,\frac{X_2}{X_1{}^2}$$

$$- \gamma_{13}A_3\,\frac{1}{X_1} - \alpha\gamma_{23}A_3 - \alpha\gamma_{12}\,\frac{X_2}{X_1{}^2}\Big\}$$

$$= \frac{r_1r_2r_3}{\alpha}\,A_3\,\frac{X_2}{X_1}\Big\{(\gamma_{12}-\alpha)\gamma_{23}A_3X_1{}^2 - (\alpha+\gamma_{13}A_3)X_1$$

$$- \alpha(\gamma_{12}+\gamma_{13}A_3)X_2\Big\}\,.$$

Now

$$\alpha(\gamma_{12}+\gamma_{13}A_3)X_2 = \alpha A_2 - (\alpha+\gamma_{13}A_3)X_1$$ (equivalent to (3.10) in text).

Thus

$$|A| = \frac{r_1r_2r_3}{\alpha} A_3 \frac{X_2}{X_1} \left\{(\gamma_{12}-\alpha)\gamma_{23}A_3X_1^2 - \alpha A_2\right\} , \tag{A9.6}$$

or using the quadratic equation defining X_1

$$|A| = \frac{r_1r_2r_3}{\alpha} A_3 \frac{X_2}{X_1} \left\{-B_1X_1 - \alpha(A_1+A_2)\right\} . \tag{A9.7}$$

In sum, an equilibrium is locally stable only if

$$(\gamma_{12}-\alpha)\gamma_{23}A_3X_1^2 - \alpha A_2 < 0 , \tag{A9.8}$$

or equivalently,

$$-B_1X_1 - \alpha(A_1+A_2) < 0 . \tag{A9.9}$$

Some general observations about stability can then be made. In particular if $\gamma_{12}-\alpha \leq 0$, then by (A9.3) the (unique) equilibrium is locally stable.

If, on the other hand, $\gamma_{12}-\alpha > 0$, then stability requires

$$X_1^2 < \frac{\alpha A_2}{(\gamma_{12}-\alpha)\gamma_{23}A_3}$$ by (A9.3).

Using the formula for X_1, this is equivalent to

$$[-B_1 \pm \sqrt{B_1{}^2-4(\gamma_{12}-\alpha)\gamma_{23}A_1A_3}\,]^2 < 4(\gamma_{12}-\alpha)\alpha\gamma_{23}A_2A_3 \ ,$$

which can be rewritten as

$$[1 \pm \sqrt{1-x}\,]^2 < \frac{A_2}{A_1} x$$

where

$$x = \frac{4(\gamma_{12}-\alpha)\gamma_{23}A_1A_3}{B_1{}^2} \leq 1$$

(note that when $\gamma_{12}-\alpha > 0$, and an equilibrium exists then $-B_1 > 0$).

Suppose now there is a double equilibrium (so $x < 1$). The equilibrium with the larger value of X_1 is stable if and only if

$$2 - x + 2\sqrt{1-x} < \frac{A_2}{A_1} x$$

$$2\sqrt{1-x} < (1 + \frac{A_2}{A_1})x - 2 \ .$$

Therefore stability requires

$$x \geq \frac{2A_1}{A_1+A_2} \qquad \text{(i.e. RHS} \geq 0)$$

and

$$4 - 4x < 4 - 4(1 + \frac{A_2}{A_1})x + (1 + \frac{A_2}{A_1})^2 x^2 \ ,$$

which simplifies to

$$0 < -4\,\frac{A_2}{A_1} + \left(\frac{A_1+A_2}{A_1}\right)^2 x$$

or

$$x > \frac{4A_1A_2}{(A_1+A_2)^2}\;.$$

Now suppose $A_1 \geq A_2$. Then

$$\frac{2A_2}{A_1+A_2} \leq 1 \leq \frac{2A_1}{A_1+A_2}$$

and $x \geq \frac{2A_1}{A_1+A_2}$ is false.

Therefore this equilibrium is unstable.

On the other hand if $A_1 < A_2$, then

$$\frac{2A_1}{A_1+A_2} < 1 < \frac{2A_2}{A_1+A_2}$$

and hence

$$\frac{2A_1}{A_1+A_2} < \frac{4A_1A_2}{A_1+A_2}\;,$$

so the equilibrium is stable if $x > \frac{4A_1A_2}{A_1+A_2}$.

The equilibrium corresponding to the smaller value of X_1 can be analyzed in a similar manner.

Stability requires

$$2 - x - 2\sqrt{1-x} < \frac{A_2}{A_1} x$$

or

$$2 - (1 + \frac{A_2}{A_1})x < 2\sqrt{1-x} \quad .$$

Thus the equilibrium is stable if

$$x > \frac{2A_1}{A_1 + A_2} \qquad \text{(i.e. LHS < 0)}$$

or if

$$4 - 4(1 + \frac{A_2}{A_1})x + (1 + \frac{A_2}{A_1})^2 \; x^2 < 4 - 4x \; ,$$

which simplifies to

$$-4\,\frac{A_2}{A_1} + (\frac{A_1 + A_2}{A_1})^2 \; x < 0$$

or

$$x < \frac{4A_1A_2}{(A_1 + A_2)^2} \quad .$$

Then in the case $A_1 > A_2$

$$x > \frac{2A_1}{A_1 + A_2} \quad \text{is not possible}$$

so stability requires

$$x < \frac{4A_1A_2}{(A_1 + A_2)^2} \quad .$$

In the case $A_1 \leq A_2$

$$\frac{2A_1}{A_1+A_2} \leq \frac{4A_1A_2}{(A_1+A_2)^2}$$

so one of the two inequalities must be true at least and the equilibrium is stable.

Finally, in the case $x = 1$, the equilibrium is stable if

$$1 < \frac{A_2}{A_1} \quad .$$

These results can be summarized as follows:

1. If $\alpha \geq \gamma_{12}$, the unique equilibrium is stable.
2. If $\alpha < \gamma_{12}$ and $x = 1$, the unique equilibrium is stable if $1 < A_2/A_1$, unstable if $1 > A_2/A_1$.
3. If $\alpha < \gamma_{12}$ and $x < 1$, then the equation defining X_1 may have two solutions but the larger root may have $X_2 < 0$. In any case
   - (a) if $A_2 = A_1$, the smaller root corresponds to a stable equilibrium and the larger root (if relevant) to an unstable equilibrium. The case includes the case of no fishing.
   - (b) if $A_1 > A_2$, the larger root (if relevant) is unstable and the smaller root is stable only if $x < 4A_1A_2/(A_1+A_2)^2$.

(c) if $A_1 < A_2$, the smaller root is stable and the larger root (if relevant) is stable only if $x > 4A_1A_2/(A_1+A_2)^2$.

A SIMULA PROGRAM FOR COMPUTATION OF EQUILIBRIUM STOCKS AND HARVEST RATES

```
BEGIN
  COMMENT
    **************************************************
    *                                                *
    *   FILNAVN PROGRAM     : KVFYSI                 *
    *   FILNAVN KOMPILERT   : KVFYSB                 *
    *   FILNAVN INNDATA     : INPUT                  *
    *   FILNAVN UTDATA      : LESES INN              *
    *   PROGRAMMERINGSSPRAK : SIMULA  1.2-353        *
    *   TYPE MASKIN         : CYBER 171MP            *
    *   OPERATIVSYSTEM      : NOS 2.0-531/528        *
    *   PROGRAMMERER        : SIGFUS KRISTMANNSSON/  *
    *                         OLA FLATEN             *
    *   INSTITUSJON         : POLLINN/IFF            *
    *   SEKSJON             : MARKED/ØKONOMI         *
    *   DATO                : 11/04/1985             *
    *   VERSJON             : 2                      *
    *                                                *
    **************************************************;

  COMMENT PROGRAMMET LESER INN HØYESTE/LAVESTE VERDI FOR EN PARAMETER
          OG VERDIER FOR 11 ANDRE. DATAENE LEGGES UT PÅ EN FIL TIL GRAFISK
          FREMSTILLING;

  REAL ARRAY PARAM(1:12);
  REAL A12,A13,A23,A1,A2,A3,G12,G13,G23,K1,K2,X1,X2,X3,W1,W2,W3,
       Y1,Y2,Y3,H1,H2,SQ,X,XMIN,XMAX,XSTEP;
  TEXT ARRAY TPARAM(1:12);
  INTEGER I,PARAMNR;
  TEXT FILNAVN;
  REF(OUTFILE)UT;

  COMMENT ******************************************
          *        H O V E D P R O G R A M         *
          ******************************************;

  FILNAVN:-BLANKS(7);
  OUTTEXT("LES INN NAVN PÅ DATAFIL");OUTIMAGE;INIMAGE;
  FILNAVN:=INTEXT(7);
  BEGIN
    UT:-NEW OUTFILE(FILNAVN.STRIP);
    UT.OPEN(BLANKS(80),0);
    FOR I:=1 STEP 1 UNTIL 12 DO
      TPARAM(I):-BLANKS(3);

    TPARAM(1):="U1";
    TPARAM(2):="U2";
    TPARAM(3):="U3";
    TPARAM(4):="R1";
    TPARAM(5):="R2";
    TPARAM(6):="R3";
    TPARAM(7):="A  ";
    TPARAM(8):="B  ";
    TPARAM(9):="K  ";
    TPARAM(10):="A12";
    TPARAM(11):="A13";
    TPARAM(12):="A23";

    PARAM(1):=0.0000;
    PARAM(2):=0.0000;
    PARAM(3):=0.0000;
    PARAM(4):=1.2704;
    PARAM(5):=1.1617;
    PARAM(6):=0.0614;
    PARAM(7):=0.90;
    PARAM(8):=0.12;
    PARAM(9):=30*10**6;
    PARAM(10):=0.125/10**6;
    PARAM(11):=0.140/10**6;
    PARAM(12):=0.310/10**6;
```

```
    FOR I:=1 STEP 1 UNTIL 12 DO
    BEGIN
      SETPOS(20);OUTINT(I,3);OUTTEXT(" ");
      OUTTEXT(TPARAM(I));OUTTEXT(" = ");
      OUTFIX(PARAM(I),12,22);OUTIMAGE;
    END FOR I;
    OUTIMAGE;OUTTEXT("NR. PÅ PARAMETER SOM SKAL FORANDRES");
    OUTIMAGE;OUTIMAGE;
    PARAMNR:=ININT;
    OUTTEXT("LES INN MIN OG MAX FOR ");
    OUTTEXT(TPARAM(PARAMNR));OUTIMAGE;OUTIMAGE;
    XMIN:=INREAL; XMAX:=INREAL;
    XSTEP:=(XMAX-XMIN)/50;

    FOR X:=XMIN STEP XSTEP UNTIL XMAX DO
    BEGIN
      PARAM(PARAMNR):=X;

      A1:=1-PARAM(1)/PARAM(4);
      A2:=1-PARAM(2)/PARAM(5);
      A3:=1-PARAM(3)/PARAM(6);

      G12:=PARAM(10)*PARAM(7)*PARAM(9)/PARAM(4);
      G13:=PARAM(11)*PARAM(7)*PARAM(8)*PARAM(9)/PARAM(4);
      G23:=PARAM(12)*PARAM(7)*PARAM(8)*PARAM(9)/PARAM(5);

      K1:=A1*A3*G23-1-G12*A2-A3*G13/PARAM(7)-A2*A3*G13;
      K2:=A3*G23*(G12/PARAM(7)-1);


      SQ:=K1**2-4*A1*K2;
      IF SQ(0 THEN
      BEGIN
        OUTTEXT("VERDI I KVADRATROT ER NEGATIV, SETTES TIL 0");OUTIMAGE;
        SQ:=0;
        UT.OUTCHAR('*');
      END ELSE
        UT.OUTCHAR(' ');

      X1:=(-K1-SQRT(SQ))/(2*K2);

      H1:=1/PARAM(7)+A2;
      H2:=1+A3*G23*X1;

      X2:=((A2-A3*G23*X1/PARAM(7))*X1)/H2;
      X3:=(A3*H1*X1)/H2;

      W1:=X1*PARAM(9)/1000;
      W2:=X2*PARAM(7)*PARAM(9)/1000;
      W3:=X3*PARAM(7)*PARAM(8)*PARAM(9)/1000;

      Y1:=PARAM(1)*W1;
      Y2:=PARAM(2)*W2;
      Y3:=PARAM(3)*W3;

      UT.OUTFIX(X,12,22);
      UT.OUTFIX(W1,3,12);UT.OUTFIX(W2,3,12);
      UT.OUTFIX(W3,3,12);UT.OUTIMAGE;
    END FOR X;

    OUTIMAGE;OUTTEXT("DATA PÅ FILEN ");OUTTEXT(FILNAVN);
    OUTIMAGE;OUTIMAGE;
  END FILNAVN;
END PROGRAM;
```

THE PARTIAL DERIVATIVES OF THE GROWTH FUNCTIONS

$$G_{ij} = \frac{\partial F_j(X_1, X_2, X_3)}{\partial X_i} \qquad (i,j = 1,2,3)$$

$$G_{11} = r_1(1 - 2X_1 - \gamma_{12}X_2 - \gamma_{13}X_3)$$

$$G_{21} = -\gamma_{12}r_1X_1$$

$$G_{31} = -\gamma_{13}r_1X_1$$

$$G_{12} = r_2X_2{}^2/X_1{}^2$$

$$G_{22} = r_2(1 - 2X_2/X_1 - \gamma_{23}X_3)$$

$$G_{32} = -\gamma_{23}r_2X_2$$

$$G_{13} = r_3X_3{}^2/(\alpha(X_1/\alpha + X_2)^2)$$

$$G_{23} = r_3X_3{}^2/(X_1/\alpha + X_2)^2$$

$$G_{33} = r_3 - 2r_3X_3/(X_1/\alpha + X_2)$$

APPENDIX 11

TWO FORTRAN PROGRAMS FOR COMPUTATION OF THE TSB-MODEL'S OPTIMAL STOCKS, SUSTAINABLE RENTS AND NET PRESENT VALUE

The OPT-program

```
      PROGRAM TEST2(INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT)
      REAL X(3),FVEC(3),XTOL,WA(51),R(14),A(6),Y(3),V(4)
      REAL B(6),C(4)
      COMMON R
      EXTERNAL FCN
C
C      A INNEHOLDER DE BIOLOGISKE PARAMETRENE I SAMME
C      REKKEFØLGE SOM PÅ ARKET.
      A(1)=1.2704
      A(2)=1.1617
      A(3)=0.0614
      A(4)=0.90
      A(5)=0.12
      A(6)=3E7
C
C      B INNEHOLDER DE ØKONOMISKE PARAMETRENE I SAMME
C      REKKEFØLGE SOM PÅ ARKET.
      B(1)=0.7926
      B(2)=2.3279
      B(3)=3.2029
      B(4)=3.7332E6
      B(5)=8.4172E6
      B(6)=2.7273E6
C
C      C INNEHOLDER DE FASTE PARAMETRENE I SAMME REKKEFØLGE
C      SOM PÅ ARKET.
      C(1)=0.125E-6
      C(2)=0.140E-6
      C(3)=0.310E-6
      C(4)=0.05
C
```

```
      R(1)=A(1)
      R(2)=A(2)
      R(3)=A(3)
      R(4)=A(4)
      R(5)=B(4)
      R(6)=B(5)
      R(7)=B(6)
      R(8)=B(1)*A(6)
      R(9)=B(2)*R(4)*A(6)
      R(10)=B(3)*R(4)*A(5)*A(6)
      R(11)=C(1)*R(4)*A(6)/R(1)
      R(12)=C(2)*R(4)*A(5)*A(6)/R(1)
      R(13)=C(3)*R(4)*A(5)*A(6)/R(2)
      R(14)=C(4)
      WRITE(2,1)
      FORMAT("GI VERDIER FOR X1,X2 OG X3")
      READ(2,*)  (X(I),I=1,3)
      TOL=SQRT(X02AAF(0.0))
      IFAIL=0
      N=3
      CALL C05NBF(FCN,N,X,FVEC,TOL,WA,51,IFAIL)
      WRITE(2,*) IFAIL
      Y(1)=R(1)*X(1)*(1-X(1)-R(11)*X(2)-R(12)*X(3))
      Y(2)=R(2)*X(2)*(1-X(2)/X(1)-R(13)*X(3))
      Y(3)=R(3)*X(3)*(1-X(3)/((X(1)/R(4))+X(2)))
      V(1)=(R(8)-R(5)/X(1))*Y(1)
      V(2)=(R(9)-R(6)/X(2))*Y(2)
      V(3)=(R(10)-R(7)/X(3))*Y(3)
      V(4)=(V(1)+V(2)+V(3))/R(14)
      WRITE(2,*)  (X(I),I=1,3)
      WRITE(2,*)  (Y(I),I=1,3)
      WRITE(2,*)  (V(I),I=1,4)
      FORMAT(F12.8)
      END
      SUBROUTINE FCN(N,X,FVEC,IFLAG)
      INTEGER N,IFLAG
      REAL X(N),FVEC(N),R(14)
      COMMON R
      FVEC(1)=R(1)*(X(1)**2*R(8)-X(1)*R(5))*(1-2*X(1)
     * -R(11)*X(2)-R(12)*X(3))+R(2)*(R(9)*X(2)**2-R(6)*X(2))
     * +(R(10)*X(3)**2-R(7)*X(3))*R(3)*R(4)*X(1)**2/
     * (X(1)+R(4)*X(2))**2
     * +R(5)*R(1)*X(1)*(1-X(1)-R(11)*X(2)-R(12)*X(3))
     * -R(14)*(R(8)*X(1)**2-R(5)*X(1))
      FVEC(2)=-R(2)*R(11)*X(2)*(R(8)*X(1)**2-R(5)*X(1))
     * +R(2)*(R(9)*X(2)-R(6))*(X(1)-2*X(2)-R(13)*X(1)*X(3))
     * +(R(10)*X(3)**2-R(7)*X(3))
     * *R(3)*R(4)**2*X(1)*X(2)/(X(1)+R(4)*X(2))**2
     * +R(6)*R(2)*(X(1)-X(2)-R(13)*X(1)*X(3))
     * -R(14)*X(1)*(R(9)*X(2)-R(6))
      FVEC(3)=-R(1)*R(12)*X(3)*(X(1)*R(8)-R(5))
     * -R(2)*R(13)*X(3)*(R(9)*X(2)-R(6))
     * +R(3)*(X(3)*R(10)-R(7))*(1-2*R(4)*X(3)/(X(1)+R(4)*X(2)))
     * +R(7)*R(3)*(1-R(4)*X(3)/(X(1)+R(4)*X(2)))
     * -(R(10)*X(3)-R(7))*R(14)
      RETURN
      END
```

The OPV-program

```
      PROGRAM TEST2(INPUT,OUTPUT,TAPE1=INPUT,TAPE2=OUTPUT)
      REAL (3),FVEC(3),XTOL,WA(51),R(14),A(6)
      REAL B(6),C(4)
      COMMON R
      EXTERNAL FCN
C
C     A INNEHOLDER DE BIOLOGISKE PARAMETRENE I SAMME
C     REKKEFØLGE SOM PÅ ARKET.
      A(1)=1.2704
      A(2)=1.1617
      A(3)=0.0614
      A(4)=0.90
      A(5)=0.12
      A(6)=3.0E7
C
C     B INNEHOLDER DE ØKONOMISKE PARAMETRENE I SAMME
C     REKKEFØLGE SOM PÅ ARKET.
      B(1)=0.5284
      B(2)=2.3279
      B(3)=3.2029
      B(4)=3.7332E6
      B(5)=8.4172E6
      B(6)=2.7273E6
C
C     C INNEHOLDER DE FASTE PARAMETRENE I SAMME REKKEFØLGE
C     SOM PÅ ARKET.
      C(1)=0.125E-6
      C(2)=0.140E-6
      C(3)=0.310E-6
      C(4)=0.05
C
C
      R(1)=A(1)
      R(2)=A(2)
      R(3)=A(3)
      R(4)=A(4)
      R(5)=B(4)
      R(6)=B(5)
      R(7)=B(6)
      R(8)=B(1)*A(6)
      R(9)=B(2)*R(4)*A(6)
      R(10)=B(3)*R(4)*A(5)*A(6)
      R(11)=C(1)*R(4)*A(6)/R(1)
      R(12)=C(2)*R(4)*A(5)*A(6)/R(1)
      R(13)=C(3)*R(4)*A(5)*A(6)/R(2)
      R(14)=C(4)
      WRITE(2,1)
 1    FORMAT("GI VERDIER FOR X1,X2 OG X3")
      READ(2,*) (X(I),I=1,3)
      TOL=SQRT(X02AAF(0.0))
      IFAIL=0
      N=3
      CALL C05NBF(FCN,N,X,FVEC,TOL,WA,51,IFAIL)
      WRITE(2,*) IFAIL
```

```
      WRITE(2,*) (X(I),I=1,3)
      END
      SUBROUTINE FCN(N,X,FVEC,IFLAG)
      INTEGER N,IFLAG
      REAL X(N),FVEC(N),R(14)

      COMMON R
      FVEC(1)=R(1)*(X(1)**2*R(8)-X(1)*R(5))*(1-2*X(1)
     *-R(11)*X(2)-R(12)*X(3))+R(2)*(R(9)*X(2)**2-R(6)*X(2))
     *+(R(10)*X(3)**2-R(7)*X(3))*R(3)*R(4)*X(1)**2/
     *(X(1)+R(4)*X(2))**2
     *+R(5)*R(1)*X(1)*(1-X(1)-R(11)*X(2)-R(12)*X(3))
     *-R(14)*(R(8)*X(1)**2-R(5)*X(1))
      FVEC(2)=-R(2)*R(11)*X(2)*(R(8)*X(1)**2-R(5)*X(1))
     *+R(2)*(R(9)*X(2)-R(6))*(X(1)-2*X(2)-R(13)*X(1)*X(3))
     *+(R(10)*X(3)**2-R(7)*X(3))
     **R(3)*R(4)**2*X(1)*X(2)/(X(1)+R(4)*X(2))**2
     *+R(6)*R(2)*(X(1)-X(2)-R(13)*X(1)*X(3))
     *-R(14)*X(1)*(R(9)*X(2)-R(6))
      FVEC(3)=-R(1)*R(12)*X(3)*(X(1)*R(8)-R(5))
     *-R(2)*R(13)*X(3)*(R(9)*X(2)-R(6))
     *+R(3)*(X(3)*R(10)-R(7))*(1-2*R(4)*X(3)/(X(1)+R(4)*X(2)))
     *+R(7)*R(3)*(1-R(4)*X(3)/(X(1)+R(4)*X(2)))
     *-(R(10)*X(3)-R(7))*R(14)
      RETURN
      END
```

Table A12. Norwegian capelin and herring catches used for fresh and frozen products 1978-80.

Year	Capelin		Herring[1]		Total		
	Quantity (metric ton)	Value ('000 Nkr)	Quantity (metric ton)	Value ('000 Nkr)	Quantity (metric ton)	Value ('000 Nkr)	Price[2] ('000 Nkr/m.t.)
1978	6924	6431	3412	13303	10336	19734	1.9092
1979	24754	25952	1670	8545	26424	34497	1.3055
1980	28314	32967	2261	6682	30575	39649	1.2968
Average price	-	-	-	-	-	-	1.503

[1] Includes winter herring and fat herring.

[2] Price equals Value divided by Quantity in the Total column.

Source: Fisheries Statistics, 1978, 1979 and 1980.

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The Economics of Predator-Prey Harvesting

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1. Introduction

The number of species within a given habitat is often large, and the way they interact may be very complex. This applies to terrestrial as well as to marine ecosystems. Nevertheless, in most cases man utilizes just a few of the species. This is probably one of the reasons why scientists often use relatively simple models to study the population dynamics and other aspects of renewable resources. Another important reason is, obviously, that simple models are easier to analyse than complex models.

Examples of predator-prey relationships are: shark – fish (D'Ancona 1926), whale – krill (May et al. 1979), sea mammals – fish (Flaaten 1988), polar bear – ringed seal (Larsen 1986a and 1986b), wolf – reindeer (Ingold 1980) and lynx – hare (May 1974). Studies of these predator-prey systems include, inter alia, mathematical, ecological, economic and anthropological aspects of the management of the resources.

Since the seminal work by Gordon (1954), theoretical bioeconomic studies have focused on the difference between open-access harvesting and socially optimal harvesting. Multispecies analyses of these kind are to be found in Quirk and Smith (1970), Anderson (1975), Clark (1976), and Silvert and Smith (1977), all of whom

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mainly use generally formulated growth equations. Hannesson (1983) utilizes Lotka-Volterra types of growth equations from Larkin (1966) to study open-access and socially-optimal harvesting of the combined resources.

The aim of this paper is to review ecological and economic aspects of predator-prey harvesting within the framework of a simple bioeconomic model. The focus is mainly on sustainable yields and comparative statics, rather than on the dynamic behaviour of the system. In addition to this simplified approach to the ecological and economic theory of predator-prey harvesting, we will also develop the conditions under which it is possible to have an increasing optimal resource stock as a function of the social rate of discount.

In the next section we review a simple but robust predator-prey model. Open-access harvesting of the resources is studied in Section 3, and optimal management is studied in Section 4. Our findings are summarized briefly in the final section.

2. Predator-Prey Relationships

It is many years since theoretical biologists started studying ecological systems by means of mathematical models (e.g., Lotka 1925 and Volterra 1928). In the classical Lotka-Volterra model, the size of the two stocks oscillates with a period determined largely by the parameters of the model, while the amplitude is determined solely by the initial conditions (May 1981, 79). The model is "structurally unstable", meaning that the slightest alteration in the functional form of the growth equations will tip the dynamics towards a stable point or towards a stable limit cycle. Structurally unstable models are considered to have no place in biology, but nevertheless the Lotka-Volterra model highlights one of the general properties of predator-prey models, namely the propensity to oscillate.

Most two-species predator-prey models analysed in the literature can be shown to have either a stable point or a stable limit cycle (May 1981, 81). A simple model capturing the essential elements of a predator-prey system is that of May et al. (1979). In this model, as opposed to the classical Lotka-Volterra model, the growth of the prey is density dependent in absence of the predator, and the predator has a positive intrinsic growth rate. The carrying capacity of the prey is constant,[2] and the model has a stable point. This predator-prey model was used by May et al. (1979) particularly to discuss the Antarctic ecosystem. Maximum sustainable yield properties of this model were studied by Beddington and May (1980) and further elaborated by Beddington and Cook (1982). The latter also investigated the stability

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[2] Larkin (1966) analyses a predator-prey model where each of the two species has its own constant carrying capacity. This expands the sustainable yield region of the system, especially for low levels of the prey stock.

properties of various harvesting regimes. Since the model is simple but still captures the essential elements of a predator-prey system, it will be used in this paper.

Let W_1 and W_2 denote the stocks of the prey and the predator, respectively. Then, the model is specified as

$$\dot{W}_1 = dW_1/dt = r_1 W_1(1 - W_1/K) - aW_1W_2, \tag{1}$$

$$\dot{W}_2 = dW_2/dt = r_2 W_2(1 - W_2/\alpha W_1), \tag{2}$$

where r_1 and r_2 are the intrinsic growth rates of the respective species. K is the carrying capacity of the total system, the level to which the prey will settle in the absence of both predator and harvest.

In case of no predators, the per capita[3] growth rate of the prey decreases from r_1, for stock levels close to zero, to zero for stock levels equal to the carrying capacity. If predators exist, the per capita growth rate for the prey equals zero for a stock level lower than the carrying capacity. The presence of predators reduces the per capita growth rate in proportion to the biomass of the predator. The predation coefficient, a, tells how much the per capita growth rate of the prey is reduced per unit of the predator. Or to put it another way, a tells which share of the prey stock one unit of the predator is consuming per unit of time. The total rate of consumption is expressed in the term aW_1W_2.

The predator's per capita growth rate decreases from r_2, when its own stock level is close to zero, to zero for a stock level equal to its own carrying capacity, which is proportional to the level of the prey stock. The proportionality coefficient is α.

The equilibrium values for the prey and the predator populations, W_1 and W_2 respectively, are obtained by letting $dW_1/dt = 0$ and $dW_2/dt = 0$ in equations (1) and (2). Without harvesting there is a unique, stable equilibrium solution:

$$W_1{}^* = K/(1+\nu), \tag{3}$$

$$W_2{}^* = \alpha K/(1+\nu), \tag{4}$$

where $\nu = a\alpha K/r_1$.

It should be noticed that the intrinsic growth rate of the predator, r_2, does not affect the equilibrium values of either of the two species. The equilibrium values of both species increase with any increase in r_1 or K, ceteris paribus. From (3) and (4) it follows:

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[3] The term "per capita" is used, even though we mean per unit of biomass.

(5) $W_2^*/W_1^* = \alpha$.

In equilibrium, α expresses the relative size of the predator stock to that of its prey.

Even though r_2 does not affect the equilibrium values of the two stocks, it is of importance to the behaviour of the system outside equilibrium. That is, r_2 will affect the time the predator will need to reach equilibrium from a higher or lower level.

We shall now, in a simple way, introduce harvesting as a factor in the model. Suppose that the fish stocks are harvested independently with constant effort per unit of time, F_i, scaled such that $F_1 = 1$ corresponds to constant catchability coefficients equal to r_i. Then the catch rates will be

(6) $h_1 = r_1 F_1 W_1$,

(7) $h_2 = r_2 F_2 W_2$.

The introduction of harvesting will influence the growth rates in (1) and (2), which will be changed to

(8) $\dot{W}_1 = r_1 W_1 (1 - W_1/K) - aW_1W_2 - r_1F_1W_1$,

(9) $\dot{W}_2 = r_2 W_2 (1 - W_2/\alpha W_1) - r_2F_2W_2$.

It may be useful to rewrite the variables W_1 and W_2 into a dimensionless form. Defining $X_1 = W_1/K$ and $X_2 = W_2/\alpha K$, we can rewrite equations (8) and (9) as

(10) $\dot{X}_1 = r_1 X_1 (1 - F_1 - X_1 - \nu X_2)$,

(11) $\dot{X}_2 = r_2 X_2 (1 - F_2 - X_2/X_1)$.

Recall that the dimensionless parameter ν is defined as $\nu = a\alpha K/r_1$.

The equilibrium properties of this ecological system depend only on the fishing efforts, F_1 and F_2, and ν. The dynamics additionally involve r_1 and r_2.

The phase-diagram for the system (10) and (11) is shown in Figure 1. The isoclines are found by setting $dX_1/dt = 0$ and $dX_2/dt = 0$ in (10) and (11). This gives

(12) $X_2 = (1/\nu)(1 - F_1 - X_1)$ for $dX_1/dt = 0$,

(13) $X_2 = (1 - F_2) X_1$ for $dX_2/dt = 0$.

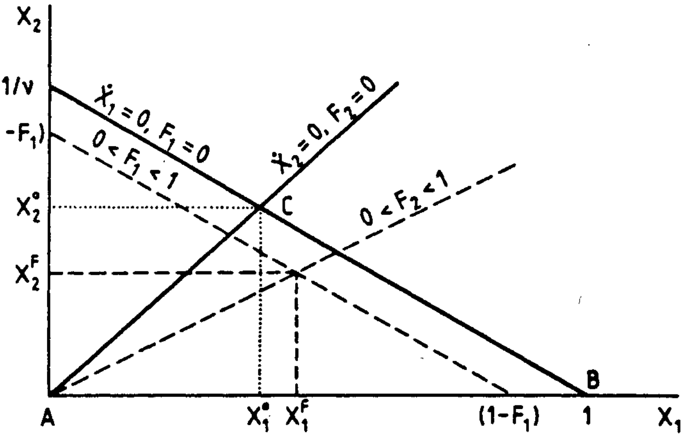

FIG. 1. The phase diagram for the predator-prey model.

If positive equilibrium levels of X_1 and X_2, denoted by X_1^F and X_2^F, exist simultaneously, they are found where the isoclines intersect:

(14) $X_1^F = (1 - F_1)/[1 + \nu(1 - F_2)]$,

(15) $X_2^F = (1 - F_1)(1 - F_2)/[1 + \nu(1 - F_2)]$.

With harvesting taken into account relative stock size is

(16) $X_2^F/X_1^F = (1 - F_2)$.

It can be seen from (14) that only for $F_1 < 1$ will there exist a positive equilibrium value of the prey. If $F_1 \geq 1$ the prey-stock will be extinct, and so of course will be the predator, as seen from (15). The latter expression shows that only for $F_2 < 1$ and $F_1 < 1$ will the predator survive.

The equilibrium values of both species, as would be expected, increase with decreasing fishing pressure on the prey, i.e. for reduced F_1. The larger prey stock results in an increased carrying capacity for the predator, thus implying a higher equilibrium level of the predator stock.

On the other hand, the effects on the prey and on the predator from decreased fishing pressure on the predator are the opposite of each other. From (14), the equilibrium value of the prey will decrease, and from (15), the equilibrium value of the predator will increase. The increased stock level for the predator means heavier predation on the prey, and thereby a reduced equilibrium level for the latter.

In Figure 1 the pre-harvesting stock levels of the prey and the predator are denoted by $X_1°$ and $X_2°$, respectively. This figure illustrates that, in the case of harvesting, the equilibrium level of the predator will always be below its pristine level. This result is independent of whether we harvest only the predator, only the prey, or both are harvested simultaneously. On the other hand, the equilibrium level of the prey will increase with the harvesting of the predator. In Figure 1, the relative strength of the predator harvesting to that of the prey is such that the net result for the prey is a higher stock level with harvesting than without. The sustainable yield area in Figure 1 is the triangle *ABC*. Combinations of stock levels outside this triangle are not possible to maintain.[4]

3. Open-Access Harvesting

In the preceding section we have seen that a predator-prey system may be capable of being harvested on a sustainable yield basis for both of the species. When the species are open-access resources, we may be interested in knowing what the equilibrium levels of the stocks will be and the corresponding harvest rates. This includes the possibilities of extinction of one or both of the stocks. It is also interesting to study the dynamics of the stocks and the harvest industries to see whether the equilibrium point is reached or not.[5] However, in this paper we shall concentrate on analysing equilibrium points by means of comparative statistics.

Given the Schaefer production function of equations (6) and (7), and assuming constant costs, c_i, per unit of rescaled effort, $E_i = r_i F_i$, the unit harvesting cost becomes

$$(17) \qquad c_i(X_i) = c_i / X_i \qquad (i = 1,2).$$

The demand for each of the two species is assumed to be independent of the price of the other one and infinitely elastic with respect to its own price. Thus, p_1 and p_2 are the constant prices of the prey and the predator, respectively.

The net profit per unit of harvest under these assumptions is[6]

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[4] Essentially all deterministic two-species models will have a bounded sustainable yield area, but it need not be a triangle. The isoclines could be curves instead of straight lines, or the area could be, e.g. a quadrangle. The latter is the case for the predator-prey model designed by Larkin (1966) where the abundance of the predator has a lower limit in the absence of the prey.

[5] Beddington and Cook (1982) have studied the stability properties of this model for various harvesting regimes, from a biological point of view.

[6] c_i includes the alternative cost of capital and labour. That is to say that normal profit on capital and normal renumeration of labour are included in the costs. The net profit in equations (18) and (19) is therefore the pure resource rent.

(18) $$b_i(X_i) = p_i - c_i/X_i \qquad (i = 1,2),$$

and the total profit for each of the species is

(19) $$\pi_i = (p_i - c_i/X_i)y_i \qquad (i = 1,2),$$

where $y_1 = h_1/K$ and $y_2 = h_2/\alpha K$ are the normalized harvest rates of the prey and the predator, respectively.

Bioeconomic equilibrium is defined as a situation in which we simultaneously have

(20) $$\pi_i = 0, \ \dot{X}_i = 0 \qquad (i = 1,2).$$

When there is zero net profit, no potential harvester has the incentive to enter the industry and no existing harvesters have the incentive to leave the business.

Let us denote the open access stock levels of the prey and the predator as X_1^∞ and X_2^∞, respectively. From equation (19) it is seen that, unless $y_i = 0$ for either of the two species, we have the following simple expression for the open access stock levels:

(21) $$X_i^\infty = c_i/p_i \qquad (i = 1,2).$$

The open-access stock level of a species depends solely on the ratio of own effort cost to own price of the yield. The corresponding harvest rates are found by substituting X_i^∞ from equation (21) into the growth equations, (10) and (11). Under our assumptions, the biological parameters have no effects on the open-access stock levels. However, they will affect the harvest rates, as seen from equations (10) and (11).

Figure 2 shows the open-access stock levels derived from the ratio of the cost of effort to the price of harvest given in equation (21). The two sloping, broken lines are the corresponding isoclines for $F_1 = F_1^\infty$ and $F_2 = F_2^\infty$, and of course they intersect at the equilibrium point.

As noted above, equation (21) is valid provided that an open access solution exists with simultaneous harvest of the prey and the predator. If that is not the case, if, for example, only one of the two species is harvested at the equilibrium, the stock level of that particular species is given by equation (21). The stock level of the other species, however, has to be found in another way. We distinguish two cases: first, the case with no harvest of the predator, and then the case with no prey harvest.

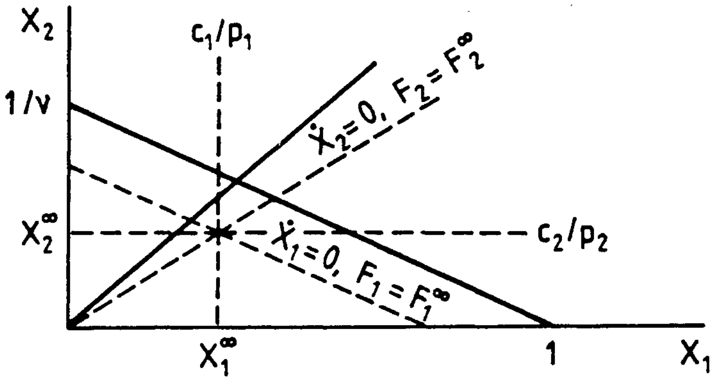

FIG. 2. Interior open-access equilibrium with simultaneous harvest of the prey and the predator.

No Predator Harvest

In general there will never be open-access harvesting of the predator if its own price is lower than the unit harvesting cost at the pristine stock level. Thus we have

$$(22) \quad p_2 < c_2/X_2^o \Rightarrow F_2 = y_2 = 0.$$

Therefore, a necessary condition for open-access harvesting of the predator is that the ratio of the cost of effort to the price of harvest must be lower than the pristine stock level. However, this is not a sufficient condition. The following example illustrates this. If the prey is a low-cost, high-price species, as shown by $X_1 = c_1/p_1$ in Figure 3, the predator stock will be reduced below its profitability level, c_2/p_2, and it will not be harvested at the equilibrium point (X_1^∞, X_2^∞).[7]

Since the predator's carrying capacity is proportional to the prey stock in this model, we will always have

$$(23) \quad X_2^\infty = X_1^\infty = c_1/p_1,$$

when the predator is unharvested, i.e. when $F_2 = 0$. This is likely to happen when the prey is "inexpensive-to-catch and valuable" compared to the predator. The

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[7] If the predator had its own, constant carrying capacity, as in Larkin (1966), the chances increase that it could economically sustain a harvest even if the prey stock is reduced through harvesting.

open-access stock level of the unharvested predator depends solely on the effort cost of prey/price of prey ratio.

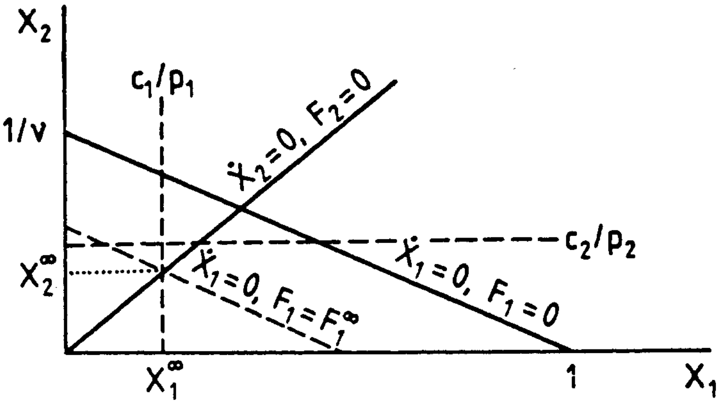

FIG. 3. Open-access equilibrium with no harvesting of the predator.

No Prey Harvest

When the prey is an " expensive-to-catch trash" species compared to the predator, the possibility emerges for this species being unharvested under open access. In this case we have

$$(24)\qquad \dot{X}_1 = 0, \qquad F_1 = 0.$$

Using equations (10), (24) and (21) for $i = 1$, we derive the open-access unharvested prey stock,

$$(25)\qquad X_1^\infty = 1 - \nu X_2^\infty = 1 - \nu c_2/p_2,$$

when the predator is harvested. Equation (25) shows that the open-access level of the prey is a linear, decreasing function of the predator stock, which is determined solely by the predator's cost of effort – own price ratio. In other words, the unharvested prey stock is greater, the more "inexpensive-to-catch and valuable" the predator is. As seen from Figure 1, harvesting of the predator increases the prey stock. Therefore, it might well happen that a prey species which is not able to support an open-access harvest industry of its own when the predator is unharvested, can do so if the predator is harvested. This is illustrated in Figure 4 where the prey's cost of effort – price of harvest ratio is c_1/p_1. With $c_2/p_2 = A$ there will be no harvest of the predator-prey system, i.e. the open-access stock levels equal the pristine levels, $X_1{}^\circ$ and $X_2{}^\circ$. If, for one reason or another, the c_2/p_2 ratio decreases from A to B, the predator becomes economical to harvest, but the prey is unharvested at the stock level $X_1{}^B$, since the equilibrium point is on the border of the sustainable yield triangle. When the c_2/p_2 ratio is further reduced to C, the predator will be harvested at the stock level $X_2{}^C$. The reduced predation pressure on the prey now makes it economical for the prey to support an open-access harvesting with the stock at the level $X_1{}^C$.

So far we have not considered the possibility of extinction. This is because the harvest function in equations (6) and (7), together with the assumption that total cost is proportional to fishing effort, imply that the unit harvesting cost in equation (17) approaches infinity when the stock approaches zero. Thus, with a constant price of harvest, the net profit per unit of harvest in equation (18) becomes negative for a positive stock level, and we have

$$(26) \qquad X_i^\infty > 0 .$$

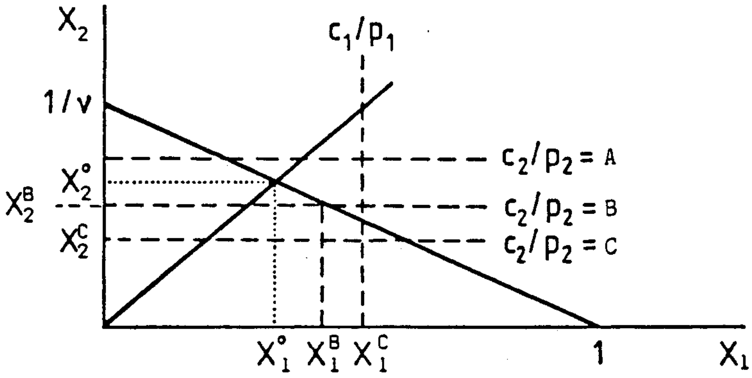

FIG. 4. Open-access equilibria with and without harvesting of the prey.

Extinction under open-access harvesting in this model only occurs either if

$$(27) \qquad c_i = 0 \qquad (i = 1,2)$$

or

(28) $p_i > c_i(0) \quad (i = 1,2)$

for one or both of the species. The expression $c_i(0)$ is the cost of harvesting the last unit of the stock. If equation (27) or (28) is fulfilled for the predator, only this species will be extinct, while in the case of extinction of the prey, the predator will be extinct as well.

4. The Optimal Harvest

Given joint management of predator and prey, we assume that the social manager's objective is to maximize the present value of the rent from the two resources. The joint rent function is

(29) $R(t) = b_1(X_1)y_1(t) + b_2(X_2)y_2(t) = \pi_1(t) + \pi_2(t),$

using the same notation as in the preceding section. The objective functionale is

(30) $PV = \int_0^\infty e^{-\delta t} R(t)dt.$

The social manager will choose the harvest rates, the *y*'s, so as to maximize the present value of the rent, given the biological restrictions implied by the growth equations, (10) and (11).[8] Rewriting the growth equations somewhat gives

(31) $\dot{X}_1 = G_1(X_1, X_2) - y_1(t), \qquad 0 \leq y_1,$

(32) $\dot{X}_2 = G_2(X_1, X_2) - y_2(t), \qquad 0 \leq y_2.$

The following notation for the first order derivatives will be used:

(33) $G_{ij} = \partial G_i(.)/\partial X_j \qquad (i, j = 1,2).$

From equations (10) and (11) we derive

(34) $G_{11} = r_1(1 - 2X_1 - \nu X_2) \gtrless 0 \text{ if } X_2 \lessgtr (1/\nu)(1 - 2X_1),$

(35) $G_{12} = -\nu r_1 X_1 < 0, \qquad G_{21} = r_2 X_2^2/X_1^2 > 0,$

---

[8] Whether one uses the effort rates, the *F*'s, or the harvest rates, the *y*'s, as the control variables, is simply a matter of convenience. In this section it is most convenient to use the latter.

(36) $\quad G_{22} = r_2(1 - 2X_2/X_1) \gtrless 0 \text{ if } X_2 \lesseqgtr X_1/2$

It is now received knowledge that if an interior solution exists, there are two joint equilibrium equations that must be satisfied at the maximum (Clark 1976, 318). They are:

(37) $\quad G_{11} + [b_2(X_2)/b_1(X_1)]\, G_{21} + [b_1{}'(X_1)/b_1(X_1)]\, G_1(.) = \delta,$

(38) $\quad G_{22} + [b_1(X_1)/b_2(X_2)]\, G_{12} + [b_2{}'(X_2)/b_2(X_2)]\, G_2(.) = \delta,$

where $b_i{}'(X_i) = db_i(X_i)/dX_i$.

The economic interpretation of the result of the predator-prey analysis is quite similar to that of the single-species model. The left-hand side of equation (37) is the prey's own rate of interest, which should equal the social rate of discount, given on the right-hand side. The first two terms on the left-hand side together form the instantaneous marginal product of the species. It consists of two parts, where the direct one, G_{11}, is equivalent to the one in a single-species model. The second part is the indirect part of the instantaneous marginal product via the predator. The last term on the left-hand side of equation (37) is the marginal stock effect; that is, the cost-reducing effect an increase in the stock level of the prey has on its own harvesting.[9] The interpretation of equation (38) is similar.

Equations (37) and (38) implicitly give the optimal equilibrium stock levels of the predator and the prey, $X_2 = X_2{}^*$ and $X_1 = X_1{}^*$, respectively.

A common economic interpretation of the singular path of single-species models can be generalized to cover the predator-prey case. Let

(39) $\quad R(X_1, X_2) = b_1(X_1)G_1(X_1, X_2) + b_2(X_2)G_2(X_1, X_2).$

$R(.)$ is the total sustainable rent associated with a sustainable harvest at given stock levels. Then

(40) $\quad \partial R(.)/\partial X_i = b_i{}'(X_i)G_i(.) + \sum_j b_j(X_j)G_{ji} \qquad (i = 1,2).$

Now equations (37) and (38) can be rewritten as

(41) $\quad (1/\delta)\partial R(.)/\partial X_i = b_i(X_i) \qquad (i = 1,2).$

---

[9] Because $b_i(X_i) = p_i - c_i(X_i)$, we have: $b_i{}'(X_i) = -c_i{}'(X_i)$.

The left-hand side is often referred to as the marginal user cost, which gives the loss in present value of sustainable harvesting when the capital asset, the resource stock, is reduced by one marginal unit. The right-hand side is the net current value of harvesting the stock at the margin. In other words, optimal harvesting of the predator-prey system requires that for each of the stocks, which can be thought of as the assets of the social manager's resource portfolio, the present value of future losses from reducing the stock through harvesting should equal the current net benefits from that harvesting.

In the single-species model the optimal stock level will always be larger than the open-access level, given a positive rate of discount. This also holds for the prey in a predator-prey model. *If* the prey stock were below the open-access level,[10] i.e. $X_1 < X_1^\infty$, equations (19) and (21) would imply $\pi_1 < 0$. The loss from harvesting the prey may be avoided simply by abolishing the harvest of this species. As shown in Section 2, this will increase the level of the prey stock, thereby increasing the carrying capacity and the surplus growth of the predator. The net profit from the harvest of the predator is therefore increased. Thus, the optimal stock level of the prey can never be below its open-access level. However, it is quite possible that the optimal predator stock level might be lower than the stock level under open-access harvesting. To see this, rewrite equations (40) and (41) for the predator:

$$(42) \qquad (1/\delta)b_1(X_1)G_{12} = b_2(X_2) - (1/\delta)\,[b_2(X_2)G_{22} + b_2'(X_2)G_2]\,.$$

The left-hand side of equation (42) is the loss in present value of sustainable harvesting of the prey when the predator stock is marginally increased. The right-hand side is the net gain of the combined current and present value of harvesting the predator, when the stock level of this species is marginally increased.

From equation (35) and what is said above it follows that the left-hand side of equation (42) always is negative when the prey is harvested. For the right-hand side we have that the last term in the parenthesis can never be negative, whereas the sign of G_{22} depends on the relative size of the predator stock compared to the prey stock, as shown in equation (36). When the prey is "inexpensive-to-catch and valuable", i.e. c_1 is low and p_1 is high, and the predation pressure on the prey is significant, i.e. ν is large, it can easily happen that b_2 on the right-hand side of equation (42) becomes negative. This is likely to happen when the predator is "expensive-to-catch trash", i.e. c_2 is high and p_2 is low. Then it will be optimal to reduce the predator stock below its open-access level, i.e. $X_2^* < X_2^\infty$. Private harvesters cannot be expected to harvest the predator resource at a loss. Therefore, the social manager would offer them a bounty to harvest a predator that is "expensive-to-catch trash" which preys on a species that is "inexpensive-to-catch and valuable".

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[10] Assuming $X_1^\infty = c_1/p_1 < X_1^0$.

It is easy to show that when the discount rate goes to infinity, the optimal stock level will approach the open-access level. If we make the reasonable assumption that $\partial R(.)/\partial X_i$ is bounded, then equation (41) implies that when $\delta \to \infty$ we have $X_i^* \to X_i^\infty$, where X_i^∞ is derived from

$$(43) \qquad p_i - c_i(X_i^\infty) = 0.$$

When the optimal predator stock level is below the open-access level, i.e. when $X_2^* < X_2^\infty$, then an increase in the discount rate will increase the optimal predator stock. In other words, we have shown that[11]

$$(44) \qquad \partial X_2^*/\partial\delta > 0 \qquad \text{when} \qquad X_2^* < X_2^\infty.$$

This result contradicts the analysis of single-species models where an increased discount rate makes it more costly to keep a large resource stock. Therefore, the optimal single-species stock is reduced when the discount rate increases. In the predator-prey model this is also the case for the prey stock, while the increase in the optimal predator stock caused by the increased discount rate helps reduce the prey to its new lower optimal stock level.[12] As noted above, a rise in the discount rate makes it more costly to keep a large prey stock; therefore a part of it is transmuted into capital in general. Another part is transmuted into predator resource capital. Thus the losses from harvesting the predator are reduced, because of the lowered unit harvesting cost and/or increased revenues.

To see how the stocks at the optimum are affected by marginal changes in prices and costs, equation (41) can be differentiated with respect to p_1, p_2, c_1, and c_2, respectively, to find the partial derivatives of the optimal stocks with respect to each of the parameters (see Flaaten 1988, ch. 7). As in the single-species model, it can be shown that the optimal level of each of the two stocks is negatively affected by an increase in the own price:

$$(45) \qquad \partial X_i^*/\partial p_i < 0 \qquad\qquad (i = 1,2).$$

Also, it can be shown that

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[11] Since the state variables in equation (41), the X_i^*s, are implicit functions of the biological and economic parameters, this result can also be found by differentiating the equation with respect to δ and solving for $\partial X_1^*/\partial\delta$ and $\partial X_2^*/\partial\delta$ (Flaaten 1988, 64–65). Such a method provides a general analysis of the effects of marginal changes in the discount rate on the optimal stocks.

[12] Hannesson (1983) asserted: "Increasing the discount rate may, at "moderate" levels, imply that the optimal standing stock of biomass increases instead of decreasing" (Hannesson 1983, 329).

It seems as if he did not notice that this only applies to the predator species.

$$(46) \qquad \partial X_i^*/\partial p_j > 0 \qquad\qquad i \neq j, \text{ when } b_2(X_2) << 0.$$

This is to say that when the predator is a great nuisance, an increase in the price of the prey will lead to a larger optimal stock of the predator, and vice versa. The case of an increase in prey price may be explained the following way. Such a price change initially augments the value of the resource capital of the prey proportionally to the price increase. However, it pays to transform some of the increased wealth into capital in general, rewarding the social manager with the interest represented by the discount rate. This transformation may be controlled directly through harvesting the prey, or indirectly by letting the predator harvest the prey. Hence, the effect of an increased price of the prey is an increased optimal stock of the predator. The investment in the predator stock is rewarded by increased revenues and reduced harvesting cost of this species. The effects of increased effort costs are the opposite of the effects of increased prices.

So far it has been implicitly assumed that the optimal solution is an interior one. This is shown in Figures 5 and 6. In the former, the star indicates the interior solution for the case when both species are "inexpensive-to-catch and valuable". In this case both stock levels are higher at the optimum than under open access. Figure 6 illustrates the case where the prey is "inexpensive-to-catch and valuable", while the predator is "expensive-to-catch trash". In this case the optimal solution is to subsidize the predator harvest so as to reduce the stock, thereby increasing the harvestable surplus production of the prey. The star in Figure 6 indicates that it is optimal to harvest at a predator stock level below the unharvested open access level. On the other hand, the optimal level of the prey stock is higher than the open-access level of this species.

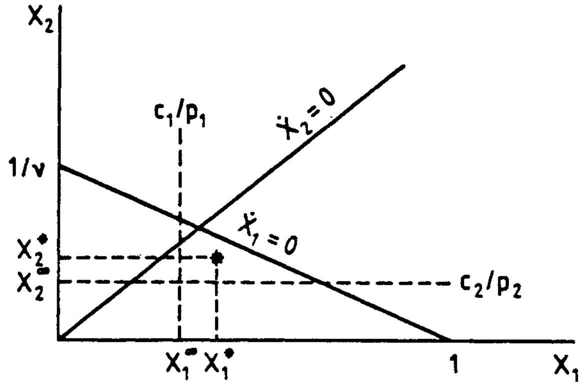

FIG. 5. Open-access and optimal interior solutions.

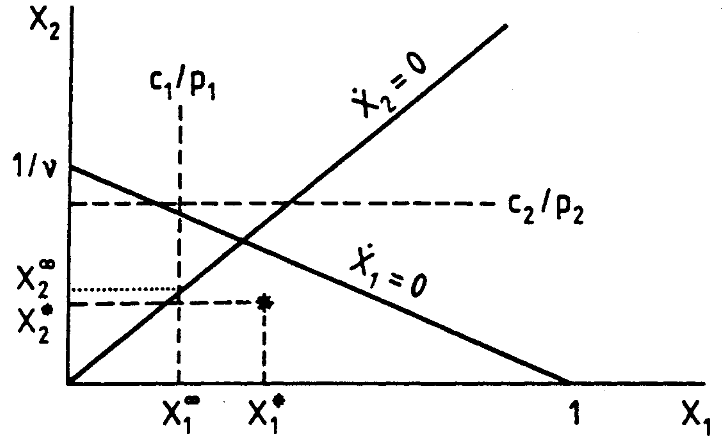

FIG. 6. Interior optimal solution and open-access solution with no harvest of the predator.

As noted above, so far the optimal solution has implicitly been assumed to be an interior one, i.e. the optimal stocks may be derived from the "golden rule" equations, (37) and (38). It is, however, possible that the optimal solution will involve no harvest of either the predator or the prey. Intuitively, it might be uneconomical to harvest the prey if it is "expensive-to-catch trash", or it might be uneconomical to harvest the predator if this species is "expensive-to-catch trash" and its predation pressure on the prey is insignificant, i.e. ν is small. The former case implies an optimal combination of stocks, indicated by the star on the prey's isocline in Figure 7, while the latter implies the combination of stocks indicated by the star on the predator's isocline in Figure 8.

To find whether the solution given by the "golden rule" equations, (37) and (38), really is the optimal solution, the following procedure may be used. First, it should be checked whether or not the golden rule solution implies positive harvest rates of both species. If it does, the solution is inside the sustainable yield triangle (ΔABC in Figure 1), and the interior solution is the solution to the maximization problem. If it does not, one of the restrictions on the harvest rates, the y's in equations (31) and (32), becomes binding.[13] Then it is necessary to substitute the relevant golden rule equation with the corresponding growth equation. For example, if the predator harvest rate becomes binding, the optimal steady state stocks are implicitly found from equations (32) and (38) ($y_2 = \dot{X}_2 = 0$ in the former). Inserting the

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[13] Mathematical analysis of constrained optimization is found in Kamien and Schwartz (1981) and Seierstad and Sydsæter (1987). Application of the theory is found in Flaaten (1988).

optimal stock levels into the prey's growth equation, equation (31), will give the harvest rate of the prey, $y_1 > 0$. This example corresponds to what is shown in Figure 8.

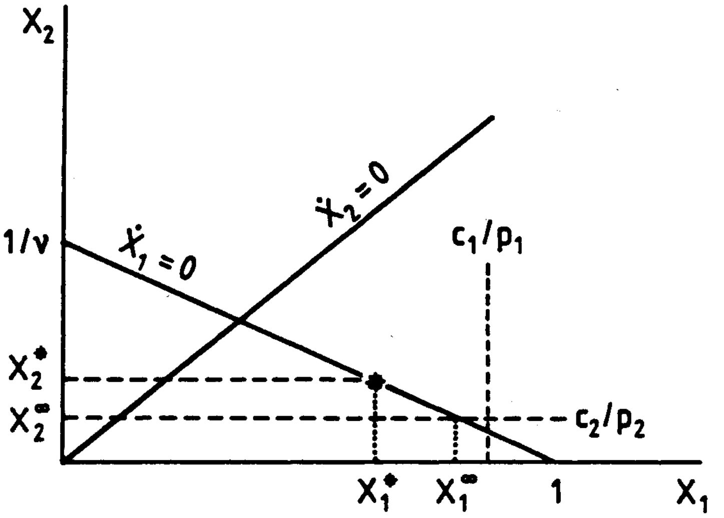

FIG. 7. Open-access and optimal stocks when the prey is "expensive-to-catch trash".

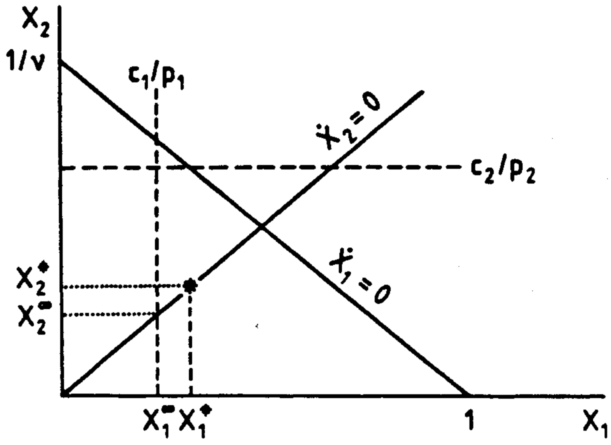

FIG. 8. Open-access and optimal stocks when the predator is "expensive-to-catch trash" with insignificant predation pressure on the prey.

5. Conclusion

In this paper we have combined a simple biological predator-prey model from May et al. (1979), reviewed in Section 2, with two economic independent harvest sectors, one for each of the species. In Section 3 we derived the equilibrium solutions of open-access harvesting of the two species simultaneously, and a corner solution with either of the species left unharvested. The ratio of cost of effort to price of harvest can be so unfavourable for both species that they are best left unharvested under an open-access regime.

Maximizing the present value of the joint economic rent from the predator and the prey may imply an interior solution with simultaneous harvest of both species, as shown in Section 4. The optimal harvest strategy could also be to harvest only the most valuable species, and leave the other unharvested. With a positive discount rate, the optimal level of the prey stock will always be larger than the open-access level. However, this need not be the case for the predator. If this species' predation pressure on the prey is significant, the predator is a low-valued species, and the prey is a high-valued species, it was shown that it may be optimal to reduce the predator stock to below its open-access level. This can be done by subsidizing the harvesters of this species.

The major finding of this paper is that the optimal predator resource stock may increase with an increase in the social rate of discount. This was shown to be the result when the predator's negative effect on the prey's growth rate is significant, the predator is "expensive-to-catch trash", and the prey is "inexpensive-to-catch and valuable".

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BIOECONOMICS OF SUSTAINABLE HARVEST OF COMPETING SPECIES *

by

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1 Introduction

Market failures in resource industries can often be classified as "problems of the commons", which means that a scarce natural resource is not imputed a rent. Analysis of open access and optimal management of common property resources already has a major place in advanced textbooks in resource economics [1], [2] and [3]. Renewable marine resources such as fish, seals and whales are, in addition to their common property nature, difficult to assess and their population dynamics are poorly known. Future management of renewable natural resources will probably place greater emphasis on species interactions. This increases the need for theoretical and applied interdisciplinary work. Theoretical exploration of the relationship between concepts and objectives in biology and economics and implications of these for management will prove valuable for interdisciplinary empirical resource management.

The theory of optimal management of renewable resources is fairly well developed within the single species context; see e.g. Munro and Scott [4] for a review of the fisheries economics literature. The policy implications of this analysis depends, to some degree, on the optimization criteria used. In single species models biological criteria such as maximum sustainable yield (MSY) and maximum yield per recruit, in general give solutions embodied in the solutions based on economic criteria, such as maximum sustainable economic yield (MEY) and maximum present value of rent (MPV) [1].

The problem of managing two or more interacting species is far more complex. Several authors have extended single species analysis to include multispecies interactions and harvesting, for example:

a) Theoretical analysis of management of competing species and predator-prey systems, e.g. Clark [1] ch. 9, Hannesson [5], May et al. [6], Silvert and Smith [7], Flaaten [13] and Getz [8]. The latter is especially concerned about the optimal path towards the long run steady state harvesting regime.

b) Analysis of harvesting interactions, e.g. Clark [1] ch. 9, Clark [2] ch. 5, Lipton and Strand [9] and European Communities [10]. The latter is especially concerned about particular fisheries and of assessment methods.

c) Applied studies on interacting species, e.g. Conrad and Adu-Asamoah [11] on competing species of tuna and Flaaten [12] on plankton feeders–fish–sea mammals interactions.

This article analyses steady state solutions in groups a) and c). Biological and economic optimization criteria are set in a common two species framework to analyse similarities and differences of their solutions. The implications of these solutions for management are compared both between them and with solutions from single species analysis.

The Gause-model [14] of two competing species is reviewed in section 2. Section 3 demonstrates how to derive the maximum sustainable yield frontier (MSF) and the locus of MSF-stocks. Section 4 shows that maximizing economic yield (rent) with positive harvesting costs and zero discount rate implies optimal stock levels above the locus of

MSF-stocks. Isorent lines prove to be ellipses in the state plane in this model. Section 5 shows that maximizing the present value of economic rent under costless harvesting with a positive discount rate implies optimal stock levels below the locus of MSF-stocks. With positive costs optimal harvesting might imply harvesting one of the species at a loss. Hence, open access harvesting of a system of competing species, using single product technology, may imply economic underexploitation of a common property resource. Section 5 also shows how the optimal stock levels are altered by changes in the exogenous social rate of discount, harvest prices and harvesting costs. Section 6 concludes the article.

2 Biological Competition

Most two species models analysed in the literature can be shown to have either a stable point or a stable limit cycle [15], p. 81. This is also the case for the Gause-model used in this paper.

Let W_1 and W_2 denote the stocks of two competing species. The model is specified as

$$\dot{W}_1 = dW_1/dt = r_1 W_1(1 - W_1/L_1) - a_1 W_1 W_2 \tag{1}$$

$$\dot{W}_2 = dW_2/dt = r_2 W_2(1 - W_2/L_2) - a_2 W_1 W_2\,, \tag{2}$$

where r_1 and r_2 are the intrinsic growth rates of the respective species.[1] L_1 and L_2 are the single species carrying capacities at which species one and two, respectively, will settle in the absence of the other species and harvest. In case of no competition, the per capita growth rate of species i, $\dot{W}_i/W_i$, decreases from r_i for stock levels close to zero, to zero for stock levels equal to its own carrying capacity. The presence of a competitor reduces the per capita growth rate in proportion to the biomass of the competitor. The competitor coefficients, a_1 and a_2, tell how much the per capita growth rate of species one and two, respectively, are reduced per unit of the other species.

To simplify the model rewrite the variables W_1 and W_2 into a dimensionless form. Defining $X_1 = W_1/L_1$ and $X_2 = W_2/L_2$ equations (1) and (2) can be rewritten as

$$\dot{X}_1 = dX_1/dt = G_1(X_1, X_2) = r_1 X_1(1 - X_1 - \alpha_1 X_2) \tag{3}$$

$$\dot{X}_2 = dX_2/dt = G_2(X_1, X_2) = r_2 X_2(1 - X_2 - \alpha_2 X_1), \tag{4}$$

where $\alpha_1 = a_1 L_2/r_1$ and $\alpha_2 = a_2 L_1/r_2$ are the dimensionless competitor parameters ($\alpha_1, \alpha_2 > 0$). These parameters tell, in a dimensionless form, how severe the competition from the other species is upon species one and two, respectively.

Harvesting is introduced in a simple way in the model by assuming that the resource stocks are harvested independently of each other, with constant effort per unit of time, F_i ($i = 1, 2$). The effort is scaled such that $F_i = 1$ corresponds to constant catchability coefficients equal to r_i. The normalized catch rates will be

$$y_1 = r_1 F_1 X_1 \tag{5}$$

$$y_2 = r_2 F_2 X_2 . \tag{6}$$

The growth rates in equations (3) and (4) now will be changed to

$$\dot{X}_1 = r_1 X_1(1 - X_1 - \alpha_1 X_2 - F_1) \tag{7}$$

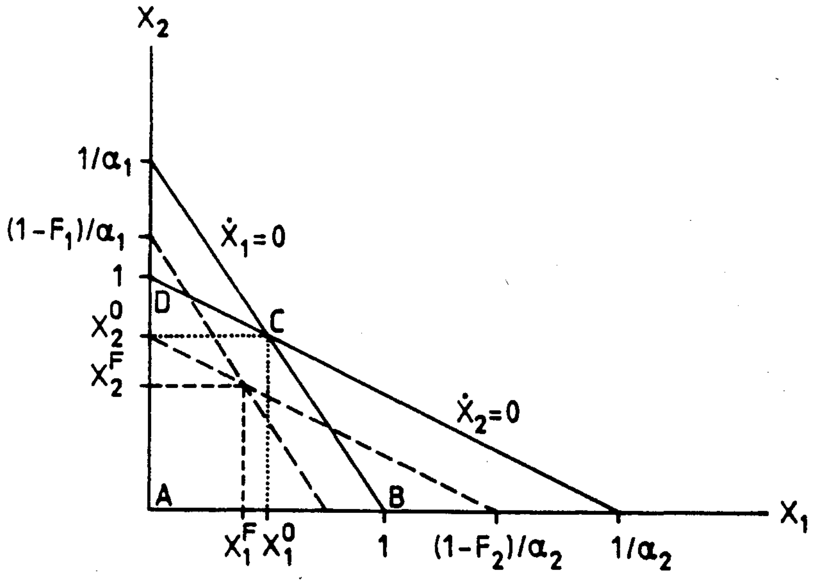

Figure 1: The phase diagram of the model of competing species.

$$\dot{X}_2 = r_2 X_2 (1 - X_2 - \alpha_2 X_1 - F_2)\,. \tag{8}$$

The equilibrium properties of this ecological system depend only on the harvest efforts, F_1 and F_2, and the competition parameters, α_1 and α_2. The intrinsic growth rates, r_1 and r_2, affect the dynamics of the system outside equilibrium. The isoclines are found from equations (7) and (8) by setting $\dot{X}_1 = 0$ and $\dot{X}_2 = 0$. This gives

$$X_2 = (1/\alpha_1)(1 - X_1 - F_1) \quad \text{for} \quad \dot{X}_1 = 0 \tag{9}$$

$$X_2 = (1 - \alpha_2 X_1 - F_2) \quad \text{for} \quad \dot{X}_2 = 0\,. \tag{10}$$

The phase diagram for the system with a stable node, C, is shown in Figure 1. The isoclines for the pristine system without harvesting ($F_i = 0$; $i = 1,2$) are shown as solid lines whereas the isoclines for the harvesting case are the dotted lines.[2] The pre-harvesting levels of the stocks are denoted X_i^0 whereas the post-harvesting levels are denoted X_i^F. Even if there is a competitive coexistence equilibrium in the pristine system, harvesting may extinct one of the species.

If positive, stable equilibrium levels of X_1 and X_2, denoted X_1^F and X_2^F, exist simultaneously, they are found where the isoclines intersect. From equations (9) and (10) follow

$$X_1^F = \frac{1-\alpha_1 - F_1 + \alpha_1 F_2}{1-\alpha_1\alpha_2} \tag{11}$$

$$X_2^F = \frac{1-\alpha_2 - F_2 + \alpha_2 F_1}{1-\alpha_1\alpha_2}. \tag{12}$$

To have an interior, stable equilibrium with both stocks positive without harvesting, it is necessary to have

$$\alpha_1 < 1 \qquad \text{and} \qquad \alpha_2 < 1. \tag{13}$$

The species which has the lowest competitor parameter, α_i, will have the highest stock level, X_i^0, at the pristine equilibrium, i.e. the equilibrium without harvesting. To have an equilibrium with both stocks being positive when just species i is harvested, it is necessary that the effort rates do not exceed the following levels

$$F_i < 1 - \alpha_i \qquad \text{and} \qquad F_j = 0\,, \qquad (i = 1 \text{ or } 2,\ i \neq j)\,. \tag{14}$$

Equations (11) and (12) show that the equilibrium level of each stock is negatively affected by the own effort rate, and positively affected by the effort rate of the competitor. This means i.a. that it is possible to increase each of the stocks above its pristine level by harvesting only the competitor. The sustainable yield area in Figure 1 is the quadrangle $ABCD$. Combinations of stock levels outside this quadrangle are not possible to maintain.[3] The sustainable yield area is of great importance as a biological restriction on the economic utilization of the competing species. This may be compared with the biomass axis in the yield-biomass diagram of single species models. To obtain a positive sustainable yield in a single species model it is necessary that the stock remains between its minimum viable level and the carrying capacity of the system. This compares to a bounded area in the biomass plane of two species models, e.g. the quadrangle $ABCD$ in Figure 1. Within the boundaries mentioned above each stock level in a single species model may produce a given sustainable yield. Likewise, in a two species model a given combination of the two stock levels within the sustainable yield area, produce a combination of sustainable yields of the two species. The concept of maximum sustainable yield (MSY), and the corresponding stock level, in single species models are of great biological importance, as well as of economic importance as a reference point for optimal management of the stock. The corresponding concept to MSY for the two species model of competition will be introduced in the next section.

3 The Maximum Sustainable Yield Frontier (MSF)

The importance of the MSY concept in biological and bioeconomic single species analysis is well known. The concept of maximum sustainable yield frontier (MSF) will be shown to be of the same importance for the two species analysis as MSY is in the single species framework. The MSF is derived[4] by maximizing the sustainable yield of one species for a constant sustainable yield of the other. This problem is equivalent to that of welfare economics: deriving the production possibility frontier by maximizing the output of one good for a specified amount of output of the other, given a fixed amount of factors of production. In a two species model of competition the limited amount of factors of production are embodied in the carrying capacities and the intrinsic growth rates. In the real world the limited factor of production can be e.g. the zooplankton communities of the sea in the case of competing fish species, and the grass of the plain in the case of grazing animals.

From equations (5)–(8) the following equilibrium harvest rates are derived, i.e. when $\dot{X}_1 = \dot{X}_2 = 0$,

$$y_1 = r_1 X_1 (1 - X_1 - \alpha_1 X_2) \tag{15}$$

$$y_2 = r_2 X_2 (1 - X_2 - \alpha_2 X_1) \,. \tag{16}$$

The problem of maximizing y_2 subject to the constraint $y_1 =$ constant, can be done using the Lagrange method. First we introduce the Lagrangian expression

$$L = r_2 X_2 (1 - X_2 - \alpha_2 X_1) - \mu (r_1 X_1 (1 - X_1 - \alpha_1 X_2) - y_1) \,. \tag{17}$$

From the necessary conditions for optimality the following quadratic equation, which implicitly gives X_2 as a function of X_1, is derived:

$$2\alpha_2 X_1^2 + 4 X_1 X_2 + 2\alpha_1 X_2^2 - (2 + \alpha_2) X_1 - (2 + \alpha_1) X_2 + 1 = 0 \,, \tag{18}$$

when $X_1 > 0$. For $X_1 = 0$ it follows immediately from maximization of y_2 in equation

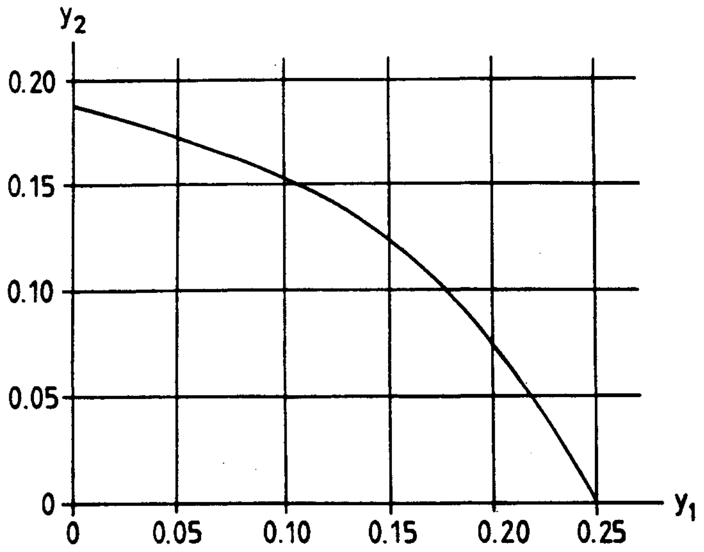

Figure 2: The maximum sustainable yield frontier (MSF) for two competing species, for parameters given in the text.

(16) that

$$X_2 = \frac{1}{2}, \qquad \text{when} \quad X_1 = 0. \tag{19}$$

Correspondingly it is easy to see that

$$X_1 = \frac{1}{2}, \qquad \text{when} \quad X_2 = 0. \tag{20}$$

For each level of X_1 we compute X_2 from (18) and the resulting yields, y_1 and y_2, are given by (15) and (16). The locus combining the yields of the two species is shown in Figure 2 for parameters $\alpha_1 = 2/3$, $\alpha_2 = 1/2$, $r_1 = 1$, $r_2 = 3/4$. In this example species one has the greatest reproductive potential, biologically speaking, as seen from the MSF in Figure 2.

The curve depicted by $X_2 = X_2^{MSF}(X_1)$, implicitly given by equations (18)–(20) we shall call the X^{MSF}-locus. The MSF will be affected by changes in r_1 and r_2, and so will the X^{MSF}-locus be, as seen from equation (18) recalling that $\alpha_i = a_i L_j / r_i$ $(i, j = 1, 2;\ i \neq j)$.

Equation (18) is an equation of a conic section of the form

$$AX_1^2 + 2BX_1X_2 + CX_2^2 + DX_1 + EX_2 + F = 0\,. \tag{21}$$

In this case $4AC < (2B)^2$, since $\alpha_1\alpha_2 < 1$ and $B = 2$, and the X^{MSF}-locus given by equation (18) is a hyperbola. Only the branch giving $0 < X_i^{MSF} < 1$ $(i = 1, 2)$ make sense, biologically speaking. The other branch is therefore excluded from the analysis.

In the $X_1 - X_2$-plane the curve of $X_2 = X_2^{MSF}(X_1)$ implicitly given by (18) will always be downward sloping since

$$\frac{dX_2}{dX_1} = -\frac{4\alpha_2 X_1 + 4X_2 - (2+\alpha_2)}{4\alpha_1 X_2 + 4X_1 - (2+\alpha_1)} < 0 \quad \text{along} \quad X_2 = X_2^{MSF}(X_1) \qquad \text{for} \quad X_1, X_2 \in [0, 0.5]\,. \tag{22}$$

Since $\lim_{X_1^{MSF} \to 1/2} X_2^{MSF} = 0$ we have

$$\lim_{X_1 \to 1/2} \left(\frac{dX_2}{dX_1}\right)^{MSF} = \frac{\alpha_2 - 2}{\alpha_1} < 0\,, \tag{23}$$

and, since $\lim_{X_1^{MSF} \to 0} X_2^{MSF} = 1/2$

$$\lim_{X_1 \to 0} \left(\frac{dX_2}{dX_1}\right)^{MSF} = \frac{\alpha_2}{\alpha_1 - 2} < 0\,. \tag{24}$$

Equations (22)–(24) show that the slope of the X^{MSF}-locus will depend on the parameters α_1 and α_2, i.e. on the slopes of the isoclines.[5]

From a biological point of view one could say that the X^{MSY}-stock level in single species models is the optimal one. The corresponding biological optimal harvesting in a two species model would be one of the inumerable combinations of stocks generating the MSF. Which point on the MSF, and the X^{MSF}-curve, is "best" is not possible to decide just from a biological reasoning. The inclusion of economics is necessary to make such a decision. This is the subject of the following two sections, and it will be shown that MSF-harvesting is the economic optimum only in special cases.

4 Maximum Sustainable Economic Yield

As demonstrated in the preceding section the competing species may be harvested on a sustainable yield basis for both of the species at inumerable combinations of the two stock levels. To see at which stock levels the system will settle under commercial harvesting, we shall study the two extreme harvesting regimes of open access harvesting and a profit maximizing sole owner. We shall mostly be concerned with solutions in the interior of the sustainable yield quadrangle.[6] The dynamic behaviour of the biological and economic system will not be considered, we shall rather concentrate on analysing equilibrium points by means of comparative statics.

Given the Schaefer harvest function of equations (5) and (6), and assuming constant costs, c_i, per unit of rescaled effort, $E_i = r_i F_i$, the unit harvesting cost becomes

$$c_i(X_i) = c_i / X_i \qquad (i = 1, 2) . \tag{25}$$

Assuming the demand for each of the two species is independent of each other and infinitely elastic with respect to its own price, p_1 and p_2 are the constant prices of species one and two, respectively. Under these assumptions the net profit per unit of harvest is

$$b_i(X_i) = p_i - c_i/X_i \qquad (i = 1,2)\,, \tag{26}$$

and the total profit from the harvest of each of the two species is

$$\pi_i(X_i) = (p_i - c_i/X_i)y_i \qquad (i = 1,2)\,. \tag{27}$$

Bioeconomic equilibrium requiers we simultaneously have[7]

$$\pi_i(X_i) = 0 \qquad \text{and} \qquad \dot{X}_i = 0 \qquad (i = 1,2)\,. \tag{28}$$

Denoting the open access stock level of the two species as X_1^{∞} and X_2^{∞}, it is seen from (27) that unless $y_i = 0$ the open access stock levels are

$$X_i^{\infty} = c_i/p_i \qquad (i = 1,2)\,. \tag{29}$$

Equation (30) also implies the standard tragedy of the commons result that price equal average costs (Cornes and Sandler [17]; Dasgupta and Heal [3]). Given our assumptions the open access stock levels solely depend on economic parameters.[8] However, the biological parameters will affect the harvest rates as seen by substituting from equations (5)–(6) and (28)–(29) into the growth equations (7) and (8).

In Figure 3 the open access equilibrium point, X^{∞}, is (arbitrarily) outside the X^{MSF}-curve, for parameters $c_1 = 1$, $c_2 = 1$, $p_1 = 4$, $p_2 = 2$, which imply $X_1^{\infty} = 1/4$ and $X_2^{\infty} = 1/2$. Note that decreasing the stock levels somewhat, to move the equilibrium point in the SW-direction from X^{∞} to the X^{MSF}, would increase biological yield from both species.

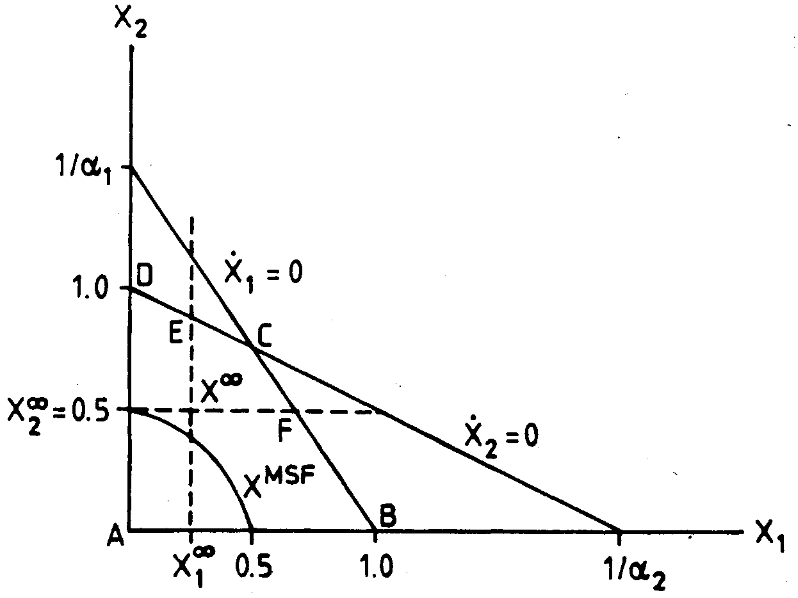

Figure 3: The areas of sustainable yield and positive profit, for parameters given in the text.

The total profit from harvesting the two species can be derived from equation (27):

$$\pi^*(X_1, X_2) = (p_1 - c_1/X_1)y_1 + (p_2 - c_2/X_2)y_2 \,. \tag{30}$$

Equation (30) gives the standard industry profit function consisting of a profit per unit expression and a steady state harvest quantity for each of the two species. Substituting for y_i from (15) and (16), inserting X_i^∞ from (29) and normalizing profit by dividing by $p_1 r_1$ in (30) gives

$$\begin{aligned} \pi(X_1, X_2) &= (X_1 - X_1^\infty)(1 - X_1 - \alpha_1 X_2) \\ &+ \beta(X_2 - X_2^\infty)(1 - X_2 - \alpha_2 X_1) \,, \end{aligned} \tag{31}$$

where $\beta = p_2 r_2 / p_1 r_1$ is the bioprice ratio, and $\pi(X_1, X_2) = \pi^*(X_1, X_2)/p_1 r_1$.

Interpreting equation (31) it should be noticed that it consists of two terms, one for each of the two species, and each term is a product of two parts. The first part, $(X_i - X_i^\infty)$, takes care of the stock effect,[9] i.e. the effect the size of the stock has on the net profit due to decreasing costs as the stock rises. The stock level must be above the open access level, X_i^∞, to yield a positive profit. The second part is the expression from which the isocline is derived. Since the isoclines are the borders of sustainable yield of the respective species, the product of the two parts can be interpreted the following way. On the one hand it pays to stay below the isoclines because that enhances biological yield, however, on the other hand it pays to have stocks above the open access levels because that reduces harvesting costs thereby increasing the unit harvesting profit. All in all there must be a tradeoff between these two effects to maximize the total profit expressed by equation (31).

Before proceeding to the problem of finding the optimal combination of the two stocks, we shall have a closer look at which combinations of stocks in Figure 3 give positive total profit. The zero profit line obviously passes through the X^∞-point, and there must also be zero profit at point C where the isoclines intersect, i.e. where the growth rates simultaneously equal zero. At point E in Figure 3 the term $(X_1 - X_1^\infty)$ equal zero and the yield from species two is zero because $(1 - X_2 - \alpha_2 X_1)$ equal zero. Both terms in equation (31) are therefore equal to zero, and so is of course the total profit. For the same reasons this also happens at point F. To see what the zero profit line looks like outside the four points X^∞, F, C and E, we start with the more general case of what the isoprofit lines in general look like.

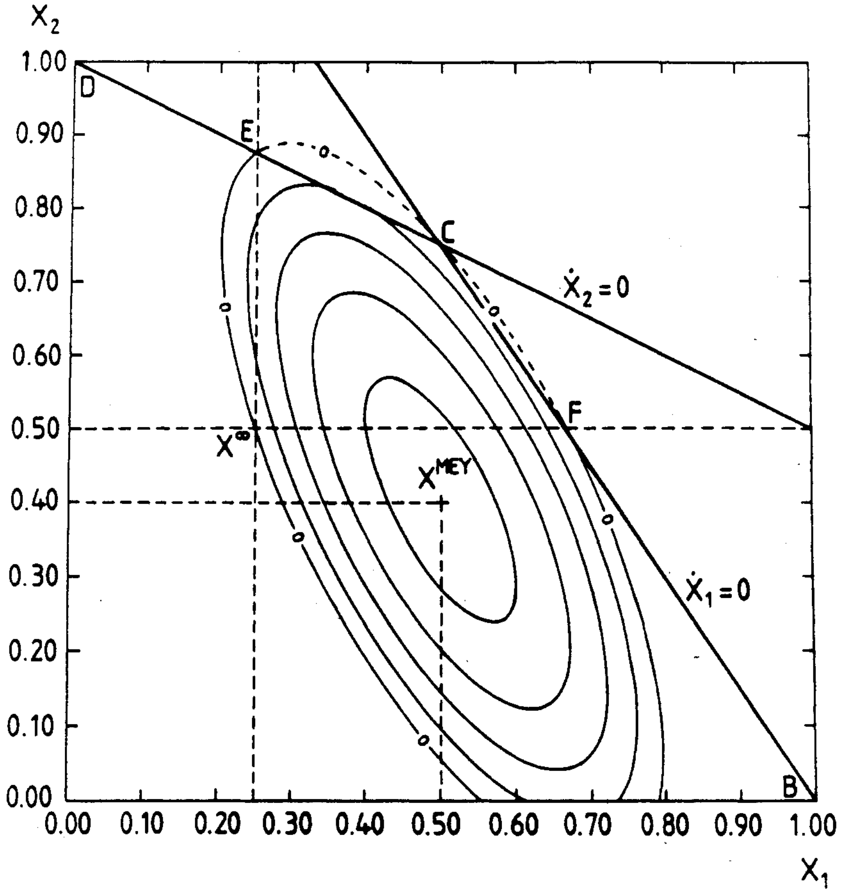

Figure 4: Isoprofit ellipses and the sustainable yield region.

Rewriting the net profit function in equation (31) gives

$$\begin{aligned}\pi(X_1,X_2) &= -X_1^2-\beta X_2^2-(\alpha_1+\beta\alpha_2)X_1X_2+(1+X_1^{\infty}+\beta\alpha_2X_2^{\infty})X_1 \\ &+ (\beta+\beta X_2^{\infty}+\alpha_1X_1^{\infty})X_2-(X_1^{\infty}+\beta X_2^{\infty})\,.\end{aligned} \tag{32}$$

For a given profit level, $\bar{\pi}$, this is an equation of a conic section, cfr. equation (21). When $4AC-(2B)^2>0$ and $B\neq 0$ this is the equation of an ellipse with axes not parallel to the abscissa and the ordinate axes.[10] The isoprofit lines are therefore ellipses in the X_1-X_2-plane. Some isoprofit lines, with the zero profit line as the outermost, are shown in Figure 4, for biological parameters used in Figure 2 and economic parameters used in Figure 3. Combinations of stocks within the zero profit ellipse, inside the sustainable yield quadrangle, give sustainable positive total profit. The size of the zero profit ellipse depends on the parameters of equation (32), i.e. on economic as well as on biological parameters. An increase in the open access stock levels, i.e. a change in point X^{∞} towards point C in Figure 4, contracts the area of profitable harvesting. Such a change eventually results in a zero profit ellipse that does not intersect the abscissa (and/or the ordinate axis). Reduction of the intrinsic growth rates also reduces the area of profitable joint harvesting of the two species.

In the single species model stock levels between the open access level, X^{∞}, and the carrying capacity are capable of giving positive sustainable profit. The lower bound, X^{∞}, is solely determined by economic and harvest-technological factors, whereas the carrying capacity is a pure biological constraint. In this model of competing species the upper bound of the profit yielding stock levels is a pure biological constraint given by the isoclines intersection point, C, in Figure 4. Also for the lower bound is there a similarity between this two species model and the single species model, the open access stock levels are solely determined by economic and technological factors. The points E and F on the zero profit ellipse correspond to a mixture of economic, technological and biological factors. However, it should be noted that these points are uniquely determined by the open access stock level, X^{∞}, and the pristine stock levels.

Assuming there is an interior solution, the necessary conditions for the maximization of the profit in equation (31) gives the following MEY-stocks:[11]

$$X_1^{MEY} = \frac{2\beta(1 + \bar{X}^{\infty}) - \gamma(\beta + \hat{X}^{\infty})}{4\beta - \gamma^2} \tag{33}$$

$$X_2^{MEY} = \frac{2(\beta + \hat{X}^{\infty}) - \gamma(1 + \bar{X}^{\infty})}{4\beta - \gamma^2}, \tag{34}$$

where $\gamma = \alpha_1 + \alpha_2\beta$, $\bar{X}^{\infty} = X_1^{\infty} + \alpha_2\beta X_2^{\infty}$, $\hat{X}^{\infty} = \alpha_1 X_1^{\infty} + \beta X_2^{\infty}$, recalling that $\beta = p_2r_2/p_1r_1$. X^{MEY} is the center of the isoprofit ellipses and the optimal long run stock levels in case of zero discount rate, and is shown in Figure 4.

Harvesting at stock levels either below X_1^{∞} *or* below X_2^{∞} can make sense in the case that one of the two species has a low economic value, expressed by p_i/c_i, and the other has a high value. The low valued species, species two in Figure 4, therefore should be harvested at a loss. To implement such a solution it is necessary to subsidize the harvest of species two to reduce this stock below its open access level. This amount of subsidy is more than offset by increased sustainable economic yield of species one. Hence, the total profit from joint harvesting of the two species increases.

From the analysis of single species biomass models is known that $X^{MEY} > X^{MSY}$ in case of stock dependent harvest costs. In the special case of costless harvesting, or no stock dependent costs, the MEY and the MSY stock levels coincides: $X^{MEY} = X^{MSY}$. In this two species context we shall show:

> **Theorem 1.** In case of two competing species and costless harvesting the combinations of stocks giving the maximum economic yield (MEY) coincides with the combinations maximizing the yield of one species for a given yield of the other (MSF).

The result stated in Theorem 1 is being proved over all possible bioprice ratios, β. The X^{MEY} is a single point for a given set of bioprices.

In the case of costless harvesting, $c_i = X_i^\infty = 0$ $(i = 1,2)$, β may be eliminated from the necessary conditions for an interior solution of the maximization of sustainable economic yield in equation (31), to give

$$2\alpha_2 X_1^2 + 4X_1X_2 + 2\alpha_1 X_2^2 - (2+\alpha_2)X_1 - (2+\alpha_1)X_2 + 1 = 0\,. \tag{35}$$

Equation (35) gives $X_2 = X_2^{MEY}(X_1)$ and is exactly the same as equation (18) which gave $X_2 = X_2^{MSF}(X_1)$. We have thereby shown Theorem 1.

Having studied the case of costless harvesting, now proceed to the case of positive harvesting costs for both species, i.e. $c_i > 0$ $(i = 1,2)$, to show, for all possible bioprice ratios,

> **Theorem 2.** In case of two competing species that are harvested independently of each other at positive harvesting costs, the MEY combinations of stocks are greater than the MSF combinations.

To show this start by eliminating β from the necessary conditions for the maximization of sustainable economic yield in equation (31), to arrive at

$$\begin{aligned} 2\alpha_2 X_1^2 &+ 4X_1X_2 + 2\alpha_1 X_2^2 - (2+\alpha_2+\alpha_2 X_1^\infty + (2-\alpha)X_2^\infty)X_1 \\ &- (2+\alpha_1+(2-\alpha)X_1^\infty + \alpha_1 X_2^\infty)X_2 + (1 + X_1^\infty + X_2^\infty) = 0\,, \end{aligned} \tag{36}$$

where $\alpha = \alpha_1\alpha_2$. This is an equation of a conic section. The constants of the two quadratic terms and that of the product term in equation (35) are the same as in the MSF-case given in equation (18), while the three others are different. This means that also the MEY combinations of the two stocks are described by a hyperbola in the case of positive harvesting costs. Since the constants of the quadratic terms and the product term are equal in the MEY and the MSF cases, the asymptotes of the hyperbolas have the same slope in these two cases. Therefore, the MEY- and the MSF-hyperbolas can not intersect. In case of the MSF we have shown in equation (20) that[12]

$$X_1^{MSF} = 1/2 \qquad \text{when} \qquad X_2^{MSF} = 0 . \tag{37}$$

To find the intersection point between the MEY-hyperbola and the abscissa, return to the profit equation (31). When $X_2 = 0$ the MEY stock of species one is found from

$$d\pi / dX_1 = 1 - 2X_1 - X_1^{\infty} = 0 , \tag{38}$$

which gives, by use of (37),

$$X_1^{MEY} = X_1^{MSF} + X_1^{\infty}/2 \qquad \text{when} \qquad X_2^{MEY} = 0 . \tag{39}$$

In other words, if species two is extinct the maximum economic yield of species one is obtained for a larger stock level than the one giving maximum sustainable yield. Since the MEY- and the MSF-hyperbolas can not intersect this implies

$$X^{MSF} < X^{MEY} , \qquad \text{when} \qquad c_i > 0 \quad (i = 1,2) , \tag{40}$$

and theorem 2 is proved.

Having focused on maximum sustainable economic yield in this section, we now proceed with the objective of maximizing present value of rent from the two competing species.

5 Maximum Present Value of Rent

Given joint management of the two competing species and assuming the objective of the management program is to maximize the present value of the rent from the resources, the objective functional is

$$PV = \int_0^\infty e^{-\delta t}\pi(X_1, X_2)dt, \tag{41}$$

where $\pi(X_1, X_2)$ is defined in (30) and δ is the social rate of discount. The social managers problem is to choose the harvest rates, the y's, so as to maximize the present value, given the biological constraints imposed by the growth equations (7) and (8). Assuming the solution is an interior one, there are two joint equilibrium equations that must be satisfied at the optimum (see [1], ch. 9.3). They can be written as

$$b_1(X_1)G_{11} + b_2(X_2)G_{21} - c_1'(X_1)G_1(X_1, X_2) = \delta b_1(X_1) \tag{42}$$

$$b_1(X_1)G_{12} + b_2(X_2)G_{22} - c_2'(X_2)G_2(X_1, X_2) = \delta b_2(X_2), \tag{43}$$

where $b_i(X_i)$ are defined in (26), $c_i'(X_i) = dc_i(X_i)/dX_i$ and $G_{ij} = \partial G_i(X_1, X_2)/\partial X_j$ $(i, j = 1, 2)$. The growth functions, $G_i(X_1, X_2)$ $(i = 1, 2)$ are defined in equations (3) and (4). At the optimum the net profit from investing in the resource capital of species one, i.e. the l.h.s. of equation (42), should equal the net profit from possibly investing the current profit at the social opportunity cost of capital, δ.

Dividing equation (42) by the social rate of discount gives, on the l.h.s., the change in the present value of the infinite horizon sustainable economic yield per unit of change in the stock level. Expressed this way the l.h.s. is often referred to as the user cost of the resource. This should equal the current profit from harvesting of one unit of the stock given on the r.h.s. Compared to the single species model the additional term on the l.h.s. of equation (42), $b_2(X_2)G_{21}$, reflects the addition to the marginal value product of species one afforded by means of species two. The interpretation of equation (43) is similar.

In single species models the optimal resource stock is above the open access stock level, whereas that need not be the case for the predator in predator–prey models (see [5] and [13]). To see that the latter result also applies to any one species in a model of competing species we rewrite equations (42)–(43) and, after having taken the derivatives, arrive at

$$(1 - X_1 - \alpha_1 X_2) - (X_1 - X_1^{\infty}) - \alpha_2\beta(X_2 - X_2^{\infty}) = \frac{\delta}{X_1 r_1}(X_1 - X_1^{\infty}) \qquad (44)$$

$$\beta(1 - X_2 - \alpha_2 X_1) - \beta(X_2 - X_2^{\infty}) - \alpha_1(X_1 - X_1^{\infty}) = \frac{\delta\beta}{X_2 r_2}(X_2 - X_2^{\infty}), \qquad (45)$$

where it is substituted for $X_i^{\infty} = c_i/p_i$ and $\beta = p_2 r_2/p_1 r_1$. The long run optimal equilibrium stocks implicitly given by equations (44) and (45) shall be denoted X_1^{MPV} and X_2^{MPV}. The first term on the l.h.s. of equation (44) will always be positive within the sustainable yield area. Assuming the optimal stock level of species two is above its open access level implies the last term on the l.h.s. of (44) is negative, included the minus sign. Now it is possible that $X_1^{MPV} < X_1^{\infty}$ which implies that the harvest of species one should be subsidized at the optimum. This result is more likely if α_1, p_1 and/or r_1 are low, or c_1 high, compared to the corresponding parameters of species two. The loss from the harvest of species one is more than offset by the increased profit from the harvest of species two which is more bioeconomic valuable. This reasoning of course also holds for the opposite case with species two being harvested at a loss.

From single species models is well known that in the extreme case of costless harvesting the optimal stock level will always be below the MSY level when the discount rate is positive. This result from the single species models shall be used to prove

Theorem 3. For costless harvesting and a positive discount rate the combination of stocks giving the maximum present value (MPV) of harvesting will be inside the locus of MSF stocks.

To prove Theorem 3 insert $c_i = X_i^\infty = 0$ $(i = 1, 2)$ into equations (44) and (45) and eliminate β. This gives the following equation

$$\begin{aligned} 2\alpha_2 X_1^2 &+ 4X_1X_2 + 2\alpha_1 X_2^2 - (2 + \alpha_2 - \alpha_2\delta_1 - 2\delta_2)X_1 \\ &- (2 + \alpha_1 - \alpha_1\delta_2 - 2\delta_1)X_2 + 1 + \delta_1\delta_2 - \delta_1 - \delta_2 = 0\,, \end{aligned} \tag{46}$$

where $\delta_i = \delta / r_i$ $(i = 1, 2)$ are the bioeconomic growth ratios. This is an equation of the quadratic form and compared with the equation of MSF stocks in (18), it is noticed that the constants of the two quadratic terms and that of the product term are equal, while the three others are different. Hence, the X^{MPV}-locus is a branch of a hyperbola with asymptotes parallel to the asymptotes of the X^{MSF} hyperbola. Therefore, they do not intersect.

To find the terminal point of the $X_{c=0}^{MPV}$-locus at the abscissa, insert $X_2 = 0$ into the

PV function in equation (41) to arrive at

$$PV = \int_0^\infty e^{-\delta t} \, X_1(1 - X_1)dt \,, \qquad X_2 = 0 \,, \qquad X_i^\infty = 0 \qquad (i = 1,2) \,. \tag{47}$$

The problem of maximizing PV is now reduced to that of a single species problem (known from Clark [1], ch. 2), and the solution is

$$X_1^{MPV} = (1 - \delta_1)/2 \,, \qquad X_2 = 0 \,, \qquad X_i^\infty = 0 \qquad (i = 1,2) \,. \tag{48}$$

Since the terminal point of the $X_{c=0}^{MPV}$ locus at the abscissa is inside the corresponding point at the X^{MSF}-locus, the conclusion is that the entire $X_{c=0}^{MPV}$-locus is inside the X^{MSF}-locus in the case of costless harvesting. Thereby Theorem 3 is proved.

From Theorems 2 and 3 and from equations (44)–(45) it now follows that in the general case of stock dependent costs and a positive social rate of discount the X^{MPV}-locus is between two borders. The inner border is determined by the stock levels, $X_{c=0}^{MPV}$, maximizing the present value of the resource rent for the zero cost case and a given (maximum) social rate of discount. The outer border is determined by the stock levels, X^{MEY}, maximizing the sustainable economic yield in the case of (maximum) stock dependent costs. For a given bioprice ratio, β, of the two species the long run optimal equilibrium stock levels are uniquely determined by a point in the $X_1 - X_2$-plane at or between these two borders, depending on the size of the harvest costs and the social rate of discount.

In single species biomass models with positive harvest costs and a positive social rate of discount, the optimal stock, X^{MPV}, approaches the MEY stock level when $\delta \rightarrow 0$, and approaches the open access stock level, X^{∞}, when $\delta \to \infty$ (see [1], ch. 2). For this two species model of competing species we can now show

> **Theorem 4.** In a model of two competing species the long run optimal equilibrium stock level for any one species may be below the open access stock level. When that is the case the optimal stock level of this particular species increases with an increase in the social rate of discount.

In this model equations (44)–(45) approaches the necessary conditions for the maximization of sustainable rent in equation (31) when $\delta \to 0$. This is to say that X^{MPV} approaches X^{MEY} when the social rate of discount approaches zero. It also follows from equations (44)–(45), after having divided by δ, that the r.h.s. must approach zero when $\delta \to \infty$, i.e. the X^{MPV} approaches the open access stock levels, X^{∞}. Thereby it is demonstrated that like in the single species model we have

$$\lim_{\delta \to 0} X_i^{MPV} = X_i^{MEY} \quad (i = 1, 2) \tag{49}$$

and

$$\lim_{\delta \to \infty} X_i^{MPV} = X_i^{\infty} \quad (i = 1, 2)\,. \tag{50}$$

This also implies

$$\frac{\partial X_i^{MPV}}{\partial \delta} > 0 \quad \text{if} \quad X_i^{PV} < X_i^{\infty} \quad (i = 1 \quad or \quad 2)\,, \tag{51}$$

and Theorem 4 is proved. The results in Theorem 4 is not possible to have in single species models, but it may happen for the predator in predator–prey models [13].

The long run optimal equilibrium stock levels are functions of biological and economic parameters. It is of interest to analyse how these stock levels are changed by marginal changes in the harvest prices and effort costs. Let us first prove

Theorem 5. For two competing species the effect of an own price increase on the long run optimal resource stock is negative, whereas an increase in the price of the other species increases the optimal stock level of the former species.

Start with the equilibrium equations (42) and (43) which may be written as

$$\frac{1}{\delta}\,\frac{\partial \pi(\cdot)}{\partial X_i} = b_i(\cdot) \qquad (i = 1, 2)\,. \tag{52}$$

With harvest price, p_i, and effort cost, c_i, as exogeneous variables, differentiating equation (26) gives

$$b_{ip} = \frac{\partial b_i(\cdot)}{\partial p_i} > 0\,, \qquad b_{ic} = \frac{\partial b_i(\cdot)}{\partial c_i} < 0 \qquad \text{and} \qquad b_{ix} = \frac{\partial b_i(\cdot)}{\partial x_i} > 0\,, \tag{53}$$

which shall be used in the analysis. Differentiating equation (52) w.r.t. p_1, rearranging somewhat and by using Cramer's rule we find

$$\frac{\partial X_1^{MPV}}{\partial p_1} = \frac{\delta b_{1p}\left(\frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x}\right)}{|D|} \tag{54}$$

$$\frac{\partial X_2^{MPV}}{\partial p_1} = \frac{-\delta b_{1p}\frac{\partial^2 \pi}{\partial X_2 \partial X_1}}{|D|}\,, \tag{55}$$

where

$$|D| = \begin{vmatrix} \frac{\partial^2 \pi}{\partial X_1^2} - \delta b_{1x} & \frac{\partial^2 \pi}{\partial X_1 \partial X_2} \\ \frac{\partial^2 \pi}{\partial X_2 \partial X_1} & \frac{\partial^2 \pi}{\partial X_2^2} - \delta b_{2x} \end{vmatrix}\,.$$

The second order conditions for the existence of an interior solution to the maximization problem in equation (41) are

$$\begin{vmatrix} \frac{\partial^2\pi}{\partial X_1^2} & \frac{\partial^2\pi}{\partial X_1 \partial X_2} \\ \frac{\partial\pi}{\partial X_2 \partial X_1} & \frac{\partial^2\pi}{\partial X_2^2} \end{vmatrix} > 0 \qquad \text{and} \qquad \frac{\partial^2\pi}{\partial X_1^2} < 0\,, \tag{56}$$

and from this follows that $\partial^2\pi/\partial X_2^2 < 0$. The second order conditions imply $|D| > 0$ because of (53). Taking the second-order derivatives of the profit function (31) w.r.t. X_i we get

$$\frac{\partial^2\pi}{\partial X_i^2} = -r_i p_i \qquad (i = 1, 2) \tag{57}$$

and

$$\frac{\partial^2\pi}{\partial X_i \partial X_j} = -\alpha_1 r_1 p_1 - \alpha_2 r_2 p_2 \qquad (i \neq j)\,. \tag{58}$$

The results in (57)–(58) imply that the second order conditions are met, and $|D| > 0$. From (54) and (55) now follows that

$$\frac{\partial X_1^{MPV}}{\partial p_1} < 0 \qquad \text{and} \qquad \frac{\partial X_2^{MPV}}{\partial p_1} > 0\,. \tag{59}$$

Since the relationship between the two species in this model is symmetric it is obvious that differentiating w.r.t. p_2 in equation (52) give similar results as in (59), hence, Theorem 5 is proved.

Using the same method as to prove Theorem 5 it is straightforward to prove

Theorem 6. For two competing species the effect of an own effort cost increase on the long run optimal resource stock is positive, whereas an increase in the effort cost of the other species reduces the optimal stock level of the former species.

From Theorems 5 and 6 we learn that a price increase (cost decrease) for one of the two species initially augments the value of the resource capital of this particular species proportionally to the price increase (cost decrease). However, it pays to transform some of the increased wealth into capital in general, rewarding the society with the interest expressed by the discount rate, and some into the resource capital of the competing species. The reward to the society from the latter investment is increased revenues and reduced harvesting costs for this competing species.

6 Conclusion

The concept of sustainable development has been widely recognized by the release of the report of the United Nations' World Commission on Environment and Development [18]. However, the interpretation of this concept is not always clear. For renewable resources such as forests, wildlife and fish it seems evident that within a multispecies framework sustainable development must take place for combinations of stocks within the sustainable yield area. Harvesting at the maximum sustainable yield frontier (MSF) may at a first glance seem evident from a biological point of view. However, from an economic point of view MSF-harvesting is optimal only in special cases, such as when there is no discounting and harvest costs are zero.

Identifying the limits of sustainable yields in ecosystems is mainly an ecological task, whereas the utilization of such a system for sustainable economic development in addition involves aspects of economics, technology and social organization. Hence, successful sustainable economic exploitation of a complicated ecosystem is a complex problem which requires a multi-disciplinary approach.

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Notes

[1] The dynamics of the model was analysed in Clark [1], ch. 6.6. We assume that there is a competitive coexistence equilibrium solution of the model.

[2] The effect of one species on another is similar to the interaction of Cournot oligopolists. The isoclines in this model of competing species are identical to the reaction functions of the Cournot model with linear demand and costs.

[3] Essentially all deterministic two species models will have a bounded sustainable yield area, but not necessarily a quadrangle. The predator–prey model used in Flaaten [13] has a triangle as the sustainable yield area.

[4] The notion of MSF was used by Flaaten [12]. The method, however, was first described and used by Beddington and May [16].

[5] Since the X^{MSF} is a hyperbola and it is downward sloping at both of the terminal points of the first quadrant, it is concave to the origin in the sustainable yield area.

[6] Solutions at the boundaries of the sustainable yield quadrangle, with only one species being harvested, may often be the case in the real world. Flaaten [13] analysed such cases, however, it will not be done in this article.

[7] It is not immediately obvious what "open access" in general means in the context of multispecies harvesting. However, recalling the assumption of technically independent

harvesting of the two competing species, open access bioeconomic equlibrium in this case means that profit on each species must be zero.

[8] In general technological parameters also affect the open access stock levels. However, in this case with the Schaefer production function and rescaled fishing effort such that the catchability coefficient equal unity, the open access stock levels in equation (29) are seemingly independent of the harvest technology.

[9] The stock effect is usually interpreted as the effect a marginal increase in the stock level of one species has on its own harvesting costs.

[10] The angle, v, between the axes of the ellipse and the coordinate system can be found from $\tan 2v = 2B/(A - C)$, according to standard geometry. In the case of the profit ellipse of equation (32) v is found from $\tan 2v = (\alpha_1 + \beta\alpha_2)/(1 - \beta)$.

[11] Graphically the MEY-stocks are found at the intersection of $X_1 = (1+\bar{X}^\infty - \gamma X_2)/2$ and $X_2 = (\beta + \hat{X}^\infty - \gamma X_1)/2$ derived from equations (33) and (34).

[12] Solutions outside the sustainable yield area are not of biological or economic interest. Therefore, only the negative root is of interest in this case.

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