European Journal of Engineering Education

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/ceee20

First-year engineering students' mathematics task performance and its relation to their motivational values and views about mathematics

Timo Tossavainen , Ragnhild Johanne Rensaa , Pentti Haukkanen , Mika Mattila & Monica Johansson

To cite this article: Timo Tossavainen , Ragnhild Johanne Rensaa , Pentti Haukkanen , Mika Mattila & Monica Johansson (2020): First-year engineering students' mathematics task performance and its relation to their motivational values and views about mathematics, European Journal of Engineering Education, DOI: 10.1080/03043797.2020.1849032

To link to this article: https://doi.org/10.1080/03043797.2020.1849032

Published online: 26 Nov 2020.

EUROPEAN JOURNAL OF ENGINEERING EDUCATION
https://doi.org/10.1080/03043797.2020.1849032

Taylor & Francis
Taylor & Francis Group

OPEN ACCESS

First-year engineering students' mathematics task performance and its relation to their motivational values and views about mathematics

Timo Tossavainen [a], Ragnhild Johanne Rensaa [b], Pentti Haukkanen [c], Mika Mattila [c] and Monica Johansson[a]

[a]Department of Arts, Communication and Education, Lulea University of Technology, Lulea, Sweden; [b]Department of Electrical Engineering, The Arctic University of Norway, Narvik, Norway; [c]Faculty of Information Technology and Communication Sciences, Tampere University, Tampere, Finland

ABSTRACT
This study investigates Finnish, Norwegian, and Swedish first-year engineering students' task performance in mathematics and examines how it relates to their motivational values and beliefs about the nature of mathematics. In a set of seven mathematical tasks, female students outperformed male students, for example, in the simplification of symbolic expressions. Our findings also show that female students set more demanding learning goals for themselves. Further, they expressed higher intrinsic motivational values, whereas male students emphasised utility values, which correlated negatively with task performance. Problem-solving and discovering structures and regularities dominate engineering students' views of mathematics. The 'toolbox' view of mathematics is the best predictor of task performance; the stronger this view is, the poorer is the task performance. However, neither motivational values nor views about mathematics are especially strong predictors of task performance. Based on the findings of this study, it is recommended that, in engineering mathematics courses, greater emphasis is placed on learning fundamental reasoning strategies and discussing the theoretical foundations of the main results. This seems to lead to a better result even in applying mathematics than if one focuses only on learning how to use ready-made mathematical tools in concrete examples.

ARTICLE HISTORY
Received 1 July 2020
Accepted 7 November 2020

KEYWORDS
Engineering student; motivation; self-efficacy; task performance; view of mathematics

1. Introduction

The transition from secondary to tertiary mathematics education is a recurring theme in mathematics education research. In the context of engineering education, recent research in this topic has concerned, for instance, success factors in transition (Bengmark, Thunberg, and Winberg 2017), and the search for the best predictors of students' performance in tertiary mathematics (Faulkner, Hannigan, and Fitzmaurice 2014). Another continuously revisited topic is gender equality, especially how to strengthen female students' foundational skills for succeeding in engineering mathematics courses (Guo et al. 2015a).

A persistent challenge for educators in engineering mathematics is that mathematics is rarely engineering students' primary interest (Kümmerer 2001; Flegg, Mallet, and Lupton 2012).

CONTACT Timo Tossavainen timo.tossavainen@ltu.se

Applicants to engineering programmes are often unaware of how mathematically demanding their education will be. Some would have chosen other studies if they had known this (Gordon and Nicholas 2013; Harris et al. 2015). Kouvela, Hernandez-Martinez, and Croft (2018, 166) claim that 'some students select their degree programme according to the exchange value that this will offer them later in the jobs market and not necessarily through any intrinsic interest in the programme itself. This eventually makes their transition harder.' Additionally, educators are not always sufficiently aware of what kind of mathematics learning strategies university mathematics freshmen engineering students actually have. A reason for this is the mathematics is taught in schools in a different way compared to that in universities. The difference concerns, e.g. expectations how students should think and communicate when solving argumentative and proving tasks. However, there are studies showing that there are quite simple means to overcome such challenges. Paying more attention to the types of discourse in teaching–learning interaction during the transitional phase has proven to be effective (Kouvela, Hernandez-Martinez, and Croft 2018).

This article sets out to provide educators with new knowledge about freshmen engineering students' knowledge and expectations of studying mathematics at university. A group ($N = 431$) of first-year engineering students from a Finnish, a Norwegian, and a Swedish technical university is investigated by focusing on their performance in a set of mathematical tasks and how the performance relates to their views of the nature of mathematics and motivation. The novelty value of this study is that, except for a paper published by the authors, there seem to be no reports outside the Nordic countries that have focused on the relationship between task performance and the views of mathematics. Moreover, the students' performance is measured at the level of individual tasks instead of course grades (cf. Bengmark, Thunberg, and Winberg 2017).

2. Theoretical framework

The present study builds on two theoretical perspectives: (1) students' epistemological beliefs on the nature of mathematics (Hofer and Pintrich 1997) and (2) their motivation values in the framework of the Expectancy–value theory (Eccles et al. 1983; Wigfield and Eccles 2000). The epistemological beliefs about the nature of mathematics concern the structure, quality, certainty, and the source of mathematical knowledge. There is evidence that such beliefs have an influence on a learner's actual competence and performance in mathematics, since they affect the perception of the mathematical task and, thus, also the learner's choice of actions in handling the task (Felbrich, Müller, and Blömeke 2008). Further, these beliefs have an effect on individuals' beliefs about themselves as learners of mathematics. Grigutsch, Raatz, and Törner (1998) studied German mathematics teachers' beliefs about the nature of mathematics, and they identified four different categories of such beliefs. Their categorisation has been used for studying student teachers' beliefs. For that purpose, Felbrich, Müller, and Blömeke (2008, 764) formulated and named the categories as follows:

- The formalism-related orientation: mathematics is viewed as an exact science that has an axiomatic basis and is developed by deduction (e.g. 'Mathematical thinking is determined by abstraction and logic').
- The scheme-related orientation: mathematics is regarded as a collection of terms, rules, and formulae (e.g. 'Mathematics is a collection of procedures and rules which precisely determine how a task is solved' ['a toolbox']).
- The process-related orientation: mathematics can be understood as a science which mainly consists of problem-solving processes and the discovery of structure and regularities (e.g. 'If one comes to grips with mathematical problems, one can often discover something new (connections, rules, and terms)').

- The application-related orientation: mathematics can be seen as a science which is relevant to society and life (e.g. 'Mathematics helps to solve daily tasks and problems').

The formalism- and scheme-related orientations underline the static nature of mathematics, a ready-made construction to be adopted and used. The process- and application-related orientations highlight discovering and developing the processes and structures needed to solve mathematical problems. A difference between the formalism- and scheme-related orientations lies in whether the focus is placed on the structure of mathematical knowledge itself or on the tasks for which the 'mathematics toolbox' is needed. The application-related orientation focuses on using mathematics to solve daily tasks and problems in society, whereas the process-related orientation appreciates problems that are interesting primarily in the context of mathematics itself. However, the process- and application-related orientations contain aspects of evolution and progress. Consequently, they represent the dynamic nature of mathematics. Clearly, these four categories do not exhaustively cover all relevant aspects of mathematics. Mathematics has a bearing on almost all sciences and throughout society, and an individual's view of mathematics depends on the context wherein mathematics is met. However, this categorisation has turned out to be useful in surveying students' beliefs about the nature of mathematics they have studied in school.

Research has shown that an individual almost always has beliefs representing several orientations simultaneously (Felbrich, Müller, and Blömeke 2008; Tossavainen et al. 2017). Therefore, it is more relevant to study a student's distribution of orientations to mathematics rather than to try to find a single orientation representing the student's view of the nature of mathematics. However, quite often an individual can identify which of the orientations best represents his or her view of mathematics. This orientation can be taken as the individual learner's primary or dominating orientation to mathematics.

The other cornerstone of our theoretical framework is the Expectancy–value theory. The aim of the theory is to explain how an individual's expectations and values affect his or her learning behaviour. Both expectancies and values represent beliefs, but they are conceptually distinguished from one another. Expectancies focus on, e.g. the chances of succeeding from the future aspect, whereas values represent the present valuations and abilities. These constructs are highly related (Wigfield and Eccles 2000, 70). In this theory, the individual's motivational values are categorised into four classes: intrinsic value, attainment value, utility value, and cost. In the context of studying mathematics, intrinsic value refers to the enjoyment of and interest in studying the subject. Attainment value represents the perceived importance of being good at mathematics, and utility value is related to the perceived usefulness of knowing mathematics for short- and long-term goals. Cost portrays how much an individual is ready to invest his or her resources in studying mathematics (Eccles et al. 1993; Tossavainen, Rensaa, and Johansson 2019).

In order to include a more general aspect of a student's self-perception and relation to mathematics, our questionnaire contains a few items measuring general self-efficacy in mathematics. Bandura (1982, 122) defines perceived self-efficacy as 'an individual's own judgement of how well one can execute courses of action required to deal with prospective situations'. If a 'prospective situation' is associated with the need to solve a single task, then self-efficacy is a task-specific notion. However, it is common to incorporate self-efficacy items into a questionnaire to measure an individual's domain-specific self-perception in a subject (Wigfield and Eccles 2000). An individual with high self-efficacy in mathematics presumably takes challenging mathematical tasks as something positive to engage with and master, whereas a person with low self-efficacy experiences such tasks merely as a burden and tries to avoid them (Bandura 2012). Not surprisingly, in many empirical studies, self-efficacy correlates with learning outcomes. A very recent study shows that self-efficacy potentially influences engineering students' learning strategies. More precisely, a student with a high sense of perceived self-efficacy tends to apply a deep approach to learning mathematics, whereas a low sense of self-efficacy seems to lead to a surface approach (Zakariya et al. 2020).

3. Research questions

This study investigates relationships between first-year engineering students' performance in mathematics, their views of mathematics, and their motivation and self-efficacy in learning mathematics. These relations are explored also from the perspective of gender. The specific questions are as follows.

(1) *How does the mathematical task performance of Nordic first-year engineering students depend on their motivational values and self-efficacy beliefs?*
(2) *How do female and male students' task performance, motivational values, self-efficacy beliefs, and views about mathematics differ from one another?*
(3) *How does students' task performance relate to the distribution of their orientations?*
(4) *To what extent can students' task performance be predicted from their orientation values?*

4. A review of the literature

Bengmark, Thunberg, and Winberg (2017) examined Swedish engineering students' motivations, beliefs, and study habits as well as their relative importance for the students' performance in the transition to tertiary mathematics. They noticed that all factors mentioned above play an important role, yet each of them alone explained less than 5% of the variation in the students' performance scores. However, after one year in university, self-efficacy had become a strong predictor of their achievement (Bengmark, Thunberg, and Winberg 2017). In spite of some apparent similarities, there are several differences between their survey and the present investigation. Engineering students from three Nordic countries are now studied focusing on the beginning of their university studies, whereas Bengmark and his colleagues followed Swedish students during the whole first year. Moreover, they measured students' performance at the level of earned credits and grades, here it is done at the level of individual tasks. Further, they base on the self-determination theory by Ryan and Deci (2000).

Alves et al. (2016) collected data from a group of Portuguese undergraduate engineering students and examined the relationships between students' self-efficacy, mathematics anxiety (i.e. a tension that interferes with the manipulation of mathematical expressions and the solving of mathematical tasks), and the perceived importance of mathematics. In this investigation, students acknowledged the importance of mathematics in their studies. The students' self-efficacy in mathematics was also relatively high and mathematics anxiety rather low. While there was no significant difference between male and female students, there were significant differences in perceived importance and anxiety between the degree programmes. This is slightly paradoxical, since the gender distributions varied significantly between the degree programmes.

Guo et al. (2015b) focused on their multi-cohort study on the relationships between motivation and variables describing students' background in predicting educational outcomes. They reported that when male and female students had similar levels of mathematical self-concept (i.e. an individual's evaluation of his or her ability in mathematics) and values, female students tended to have better mathematics results and higher educational aspirations. Similar results have been reported from some other cultures (Vitasari et al. 2010).

The role of intrinsic motivation has been investigated, for instance, by Tossavainen and Juvonen (2015) and Kosiol, Rach, and Ufer (2019). The results in both these studies indicate that interest in mathematics is a strong predictor for more general student satisfaction and motivation. The latter study also points out that, in the transition from upper secondary mathematics to tertiary mathematics, interest correlates only weakly with the cognitive prerequisites. There are also studies that focus on utility value. Based on data collected from Australian first-year engineering students, Flegg, Mallet, and Lupton (2012) noticed that a majority of students acknowledged the applicability and importance of mathematics to their studies and career. However, there was a large variation in how and at what level the relevance was acknowledged. One explanation for this is that mathematics is only rarely a central motive for engineering students (Kümmerer 2001; Flegg, Mallet, and Lupton 2012).

According to Harris et al. (2015), engineering students seek the usefulness of mathematics, yet their experience from engineering mathematics courses is that the teaching of mathematics is decontextualised and does not provide students with a use-value perspective. The students' opinion was that mathematics is discussed only with a few references to engineering. In their view, the absence of relevant examples is inherent in these courses. However, Harris et al. (2015) also pointed to that engineering students may change their view of mathematics during their years at university. In school, good grades in mathematics have merely a short-term exchange value; they give a student an entrance to an 'elite' university. At university, some students start to speak about mathematics in terms of support to their engineering studies and long-term prospects of getting well-paid jobs (Harris et al. 2015).

5. Method

5.1. Participants and data collection

The participants in this cross-sectional study (N = 431) were first-year engineering students from three Nordic universities. The number of students was 272 from Finland, 71 from Norway, and 88 from Sweden. The Norwegian and Swedish universities are medium-sized and situated in northern and sparsely populated areas, whereas the Finnish university is one of the most popular and largest universities in Finland, and it is situated in a densely populated area in southern Finland. The sizes of the national cohorts represent well the relative sizes of the participating universities. The samples represent 17% (NOR), 18% (SWE), and 32% (FIN) of all first-year engineering students at these universities. The participants represented a total of 22 different engineering programmes. The number of female and male students was 118 (27%) and 305 (71%), respectively. Eight students chose the alternative 'Other/I do not want to answer'. When studying the data from the gender perspective, the latter individuals were excluded because of the small group size.

The data were collected at each university from the first compulsory course in engineering mathematics. This course is similar in each country and comprises topics mainly from calculus. It is conducted in the beginning of the Autumn semester in all three countries. The data were collected during the first two weeks of the course using an anonymous questionnaire. The questionnaire was estimated to take 60 min to complete, but only a few students spent that much time. The use of calculators and other computing devices was allowed, but students were advised to show their own knowledge and not, for example, to seek help from others via social media. Participation was voluntary and completely independent of the examination of the course. To encourage students to take the questionnaire, students were notified in the introduction of the questionnaire that their responses would be handled anonymously and confidentially and used only for research purposes and for improving their mathematics courses.

5.2. Questionnaire

The questionnaire consisted of three sections. The first section surveyed the student's background; the second one the student's view of the nature of mathematics, self-efficacy, and motivational values; and the third one the student's performance in seven mathematical tasks. The statements concerning the orientations were adapted from the previous works by Felbrich, Müller, and Blömeke (2008), and Tossavainen et al. (2017). Following the example of Wigfield and Eccles (2000), there were two statements for each motivational value and orientation. Further, the respondent was asked to choose from four given metaphors that correspond best to his/her view of the nature of mathematics.

The first versions of the tasks in the third section were designed by two authors. Following feedback from a group of experienced university lecturers of engineering mathematics courses, some simplifications of the tasks were made. The final version of the questionnaire contained three straightforward calculation tasks (4.1–4.3, see Appendix), which involve percentages and the simplification of symbolic expressions, and four somewhat more challenging tasks (4.4–4.7, see Appendix). Task 4.4 was designed to be solved by providing either a counterexample or an explanation as to why the given rule is not sufficient to guarantee that (a_n) is increasing. In order to get a full score, the student had to consider the case $a_1 < 0$. In Task 4.5, the respondent had to apply mathematical knowledge about increasing and decreasing functions to determine whether a medicine is effective or not, i.e. in order to solve a problem that is relevant to society. A detailed proof was not required, but a full score presumed a solution explicitly stating that the number of bacteria tends to zero when the medicine is applied long enough. Task 4.6 focuses on the definition of a decreasing function. For a full score, the solution had to communicate a relevant definition and analyse the given function correctly in terms of the definition. Task 4.7 concerns the construction of a function satisfying the given conditions. Both verbal and graphical solutions were accepted as long as a sufficient explanation was provided. An example of applicable functions is the constant function $f(x) = -0.5$.

5.3. Limitations of study

As the participation in the study was voluntary and without a reward, the response rate was reduced to some extent. It is anticipated that less motivated students may have been less willing to participate. This may have caused some selection bias. Another limitation is that the Finnish university has far more applicants per place compared to the Swedish and Norwegian universities. This implies that entrance to the Finnish university requires better marks in upper secondary school compared to the other two universities. Hence it was likely that the Finnish participants would perform better, which they also did. On the other hand, the relative size of the Finnish sample is approximately 1.8 times larger than that of the other two samples. This reduces the qualitative difference between the samples because, due to its size, the Finnish sample necessarily contains also less motivated and less gifted students.

A possible source for a minor analytical bias is the fact that the first versions of some tasks were adapted from Finnish upper secondary learning materials. This does not necessarily mean that the final versions were more challenging for the Swedish and Norwegian students, but it is acknowledged that the areas of focus and formulation of tasks may slightly vary across the textbooks used in the participating countries. Even small changes in formulation or design may create a feeling of unfamiliarity with a task, which may induce a remarkable change in the performance of students (Tossavainen, Haukkanen, and Pesonen 2013; Tossavainen et al. 2015). The above-mentioned limitations are considered in the analyses and this study does not draw any conclusions that assume comparisons between the national cohorts.

5.4. Analyses

Each student's responses to Tasks 4.1–4.7 were independently evaluated by two authors on the following scale: 0 = 'No answer/almost completely wrong answer', 1 = 'A relevant strategy but major mistakes or deficiencies/a correct answer without justification', 2 = 'A relevant strategy but at most two minor mistakes or deficiencies in the argument', 3 = 'A correct answer with a sufficient justification'. Crohnbach's alphas for the assessments of the tasks vary from 0.81 to 0.96, indicating the very high internal consistency of the assessments. The final scores were computed as the means of individual evaluations. Hence the maximum of the sum scores for the tasks is 21.

The data were analysed using SPSS Statistics version 25. In addition to computing standard descriptive measures for the concerned variables, the following methods were applied: Student's independent samples and paired samples t-tests, one-way analysis of variance (One-way ANOVA) with post hoc tests, Pearson correlation analysis, partial correlation analysis, and linear regression analysis. Finally, it is noted that the number of participants varies slightly across the tables in the following section. This is because some students' responses were empty or hard to read for a couple of items. These responses have been excluded from those analyses.

6. Results

Table 1 summarises how male and female students performed in the seven mathematical tasks. The students' motivational values and self-efficacy beliefs are reported in Table 2, and a summary of their views of the nature of mathematics is given Table 3. Tables 4 and 5 show how the students' task performance is related to their motivational values and self-efficacy, and Tables 6 and 7 display how the task performance relates to the distribution of the students' views of the nature of mathematics. Tables 8 and 9 answer to what extent the students' task performance can be predicted from their views of the nature of mathematics and their motivational values.

The 'Null answers' rows in Table 1 indicate the portion of students who either did not provide an answer or whose answer was evaluated to be seriously wrong in the respective tasks. Table 1 suggests that the participating students' performance was not very good given that it would have been easy to score nine points just by simplifying two elementary expressions and by solving a standard problem on percentages. Most participants correctly solved the simple tasks based on percentages (4.1) and simplified powers (4.3), but they were not as good at manipulating rational expressions (4.2). Half of the students failed to analyse the sufficient criteria for an increasing sequence (4.4). Further, a third of the students were not able to apply mathematics to solve a concrete problem on the effectiveness of a medicine (4.5), although a detailed explanation was not assumed. Similarly, a third could not give a definition of a decreasing function (4.6). Only 23 out

Table 1. Performance in Tasks 4.1–4.7 (N = 422–423).

	4.1	4.2	4.3	4.4	4.5	4.6	4.7	Sum
Male (N = 305)								
Mean	2.25	1.48	2.09	0.53	1.12	1.36	0.47	9.30
Std. dev.	1.15	1.23	1.13	0.84	0.99	1.13	0.88	4.57
Null answers (%)	14	29	16	54	31	30	71	6
Female (N = 118)								
Mean	2.33	1.73	2.34	0.61	1.20	1.54	0.40	10.15
Std. dev.	1.17	1.28	1.01	0.87	0.91	1.15	0.80	4.77
Null answers (%)	15	26	9	41	20	25	72	6

Table 2. Means and standard deviations of self-efficacy and motivational value items (Scale: 1–5, N_{male} = 303–305, N_{female} = 117–118).

	$\bar{x}_{male}$	d	$\bar{x}_{female}$	d	t	p
2.5. In school, I was good at mathematics (S)	3.81	0.93	3.96	0.94	1.44	>.05
2.6. In school, I was able to understand most of the mathematics (S)	3.94	0.85	4.05	0.90	1.21	>.05
2.7. I really like mathematics (I)	3.63	0.85	3.85	0.90	2.28	<.05
2.8. I am motivated to study maths mainly because it is useful to other studies (U)	3.59	0.93	3.14	0.95	−4.35	<.001
2.9. I want to succeed as well as possible (A)	4.42	0.65	4.29	0.73	−1.82	>.05
2.10. I would suspend a hobby in order to succeed in a maths exam (C)	3.67	0.94	3.31	1.03	−3.36	<.01
2.11. I would do extra exercises to guarantee that I succeeded well (C)	3.89	0.83	4.06	0.73	1.97	<.05
2.12. I would study maths voluntarily because every engineer must know it (U)	3.92	0.84	3.70	0.96	−2.19	<.05
2.13. If I get a low grade in mathematics, I want to take the exam again (A)	3.13	1.05	3.30	1.04	1.51	>.05
2.14. Mathematics is full of interesting problems and results (I)	3.95	0.91	3.73	0.80	−2.49	<.05

Table 3. Descriptive statistics on students' views of the nature of mathematics (Scale: 1–5, N_{male} = 298–300, N_{male} = 116–118).

	$\bar{x}_{male}$	d	$\bar{x}_{female}$	d	t	p
3.1. Mathematics is about describing the real world (A)	4.04	0.81	3.81	0.86	−2.49	<.05
3.2. It is not mathematics if it cannot be proved theoretically (F)	3.30	0.89	3.46	0.92	1.62	>.05
3.3. Mathematics is a collection of formulas and concepts (S)	3.11	1.00	3.37	0.93	2.43	<.05
3.4. Mathematics is problem solving (*P*)	3.72	0.92	3.89	0.89	1.69	>.05
3.5. The purpose of mathematics is to maintain functionality in society (A)	3.29	0.92	3.19	0.89	−1.00	>.05
3.6. Mathematics is about discovering structures and regularities (*P*)	3.75	0.78	3.59	0.76	−1.87	>.05
3.7. The main task of mathematics is to give correct rules for calculations (S)	3.19	1.04	3.15	0.86	−0.34	>.05
3.8. In mathematics, all concepts must be defined in a precise and clear way (F)	4.01	0.82	4.07	0.77	0.70	>.05

Table 4. Pearson correlations between motivational values, self-efficacy, and sum scores in Tasks 4.1–4.7 (N = 431).

	Scores	Self-efficacy	Intrinsic	Attainment	Utility	Cost
Scores	1	0.41**	0.28**	−0.02	−0.16	−0.17*
Self-efficacy		1	0.47**	0.17*	−0.01	−0.15*
Intrinsic			1	0.34**	0.18**	0.08
Attainment				1	0.25**	0.37**
Utility					1	0.27**
Cost						1

*p < .01, **p < .001.

Table 5. Partial correlations between motivational values and sum scores in Tasks 4.1–4.7 when controlled for self-efficacy (N = 431).

	Scores	Intrinsic	Attainment	Utility	Cost
Scores	1	0.12*	0.11*	−0.18**	−0.13*
Intrinsic		1	0.30**	0.21**	0.18**
Attainment			1	0.24**	0.40**
Utility				1	0.26**
Cost					1

*p < .05, **p < .001.

Table 6. Pearson correlations between orientations and sum scores in Tasks 4.1–4.7 (N = 421–431).

	Scores	Formalism	Scheme	Process	Application
Scores	1	0.06	−0.23**	−0.03	0.08
Formalism (3.68)		1	0.10*	0.07	0.05
Scheme (3.19)			1	0.21**	0.04
Process (3.78)				1	0.17*
Application (3.63)					1

*p < .05, **p < .001.

Table 7. Mean scores in Tasks 4.1–4.7 across the metaphor categories (N = 399).

	4.1	4.2	4.3	4.4	4.5	4.6	4.7	Sum
Toolbox (N = 138)	2.39	1.47	2.20	0.47	1.27	1.41	0.41	9.60
Applications (N = 43)	2.15	1.45	2.20	0.37	1.14	1.45	0.29	9.06
Problem solving (N = 165)	2.32	1.67	2.17	0.62	1.07	1.39	0.40	9.64
Exact reasoning (N = 53)	2.35	1.93	2.55	0.87	1.40	1.96	0.91	11.96

Table 8. Significant predictors for the scores in Tasks 4.1–4.7.

	4.1	4.2	4.3	4.4	4.5	4.6	4.7	Sum
Formalism								
Scheme			−0.21	−0.12	−0.14	−0.20	−0.16	−0.23
Process	0.11							
Application	−0.11				0.14			0.12
R^2	0.02	–	0.04	0.01	0.04	0.04	0.02	0.07

Table 9. The regression model for the sum scores with motivational values and self-efficacy as possible predictors.

	Beta	t	*p*
Self-efficacy	0.33	6.79	<.001
Intrinsic	0.20	3.87	<.001
Attainment	−0.11	−2.33	<.05
Utility	−0.17	−3.86	<.001
Cost	–		

R^2 = 0.22, $F(4,425) = 30.43$.

of 463 students were able to construct an example of a function that satisfies the given criteria in Task 4.7. In Task 4.3, the female students' mean is significantly higher than that of the male students ($t(421) = 2.10$, $p < .05$). If the significance level is raised to 0.10, then the mean difference is significant also in Task 4.2 ($t(421) = 1.89$, $p = .06$) and in the sum scores ($t(421) = 1.71$, $p = .09$). In both cases, the mean is higher for female students.

Table 2 summarises the respondents' motivational values and their self-efficacy in mathematics. The letter at the end of each item refers to the latent variable that is operationalised by the statement. They are S = self-efficacy, I = intrinsic value, U = utility value, A = attainment value, and C = cost. In order to increase the readability of Table 2, the statements are not presented exactly as they appear in the questionnaire, but in condensed form (cf. the Appendix).

In Table 2, the mean difference is statistically significant in 2.7–8, 2.10–12, and 2.14. The degree of freedom is smaller in 2.12 than in the other items since, by the Levene's test, the variances in the groups cannot be assumed to be equal. These differences reveal that the first-year engineering students' motivational values are not completely similar across the genders. Female students' means are higher in items 2.7 and 2.11, which represent the joy of studying mathematics and the willingness to invest extra time in learning mathematics. Male students express higher values in 2.8, 2.10, 2.12, and 2.14, which stand for utility value, interest in mathematics, and cost by being willing to (temporarily) suspend a hobby in order to succeed in an exam. Especially in utility value and cost, the mean difference can already be said to have a practical significance; the effect size (Cohen's d) is 0.48 for 2.8 and 0.37 for 2.10. In 2.5 and 2.6, the means are higher for female students, yet the differences are not statistically significant.

Table 3 summarises the descriptive statistics of statements measuring male and female students' views of the nature of mathematics. The mean differences are statistically significant in 3.1 and 3.3. In 3.6, the difference is significant at the $p < .10$ level. Here the differences are interpreted to mean that male students associate mathematics more strongly or more often with the real world and its phenomena compared to female students. Further, the female students' scheme-related orientation is a little stronger than that of their male peers. By and large, the respondents' view of the nature of mathematics is quite strongly associated with describing the real world (3.1), but they also acknowledge the activities related to problem-solving (3.4), the discovery of structures and regularities in general (3.6), and the need for precision and clarity in using mathematical notions (3.8). On the other hand, the role of proving is clearly more marginal to them, e.g. the mean differences between 3.2 and 3.4 and 3.2 and 3.8 are highly significant (($t(423) = -7.24$, $p < .001$) and ($t(423) = -13.22$, $p < .001$), respectively).

In order to study how the respondents' sum scores, motivational values, and self-efficacy beliefs are related, the values and self-efficacy have been computed as the means of the corresponding statements given in Table 2. The Pearson correlation coefficients are shown in Table 4. Table 5 contains the partial correlation coefficients with self-efficacy as the control variable. In other words, it shows how motivational values and task performance are related to one another when students are equalised with respect to self-efficacy.

Table 4 shows that self-efficacy is the variable that has the highest correlation with the success in the given mathematical tasks. Moreover, the correlation is positive, which indicates that task performance depends directly on self-efficacy. Of the motivational values, intrinsic value has the highest correlation with the sum scores. Table 5 shows the order of the dependence of task performance on the motivational values when the effect of self-efficacy is controlled. The most interesting and also rather surprising finding is that utility value has the highest correlation with the sum scores and the correlation is negative. On the other hand, the correlations between scores and the motivational values are quite weak in both tables. In other words, task performance does not depend on a single motivational value. As a matter of fact, there is no significant correlation between the sum scores and the sum variable computed as the mean of the four value variables in Table 4. This underlines the relevance of self-efficacy for task performance.

For Table 6 and beyond, the sum variables for each orientation have been computed as the mean of the Likert scales representing the orientation. The mean values of these constructs are also given in the parentheses after the name of each sum variable. They show that the participants' orientations are quite evenly distributed, except for the scheme-related orientation. In Bonferroni's post hoc test, the mean of this orientation is statistically significantly weaker than the means of the other orientations ($F(3,1692) = 46.60$, $p < .001$).

In general, the correlation coefficients in Table 6 are not high, yet two correlations are statistically significant at the $p < .001$ level. Further, it is somewhat surprising that the correlation between the success in tasks and the scheme-related orientation is negative. As this orientation represents views of mathematics as a collection of procedures and rules, this correlation suggests that students who have a broader view of mathematics than seeing it merely as a toolbox succeed better in solving various mathematical tasks.

To complete our answer to the third question, the distributions of mean scores in the tasks across the categories that are due to the students' metaphor of mathematics have been recorded in Table 7.

In Bonferroni's post hoc test, there are statistically significant mean differences in Tasks 4.3 ($p < .05$), 4.5 ($p < .05$), 4.6 ($p < .05$), 4.7 ($p < .01$), and in the sum scores ($p < .01$). In all of these items, the group of students who chose the metaphor 'Exact reasoning' has the highest mean, whereas those who chose 'Applications' or 'Problem-solving' have the lowest mean.

The last research question has been answered by recording the results from the linear regression analyses (method: stepwise). For better readability, Table 8 shows only the standardised coefficients that are significant at the $p < .05$ level and the R^2, the coefficient of determination. The latter notion measures the proportion of the variance in the dependent variable (sum scores) that is predictable from the independent variables (the orientation categories).

Table 8 shows that the sum variable standing for the scheme-related orientation is a significant predictor for success in each of tasks 4.3–4.7 and, consequently, for the sum scores, too. Further, the application-related orientation also predicts success in a couple of the tasks and in the whole set of tasks. The sum variable of process-related orientation is a significant predictor in 4.1. In 4.2, none of the orientation constructs is a significant predictor. The coefficient of determination is relatively low in each regression model. This is quite typical for educational studies; one or two independent variables only rarely have enough power to explain a major part of the variance of a variable measuring learning or performance in a task. To provide a complementary perspective for these results, we record also a linear regression model to predict the sum scores from self-efficacy and the sum variables of the motivational values given in Table 4.

In light of Tables 4 and 5, it is not surprising that self-efficacy and intrinsic value are the strongest predictors in Table 9. A fact that can be especially relevant is that utility value is also a highly significant predictor. However, the coefficients for utility value and attainment value are negative.

7. Discussion and conclusions

Table 1 summarises the participants' performance in each task separately for male and female respondents. It shows that female students performed somewhat better than male students did. In addition, in the self-efficacy items (2.5 and 2.6) in Table 2, the means were higher for female students. This finding is worth acknowledging, since many previous studies suggest that female students' self-efficacy in mathematics is lower than that of male students (Sax et al. 2015). There are also studies that associate mathematics anxiety with gender, so that female students suffer more anxiety dealing with mathematical tasks compared to male students (Alves et al. 2016). The present results show that this may not be the case in every culture. Our findings support Guo et al. (2015b), who have pointed to that female students tend to have higher mathematics achievement and educational aspirations compared to male students if they have similar levels of mathematics self-concept and values. In general, the participating students' task performance was satisfactory. They were able to solve the basic tasks (4.1–4.3) and able to apply their mathematical knowledge when a precise explanation was not required (4.5). Their mean scores in the other tasks are quite low, but the standard deviations are relatively high. This indicates that a minority of students performed well also in these tasks. However, the tasks requesting students to define the notion of decreasing function or to construct a monotone function with a couple of quite elementary properties were too challenging for most participants.

Table 2 shows some significant gender differences in motivational values. In light of the previous research (Vitasari et al. 2010; Sax et al. 2015; Guo et al. 2015a), it is a significant and less often reported result that Nordic female first-year engineering students express at least equally high levels of motivation as their male peers. Bengmark, Thunberg, and Winberg (2017) did not focus on the gender perspective in their study, but a common finding in their study and this study is that self-efficacy is strongly related to task performance. In their data, this is shown at the end of the first year at university; in our data, it is visible already at the beginning of the first year.

Concerning Table 3, it is interesting to observe that male students agreed more strongly with the claim that mathematics is full of interesting problems and results (2.14), but female students expressed more strongly that they really like mathematics (2.7). These differences seem to summarise a more general phenomenon: male students are interested in results and the usefulness of mathematics, whereas female students enjoy succeeding in learning mathematics and mathematical thinking in general. Female students seem to set more demanding learning goals for themselves, too (2.11). Another finding that has a novelty value is related to Tables 4, 5, and 9. Utility and attainment values correlated negatively with the task performance. This suggests that a strong extrinsic motivation does not help to achieve better results in mathematical tasks but, actually, affects in the opposite direction.

Concerning the effects of the epistemological beliefs about the nature of mathematics on task performance, it is noteworthy how the mean value and the correlation coefficients related to the scheme-related orientation differ from those related to other orientations. At the outset of this study, the authors hypothesised that some orientations might have an especially significant role in predicting engineering students' performance in certain kinds of mathematical tasks. Tables 6–8 support this hypothesis, but the relation is quite unexpected: the weaker the scheme-related orientation is, the better is the task performance. On the other hand, this effect is not very strong. It is anyway concluded that putting emphasis on the 'toolbox' view of mathematics in teaching mathematics does not improve the learning results – it may be even counter-productive. An indication of the detrimentally emphasised 'toolbox' view is that students ask for concrete examples of how to apply mathematical results in various situations instead of showing interest in discussing the theoretical foundations of these results. The authors' experience is that this phenomenon is quite common, especially in lectures immediately before course exams, cf. Tossavainen et al. (2020).

When it comes to the motivational values, one might expect that a clear indication of readiness to invest extra time to studying mathematics would yield high task performance. However, in Table 9, cost is not a significant predictor for the sum scores. Another interesting detail in the same table is that attainment value is a significant predictor for the sum scores, although there is not a significant correlation between the sum scores and attainment value in Table 4. This outcome is not a contradiction or an error because regression models are based on partial correlations. For instance, the partial correlation between the sum scores and Attainment is −0.13 ($p < .01$) when intrinsic value is controlled for.

In conclusion, Nordic students arrive at university with quite good levels of motivation and self-efficacy, and with rather positive prior experiences from studying mathematics. They are also aware of the versatility and multidimensional nature of mathematics, yet they focus on solving problems and discovering regularities more than on other aspects of mathematics. Some of the students seem to have a little over-optimistic conception of their knowledge about or motivation for mathematics, but putting more emphasis on discussing the fundamental concepts and their role in mathematics and supporting students in developing reasoning skills already in the early stage of transition from upper secondary mathematics to engineering mathematics should result in better performance.

Acknowledgement

The authors like to thank Pål Gunnar Ellingsen for carefully reading the manuscript and providing useful comments. The authors also want to thank Matthew James for proofreading the text.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Timo Tossavainen received his PhD in mathematics from Jyväskylä University, Finland. Currently, he is professor of mathematics education at Lulea University of Technology in Sweden.

Ragnhild Johanne Rensaa is professor of mathematics and vice-dean of education at The Arctic University of Norway, campus Narvik.

Pentti Haukkanen received his PhD in mathematics from University of Tampere, Finland. Today, he works as university lecturer of mathematics at Tampere University in Finland.

Mika Mattila has got a PhD in mathematics from University of Tampere and he is university teacher of mathematics at Tampere University.

Monica Johansson received her PhD in mathematics education at Lulea University of Technology, where she works as university lecturer. All authors share a wide interest in both mathematics and mathematics education.

ORCID

Timo Tossavainen http://orcid.org/0000-0002-7494-4632
Ragnhild Johanne Rensaa http://orcid.org/0000-0003-3275-4483
Pentti Haukkanen http://orcid.org/0000-0002-1723-1193
Mika Mattila http://orcid.org/0000-0001-7946-6433

References

Alves, M., C. S. Rodrigues, A. M. A. C. Rocha, and C. Coutinho. 2016. "Self-efficacy, Mathematics' Anxiety and Perceived Importance: an Empirical Study with Portuguese Engineering Students." *European Journal of Engineering Education* 41 (1): 105–121. doi:10.1080/03043797.2015.1095159.

Bandura, A. 1982. "Self-efficacy Mechanism in Human Agency." *American Psychologist* 37 (2): 122–147. doi:10.1037/0003-066X.37.2.122.

Bandura, A. 2012. "On the Functional Properties of Perceived Self-Efficacy Revisited." *Journal of Management* 38 (1): 9–44. doi:10.1177/0149206311410606.

Bengmark, S., H. Thunberg, and T. M. Winberg. 2017. "Success-Factors in Transition to University Mathematics." *International Journal of Mathematical Education in Science and Technology* 48 (7): 988–1001. doi:10.1080/0020739X.2017.1310311.

Eccles, J. S., T. F. Adler, R. Futterman, S. B. Goff, C. M. Kaczala, J. Meece, and C. Midgley. 1983. "Expectancies, Values and Academic Behaviors." In *Achievement and Achievement Motives*, edited by J. T. Spence, 75–146. San Francisco, CA: W. H. Freeman.

Eccles, J., A. Wigfield, R. D. Harold, and P. Blumenfeld. 1993. "Age and Gender Differences in Children's Self- and Task Perceptions During Elementary School." *Child Development* 64 (3): 830–847. doi:10.2307/1131221.

Faulkner, F., A. Hannigan, and O. Fitzmaurice. 2014. "The Role of Prior Mathematical Experience in Predicting Mathematics Performance in Higher Education." *International Journal of Mathematical Education in Science and Technology* 45 (5): 648–667. doi:10.1080/0020739X.2013.868539.

Felbrich, A., C. Müller, and S. Blömeke. 2008. "Epistemological Beliefs Concerning the Nature of Mathematics among Teacher Educators and Teacher Education Students in Mathematics." *ZDM Mathematics Education* 40: 763–776. doi:10.1007/s11858-008-0153-5.

Flegg, J., D. Mallet, and M. Lupton. 2012. "Students' Perceptions of the Relevance of Mathematics in Engineering." *International Journal of Mathematical Education in Science and Technology* 43 (6): 717–732. doi:10.1080/0020739X.2011.644333.

Gordon, S., and J. Nicholas. 2013. "Students' Conceptions of Mathematics Bridging Courses." *Journal of Further and Higher Education* 37 (1): 109–125. doi:10.1080/0309877X.2011.644779.

Grigutsch, S., U. Raatz, and G. Törner. 1998. "Einstellungen gegenüber mathematik bei mathematiklehrern [Mathematics Teachers' Attitudes Towards Mathematics]." *Journal für Mathematikdidaktik* 19: 3–45. doi:10.1007/BF03338859.

Guo, J., H. W. Marsh, P. D. Parker, A. J. S. Morin, and A. S. Yeung. 2015b. "Expectancy-value in Mathematics, Gender and Socioeconomic Background as Predictors of Achievement and Aspirations: A Multi-Cohort Study." *Learning and Individual Differences* 37 (January): 161–168. doi:10.1016/j.lindif.2015.01.008.

Guo, J., P. D. Parker, H. W. Marsh, and A. J. S. Morin. 2015a. "Achievement, Motivation, and Educational Choices: A Longitudinal Study of Expectancy and Value Using a Multiplicative Perspective." *Developmental Psychology* 51 (8): 1163–1176. doi:10.1037/a0039440.

Harris, D., L. Black, P. Hernandez-Martinez, B. Pepin, and J. Williams, with the TransMaths Team. 2015. "Mathematics and its Value for Engineering Students: What Are the Implications for Teaching?" *International Journal of Mathematical Education in Science and Technology* 46 (3): 321–336. doi:10.1080/0020739X.2014.979893.

Hofer, B., and P. R. Pintrich. 1997. "The Development of Epistemological Theories: Beliefs About Knowledge and Knowing and Their Relation to Learning." *Review of Educational Research* 67 (1): 88–140. doi:10.3102/00346543067001088.

Kosiol, T., S. Rach, and S. Ufer. 2019. "(Which) Mathematics Interest is Important for a Successful Transition to a University Study Program?" *International Journal of Science and Mathematics Education* 17: 1359–1380. doi:10.1007/s10763-018-9925-8.

Kouvela, E., P. Hernandez-Martinez, and T. Croft. 2018. ""This is What you Need to be Learning": an Analysis of Messages Received by First-Year Mathematics Students During Their Transition to University." *Mathematics Education Research Journal* 30 (2): 165–183. doi:10.1007/s13394-017-0226-2.

Kümmerer, B. 2001. "Trying the Impossible: Teaching Mathematics to Physicists and Engineers." In *The Teaching and Learning of Mathematics at University Level: An ICMI Study*, edited by D. Holton, 321–334. Dordrecht: Kluwer Academic Publishers.

Ryan, R. M., and E. L. Deci. 2000. "Intrinsic and Extrinsic Motivations: Classic Definitions and new Directions." *Contemporary Educational Psychology* 25 (1): 54–67. doi:10.1006/ceps.1999.1020.

Sax, L. J., M. A. Kanny, T. A. Riggers-Piehl, H. Whang, and L. N. Paulson. 2015. ""But I'm Not Good at Math": The Changing Salience of Mathematical Self-Concept in Shaping Women's and Men's STEM Aspirations." *Research in Higher Education* 56 (8): 813–842. doi:10.1007/s11162-015-9375-x.

Tossavainen, T., J. Gröhn, L. Heikkinen, A. Kaasinen, and A. Viholainen. 2020. "University Mathematics Students' Study Habits and use of Learning Materials." *LUMAT: International Journal on Math, Science and Technology Education* 8 (1): 252–270. doi:10.31129/LUMAT.8.1.1317.

Tossavainen, T., P. Haukkanen, and M. Pesonen. 2013. "Different Aspects of the Monotonicity of a Function." *International Journal of Mathematical Education in Science and Technology* 44 (8): 1117–1130. doi:10.1080/0020739X.2013.770088.

Tossavainen, T., and A. Juvonen. 2015. "Finnish Primary and Secondary School Students' Interest in Music and Mathematics Relating to Enjoyment of the Subject and Perception of the Importance and Usefulness of the Subject." *Research Studies in Music Education* 37 (1): 107–121. doi:10.1177/1321103X15589259.

Tossavainen, T., R. J. Rensaa, and M. Johansson. 2019. "Swedish First-Year Engineering Students' Views of Mathematics, Self-Efficacy, and Motivation and Their Effect on Task Performance." *International Journal of Mathematical Education in Science and Technology*. Published online. doi:10.1080/0020739X.2019.1656827.

Tossavainen, T., P. Väisänen, J. K. Merikoski, T. Lukin, and H. Suomalainen. 2015. "A Survey on the Performance of Finnish Students' Arithmetical Skills and the Role of Motivation." *Education Research International*. Article ID 213429. Published online. doi:10.1155/2015/213429

Tossavainen, T., A. Viholainen, M. A. Asikainen, and P. E. Hirvonen. 2017. "Explorations of Finnish Mathematics Students' Beliefs About the Nature of Mathematics." *Far East Journal of Mathematical Education* 17 (3): 105–120. doi:10.17654/ME017030105.

Vitasari, P., T. Herawan, M. N. A. Wahab, A. Othman, and S. K. Sinnadurai. 2010. "Exploring Mathematics Anxiety among Engineering Students." *Procedia – Social and Behavioral Sciences* 8: 482–489. doi:10.1016/j.sbspro.2010.12.066.

Wigfield, A., and J. S. Eccles. 2000. "Expectancy–Value Theory of Achievement Motivation." *Contemporary Educational Psychology* 25 (1): 68–81. doi:10.1006/ceps.1999.1015.

Zakariya, Y. F., H. K. Nilsen, S. Goodchild, and K. Bjørkestøl. 2020. "Self-efficacy and Approaches to Learning Mathematics among Engineering Students: Empirical Evidence for Potential Causal Relations." *International Journal of Mathematical Education in Science and Technology*. Published online. doi:10.1080/0020739X.2020.1783006.