Newton's Second Law to go.

Students should have a strong conceptual understanding of the connection between force and acceleration before they start working with applications and problem solving. This has been discussed in the literature.[1,2] During these days of flipped classroom instruction, an at-home activity could be especially valuable. I will describe an experiment with very simple equipment that students could do on their own. To give students a feeling of the connection between force and acceleration, it has been quite common to let them draw blocks or carts across the table with a rubber band or a metal spring. But keeping the rubber band or spring at constant length can be quite a challenge. Instead I was looking for some kind of dynamics cart track made from very simple equipment that students have at home.

A kitchen dynamics cart track

The main parts of the equipment are a piece of cardboard, soda cans, a smartphone and weights, as shown in Fig. 1. In the experiment shown we used small chocolate bars with a mass of 25 g each as weights, but anything that could be placed fairly sturdily on the "cart" could be used. The cart is made from a piece of stiff cardboard about 30 cm long. If it is difficult to find a stiff piece of cardboard, one could fold and glue together a double piece. It would be wise to introduce this experiment some time beforehand so that students could have time to think about equipment they would need. It can also be worth to consider if students should be offered to borrow equipment like metal nuts, (which we have sometimes used instead of chocolate bars) rubber bands, string etc.
The cart rolls very easily on the two empty soda cans. Soda cans are quite light and they are sturdy. In our case the mass of the two soda cans amounted to about 10% of the total mass of the weights, phone and cart. Thus the mass of the soda cans should not contribute too much to the inertia of the system. In the Discussion section we will show that the cans can be said to contribute with only half their mass to the inertia of the system. The phone was fastened to the cardboard with rubber bands. The rubber bands must be placed so that they do not touch the cans as the cardboard rolls forward. The weights are glued to the cardboard with double-sided tape. Instead of a pulley at the edge of the table, the string slides easily over two full soda cans. The two full cans are held closely together by rubber bands, and this makes them not roll off the table. The coupled cans are almost immoveable. The author would be very glad to hear from readers who have other ideas for a "low friction pulley" made from very simple and common material For the smartphone's safety, something soft could be placed in front of the full cans. Here we used a loose-leaf binder. The weights are distributed on the cardboard so that it does not tip during the forward motion. Something should be placed under the edge of the table to stop the falling weights. We used a cardboard box as shown in Fig. 2. The string that can be glimpsed at the right end of the cardboard in Fig. 1 is just for making the release of the cardboard more controlled.

The experiment

The intention of the experiment is to give students an introductory experience with the connection between net force and acceleration when the mass is kept constant. That is $a \propto F$. The trick we use to keep the mass constant in this experiment, is to move weights from the cart to the basket or hook at the end of the string, as shown in Fig. 2 Then the force that accelerates the weights on the cart and the weights in the basket has increased. But the total number of weights to be accelerated is the same. This technique of moving weights is well known from more standard experiments with Atwood's machine.[3] When introducing the experiment beforehand the teacher should emphasize that the same force is needed to accelerate a mass whether it is horizontally or vertically. All the time the weight of the hanging chocolates is to accelerate the whole system, that is all the chocolate bars, cart and phone. We recommend the use of 4 or 5 weights. We also recommend to start with few weights in the basket. Then the acceleration is small and you get some experience before the larger accelerations occur. In any case students should be warned that they must take their time and practice. It is my experience that students are more patient and creative when they work with an experiment at home than when they work in the lab. Students are often to rushed in the lab.

The measurements must be repeated for each weight distribution until you get a plateau of relatively constant acceleration. Then the value of the acceleration is read from the graph. Figure 3 shows an example. Here we used quite small metal nuts as weights and they were moved two by two from the cart to the basket. The measurements were made with the app Vernier Graphical Analysis.[4] The results for the different weight distributions are presented in Fig. 4. Here we made three measurements for each weight distribution. We added a trendline of the form $y = constant \cdot x$ to get an impression of the proportionality. We can see that many of the data points overlap quite well. It was not the point of this introductory experiment on the connection between force and acceleration to introduce the standard units, therefore we have used *number of nuts* as the unit on the horizontal axis.

Discussion

We have disregarded two effects that could be important in this experiment, the inertia of the rolling cans and the friction between the string and the full cans. When I first tested this experiment, I was pleased to see that the results showed quite a high degree of proportionality of F and a. Concerning the rolling cans, both their linear and angular accelerations would be proportional to the net force acting on them. Just to see how well the values in Fig. 4 agree with the quantitative version of Newton's second law $a = \frac{1}{M}F$, we could look at the slope of the trendline in the figure. The slope is 0.459 and the mass of a nut was 16.6 g. Using g = 9.81m/s^2, this gives $a = kF$ where k = 2.82 kg^{-1}. The total mass of nuts, phone and cart in this case was M = 0.291 kg or $1/M$ = 3.43 kg^{-1}. We see that the deviation is quite large. In these experiments it typically lies in the range 15-20%.

The effect of the inertia of the rolling cans could be calculated relatively easily as shown in the following derivation. This derivation is not intended for the students, it is just background information for the teacher. Of course, if later in the semester students will work with rotational dynamics, they could go back to this problem. I think the solution involves a nice application of Newton's third law. In Fig. 5 we see that m_1 is the mass of the cart including weights, m is the mass of the empty cans (for the sake of simplicity only one rolling can is shown in the figure), m_2 is the mass of the weight at the end of the string and $\boldsymbol{g}$ is the acceleration due to gravity. The horizontal force on the can from the cart is $\boldsymbol{S}$, and the horizontal force on the cart from the can is $-\boldsymbol{S}$.

Considering m_1 and m_2 as our system, we have for the acceleration a

$$m_2 g - S = (m_1 + m_2)a. \qquad (1)$$

Let α be the angular acceleration of the cylinder. Taking the torque around the contact point P gives

$$2rS = I_P \alpha, \qquad (2)$$

where r is the radius of the cylinder and I_P is the moment of inertia around the contact line. From the parallel axis theorem we have

$$I_P = I_{CM} + mr^2 = 2mr^2. \qquad (3)$$

Since the cart rolls without slipping on the cylinder $\alpha = a/2r$. Equation (2) then gives

$$S = \frac{ma}{2} \qquad (4)$$

Substituting into Eq. (1) gives

$$a = \frac{m_2 g}{m_1 + m_2 + \frac{m}{2}} \qquad (5)$$

Thus we may say that the cans contribute with only half their mass to the inertia of the cart. This indicates that the inertia of the cans cannot be the main explanation of the deviation. Therefore friction between the string and the full cans must be an important factor.
Since this is not an experiment for studying the quantitative relationship in Newton's second law, but for studying the functional relationship, we have not elucidated this question any further

Summary
In this article we have described a very simple home-made dynamics cart track that can be used to study the functional relationship between net force and acceleration. It can be used as a take-home experiment for students. In that case, it would be natural that students include a video when they report from their experiment. This experiment should give students a better conceptual understanding of the connection between force and acceleration before they start working with applications and problem solving.

References

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