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## **Assessment of the Remaining Carbon Budget: Incorporating Arctic Amplification in a Simple Response Model**

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“Even the smallest person can change the course of the future.”

- Galadriel



## Abstract

Remaining carbon budgets (RCBs) quantify the total amount of CO<sub>2</sub> that can still be emitted into the atmosphere while keeping the global mean surface temperature below a specific target. However, there is significant uncertainty in RCBs estimates. This thesis develops a Simple Response Model (SRM) to explore the uncertainties in RCBs. We use temperature response functions estimated from multi-box energy-balance models fitted to 4xCO<sub>2</sub> runs of 14 Earth System Models (ESMs) to analyze likelihood plots of RCBs. To validate the SRM, we compare the temperature projections with those from the Model for the Assessment of Greenhouse Gas Induced Climate Change (MAGICC). Incorporation of RCBs for Arctic temperature is implemented through an amplification factor  $T_{\text{arc}} = 0.10 + 2.23T_{\text{glob}}$  estimated from NASA datasets. The SRM can incorporate non-linear permafrost feedback as a hyperbolic tangent function. Our results are in line with standard RCB estimates of 580 GtCO<sub>2</sub> for the 1.5°C target but find the probabilistic range (90% probability to 10%) to be between 1.2°C-1.9°C, and increases to a range of 1.3°C-2.4°C when including non-linearities for the same RCB. The uncertainty in the budgets increase significantly with less ambitious targets. Uncertainty in Arctic temperature are of particular interest due to the risk of triggering an irreversible transition of the Greenland Ice Sheet. Our SRM agrees well with MAGICC, which validates the accuracy of RCBs in our likelihood plots.



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# 1 Introduction

The climate system is equally fascinating as it is terrifying. Today it is changing more rapidly than expected, and it affects us all. We know from paleoclimate records that the climate has undergone considerable changes in the past; however, these changes have been solely due to natural phenomena such as variations in the Earth's orbit or the Sun's energy output. Both historical observations and simulations of climate models shows constant solar radiation and even cooling in the top of the atmosphere; thus, the Sun cannot be the reason for the increasing temperature. The models are unable to reproduce the temperature increase without including an increase in greenhouse gases (GHG's). It is clear (above 95 per cent probability) that there is a causal link between anthropogenic emissions and the current unprecedented temperature trend<sup>1</sup>.

A simple way of understanding the climate system is by considering the radiative energy balance of the Earth and construct an energy balance model (EBM). The EBMs describes the energy fluxes from incoming shortwave solar radiation,  $E_{in}$ , and the outgoing longwave radiation emitted from Earth,  $E_{out}$ . If  $E_{in}$  is greater than  $E_{out}$ , the temperature rises, and vice versa if the opposite state occurs. In the last decades climate models have become increasingly complex, and today's Earth System Models (ESMs) include a wide range of physical and biological processes. These models are used by the Intergovernmental Panel on Climate Change (IPCC) to explore different future scenarios and provide temperature projections. Much research in recent years have used data from the climate models in Phase 5 of the Coupled Model Intercomparison Project (CMIP5), and are also central in the latest IPCC report. Besides temperature projections, climate models can be used to estimate carbon budgets with respect to specified temperature targets. Hence, they can guide policymaking for climate change mitigation.

The aim for the research in this thesis is to develop a Simple Response Model (SRM) that explores the uncertainty in remaining carbon budgets. To validate the SRM, we compare the temperature projections with those from the Model for the Assessment of Greenhouse Gas Induced Climate Change (MAGICC), which is an acknowledged reduced complexity model that allows evaluation of multiple mitigation scenarios. However, our SRM is even simpler than MAGICC, and allows for probabilistic studies. Though the model is simple, it can incorporate non-linear feedbacks such as permafrost thawing.

The thesis is written as a part of a research project with my fellow students Andreas Rostrup Martinsen and Endre Falck Mentzoni. The project was supervised by Martin Rypdal. The three theses from the project have overlapping topics since much of the work was carried out in collaboration, as instructed by our supervisor.

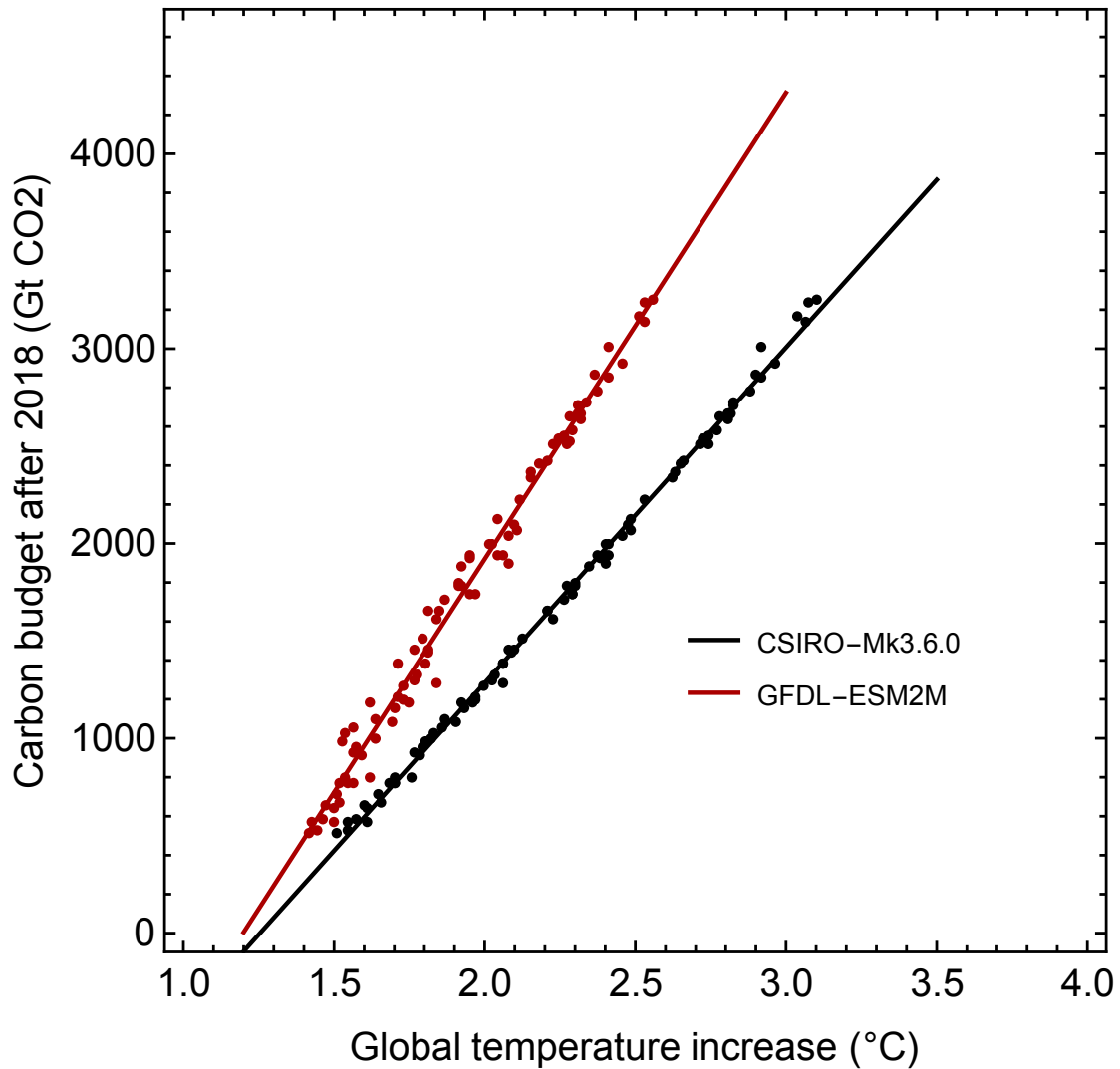


## 2 Background and Theory

### 2.1 RCB/TCRE

The remaining carbon budget (RCB) is a concept that has been a popular tool of guidance since the IPCC Fifth Assessment Report (AR5). Rogelj *et al.*<sup>2</sup> define it as the finite total amount of CO<sub>2</sub> that can be emitted into the atmosphere by human activities while holding global warming to the desired temperature limit. In the AR5, the IPCC establish evidence of a linear relationship between the global temperature increase and anthropogenic CO<sub>2</sub> emissions, and this was the motivation for introducing the concept of an RCB. Used together with the temperature targets from the Paris Agreement it is possible to estimate how much CO<sub>2</sub> we may emit before exceeding the limit, and more importantly when and how to reach net-zero emission targets, which is unavoidable if we are to limit the global temperature increase to acceptable limits.

The RCB is estimated using the transient climate response to cumulative emissions of CO<sub>2</sub> (TCRE), which is the global mean surface temperature (GMST) change per 1000 GtC of anthropogenic CO<sub>2</sub> emissions<sup>3</sup>. By simulating emission scenarios with an ESM, the linear relationship of the cumulated carbon emissions and the corresponding temperature increase illustrates the ratio, as presented in Figure 1. Finding the best linear fit of the relationship yields a more interpretable and more understandable illustration to estimate the RCB for multiple temperature targets.



**Figure 1:** Illustration of estimated Transient Climate Response to cumulative Emissions of CO<sub>2</sub> (TCRE) for two different climate models. 86 different SSP trajectories ran with an ESM estimates the temperature responses used to assess the TCREs for the respective models, as separated by color. The implementation of the TCRE in the SRM (section 3.1.4) uses this procedure though in a more comprehensive matter.

## 2.2 Radiative forcing

The Earth is continuously receiving energy from the Sun, though not all of it reaches the surface. A portion of the radiation is reflected into space by clouds, the atmosphere, and the surface; the factors reflecting incoming solar radiation is the planetary albedo. Some of the incoming solar radiation reaches the surface and heats the planet. Depending on the surface temperature, Earth is emitting heat in order to maintain energy balance. Since some radiation is

absorbed by clouds, aerosols, and GHGs, the effective outgoing radiation is dependent on temperature affected by changes in atmospheric composition.

The difference in incoming and outgoing energy at the top of the atmosphere (TOA) can be defined as the radiative forcing (RF). Forcing is usually expressed in watts per square meter ( $\text{Wm}^{-2}$ )<sup>4</sup>. If the Earth receives more incoming solar radiation than it emits outgoing radiation, the forcing is defined as positive. Negative forcing will lead to more outgoing energy than incoming and will result in a negative forcing and a cooling effect.

Radiative forcing can be modified through feedback mechanisms. A climate feedback is a process which is affected by the forcing, and which in turn amplifies or reduces the original forcing. An amplifying effect is called a positive feedback, for example, the ice-albedo feedback: a warmer Earth causes melting of sea ice, dark ocean increases the energy absorption, more ice melts, and the Earth warms even more. An example of a negative feedback is increased cloud cover: a warmer atmosphere can hold more water, increasing cloud cover, reflecting more incoming radiation, and increasing the albedo which slows the warming.

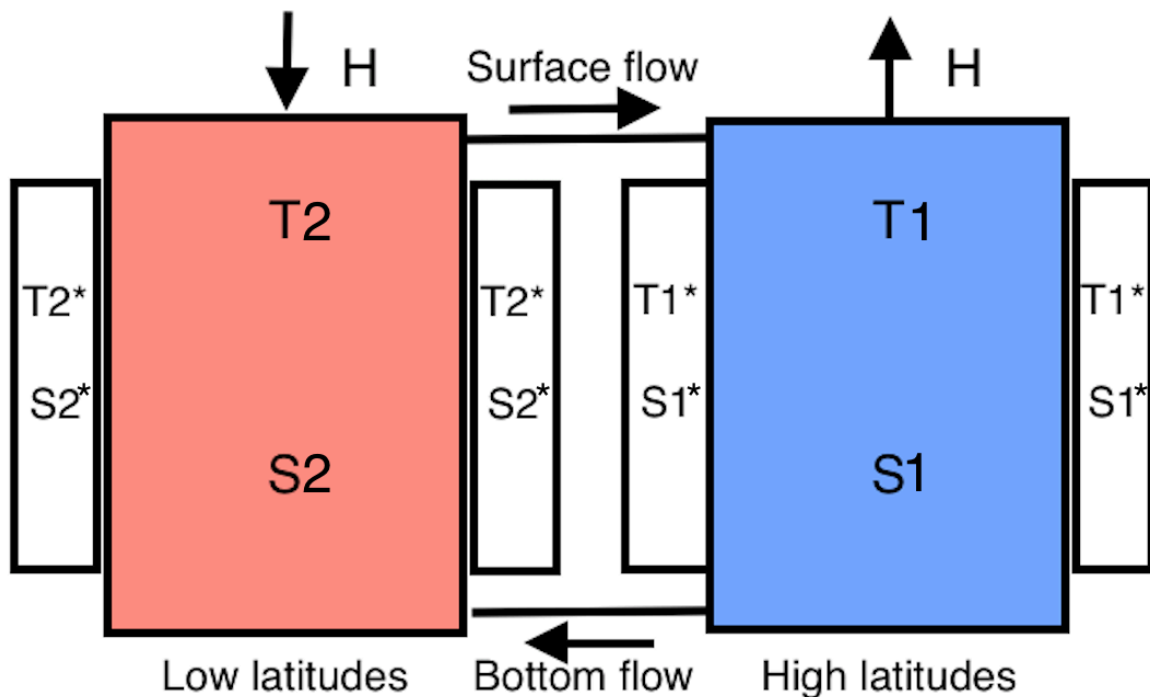
## **2.3 Climate models/Box models**

### **2.3.1 Box models**

When the planet warms, the heat is distributed through different processes. For instance, the oceans mainly distribute energy through a global circulation pattern (the conveyor belt) driven primarily by density gradients in temperature and salinity. The pattern is known as the thermohaline circulation (THC)<sup>5</sup>. Warmer, less dense surface water is transported from low equatorial latitudes to the high polar latitude regions. Reaching the polar region, the warm surface water cools, becoming denser and sinks below the less dense water before flowing towards low latitudes again. In the high latitudes there is downwelling, whereas the opposite is happening in low latitudes; the cold, dense water warms up, gets less dense and rises towards the surface.

The complexity of the climate system comes from multiple components interacting with each other. The challenge for climate models is to compute all these interconnected components. The solution is to partition the system into multiple boxes where compute approximate the

dynamics in each box. A model can be coarse, with few boxes such as a typical two-box model to study the ocean circulation between lower and higher latitudes. Such a system would consider each region as just a reservoir of saltwater with a density-driven circulation, thus makes it easier to explore the flux between the boxes, as shown in Figure 2<sup>5</sup>.



**Figure 2:** Representation of a two-box model describing the overturning ocean circulation in the North Atlantic, where high latitudes are the polar region and low latitudes are the equatorial region. T and S are temperature and salinity marked with 1 and 2 for the different ocean basins, and T\* and S\* are the temperature and salinity of surrounding ocean. H is the salt flux to account for the domination of evaporation and precipitation in the respective boxes. The boxes connect by a bottom flow and a surface flow and combined they represent the overturning circulation driven by temperature and salinity.

### 2.3.2 Energy balance models

The simplest way of modelling the climate system is by approximating the radiative energy balance of the Earth. We call such models the energy balance models (EBMs). The EBMs describe the energy fluxes by measuring incoming shortwave solar radiation, incoming energy  $E_{in}$ , and outgoing longwave radiation emitted from Earth,  $E_{out}$ . If  $E_{in}$  is greater than  $E_{out}$ , the temperature rises, and vice versa in the opposite circumstance. Mathematically, this can be expressed as:

$$AC\Delta T = A(E_{in} - E_{out})\Delta t \quad (1)$$

Here  $C$  is the heat capacity of Earth which is the energy needed to raise the temperature by one degree. However, for simplicity, we assume a constant capacity of the entire globe, i.e., an average (or effective) value. Since the temperature  $T$  is dependent on the time, the amount of energy required to reach  $T$  after a time interval  $\Delta t$  is  $AC\Delta T$  where  $A$  is the surface area of the planet<sup>5</sup>. From the Sun's perspective, Earth's surface looks like a flat disc with a radius  $R$ , and thus surface area is  $\pi R^2$ . The incoming solar radiation per unit time is  $\pi R^2 S_0$ , where  $S_0$  is the solar constant. The co-albedo,  $1-\alpha$ , describes the absorbed energy that the albedo does not reflect, where  $\alpha$  is the planetary albedo. Since the Earth is a sphere, the surface area is  $4\pi R^2$ , and by assuming uniformly distribution of energy, the incoming solar energy per unit area per unit time is:

$$\frac{(1 - \alpha)\pi R^2 S_0}{4\pi R^2}$$

Hence, abbreviating  $Q = \frac{1}{4} S_0$  the incoming energy to Earth per unit area per unit time yields incoming energy to the system as:

$$E_{in} = (1 - \alpha)Q$$

The longwave outgoing radiation from Earth depends on temperature and the assumption of the Earth radiating as a black body. The energy emitted per unit area per unit time follows the Stefan-Boltzmann law<sup>5</sup>:

$$E_{out} = \sigma T^4$$

Here,  $\sigma$  is the Stefan's constant,  $\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ . Dividing by  $\Delta t$ , canceling  $A$ , and with the expressions for  $E_{in}$  and  $E_{out}$ , Equation 1 yields the EBM as the differential equation:

$$C \frac{dT}{dt} = (1 - \alpha)Q - \sigma T^4 \quad (2)$$

At equilibrium, with the absence of forcing, the left-hand side of equation two is zero, and the equation solves for  $T$ . However, this predicts colder Earth than what we observe. The difference in temperature is due to the greenhouse effect which the factor  $e$ , emittance, that accounts for

the efficiency of which the Earth emits radiant energy<sup>5</sup>. After adding the emittance factor, Equation 2 yields a final energy balance equation at equilibrium shown in Equation 3. A climate model described as in Equation 3 is the most basic climate model, often called the zero-dimensional EBM, since there are no spatial variations included.

$$C \frac{dT}{dt} = (1 - \alpha)Q - \epsilon\sigma T^4 \quad (3)$$

### 2.3.3 Advanced climate models

With the progress in modern technology, climate models have evolved accordingly. Improved technology has made it possible to simulate more components of the climate system, such as the carbon cycle, ocean ecology, atmospheric chemistry, rainfall, carbon sinks, along with other climate factors. These advanced models are the ESMs. In combination with Integrated Assessment Models (IAMs), they are used to project future greenhouse gas emissions, climate, economic growth, energy use<sup>6</sup>. IAMs provide information for decision-makers by analyzing the economic, physical and social aspects of the climate as well as an estimate of the benefits of policy options. The use of IAMs and ESMs are discussed further in Section 2.5.

## 2.4 Non-linear forcing effect validated through iteration

We implement a non-linear forcing effect in Section 3.2 to describe non-linearities such as thawing of permafrost. The forcing effect applies by adding an extra forcing parameter and is solved by iteration. This section prove that the method is valid and derives it from a one-box model.

$$C \frac{dT}{dt} = -\lambda T + f(t)$$

It can be rewritten as

$$\mathcal{L}T = f(t), \quad (4)$$

where

$$\mathcal{L} = C \frac{d}{dt} + \lambda.$$

The above equation implies that

$$(\mathcal{L}T)(t) = C \frac{dT}{dt} + \lambda T.$$

The linear differential operator generalizes such that the model may be describe an N-box model. The climate models used in the SRM are on the form in Equation 4 and the equation is solved by finding a function  $\tilde{G}(t)$  such that

$$(\mathcal{L}\tilde{G})(t) = \delta(t)$$

since a solution to Equation 4 can then be written as

$$T(t) = \int_0^{\infty} \tilde{G}(t-s)f(s)ds.$$

Here,  $\tilde{G}(t) = G(t)\theta(t)$ . This solution is proven as follows:

$$(\mathcal{L}T)(t) = \mathcal{L} \int_0^{\infty} \tilde{G}(t-s)f(s)ds$$

$$(\mathcal{L}T)(t) = \int_0^{\infty} (\mathcal{L}\tilde{G})(t-s)f(s)ds$$

$$(\mathcal{L}T)(t) = \int_0^{\infty} \delta(t-s)f(s)ds$$

$$(\mathcal{L}T)(t) = f(t).$$

For the non-linear equation

$$\mathcal{L}T = f(t) + F(T) \quad (5)$$

we can transform it to an integral-equation:

$$T(t) = \int_0^{\infty} \tilde{G}(t-s)[f(s) + F(T(s))]ds.$$

The proof:

$$(\mathcal{L}T)(t) = \int_0^{\infty} [\mathcal{L}\tilde{G}(t-s)][f(s) + F(T(s))]ds$$

$$(\mathcal{L}T)(t) = \int_0^{\infty} \delta(t-s)[f(s) + F(T(s))]ds$$

$$(\mathcal{L}T)(t) = f(t) + F(T(t)).$$

By defining an integral operator

$$RT = \int_0^{\infty} \tilde{G}(t-s)[f(s) + F(T(s))]ds$$

Equation 5 becomes equivalent to:

$$RT = T \quad (6)$$

Equation 6 is solved by iteration as follows:

$$T_{n+1} = R(T_n)$$

The contraction mapping principle theorem states that the iteration converges if



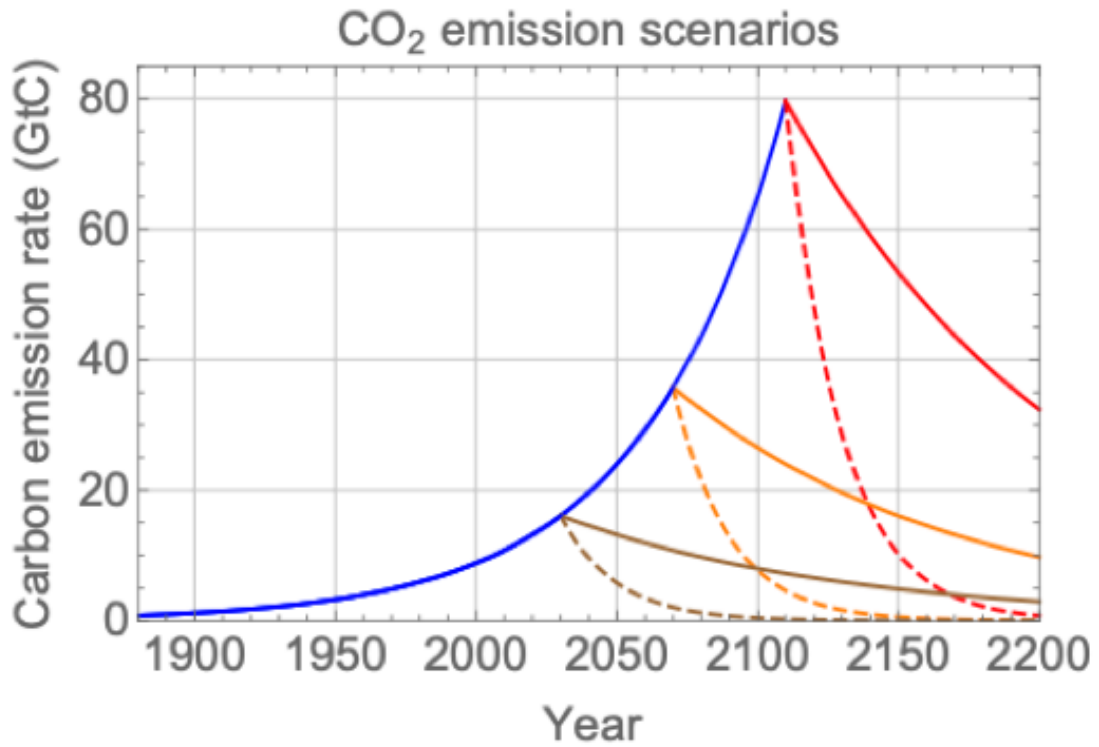
$$\|R(T) - R(T')\| = c\|T - T'\| \forall T, T'$$

For some  $c \in (0,1)$ . Here,  $\|\cdot\|$  might be whichever norm for  $T$ , e.g.,  $\|T\| = \sup_{0 \leq t \leq t_{\max}}$ . The theorem is an adequate condition for the convergence, though not necessary. In this project we assess the convergence numerically.

## 2.5 RCP/SSP

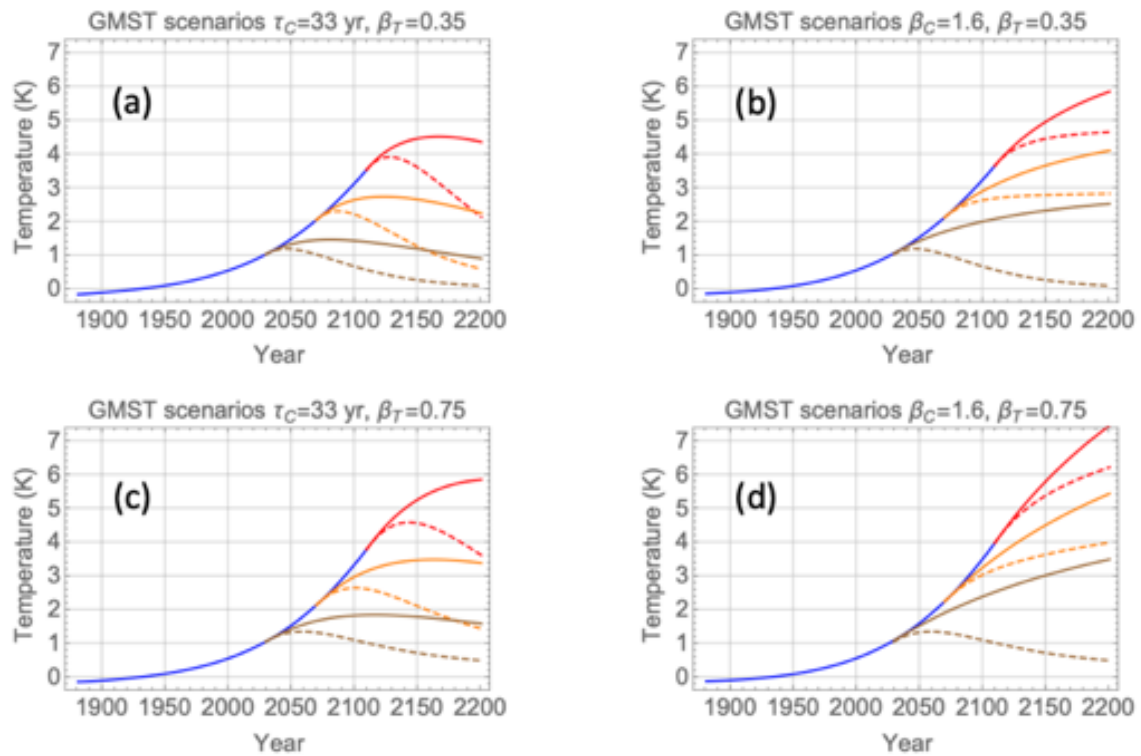
### 2.5.1 Representative Concentration Pathways

Climate models describe processes, components and interactions of the climate system. The magnitude of anthropogenic emissions and land-use change that the models include depends on technological change, economic growth, population growth and urbanization, which has to be attained from IAMs to create different scenarios. From these scenarios one has constructed Representative Concentration Pathways (RCPs)<sup>7</sup>. These pathways quantify the concentration of GHGs in the atmosphere from now until 2100. The associated forcing is obtained from ESMs. Previous scenarios were more limited than the RCPs since they were based on fixed storylines and then had to address climate impacts<sup>7</sup>. Therefore, multiple socio-economic scenarios may correspond to the same forcing, which enables the assessment of various climate policies and technologies compatible with a set climate target. For example, RCP8.5 is a scenario where we continue our current level of emissions and our relatively conservative mitigation policies. In order to reach the 1.5°C or 2°C targets, there is broad consensus that we need to start mitigating as soon as possible. In Figure 3, there are six different emission scenarios based on the RCP8.5 (the blue curve), where the different colors represent different start times for a mitigation policy and the solid lines, and the dashed lines, are 1% (50% over 70 years) and 5% (50% over 13.5 years) annual emission reductions, respectively<sup>8</sup>. The radical approach (5%) considers to be the limit of what is economically sustainable, and 1% reduction is the most feasible.



**Figure 3:** The blue line is an exponential fitted emission scenario close to the RCP8.5 scenario which represents the baseline scenario, with six different emission scenarios with two types of mitigation action applied. The solid lines is the 1% reduction, and the dashed is the 5%. Reproduced from Rypdal<sup>8</sup>.

The different emission scenarios give the atmospheric CO<sub>2</sub> concentration, which in turn gives the temperature responses shown in Figure 4<sup>8</sup>.

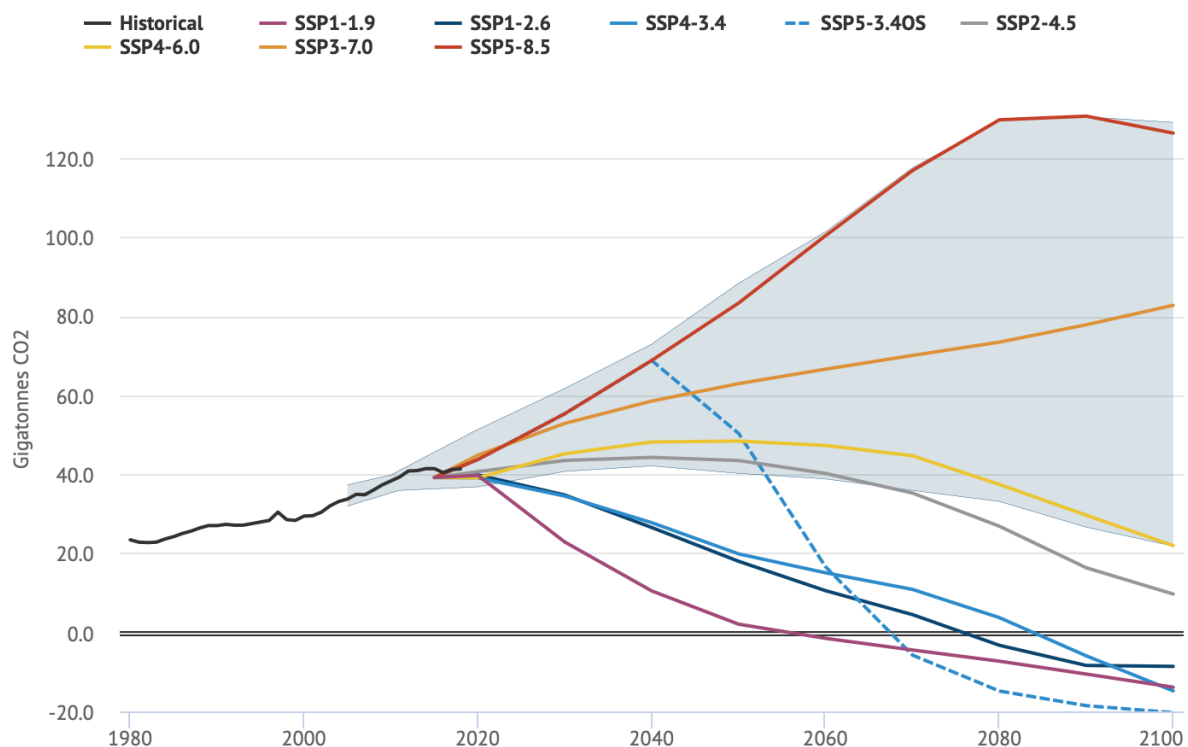


**Figure 4:** Temperature responses to the emission scenarios where the colors correspond to Figure 3. (b) and (d) are the most relevant due to the considered response kernel of 300 years (elimination rate of atmospheric, anthropogenic CO<sub>2</sub>), where (a) and (c) have a response kernel of 33 years. (b) and (d) have a different  $\beta_T$ , which is the corresponding low and high inertia (memory) in the climate system. Reproduced from Rypdal<sup>8</sup>.

From panels (b) and (d) in Figure 4, we see that the climate inertia is of relatively little importance in these scenarios. They also show that the 1.5°C and 2°C targets are only achievable by initiating a radical mitigation scheme, and it must happen within the next decade.

## 2.5.2 Shared Socio-economic Pathways

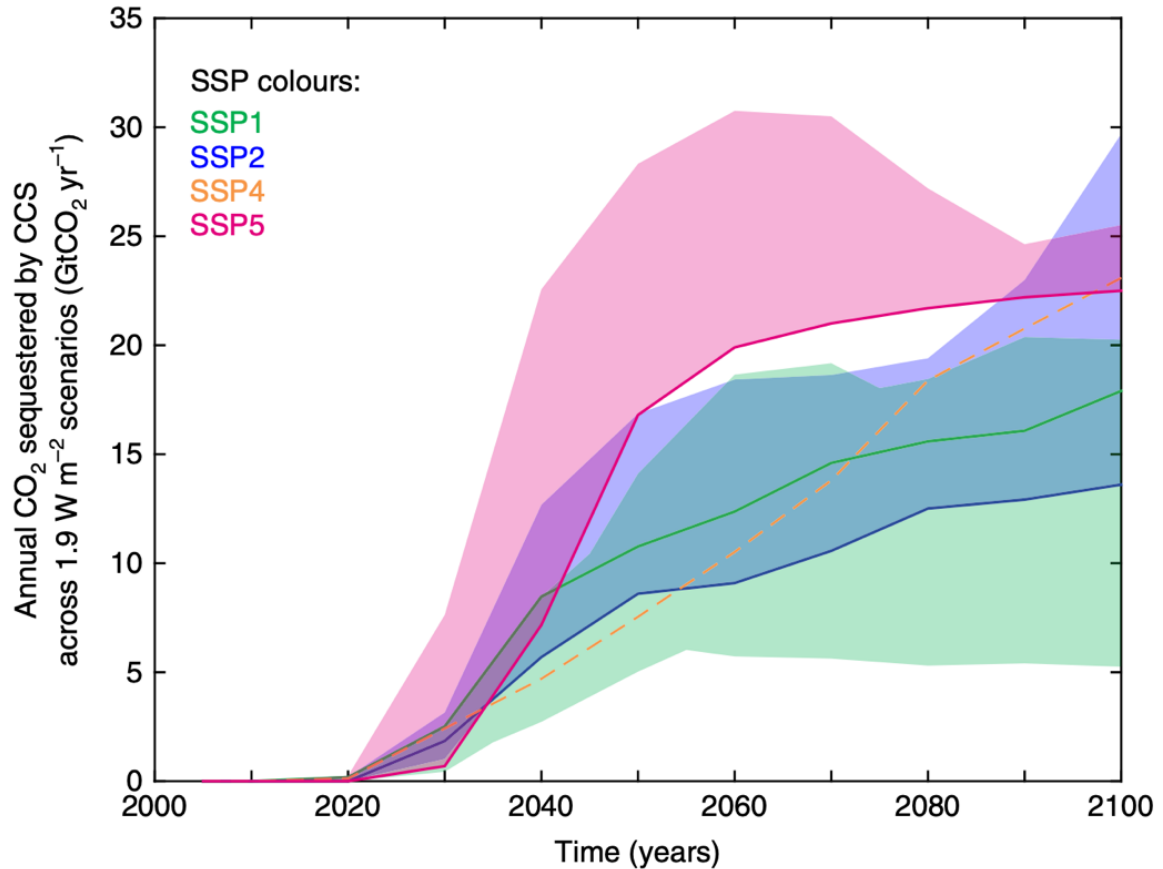
The CMIP5 is used in the AR5 to provide different consistent climate forcing scenarios. Recently, Shared Socioeconomic Pathways (SSPs) have been developed to describe socioeconomic effects as well as changes to the physical Earth system<sup>9,10</sup>. The SSPs will contribute to the AR6 as input to the climate models in CMIP6, which is under development. As the results from around half of the CMIP6 models have been published, the trend seems to be higher climate sensitivities leading to greater warming<sup>11</sup>. The different scenarios are: the green road SSP1<sup>12</sup> (sustainable pathway), middle-of-the-road development SSP2<sup>13</sup> (as we are), the rocky road SSP3<sup>14</sup> (regional rivalry), a road divided SSP4<sup>15</sup> (inequality), and taking the highway SSP5<sup>16</sup> (fossil-fueled development); shown in Figure 5<sup>11</sup>. The SSPs project future GHG emissions and energy use by using IAMs<sup>10</sup>.



**Figure 5:** The different lines represent future CO<sub>2</sub> emissions estimated from SSP trajectories ran through climate models in the CMIP6 and differentiated by colors, as explained in the legend. The range of no-policy narratives from the SSPs are represented by the shaded area, and the black line is historical emissions. Reproduced from Hausfather<sup>11</sup>.

The scenarios reaching the lowest climate goal are dependent on large-scale carbon dioxide removal (CDR) to be able to limit the temperature increase. Hence the most significant contributor would be a form of CCS due to its potential and already existing knowledge. The most prominent form of CCS could be BECCs (bioenergy with carbon capture and storage)<sup>10</sup> due to its reduced climate risk compared to fossil-CCS regarding leakage and capture. BECCS is a process of capturing CO<sub>2</sub> released from the industrial processes using biomass to develop bioenergy and then capture and store it.

The process seems identical to the fossil-CCS stages described later in section 4.2. However, the capture process is more straightforward when the emitted GHGs only consist of CO<sub>2</sub> and not residual gases like methane. Figure 6<sup>10</sup> illustrates the essential part CCS has in the path towards a sustainable climate, and it also illustrates the increasing degree of sequestration dependent on the pathway we choose. An increased commitment towards equality, global environmental goals, and lower resource and intensity result in the lowest deployment of BECCS



**Figure 6:** The different colors represent the annual amount of sequestered CO<sub>2</sub> required to achieve different SSP scenarios and the range of them. It illustrates the importance and the expected development of CCS in the different scenarios. Reproduced from Rogelj *et al.*<sup>10</sup>

of the scenarios achieving 1.9 W m<sup>-2</sup>; 150-700 Gt CO<sub>2</sub>. In comparison, the SSP2 (follow historical patterns) rely on 400-975 Gt CO<sub>2</sub>, and the SSP5 requires 950-1200 Gt CO<sub>2</sub> of BECCS deployment<sup>10</sup>. The estimates of SSP2 and SSP5 might be dangerously high if the technology does not get as effective as expected, thus creating a higher probability of approaching tipping points such as the melting of permafrost or the disruption of the thermohaline circulation. Industries with an elevated quantity of emissions may see the CCS as an incentive to keep emitting since the technology might reduce the emissions sufficiently enough to avoid a climate target.



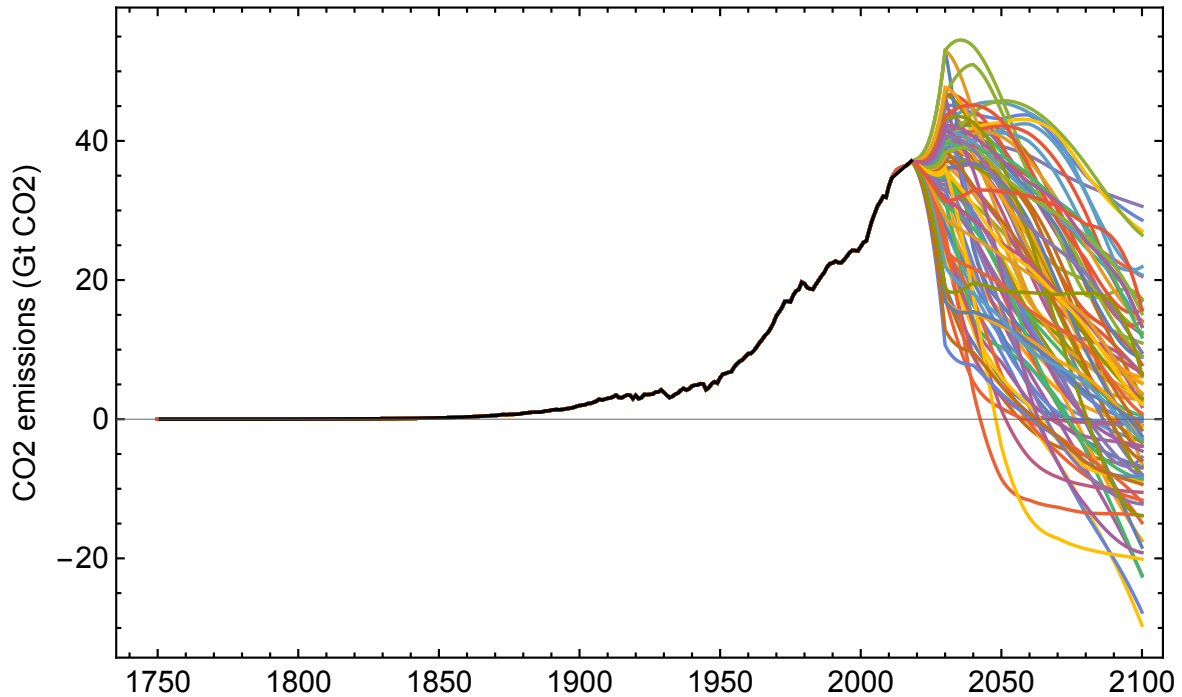
## 3 Modeling and results

### 3.1 Simple Response Model

#### 3.1.1 Emissions

The Paris Agreement aims at holding global warming to well below 2 degrees Celsius and to pursue efforts to limit it to 1.5 degrees Celsius. There is broad agreement in the climate science community that a reduction of emissions is absolutely necessary to be able to limit the global temperature increase. However, there are substantial uncertainties regarding values of the carbon budgets for different temperature targets. Thus, our motivation for this research project is to develop a simple response model and explore scenarios avoiding or barely reaching the temperature targets in the Paris Agreement. The advantages of the model are the ability to run a large number of runs with varying parameters and over a broader range of scenarios.

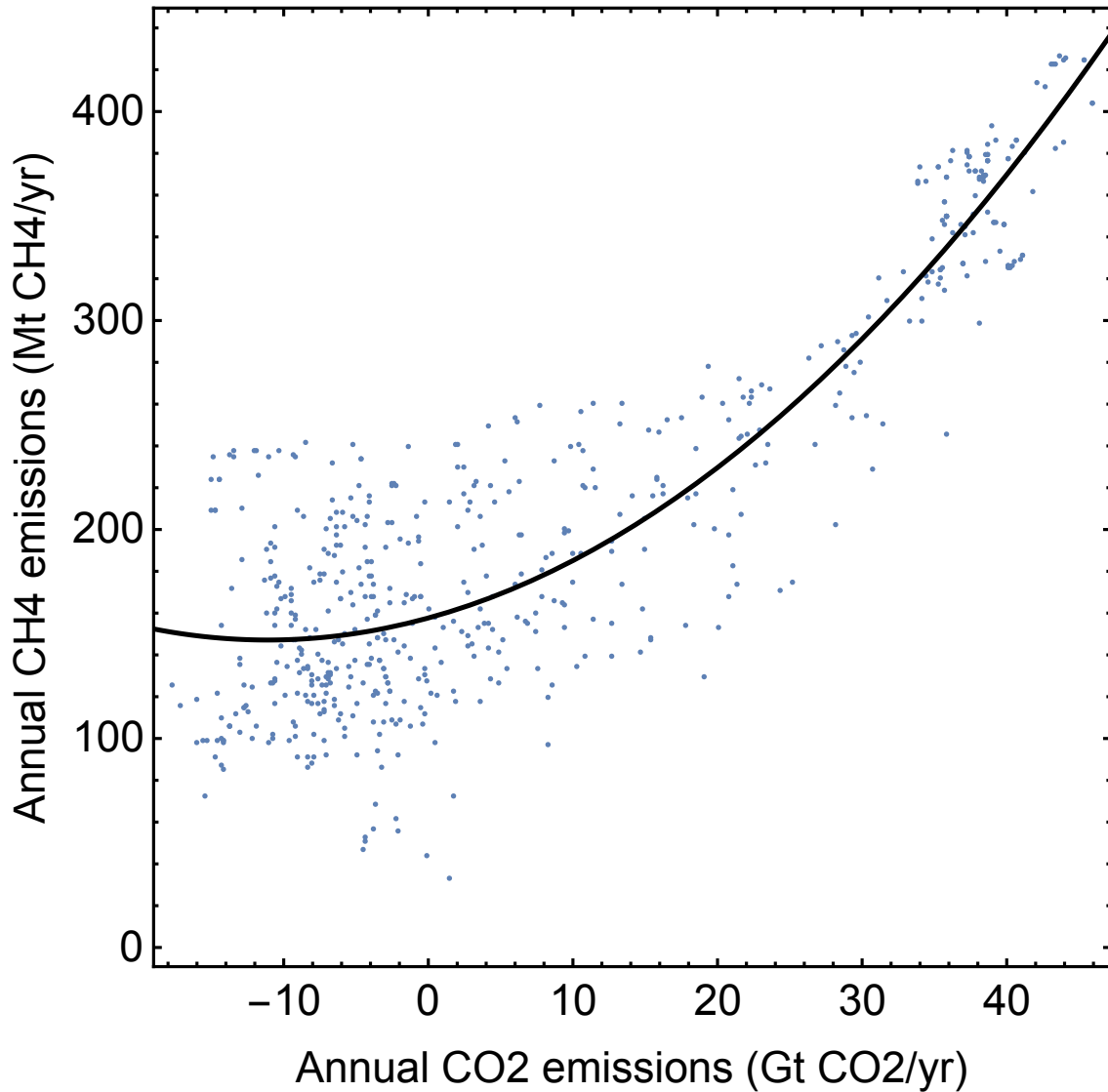
The SRM takes emission scenarios to temperature increase relative to the pre-industrial period. In our analysis we have disregarded scenarios where emissions increase through the 21st century and are far off the targets in the Paris Agreement. We end up analyzing emission scenarios from 86 out of 127 SSPs from the SSP Public Database<sup>9</sup>. Historical temperatures are reproduced using data of anthropogenic CO<sub>2</sub> emissions from 1750-2018, and these include fossil-fuel emissions, industrial emissions, and land use impacts<sup>17</sup>. The 86 future emission scenarios are shown in Figure 7.



**Figure 7:** The 86 scenarios assed in the SRM after disregarding the emission scenarios increasing emissions throughout the 21<sup>st</sup> century.

For historical data we assume that methane emissions are proportional to the CO<sub>2</sub> emissions in such a way that the respective emissions correspond to the observed emissions equal to 2019 level. Hence  $E_m(t) = aE(t)$  where  $E_m(t)$  is methane emissions,  $a$  denotes a factor defining the relationship between methane and CO<sub>2</sub>, and  $E(t)$  denotes CO<sub>2</sub> emissions. The assumption of a constant relationship between methane and CO<sub>2</sub> from 2019-2100 is suitable as long as CO<sub>2</sub> emissions are high enough. In Figure 8, the methane response to CO<sub>2</sub> is represented as a scatterplot with a calculation of the best-fitted quadratic function to show the trend as CO<sub>2</sub> emissions are reduced towards zero or become net negative. The dataset used is from the IAMC 1.5°C Scenario Explorer and Data hosted by IIASA, release 1.1<sup>18,19</sup>. The relationship  $E_m(t) = aE(t)$  implies that methane emissions decrease with the same amount as  $E(t)$ . However, the trend flattens as CO<sub>2</sub> emissions approach zero or become net negative, implying methane emissions will continue independently of CO<sub>2</sub> emissions. Indeed, there are emissions sources such as industrial agriculture, landfills and waste ensuring a supply of methane to the atmosphere regardless of CO<sub>2</sub> emissions. Thus, the assumption of a linear relationship between methane and CO<sub>2</sub> may not be an ideal approximation, and we use the fitted quadratic function in our analyses.



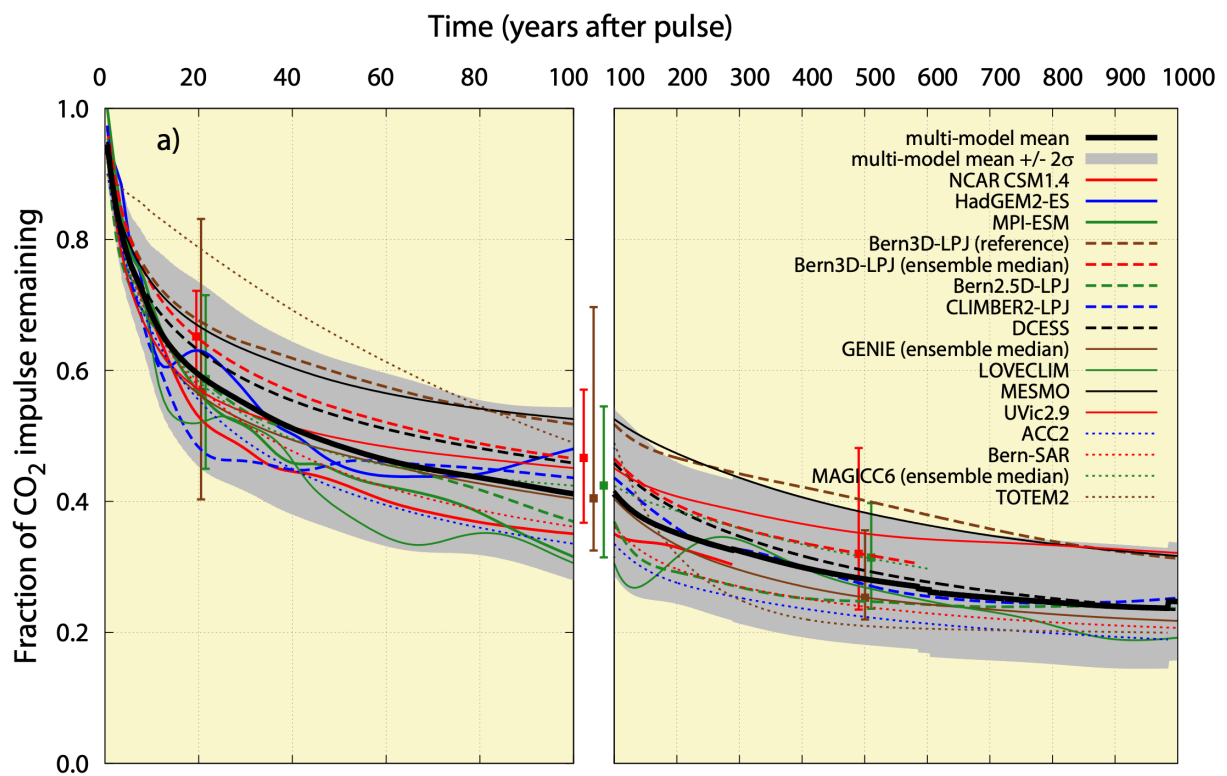


**Figure 8:** Presents the relationship between CO<sub>2</sub> and methane and visualized with a line of best fit for interpretation. The relationship is approximately linear with annual CO<sub>2</sub> emissions corresponding to today's level. As annual CO<sub>2</sub> emissions decrease, methane emissions flattens, even when surpassing net zero emissions.

Aerosol emissions are assumed to be proportional to CO<sub>2</sub> emissions  $E_a(t) = bE(t)$  where  $E_a(t)$  denotes aerosol emissions,  $b$  denotes the constant defining the relationship with CO<sub>2</sub>. Aerosols are tiny solid particles, or droplets, suspended in the atmosphere, which affect the radiation balance both directly and indirectly. Their lifetime varies from a few hours to two weeks, and they are of both natural and anthropogenic origin. Hence the relationship  $E_a(t) = bE(t)$  is only valid as long as net CO<sub>2</sub> emissions are positive. Since there exist natural aerosols independently of CO<sub>2</sub> we assume a constant forcing for net negative CO<sub>2</sub> emissions of  $-0.4 \text{ W m}^{-2}$ , which is consistent with the best estimate of  $-0.354 \text{ W m}^{-2}$  in AR5<sup>4</sup>.

### 3.1.2 Atmospheric Concentrations

For an emission scenario  $E(t)$  (from the SSPs), we can compute the atmospheric concentration of CO<sub>2</sub> in 2100 by using a linear response to model removal of carbon from the atmosphere. The carbon module in our SRM uses four exponential response functions. The linear combination of exponential functions are fitted to runs in the carbon cycle-climate model intercomparison project<sup>20</sup>. We made fits to the multi-model mean, and to plus/minus one standard deviation. The carbon-cycle model experiments were set up with a 100 GtC pulse added to the existing concentration of CO<sub>2</sub>, and the response functions describe the decay and saturation of the pulse<sup>20</sup>, as illustrated in Figure 9.



**Figure 9:** The carbon cycle-climate model intercomparison project done by Joos *et al.* The carbon model in the SRM derives from this project by fitting four exponential functions, though with plus/minus one standard deviation instead of plus/minus two as the project does. The intercomparison project add a 100 GtC pulse and quantifies how much CO<sub>2</sub> remains. All the different climate models involved are labeled in the legend. The figure is reproduced from Joos *et al.*<sup>20</sup>.

The response function of our carbon model  $G_{CAR}$  has the form

$$G_{CAR}(t) = 0.47 \left( c_0 + c_1 e^{-\frac{t}{\tau_1}} + c_2 e^{-\frac{t}{\tau_2}} + c_3 e^{-\frac{t}{\tau_3}} + c_4 e^{-\frac{t}{\tau_4}} \right) \quad (7)$$

Where 0.47 is the airborne fraction which is the fraction of anthropogenic carbon emissions which remain in the atmosphere after natural processes have absorbed a portion<sup>21</sup>. The coefficients  $c_i$  (normalized to sum to 1) are the parameters giving the best estimate to the pulse experiments, and  $\tau_i$  are the time scales of 1, 10, 100 and 1000 years which we have chosen to span the scales of variability in the carbon cycle. For instance, carbon sink through plants is a swift cycle where the time scale is about a year, whereas transfer of carbon from the ocean surface to the deep ocean has a time scale of 1000 years.

The linear response model for the atmospheric concentration of CO<sub>2</sub> and methane adds the pre-industrial levels to the convolution integrals. Hence, the following equations yields the estimated concentrations:

$$C = C_0 + c_{CAR} \int_{-\infty}^t G_{CAR}(t-s)E(s)ds \quad (8)$$

$$M = M_0 + c_{MET} \int_{-\infty}^t G_{MET}(t-s)E_m(s)ds \quad (9)$$

Here,  $C$  and  $M$  are concentrations of CO<sub>2</sub> and methane respectively,  $C_0 = 280$  ppm and  $M_0 = 700$  ppb are the pre-industrial baseline concentrations. The concentration responses are assumed to be translation invariant, i.e. the response of  $C$  and  $M$  are thought to respond in the same manner today as in the future<sup>8</sup>.

We use a separate response function to estimate the methane response,  $G_{MET}$ , which is a single exponential function:

$$G_{MET}(t) = d e^{-\frac{t}{\tau}} \quad (10)$$

Here,  $d$  is the parameter giving the best fit of the response,  $\tau$  is the general time scale of methane in the atmosphere and is estimated to 12.4 years. The lifetime of methane, excluding

interactions in the atmosphere, is 10 years, though, since methane emissions have positive feedback on the concentration of itself, due to interactions with OH generating a more prolonged lifetime, its characteristic time scale is 12.4 years<sup>4</sup>.

### 3.1.3 Forcing

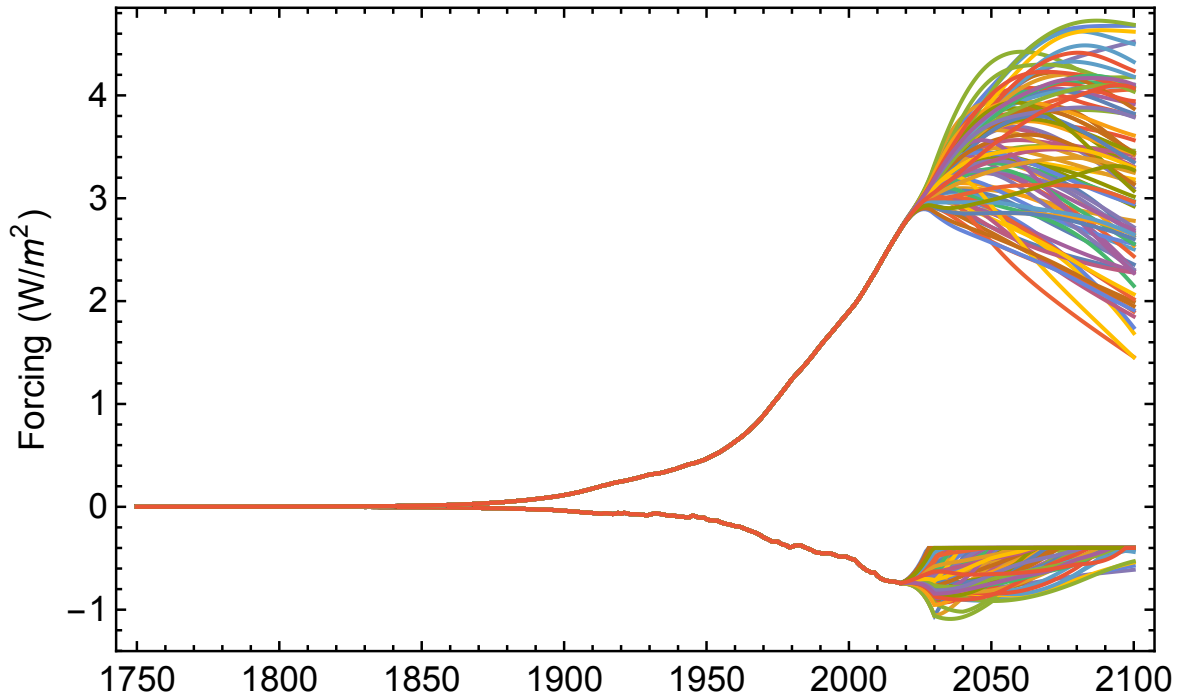
Radiative forcing associated with changing concentrations of CO<sub>2</sub> and methane is estimated through standard relations as described in section 3.1.2. Aerosols forcing, however, is only dependent on aerosol emissions. Since the lifespan of aerosols is a few hours to two weeks, it is assumed to be no cumulative concentration of aerosols. Therefore, our SRM uses a simplification where the aerosol forcing is proportional to the emissions, i.e.  $F_A \propto E_a$ . As aerosol forcing is highly uncertain, this simplification might be as suitable as any other incorporation. Myhre *et al.*<sup>22</sup> estimates the relation between  $C$  and the radiative forcing from CO<sub>2</sub> through the logarithmic relationship in Equation 11.

$$F_{\text{CO}_2}(t) = 5.35 \ln \left( 1 + \frac{C - 280 \text{ ppm}}{280 \text{ ppm}} \right) \text{Wm}^{-2} \quad (11)$$

The standard algebraic relation from Myhre *et al.*<sup>22</sup> estimates the radiative forcing from methane concentration, as shown in Equation 12. We do not take nitrogen-methane interactions<sup>22</sup> into account:

$$F_{\text{MET}}(t) = k(\sqrt{M} - \sqrt{700 \text{ ppb}}) \text{Wm}^{-2} \quad (12)$$

Here,  $k$  is a constant which incorporates methane's potency as a GHG. The total anthropogenic radiative forcing is the combination of CO<sub>2</sub>, methane and aerosols and becomes  $F_{\text{TOT}}(t) = F_{\text{CO}_2}(t) + F_{\text{MET}}(t) + F_A(t)$ , as illustrated in Figure 10.

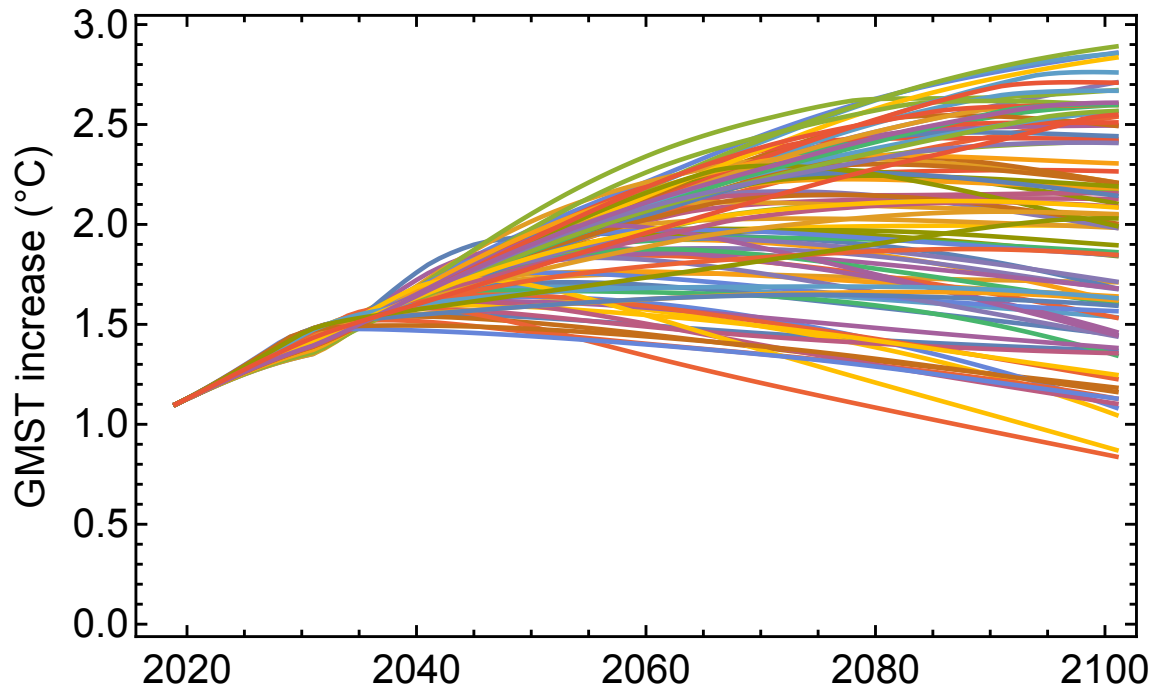


**Figure 10:** The red lines from 1751 to 2018 are the historic observed forcing ( $\text{W m}^{-2}$ ) of CO<sub>2</sub> and methane (upper line after split), and aerosols (lower line after split). From 2019 all the colored lines are multiple scenarios for one climate model, where the upper collection is methane and CO<sub>2</sub> forcing while the lower is aerosol forcing. The resulting global mean surface temperature (GMST) increase is shown in Figure 11.

For a time-dependent forcing, the estimate of the temperature response is given by convolving the total anthropogenic forcing with an exponential response function, as shown in Equation 13.

$$T(t) = \int^t G_{\text{CLI}}(t-s)F_{\text{TOT}}(s)ds \quad (13)$$

Here,  $G_{\text{CLI}}$  is a response that emulates one of the climate models in the CMIP5 ensemble. More precisely, In the SRM we use response function estimates from multi-box energy-balance models fitted to 4xCO<sub>2</sub> runs of 14 different ESMs in the CMIP5 ensemble. The parameters from the 14 box-models are obtained from Cummins *et al*<sup>23</sup>. The ensemble of emission scenarios in the SSPs provide multiple storylines towards reaching specific temperature targets. Each of these emission scenarios are used as input in 14 different ESM emulators. Figure 11 shows the projected temperature response from Equation 13 for one ESM emulator.



**Figure 11:** Visualization of elimination of model-observation differences by running the simulations from the same point in time. Here showing the temperature response of the 86 scenarios assessed for one climate model.

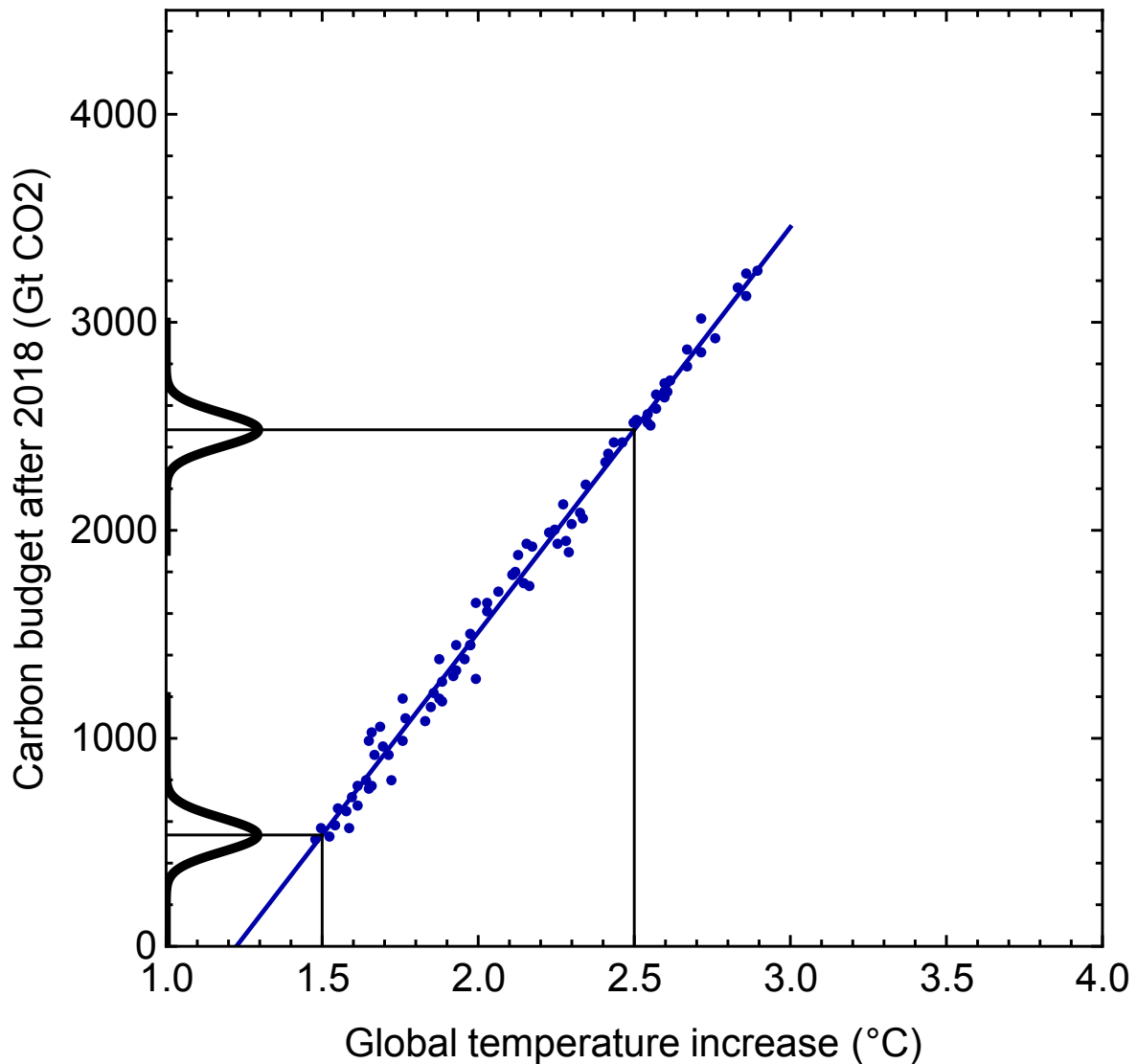
### 3.1.4 TCRE and carbon budget probability

In addition to varying the climate model, we assess each of the 86 positive emission scenarios by using both the upper and the lower limit of carbon module. Moreover, internal variability (described in section 3.1.5) incorporated in the climate models. The TCRE from the individual CMIP5 emulators gives the remaining carbon budget estimate from that specific ESM (Figure 1). However, our estimate of the RCB, and the uncertainty around it, is obtained by systematically assessing the budgets from all the climate models in the ensemble.

The uncertainty in the budgets comes from different factors. One of them is the uncertainty in how the TCRE responds to a high concentration of CO<sub>2</sub> emissions since it is not constant above 2000 GtC (and even slightly less constant above 1500<sup>24</sup>). Another factor is the uncertainty in the climate sensitivity. The relevant measure of climate sensitivity for the TCRE is the transient climate response (TCR), which the IPCC defines as the change in global mean surface temperature at the time of CO<sub>2</sub> doubling following a gradual increase in concentrations<sup>24</sup>. The TCR varies from 1.0°C to 2.5°C within the CMIP5 ensemble, which results in a TCRE variation range of 0.8°C to 2.5°C per 1000 GtC<sup>24</sup>, or 0.2°C-0.7°C per 1000 GtCO<sub>2</sub><sup>25</sup>. The TCRE is also

dependent on the airborne fraction at the time of the TCR, though it is the latter which is the most significant uncertainty<sup>26</sup>.

We apply the method of Cox *et al.*<sup>27</sup> to estimate the probability density functions (PDF) of the carbon budgets on each climate model in CMIP5. To visualize we plotted the PDF for temperature targets of 1.5°C and 2.5°C given a single model (Figure 12). Cox *et al.*<sup>27</sup> use a Gaussian distribution of the PDF, which is the AR5 consensus, though with limited evidence to support it<sup>25</sup>. One should examine other distributions to investigate the impact, which Peters *et al.*<sup>28</sup> did with a lognormal distribution that resulted in a budget variation of  $\pm 250$  GtCO<sub>2</sub> for a 2°C threshold. However, even with a known distribution of the TCRE, the carbon budgets would remain highly uncertain due to path-dependent non-CO<sub>2</sub> emissions, and the complexity of including negative emissions when estimating the carbon budget<sup>29</sup>. The uncertainty in the carbon budgets is an essential part of our research project and is addressed in the following.



**Figure 12:** Estimate of the probability density function (PDF) of the carbon budgets to the TCRE of a single ESM. The PDF plotted on the y-axis is a Gaussian distribution (AR5 consensus) from Cox <sup>27</sup> and done for 1.5°C and 2.5°C.

### 3.1.5 Uncertainty in RCBs

In the following we explore the uncertainty of the TCRE for the different ESMs, and how the uncertainty affects the estimate of the carbon budget, in a systematic manner. Our likelihood plots show RCBs between 0 and 4000 GtCO<sub>2</sub> with steps of 100 GtCO<sub>2</sub>. They display estimates of temperature responses from 1.1°C to 4.0°C, with computations carried out for every 0.01°C. Our motivation is to explore the uncertainty of aiming for a specific carbon budget, and to explore how the carbon-budget-uncertainty vary with less ambitious temperature targets.

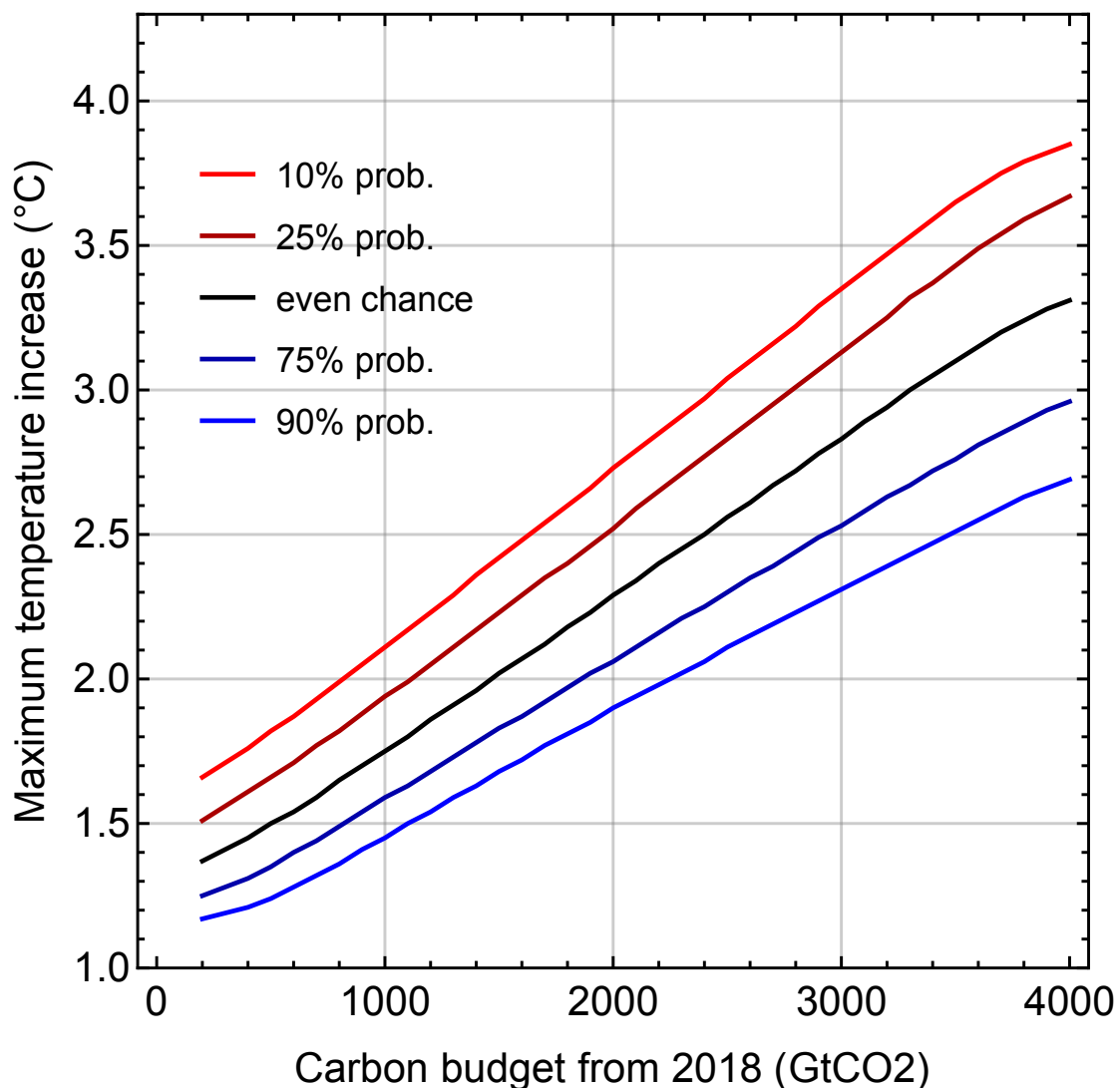
We investigate the causes of uncertainty in the RCB by constructing three different likelihood plots: one assessing uncertainty in the 14 ESMs in the CMIP5 ensemble, another assessing the



sensitivity to variations in the carbon module, and a third, assessing the uncertainty from internal variability at decadal time scales in the global (and Arctic) temperatures. Combining the effects of these three factors gives our fourth likelihood plot, displaying the total uncertainty in the RCBs for each target temperature.

### 3.1.5.1 Climate models

The climate model uncertainty explains the variation in the TCRE from inter-model variations within the CMIP5 ensemble. We assess this by using the mean carbon model,  $G_{CAR}$  and no internal variability. The likelihood plot is shown in Figure 13. The figure shows that uncertainty grows as the carbon budget increases.

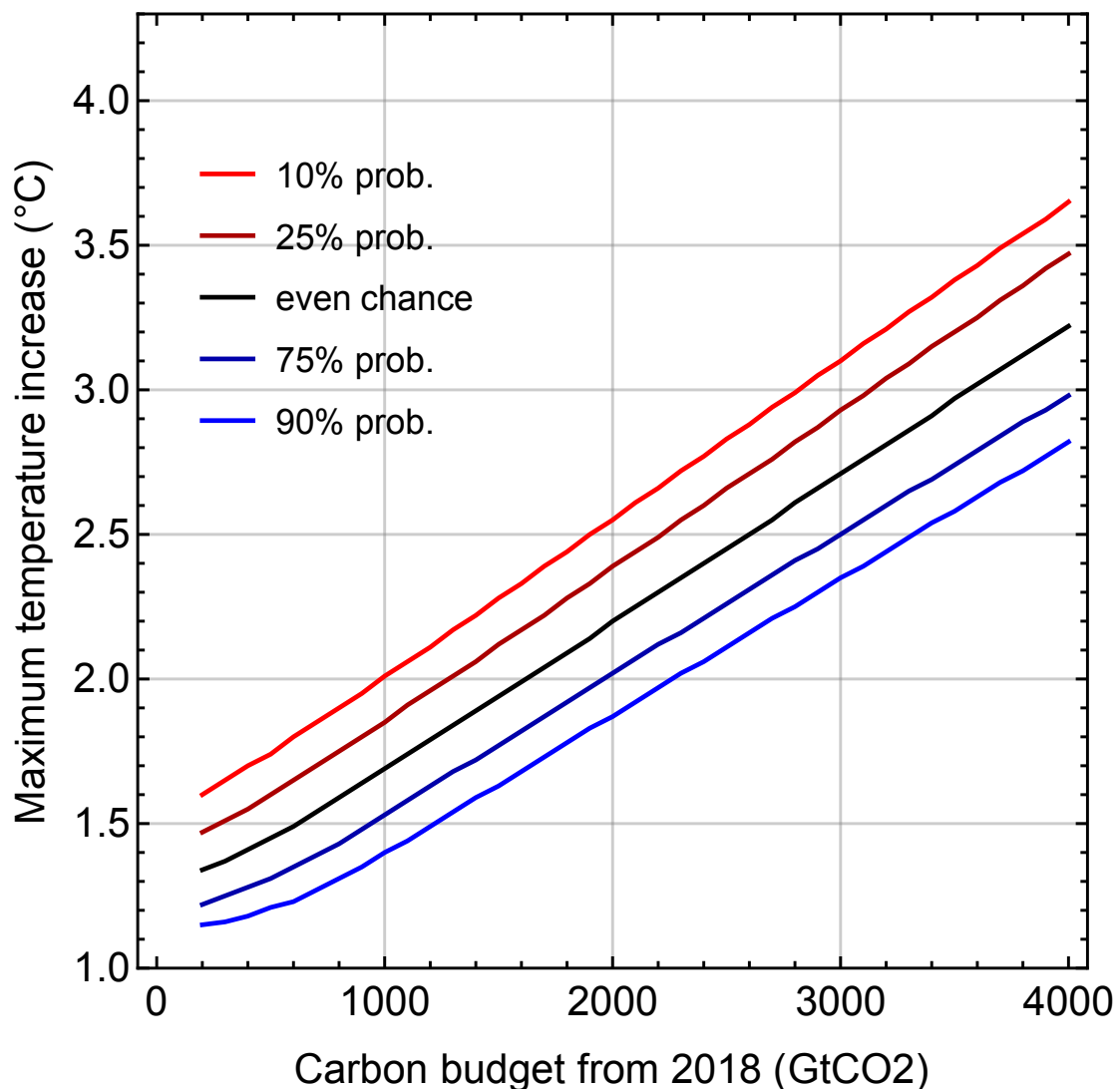


**Figure 13:** Likelihood plot of the RCB to the climate models used from the CMIP5 ensemble. The estimates are run without internal variability and with a mean carbon model to isolate the models and explore the uncertainty in them. The different lines represent the probability of maximum temperature increase as a response to different carbon budgets. The plot considers RCB between 0-4000 GtCO<sub>2</sub> and temperature responses from 1.1°C to 4°C with steps of 0.01°C.

### 3.1.5.2 Carbon model

The uncertainty in the carbon model is evaluated by running the two carbon models representing the plus/minus one standard deviation range in the carbon cycle-climate model intercomparison project. We use the mean of the 14 ESMs and disregard internal variability. The results are shown in Figure 14.

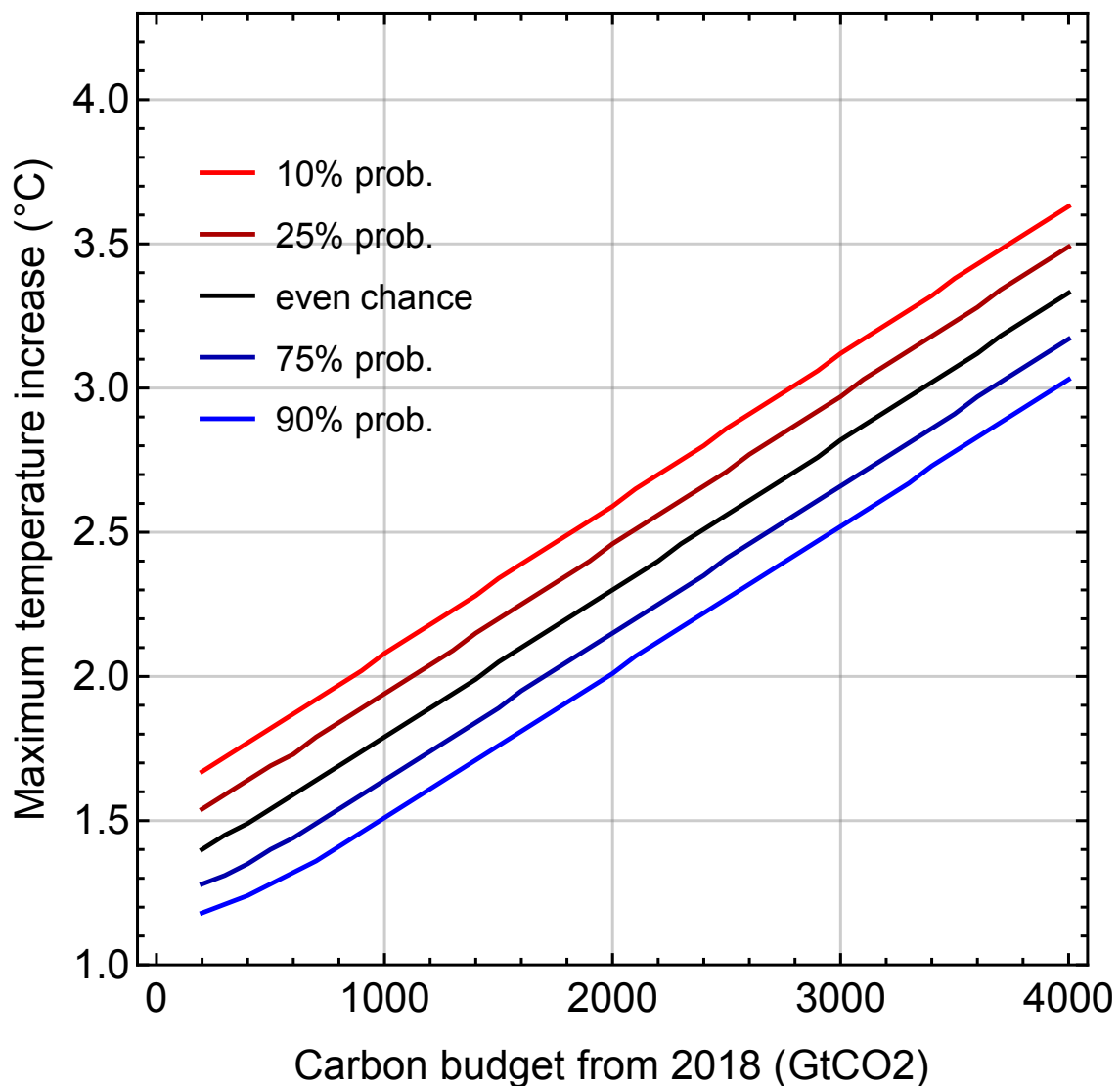
The uncertainty in carbon model used in the SRM appears to be approximately constant as the carbon budget increases, which indicates that the carbon model does not cause a significant target-dependent uncertainty in the RCB.



**Figure 14:** Likelihood plot of the RCB when estimates is calculated with a mean of the 14 ESM's, two carbon models representing plus/minus one standard deviation range, and excluding internal variability. The result isolates the impact on the uncertainty from the carbon models. The different lines represent the probability of maximum temperature increase as a response to different carbon budgets. The plot considers RCB between 0-4000 GtCO<sub>2</sub> and temperature responses from 1.1°C to 4°C with steps of 0.01°C.

### 3.1.5.3 Decadal internal variability

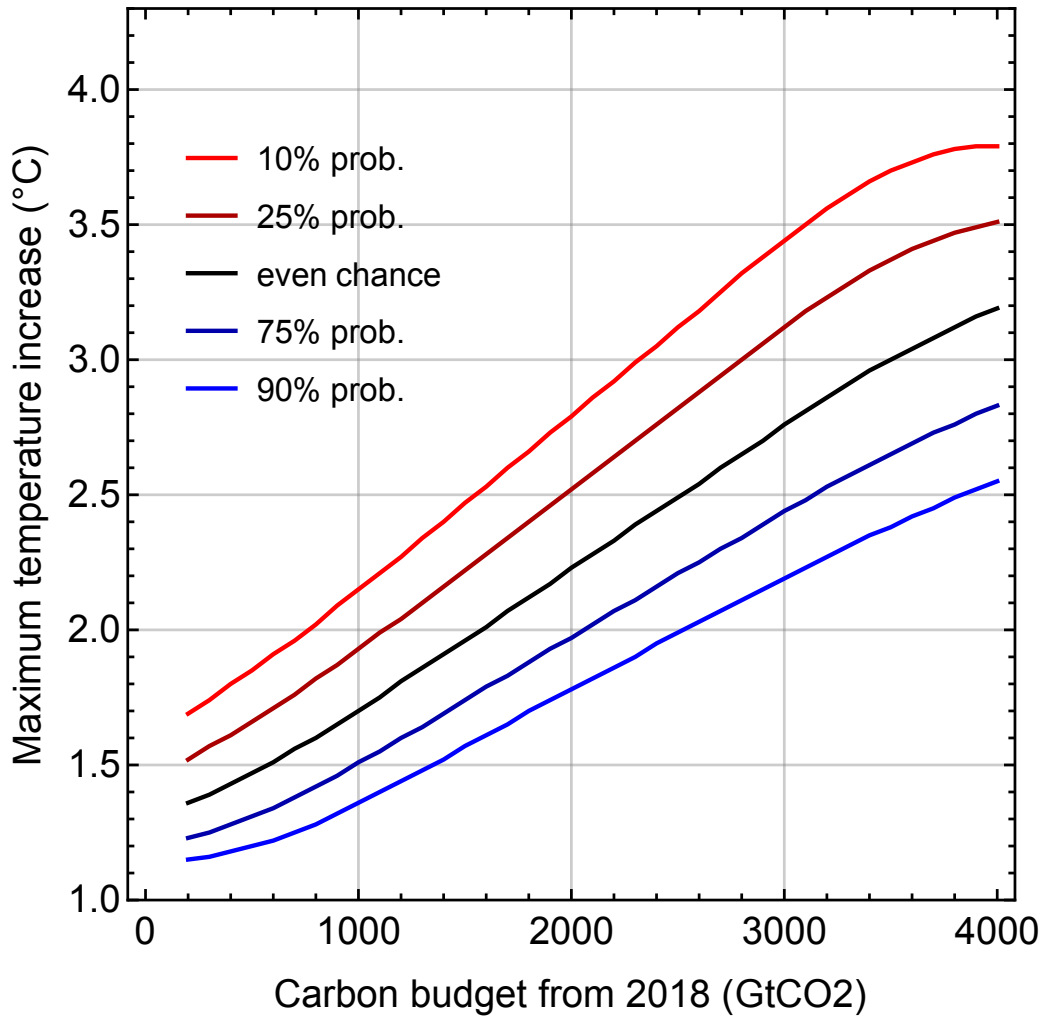
We incorporate internal decadal variability through and additive Gaussian white noise (AWGN) in the convolution integrals for the 14 climate models, and applying a 10-yr moving average to the stochastic component of the climate variability. To isolate the effect of internal variability on RCB uncertainty, we use the mean climate module and the mean carbon module. The result is shown in Figure 15. As with the carbon model, the internal variability clearly show a near constant value across different targets.



**Figure 15:** Likelihood plot of the RCB when estimates is calculated with a mean of the 14 ESM's, a mean carbon model, and includes decadal internal variability by applying Additive White Gaussian Noise (AWGN) to each of the 14 climate models. The result isolates the uncertainty from the internal variability. The different lines represent the probability of maximum temperature increase as a response to different carbon budgets. The plot considers RCBs between 0-4000 GtCO<sub>2</sub> and temperature responses from 1.1°C to 4°C with steps of 0.01°C.

#### **3.1.5.4 Combined uncertainty**

Combining all 14 ESM, two carbon models (upper and lower), and internal decadal variability, we obtain the total uncertainty in the RCB. The likelihood plot in Figure 16 shows the increase in the uncertainty with increasing budgets, and by reviewing the uncertainty factors from section we know that this effect is caused by uncertainty in the climate models. In Figure 16, the uncertainty increases with less ambitious mitigation targets, that is with increasing carbon budgets. A budget with an even chance of achieving maximum temperature increase of 1.5°C RCB=580 GtCO<sub>2</sub> yields an uncertainty range (90% probability to 10%) of 1.2°C to 1.9°C, difference of 0.7°C, while for RCB=4000 GtCO<sub>2</sub> has a range between 2.55°C to 3.8°C, 1.25°C difference. The RCB for a specific temperature response is often estimated with 50 per cent probability which increases the uncertainty range the higher the response is. For example, with 50 per cent probability, the carbon budget of approximately 700 GtCO<sub>2</sub> will lead to a temperature response of 1.6°C. However, there is a 10 per cent chance of exceeding 2.0°C, a 25% temperature increase. Such an increase may be catastrophic to the climate system.



**Figure 16:** Likelihood plot of the uncertainty in the RCB with all factors included; 14 ESM's, two carbon models representing plus/minus one standard deviation range, and decadal internal variability. The different lines represent the probability of maximum temperature increase as a response to different carbon budgets. The plot considers RCB between 0-4000 GtCO<sub>2</sub> and temperature responses from 1.1°C to 4°C with steps of 0.01°C.

## 3.2 Non-linear feedbacks

The most uncertain aspect of climate models is the modelling of non-linearities in the climate system. There are non-linearities in all dynamical interactions in the Earth's climate system and, as Rial *et al.*<sup>30</sup> describes, it includes the natural spheres (e.g., atmosphere, hydrosphere, biosphere and geosphere) and the anthroposphere (e.g., society, economy, culture)<sup>30</sup>. Non-linear dynamic climate drivers, such as the thermohaline circulation, may have multiple equilibriums and thresholds, which may result in rapid changes in the climate system<sup>30</sup>.

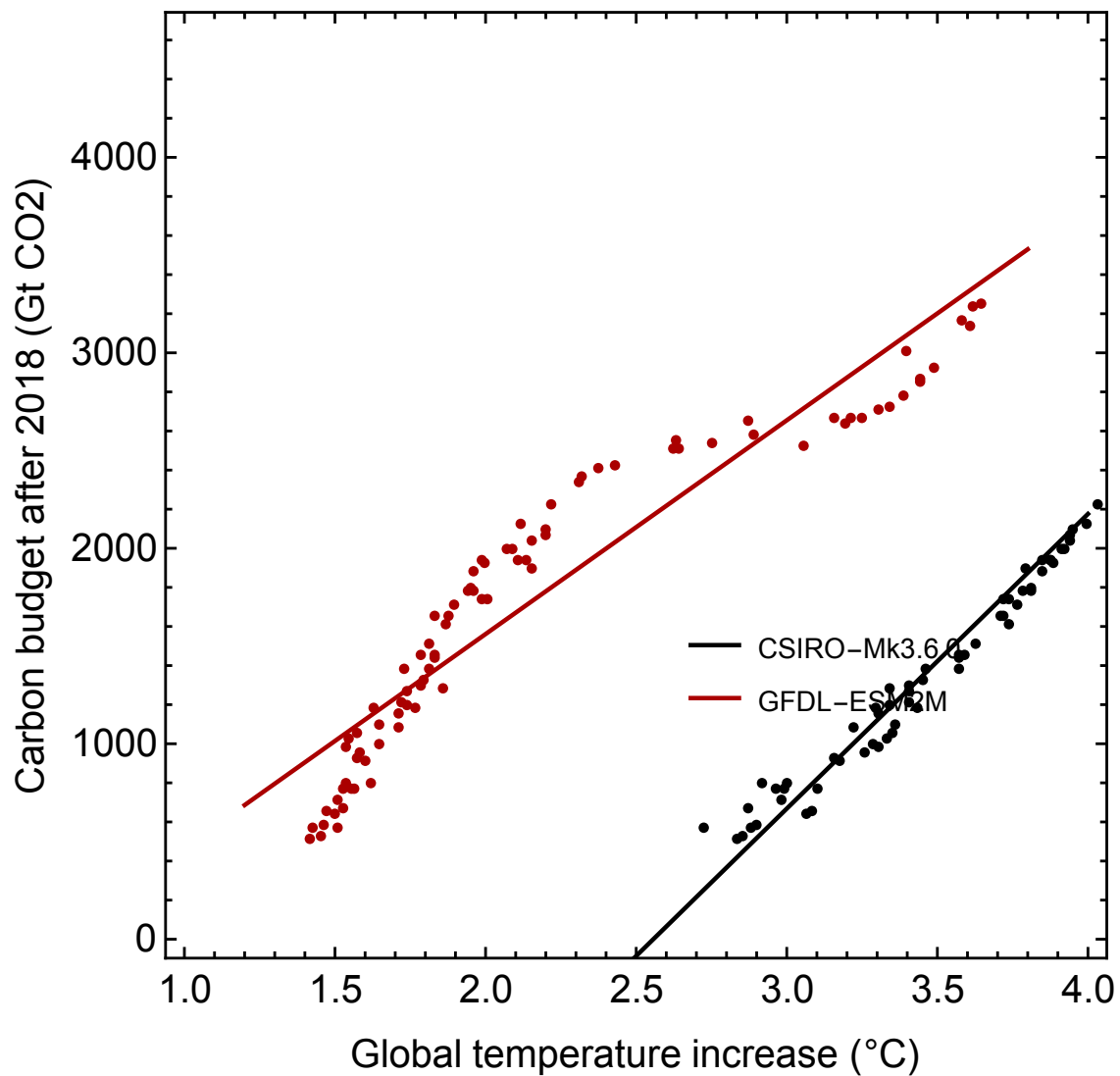
Since our research project includes the incorporation of Arctic amplification into RCBs, non-linear climate responses are of great interest, and in particular Earth-system feedbacks relating to the melting of permafrost. This effect may already be contributing to climate change. Permafrost carbon, as a result of perennially frozen ground, may contain twice as much carbon as the current atmospheric concentration<sup>31</sup>.

A warming climate thaws the permafrost causing positive feedback that may trigger a tipping - point-like warming threshold, which we already may have crossed. Due to the importance of non-linearities, Endre Falck Mentzoni focused his theses work on the implementation of non-linear (in this case: temperature-dependent) forcing in the SRM<sup>32</sup>. Since non-linear feedbacks in the climate system are a source of such high uncertainty, we explored a range of parametrized functions. We assessed both linear and non-linear functions, but these modelling choices is not derived from physical principles and should only be interpreted as illustrations. We hope that future work can incorporate simple, but principle-based, modelling of state-dependent forcing so that Earth system feedbacks can be included in RCBs.

Our model assumes that there is already an effect of thawing permafrost on the climate system<sup>33</sup>. The forcing function should, therefore, include the ongoing effect on the climate system and then increases more rapidly until it reaches a threshold triggering an abrupt release of GHGs and increasing the radiative forcing. We can use a hyperbolic tangent to model this:

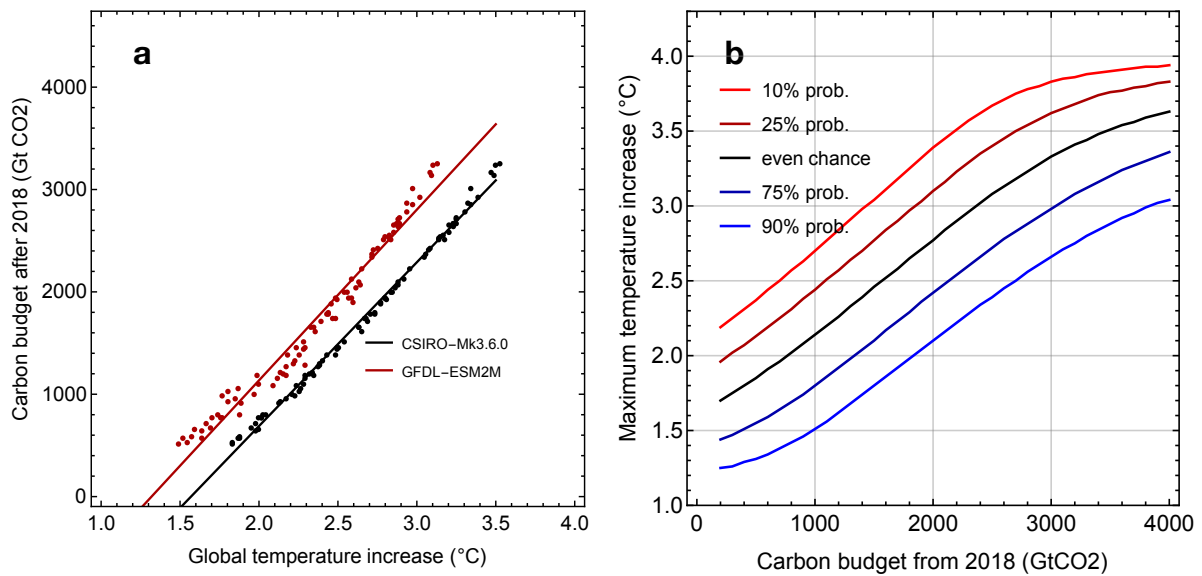
$$F(T) = \frac{m}{2} \left( 1 + \text{Tanh} \left( \frac{T - T_c}{s} \right) \right)$$

Here,  $m$  is the magnitude of the forcing specified in  $\text{Wm}^{-2}$ ,  $T_c$  is the threshold temperature for triggering the effect, and  $s$  describes how rapid the change of forcing as temperature increases. Deciding on the most reasonable parameters is a challenging task, and beyond the scope of this thesis. However, in order for our RCB analysis to be valid, the parameters choices be such that the linearity in the TCRE is maintained. If not, the budget estimates do not rely on valid TCREs. Parameters considered are  $m$  of  $1 \text{ Wm}^{-2}$  and  $2 \text{ Wm}^{-2}$ , thresholds of  $2$  and  $3^\circ\text{C}$ , and  $s$ -values of  $0.5$  and  $1^\circ\text{C}$ . Table A.1 in Appendix A presents the result of the analysis and which parameters that keep the TCRE valid and allow us to estimate the RCB. Parameters of  $m=2 \text{ Wm}^{-2}$ ,  $T_c=3^\circ\text{C}$  and  $s=0.5^\circ\text{C}$  had a too strong effect on the TCRE, as shown in Figure 17. Here the linearity of the TCRE of the GFDL-model is interrupted and therefore discarded as it does not make sense to estimate the RCB.



**Figure 17:** Illustrates non-linear forcing effect applied to the TCRE's in Figure 1, where the parameters of the non-linear effect correspond to Equation A7 in Appendix A. The TCRE's is less linear than in Figure 1.

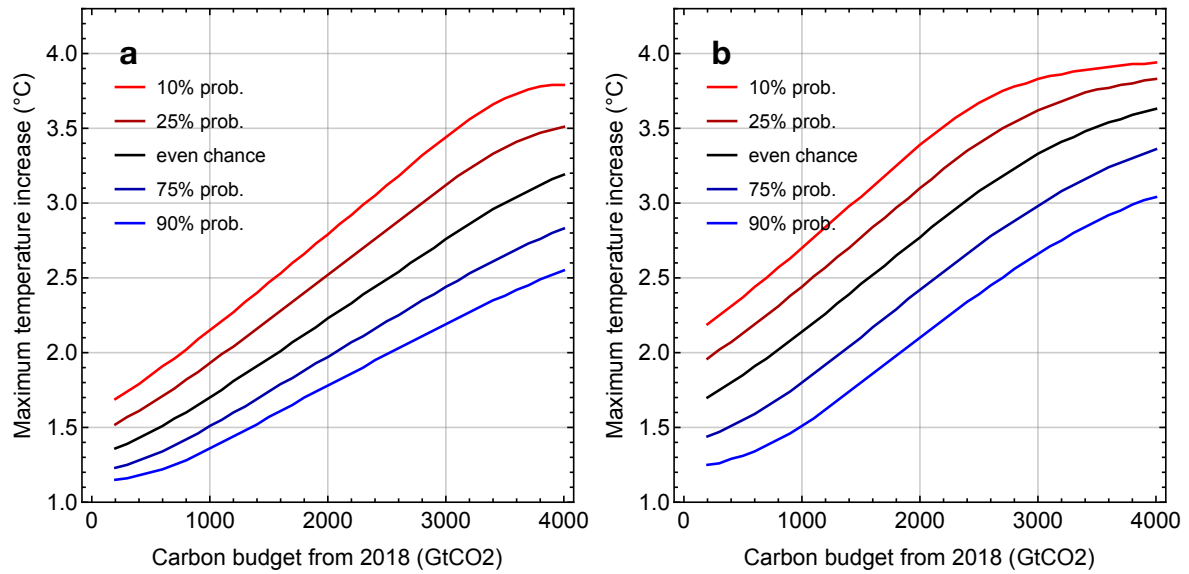
We found the parameters  $m=1 \text{ Wm}^{-2}$ ,  $T_c=2^\circ\text{C}$ , and  $s=0.5^\circ\text{C}$  to work well as an illustration. The abrupt forcing is not strong enough to disrupt the linearity of the TCRE and the steepness of the transition is sufficient to describe an abrupt release of permafrost carbon. The associated TCRE is shown in Figure 18.



**Figure 18:** Illustration of the non-linear forcing effect corresponding to Equation A1 in Appendix A. (a), TCRE's from Figure 1 with non-linear forcing effect. (b), Likelihood plot of the RCB estimated from the TCRE's in (a). Colored lines represent the probabilities of temperature responses to the RCBs. The estimated temperature responses range from 1.1°C to 4°C.

The RCB estimate from the TCRE in Figure 18(b) illustrates a clear difference from the RCB without implementation of non-linearities. Figure 19 illustrates a comparison between linear and non-linear RCBs, which demonstrates a larger uncertainty in the latter and the spread is high already in the small carbon budgets. Figure 19(a) indicates that the maximum temperature increase for RCB=580 GtCO<sub>2</sub> (1.5°C target) is greater than 1.2°C, with a 90 per cent probability. By including the implementation of non-linearity (Figure 19(b)), this number changes to 1.3°C. The difference is more significant in the high-end of the confidence intervals. For RCB=580 GtCO<sub>2</sub> the maximum temperature response is below 1.9°C with 10% certainty, but with the non-linear term this number changes to 2.4°C. This effect gives a broader uncertainty range in the RCB likelihood plot when including the non-linear forcing effect, 1.1°C range, than in the likelihood plot without non-linearities, 0.7°C.



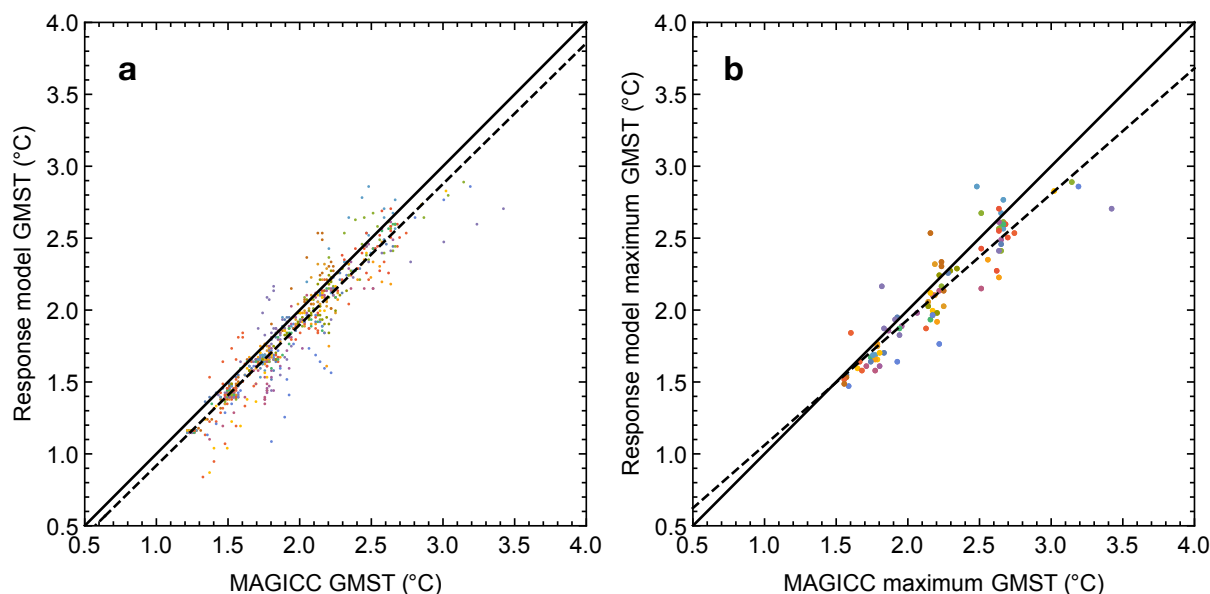


**Figure 19:** Uncertainty in the RCB including 14 ESM's, two carbon models representing plus/minus one standard deviation range, and decadal internal variability. (a), Same as Figure 16(b), Non-linear forcing effect added to (a) by implementing Equation 14 with parameters  $m=1 \text{ Wm}^{-2}$ ,  $T_c=2 \text{ }^\circ\text{C}$ , and  $s=0.5 \text{ }^\circ\text{C}$ . Plot (a) and (b) consider RCBs between 0-4000 GtCO<sub>2</sub> and temperature responses from 1.1°C to 4°C with steps of 0.01°C.

The considerable difference between the different RCB estimates highlights the importance of ambitious mitigation strategies. Our results show that uncertainty in the RCBs are greater than previously reported, and that non-linear feedback in particular, may increase uncertainties even for the most ambitious targets.

### 3.3 MAGICC comparison and SRM validation

MAGICC is an acknowledged reduced-complexity climate model that allows quick evaluation of multiple mitigation scenarios. Our SRM is an even simpler model, but simplicity does not imply less accuracy. In this thesis, Andreas Rostrup Martinsen compares results from our SRM with the results of MAGICC by extracting existing MAGICC temperature responses from the SSP simulations<sup>34</sup>. Martinsen applies the same method as in the SRM to the MAGICC simulations and disregards emission scenarios where emissions increase throughout the 21st century. Hence, he compares 86 different scenarios and Figure 20 shows GMST in MAGICC and the SRM, as well as a comparison of the maximum GMST increases. Figure 20(a) illustrates the correspondence between the models where the solid line is a 1:1 slope to visualize how a perfect linear relationship would look, and the dashed line is the best-fitted data. The SRM is consistent with MAGICC with a small underestimate. The agreement between the SRM and MAGICC



**Figure 20:** Comparison of temperature responses from MAGICC and the SRM. (a), The solid line mimics a perfect linear relationship (correlation of 1) and the dashed line illustrates the SRMs magnitude of correlation with MAGICC. The data is extracted from MAGICC estimates for every decade from 2020 to 2100 using the 86 SSP scenarios as used in SRM. (b), Same (a) but for maximum temperature response.

validates the SRM since MAGICC in some sense is the benchmark of reduced complexity models. Analyzing difference between the two models has not been a priority in this research project. However, it is reasonable to assume the difference might be related to the more comprehensive implementation of forcing and other climate drivers in the MAGICC. A review of the model differences can be found in Martinsen<sup>34</sup>.

Since our RCB analysis relates to maximum temperature increases, also make a comparison with MAGICC for those (Figure 20(b)). The solid line and the dashed line in Figure 20(b) are as in Figure 20(a) and illustrates that the SRM is consistent with MAGICC for temperatures between 1.5°C and 2°C. Above 2°C there is, perhaps, a slight bias. However, there is still a high degree of agreement between the two models. With only 86 data points available, the majority between 1.5°C and 2.8°C, it is difficult to conclude too much from the comparison of maximum GMSTs.

## **3.4 Arctic Amplification**

The Arctic plays an important role in the dynamics of our climate. It is also home to fragile ecosystems. The ice-albedo feedback is a strong positive feedback, and together with other positive feedbacks, it causes the temperature in the Arctic to increase much faster than the rest of the world<sup>35</sup>.

### **3.4.1 Climate indicators**

#### **3.4.1.1 Air temperature**

Box *et al.*<sup>36</sup> compares the air temperature change in the Arctic to the global (here Northern Hemisphere) temperature for the period 1971 to 2017 to derive an Arctic amplification factor. They found a factor of 2.4.

#### **3.4.1.2 Permafrost and carbon cycle**

Atmospheric carbon interacts with chemical components in the climate system, storing it or dissolving it in rocks, plants, ocean and other carbon sinks. The carbon flows between reservoirs through the carbon cycle where flux changes in one reservoir affect the flux in another. As a consequence, the cycle has a limit of how much carbon it can contain before being saturated. Exceeding the saturation limit of the carbon cycle will cause the carbon gases to transpire into the atmosphere and contribute to the global temperature increase<sup>37</sup>. The global climate changes are affecting the thermohaline circulation, which disrupts the energy transport from lower latitudes to higher latitudes. An abundance of energy will increase the energy transported towards the Arctic, which may disturb the Arctic carbon cycle by thawing the permafrost<sup>36</sup>. When permafrost thaws the carbon stored in plants and animals starts to decay, which releases CO<sub>2</sub> and CH<sub>4</sub> into the saturated carbon cycle, causing the temperature to increase.

#### **3.4.1.3 Arctic sea and land ice**

The albedo of the Arctic has a significant cooling effect. Satellite data from 1979 to 2019 shows that the average monthly Arctic sea ice extent is on a declining with a rate of 12.85 per cent per decade, declining from 7.05 million square km in 1979, to 4.32 million square km in 2019<sup>38</sup>. The rapid decline leads to decreasing albedo and causing more melting. However, if the Arctic is warming up, it could also have a negative feedback effect. A prolonged growing season has

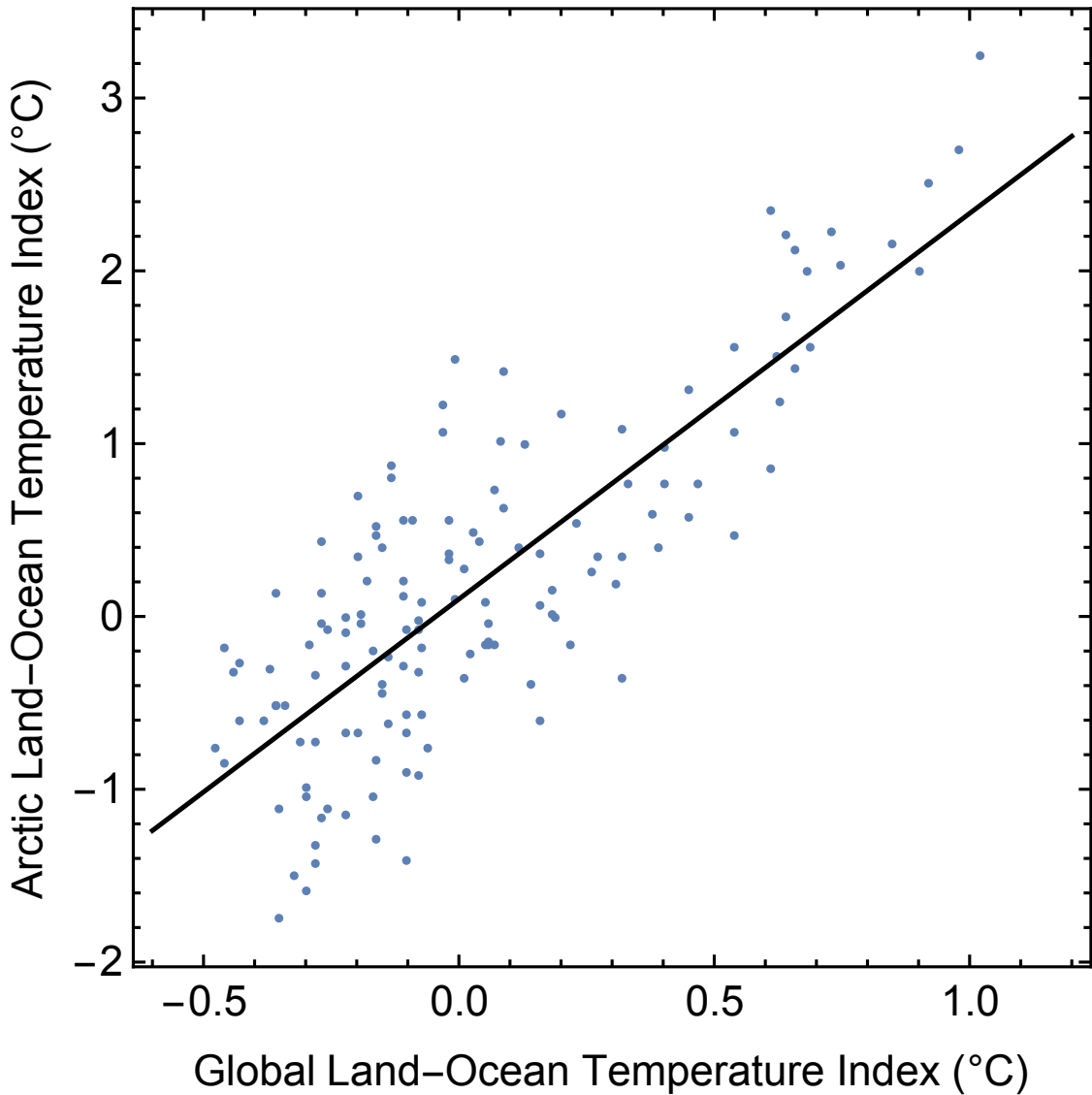
a positive effect on how long the plants can survive and therefore absorb more carbon. Nonetheless, most evidence reports a more substantial overall positive feedback in the Arctic.

#### **3.4.1.4 Clouds**

Clouds are arguably be the most uncertain components in the climate system with respect to impact on warming. They interact and affect the radiation balance both positively and negatively. They cool the climate by reflecting incoming sunlight, increasing the planetary albedo. However, they also absorb heat emitted by the Earth and therefore keeping the surface warm, The effect varies and is highly uncertain, though the net effect of clouds on global scale estimates a cooling contribution of  $-20 \text{ Wm}^{-2}$  according to the AR5<sup>39</sup>. Nevertheless, the situation in the Arctic seems to be opposite. The clouds are believed to have a net warming effect on the region, mainly due to the lack of incoming radiation in winter. In this period the effect from clouds would only be absorbing heat from the Earth and resulting in warming of the surface implying more clouds leads to more heat<sup>40</sup>. The feedback effect from clouds is still quite uncertain as reports also suggest that more cloud form during the melting season which increases the albedo and partly compensates the albedo decrease from sea ice loss<sup>41</sup>.

#### **3.4.2 Incorporation and results**

The estimate of the Arctic amplification factor is obtained from NASA, i.e., the GISS surface temperature analysis (GISTEMP) of global temperatures and zonal northern hemisphere temperatures from 1880-2019<sup>42, 43</sup>. The Arctic is most commonly defined as the region above the Arctic Circle, i.e., north of latitude  $66^{\circ}34' \text{ N}$ <sup>44</sup>. However, the northernmost zone from NASA covers 64 – 90N, and for convenience use this definition of the Arctic. The GISTEMP uses a land-ocean temperature index (LOTI) which is a result combining land-surface air and sea-surface water temperature anomalies. This method is used since landmasses only cover one-third of the planet, accuracy can be increased by using weather stations capable of measuring surface air temperature (SAT) in combination with the methods of measuring ocean temperatures. Sea surface temperatures (SSTs) are available from ship and buoys, and more recently also from satellites. Since air cools and warms much faster than water, SATs and SSTs have different properties, but the anomalies are very similar under anthropogenic warming<sup>42, 43</sup>. The GISS data uses a base period from 1951-1980, and the measurements have good global coverage in this period.



**Figure 21:** Presents the relationship between global LOTI and Arctic LOTI, both in 0.01 degrees Celsius. The blue dots are the correlated temperature indexes and the black line is the linear regression, used to calculate the amplification factor.

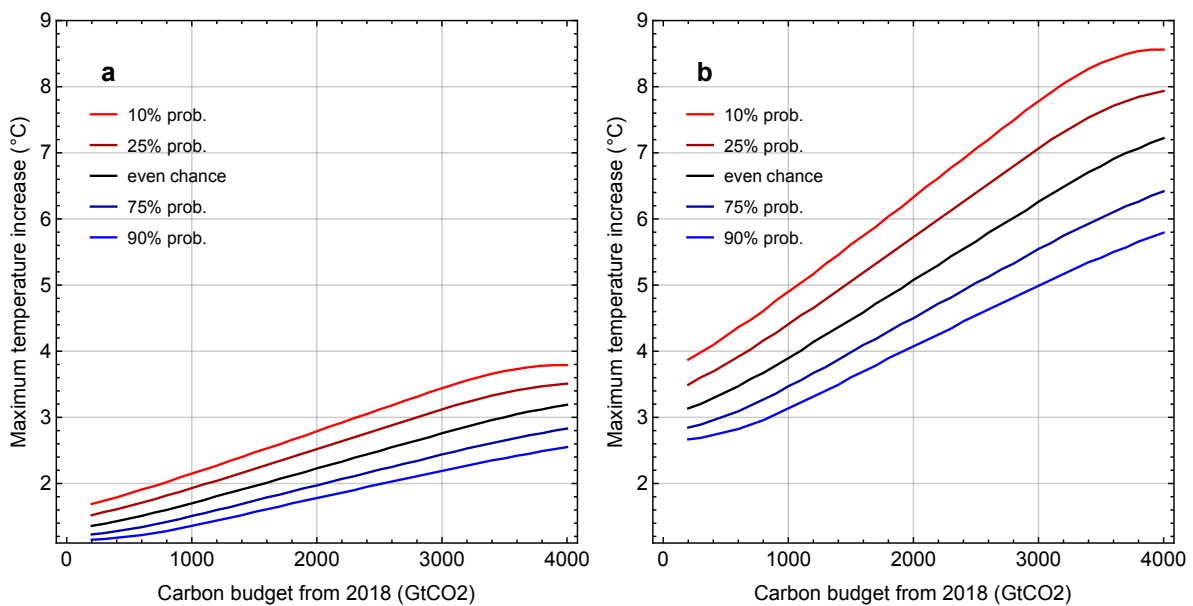
Plotting global temperatures against Arctic temperatures (Figure 21) and using linear regression to estimate the amplification factor we obtained a value of 2.23. More precisely:

$$T_{\text{arc}} = 0.100 + 2.23T_{\text{glob}} \quad (15)$$

Here  $T_{\text{arc}}$  is the arctic temperature corresponding to the global temperature  $T_{\text{glob}}$ .

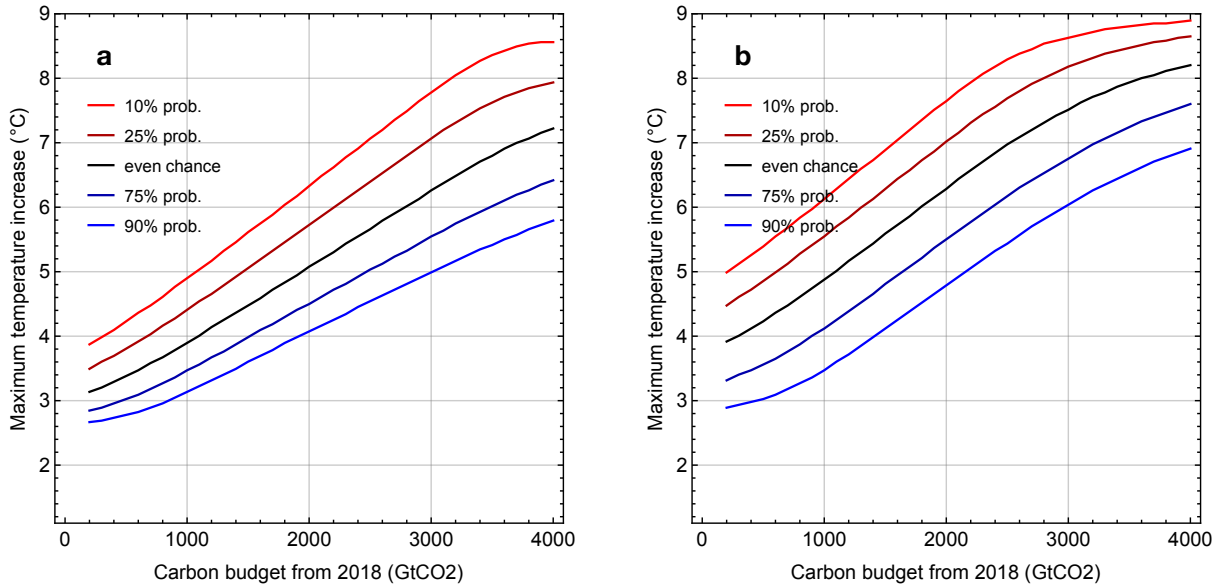
The data used from NASA differs slightly from the standard definition of the Arctic, as previously mentioned. However, the  $6^\circ$  not accounted for appears to have low significance as the IPCC defines the Arctic as  $67^\circ$  with an AA range of 2.2-2.4<sup>24</sup>.

To analyze the impact the Arctic amplification has on the likelihood plot of the remaining carbon budget, we simply substitute the global temperature parameter with an arctic temperature parameter,  $T_{arc}$ , shown in Equation 15, as a function of the global temperature  $T_{glob}$ . The result in Figure 22 clearly shows that the uncertainty is increasing with less ambitious mitigation targets. Aspiring to limit the remaining carbon budget to 580 GtCO<sub>2</sub>, which is the RCB to limit warming to 1.5°C with a 50% probability<sup>19</sup>, yields a 10% probability of reaching 1.9°C. However, the impacts are more severe in the Arctic as limiting the RCB to 580 GtCO<sub>2</sub> yields 50% of reaching 3.5°C and a probability of 10% of reaching 4.35°C.



**Figure 22:** Comparison of global RCB and Arctic RCB. (a), RCB estimated by the SRM with 14 ESMs, two carbon models representing  $\pm$  one standard deviation range, and decadal internal variability, which is the same as Figure 16 but with plot range extended to 9°C. (b), Same plot as (a) but estimated with the Arctic Amplification factor from Equation 15. The plot legends illustrate the probability of achieving specific temperatures related to the RCB.

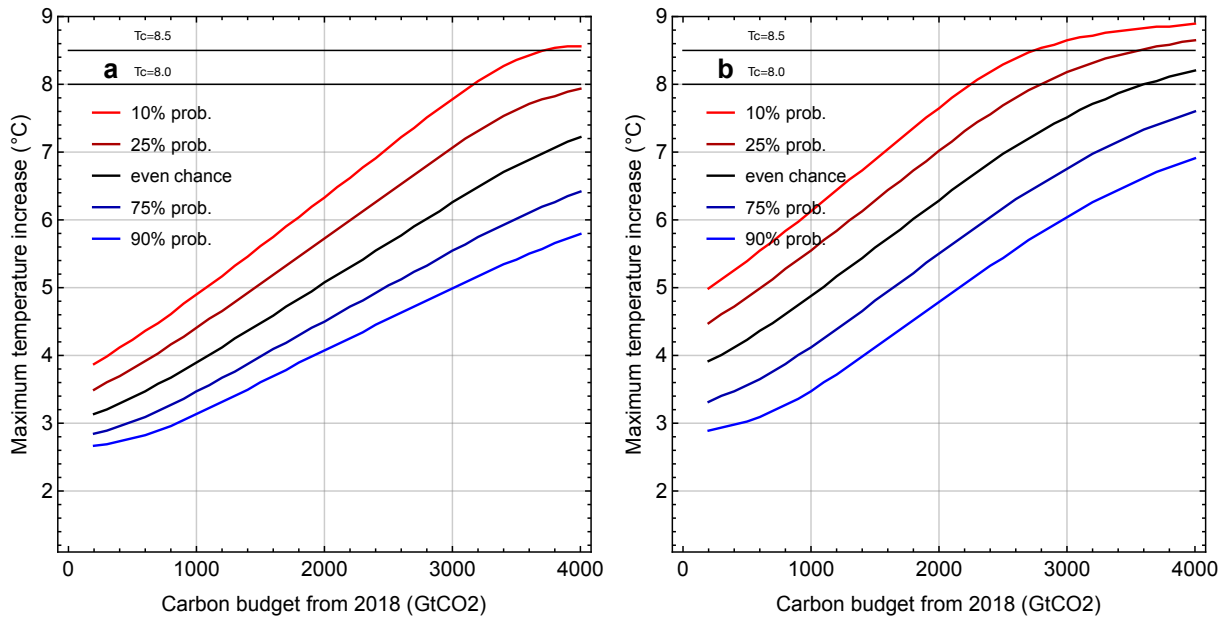
Since thawing of permafrost may cause a strong feedback loop in the Arctic, which is not unlikely with the estimated temperature response mentioned above, we include the non-linear effect from section 3.2 ( $m=1 \text{ Wm}^{-2}$ ,  $T_c=2 \text{ }^\circ\text{C}$ , and  $s=0.5 \text{ }^\circ\text{C}$ ) and assumes a stronger temperature increase. Figure 23 shows a comparison of a linear forcing effect in the Arctic to a model that includes non-linear forcing. Figure 23(a) is the estimated RCB when with AA (same as Figure 22(b)), and Figure 23(b) is the equivalent likelihood plot when including non-linear forcing. The same wide uncertainty range as seen in Figure 18 (in section 3.2) applies to the Arctic. There is a spread from across the different budgets and there are generally higher temperature



**Figure 23:** Likelihood plots of estimated RCBs including AA from Equation 15, and the implementation of the non-linear forcing effect from section 3.2. (a), Same as Figure 22(b). (b), Non-linearities implemented to (a) with parameters corresponding to Equation A7 in Appendix A.

responses to the budgets, for all probability bounds. The 90% probable temperature response in Figure 23(b) extend to a higher maximum temperature: RCB=4000 GtCO<sub>2</sub> yields 6.9 °C, compared to the 5.8°C without any non-linear effects. This is an increase of approximately 19%.

In addition, we illustrate an assumed temperature threshold for an irreversible transition of the Greenland Ice Sheet (GIS) and its substantial impacts on the global sea level and ocean circulations such as the Atlantic Meridional Overturning Circulation. We speculate that critical temperature during the June-July-August season could be between  $T_c = 8^\circ\text{C}$  and  $T_c = 8.5^\circ\text{C}$  as presented in Figure 24, which can be reached within the next two or three decades.



**Figure 24:** Illustration of temperature thresholds of the Greenland Ice Sheet added to the estimated RCB including AA. (a), Assumed thresholds for a tipping point for the Greenland Ice Sheet added to Figure 22(b), where the two horizontal solid lines at the top of the plot represents the boundaries of the critical temperature interval. (b), Same as (a) but applied to Figure 23(b), which includes non-linear forcing effects.



## 4 Mitigation and adaptation

### 4.1 Climate inertias

The economic inertia described in Section 4.2 is one of several affecting the climate system. Two other inertias are the carbon cycle inertia and the thermal inertia. The Earth's carbon cycle has a limit of how much it can absorb, which means that an abundance of CO<sub>2</sub> in the atmosphere can saturate the cycle. The CO<sub>2</sub> has a lifespan of several hundred years, and the amount which is not absorbed will continue to contribute to the warming of the climate system as long as it is in the atmosphere. Even if we were to reduce all emissions to zero tomorrow, the atmospheric CO<sub>2</sub> concentration would not be affected straight away. It would take time for the concentration to respond to a reduction in emissions due to the lifetime and the saturation of the carbon cycle. Besides, the temperature response will experience a delay due to the thermal inertia of the ocean. The ocean can absorb enormous amounts of heat due to its high heat capacity, which creates thermal inertia within the climate system and resulting in a temperature lag. It takes time from when the climate system is affected by an imbalance until it adequately responds to it and sets for a new equilibrium in which will increase the temperature on the planet.

Combining the three inertias mentioned makes it challenging to reach an emission reduction of 7.6% per year starting from 2020 to reach the optimistic 1.5°C. The more realistic 2°C target demands emission output of maximum 40 Gt CO<sub>2</sub> in 2030, which requires emission reduction of 2.7% from 2020<sup>45</sup>.

### 4.2 Decarbonization

Even if the anthropogenic GHG emissions to the atmosphere become zero, the climate would still take time to respond fully. The mitigation required to reach the temperature targets of 1.5 degrees Celsius or 2 degrees Celsius depends on improvements in the technology sector and negative-emission strategies.

Specific characteristics become essential when addressing the 1.5°C and 2°C targets, especially improvements in energy efficiency. Future emissions and future energy demand are highly correlated, and it will be essential to supply the demand with less effort. Scenarios limiting temperature increase to 1.5°C have an average energy demand of 400 EJ (quintillion, 10<sup>18</sup>, Joules)

per year, increasing from today's 350 EJ per year<sup>46</sup>. These scenarios of only 50 EJ increase compared to today's consumption exemplify much lower energy intensity (measure of energy inefficiency), which will substantially reduce mitigation costs<sup>46</sup>. The energy demand goal of 400 EJ without exceeding RCBs is doable by committing to renewable energies, which a study finds to be almost equally efficient as fossil fuels<sup>47</sup>.

Brockway *et al.*<sup>47</sup> apply the most popular metric for net energy analysis: energy-return-on-investment (EROI). They define EROI as Kunz *et al.*<sup>48</sup>, as a ratio that divides the total energy output by the energy input and applies it to calculate the EROI of fossil fuels to compare it to the EROI of renewables. Published standard estimates of EROI for fossil fuels are approximately 30<sup>47</sup>. However, the ratio of 30:1 represents primary-state estimates, that is, fossil fuels not accounting for the refinery process and when the energy enters the economy in the form of, for example, electricity or petrol. The estimates of the final stage of fossil fuels have approximately an EROI ratio of 6:1, which is the level where renewable energy is competitive<sup>47</sup>. The declining EROI ratio of fossil fuels is dangerously close to the net energy cliff, the situation where EROI rapidly decreases (below 5:1 for fossil fuels) and restricts energy to society since the production of new energy consumes all available energy<sup>47,49</sup>. Brockway *et al.*<sup>47</sup> find the low EROI ratio of fossil fuels, 5:1, to be closer to a net energy cliff than expected, which signifies a transition to renewables since the cliff is non-linear. Meaning a reduction in available net energy is more significant in a scenario when EROI drops from 10:1 to 5:1, unlike a scenario of 40:1 to 20:1. The study of Brockway *et al.* used data from the International Energy Agency (IEA) to analyze the development of the final stage EROI ratio of fossil fuels from 1995 to 2011. The results were a reduction of 23 per cent over the 16 years, which eventually will constrain the available net energy. An option to counteract the declining EROI would be to increase the production of fossil fuels. However, doing so would be destructive from a climate change perspective as more production would increase GHG emissions in the atmosphere and is most likely not feasible due to the shortage of abundant and easy-to-extract fossil fuels<sup>47,50</sup>.

Since renewables and fossil fuels have approximately equal EROI level, the transition to renewables might be less troublesome, and more profitable, than previous estimates, especially when considering the potential of increased EROI of renewables when the infrastructure is in place, and fossil fuels begin the phaseout. The temperature targets from Paris is achievable by combining a green transition with other mitigation techniques such as negative emissions.

In less ambitious mitigation strategies, negative emissions are a crucial technology to apply along with renewable energy to achieve the 1.5°C target. The technology will have an essential contribution in sectors where it is hard to decarbonize, such as transportation, especially containerships and other large vessels and vehicles, and industries such as cement, steel, and power plants. However, the technology may not be capable enough today. It might cause a false safety net in existing carbon budgets which achieves the temperature targets depending on sufficient negative emission strategies. Such overly dependent scenarios would be catastrophic from the climate change perspective. The most common technology referred to as negative emissions are carbon capture storage (CCS) whose purpose is to limit the amount of CO<sub>2</sub> emissions into the atmosphere by capturing CO<sub>2</sub> and then storing it. First, the CO<sub>2</sub> is captured from the emissions from power plants and industrial production processes, or it can be separated directly from the natural gas during an industrial process. The latter is the easiest one since it is quite challenging to capture CO<sub>2</sub> from the already processed emissions (waste gas). The captured CO<sub>2</sub> is then compressed to a liquid state and is ready for transport<sup>51</sup>. Secondly, the liquid gas is transported by ship or pipelines to the storage location. Transport by pipelines is preferable for larger quantities and shorter distances, whereas ship is preferable for smaller quantities and larger distances<sup>51</sup>. The final stage is storing the CO<sub>2</sub> safely under the Earth surface so that it does not leak into the atmosphere. The oil industry uses a standard method where they inject the compressed CO<sub>2</sub> deeper than 1 km into the seafloor of old stable oil reservoirs. The suitable formations must be porous rock layers which can store the CO<sub>2</sub> with solid rock layer(s), cap rocks, such that it does not leak.

The simple solution is stating that the policies and NDCs should be stricter. However, it is easier said than done due to the uncertainty around the feasibility of achieving the required level of decarbonization. Rapid decarbonization depends on the willingness of the society to replace already existing capital with sustainable, climate-friendly investments on a massive scale<sup>52</sup>. Different inertias limit the realizable mitigation pathways within the economic system, such as long lifetimes in the energy sector<sup>53</sup>. There are power plants around the world today that are more than 30 years old (some over 50 years) with no intention of reducing emissions. They have to be replaced by more effective power plants which operate more sustainable, preferably renewable if we are to reach the climate goals. The problem is that these renewable power plants will take time to build and the relatively new power plants will continue to emit greenhouse gases (GHGs) far into the future. Implementation of new policies that require new sustainable solutions and do not approve the existing infrastructure might lead to stranded assets. Stranded

assets are defined by Bos and Gupta<sup>54</sup> as assets that lose economic value well ahead of their anticipated useful life, whether that is a result of changes in legislation, market forces, disruptive innovation, societal norms, or environmental shocks. This is the economic inertia of the climate system.

The IPCC estimates with 50 per cent probability that to keep the temperature below 2°C the cumulative carbon budget between 2011-2050 must be limited to 1100 GtCO<sub>2</sub><sup>55</sup>. However, the remaining fossil fuel reserves are about 2.7 times larger than this amount. McGlade and Ekins<sup>56</sup> investigated how the 2011-2050 budget limits the fossil fuel production on a global scale, where their results indicates that 33% of oil, 49% of gas, and 82% of coal reserves should remain unused.

### **4.3 Emission gap**

The global surface temperature increase relative to pre-industrial temperatures has increased by 1.1°C, and we have to be careful not to exceed the temperature targets from the Paris Agreement. There exists a gap between what we need to do and what we say we will do to limit a global climate change. This difference is called the emission gap. Every year the United Nations Environment Programme (UNEP) investigates the status of the commitments of the participants of the Paris Agreement, the NDCs (nationally determined contributions). The 2019 report states that we are failing to reach the 1.5°C target and that we are on the brink of missing the opportunity to limit global warming to 1.5°C<sup>45</sup>. They estimate, based on today's NDCs, the emissions in 2030 to be 56 Gt CO<sub>2</sub> per year when they should be 25 Gt CO<sub>2</sub> yr<sup>-1</sup> to limit the temperature increase to 1.5°C<sup>45</sup>. It is still possible to limit the temperature to this target. However, it requires more stringent commitments and policies that can deliver a global emission reduction of 7.6% per year between 2020 and 2030<sup>45</sup>. There is a broad agreement in the scientific community that by delaying action it will make it almost impossible to reach the 1.5°C target, and the yearly emission reduction will be more extensive each year by doing so. A delay will inflict higher price tags of mitigation, infrastructure adaption, food security along with other socioeconomic factors. If mitigation schemes had started in 2010 to achieve a 2°C or a 1.5°C increase, the yearly mitigation cost would have been 0.7% and 3.3%, respectively, which would be far more feasible and economically sustainable. The participants of the Paris Agreement must strengthen their NDCs by introducing stricter emission-reduction targets, raise the carbon prices (should

be 3-5 times higher than today<sup>46</sup>), reducing fossil fuel subsidies and ensure added investments in renewable energy.



## 5 Conclusion and Future Work

### 5.1 Conclusion

Using SSP trajectories and box-model ESM emulators, we use our SRM to model uncertainty in RCBs. The SRM allows quick evaluation of multiple mitigation scenarios in the same manner as MAGICC, and it is simple to change and evaluate parameters. The temperature response from the SRM agrees well with MAGICC (Section 3.3) which validates the estimates from SRM since MAGICC is the benchmark of reduced complexity models in the climate science community. It implies that the simplicity of our SRM does not result in less accuracy and the simplicity makes it easy to comprehend.

The uncertainty in the estimated RCBs increase significantly with less ambitious mitigation targets (Section 3.1), e.g., efforts of achieving 1.5°C have one tenth of a chance of exceeding the temperature target by 27%. Due to the uncertainty in global RCBs, we analyze the RCBs for Arctic temperatures by adding an estimated Arctic amplification factor and evaluate the significantly higher temperature responses. We found the amplification factor to cause 2.2 times higher temperatures in the Arctic, which might trigger Arctic tipping points. Examples of non-linear forcing give increases of around 20% for the 90-percentile for temperatures by the end of this century. We believe that this substantial increase is high enough for an irreversible transition of the stability of the Greenland ice sheet. It affects feedback mechanisms associated with the albedo decrease, sea-level rise, and disruption of ocean circulation patterns.

The world is missing the opportunity of limiting GMST increase to 1.5°C as the estimates of emissions in 2030 is 56 Gt CO<sub>2</sub> per yr when they should be 25 Gt CO<sub>2</sub> pr yr. It requires global mitigation levels of 7.6% between 2020-2030 to achieve the 1.5°C target, which requires higher energy efficiency with today's energy sources not to exceed RCBs. Renewable energies are a solution to achieve the increasing energy demand and at the same time, reduce global emissions. Recent studies estimate incorrect implementation of fossil fuel EROI in economy-energy models as the EROI at the final energy stage is equal, or even lower, compared to renewable energy EROI. Therefore, there is an urgent need for applying these estimates and focus on renewable energy since future mitigation policies and infrastructure investments are decided today.

## 5.2 Future Work

Further development of the SRM should explore a dynamical emission relation between CO<sub>2</sub> emissions and methane emissions.

Another approach for further development is to use response functions estimated from CMIP6 models in contrast to the 14 ESMs from the CMIP5 ensemble used in the current SRM. The climate sensitivity in the CMIP6 models appears to be higher and contributing to more extensive temperature responses. Therefore, our SRM produces conservative estimates of the uncertainty in the RCB, whereas the actual uncertainty might be higher.

This study does not consider other forcing factors than CO<sub>2</sub>, methane and aerosols. Implementation of other GHG's than CO<sub>2</sub> and methane, and more comprehensive implementation of aerosols, may result in more extensive estimates. However, the implementation must be simple enough to maintain the simplicity of the SRM.



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## Appendix A

### Additional plots for testing different non-linear implementation

We examined multiple equations to incorporate the non-linear forcing effect, with both non-linear and linear equations as listed in Table A1. Equations A1-A8 estimate the non-linear effect by the hyperbolic tangent function as in Section 3.2:

$$F(T) = \frac{m}{2} \left( 1 + \text{Tanh} \left( \frac{T - T_c}{s} \right) \right) \quad (A1 - A8)$$

The linear forcing effect has the form:

$$L \times T \quad (A9 - A11)$$

Here, L is the linear forcing factor with units  $\text{Wm}^{-2} (\text{°C})^{-1}$  and T is the GMST.

Figures A.1-A.11 illustrates the application of the different non-linear forcing effects, and the caption states which equations used.

**Table A1:** List of different non-linear equation tests. M is magnitude, T is threshold, S is steepness, and L is the linear forcing factor. The far-right column explains which equations that breaks the linearity of the TCRE and yields invalid results.

Equation #	M(Wm <sup>-2</sup> )	T <sub>c</sub> (°C)	S(°C)	L(Wm <sup>-2</sup> (°C) <sup>-1</sup> )	Dis- carded
A1	1	2	0.5		No
A2	1	2	1		No
A3	1	3	0.5		No
A4	1	3	1		No
A5	2	2	0.5		No
A6	2	2	1		Yes
A7	2	3	0.5		Yes
A8	2	3	1		No
A9				0.1	No
A10				0.2	No
A11				0.45	No

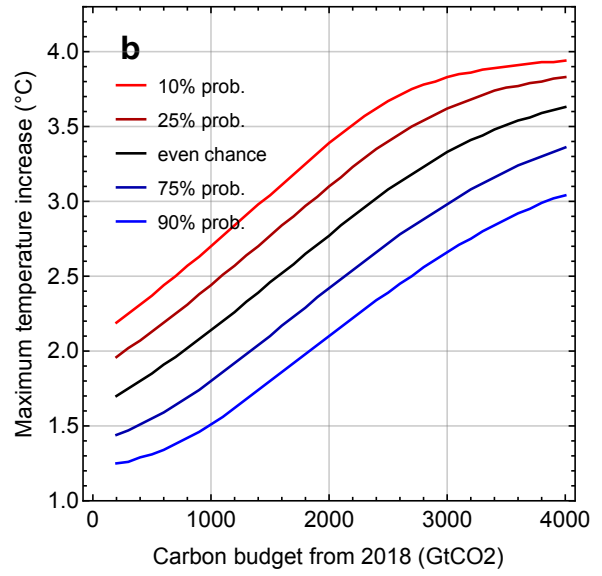
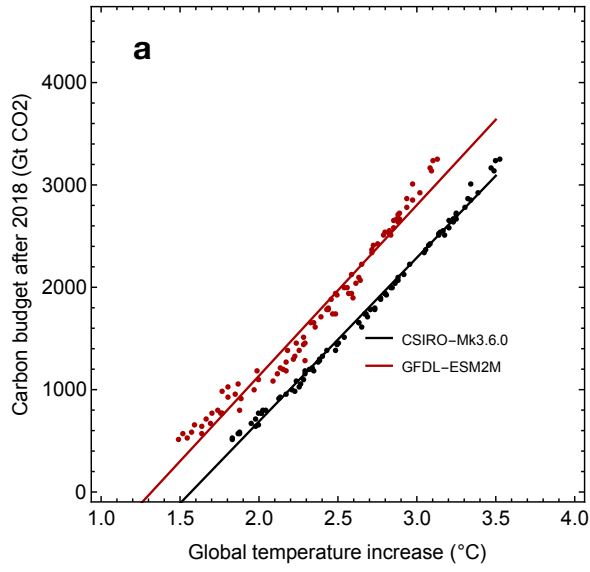


Figure A.1: 1-2-0.5

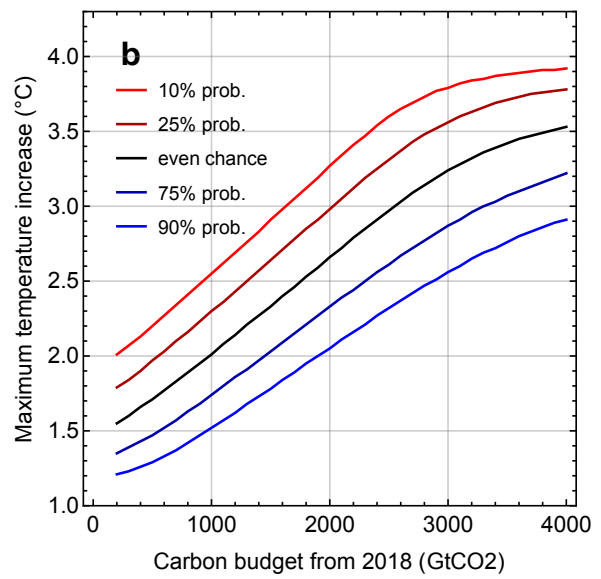
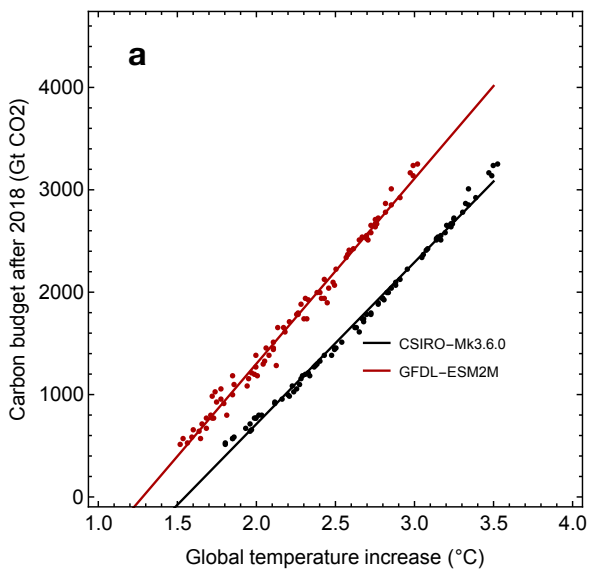


Figure A.2: 1-2-1

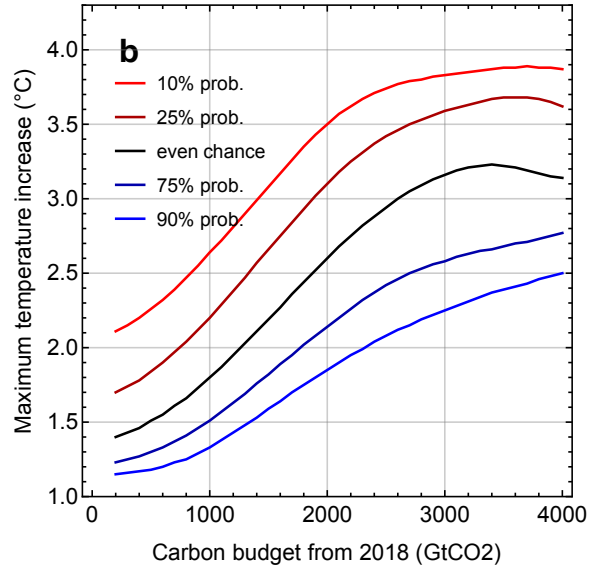
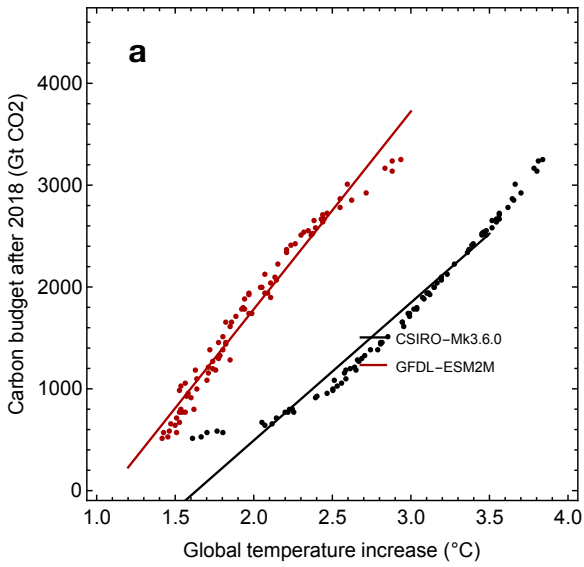


Figure A.3: 1-3-0.5

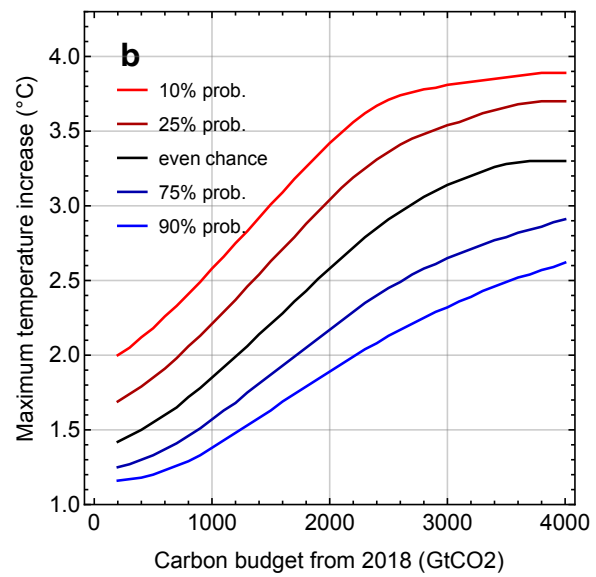
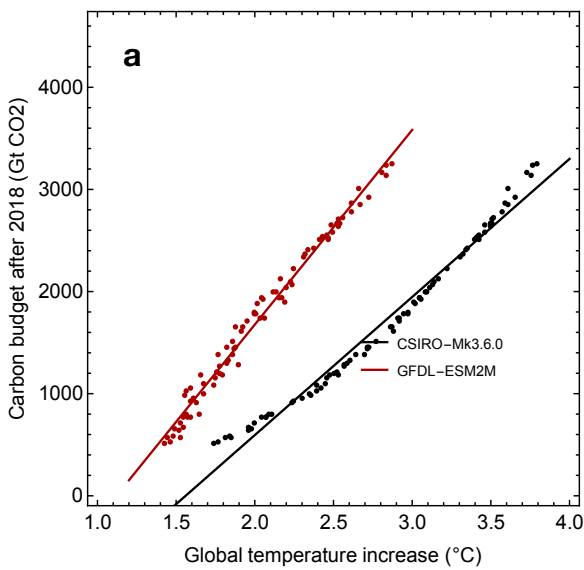


Figure A.4: 1-3-1



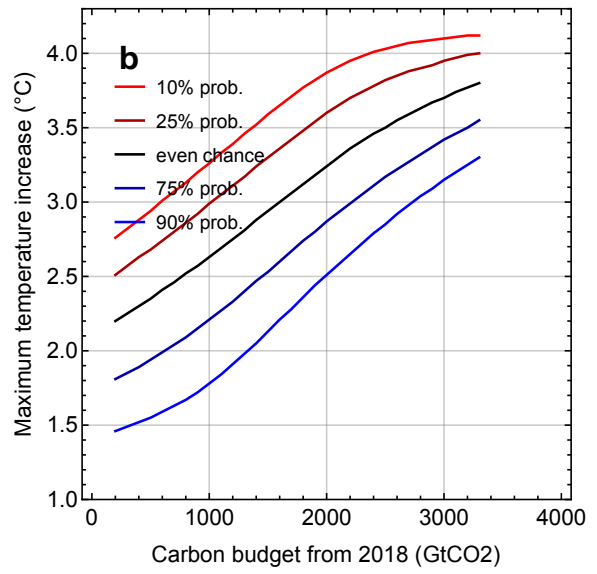
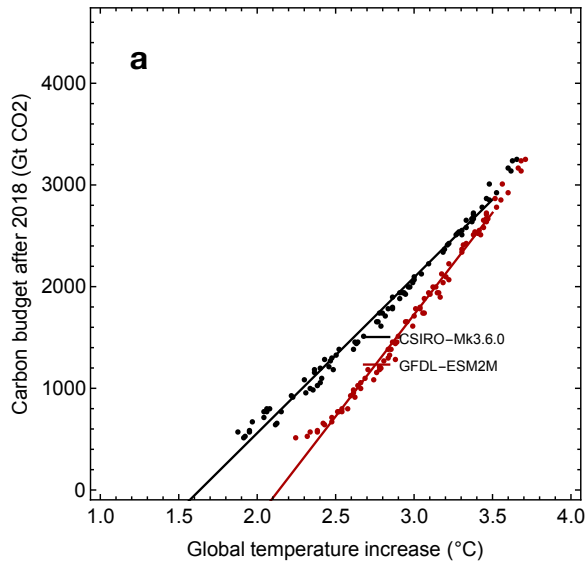


Figure A.5: 2-2-0.5

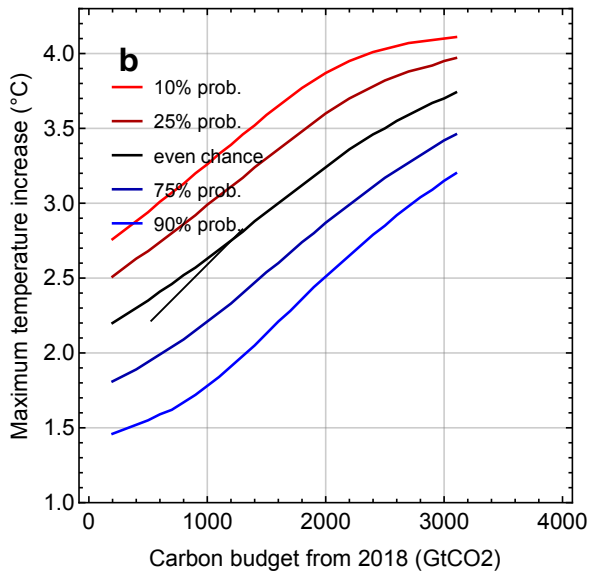
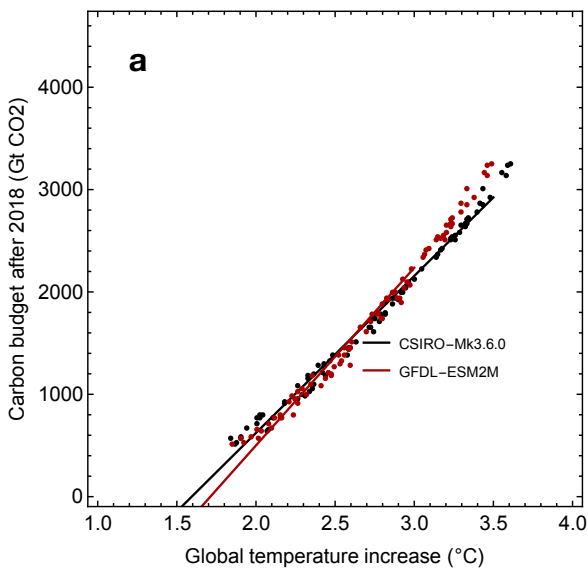


Figure A.6: 2-2-1

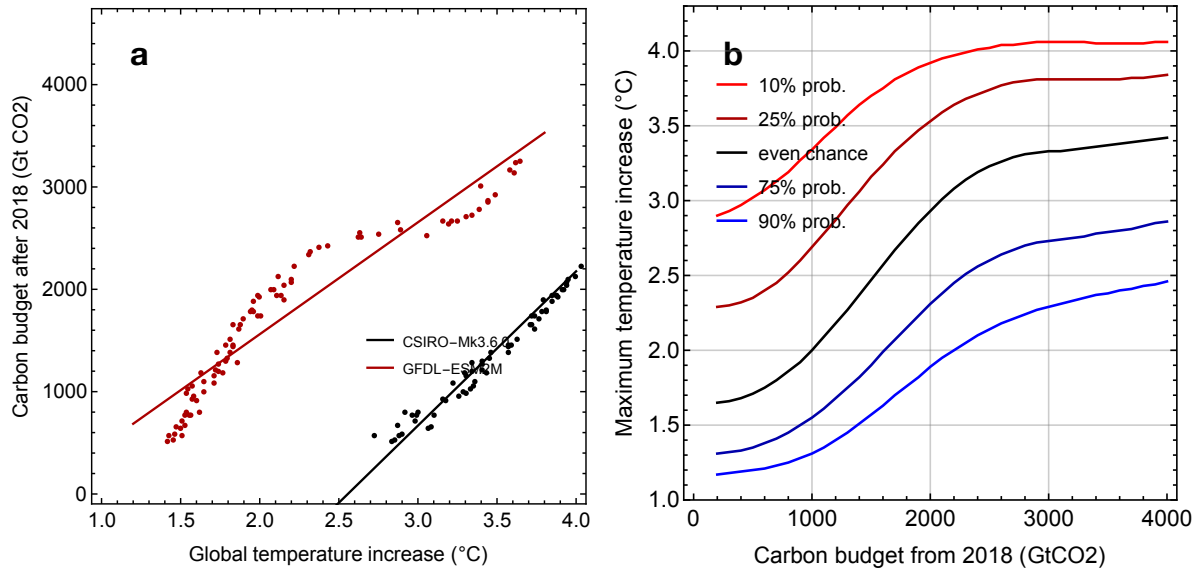


Figure A.7: 2-3-0.5

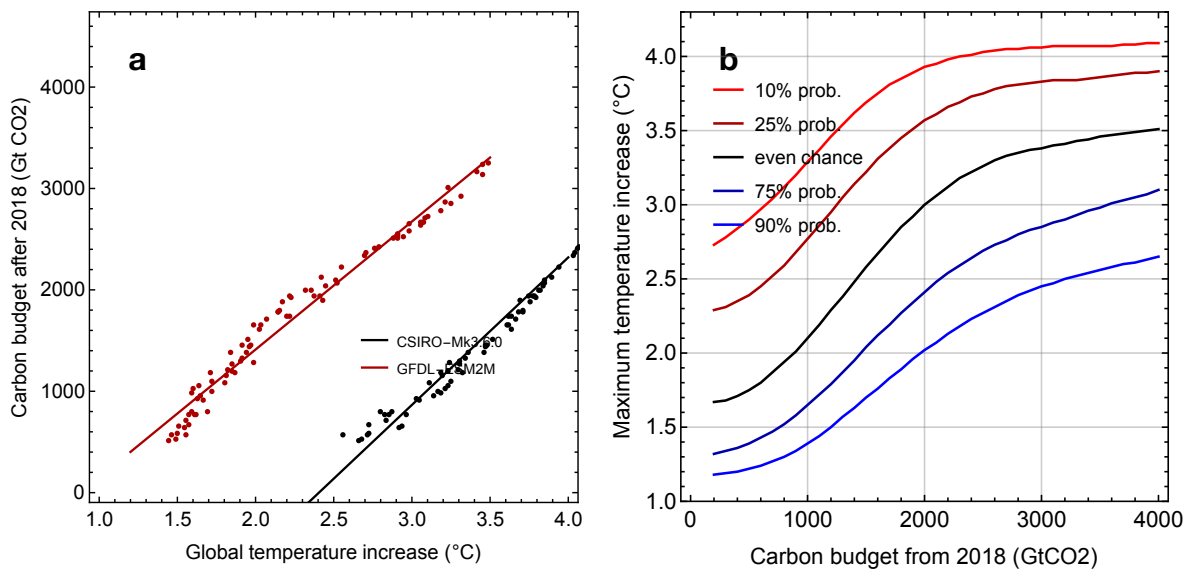


Figure A.8: 2-3-1

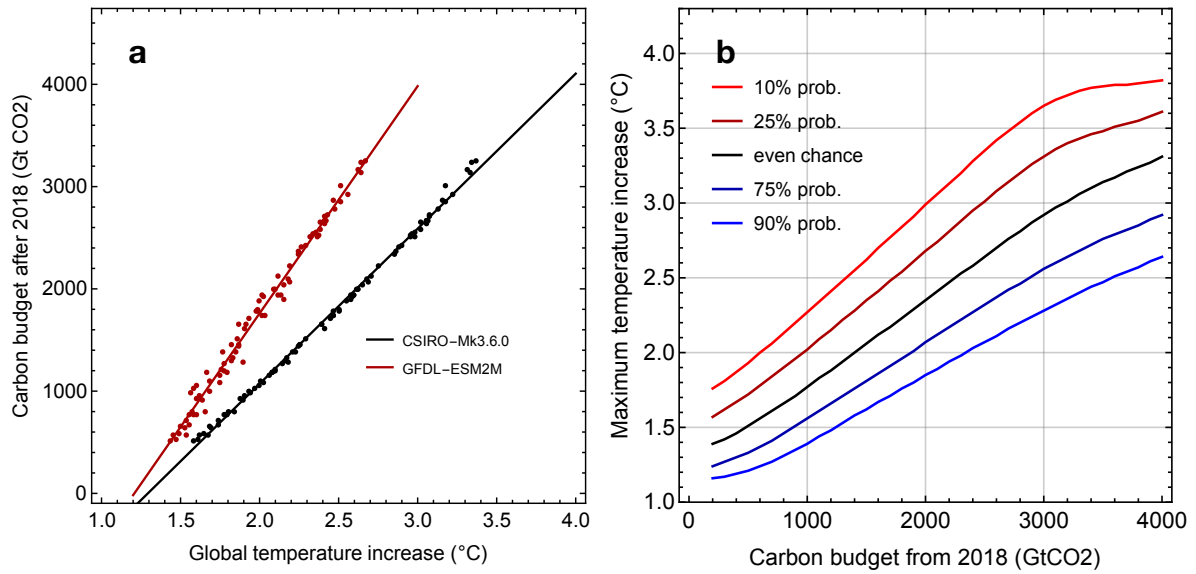


Figure A.9: 0.1T

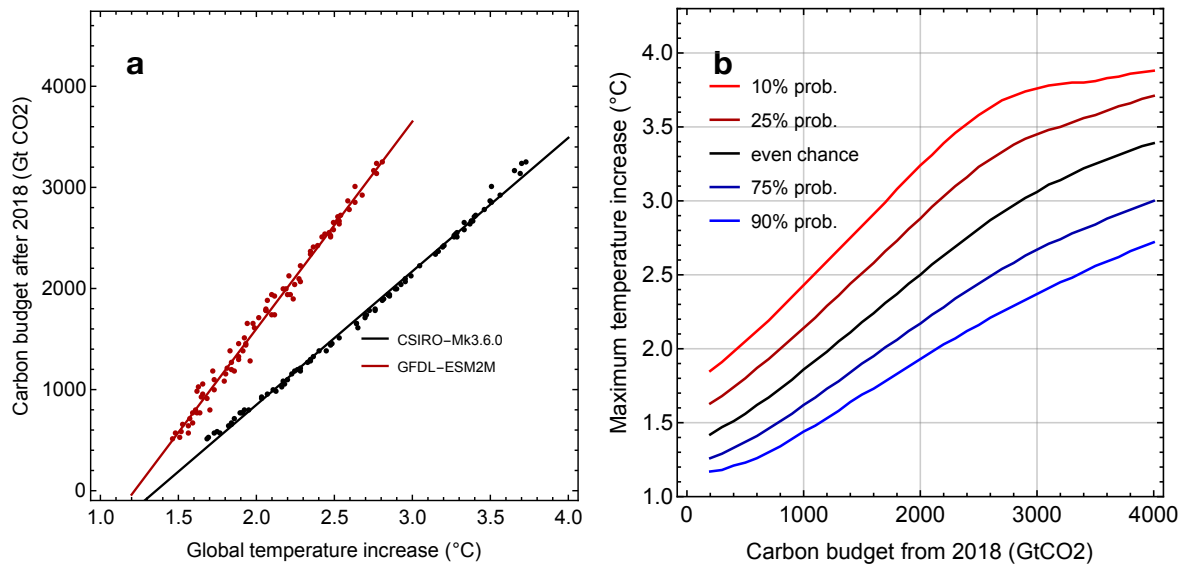


Figure A.10: 0.2T

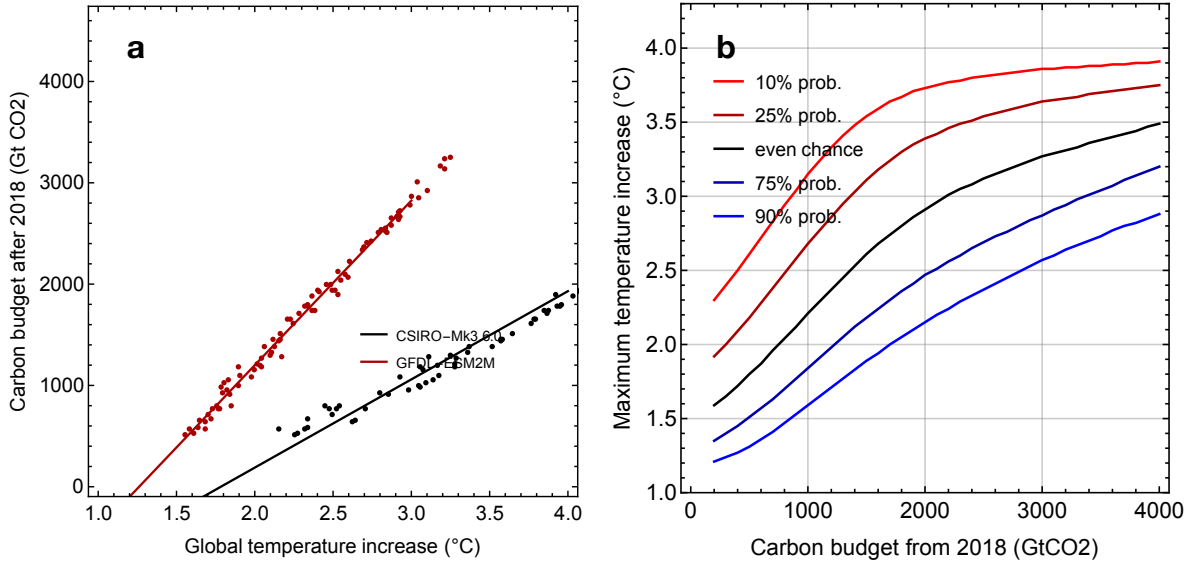


Figure A.11: 0.45T

**Additional figures illustrating different components of the SRM without non-linear forcing (Figure A.12 is equal independent of non-linearities):**

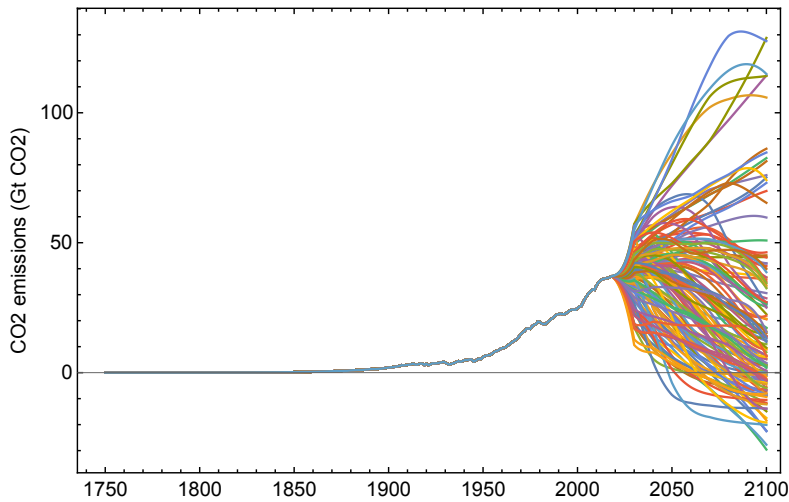


Figure A.12: All 127 scenarios

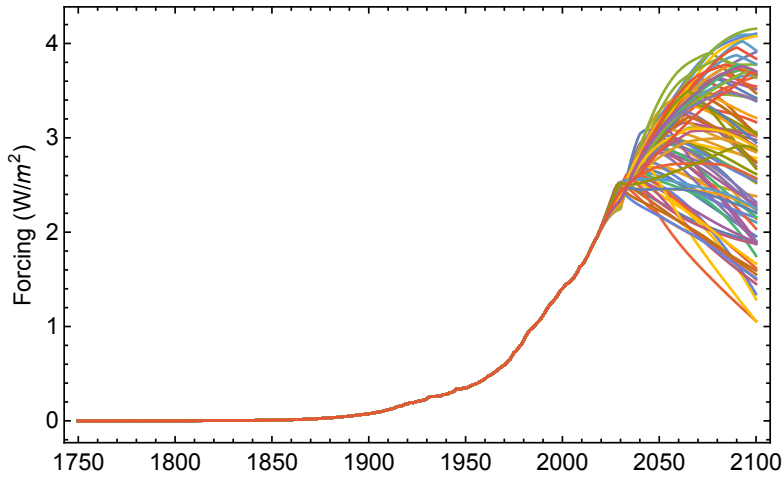


Figure A.13: Combined forcing

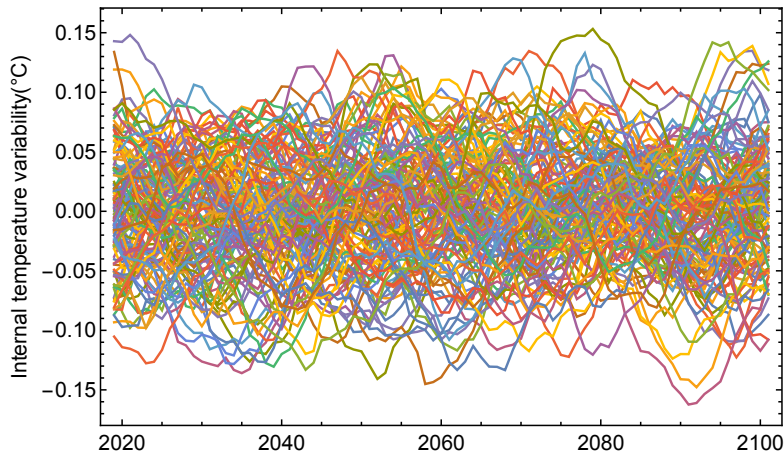


Figure A.14: Noise to estimate internal variability

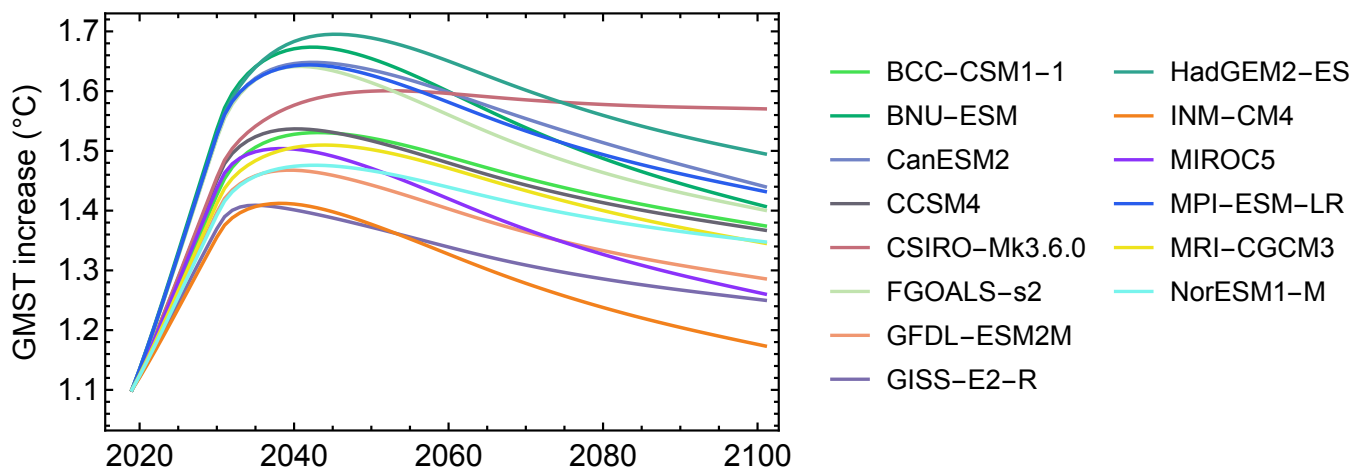


Figure A.15: 14 ESMs

Additional figures illustrating different components of the SRM including non-linear forcing for both equations (Equation A1-A8 and Equation A9-A11). Figure caption describes parameter choices and equation.

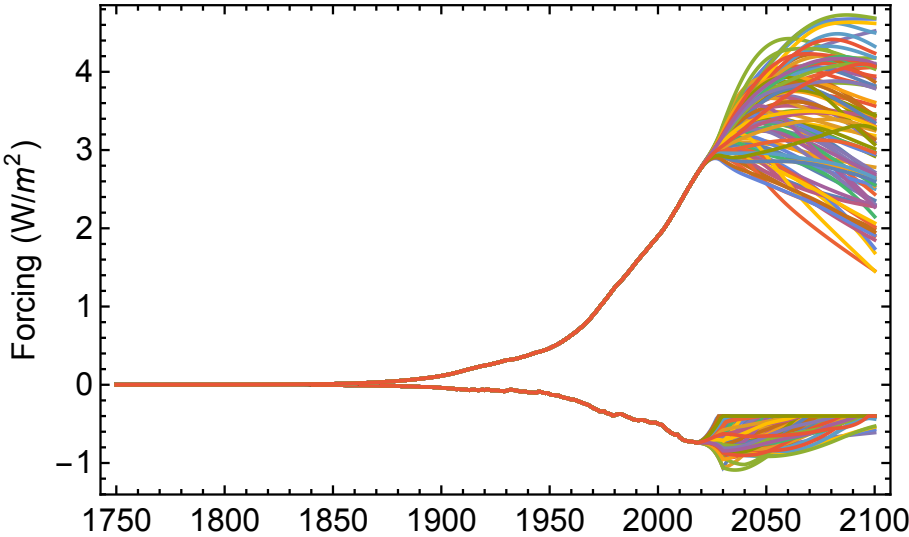


Figure A.16: 1-2-0.5

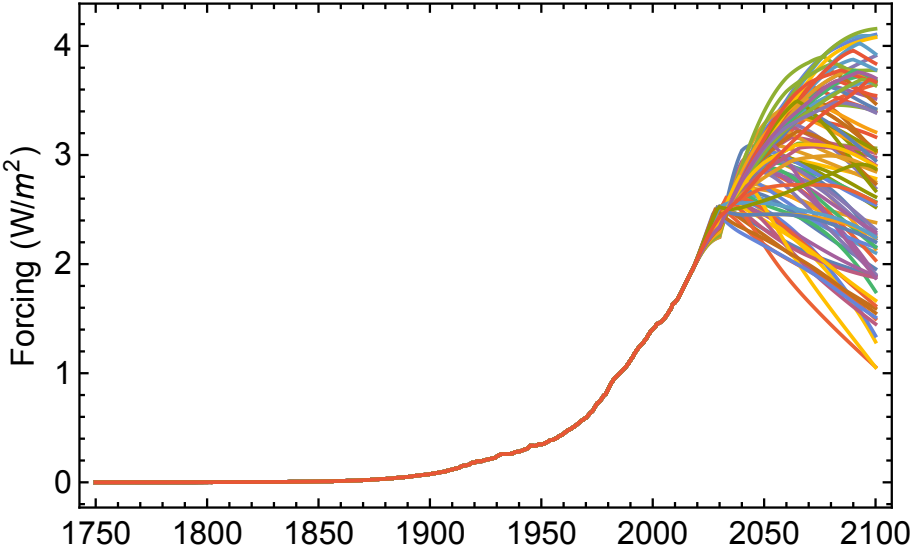


Figure A.17: 1-2-0.5

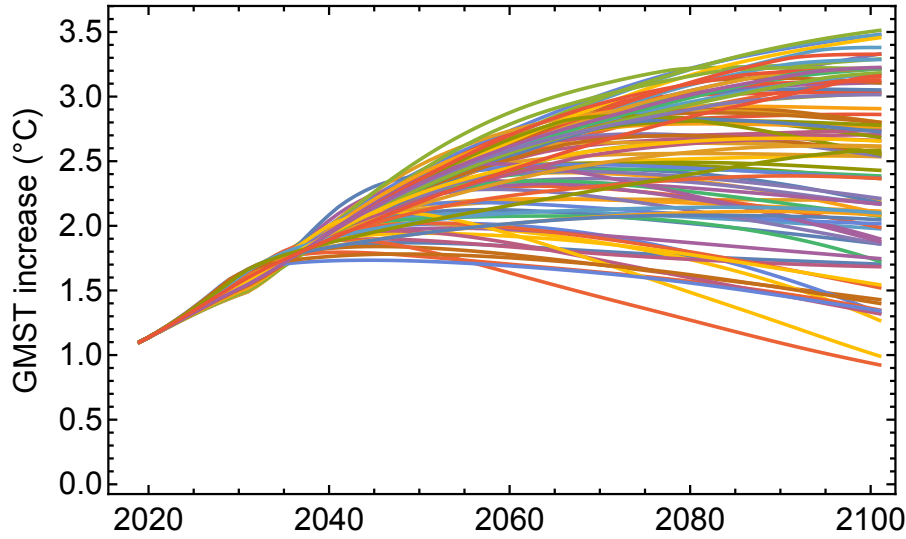


Figure A.18: 1-2-0.5

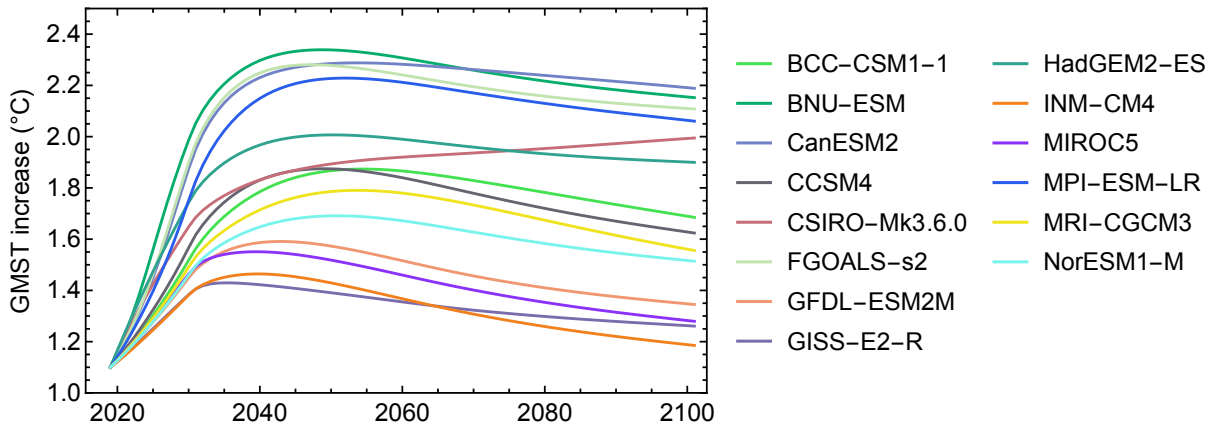


Figure A.19: 1-2-0.5

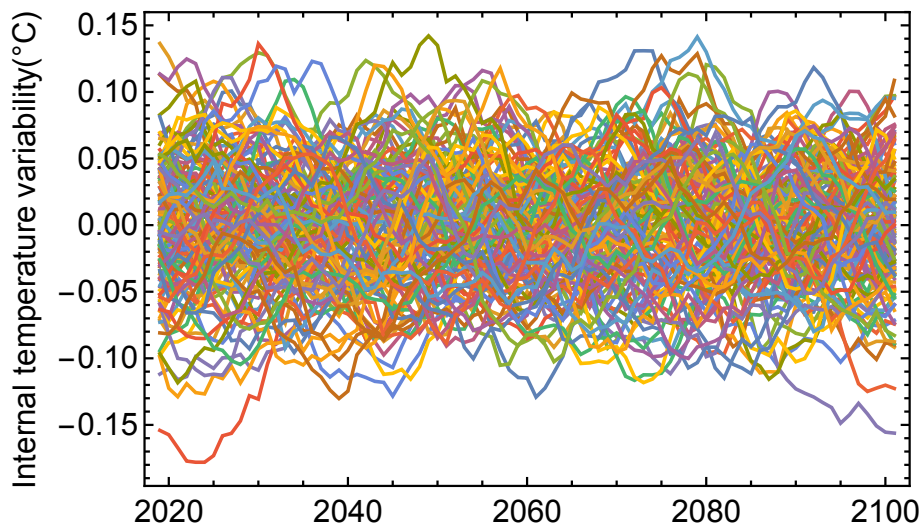


Figure A.20: 1-2-0.5

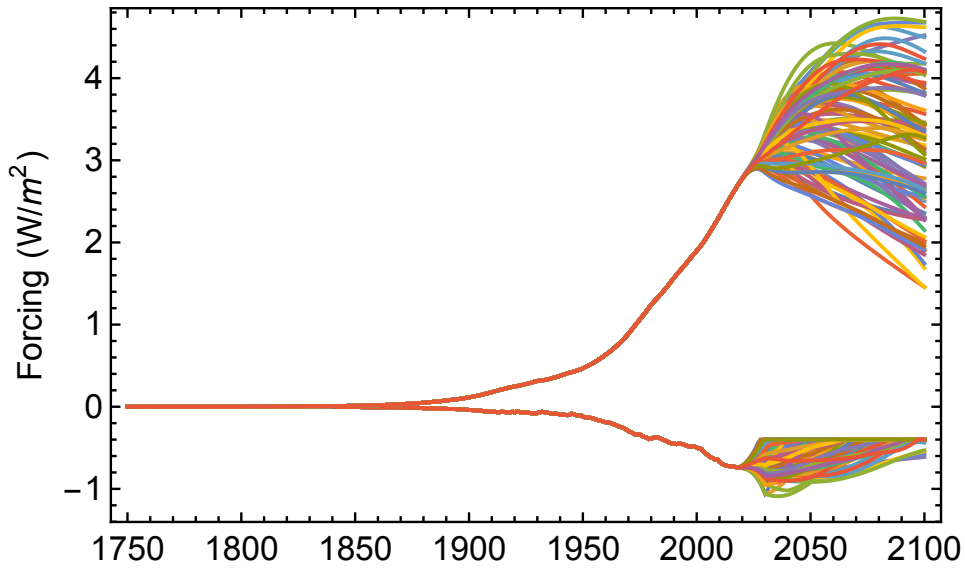


Figure A.21: 0.2T

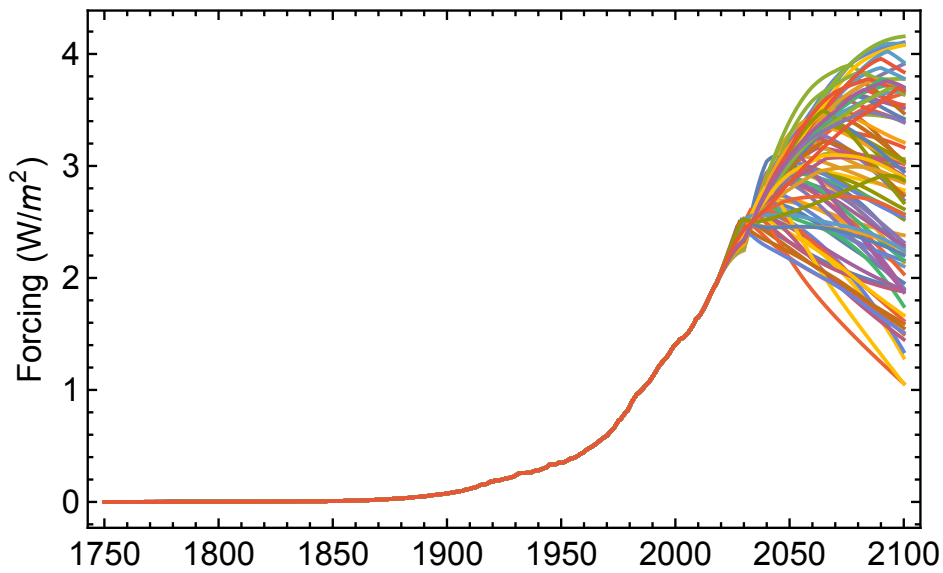


Figure A.22: 0.2T



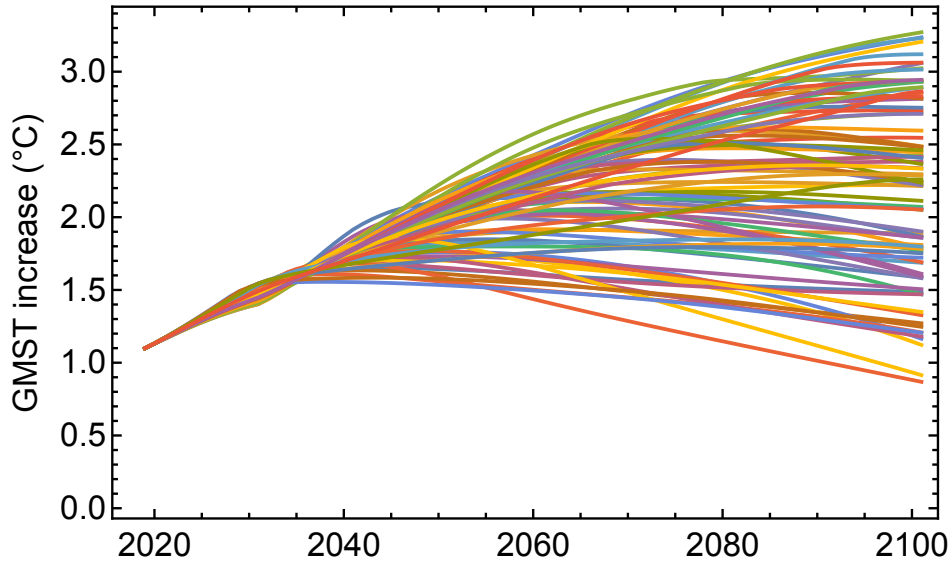


Figure A.23: 0.2T

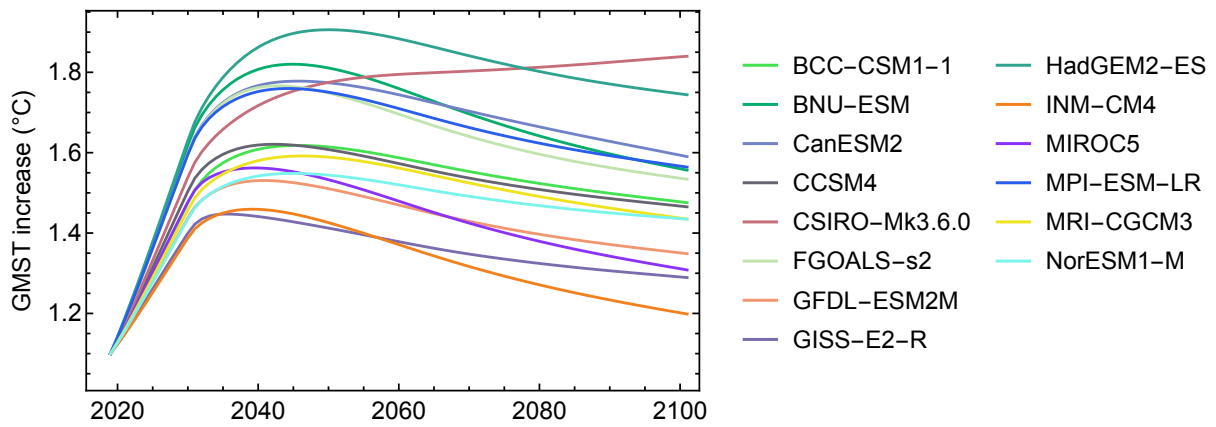


Figure A.24: 0.2T

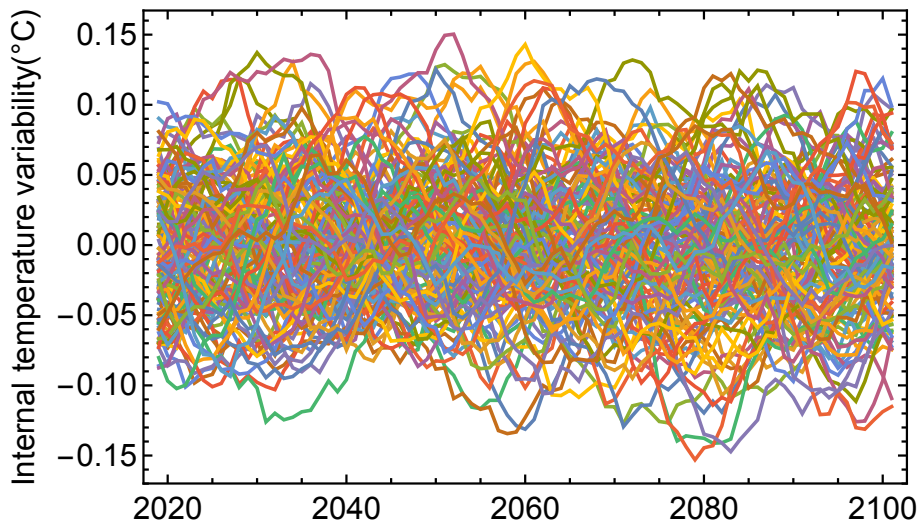
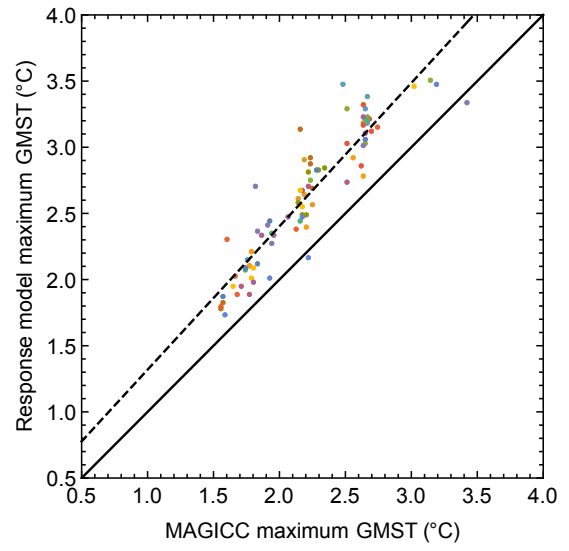
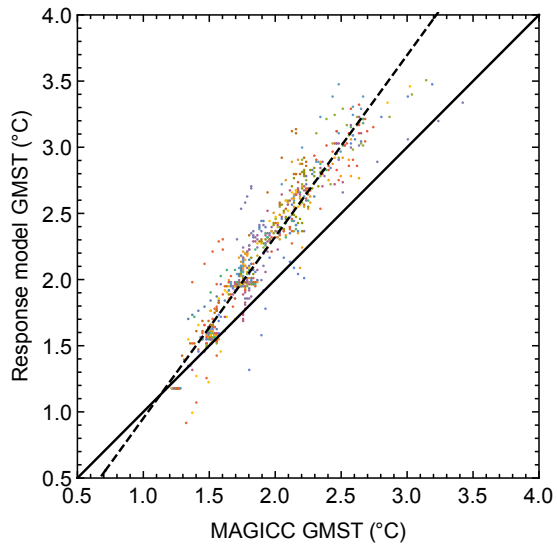


Figure A.25: 0.2T



**Figure A. 26:** Non-linear parameters 1-2-0.5 of SRM compared to MAGICC, Same denotation as Figure 20 in Section 3.3

# Appendix B

## Mathematica code for the SRM

The following code is used for plotting the linear SRM as described in Section 3.1-3.5. Produced in Mathematica 12.0.0.0 in collaboration with research partners Andreas Rostrup Martinsen, Endre Falck Mentzoni and supervision from Martin Rypdal. Platform: Mac OS X x86 (64-bit). macOS Catalina: Version 10.15.3.

```
In[283]:= SetDirectory["OneDrive - UiT Office 365"];  
Z = Import["SSP_IAM_V2_201811.csv"];  
Z = Map[StringSplit[#, ","] &, Z];
```

```
In[5]:= hh=157.65890684920566`+1.8942819330281027`zz+0.08520850267749702`zz2;
```

```
In[6]:= Z[[1]]  
Out[6]= {{MODEL, "SCENARIO", "REGION", "VARIABLE", "UNIT",  
2005, 2010, 2020, 2030, 2040, 2050, 2060, 2070, 2080, 2090, 2100}}
```

```
In[7]:= RR=Table[Z[[k]][[1]][[4]],{k,1,Length[Z]}];  
Union[RR]
```

```
Out[8]= {"Agricultural Demand|Crops", "Agricultural Demand|Crops|Energy",  
"Agricultural Demand|Livestock", "Agricultural Production|Crops|Energy",  
"Agricultural Production|Crops|Non-Energy", "Agricultural Production|Livestock", "Capacity|Electricity",  
"Capacity|Electricity|Biomass", "Capacity|Electricity|Coal", "Capacity|Electricity|Gas",  
"Capacity|Electricity|Geothermal", "Capacity|Electricity|Hydro", "Capacity|Electricity|Nuclear",  
"Capacity|Electricity|Oil", "Capacity|Electricity|Other", "Capacity|Electricity|Solar", "Capacity|Electricity|Solar|CSP",  
"Capacity|Electricity|Solar|PV", "Capacity|Electricity|Wind", "Capacity|Electricity|Wind|Offshore",  
"Capacity|Electricity|Wind|Onshore", "Consumption", "Diagnostics|MAGICC6|Concentration|CH4",  
"Diagnostics|MAGICC6|Concentration|CO2", "Diagnostics|MAGICC6|Concentration|N2O",  
"Diagnostics|MAGICC6|Forcing", "Diagnostics|MAGICC6|Forcing|Aerosol", "Diagnostics|MAGICC6|Forcing|CH4",  
"Diagnostics|MAGICC6|Forcing|CO2", "Diagnostics|MAGICC6|Forcing|F-Gases",  
"Diagnostics|MAGICC6|Forcing|Kyoto Gases", "Diagnostics|MAGICC6|Forcing|N2O",  
"Diagnostics|MAGICC6|Temperature|Global Mean", "Emissions|BC", "Emissions|CH4",  
"Emissions|CH4|Fossil Fuels and Industry", "Emissions|CH4|Land Use", "Emissions|CO",  
"Emissions|CO2", "Emissions|CO2|Carbon Capture and Storage",  
"Emissions|CO2|Carbon Capture and Storage|Biomass", "Emissions|CO2|Fossil Fuels and Industry",  
"Emissions|CO2|Land Use", "Emissions|F-Gases", "Emissions|Kyoto Gases", "Emissions|N2O",  
"Emissions|N2O|Land Use", "Emissions|NH3", "Emissions|NOx", "Emissions|OC", "Emissions|Sulfur",  
"Emissions|VOC", "Energy Service|Transportation|Freight", "Energy Service|Transportation|Passenger",  
"Final Energy", "Final Energy|Electricity", "Final Energy|Gases", "Final Energy|Heat",  
"Final Energy|Hydrogen", "Final Energy|Industry", "Final Energy|Liquids",  
"Final Energy|Residential and Commercial", "Final Energy|Solar", "Final Energy|Solids",  
"Final Energy|Solids|Biomass", "Final Energy|Solids|Biomass|Traditional", "Final Energy|Solids|Coal",  
"Final Energy|Transportation", "GDP|PPP", "Harmonized Emissions|BC",  
"Harmonized Emissions|CH4|Fossil Fuels and Industry", "Harmonized Emissions|CH4|Land Use",  
"Harmonized Emissions|CO", "Harmonized Emissions|CO2|Fossil Fuels and Industry",  
"Harmonized Emissions|CO2|Land Use", "Harmonized Emissions|F-Gases", "Harmonized Emissions|Kyoto Gases",  
"Harmonized Emissions|NH3", "Harmonized Emissions|NOx", "Harmonized Emissions|OC",  
"Harmonized Emissions|Sulfur", "Harmonized Emissions|VOC", "Land Cover|Built-up Area",  
"Land Cover|Cropland", "Land Cover|Forest", "Land Cover|Pasture", "Population", "Price|Carbon", "Primary Energy",  
"Primary Energy|Biomass", "Primary Energy|Biomass|Traditional", "Primary Energy|Biomass|w/ CCS",  
"Primary Energy|Biomass|w/o CCS", "Primary Energy|Coal", "Primary Energy|Coal|w/ CCS",  
"Primary Energy|Coal|w/o CCS", "Primary Energy|Fossil", "Primary Energy|Fossil|w/ CCS",  
"Primary Energy|Fossil|w/o CCS", "Primary Energy|Gas", "Primary Energy|Gas|w/ CCS",  
"Primary Energy|Gas|w/o CCS", "Primary Energy|Geothermal", "Primary Energy|Hydro",  
"Primary Energy|Non-Biomass Renewables", "Primary Energy|Nuclear", "Primary Energy|Oil",
```

```
"Primary Energy|Oil|w/ CCS", "Primary Energy|Oil|w/o CCS", "Primary Energy|Other",
"Primary Energy|Secondary Energy Trade", "Primary Energy|Solar", "Primary Energy|Wind",
"Secondary Energy|Electricity", "Secondary Energy|Electricity|Biomass",
"Secondary Energy|Electricity|Biomass|w/ CCS", "Secondary Energy|Electricity|Biomass|w/o CCS",
"Secondary Energy|Electricity|Coal", "Secondary Energy|Electricity|Coal|w/ CCS",
"Secondary Energy|Electricity|Coal|w/o CCS", "Secondary Energy|Electricity|Gas",
"Secondary Energy|Electricity|Gas|w/ CCS", "Secondary Energy|Electricity|Gas|w/o CCS",
"Secondary Energy|Electricity|Geothermal", "Secondary Energy|Electricity|Hydro",
"Secondary Energy|Electricity|Non-Biomass Renewables", "Secondary Energy|Electricity|Nuclear",
"Secondary Energy|Electricity|Oil", "Secondary Energy|Electricity|Solar", "Secondary Energy|Electricity|Wind",
"Secondary Energy|Gases", "Secondary Energy|Gases|Biomass", "Secondary Energy|Gases|Coal",
"Secondary Energy|Gases|Natural Gas", "Secondary Energy|Heat", "Secondary Energy|Heat|Geothermal",
"Secondary Energy|Hydrogen", "Secondary Energy|Hydrogen|Biomass",
"Secondary Energy|Hydrogen|Biomass|w/ CCS", "Secondary Energy|Hydrogen|Biomass|w/o CCS",
"Secondary Energy|Hydrogen|Electricity", "Secondary Energy|Liquids", "Secondary Energy|Liquids|Biomass",
"Secondary Energy|Liquids|Biomass|w/ CCS", "Secondary Energy|Liquids|Biomass|w/o CCS",
"Secondary Energy|Liquids|Coal", "Secondary Energy|Liquids|Coal|w/ CCS",
"Secondary Energy|Liquids|Coal|w/o CCS", "Secondary Energy|Liquids|Gas",
"Secondary Energy|Liquids|Gas|w/ CCS", "Secondary Energy|Liquids|Gas|w/o CCS",
"Secondary Energy|Liquids|Oil", "Secondary Energy|Solids", "VARIABLE"}
```

```
In[9]:= RRR=Table[Z[[k]][[1]][[3]],{k,1,Length[Z]}];
Union[RRR]
```

```
Out[10]= {"R5.2ASIA", "R5.2LAM", "R5.2MAF", "R5.2OECD", "R5.2REF", "REGION", "World"}
```

```
In[11]:= co2pos1=Position[RR,?{#=="Emissions|CO2|Fossil Fuels and Industry\ ""&}];
co2pos2 = Position[RR, _ ? {# == "Emissions|CO2|Land Use\ "" &}];
co2pos3 = Position[RRR, _ ? {# == "World\ "" &}];
```

```
In[14]:= ppos1=Intersection[co2pos3,co2pos1];
ppos2 = Intersection[co2pos3, co2pos2];
```

```
In[16]:= Extract[Z,co2pos1][[1]]
```

```
Out[16]= {{AIM/CGE, "SSP1-19", "R5.2ASIA", "Emissions|CO2|Fossil Fuels and Industry", "Mt CO2/yr", 8985.6725,
10008.8152, 11790.747500000001,
6131.6627, 3271.4353000000006, 1678.8029, 638.87, 259.4755, 82.29590000000003, -7.935300000000105, -
103.9171}}
```

```
In[17]:= em1=ToExpression[Map[Drop[Flatten[#,7]&,Extract[Z,ppos1]]];
em2 = ToExpression[Map[Drop[Flatten[#, 7] &, Extract[Z, ppos2]]];
```

```
In[19]:= ListPlot[em1,PlotRange→All,Joined→True]
```

```
In[20]:= Length[em1]
```

```
Out[20]= 127
```

```
In[21]:= emissions=Map[#[[1;;2]]&,ToExpression[Map[StringSplit[#] &, Drop[ReadList["emissionsCO2.txt",
String], 31]]];
emissions = Table[{emissions[[i, 1]], (44 / 12) * emissions[[i, 2]] / 1000.}, {i, 1, Length[emissions]};
ListPlot[emissions, Joined → True, PlotStyle → {Black, Thick}, PlotRange → All, Axes → False, Frame → True,
FrameStyle → Directive[14, Black],
FrameLabel → {"year", "CO2 emissions (Gt CO/yr)"}]
(*historical emissions*)
```

```
EM = Join[emissions, {{2018, 37.1}}];
data2 = Table[Prepend[Table[{t, Interpolation][Join[EM, Transpose[{{2030, 2040, 2050, 2060, 2070, 2080,
2090, 2100}, 0.001 * Drop[em1[[k]], 1]]][t]], {t, 1751, 2100}], {1750, 0}], {k, 1, Length[em1]}; totliste = Ta-
ble[data2[[k]][[All, 2]], {k, 1, Length[data2]}];
PLAll = ListPlot[data2, Joined → True, Frame → True, FrameStyle → Directive[Black, 14], PlotRange → All,
FrameLabel → {None, "CO2 emissions (Gt CO2)"}]
```

```

positivepaths = Table[DeleteCases[Map[# * UnitStep[#] &, totliste[[k]][[269 ;; 351]]], _? (# == 0 &)], {k, 1,
Length[totliste]}; ListPlot[positivepaths, Joined → True, PlotRange → All]
(*Before removal of exceedance scenarios*)

```

```

In[29]:= RCBliste2=Map[Plus@@#&,positivepaths];

```

```

In[175]:= p1=Position[RCBliste2,_?(<3300&)];

```

```

In[31]:= maxtemp=Map[Max[#]&,templiste];
maxtemp2 = Map[Max[#] &, utempliste];
maxtemp3 = Map[Max[#] &, lowtempliste];

```

```

PL1 = ListPlot[Extract[data2, p1], Joined → True, Frame → True, FrameStyle → Directive[Black, 14], PlotRange
→ All];

```

```

PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];

```

```

FFC = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["", 18],
Scaled[{0.1, 0.9}]]]

```

```

(*After removal of exceedance scenarios*)

```

```

In[37]:= PL1=ListPlot[data2[[1]],Joined→True,Frame→True, FrameStyle → Directive[Black, 14], PlotStyle →
Darker[Blue]];

```

```

PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];

```

```

FFA = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["a", 18],
Scaled[{0.1, 0.9}]]]

```

```

In[40]:= n=Length[data2[[1]];

```

```

futuretime = 2100 - 2020;

```

```

τmetan = 12.4;

```

```

In[43]:= (* Carbon model *)

```

```

τ1=1;

```

```

τ2=10;

```

```

τ3 = 100;

```

```

τ4 = 1000;

```

```

c1mean = 0.152;

```

```

c2mean = 0.246;

```

```

c4mean = 0.134;

```

```

c5mean = 0.194;

```

```

Gmean = (12/44)*0.47*(c1mean * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]
+ c2mean * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1mean - c2mean - c4mean -
c5mean)* Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]
+ c4mean * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + Table[c5mean * UnitStep[i - j], {i, 1, n}, {j,
1, n}]);

```

```

In[52]:= (* Carbon models *)

```

```

c1upper = 0.11;

```

```

c2upper = 0.212;

```

```

c4upper = 0.106;

```

```

c5upper = 0.262;

```

```

c1lower = 0.18;

```

```

c2lower = 0.296;

```

```

c4lower = 0.122;

```

```

c5lower = 0.148;

```

```

Glower = (12 / 44) * 0.47 * (c1lower * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c2lower * Ta-
ble[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1lower - c2lower - c4lower - c5lower) * Ta-
ble[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c4lower * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1,
n}, {j, 1, n}] + Table[c5lower * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

```

```

Gupper = (12/44)*0.47*(c1upper * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c2upper * Ta-
ble[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1upper - c2upper - c4upper - c5upper) *

```

```
Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c4upper * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + Table[c5upper * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
```

```
In[62]:=
(*Optimal Estimation of Stochastic Energy Balance Model Parameters *)
```

```
In[63]:= (* Climate models *)
models = ReadList["CMIP5parameters.txt", String];
models = Delete[models, {{5}, {12}}];
boxes = StringSplit[models][[All, 2]];
Klimaliste = {};
Γliste = {};
σ2liste = {};
Monitor[
Do[
```

```
Clear[A];
modelnr = p; If[boxes[[p]] == "2",
```

```
{C1, C2, κ1, κ2, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];
```

```
A = {{-(κ1+ κ2) /C1, κ2/C1}, {κ2/C2, -κ2/C2}};
g = (MatrixExp[t A].{1 / C1, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];
```

```
If[ boxes[[p]] == "3",
```

```
{C1, C2, C3, κ1, κ2, κ3, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];
```

```
A = {{-(κ1+ κ2) /C1, κ2/C1, 0}, {κ2/C2, -(κ2+κ3)/C2, κ3/C2},{0, κ3/C3, - κ3/C3}};
```

```
g = (MatrixExp[t A].{1 / C1, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];
```

```
If [ boxes[[p]] == "4",
```

```
{C1, C2, C3, C4, κ1, κ2, κ3, κ4, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];
```

```
A = {{-(κ1+κ2)/C1, κ2/C1, 0, 0} , {κ2/C2, -(κ2+κ3)/C2, κ3/C2, 0},{ 0, κ3/C3, -(κ3+κ4)/C3, κ4/C3},
{0, 0, κ4/C4, -κ4/C4}};
```

```
g = (MatrixExp[t A].{1 / C1, 0, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];
```

```
Γliste = Append[Γliste, Γ];
σ2liste = Append[σ2liste, σ2];
, {p, 1, Length[models]}
];
, {p, boxes[[p]]}
];
```

```
In[70]:= RCBliste={};
totliste = {};
```

```

templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
Δfaeroliste = {};
Δfghgliste = {};
Δfliste = {};
noiseliste = {};

Monitor[
Do[
tot = data2[[u]][[All, 2]]; meanco2 = Gmean.tot + 280;

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 11.9 tunes 2019 methane emissions in 2019 to 440 Tg Methane *) ;
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.34 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];

Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfaer = -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]];
Δfaer2 = Drop[Δfaer, Length[EM]];
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;

Δfliste = Append[Δfliste, Δf];
Δfaeroliste = Append[Δfaeroliste, Δfaer];
Δfghgliste = Append[Δfghgliste, Δfco2 + Δfmetan];

Tliste = {};
Do[
T2 = Klimaliste[[p]].Δf;
noise = σ2liste[[p]] * (Klimaliste[[p]].RandomReal[NormalDistribution[0, 1], Length[Δf]]);
noise = Drop[noise, 268 - 20];
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2 = 1.1 + T2 - T2[[1]];
noiseliste = Append[noiseliste, noise]; Tliste = Append[Tliste, T2];
, {p, 1, Length[models]};

middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]}];
RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}
, u];

```

```
In[183]:= Length[noise]
Out[183]= 103
```

```
In[82]:= Length[T2]
Out[82]= 83
```

```
In[83]:= window=10;
noiselist2 = Table[MovingAverage[noiselist[[i]], window][[1 ;; Length[T2]]], {i, 1, Length[noiselist]};
noiselist2 = Transpose[Partition[noiselist2, 14]];
```

```
In[86]:= Length[noiselist2]
Out[86]= 14
```

```
In[87]:= Dimensions[noiselist2]
Out[87]= {14, 127, 83}
```

```
In[88]:= Length[noiselist2[[1]]]
Out[88]= 127
```

```
PLNoise = ListPlot[Map[Transpose[{2018 + Range[Length[templiste[[1]]], #]} &, noiselist2[[3]], Joined → True];
FFNoise = Show[PLNoise, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], Joined → True, FrameLabel → {None, "Internal temperature variability(°C)"}, Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]]]
(*Plot of internal variability*)
```

```
In[90]:= FFE=ListPlot[Map[Transpose[{2018+Range[Length[templiste[[1]]],#]}&, Extract[templiste, p1]], PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "GMST increase (°C)"}, Epilog → Inset[Style["e", 18], Scaled[{0.1, 0.9}]]]
(*Temperature response*)
```

```
In[121]:= PL3=ListPlot[Map[Transpose[{1749+Range[Length[Δfaeroliste[[1]]],#]}&, Extract[Δfliste, p1]], Joined → True];
FF3 = Show[PL3, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "Forcing (W/m2)"}, Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]]]
(*Combined forcing for both ghg's and aerosols*)
```

```
In[150]:= PL1=ListPlot[Map[Transpose[{1749+Range[Length[Δfaeroliste[[1]]],#]}&, Extract[Δfaeroliste, p1]], Joined → True];
PL2 = ListPlot[Map[Transpose[{1749 + Range[Length[Δfaeroliste[[1]]], #]} &, Extract[Δfghgliste, p1]], Joined → True];
FFD = Show[{PL1, PL2}, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "Forcing (W/m2)"}, Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]]]
(*Split forcing for ghg's and aerosols*)
```

```
In[94]:=modellfarger={,,,,,,,,,,,,};
```

```
In[95]:= pan=LineLegend[modellfarger,Map[StringSplit[#]&,models][[All,1]]]
```

```
Out[95]= BCC-CSM1-1 BNU-ESM CanESM2 CCSM4 CSIRO-Mk3.6.0 FGOALS-s2 GFDL-ESM2M GISS-E2-R HadGEM2-ES INM-CM4 MIROC5 MPI-ESM-LR MRI-CGCM3 NorESM1-M
```

```
FFB = ListPlot[Map[Transpose[{2018 + Range[Length[alltliste[[1]]], #]} &, alltliste[[1 ;; 14]]], PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "GMST increase (°C)"}, Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]], PlotStyle → Map[{#} &, modellfarger]]
(*Plot for temperature response, 1 scenario and 14 ESMs*)
```

```
In[ ]:= Grid[{{Show[FFA,ImageSize→400],Show[FFB,ImageSize→400],pan}}]
(*Grid plot for one scenario, 14 ESMs*)
```

```
Grid[{{Show[FFC, ImageSize → 400, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]]], Show[FFD, ImageSize → 400, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]]], Show[FFE, ImageSize → 400, Epilog → Inset[Style["c", 18], Scaled[{0.1, 0.9}]]]}}]
(*Grid plot for emissions, forcing and GMST for 86 scenarios and 1 esm*)
```



```

maxtemp = Map[Max[#] &, templiste]; (*mean*)
maxtemp2 = Map[Max[#] &, uptempliste]; (*+1sd*)
maxtemp3 = Map[Max[#] &, lowtempliste]; (*-1sd*)

```

```

In[ ]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1], AspectRatio → 1, PlotRange → All];
PL3 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Red];
PL4 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Red];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz]; PL2 = Plot[gg, {zz, 1.2, 3}];
gg2 = Fit[Extract[Transpose[{maxtemp2, RCBliste2}], p1], {zz, 1}, zz]; PL4 = Plot[gg2, {zz, 1.2, 3}];
Show[{PL1, PL2}, PlotRange → All]
(*TCRE plot w/o pdf*)

```

```

In[ ]:= pairs=Extract[Transpose[{maxtemp,RCBliste2}],p1];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];

```

```

σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];

```

```

In[ ]:= pdf=(PDF[NormalDistribution[gg,σf[zz]]][p])/zz→1.5;
pdf2 = (PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → 2.5;
Plot[{pdf, pdf2}, {p, 0, 5200}, PlotRange → All]

```

```

In[ ]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1],AspectRatio→1, PlotRange → {{1, 4}, {0,
4000}}, PlotStyle → Darker[Blue]];
PL2 = Plot[gg, {zz, 1, 3}, PlotStyle → Darker[Blue]];
l1 = Graphics[{Black, Line[{{1.5, 0}, {1.5, gg /. zz → 1.5}}]};
l2 = Graphics[{Black, Line[{{1.5, gg /. zz → 1.5}, {1, gg /. zz → 1.5}}]}; inset = ParametricPlot[{1 + 60 * pdf, p},
{p, 0, 1200}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];

```

```

l11 = Graphics[{Black, Line[{{2.5, 0}, {2.5, gg /. zz → 2.5}}]};
l22 = Graphics[{Black, Line[{{2.5, gg /. zz → 2.5}, {1, gg /. zz → 2.5}}]}; inset2 = ParametricPlot[{1 + 60 * pdf2,
p}, {p, 1900, 3000}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];

```

```

FA = Show[{PL1, PL2, l1, l2, l11, l22, inset, inset2}, Axes → False, Frame → True, FrameStyle → Directive[Black,
14], FrameLabel → {"global temperature increase (°C)", "carbon budget after 2018 (Gt CO2)"}, Epilog → In-
set[Style["a", 18], Scaled[{0.1, 0.9}], ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]
(*TCRE with pdf, 1 ESM*)

```

## ALL CLIMATE MODELS

```

In[ ]:= maxtemp=Partition[Map[Max[#]&,alltliste],14][[All,5]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Black];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 3.5}, PlotStyle → Black];
QL1 = Show[{PL1, PL2}, PlotRange → All];
maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, 7]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Darker[Red]];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz]; PL2 = Plot[gg, {zz, 1.2, 3}, PlotStyle →
Darker[Red]];
QL2 = Show[{PL1, PL2}, PlotRange → All];
FB = Show[{QL1, QL2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel →
{"global temperature increase (°C)", "carbon budget after 2018 (Gt CO2)"}, Epilog → {Inset[Style["b", 18],
Scaled[{0.1, 0.9}], Inset[LineLegend[{Black, Darker[Red]}, {"CSIRO-Mk3.6.0", "GFDL-ESM2M"}], Scaled[{0.7,

```

```
0.3]]], ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]
```

```
In[ ]:= Grid[{{FA,FB}}  
(*TCREs for two ESM's*)
```

```
(*MEAN TCRE CALCULATION*)
```

```
In[ ]:= maxtemp1=Partition[Map[Max[#]&,alltliste],14][[All,1]];  
maxtemp2 = Partition[Map[Max[#] &, alltliste], 14][[All, 2]];  
maxtemp3 = Partition[Map[Max[#] &, alltliste], 14][[All, 3]];  
maxtemp4 = Partition[Map[Max[#] &, alltliste], 14][[All, 4]];  
maxtemp5 = Partition[Map[Max[#] &, alltliste], 14][[All, 5]];  
maxtemp6 = Partition[Map[Max[#] &, alltliste], 14][[All, 6]];  
maxtemp7 = Partition[Map[Max[#] &, alltliste], 14][[All, 7]];  
maxtemp8 = Partition[Map[Max[#] &, alltliste], 14][[All, 8]];  
maxtemp9 = Partition[Map[Max[#] &, alltliste], 14][[All, 9]];  
maxtemp10 = Partition[Map[Max[#] &, alltliste], 14][[All, 10]];  
maxtemp11 = Partition[Map[Max[#] &, alltliste], 14][[All, 11]];  
maxtemp12 = Partition[Map[Max[#] &, alltliste], 14][[All, 12]];  
maxtemp13 = Partition[Map[Max[#] &, alltliste], 14][[All, 13]];  
maxtemp14 = Partition[Map[Max[#] &, alltliste], 14][[All, 14]];
```

```
ggm1 = Fit[Extract[Transpose[{maxtemp1, RCBliste2}], p1], {zz, 1}, zz];  
ggm2 = Fit[Extract[Transpose[{maxtemp2, RCBliste2}], p1], {zz, 1}, zz];  
ggm3 = Fit[Extract[Transpose[{maxtemp3, RCBliste2}], p1], {zz, 1}, zz];  
ggm4 = Fit[Extract[Transpose[{maxtemp4, RCBliste2}], p1], {zz, 1}, zz];  
ggm5 = Fit[Extract[Transpose[{maxtemp5, RCBliste2}], p1], {zz, 1}, zz];  
ggm6 = Fit[Extract[Transpose[{maxtemp6, RCBliste2}], p1], {zz, 1}, zz];  
ggm7 = Fit[Extract[Transpose[{maxtemp7, RCBliste2}], p1], {zz, 1}, zz];  
ggm8 = Fit[Extract[Transpose[{maxtemp8, RCBliste2}], p1], {zz, 1}, zz];  
ggm9 = Fit[Extract[Transpose[{maxtemp9, RCBliste2}], p1], {zz, 1}, zz];  
ggm10 = Fit[Extract[Transpose[{maxtemp10, RCBliste2}], p1], {zz, 1}, zz];  
ggm11 = Fit[Extract[Transpose[{maxtemp11, RCBliste2}], p1], {zz, 1}, zz];  
ggm12 = Fit[Extract[Transpose[{maxtemp12, RCBliste2}], p1], {zz, 1}, zz];  
ggm13 = Fit[Extract[Transpose[{maxtemp13, RCBliste2}], p1], {zz, 1}, zz];  
ggm14 = Fit[Extract[Transpose[{maxtemp14, RCBliste2}], p1], {zz, 1}, zz];
```

```
PLm5 = ListPlot[Extract[Transpose[{maxtemp5, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle  
→ Black];  
PLm7 = ListPlot[Extract[Transpose[{maxtemp7, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle  
→ Darker[Red]];  
PLm7dot = Plot[ggm7, {zz, 1.2, 3}, PlotStyle → Darker[Red]]; PLm5dot = Plot[ggm5, {zz, 1.2, 3}, PlotStyle →  
Black];
```

```
meanTCRE=  
(ggm1+ggm2+ggm3+ggm4+ggm5+ggm6+ggm7+ggm8+ggm9+ggm10+ggm11+ggm12+ggm13+ggm14)/14;
```

```
PLmeanTCRE = Plot[meanTCRE, {zz, 1.2, 3}, PlotStyle → {Black, Dashed}];  
QLmeanTCRE = Show[{PLm5dot, PLm5, PLm7dot, PLm7, PLmeanTCRE}, Axes → False, Frame → True, Frame-  
Style → Directive[Black, 14], AspectRatio → 1, FrameLabel → {"global temperature increase (°C)", "carbon  
budget after 2018 (Gt CO2)"}, ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}, Epilog → Inset[LineL-  
egend[{{Black, {Black, Dashed}, Darker[Red]}, {"CSIRO-Mk3.6.0", "Mean TCRE", "GFDL-ESM2M"}], Scaled[{{0.7,  
0.3}}]]]  
(*mean TCRE plot*)
```

PDF estimation

```
In[ ]:= smliste={};  
tliste = {};  
Monitor[ Do[
```

```
pdfliste = {}; Do[  
maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, kk]];
```

```

gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
pairs = Extract[Transpose[{maxtemp, RCBliste2}], p1];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
, {kk, 1, 14}];

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x]; sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];

```

```

In[ ]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y / ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]
In[ ]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y / ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

```

```

In[ ]:= bliste={};
Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]]; y = y / ((Plus@@y) * Δt);
t1 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.90 &)]]][[1]] - 1];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.75 &)]]][[1]] - 1];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.5 &)]]][[1]] - 1];
t4 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.25 &)]]][[1]] - 1];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.10 &)]]][[1]] - 1];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}];
, {budget, 200, 4000, 100}];

```

```

In[ ]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};

```

```

In[ ]:= GGA=ListPlot[{Transpose[{bliste[[All,1]],bliste[[All,2]]}], Transpose[{bliste[[All, 1]], bliste[[All, 3]]}],
Transpose[{bliste[[All, 1]], bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], bliste[[All, 5]]}], Trans-
pose[{bliste[[All, 1]], bliste[[All, 6]]}], Joined → True, AspectRatio → 1, PlotRange → {1, 4}, Axes → False,
Frame → True, FrameStyle → Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines → Auto-
matic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)"}, PlotLeg-
ends → Placed[{"10% prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05, 0.7}], {0,
0.5}}]]]

```

(\*RCB plot one carbon, 14 ESMs\*)

## TWO CARBON MODELS, MEAN CLIMATE MODEL

```

In[ ]:= RCBliste={};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
Monitor[
Do[

```

```

tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emissions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
Δfaer = -0.02tot; (* aerosols *)
Δfaer1 = Δfaer[[1 ;; Length[EM]]];
Δfaer2 = Drop[Δfaer, Length[EM]];
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2]; (*-0.4 asymptote*)
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;

Tliste = {};
Do[
(*T2=Klimaliste[[p]].Δf;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268]; T2=1.0+T2-T2[[1]];
Tliste=Append[Tliste,T2];*)

T2 = Klimaliste[[p]].Δfupper;
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268]; T2=1.1+T2-T2[[1]];
Tliste = Append[Tliste, T2];

T2 = Klimaliste[[p]].Δflower;
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268]; T2=1.0+T2-T2[[1]];
Tliste = Append[Tliste, T2];
, {p, 1, Length[models]};

Tlisteupper = Partition[Tliste, 2][[All, 1]];
Tlistelower = Partition[Tliste, 2][[All, 2]];
meanupper = Map[Mean[#] &, Transpose[Tlisteupper]];
meanlower = Map[Mean[#] &, Transpose[Tlistelower]];
Tliste = {meanupper, meanlower};

middel = Table[Mean[Transpose[Tliste][[i]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]] + StandardDeviation[Transpose[Tliste][[i]], {i, 1, Length[Transpose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]] - StandardDeviation[Transpose[Tliste][[i]], {i, 1, Length[Transpose[Tliste]]}];

RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];

```

```
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}
,u]
```

```
In[ ]:= Dimensions[alltliste]
Out[ ]:= {254, 83}
```

```
In[ ]:= 3556/14
Out[ ]:= 254
```

```
In[ ]:= CM = {};
cm1 =
Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2], p1][[kk]], 2][[All, 1]], {kk, 1, Length[p1]}];
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2], p1][[kk]], 2][[All, 2]], {kk, 1,
Length[p1]}];
```

```
Do[
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]];
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]];
, {j, 1, Length[Transpose[cm2]]}];
```

```
In[ ]:= ListPlot[{CM[[1]],CM[[2]]}]
```

```
In[ ]:= smliste={};
tliste = {};
```

```
Monitor[ Do[
pdfliste = {}; Do[
gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]])]^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
, {kk, 1, 2}];
```

```
g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];
```

```
In[ ]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y / ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]
```

```
In[ ]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y / ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]
```

```
In[ ]:= bliste={};
Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]]; y = y / ((Plus@@y) * Δt);
t1 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.90 &)]]][[1]] - 1];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.75 &)]]][[1]] - 1];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.5 &)]]][[1]] - 1];
t4 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.25 &)]]][[1]] - 1];
```

```
t5 = t1 + FoldList[Plus, 0, y * Δt, _? (# > 0.10 &)] [[1]] - 1];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}]; , {budget, 200, 4000, 100}]
```

```
ln["farger" := farger = {Red, Darker[Red], Black, Darker[Blue], Blue};
```

```
ln["a" := a = ListPlot[{Transpose[{bliste[[All, 1]], bliste[[All, 2]]}], Transpose[{bliste[[All, 1]], bliste[[All, 3]]}], Trans-
pose[{bliste[[All, 1]], bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], bliste[[All, 5]]}], Transpose[{bliste[[All, 1]],
bliste[[All, 6]]}], Joined → True, AspectRatio → 1, PlotRange → {1, 4}, Axes → False, Frame → True, FrameStyle
→ Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines → Automatic, FrameLabel → {"Carbon
budget from 2018 (GtCO2)", "Maximum temperature increase (°C)"}, PlotLegends → Placed[{"10% prob.",
"25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05, 0.7}], {0, 0.5}}]; l = Graphics[{Black,
Line[{{1294, 1}, {1294, 2.5}}]};
GGB = Show[{a}]
```

(\*RCB estimate two carbons, mean ESM\*)

## INTERNAL VARIABILITY

```
RCBliste = {};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
```

```
Monitor[ Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)
```

```
(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emissions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];
```

```
Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
(* aerosols *)
Δfaer = -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]]; Δfaer2 = Drop[Δfaer, Length[EM]]; Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;
```

```
Tliste = {};
Do[
T2 = Klimaliste[[p]].Δf;
T2 = T2 * (Tliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2 = 1.1 + T2 - T2[[1]];
```

```
Tliste = Append[Tliste, T2 + StandardDeviation[Flatten[noiseliste2[[p]]]];
, {p, 1, Length[models]}];
```

```
Tlisteupper = Partition[Tliste, 2][[All, 1]];
Tlistelower = Partition[Tliste, 2][[All, 2]];
mupper = {};
mlower = {};
```

```
Do[
meanupper = Map[Mean[#] &, Transpose[Tlisteupper]] + StandardDeviation[Flatten[noiseliste2[[p]]]];
meanlower = Map[Mean[#] &, Transpose[Tlistelower]] - StandardDeviation[Flatten[noiseliste2[[p]]]];
mupper = Append[mupper, meanupper]; mlower = Append[mlower, meanlower]; , {p, 1, 14}];
Tliste = Join[mupper, mlower];
```

```
middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]};
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1, Length[Trans-
pose[Tliste]]};
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1, Length[Trans-
pose[Tliste]]};
RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}
,u]
```

```
In[ ]:= Dimensions[alltliste]
Out[ ]:= {3556, 83}
```

```
In[ ]:= CM = {};
cm1 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[ All, 1]], {kk, 1,
Length[p1]}];
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[ kk]], 2][[All, 2]], {kk, 1,
Length[p1]}];
```

```
Do[ CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]];
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]]; , {j, 1, Length[Trans-
pose[cm2]]}];
```

```
In[ ]:= ListPlot[{CM[[1]],CM[[2]]}]
```

```
In[ ]:= smliste={};
tliste = {};
Monitor[ Do[
```

```
pdfliste = {}; Do[
gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
,{kk,1,2*14}];
```

```
g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
```

```

tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01};
, target];

```

```

In[ ]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

```

```

In[ ]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

```

```

In[ ]:= bliste={};
Do[ Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt);
t1 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.90 &)]]][[1]] - 1];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.75 &)]]][[1]] - 1];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.5 &)]]][[1]] - 1];
t4 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.25 &)]]][[1]] - 1];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.10 &)]]][[1]] - 1];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}];
, {budget, 200, 4000, 100}]

```

```

In[ ]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};

```

```

In[ ]:= a=ListPlot[{Transpose[{bliste[[All,1]],bliste[[All,2]]}], Transpose[{bliste[[All, 1]], bliste[[All, 3]]}], Trans-
pose[{bliste[[All, 1]], bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], bliste[[All, 5]]}], Transpose[{bliste[[All, 1]],
bliste[[All, 6]]}], Joined → True, AspectRatio → 1, PlotRange → {1, 4.0}, Axes → False, Frame → True, Frame-
Style → Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines → Automatic, FrameLabel →
{"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)", PlotLegends → Placed[{"10%
prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05, 0.7}], {0, 0.5}}]};
l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]};
GGC = Show[{a}]
(*RCB estimate internal variability*)

```

## COMBINATION BETWEEN TWO CARBON MODELS, 14 ESMs AND INTERNAL VARIABILITY

```

RCBliste = {};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};

```

```

Monitor[ Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

```

```

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emmissions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

```



```

Gmetan = 0.34 * Table[Exp[- (i - j) /  $\tau$ metan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];

 $\Delta$ fmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
 $\Delta$ fco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
 $\Delta$ fco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
 $\Delta$ fco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
(* aerosols *)
 $\Delta$ faer = -0.02tot;
 $\Delta$ faer1 =  $\Delta$ faer[[1 ;; Length[EM]]];  $\Delta$ faer2 = Drop[ $\Delta$ faer, Length[EM]];  $\Delta$ faer2 = Map[Min[-0.4, #] &,  $\Delta$ faer2];
 $\Delta$ faer = Join[ $\Delta$ faer1,  $\Delta$ faer2];
 $\Delta$ f =  $\Delta$ fco2 +  $\Delta$ faer +  $\Delta$ fmetan;
 $\Delta$ fupper =  $\Delta$ fco2upper +  $\Delta$ faer +  $\Delta$ fmetan;
 $\Delta$ flower =  $\Delta$ fco2lower +  $\Delta$ faer +  $\Delta$ fmetan;

Tliste = {};
Do[

T2 = Klimaliste[[p]]. $\Delta$ fupper;
T2 = T2 * ( $\Gamma$ liste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2 = 1.1 + T2 - T2[[1]];
Tliste = Append[Tliste, T2 + StandardDeviation[Flatten[noisaliste2[[p]]]]];

(*T2=Klimaliste[[p]]. $\Delta$ fupper;
T2=T2*( $\Gamma$ liste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.1+T2-T2[[1]]; Tliste=Append[Tliste,T2-StandardDeviation[Flatten[noisaliste2[[p]]]]];*)

(*T2=Klimaliste[[p]]. $\Delta$ flower;
T2=T2*( $\Gamma$ liste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.0+T2-T2[[1]]; Tliste=Append[Tliste,T2+StandardDeviation[Flatten[noisaliste2[[p]]]]];*)

T2 = Klimaliste[[p]]. $\Delta$ flower;
T2 = T2 * ( $\Gamma$ liste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2 = 1.0 + T2 - T2[[1]];
Tliste = Append[Tliste, T2 - StandardDeviation[Flatten[noisaliste2[[p]]]]];

, {p, 1, Length[models]};

middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]};
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]};
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]};
RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];

alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}
,u]

In[ ]:= Dimensions[alltliste]
Out[ ]:= {3556, 83}

```

```

In[ ]:= CM = {};
cm1 = Table[ Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[All, 1]], {kk, 1,
Length[p1]};
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[All, 2]], {kk, 1,
Length[p1]};

Do[
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]];
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]];
, {j, 1, Length[Transpose[cm2]]};

In[ ]:= ListPlot[{CM[[1]],CM[[2]]}]

In[ ]:= smliste={};
tliste = {};
Monitor[ Do[
pdfliste = {}; Do[

gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]])]^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
,{kk,1,2*14}];

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];

In[ ]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[ ]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[ ]:= bliste={};
Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt);
t1 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.90 &)]]][[1]] - 1];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.75 &)]]][[1]] - 1];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.5 &)]]][[1]] - 1];
t4 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.25 &)]]][[1]] - 1];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.10 &)]]][[1]] - 1];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}];
, {budget, 200, 4000, 100}]

In[ ]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};

In[ ]:= a=ListPlot[{Transpose[{bliste[[All,1]],bliste[[All,2]]}], Transpose[{bliste[[All, 1]], bliste[[All, 3]]}], Trans-
pose[{bliste[[All, 1]], bliste[[All, 4]]}], Transpose[{bliste[[All, 1]], bliste[[All, 5]]}], Transpose[{bliste[[All, 1]],

```

```

bliste[[All, 6]]]], Joined → True, AspectRatio → 1, PlotRange → {1, 4.0}, Axes → False, Frame → True, Frame-
Style → Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines → Automatic, FrameLabel →
{"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)", PlotLegends → Placed[{"10%
prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05, 0.7}], {0, 0.5}}]];
l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]}}];
GGD = Show[{a}

```

(\*RCB estimate for 2 carbon models, 14 ESMs and internal variability\*)

(\*Comparison plot\*)

(\*GGA = all climate - GGB = mean climate + 2 carbon - GGC = mean climate + mean carbon + internal - GGD = All climate + 2 carbon + internal\*)

```

Grid[{{
Show[GGA, PlotRange → {{0, 4000}, {1.1, 4.3}}, ImageSize → 380, Epilog → Inset[Style["a", 18], Scaled[{0.1,
0.9}]],
Show[GGB, PlotRange → {{0, 4000}, {1.1, 4.3}}, ImageSize → 380, Epilog → Inset[Style["b", 18], Scaled[{0.1,
0.9}]]],

```

```

{Show[GGC, PlotRange → {{0, 4000}, {1.1, 4.3}}, ImageSize → 380, Epilog → Inset[Style["c", 18], Scaled[{0.1,
0.9}]]],
Show[GGD, PlotRange → {{0, 4000}, {1.1, 4.3}}, ImageSize → 380, Epilog → Inset[Style["d", 18], Scaled[{0.1,
0.9}]]]}}]

```

(\*Grid plot \*)

## CODE FOR IMPLEMENTATION OF NON-LINEAR FRAMEWORK.

```
In[283]:= SetDirectory["OneDrive - UiT Office 365"];  
Z = Import["SSP_IAM_V2_201811.csv"];  
Z = Map[StringSplit[#, ", "] &, Z];
```

```
In[5]:= hh=157.65890684920566`+1.8942819330281027`zz+0.08520850267749702`zz2;
```

```
In[6]:= Z[[1]]  
Out[6]= {{MODEL, "SCENARIO", "REGION", "VARIABLE", "UNIT",  
2005, 2010, 2020, 2030, 2040, 2050, 2060, 2070, 2080, 2090, 2100}}
```

```
In[7]:= RR=Table[Z[[k]][[1]][[4]],{k,1,Length[Z]}];  
Union[RR]
```

```
Out[8]= {"Agricultural Demand|Crops", "Agricultural Demand|Crops|Energy",  
"Agricultural Demand|Livestock", "Agricultural Production|Crops|Energy",  
"Agricultural Production|Crops|Non-Energy", "Agricultural Production|Livestock", "Capacity|Electricity",  
"Capacity|Electricity|Biomass", "Capacity|Electricity|Coal", "Capacity|Electricity|Gas",  
"Capacity|Electricity|Geothermal", "Capacity|Electricity|Hydro", "Capacity|Electricity|Nuclear",  
"Capacity|Electricity|Oil", "Capacity|Electricity|Other", "Capacity|Electricity|Solar", "Capacity|Electricity|Solar|CSP",  
"Capacity|Electricity|Solar|PV", "Capacity|Electricity|Wind", "Capacity|Electricity|Wind|Offshore",  
"Capacity|Electricity|Wind|Onshore", "Consumption", "Diagnostics|MAGICC6|Concentration|CH4",  
"Diagnostics|MAGICC6|Concentration|CO2", "Diagnostics|MAGICC6|Concentration|N2O",  
"Diagnostics|MAGICC6|Forcing", "Diagnostics|MAGICC6|Forcing|Aerosol", "Diagnostics|MAGICC6|Forcing|CH4",  
"Diagnostics|MAGICC6|Forcing|CO2", "Diagnostics|MAGICC6|Forcing|F-Gases",  
"Diagnostics|MAGICC6|Forcing|Kyoto Gases", "Diagnostics|MAGICC6|Forcing|N2O",  
"Diagnostics|MAGICC6|Temperature|Global Mean", "Emissions|BC", "Emissions|CH4",  
"Emissions|CH4|Fossil Fuels and Industry", "Emissions|CH4|Land Use", "Emissions|CO",  
"Emissions|CO2", "Emissions|CO2|Carbon Capture and Storage",  
"Emissions|CO2|Carbon Capture and Storage|Biomass", "Emissions|CO2|Fossil Fuels and Industry",  
"Emissions|CO2|Land Use", "Emissions|F-Gases", "Emissions|Kyoto Gases", "Emissions|N2O",  
"Emissions|N2O|Land Use", "Emissions|NH3", "Emissions|NOx", "Emissions|OC", "Emissions|Sulfur",  
"Emissions|VOC", "Energy Service|Transportation|Freight", "Energy Service|Transportation|Passenger",  
"Final Energy", "Final Energy|Electricity", "Final Energy|Gases", "Final Energy|Heat",  
"Final Energy|Hydrogen", "Final Energy|Industry", "Final Energy|Liquids",  
"Final Energy|Residential and Commercial", "Final Energy|Solar", "Final Energy|Solids",  
"Final Energy|Solids|Biomass", "Final Energy|Solids|Biomass|Traditional", "Final Energy|Solids|Coal",  
"Final Energy|Transportation", "GDP|PPP", "Harmonized Emissions|BC",  
"Harmonized Emissions|CH4|Fossil Fuels and Industry", "Harmonized Emissions|CH4|Land Use",  
"Harmonized Emissions|CO", "Harmonized Emissions|CO2|Fossil Fuels and Industry",  
"Harmonized Emissions|CO2|Land Use", "Harmonized Emissions|F-Gases", "Harmonized Emissions|Kyoto Gases",  
"Harmonized Emissions|NH3", "Harmonized Emissions|NOx", "Harmonized Emissions|OC",  
"Harmonized Emissions|Sulfur", "Harmonized Emissions|VOC", "Land Cover|Built-up Area",  
"Land Cover|Cropland", "Land Cover|Forest", "Land Cover|Pasture", "Population", "Price|Carbon", "Primary Energy",  
"Primary Energy|Biomass", "Primary Energy|Biomass|Traditional", "Primary Energy|Biomass|w/ CCS",  
"Primary Energy|Biomass|w/o CCS", "Primary Energy|Coal", "Primary Energy|Coal|w/ CCS",  
"Primary Energy|Coal|w/o CCS", "Primary Energy|Fossil", "Primary Energy|Fossil|w/ CCS",  
"Primary Energy|Fossil|w/o CCS", "Primary Energy|Gas", "Primary Energy|Gas|w/ CCS",  
"Primary Energy|Gas|w/o CCS", "Primary Energy|Geothermal", "Primary Energy|Hydro",  
"Primary Energy|Non-Biomass Renewables", "Primary Energy|Nuclear", "Primary Energy|Oil",  
"Primary Energy|Oil|w/ CCS", "Primary Energy|Oil|w/o CCS", "Primary Energy|Other",  
"Primary Energy|Secondary Energy Trade", "Primary Energy|Solar", "Primary Energy|Wind",  
"Secondary Energy|Electricity", "Secondary Energy|Electricity|Biomass",
```

```
"Secondary Energy|Electricity|Biomass|w/ CCS", "Secondary Energy|Electricity|Biomass|w/o CCS",
"Secondary Energy|Electricity|Coal", "Secondary Energy|Electricity|Coal|w/ CCS",
"Secondary Energy|Electricity|Coal|w/o CCS", "Secondary Energy|Electricity|Gas",
"Secondary Energy|Electricity|Gas|w/ CCS", "Secondary Energy|Electricity|Gas|w/o CCS",
"Secondary Energy|Electricity|Geothermal", "Secondary Energy|Electricity|Hydro",
"Secondary Energy|Electricity|Non-Biomass Renewables", "Secondary Energy|Electricity|Nuclear",
"Secondary Energy|Electricity|Oil", "Secondary Energy|Electricity|Solar", "Secondary Energy|Electricity|Wind",
"Secondary Energy|Gases", "Secondary Energy|Gases|Biomass", "Secondary Energy|Gases|Coal",
"Secondary Energy|Gases|Natural Gas", "Secondary Energy|Heat", "Secondary Energy|Heat|Geothermal",
"Secondary Energy|Hydrogen", "Secondary Energy|Hydrogen|Biomass",
"Secondary Energy|Hydrogen|Biomass|w/ CCS", "Secondary Energy|Hydrogen|Biomass|w/o CCS",
"Secondary Energy|Hydrogen|Electricity", "Secondary Energy|Liquids", "Secondary Energy|Liquids|Biomass",
"Secondary Energy|Liquids|Biomass|w/ CCS", "Secondary Energy|Liquids|Biomass|w/o CCS",
"Secondary Energy|Liquids|Coal", "Secondary Energy|Liquids|Coal|w/ CCS",
"Secondary Energy|Liquids|Coal|w/o CCS", "Secondary Energy|Liquids|Gas",
"Secondary Energy|Liquids|Gas|w/ CCS", "Secondary Energy|Liquids|Gas|w/o CCS",
"Secondary Energy|Liquids|Oil", "Secondary Energy|Solids", "VARIABLE")
```

```
In[9]:= RRR=Table[Z[[k]][[1]][[3]],{k,1,Length[Z]}];
Union[RRR]
```

```
Out[10]= {"R5.2ASIA", "R5.2LAM", "R5.2MAF", "R5.2OECD", "R5.2REF", "REGION", "World"}
```

```
In[11]:= co2pos1=Position[RR,_(#=="\Emissions|CO2|Fossil Fuels and Industry\"&)];
co2pos2 = Position[RR,_(#=="\Emissions|CO2|Land Use\"&)];
co2pos3 = Position[RRR,_(#=="\World\"&)];
```

```
In[14]:= ppos1=Intersection[co2pos3,co2pos1];
ppos2 = Intersection[co2pos3, co2pos2];
```

```
In[16]:= Extract[Z,co2pos1][[1]]
```

```
Out[16]= {{AIM/CGE, "SSP1-19", "R5.2ASIA", "Emissions|CO2|Fossil Fuels and Industry", "Mt CO2/yr", 8985.6725,
10008.8152, 11790.747500000001,
6131.6627, 3271.4353000000006, 1678.8029, 638.87, 259.4755, 82.29590000000003, -7.935300000000105, -
103.9171}}
```

```
In[17]:= em1=ToExpression[Map[Drop[Flatten[#],7]&,Extract[Z,ppos1]]];
em2 = ToExpression[Map[Drop[Flatten[#], 7] &, Extract[Z, ppos2]]];
```

```
In[19]:= ListPlot[em1,PlotRange→All,Joined→True]
```

```
In[20]:= Length[em1]
Out[20]= 127
```

```
In[21]:= emissions=Map[#[[1;;2]]&,ToExpression[Map[StringSplit[#] &, Drop[ReadList["emissionsCO2.txt",
String], 31]]];
emissions = Table[{emissions[[i, 1]], (44 / 12) * emissions[[i, 2]] / 1000.}, {i, 1, Length[emissions]}];
ListPlot[emissions, Joined → True, PlotStyle → {Black, Thick}, PlotRange → All, Axes → False, Frame → True,
FrameStyle → Directive[14, Black],
FrameLabel → {"year", "CO2 emissions (Gt CO2/yr)"}]
(*historical emissions*)
```

```
EM = Join[emissions, {{2018, 37.1}}];
data2 = Table[Prepend[Table[{t, Interpolation[Join[EM, Transpose[{{2030, 2040, 2050, 2060, 2070, 2080,
2090, 2100}, 0.001 * Drop[em1[[k]], 1]]][t]], {t, 1751, 2100}], {1750, 0}], {k, 1, Length[em1]}]; totliste = Ta-
ble[data2[[k]][[All, 2]], {k, 1, Length[data2]}];
PLAll = ListPlot[data2, Joined → True, Frame → True, FrameStyle → Directive[Black, 14], PlotRange → All,
FrameLabel → {None, "CO2 emissions (Gt CO2)"}]
positivepaths = Table[DeleteCases[Map[# * UnitStep[#] &, totliste[[k]][[269 ;; 351]], _? (# == 0 &)], {k, 1,
Length[totliste]}]; ListPlot[positivepaths, Joined → True, PlotRange → All]
(*Before removal of exceedance scenarios*)
```

```
In[29]:= RCBliste2=Map[Plus@@#&,positivepaths];
```

```
In[175]:= p1=Position[RCBliste2,_?{#<3300&}];
```

```
In[31]:= maxtemp=Map[Max[#]&,templiste];  
maxtemp2 = Map[Max[#] &, utempliste];  
maxtemp3 = Map[Max[#] &, lowtempliste];
```

```
PL1 = ListPlot[Extract[data2, p1], Joined → True, Frame → True, FrameStyle → Directive[Black, 14], PlotRange  
→ All];  
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];  
FFC = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["", 18],  
Scaled[{0.1, 0.9}]]]  
(*After removal of exceedance scenarios*)
```

```
In[37]:= PL1=ListPlot[data2[[1]],Joined→True,Frame→True, FrameStyle → Directive[Black, 14], PlotStyle →  
Darker[Blue]];  
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];  
FFA = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["a", 18],  
Scaled[{0.1, 0.9}]]]
```

```
In[40]:= n=Length[data2[[1]]];  
futuretime = 2100 - 2020;  
τmetan = 12.4;
```

```
In[43]:= (* Carbon model *)
```

```
τ1=1;  
τ2=10;  
τ3 = 100;  
τ4 = 1000;  
c1mean = 0.152;  
c2mean = 0.246;  
c4mean = 0.134;  
c5mean = 0.194;
```

```
Gmean = (12/44)*0.47*(c1mean * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]  
+ c2mean * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1mean - c2mean - c4mean -  
c5mean)* Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]  
+ c4mean * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + Table[c5mean * UnitStep[i - j], {i, 1, n}, {j,  
1, n}]);
```

```
In[52]:= (* Carbon models *)
```

```
c1upper = 0.11;  
c2upper = 0.212;  
c4upper = 0.106;  
c5upper = 0.262;  
c1lower = 0.18;  
c2lower = 0.296;  
c4lower = 0.122;  
c5lower = 0.148;
```

```
Glower = (12 / 44) * 0.47 * (c1lower * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
c2lower * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
(1 - c1lower - c2lower - c4lower - c5lower) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
c4lower * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
Table[c5lower * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);
```

```
Gupper = (12/44)*0.47*(c1upper * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
c2upper * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
(1 - c1upper - c2upper - c4upper - c5upper) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c4up-  
per * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
Table[c5upper * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);
```

```
In[62]:=
(*Optimal Estimation of Stochastic Energy Balance Model Parameters *)
```

```
In[63]:= (* Climate models *)
models = ReadList["CMIP5parameters.txt", String];
models = Delete[models, {{5}, {12}}];
boxes = StringSplit[models][[All, 2]];
Klimaliste = {};
Γliste = {};
σ2liste = {};
Monitor[
Do[
```

```
Clear[A];
modelnr = p; If[boxes[[p]] == "2",
```

```
{C1, C2, κ1, κ2, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];
```

```
A = {{-(κ1+ κ2) /C1, κ2/C1}, {κ2/C2, -κ2/C2}};
g = (MatrixExp[t A].{1 / C1, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];
```

```
If[ boxes[[p]] == "3",
```

```
{C1, C2, C3, κ1, κ2, κ3, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];
```

```
A = {{-(κ1+ κ2) /C1, κ2/C1, 0}, {κ2/C2, -(κ2+κ3)/C2, κ3/C2},{0, κ3/C3, - κ3/C3}};
```

```
g = (MatrixExp[t A].{1 / C1, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];
```

```
If [ boxes[[p]] == "4",
```

```
{C1, C2, C3, C4, κ1, κ2, κ3, κ4, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]]], 2]];
```

```
A = {{-(κ1+κ2)/C1, κ2/C1, 0, 0} , {κ2/C2, -(κ2+κ3)/C2, κ3/C2, 0},{ 0, κ3/C3, -(κ3+κ4)/C3, κ4/C3},
{0, 0, κ4/C4, -κ4/C4}};
```

```
g = (MatrixExp[t A].{1 / C1, 0, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];
```

```
Γliste = Append[Γliste, Γ];
σ2liste = Append[σ2liste, σ2];
, {p, 1, Length[models]}
];
, {p, boxes[[p]]}
];
```

```
(*Nonlin parameter changes*)
```

```

styrke = 1; (*w/m^2*)
terskel = 2; (*grader*)
bratthet = 0.5;
Plot[styrke * 0.5 * (1 + Tanh[(T - terskel) / bratthet]), {T, 0, 4}]
(*Test plot to visualise the non-linear forcing*)

```

```

RCBliste={};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
Δfaeroliste = {};
Δfghgliste = {};
Δfliste = {};
noiseliste = {};

```

```

Monitor[ Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 11.9 tunes 2019 methane emmissions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

```

```

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.34 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);

```

```

Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfaer = -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]];
Δfaer2 = Drop[Δfaer, Length[EM]];
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfliste = Append[Δfliste, Δf];
Δfaeroliste = Append[Δfaeroliste, Δfaer];
Δfghgliste = Append[Δfghgliste, Δfco2 + Δfmetan];

```

```

Tliste = {}; Do[
T2 = Klimaliste[[p]].Δf;

```

```

(*nonlin loop*)
Do[
T2 = Klimaliste[[p]].(Δf + styrke * 0.5 * (1 + Tanh[(T2 - terskel) / bratthet]));, {10}; (*number of iterations*)
noise = σ2liste[[p]] * (Klimaliste[[p]].RandomReal[NormalDistribution[0, 1], Length[Δf]]);
noise = Drop[noise, 268 - 20];
T2 = T2 * (Tliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268]; T2=1.1+T2-T2[[1]];
noiseliste = Append[noiseliste, noise];
Tliste = Append[Tliste, T2];
, {p, 1, Length[models]};

```

```

middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]};
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1, Length[Trans-
pose[Tliste]]};

```

```

lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1, Length[Trans-
pose[Tliste]]};

```



```

RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot]; alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower]; , {u, 1, Length[data2]}]
, u];

```

```

In[258]:= Length[noise]
Out[ ]:= 103

```

```

In[259]:= Length[T2]
Out[ ]:= 83

```

```

In[260]:= window=10;
noisaliste2 = Table[MovingAverage[noisaliste[[i]], window][[1 ;; Length[T2]]], {i, 1, Length[noisaliste]}];
noisaliste2 = Transpose[Partition[noisaliste2, 14]];

```

```

In[263]:= Length[noisaliste2]
Out[ ]:= 14

```

```

In[264]:= Dimensions[noisaliste2]
Out[ ]:= {14, 127, 83}

```

```

In[265]:= Length[noisaliste2[[1]]]
Out[ ]:= 127

```

```

PLNoise = ListPlot[Map[Transpose[{2018 + Range[Length[templiste[[1]]], #}] &, noisaliste2[[3]]], Joined → True];
FFNoise = Show[PLNoise, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], Joined → True, FrameLabel → {None, "Internal temperature variability(°C)"}, Epilog → Inset[Style["", 18], Scaled[{0.1, 0.9}]]]

```

```

FFE=ListPlot[Map[Transpose[{2018+Range[Length[templiste[[1]]],#}]&, Extract[templiste, p1]], PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "GMST increase (°C)"}, Epilog → Inset[Style["e", 18], Scaled[{0.1, 0.9}]]]
PL1 = ListPlot[Map[Transpose[{1749 + Range[Length[Δfaeroliste[[1]]], #}] &, Extract[Δfaeroliste, p1]], Joined → True];
PL2 = ListPlot[Map[Transpose[{1749 + Range[Length[Δfaghliste[[1]]], #}] &, Extract[Δfaghliste, p1]], Joined → True];
FFD = Show[PL1, PL2], PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "Forcing (W/m²)"}, Epilog → Inset[Style["d", 18], Scaled[{0.1, 0.9}]]]

```

```

In[267]:=

```

```

In[271]:=modellfarger={,,,,,,,,,,,,,};
In[272]:= pan=LineLegend[modellfarger,Map[StringSplit[#]&,models][[All,1]]]

```

```

In[273]:= FFB=ListPlot[Map[Transpose[{2018+Range[Length[alltliste[[1]]],#}]&, alltliste[[1 ;; 14]]], PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None, "GMST increase (°C)"}, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]], PlotStyle → Map[{#} &, modellfarger]]

```

```

In[ ]:= Grid[{{Show[FFA,ImageSize→400],Show[FFB,ImageSize→400],pan}}]

```

```

In[ ]:= Grid[{{Show[FFC,ImageSize→400, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]]], Show[FFD, ImageSize → 400, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]]], Show[FFE, ImageSize → 400, Epilog → Inset[Style["c", 18], Scaled[{0.1, 0.9}]]]}}]

```

```

In[ ]:= maxtemp=Map[Max[#]&,templiste];
maxtemp2 = Map[Max[#] &, uptempliste];
maxtemp3 = Map[Max[#] &, lowtempliste];

```

```

In[]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1], AspectRatio → 1, PlotRange → All];
PL3 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Red];
PL4 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Red];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz]; PL2 = Plot[gg, {zz, 1.2, 3.5}];
gg2 = Fit[Extract[Transpose[{maxtemp2, RCBliste2}], p1], {zz, 1}, zz]; PL4 = Plot[gg2, {zz, 1.2, 3}];
Show[{PL1, PL2}, PlotRange → All]

```

```

In[]:= pairs=Extract[Transpose[{maxtemp,RCBliste2}],p1];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];

```

```

In[]:= pdf=(PDF[NormalDistribution[gg,σf[zz]]][p])/ .zz→2.0;
pdf2 = (PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → 3.0;
Plot[{pdf, pdf2}, {p, 0, 5200}, PlotRange → All]

```

```

PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → {{1, 4}, {0, 4000}}, PlotStyle → Darker[Blue]];

```

```

PL2 = Plot[gg, {zz, 1, 3.5}, PlotStyle → Darker[Blue]];
l1 = Graphics[{Black, Line[{{2.0, 0}, {2.0, gg /. zz → 2.0}}]};
l2 = Graphics[{Black, Line[{{2.0, gg /. zz → 2.0}, {1, gg /. zz → 2.0}}]}; inset = ParametricPlot[{1 + 60 * pdf, p}, {p, 400, 1200}, Axes → False, PlotStyle → {Black, Thickness[0.01]}]; (*{p, budget low, budget high}*)
l11 = Graphics[{Black, Line[{{3.0, 0}, {3.0, gg /. zz → 3.0}}]};
l22 = Graphics[{Black, Line[{{3.0, gg /. zz → 3.0}, {1, gg /. zz → 3.0}}]}; inset2 = ParametricPlot[{1 + 60 * pdf2, p}, {p, 1900, 2700}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];(*{p, budget low, budget high}*)

```

```

FA = Show[{PL1, PL2, l1, l2, l11, l22, inset, inset2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {"Global temperature increase (°C)", "Carbon budget after 2018 (Gt CO2)"}, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]], ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]
(*TCRE for 1 ESM, 86 scenarios*)

```

## ALL CLIMATE MODELS

(\*Just to get the TCRE plots\*)

```

maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, 5]]; PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Black];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 3.5}, PlotStyle → Black];
QL1 = Show[{PL1, PL2}, PlotRange → All];

```

```

maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, 7]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle → Darker[Red]];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 3.5}, PlotStyle → Darker[Red]];
QL2 = Show[{PL1, PL2}, PlotRange → All];
FB = Show[{QL1, QL2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {"Global temperature increase (°C)", "Carbon budget after 2018 (Gt CO2)"}, Epilog → {Inset[Style["b", 18], Scaled[{0.1, 0.9}]], Inset[LineLegend[{Black, Darker[Red]}, {"CSIRO-Mk3.6.0", "GFDL-ESM2M"}], Scaled[{0.7, 0.3}]]}, ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]

```

```

In[]:= Grid[{{FA,FB}}]
(*Grid plot of TCRE's *)

```

## ALL CLIMATE MODELS, 2 CARBON MODELS AND INTERNAL VARIABILITY

```

RCBliste = {};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};

Monitor[ Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;
meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emissions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];

Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
(* aerosols *)
Δfaer = -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]];
Δfaer2 = Drop[Δfaer, Length[EM]];
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;

Tliste = {};
Do[
T2 = Klimaliste[[p]].Δfupper;

(*nonlin loop*)
Do[
T2 = Klimaliste[[p]].
(Δfupper + styrke * 0.5 * (1 + Tanh[(T2 - terskel) / bratthet]));
, {10}]; (*number of iterations*)

T2 = T2 * (Tliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.1+T2-T2[[1]];
Tliste = Append[Tliste, T2 + StandardDeviation[Flatten[noiselist2[[p]]]];

(*T2=Klimaliste[[p]].Δfupper;
T2=T2*(Tliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.1+T2-T2[[1]]; Tliste=Append[Tliste,T2-StandardDeviation[Flatten[noiselist2[[p]]]];*)

```

```
(*T2=Klimaliste[[p]].Δflower;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.0+T2-T2[[1]]; Tliste=Append[Tliste,T2+StandardDeviation[Flatten[noisaliste2[[p]]]];*)
```

```
T2 = Klimaliste[[p]].Δflower;
(*nonlin loop*)
Do[
T2 = Klimaliste[[p]].
(Δflower + styrke * 0.5 * (1 + Tanh[(T2 - terskel) / bratthet]));
, {10}]; (*number of iterations*)
```

```
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268]; T2=1.0+T2-T2[[1]];
```

```
Tliste = Append[Tliste, T2 - StandardDeviation[Flatten[noisaliste2[[p]]]]];
, {p, 1, Length[models]}];
```

```
middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]};
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1, Length[Trans-
pose[Tliste]]};
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1, Length[Trans-
pose[Tliste]]}; RCB = Plus @@ Drop[tot, 270];
```

```
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}
,u]
```

```
In[ ]:= Dimensions[alltliste]
Out[ ]:= {3556, 83}
```

```
In[ ]:= CM = {};
cm1 = Table[ Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[ All, 1]], {kk, 1,
Length[p1]}];
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[ kk]], 2][[All, 2]], {kk, 1,
Length[p1]}];
```

```
Do[
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]];
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]];
, {j, 1, Length[Transpose[cm2]]}];
```

```
In[ ]:= ListPlot[{CM[[1]],CM[[2]]}]
```

```
In[ ]:= smliste={};
tliste = {};
```

```
Monitor[ Do[
pdfliste = {}; Do[
```

```
gg = Fit[Map[Reverse[#] &, CM[[kk]]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
```

```

pdfliste = Append[pdfliste, pdf];
,{kk,1,2*14};

g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01};
, target];

In[ ]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[ ]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

In[ ]:= bliste={};

Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt);

t1= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.90 &)]]][[1]] - 1];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.75 &)]]][[1]] - 1];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.5 &)]]][[1]] - 1];
t4= tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.25 &)]]][[1]] - 1];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.10 &)]]][[1]] - 1];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}];
, {budget, 200, 4000, 100}]

In[ ]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};
a = ListPlot[{Transpose[{bliste[[All, 1]], bliste[[All, 2]]}],

Transpose[{bliste[[All, 1]], bliste[[All, 3]]}], Transpose[{bliste[[All, 1]], bliste[[All, 4]]}], Trans-
pose[{bliste[[All, 1]], bliste[[All, 5]]}], Transpose[{bliste[[All, 1]], bliste[[All, 6]]}], Joined → True, AspectRatio
→ 1, PlotRange → {1, 4.0}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], PlotStyle → Ta-
ble[farger[[i]], {i, 1, 5}], GridLines → Automatic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maxi-
mum temperature increase (°C)"}, PlotLegends → Placed[{"10% prob.", "25% prob.", "even chance", "75%
prob.", "90% prob."}, {Scaled[{0.05, 0.7}], {0, 0.5}}]; l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]};
GGD = Show[{a}, PlotRange → {{0, 4000}, {1, 4.3}}]

(*RCB estimates for non-linear framework *)

```

## CODE FOR IMPLEMENTATION OF THE ARCTIC AMPLIFICATION FACTOR

```
In[283]:= SetDirectory["OneDrive - UiT Office 365"];  
Z = Import["SSP_IAM_V2_201811.csv"];  
Z = Map[StringSplit[#, ", "] &, Z];
```

```
In[5]:= hh=157.65890684920566`+1.8942819330281027`zz+0.08520850267749702`zz2;
```

```
In[6]:= Z[[1]]  
Out[6]= {{MODEL, "SCENARIO", "REGION", "VARIABLE", "UNIT",  
2005, 2010, 2020, 2030, 2040, 2050, 2060, 2070, 2080, 2090, 2100}}
```

```
In[7]:= RR=Table[Z[[k]][[1]][[4]],{k,1,Length[Z]}];  
Union[RR]
```

```
Out[8]= {"Agricultural Demand|Crops", "Agricultural Demand|Crops|Energy",  
"Agricultural Demand|Livestock", "Agricultural Production|Crops|Energy",  
"Agricultural Production|Crops|Non-Energy", "Agricultural Production|Livestock", "Capacity|Electricity",  
"Capacity|Electricity|Biomass", "Capacity|Electricity|Coal", "Capacity|Electricity|Gas",  
"Capacity|Electricity|Geothermal", "Capacity|Electricity|Hydro", "Capacity|Electricity|Nuclear",  
"Capacity|Electricity|Oil", "Capacity|Electricity|Other", "Capacity|Electricity|Solar", "Capacity|Electricity|Solar|CSP",  
"Capacity|Electricity|Solar|PV", "Capacity|Electricity|Wind", "Capacity|Electricity|Wind|Offshore",  
"Capacity|Electricity|Wind|Onshore", "Consumption", "Diagnostics|MAGICC6|Concentration|CH4",  
"Diagnostics|MAGICC6|Concentration|CO2", "Diagnostics|MAGICC6|Concentration|N2O",  
"Diagnostics|MAGICC6|Forcing", "Diagnostics|MAGICC6|Forcing|Aerosol", "Diagnostics|MAGICC6|Forcing|CH4",  
"Diagnostics|MAGICC6|Forcing|CO2", "Diagnostics|MAGICC6|Forcing|F-Gases",  
"Diagnostics|MAGICC6|Forcing|Kyoto Gases", "Diagnostics|MAGICC6|Forcing|N2O",  
"Diagnostics|MAGICC6|Temperature|Global Mean", "Emissions|BC", "Emissions|CH4",  
"Emissions|CH4|Fossil Fuels and Industry", "Emissions|CH4|Land Use", "Emissions|CO",  
"Emissions|CO2", "Emissions|CO2|Carbon Capture and Storage",  
"Emissions|CO2|Carbon Capture and Storage|Biomass", "Emissions|CO2|Fossil Fuels and Industry",  
"Emissions|CO2|Land Use", "Emissions|F-Gases", "Emissions|Kyoto Gases", "Emissions|N2O",  
"Emissions|N2O|Land Use", "Emissions|NH3", "Emissions|NOx", "Emissions|OC", "Emissions|Sulfur",  
"Emissions|VOC", "Energy Service|Transportation|Freight", "Energy Service|Transportation|Passenger",  
"Final Energy", "Final Energy|Electricity", "Final Energy|Gases", "Final Energy|Heat",  
"Final Energy|Hydrogen", "Final Energy|Industry", "Final Energy|Liquids",  
"Final Energy|Residential and Commercial", "Final Energy|Solar", "Final Energy|Solids",  
"Final Energy|Solids|Biomass", "Final Energy|Solids|Biomass|Traditional", "Final Energy|Solids|Coal",  
"Final Energy|Transportation", "GDP|PPP", "Harmonized Emissions|BC",  
"Harmonized Emissions|CH4|Fossil Fuels and Industry", "Harmonized Emissions|CH4|Land Use",  
"Harmonized Emissions|CO", "Harmonized Emissions|CO2|Fossil Fuels and Industry",  
"Harmonized Emissions|CO2|Land Use", "Harmonized Emissions|F-Gases", "Harmonized Emissions|Kyoto Gases",  
"Harmonized Emissions|NH3", "Harmonized Emissions|NOx", "Harmonized Emissions|OC",  
"Harmonized Emissions|Sulfur", "Harmonized Emissions|VOC", "Land Cover|Built-up Area",  
"Land Cover|Cropland", "Land Cover|Forest", "Land Cover|Pasture", "Population", "Price|Carbon", "Primary Energy",  
"Primary Energy|Biomass", "Primary Energy|Biomass|Traditional", "Primary Energy|Biomass|w/ CCS",  
"Primary Energy|Biomass|w/o CCS", "Primary Energy|Coal", "Primary Energy|Coal|w/ CCS",  
"Primary Energy|Coal|w/o CCS", "Primary Energy|Fossil", "Primary Energy|Fossil|w/ CCS",  
"Primary Energy|Fossil|w/o CCS", "Primary Energy|Gas", "Primary Energy|Gas|w/ CCS",  
"Primary Energy|Gas|w/o CCS", "Primary Energy|Geothermal", "Primary Energy|Hydro",  
"Primary Energy|Non-Biomass Renewables", "Primary Energy|Nuclear", "Primary Energy|Oil",  
"Primary Energy|Oil|w/ CCS", "Primary Energy|Oil|w/o CCS", "Primary Energy|Other",  
"Primary Energy|Secondary Energy Trade", "Primary Energy|Solar", "Primary Energy|Wind",  
"Secondary Energy|Electricity", "Secondary Energy|Electricity|Biomass",  
"Secondary Energy|Electricity|Biomass|w/ CCS", "Secondary Energy|Electricity|Biomass|w/o CCS",  
"Secondary Energy|Electricity|Coal", "Secondary Energy|Electricity|Coal|w/ CCS",  
"Secondary Energy|Electricity|Coal|w/o CCS", "Secondary Energy|Electricity|Gas",
```

```
"Secondary Energy|Electricity|Gas|w/ CCS", "Secondary Energy|Electricity|Gas|w/o CCS",
"Secondary Energy|Electricity|Geothermal", "Secondary Energy|Electricity|Hydro",
"Secondary Energy|Electricity|Non-Biomass Renewables", "Secondary Energy|Electricity|Nuclear",
"Secondary Energy|Electricity|Oil", "Secondary Energy|Electricity|Solar", "Secondary Energy|Electricity|Wind",
"Secondary Energy|Gases", "Secondary Energy|Gases|Biomass", "Secondary Energy|Gases|Coal",
"Secondary Energy|Gases|Natural Gas", "Secondary Energy|Heat", "Secondary Energy|Heat|Geothermal",
"Secondary Energy|Hydrogen", "Secondary Energy|Hydrogen|Biomass",
"Secondary Energy|Hydrogen|Biomass|w/ CCS", "Secondary Energy|Hydrogen|Biomass|w/o CCS",
"Secondary Energy|Hydrogen|Electricity", "Secondary Energy|Liquids", "Secondary Energy|Liquids|Biomass",
"Secondary Energy|Liquids|Biomass|w/ CCS", "Secondary Energy|Liquids|Biomass|w/o CCS",
"Secondary Energy|Liquids|Coal", "Secondary Energy|Liquids|Coal|w/ CCS",
"Secondary Energy|Liquids|Coal|w/o CCS", "Secondary Energy|Liquids|Gas",
"Secondary Energy|Liquids|Gas|w/ CCS", "Secondary Energy|Liquids|Gas|w/o CCS",
"Secondary Energy|Liquids|Oil", "Secondary Energy|Solids", "VARIABLE"}

```

```
In[9]:= RRR=Table[Z[[k]][[1]][[3]],{k,1,Length[Z]}];
Union[RRR]
```

```
Out[10]= {"R5.2ASIA", "R5.2LAM", "R5.2MAF", "R5.2OECD", "R5.2REF", "REGION", "World"}
```

```
In[11]:= co2pos1=Position[RR,?{#=="Emissions|CO2|FossilFuelsandIndustry\ ""&}];
co2pos2 = Position[RR, _ ? {# == "Emissions|CO2|Land Use\ "" &}];
co2pos3 = Position[RRR, _ ? {# == "World\ "" &}];
```

```
In[14]:= ppos1=Intersection[co2pos3,co2pos1];
ppos2 = Intersection[co2pos3, co2pos2];
```

```
In[16]:= Extract[Z,co2pos1][[1]]
```

```
Out[16]= {{AIM/CGE, "SSP1-19", "R5.2ASIA", "Emissions|CO2|Fossil Fuels and Industry", "Mt CO2/yr", 8985.6725,
10008.8152, 11790.747500000001,
6131.6627, 3271.4353000000006, 1678.8029, 638.87, 259.4755, 82.29590000000003, -7.935300000000105, -
103.9171}}
```

```
In[17]:= em1=ToExpression[Map[Drop[Flatten[#,7]&,Extract[Z,ppos1]]];
em2 = ToExpression[Map[Drop[Flatten[#, 7] &, Extract[Z, ppos2]]];
```

```
In[19]:= ListPlot[em1,PlotRange->All,Joined->True]
```

```
In[20]:= Length[em1]
```

```
Out[20]= 127
```

```
In[21]:= emissions=Map[#[[1;;2]]&,ToExpression[Map[StringSplit[#] &, Drop[ReadList["emissionsCO2.txt",
String], 31]]];
emissions = Table[{emissions[[i, 1]], (44 / 12) * emissions[[i, 2]] / 1000.}, {i, 1, Length[emissions]};
ListPlot[emissions, Joined -> True, PlotStyle -> {Black, Thick}, PlotRange -> All, Axes -> False, Frame -> True,
FrameStyle -> Directive[14, Black],
FrameLabel -> {"year", "CO2 emissions (Gt CO/yr)"}]
(*historical emissions*)
```

```
EM = Join[emissions, {{2018, 37.1}}];
data2 = Table[Prepend[Table[{t, Interpolation][Join[EM, Transpose[{{2030, 2040, 2050, 2060, 2070, 2080,
2090, 2100}, 0.001 * Drop[em1[[k], 1]]]]][t]], {t, 1751, 2100}], {1750, 0}], {k, 1, Length[em1]}; totliste = Ta-
ble[data2[[k]][[All, 2]], {k, 1, Length[data2]};
PLAll = ListPlot[data2, Joined -> True, Frame -> True, FrameStyle -> Directive[Black, 14], PlotRange -> All,
FrameLabel -> {None, "CO2 emissions (Gt CO2)"}]
```

```
positivepaths = Table[DeleteCases[Map[# * UnitStep[#] &, totliste[[k]][[269 ;; 351]]], _ ? {# == 0 &}], {k, 1,
Length[totliste]}; ListPlot[positivepaths, Joined -> True, PlotRange -> All]
(*Before removal of exceedance scenarios*)
```

```
In[29]:= RCBliste2=Map[Plus@@#&,positivepaths];
```

```
In[175]:= p1=Position[RCBliste2,?(#<3300&)];
```

```
In[31]:= maxtemp=Map[Max[#]&,templiste];  
maxtemp2 = Map[Max[#] &, uptempliste];  
maxtemp3 = Map[Max[#] &, lowtempliste];
```

```
PL1 = ListPlot[Extract[data2, p1], Joined → True, Frame → True, FrameStyle → Directive[Black, 14], PlotRange  
→ All];  
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];  
FFC = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["", 18],  
Scaled[{0.1, 0.9}]]]  
(*After removal of exceedance scenarios*)
```

```
In[37]:= PL1=ListPlot[data2[[1]],Joined→True,Frame→True, FrameStyle → Directive[Black, 14], PlotStyle →  
Darker[Blue]];  
PL2 = ListPlot[EM, PlotStyle → Black, Joined → True];  
FFA = Show[{PL1, PL2}, FrameLabel → {None, "CO2 emissions (Gt CO2)"}, Epilog → Inset[Style["a", 18],  
Scaled[{0.1, 0.9}]]]
```

```
In[40]:= n=Length[data2[[1]]];  
futuretime = 2100 - 2020;  
τmetan = 12.4;
```

```
In[43]:= (* Carbon model *)  
τ1=1;  
τ2=10;  
τ3 = 100;  
τ4 = 1000;  
c1mean = 0.152;  
c2mean = 0.246;  
c4mean = 0.134;  
c5mean = 0.194;
```

```
Gmean = (12/44)*0.47*(c1mean * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]  
+ c2mean * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + (1 - c1mean - c2mean - c4mean -  
c5mean)* Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]  
+ c4mean * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + Table[c5mean * UnitStep[i - j], {i, 1, n}, {j,  
1, n}]);
```

```
In[52]:= (* Carbon models *)  
c1upper = 0.11;  
c2upper = 0.212;  
c4upper = 0.106;  
c5upper = 0.262;  
c1lower = 0.18;  
c2lower = 0.296;  
c4lower = 0.122;  
c5lower = 0.148;
```

```
Glower = (12 / 44) * 0.47 * (c1lower * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
c2lower * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
(1 - c1lower - c2lower - c4lower - c5lower) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
c4lower * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
Table[c5lower * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);
```

```
Gupper = (12/44) * 0.47 * (c1upper * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
c2upper * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
(1 - c1upper - c2upper - c4upper - c5upper) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c4up-  
per * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +  
Table[c5upper * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);
```

```
In[62]:=  
(*Optimal Estimation of Stochastic Energy Balance Model Parameters *)
```



```

In[63]:= (* Climate models *)
models = ReadList["CMIP5parameters.txt", String];
models = Delete[models, {{5}, {12}}];
boxes = StringSplit[models][[All, 2]];
Klimaliste = {};
GammaListe = {};
sigma2Liste = {};
Monitor[
Do[

Clear[A];
modelnr = p; If[boxes[[p]] == "2",

{C1, C2, kappa1, kappa2, sigma1, Gamma, sigma2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];

A = {{-(kappa1 + kappa2) / C1, kappa2 / C1}, {kappa2 / C2, -kappa2 / C2}};
g = (MatrixExp[t A].{1 / C1, 0})[[1]];
Gklima = Table[Chop[(g /. t -> (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If[ boxes[[p]] == "3",

{C1, C2, C3, kappa1, kappa2, kappa3, sigma1, Gamma, sigma2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];

A = {{-(kappa1 + kappa2) / C1, kappa2 / C1, 0}, {kappa2 / C2, -(kappa2 + kappa3) / C2, kappa3 / C2}, {0, kappa3 / C3, -kappa3 / C3}};

g = (MatrixExp[t A].{1 / C1, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t -> (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If [ boxes[[p]] == "4",

{C1, C2, C3, C4, kappa1, kappa2, kappa3, kappa4, sigma1, Gamma, sigma2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];

A = {{-(kappa1 + kappa2) / C1, kappa2 / C1, 0, 0}, {kappa2 / C2, -(kappa2 + kappa3) / C2, kappa3 / C2, 0}, {0, kappa3 / C3, -(kappa3 + kappa4) / C3, kappa4 / C3},
{0, 0, kappa4 / C4, -kappa4 / C4}};

g = (MatrixExp[t A].{1 / C1, 0, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t -> (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

GammaListe = Append[GammaListe, Gamma];
sigma2Liste = Append[sigma2Liste, sigma2];
, {p, 1, Length[models]}
];
, {p, boxes[[p]]}
];

```

(\*Nonlin parameter changes\*)

```

styrke = 1; (*w/m^2*)
terskel = 2; (*grader*)
bratthet = 0.5;

```

```
Plot[styrke * 0.5 * (1 + Tanh[(T - terskel) / bratthet]), {T, 0, 4}]
(*Test plot to visualise the non-linear forcing*)
```

```
in[]:= RCBliste={};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};
Δfaeroliste = {};
Δfghgliste = {};
Δfliste = {};
noiseliste = {};
```

```
Monitor[
```

```
Do[
tot = data2[[u]][[All, 2]]; meanco2 = Gmean.tot + 280;
(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 11.9 tunes 2019 methane emmissions in 2019 to 440 Tg Methane *)
del2 = hh /. zz → tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];
```

```
Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.34 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
```

```
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfaer = -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]];
Δfaer2 = Drop[Δfaer, Length[EM]];
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfliste = Append[Δfliste, Δf];
Δfaeroliste = Append[Δfaeroliste, Δfaer];
Δfghgliste = Append[Δfghgliste, Δfco2 + Δfmetan];
```

```
Tliste = {};
Do[
T2 = Klimaliste[[p]].Δf;
```

```
(*nonlin nonlin loop*) (*Do[
T2=Klimaliste[[p]].(Δf+styrke*0.5*(1+Tanh[(T2-terskel)/bratthet])); ,{10}];*)
(*nonlin lin loop*)
```

```
Do[
T2 = Klimaliste[[p]].(Δf + 0.2 T2); , {10}];
noise = σ2liste[[p]] * (Klimaliste[[p]].RandomReal[NormalDistribution[0, 1], Length[Δf]]);
noise = Drop[noise, 268 - 20];
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35; T2 = Drop[T2, 268]; T2=1.1+T2-T2[[1]];
noiseliste = Append[noiseliste, noise]; Tliste = Append[Tliste, T2];
, {p, 1, Length[models]}];
```

```
middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]};
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]]], {i, 1, Length[Trans-
pose[Tliste]]};
```

```
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]]], {i, 1, Length[Trans-
pose[Tliste]]};
```

```
RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
```

```

totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}
, u];

In[ ]:= Length[noise]
Out[ ]:= 103
In[ ]:= Length[T2]
Out[ ]:= 83

In[ ]:= window=10;
noisaliste2 = Table[MovingAverage[noisaliste[[i]], window][[1 ;; Length[T2]]], {i, 1, Length[noisaliste]}];
noisaliste2 = Transpose[Partition[noisaliste2, 14]];

In[ ]:= Length[noisaliste2]
Out[ ]:= 14
In[ ]:= Dimensions[noisaliste2]
Out[ ]:= {14, 127, 83}
In[ ]:= Length[noisaliste2[[1]]]
Out[ ]:= 127

In[ ]:= ListPlot[noisaliste2[[3]],Joined→True]
In[ ]:= FFE=ListPlot[Map[Transpose[{{2018+Range[Length[templiste[[1]]],#}]&, Extract[templiste, p1]],
PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel
→ {None, "GMST increase (°C)"}, Epilog → Inset[Style["e", 18], Scaled[{0.1, 0.9}]]]

In[ ]:= PL1=ListPlot[Map[Transpose[{{1749+Range[Length[Δfaeroliste[[1]]],#}]&, Extract[Δfaeroliste, p1]],
Joined → True];
PL2 = ListPlot[Map[Transpose[{{1749 + Range[Length[Δfaeroliste[[1]]], #}] &, Extract[Δfghgliste, p1]], Joined
→ True];

FFD = Show[{PL1, PL2}, PlotRange → All, Joined → True, Axes → False, Frame → True, FrameStyle → Di-
rective[Black, 14], FrameLabel → {None, "forcing"},Epilog → Inset[Style["d", 18], Scaled[{0.1, 0.9}]]]

modellfarger={,,,,,,,,,,,,,,,,,,,,,};
In[ ]:= pan=LineLegend[modellfarger,Map[StringSplit[#]&,models][[All,1]]]
Out[ ]:= BCC-CSM1-1 BNU-ESM CanESM2 CCSM4 CSIRO-Mk3.6.0 FGOALS-s2 GFDL-ESM2M GISS-E2-R
HadGEM2-ES INM-CM4 MIROC5 MPI-ESM-LR MRI-CGCM3 NorESM1-M

In[ ]:= FFB=ListPlot[Map[Transpose[{{2018+Range[Length[alltliste[[1]]],#}]&, alltliste[[1 ;; 14]]], PlotRange →
All, Joined → True, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel → {None,
"GMST increase (°C)"},Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]], PlotStyle → Map[{#} &, modellfarger]]

In[ ]:= Grid[{{Show[FFA,ImageSize→400],Show[FFB,ImageSize→400],pan}}

In[ ]:= Grid[{{Show[FFC,ImageSize→400, Epilog → Inset[Style["a", 18], Scaled[{0.1, 0.9}]]], Show[FFD, ImageSize
→ 400, Epilog → Inset[Style["b", 18], Scaled[{0.1, 0.9}]]], Show[FFE, ImageSize → 400, Epilog → Inset[Style["c",
18], Scaled[{0.1, 0.9}]]]]}}

In[ ]:= maxtemp=Map[Max[#]&,templiste];
maxtemp2 = Map[Max[#] &, uptempliste];
maxtemp3 = Map[Max[#] &, lowtempliste];

In[ ]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1], AspectRatio → 1, PlotRange → All];

PL3 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Red];
PL4 = ListPlot[Extract[Transpose[{maxtemp2, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Red];

```

```

gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz]; PL2 = Plot[gg, {zz, 1.2, 3}];
gg2 = Fit[Extract[Transpose[{maxtemp2, RCBliste2}], p1], {zz, 1}, zz]; PL4 = Plot[gg2, {zz, 1.2, 3}];
Show[{PL1, PL2}, PlotRange → All]

```

```

In[]:= pairs=Extract[Transpose[{maxtemp,RCBliste2}],p1];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]]])^2 / (Length[pairs] * σx^2)];

```

```

In[]:= pdf=(PDF[NormalDistribution[gg,σf[zz]]][p])/zz→1.5;
pdf2 = (PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → 2.5;
Plot[{pdf, pdf2}, {p, 0, 5200}, PlotRange → All]

```

```

In[]:= PL1=ListPlot[Extract[Transpose[{maxtemp,RCBliste2}],p1],AspectRatio→1, PlotRange → {{1, 4}, {0,
4000}}, PlotStyle → Darker[Blue]];
PL2 = Plot[gg, {zz, 1, 3}, PlotStyle → Darker[Blue]];
l1 = Graphics[{Black, Line[{{1.5, 0}, {1.5, gg /. zz → 1.5}}]};
l2 = Graphics[{Black, Line[{{1.5, gg /. zz → 1.5}, {1, gg /. zz → 1.5}}]}; inset = ParametricPlot[{1 + 60 * pdf, p},
{p, 0, 1200}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];

```

```

l11 = Graphics[{Black, Line[{{2.5, 0}, {2.5, gg /. zz → 2.5}}]};
l22 = Graphics[{Black, Line[{{2.5, gg /. zz → 2.5}, {1, gg /. zz → 2.5}}]}; inset2 = ParametricPlot[{1 + 60 * pdf2,
p}, {p, 1900, 3000}, Axes → False, PlotStyle → {Black, Thickness[0.01]}];

```

```

FA = Show[{PL1, PL2, l1, l2, l11, l22, inset, inset2}, Axes → False, Frame → True, FrameStyle → Directive[Black,
14], FrameLabel → {"global temperature increase (°C)", "carbon budget after 2018 (Gt CO2)"}, Epilog → In-
set[Style["a", 18], Scaled[{0.1, 0.9}], ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}]

```

```

In[]:= maxtemp=Partition[Map[Max[#]&,alltliste],14][[All,5]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Black];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 4}, PlotStyle → Black];
QL1 = Show[{PL1, PL2}, PlotRange → All];

```

```

maxtemp = Partition[Map[Max[#] &, alltliste], 14][[All, 7]];
PL1 = ListPlot[Extract[Transpose[{maxtemp, RCBliste2}], p1], AspectRatio → 1, PlotRange → All, PlotStyle →
Darker[Red]];
gg = Fit[Extract[Transpose[{maxtemp, RCBliste2}], p1], {zz, 1}, zz];
PL2 = Plot[gg, {zz, 1.2, 3.5}, PlotStyle → Darker[Red]];
QL2 = Show[{PL1, PL2}, PlotRange → All];
FB = Show[{QL1, QL2}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], FrameLabel →
{"global temperature increase (°C)", "carbon budget after 2018 (Gt CO2)"}, Epilog → {Inset[Style["b", 18],
Scaled[{0.1, 0.9}], Inset[LineLegend[{Black, Darker[Red]}, {"CSIRO-Mk3.6.0", "GFDL-ESM2M"}],

```

```

Scaled[{0.7, 0.3}], ImageSize → 400, PlotRange → {{1, 4}, {0, 4500}}] In[]:= Grid[{{FA,FB}}]

```

## 14 ESMS FROM CMIP5, 2 CARBONMODELS FROM SRM, INTERNAL VARIABILITY

```

RCBliste = {};
totliste = {};
templiste = {};
uptempliste = {};
lowtempliste = {};
alltliste = {};

```

```

Monitor[
Do[
tot = data2[[u]][[All, 2]];
meanco2 = Gmean.tot + 280;

```

```

meanco2upper = Gupper.tot + 280;
meanco2lower = Glower.tot + 280; (* forcing *)

```

```

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 3.0 tunes 2019 methane emissions in 2019 to 440 Tg Methane *)
del2 = hh /. zz -> tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

```

```

Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.35 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfco2upper = 5.35 Log[1 + (meanco2upper - 280) / 280]; (* CO2 til forcing*)
Δfco2lower = 5.35 Log[1 + (meanco2lower - 280) / 280]; (* CO2 til forcing*)
(* aerosols *)
Δfaer = -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]];
Δfaer2 = Drop[Δfaer, Length[EM]];
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfupper = Δfco2upper + Δfaer + Δfmetan;
Δflower = Δfco2lower + Δfaer + Δfmetan;

```

```

Tliste = {};
Do[
T2 = Klimaliste[[p]].Δfupper;

```

```

(*nonlin loop*) (*Do[
T2=Klimaliste[[p]].(Δfupper+styrke*0.5*(1+Tanh[(T2-terskel)/bratthet]));
,{10}];*)

```

```

(*nonlin lin loop*) Do[
T2 = Klimaliste[[p]].(Δfupper + 0.2 T2);
, {10}];

```

```

T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.1+T2-T2[[1]];
Tliste = Append[Tliste, T2 + StandardDeviation[Flatten[noiseliste2[[p]]]]];

```

```

(*T2=Klimaliste[[p]].Δfupper;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.1+T2-T2[[1]]; Tliste=Append[Tliste,T2-StandardDeviation[Flatten[noiseliste2[[p]]]]];*)

```

```

(*T2=Klimaliste[[p]].Δflower;
T2=T2*(Γliste[[p]]/Log[4.])/5.35;
T2=Drop[T2,268];
T2=1.0+T2-T2[[1]]; Tliste=Append[Tliste,T2+StandardDeviation[Flatten[noiseliste2[[p]]]]];*)

```

```

T2 = Klimaliste[[p]].Δflower;

```

```

(*nonlin loop*) (*Do[
T2=Klimaliste[[p]].(Δflower+styrke*0.5*(1+Tanh[(T2-terskel)/bratthet]));
,{10}];*)

```

```

(*nonlin lin loop*) Do[
T2 = Klimaliste[[p]].(Δflower + 0.2 T2);
, {10}];

```

(\* Comment out everything between: Do[ T2=Klimaliste[[p]].Δfupper; to here to look at linear comparison\*)

```
T2 = T2 * (Tliste[[p]] / Log[4.]) / 5.35;
T2 = Drop[T2, 268];
T2=1.0+T2-T2[[1]];
Tliste = Append[Tliste, T2 - StandardDeviation[Flatten[noisaliste2[[p]]]];
, {p, 1, Length[models]};

middel = Table[Mean[Transpose[Tliste][[i]], {i, 1, Length[Transpose[Tliste]]}];
upper = Table[Mean[Transpose[Tliste][[i]] + StandardDeviation[ Transpose[Tliste][[i]], {i, 1, Length[Trans-
pose[Tliste]]}];
lower = Table[Mean[Transpose[Tliste][[i]] - StandardDeviation[ Transpose[Tliste][[i]], {i, 1, Length[Trans-
pose[Tliste]]}]; RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
```

```
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}
,u]
```

```
In[ ]:= Dimensions[alltliste]
Out[ ]:= {3556, 83}
```

```
In[ ]:= CM = {};
cm1 = Table[ Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[kk]], 2][[ All, 1]], {kk, 1,
Length[p1]};
cm2 = Table[Partition[Extract[Partition[Map[Max[#] &, alltliste], 2 * 14], p1][[ kk]], 2][[All, 2]], {kk, 1,
Length[p1]};
```

```
Do[
CM = Append[CM, Transpose[{Extract[RCBliste2, p1], Transpose[cm1][[j]]}]; CM = Append[CM, Trans-
pose[{Extract[RCBliste2, p1], Transpose[cm2][[j]]}]; , {j, 1, Length[Transpose[cm2]]};
```

```
In[ ]:= ListPlot[{CM[[1]],CM[[2]]}]
```

```
In[ ]:= smliste={};
tliste = {};
```

```
Monitor[ Do[
pdfliste = {};
Do[
gg = Fit[Map[Reverse[#] &, CM[[kk]], {zz, 1}, zz];
pairs = Map[Reverse[#] &, CM[[kk]];
error = pairs[[All, 2]] - (gg /. zz → pairs[[All, 1]]);
S = Sqrt[(Plus @@ (error^2)) / (Length[pairs] - 2)];
σx = StandardDeviation[pairs[[All, 1]]];
σf[x_] := S * Sqrt[1 + 1 / Length[pairs] + (x - Mean[pairs[[All, 1]])^2 / (Length[pairs] * σx^2)];
pdf = Chop[(PDF[NormalDistribution[gg, σf[zz]]][p]) /. zz → target];
pdfliste = Append[pdfliste, pdf];
,{kk,1,2*14}];
```

```
g = Mean[pdfliste];
smooth = Convolve[PDF[NormalDistribution[0, 400]][p], g, p, x];
sm = smooth /. x → Range[7000];
smliste = Append[smliste, sm];
tliste = Append[tliste, target];
, {target, 1.1, 4.0, 0.01}];
, target];
```

```
In[ ]:= (*forsterkningsfaktor a+bT: 0.10034007260683281`+2.2321837475237376` x*)
```

```

In[ ]:= a=0.10034;
b = 2.23218;
Δtarc = a+b* Δt;

```

```

In[ ]:= budget=500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

```

```

In[ ]:= budget=1500;
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt); ListPlot[Transpose[{tliste, y}], Joined → True]

```

```

In[ ]:= bliste={};
Do[
Δt = tliste[[2]] - tliste[[1]];
y = Transpose[smliste][[budget]];
y = y/ ((Plus@@y) * Δt);
t1 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.90 &)]]][[1]] - 1];
t2 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.75 &)]]][[1]] - 1];
t3 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.5 &)]]][[1]] - 1];
t4 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.25 &)]]][[1]] - 1];
t5 = tliste[[First[Position[FoldList[Plus, 0, y * Δt], _? (# > 0.10 &)]]][[1]] - 1];
bliste = Append[bliste, {budget, t1, t2, t3, t4, t5}]; , {budget, 200, 4000, 100}

```

```

In[ ]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};
aa = ListPlot[Transpose[{bliste[[All, 1]], bliste[[All, 2]]}],

```

```

Transpose[{bliste[[All, 1]], bliste[[All, 3]]}, Transpose[{bliste[[All, 1]], bliste[[All, 4]]}, Trans-
pose[{bliste[[All, 1]], bliste[[All, 5]]}, Transpose[{bliste[[All, 1]], bliste[[All, 6]]}], Joined → True, AspectRatio
→ 1, PlotRange → {{1, 9}}, Axes → False, Frame → True, FrameStyle → Directive[Black, 14], PlotStyle → Ta-
ble[farger[[i]], {i, 1, 5}], GridLines → Automatic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maxi-
mum temperature increase (°C)"}, PlotLegends → Placed[{"10% prob.", "25% prob.", "even chance", "75%
prob.", "90% prob."}, {Scaled[{0.05, 0.7}], {0, 0.5}}]; l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]};
GGD = Show[aa]

```

```

In[ ]:= Tclow=Graphics[{Black,Line[{{0,8},{4000,8}}]}; (*Greenland paper critical values*)
Tcupper = Graphics[{Black, Line[{{0, 8.5}, {4000, 8.5}}]};

```

```

In[ ]:= farger={Red,Darker[Red],Black,Darker[Blue],Blue};
aaa = ListPlot[{Transpose[{bliste[[All, 1]], a + b * bliste[[All, 2]]}, Transpose[{bliste[[All, 1]], a + b * bliste[[All,
3]]}, Transpose[{bliste[[All, 1]], a + b * bliste[[All, 4]]}, Transpose[{bliste[[All, 1]], a + b * bliste[[All, 5]]},
Transpose[{bliste[[All, 1]], a + b * bliste[[All, 6]]}], Joined → True, AspectRatio → 1, PlotRange → {{1, 9}}, Axes
→ False, Frame → True, FrameStyle → Directive[Black, 14], PlotStyle → Table[farger[[i]], {i, 1, 5}], GridLines →
Automatic, FrameLabel → {"Carbon budget from 2018 (GtCO2)", "Maximum temperature increase (°C)"},
PlotLegends → Placed[{"10% prob.", "25% prob.", "even chance", "75% prob.", "90% prob."}, {Scaled[{0.05,
0.7}], {0, 0.5}}]; l = Graphics[{Black, Line[{{1294, 1}, {1294, 2.5}}]};
GGD = Show[aaa, Tclow, Tcupper}, PlotRange → {{0, 4000}, {1.1, 10}}]

```

```

In[ ]:= Max[a+b*bliste[[All,2]]]
Out[ ]:= 8.7612

```

## COMPARISON

```

In[ ]:= Grid[{{Show[aa,PlotRange→{{0,4000},{1.1,9}}, ImageSize → 400, Epilog → Inset[Style["a", 18], Scaled[{0.1,
0.9}]]], Show[aaa, PlotRange → {{0, 4000}, {1.1, 9}}, ImageSize → 400, Epilog → Inset[Style["b", 18],
Scaled[{0.1, 0.9}]]]
}}]

```

## ESTIMATION OF THE ARCTIC AMPLIFICATION FACTOR

(\* GLOBAL LAND-OCEAN TEMPERATURE\*)

SetDirectory["OneDrive - UiT Office 365"];

global = Drop[Drop[Import["GLB.Ts+dSST.csv", {"Data", All, 14}], 2], - 1];

In[ ]:= PL1=ListPlot[global,Joined→True,PlotRange→All, DataRange→{1880, 1880 + Length[global]}

(\*base period 1951-1980\*)

In[ ]:= global;

(\* ANNUAL MEAN LAND-OCEAN TEMPERATURE 64N-90N (ARCTIC IS 66.34N)\*)

arctic = Drop[Import["ZonAnn.Ts+dSST.csv", {"Data", All, 8}], 1];

In[ ]:= PL2=ListPlot[arctic,Joined→True,PlotRange→All, PlotStyle→Orange, DataRange→{1880, 1880 + Length[arctic]}

(\*base period 1951-1980\*)

In[ ]:= Show[{PL1,PL2},PlotRange→All]

(\* COMPARISON\*)

comp = Transpose[{global, arctic}];

In[ ]:= compPlot=ListPlot[comp,PlotRange→All, AxesLabel→{"global", "arctic"}, AspectRatio→1]

In[ ]:= lm=LinearModelFit[comp,x,x](\*forsterkningsfaktor=2.23218\*)

Out[ ]:= FittedModel 0.10034 + 2.23218 x

In[ ]:= Fit[comp,{zz,1},zz]

Out[ ]:= 0.10034+2.23218zz

In[ ]:= fit=Plot[lm[x],{x,-3,5}];

In[ ]:= Show[compPlot,fit]



## CODE FOR COMPARISON OF A SIMPLE RESPONSE MODEL TO MAGICC6.

```
SetDirectory["OneDrive - UiT Office 365"] M = Import["SSP_IAM_V2_201811.csv"];
M[[1]]
Out[]:= {MODEL,"SCENARIO","REGION","VARIABLE","UNIT",2005,2010,2020,2030,2040,2050,2060
,2070,2080,2090,2100}

In[]:= M=Map[StringSplit[#,","]&,M];
In[]:= TT=Table[M[[k]][[1]][[4]],{k,1,Length[M]}; Union[TT];
TTT = Table[M[[k]][[1]][[3]], {k, 1, Length[M]};

In[]:= MAG=Position[TT,_(#=="\Diagnostics|MAGICC6|Temperature|GlobalMean\ ""&)];
In[]:= MAGpos=Position[TTT,_(#=="\World\ ""&)];
In[]:= mpos1=Intersection[MAGpos,MAG]; Extract[M, MAG][[1]];
In[]:= temp1=ToExpression[Map[Drop[Flatten[#],6]&,Extract[M,mpos1]]]; ListPlot[temp1, Joined → True]
```

### SRM MODEL

```
In[]:= hh=157.65890684920566`+1.8942819330281027`zz+0.08520850267749702`zz2;
In[]:= RR=Table[M[[k]][[1]][[4]],{k,1,Length[M]};
Union[RR];
In[]:= RRR=Table[M[[k]][[1]][[3]],{k,1,Length[M]};

Union[RRR]
Out[]:= {"R5.2ASIA", "R5.2LAM", "R5.2MAF", "R5.2OECD", "R5.2REF", "REGION", "World"}

In[]:= co2pos1=Position[RR,_(#=="\Emissions|CO2|FossilFuelsandIndustry\ ""&)];
co2pos2 = Position[RR, _ ? (# == "\Emissions|CO2|Land Use\ "" &)];
co2pos3 = Position[RRR, _ ? (# == "\World\ "" &)];

In[]:= ppos1=Intersection[co2pos3,co2pos1];
ppos2 = Intersection[co2pos3, co2pos2];

In[]:= Extract[M,co2pos1][[1]]

Out[]:= {{AIM/CGE, "SSP1-19", "R5.2ASIA", "Emissions|CO2|Fossil Fuels and Industry",
"Mt CO2/yr", 8985.6725, 10008.8152, 11790.747500000001,
6131.6627, 3271.4353000000006, 1678.8029, 638.87, 259.4755, 82.29590000000003, -7.935300000000105, -
103.9171}}
```

```
In[]:= em1=ToExpression[Map[Drop[Flatten[#],7]&,Extract[M,ppos1]]];
em2 = ToExpression[Map[Drop[Flatten[#], 7] &, Extract[M, ppos2]]];

In[]:= ListPlot[em1,PlotRange→All,Joined→True]
emissions = Map[#[[1 ;; 2]] &, ToExpression[ Map[StringSplit[#] &, Drop[ReadList["emissionsCO2.txt", String],
31]]]; In[]:= emissions=Table[{emissions[[i,1]],(44/12)*emissions[[i,2]]/1000.}
, {i, 1, Length[emissions]};

In[]:= EM=Join[emissions,{{2018,37.1}}];
data2 = Table[Prepend[Table[{t, Interpolation[Join[EM, Transpose[{{2030, 2040, 2050, 2060, 2070, 2080,
2090, 2100}, 0.001 * Drop[em1[[k]], 1]]]]][t]], {t, 1751, 2100}], {1750, 0}], {k, 1, Length[em1]};
totliste = Table[data2[[k]][[All, 2]], {k, 1, Length[data2]};

In[]:= positivepaths= Table[DeleteCases[Map[# * UnitStep[#] &, totliste[[k]][[269 ;; 351]]], _?(#
==0&)],{k,1,Length[totliste]}; ListPlot[positivepaths, Joined → True]
```

```

In[ ]:= RCBliste2=Map[Plus@@#&,positivepaths];
In[ ]:= p1=Position[RCBliste2,_(#<3300&)];
In[ ]:= ListPlot[data2,Joined→True,PlotRange→All]

```

```

In[ ]:= n=Length[data2[[1]]];
futuretime = 2100 - 2020;
τmetan = 12.4;

```

```

In[43]:= (* Carbon model *)
τ1=1;
τ2=10;
τ3 = 100;
τ4 = 1000;
c1mean = 0.152;
c2mean = 0.246;
c4mean = 0.134;
c5mean = 0.194;

```

```

Gmean = (12/44)*0.47*(c1mean * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]+
c2mean * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1mean - c2mean - c4mean - c5mean)* Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}]+
c4mean * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5mean * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

```

```

In[52]:= (* Carbon models *)
c1upper = 0.11;
c2upper = 0.212;
c4upper = 0.106;
c5upper = 0.262;
c1lower = 0.18;
c2lower = 0.296;
c4lower = 0.122;
c5lower = 0.148;

```

```

Glower = (12 / 44) * 0.47 * (c1lower * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c2lower * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1lower - c2lower - c4lower - c5lower) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c4lower * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5lower * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

```

```

Gupper = (12/44) * 0.47 * (c1upper * Table[Exp[- (i - j) / τ1] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
c2upper * Table[Exp[- (i - j) / τ2] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
(1 - c1upper - c2upper - c4upper - c5upper) * Table[Exp[- (i - j) / τ3] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] + c4up-
per * Table[Exp[- (i - j) / τ4] * UnitStep[i - j], {i, 1, n}, {j, 1, n}] +
Table[c5upper * UnitStep[i - j], {i, 1, n}, {j, 1, n}]);

```

```

In[62]:=
(*Optimal Estimation of Stochastic Energy Balance Model Parameters *)

```

```

In[63]:= (* Climate models *)
models = ReadList["CMIP5parameters.txt", String];
models = Delete[models, {{5}, {12}}];
boxes = StringSplit[models][[All, 2]];
Klimaliste = {};
Γliste = {};
σ2liste = {};
Monitor[
Do[

```

```

Clear[A];
modelnr = p; If[boxes[[p]] == "2",

```

```

{C1, C2, κ1, κ2, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];

```

```

A = {{-(κ1+ κ2) /C1, κ2/C1}, {κ2/C2, -κ2/C2}};
g = (MatrixExp[t A].{1 / C1, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If[ boxes[[p]] == "3",

{C1, C2, C3, κ1, κ2, κ3, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];

A = {{-(κ1+ κ2) /C1, κ2/C1, 0}, {κ2/C2, -(κ2+κ3)/C2, κ3/C2},{0, κ3/C3, - κ3/C3}};

g = (MatrixExp[t A].{1 / C1, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

If [ boxes[[p]] == "4",

{C1, C2, C3, C4, κ1, κ2, κ3, κ4, σ1, Γ, σ2} = ToExpression[Drop[StringSplit[models[[modelnr]], 2]];

A = {{-(κ1+κ2)/C1, κ2/C1, 0, 0} , {κ2/C2, -(κ2+κ3)/C2, κ3/C2, 0},{ 0, κ3/C3, -(κ3+κ4)/C3, κ4/C3},
{0, 0, κ4/C4, -κ4/C4}};

g = (MatrixExp[t A].{1 / C1, 0, 0, 0})[[1]];
Gklima = Table[Chop[(g /. t → (i - j)) * UnitStep[i - j]], {i, 1, n}, {j, 1, n}];
Klimaliste = Append[Klimaliste, Gklima];
];

Γliste = Append[Γliste, Γ];
σ2liste = Append[σ2liste, σ2];
, {p, 1, Length[models]}
];
, {p, boxes[[p]]}
];

(*Nonlin parameter changes*)

styrke = 1; (*w/m^2*)
terskel = 2; (*grader*)
bratthet = 0.5;
Plot[styrke * 0.5 * (1 + Tanh[(T - terskel) / bratthet]), {T, 0, 4}]
(*Test plot to visualise the non-linear forcing*)

RCBliste = {}; totliste = {}; templiste = {}; uptempliste = {}; lowtempliste = {}; alltliste = {}; Δfaeroliste = {};
Δfghliste = {}; Δfliste = {}; noiseliste = {}; Monitor[

Do[
tot = data2[[u]][[All, 2]]; meanco2 = Gmean.tot + 280;

(* metan *)
del1 = 11.9 * tot[[1 ;; Length[EM]]];
(* The factor 11.9 tunes 2019 methane emmissions in 2019 to 440 Tg Methane *) del2 = hh /. zz →
tot[[Length[EM] + 1 ;; Length[tot]]];
del2 = Last[del1] + (del2 - First[del2]) * (Last[del1] - Last[del2]) / (First[del2] - Last[del2]);
metemis = Join[del1, del2];

```

```
Gmetan = 0.34 * Table[Exp[- (i - j) / τmetan] * UnitStep[i - j], {i, 1, n}, {j, 1, n}];
(* The factor 0.34 tunes 2019 methane concentration to around 1880 ppb *)
metan = Map[Max[#, 0] &, 700 + Gmetan.metemis];
```

```
Δfmetan = 0.036 * (Sqrt[metan] - Sqrt[700]);
Δfco2 = 5.35 Log[1 + (meanco2 - 280) / 280]; (* CO2 til forcing*)
Δfaer = -0.02tot;
Δfaer1 = Δfaer[[1 ;; Length[EM]]];
Δfaer2 = Drop[Δfaer, Length[EM]];
Δfaer2 = Map[Min[-0.4, #] &, Δfaer2];
Δfaer = Join[Δfaer1, Δfaer2];
Δf = Δfco2 + Δfaer + Δfmetan;
Δfliste = Append[Δfliste, Δf];
Δfaeroliste = Append[Δfaeroliste, Δfaer];
Δfghgliste = Append[Δfghgliste, Δfco2 + Δfmetan];
```

```
Tliste = {}; Do[ T2 = Klimaliste[[p]].Δf;
```

```
(* Remove this part to run the linear SRM comparison*)
```

```
Do[
T2 = Klimaliste[[p]].(Δf + 0.2 T2);
, {10}];
```

```
(* End of part *)
```

```
noise = σ2liste[[p]] * (Klimaliste[[p]].RandomReal[NormalDistribution[0, 1], Length[Δf]]);
noise = Drop[noise, 268 - 20];
T2 = T2 * (Γliste[[p]] / Log[4.]) / 5.35; T2 = Drop[T2, 268]; T2=1.1+T2-T2[[1]];
noiseliste = Append[noiseliste, noise]; Tliste = Append[Tliste, T2];
, {p, 1, Length[models]}];
```

```
middel = Table[Mean[Transpose[Tliste][[i]]], {i, 1, Length[Transpose[Tliste]]};
upper = Table[Mean[Transpose[Tliste][[i]]] + StandardDeviation[ Transpose[Tliste][[i]], {i, 1, Length[Trans-
pose[Tliste]]};
lower = Table[Mean[Transpose[Tliste][[i]]] - StandardDeviation[ Transpose[Tliste][[i]], {i, 1, Length[Trans-
pose[Tliste]]};
```

```
RCB = Plus @@ Drop[tot, 270];
RCBliste = Append[RCBliste, RCB];
totliste = Append[totliste, tot];
alltliste = Join[alltliste, Tliste];
templiste = Append[templiste, middel];
uptempliste = Append[uptempliste, upper];
lowtempliste = Append[lowtempliste, lower];
, {u, 1, Length[data2]}
, u];
```

```
In[ ]:= Length[noise]
Out[ ]:= 103
```

```
In[ ]:= Length[T2]
Out[ ]:= 83
```

```
In[ ]:= window=10;
noiseliste2 = Table[MovingAverage[noiseliste[[i]], window][[1 ;; Length[T2]]], {i, 1, Length[noiseliste]}];
noiseliste2 = Transpose[Partition[noiseliste2, 14]];
```

```
In[ ]:= Length[noiseliste2]
Out[ ]:= 14
```

```
In[ ]:= Dimensions[noiseliste2]
Out[ ]:= {14, 127, 83}
```

```
In[ ]:= Length[noisaliste2[[1]]]
Out[ ]:= 127
```

```
In[ ]:= ListPlot[noisaliste2[[3]],Joined→True]
In[ ]:= ListPlot[Extract[templiste,p1],Joined→True]
In[ ]:= ListPlot[Extract[temp1,p1],Joined→True,Axes→False,Frame→True,FrameStyle→Directive[Black,14],
FrameLabel→{None,"GMST increase (°C)"}]
```

```
In[ ]:= OURMODEL=Extract[templiste,p1];
MAGICC = Extract[temp1, p1];
```

```
In[ ]:= Length[OURMODEL]
Out[ ]:= 86
In[ ]:= Length[MAGICC]
Out[ ]:= 86
```

```
In[ ]:= ListPlot[{OURMODEL[[1]],MAGICC[[1]]}]
```

```
In[ ]:= parliste={};
```

```
Do[
x = Drop[MAGICC[[i]], 1];
y = OURMODEL[[i]][[3, 13, 23, 33, 43, 53, 63, 73, 83]]; par = Transpose[{x, y}];
parliste = Append[parliste, par];
, {i, 1, Length[OURMODEL]}]
```

```
In[ ]:= y=OURMODEL[[1]][[3,13,23,33,43,53,63,73,83]]
Out[ ]:= {1.16787, 1.54466, 1.64649, 1.64045, 1.60513, 1.56572, 1.53297, 1.50672, 1.48457}
PL1 = ListPlot[parliste, AspectRatio → 1, PlotRange → {{0.5, 4}, {0.5, 4}}]; PL2 = Plot[zz, {zz, 0.5, 4}, PlotStyle →
Black];
gg = Fit[Partition[Flatten[parliste], 2], {zz, 1}, zz];
PL3 = Plot[gg, {zz, 0.5, 4}, PlotStyle → {Black, Dashed}];
Show[{PL1, PL2, PL3}, FrameStyle → Directive[Black, 14], Axes → False, Frame → True, FrameLabel →
{"MAGICC GMST (°C)", "Response model GMST (°C)"}]
(* All 774 datapoints *)
In[ ]:= parliste2={};
```

```
Do[
x = Drop[MAGICC[[i]], 1];
y = OURMODEL[[i]][[3, 13, 23, 33, 43, 53, 63, 73, 83]]; par = {Max[x], Max[y]};
parliste2 = Append[parliste2, par];
, {i, 1, Length[OURMODEL]}];
```

```
PL1 = ListPlot[Map[{#} &, parliste2],
AspectRatio → 1, PlotRange → {{0.5, 4}, {0.5, 4}}];
```

```
PL2 = Plot[zz, {zz, 0.5, 4}, PlotStyle → Black];
gg = Fit[parliste2, {zz, 1}, zz];
PL3 = Plot[gg, {zz, 0.5, 4}, PlotStyle → {Black, Dashed}]; Show[{PL1, PL2, PL3}, FrameStyle → Directive[Black,
14], Axes → False, Frame → True, FrameLabel → {"MAGICC maximum GMST (°C)", "Response model maximum
GMST (°C)"}]
(* Max temperature comparison for 86 scenarios *)
```

