Merger and bilateral bargaining: A note

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Abstract

In a context of bilateral bargaining between an upstream supplier and several downstream buyers, this note determines the conditions under which two buyers have an incentive to merge depending on whether (i) the bargaining process is simultaneous or sequential and (ii) the post merger buyer becomes pivotal or not. We also determine conditions under which the players will prefer to bargain simultaneously or in sequence.

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1 Introduction

When a seller engages in bilateral bargaining with multiple buyers over the payment corresponding to the quantity purchased, one can imagine bargaining structures that are simultaneous as well as those in which negotiations will be conducted in sequence. In the theoretical literature both types of procedure have been investigated: Chipty and Snyder (1999) and Raskovich (2003) are examples of simultaneous bargaining in this context, whereas Stole and Zwiebel (1996) conduct their analysis in a sequential framework. Indeed, Raskovich (2003) presents a numerical example in his model such that the seller involved is always better off with simultaneous bargaining. In this note we present more general conditions under which this result will and will not hold. Part of the focus of the inaugural work of Chipty and Snyder (1999) is on buyer merger with simultaneous bargaining over supply contracts; they divide the incentives into the standard effect that the merger may have on upstream and downstream efficiency, and a new channel reflecting the bargaining position of the merging buyers vis-a-vis the supplier. Raskovich (2003) refines these results by introducing the notion that some buyers may be pivotal in the sense that effective production relies on agreement with them. A merged firm that become pivotal always tends to have its bargaining power reduced since it has to pay a higher price for its supply.[1] We retain the possibility of merged buyers in our analysis, and analyse how bargaining power is affected by the type of negotiation that is conducted.

In contrast to the simultaneous bargaining conducted in the analyses of Chipty and Snyder (1999) and Raskovich (2003), Stole and Zwiebel (1996) examine a sequential bilateral bargaining process between a single firm and employees in the context of non-binding labour contracts. Non-binding contracts are terminable at will and after the failure of one round of bargaining or the termination of an employee, all the other workers can engage in renegotiation. They show that the non cooperative equilibrium outcome in which there is no incentive for agents to re-open negotiation is equivalent to the Shapley values of a corresponding cooperative game without introducing any form of randomization over the player order and expectations of payoffs over all ordering. They underscore that this result holds for any bargaining order, and this is a result that we utilise in our analysis. Chipty and Snyder (1999) relax the assumption of unenforceable contracts, so that the parties that reach an agreement in the bargaining phase are bound by their contract.

[1] Adilov and Alexander (2006) show that this results does not necessarily hold when the initial bargaining positions are not symmetric.

Building on the work of Stole and Zwiebel (1996), Chipty and Snyder (1999) and Raskovich (2003) both assume that bilateral negotiations are simultaneous and are over the marginal contribution of each buyer to the supplier's gross surplus. In every negotiation each buyer believes the supplier will reach efficient agreements with all other buyers and so considers itself as the marginal buyer. Each negotiated deal is "on the margin". Hence, when the contribution of a buyer to the supplier's surplus is greater (lower) than the inframarginal buyer's contribution, it has no incentive to merge (to remain unintegrated) and prefers to bargain over its marginal (average) contribution. According to Raskovich (2003), two points justify the simultaneous bilateral bargaining with respect to sequential bargaining.[2] The first is the numerical illustration mentioned above in which the supplier has fixed cost and where the supplier is always better off in simultaneous negotiation. Additionally, each buyer does not want to be the last to bargain since in that case they will have to cover the fixed cost. This result is not derived from a general specification of the supplier's gross surplus function, however, and we investigate its generality here. The second reason concerns the fact that in sequential bargaining contracts are binding and irrevocable. If we assume that every buyer has the same probability of participating in the final round of negotiations, the sequential configuration considered briefly by Raskovich (2003) corresponds to the Shapley value framework.[3]

The novelty of the work by Chipty and Snyder (1999) and Raskovich (2003) is their attention to the effect that buyer merger may have on the bargaining power of those involved. In spite of the reservations made in the latter paper, we consider simultaneous and sequential bargaining frameworks and show how these affect firm payoffs and the incentives of buyers to merge in the downstream market. This is useful information to the upstream seller if it is the case that it can decide which type of bargaining to enforce. Antitrust and regulatory agencies would also be interested in these effects. We show that whether firms prefer the sequential or simultaneous bargaining setup depends crucially on the shape of the seller's gross surplus function.

Section 2 presents the basic model. Sections 3 and 4 present the simultaneous and the sequential negotiation processes and solve for the equilibrium payoffs and the merger conditions in each case. Brief conclusions are offered in section 5.

[2] Let us remark that this section page 411-412 has been added to the 2001 working paper (available at SSRN: http://ssrn.com/abstract=288274).

[3] Hence we use the term sequential bargaining or Shapley value interchangeably.

2 The Model

Based on Chipty and Snyder (1999) and Raskovich (2003), we consider one supplier producing a homogeneous good demanded by n buyers $i = 1, ..., n$. Let $v_i(q_i)$ be the gross surplus obtained by buyer i when it purchases q_i units of the good, $V(Q)$ the gross surplus obtained by the supplier from production with $Q = \sum_{i=1}^{n} q_i$ the total supply of the good. The sequence of events is the following: At stage one, the supplier enters negotiations with each of the buyers separately. Negotiations are over the payment T_i paid by the buyer to the supplier for the traded quantity q_i. At stage two, the supplier undertakes production and enforces the contracts when costs are covered: $V(Q) + \sum_{i=1}^{n} T_i \geq 0$, and does not produce if this inequality fails, in which case all payoffs are zero. The net surpluses of buyer i and the seller are

$$\begin{aligned} NS_{B_i} &= v_i(q_i) - T_i \text{ for } i = 1, .., n & (1) \\ NS_S &= V(Q) + \sum_{i=1}^{n} T_i & (2) \end{aligned}$$

Following the previous papers, an *s-equilibrium* refers to the pre-merger equilibrium, in which the supplier bargains bilaterally with n separate buyers. The *m-equilibrium* refers to the post-merger equilibrium in which the supplier faces $n-1$ buyers, the merged buyer denoted by 12 and the remaining buyers. When buyers are separate entities during the negotiations, the vector of equilibrium quantities purchased by the buyers is denoted by $q^s = (q_1^s, ...q_n^s)$ where q_i^s maximizes the bilateral joint total surplus with $Q_{-i}^s = \sum_{j \neq i}^{n} q_j^s$

$$q_i^s = \arg\max_x \left[V\left(Q_{-i}^s + x\right) + v_i(x) \right], \ i = 1, .., n$$

However when two buyers, namely 1 and 2 are merged the vector of quantities purchased by the buyers $q^m = (q_{12}^m, q_3^m ... q_n^m)$ is the solution of

$$\begin{aligned} q_{12}^m &= \arg\max_x \left[V\left(Q_{-12}^m + x\right) + v_{12}(x) \right] \\ q_i^m &= \arg\max_x \left[V\left(Q_{-i}^m + x\right) + v_i(x) \right], \ i = 3, .., n \end{aligned}$$

Let us note that $q_i^m \neq q_i^s$ for $i = 3, ..n$ if $q_1^s + q_2^s \neq q_{12}^s$. As in Raskovich (2003), given the transfer paid by the buyers except i, $T_{-i} = \sum_{j \neq i} T_j$ and the quantity they purchase Q_{-i}, a buyer is pivotal to the supplier's production decision if and only if

$$\begin{aligned} V(Q_{-i}) + T_{-i} &< 0 & (3) \\ \max_x \left(V\left(Q_{-i} + x\right) + v_i(x) \right) + T_{-i} &> 0 \end{aligned}$$

The first condition states that production is not possible if buyer i is not involved. The second condition states that there are joint gains to reaching an agreement. To simplify the notation, we set $v_i(q_i) = v_i$.[4]

3 Simultaneous bilateral negotiation

The bargaining is over the transfer that the buyer i has to pay to the seller for the purchase of a given amount q_i corresponding to the incremental surplus generated by their trade. We have to consider two kinds of bargaining depending on whether it concerns a non pivotal or a pivotal buyer. In contrast to the previous work, we assume an infinite alternating offers framework for the bargaining protocol. With common discount factor $1 \geq \delta > 0$, the discounted utility functions for a non pivotal buyer and a seller takes the form

$$\begin{aligned} U_{B_i}^{sim}(T_i, \tau) &= \delta^{\tau}(v_i - T_i) \quad (4) \\ U_S^{sim}(T_i, \tau) &= \delta^{\tau}(\Delta V_i + T_i) \quad (5) \end{aligned}$$

for an agreement reached at time τ. S only considers the incremental gross surplus $\Delta V_i = V(Q) - V(Q_{-i})$ without taking into account the transfers paid by the other buyers. However when the seller faces a pivotal buyer, its discounted utility function is

$$U_S^{sim}(T_i + T_{-i}, \tau) = \delta^{\tau}(V(Q) + T_i + T_{-i}) \quad (6)$$

Let us explain the simultaneous bargaining process between one seller and 3 buyers where buyer 1 is a pivotal. In its negotiation with B_1, S bargains over T_1 but he only considers the gross surplus $V(Q)$ since by definition $V(Q_{-1}) + T_{-1} < 0$ and takes into account the payments T_2 and T_3. In its negotiation with B_2, the bargain is over T_2 and S only considers the incremental gross surplus ΔV_2 without taking into account T_1 and T_3. When negotiating with B_3 over T_3, S only considers the incremental gross surplus ΔV_3 without taking into account T_1 and T_2. In a non-pivotal bargaining, the Rubinstein solution is given by the program

$$\begin{aligned} \Delta V_i + T_i^{B_i} &= \delta\left(\Delta V_i + T_i^S\right) \\ v_i - T_i^S &= \delta\left(v_i - T_i^{B_i}\right) \end{aligned}$$

[4] We use the assumption $v_i(q_i)$ as in Chipty and Snyder, whereas Raskovich allows for the form $v_i(q_i, q_{-i})$.

where the superscript to the transfer denotes the identity of the player making the offer. Assuming that S makes the first offer in all rounds of bargaining leads to the following transfer:

$$T_i^S = \frac{v_i - \delta \Delta V_i}{(1+\delta)}$$

More generally, we obtain the equilibrium payment and payoff

- when B_i is not pivotal (np)

$$T_{i(np)}^{sim} = \frac{1}{(1+\delta)} v_i - \frac{\delta}{(1+\delta)} \Delta V_i \tag{7}$$

$$NS_{B_{i(np)}}^{sim} = \frac{\delta}{(1+\delta)} (v_i + \Delta V_i) \tag{8}$$

- When B_i is pivotal (p)

$$T_{i(p)}^{sim} = \frac{1}{1+\delta} v_i - \frac{\delta}{1+\delta} (V(Q) + T_{-i}) \tag{9}$$

$$NS_{B_{i(p)}}^{sim} = \frac{\delta}{(1+\delta)} (v_i + V(Q) + T_{-i}) \tag{10}$$

From (8) a non pivotal buyer gets a share of the increment to downstream surplus v_i and upstream surplus ΔV_i generated by its trading with the supplier. For a pivotal buyer, its payoff (10) depends on all the payments T_{-i} made by the other non pivotal buyers given by (7). Under the pivotal condition (3), the net surplus of a non pivotal buyer given by (8) exceeds the surplus of a pivotal buyer (10) since the payment paid by a non pivotal (7) is lower than the payment paid by a pivotal (9). It also implies that the seller is always better off when facing n buyers including one pivotal buyer rather than n non pivotal buyers. When the seller faces n non pivotal buyers, its payoff is

$$NS_{S(np)}^{sim} = V(Q^s) + \sum_{i=1}^{n} T_{i(np)}^{sim} \tag{11}$$

and when it faces one pivotal buyer and $(n-1)$ non pivotal buyers

$$NS_{S(p)}^{sim} = V(Q^s) + T_{i(p)} + T_{-i(p)} \tag{12}$$

which implies

$$NS_{S(p)}^{sim} > NS_{S(np)}^{sim} \Leftrightarrow T_{i(p)} > T_{i(np)}$$

Buyer 1 and 2 have an incentive to merge if and only if

$$NS^{sim}_{B^m_{12(np)}} > NS^{sim}_{B^s_{1(np)}} + NS^{sim}_{B^s_{2(np)}} \Leftrightarrow DE + UE + BP > 0 \tag{13}$$

with

$$\begin{aligned} DE &= v^m_{12} - v^s_1 - v^s_2 \quad (14) \\ UE &= \Delta V^m_{-12} - \left[V\left(Q^s\right) - V\left(Q^s_{-1-2}\right)\right] \quad (15) \\ BP &= \left[V\left(Q^s_{-2}\right) - V\left(Q^s_{-1-2}\right)\right] - \Delta V^s_{-1} \quad (16) \end{aligned}$$

Following Chipty and Snyder (1999), the downstream efficiency (DE) term measures the effect of the merger on the merging buyer's gross surplus. $DE > 0$ when the merger leads to a fixed-cost saving or a reduction in marginal costs. The upstream efficiency (UE) term measures the indirect effect of the merger on the supplier's gross surplus. The last term BP for bargaining power captures the effect of the merger on the merging buyer's bargaining position vis-à-vis the supplier. The sign of this term depends on the shape of the function $V()$, i.e. $BP > (<) 0$ when $V(Q)$ concave (convex).[5] If the supplier's gross surplus function is concave, incremental surplus is low at the margin, so buyers tend to gain by merging and bargaining jointly, thereby making their purchases more inframarginal. Conversely, if the supplier's gross surplus is convex, incremental surplus is high at the margin, so a buyer merger tends to worsen the merging buyers' bargaining position.

When a buyer merger creates a pivotal buyer, the condition for merging

$$NS^{sim}_{B^m_{12(p)}} > NS^{sim}_{B^s_{1(np)}} + NS^{sim}_{B^s_{2(np)}} \tag{17}$$

is more strict ($NS^{sim}_{B^m_{12(np)}} > NS^{sim}_{B^m_{12(p)}}$) since a new negative term is added to the term BP

$$BP = \left[V\left(Q^s_{-2}\right) - V\left(Q^s_{-1-2}\right)\right] - \Delta V^s_{-1} + \left(V\left(Q^m_{-12}\right) + T^m_{-12}\right) \tag{18}$$

with T^m_{-12} given by (7). This corresponds to the result shown by Raskovich (2003; equation 12.3).

4 Sequential bilateral bargaining

As discussed in the introduction, the results of Stole and Zwiebel (1996) lead us to conclude that the outcome of sequential bargaining over the incremental

[5] See Chipty and Snyder (1999) Proposition 2.

surplus added by each buyer in all possible orders is given by the Shapley value. With n buyers, the transfer for a non pivotal buyer i is

$$T_{i(np)}^{\text{seq}} = \frac{1}{(1+\delta)} v_i - \frac{\delta}{(1+\delta)} \Gamma_i \tag{19}$$

with Γ_i the shapley value defined by

$$\Gamma_i = \sum_{i \in S \subseteq Q} \frac{(s-1)!(n-s)!}{n!} \left[V(S) - V(S - \{i\}) \right]$$

where S is a subset of Q and s is the size of S. The net surplus of buyer i is then equal to

$$NS_{B_{i(np)}}^{\text{seq}} = \frac{\delta}{(1+\delta)} v_i + \frac{\delta}{(1+\delta)} \Gamma_i \tag{20}$$

The net surplus of the seller is given by

$$NS_{S(np)}^{\text{seq}} = V(Q^s) + \sum_{i=1}^{n} T_{i(np)}^{\text{seq}} \tag{21}$$

To determine the conditions under which a non pivotal buyer prefers to be involved in simultaneous rather than in sequential bargaining, we have to compare $NS_{B_i}^{\text{seq}}$ with $NS_{B_i}^{sim}$. Using (20)-(8) yields

$$NS_{B_{i(np)}^s}^{\text{seq}} - NS_{B_{i(np)}^s}^{sim} = \Gamma_i^s - \Delta V_i^s$$

From (11) and (21), we have for the seller

$$NS_{S(np)}^{sim} - NS_{S(np)}^{\text{seq}} = \sum_{i=1}^{n} \left(\Gamma_i^s - \Delta V_i^s \right)$$

We obtain the following proposition

Proposition 1 *A non-pivotal buyer prefers to be involved in sequential rather than in simultaneous bargaining when the gross surplus function of the supplier $V(.)$ is concave. The seller is better off with simultaneous bargaining when $V(.)$ is concave. The opposite result holds if $V(.)$ is convex.*

Proof. It is easy to show that

- $V(.)$ concave implies $\Gamma_i^s > \Delta V_i^s$
- $V(.)$ convex implies $\Gamma_i^s < \Delta V_i^s$

■

When $V(.)$ is concave, a non pivotal buyer will make a lower payment which increases its payoff but decreases the seller's payoff.

Merger will be beneficial if for two buyers 1 and 2, we have

$$NS^{\text{seq}}_{B^m_{12(np)}} > NS^{\text{seq}}_{B^s_{1(np)}} + NS^{\text{seq}}_{B^s_{2(np)}} \Leftrightarrow DE + \Gamma^m_{12} - (\Gamma^s_1 + \Gamma^s_2) > 0 \tag{22}$$

Let us remark that Proposition 1 remains valid for the seller when a merger occurs but does not create a pivotal buyer. The seller prefers to be involved in simultaneous rather than in sequential bargaining when $V(.)$ is concave. From (11) and (21), the condition $NS^{sim}_{S(np)} > NS^{\text{seq}}_{S(np)}$ implies $(\Gamma^m_{12} - \Delta V^m_{12}) + (\sum_{i=3}^n \Gamma^m_i - \sum_{i=3}^n \Delta V^m_i) > 0$ which is verified for $V(.)$ concave.

The comparison of the merger incentives in both bargaining configurations gives the following proposition.

Proposition 2 *With non-pivotal buyers, the merger condition in the sequential bargaining structure is more restrictive than in the simultaneous one when the following condition holds*

$$\left| NS^{\text{seq}}_{B^m_{12}} - NS^{sim}_{B^m_{12}} \right| > \left| \sum_{i=1}^{2} \left(NS^{\text{seq}}_{B^s_i} - NS^{sim}_{B^s_i} \right) \right|$$

Proof. From (22) and (13), we obtain

$$\Gamma^m_{12} - (\Gamma^s_1 + \Gamma^s_2) > UE + BP \Leftrightarrow \left(\Gamma^m_{12} - \Delta V^m_{-12}\right) > \sum_{i=1}^{2} \left(\Gamma^s_i - \Delta V^s_{-i}\right) \tag{23}$$

■

Note that all the terms in brackets of (23) are positive when $V(.)$ is concave and negative when $V(.)$ is convex. This explains why the condition in Proposition 2 is expressed in terms of absolute value.

Now we assume that after merging the new buyer denoted by B_{12} becomes pivotal. The transfer and net surplus of the pivotal buyer post merger are given by

$$\begin{aligned} T^{\text{seq}}_{i(p)} &= \frac{1}{(1+\delta)} v^m_i - \frac{\delta}{(1+\delta)} \left(V(Q^m) + T^m_{-i}\right) \\ NS^{\text{seq}}_{B_{i(p)}} &= \frac{\delta}{(1+\delta)} \left(v^m_i + V(Q^m) + T^m_{-i}\right) \end{aligned}$$

Hence, if the merged buyer becomes pivotal, the merging condition $NS^{\text{seq}}_{B^m_{12(p)}} > NS^{\text{seq}}_{B^s_{1(np)}} + NS^{\text{seq}}_{B^s_{2(np)}}$ leads to

$$DE + \left[\Delta V^m_{-12} - \Gamma^s_1 - \Gamma^s_2\right] + \left(V(Q^m_{-12}) + T^m_{-12}\right) > 0 \tag{24}$$

with T^m_{-12} given by (19) where the number of buyers n is adjusted in the expression for the shapley value. Since the last term in brackets is negative, becoming pivotal after the merger reduces the incentive to merge. The comparison of the merger incentives in both bargaining cases gives the following proposition.

Proposition 3 *When the merged buyers become pivotal in the post-merger situation, the merger condition in the sequential case will be more (less) restrictive when $V(.)$ is concave (convex).*

Proof. Using (24) and (14)-(15)-(18), we have

$$\begin{aligned}
\frac{\delta}{(1+\delta)}\left(\Gamma^m_{-12} - \Delta V^m_{-12}\right) + \left(\Gamma^s_1 - \Delta V^s_{-1}\right) + \left(\Gamma^s_2 - \Delta V^s_{-2}\right) &> 0 \\
\frac{\delta}{(1+\delta)}\left(NS^{\text{seq}}_{B^m_{-12}} - NS^{sim}_{B^m_{-12}}\right) + \left(NS^{\text{seq}}_{B^s_1} - NS^{sim}_{B^s_1}\right) + \left(NS^{\text{seq}}_{B^s_2} - NS^{sim}_{B^s_2}\right) &> 0
\end{aligned}$$

■

This inequality holds always for $V(.)$ concave since all the terms in brackets are positive. Becoming pivotal after a buyer merger decreases the participants' payoff since it has to make a higher payment; this is exacerbated when $V(.)$ is concave because the other buyers pay less in the sequential bargain.

5 Conclusion

The note has determined the conditions under which an upstream supplier prefers to engage in simultaneous or sequential bargaining with several downstream buyers. The results depend crucially on the comparison between the average of the inframarginal contributions and the average contribution. We also determine the impact of the bargaining protocol on the incentive for two buyers to merge.

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