



The Impact of Turbulence on the Ionosphere and Magnetosphere

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An important property associated with turbulence in plasmas and fluids is anomalous transport. Plasma, being a good conductor, can in addition be affected by turbulence causing an anomalous resistivity that can significantly exceed its classical counterpart. While turbulent transport may be adequately described in configuration space, some aspects of the anomalous resistivity are best accounted for in phase space. Kinetic phenomena like electron and ion phase space vortices can thus act as obstacles for the free flow of slow charged particles. Plasma instabilities and large amplitude plasma waves are candidates for contributions to the anomalous resistivity by generating such structures. Langmuir waves can be relevant, but also others, such as upper- as well as lower-hybrid waves in magnetized plasmas. Often these anomalous resistivity effects can be small, but due to the large spatial and temporal scales involved in space plasmas, planetary ionosphere and magnetosphere in particular, even such moderate effects can be important. This mini-review is discussing elements of the description of plasma turbulence with particular attention to wave phenomena that contribute to anomalous resistivity and diffusion. Turbulence effects can have relevance for space weather phenomena as well, where ground based and airborne activities relying on for instance Global Positioning and Global Navigation Satellite Systems are influenced by plasma conditions in geospace.

Keywords: plasma, turbulence, ionosphere, anomalous resistivity, nonlinear waves, diffusion

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1 INTRODUCTION

Plasmas, magnetized plasmas in particular, can support a variety of wave phenomena, electromagnetic as well as electrostatic. These waves can be excited in laboratory experiments, and are frequently found to be generated by some instability mechanisms in naturally occurring plasmas, in the Earth's ionosphere and magnetosphere in particular, as described by, e.g., Shawhan (1979), Shawhan (1985). Informative summary figures can be found in these works. Controlled laboratory plasma studies often assume conditions where harmonic wave phenomena are excited, but these are rarely met in natural environments where turbulent states are more likely to develop. For some conditions further developed in this review, such turbulent stages can have similarities to classical hydrodynamic turbulence.

In neutral fluids and gases, 'strongly' turbulent states often develop, while in plasmas, turbulence is often observed to be 'weak'. For discussing this distinction, we consider a nonlinear model wave equation (Dupree, 1969; Similon and Sudan, 1990; Galtier, 2009) in the form

$$\frac{\partial Q_\ell(\mathbf{k}, t)}{\partial t} + i\omega(\mathbf{k})Q_\ell(\mathbf{k}, t) = \sum_{m,n} \sum_{\mathbf{k}'} M_{\ell,m,n}(\mathbf{k}, \mathbf{k}') Q_m(\mathbf{k}', t) Q_n(\mathbf{k} - \mathbf{k}', t). \quad (1)$$

The indices $\{\ell, m, n\} \in \{x, y, z\}$ label components of the complex vector $\mathbf{Q}(\mathbf{k}, t)$. The nonlinear term in the right hand side contains the coupling coefficients $M_{\ell,m,n}$ between components for wavenumbers \mathbf{k} and \mathbf{k}' . Any quadratically nonlinear partial differential equation with a first order time derivative can be brought in the form of Eq. 1 by a Fourier series representation of the variables in configuration space. One such example is the Navier-Stokes equation (Kollmann, 2019) where Q_ℓ represents the incompressible fluid velocity component u_ℓ , and $i\omega(\mathbf{k}) \rightarrow k^2\nu$, where ν is the fluid kinematic viscosity. For plasma waves, on the other hand, Q_ℓ can represent the electric field component E_ℓ , while $\omega(\mathbf{k})$ is a linear dispersion relation, which may be complex for some wave vector ranges for linearly unstable systems. We let \mathcal{M} be a ‘representative’ value of $M_{\ell,m,n}$, for the Navier-Stokes equation we have, $\mathcal{M}_{\text{NS}} \sim k$. An effective Reynolds number \mathcal{R} can be constructed as $\mathcal{R} = \mathcal{M}Q/\omega(k)$ i.e., the ratio of two time-scales, one originating from the nonlinear part in the right hand side and the other representing the linear part in the left hand side. When Eq. 1 represents the Navier-Stokes equation, we find $\mathcal{R}_{\text{NS}} \approx U/(K\nu)$ where U is a ‘representative’ velocity and K a ‘representative’ wavenumber. For a pipe flow, U would be the injected flow velocity and $K \sim 1/D$ the inverse diameter of the pipe. When $U \gg \nu/D$, we expect nonlinearity to dominate and strong turbulence to develop. For fluids and gases, where ν is generally small, this condition is easily fulfilled and we can anticipate that strongly turbulent states can be obtained. In the limit where $\omega(k)$ is negligible, the dynamics of fluids or gases is solely due to nonlinear effects. For the Navier-Stokes equation, this is the case for the limit where $k^2\nu$ is negligible. For a plasma, the situation is frequently the opposite (Dupree, 1969), the linear part $\omega(k)$ is often large. For Langmuir waves $\omega(k) \approx \omega_{pe}$, the electron plasma frequency. The corresponding plasma Reynolds number is $\mathcal{R}_p \approx \mathcal{E}\mathcal{M}_p/\omega(K)$, with \mathcal{E} being a representative electrostatic electric field value, and K a representative wavenumber to be determined by some externally imposed parameters, e.g., the geometry. \mathcal{R}_p is often a small number, resulting in a weakly turbulent state (Dupree, 1969; Pécseli, 2016). Models for weak turbulence have been developed for plasma sciences (Kadomtsev, 1965; Nicholson, 1983; Horton, 1985; Kono and Škorić, 2010). Depending on the plasma conditions, the magnetization for instance, both strongly and weakly turbulent conditions can be found.

Diagnostic tools involving two-point measurements that can distinguish strong and weak turbulence have been developed (Iwama et al., 1979; Beall et al., 1982; Pécseli, 2015). In some cases, the linear dispersion relation can be identified in the (ω, \mathbf{k}) -space, thus demonstrating the importance of the $\omega(\mathbf{k})$ term in Eq. 1, while for strongly turbulent conditions a similar analysis shows enhanced amplitudes for a wide range of wave vectors with no discernible frequency-wavenumber relation.

Classical physics, and thus also plasma physics, used to rest on a two-fold basis: theory and observations, the latter including results from controlled laboratory experiments. With the development of

efficient high-speed computing, it has become possible to make numerical simulations on a level competing with laboratory results. Modern physics, also plasma physics, now rests on a basis of three pillars, theory, experiment and numerical simulations (Post and Votta, 2005; Hut, 2006). Use of advanced high performance computing techniques allow massively parallel computations involving tens of thousands of cores. Modern numerical simulations can in many cases serve as a substitute for experiments. Fine details in the variations of the magnetospheric plasma can now be resolved in numerical simulations (von Alfthan et al., 2014). Predictions of such details can be important for instance for space weather phenomena (Morley, 2020) that can have effects being important for activities on ground. Scintillations in the plasma index of refraction affecting Global Positioning Systems (GPS) and Global Navigation Satellite Systems (GNSS) offer examples (Jin et al., 2020). The energy source for intense space weather phenomena is often found in solar outbursts that give rise to fluctuations in the Earth’s plasma environment (Sato et al., 2019). Numerical simulations can also give support when interpreting observations made by instrumented spacecraft. If the simulations reproduce the observed data, it can be assumed that the numerical results can be trusted also for information not directly accessible for confirmation by measurements.

2 TURBULENT SPECTRA

2.1 Strong Turbulence in Neutral Fluids

Fully developed strong turbulence in incompressible neutral flows develops a universal continuous power spectrum $F(k)$ for the velocity fluctuations. Ignoring intermittency effects, a spectrum characterizing an inertial subrange for homogeneous and isotropic conditions is found to follow a power law $\sim C_K \epsilon^{2/3} k^{-5/3}$, the Kolmogorov-Obukhov spectrum, where ϵ is the energy dissipation rate per unit fluid mass, and C_K the Kolmogorov constant (Sreenivasan, 1995) which has to be determined experimentally. For shorter wavelengths, Heisenberg (1948) predicted the existence of a viscous spectral subrange following a $\sim k^{-7}$ power law (Heisenberg and Taylor, 1948; Beran, 1968). Although some numerical and experimental studies seem to offer support for this result, it is unlikely to be correct (Landahl and Mollo-Christensen, 1992) since it predicts that, for instance, the integral for spatial derivative spectrum (Bendat, 1958) of m th order, $\int_0^\infty k^{2m} F(k) dk$, diverges for $m \geq 3$. This would imply that the flow develops singular third and higher order spatial derivatives. There are no indications that the Navier-Stokes equation has this property for three-dimensional turbulence with ‘smooth’ initial conditions (Sulem et al., 1983; Constantin, 1991), and there is no natural wavenumber associated with the Navier-Stokes equation to give an upper limit for a viscous subrange. For turbulence confined to two-spatial dimensions, the situation might be different. The classical Kolmogorov-Obukhov result implicitly assumes the energy dissipation ϵ to be a deterministic quantity. In reality, it is randomly fluctuating so that the dissipation can take place in ‘hot spots’ i.e., within intermittent spatial regions and time periods (Novikov, 1990; Davidson, 2004; Kollmann, 2019).

When additional energy is injected into a three-dimensional turbulent field, classical turbulence models predicts the energy to cascade toward smaller scales by a ‘vortex-stretching’ mechanism (Tennekes and Lumley, 1972). One feature of two-dimensional turbulence is that energy can cascade toward *larger* scales. Convincing observations of this process seems to be missing in space plasmas, but it has been amply demonstrated by numerical simulations. Turbulence models in two spatial dimensions (Hasegawa and Mima, 1978; Kraichnan and Montgomery, 1980; Gruzinov and Pogutse, 1986) are relevant for low frequency phenomena confined to a plane $\perp \mathbf{B}$ in magnetized plasmas.

2.2 Strong Turbulence in Plasmas

One important parameter classifying plasma conditions is β , the ratio of plasma to magnetic pressure. Note that $\beta \approx (C_s/V_A)^2$, the square ratio of the ion acoustic sound speed to the Alfvén velocity. In the near Earth environment we usually have $\beta \ll 1$. For larger β -values, as in the Solar wind, a coupling between transverse electromagnetic perturbations and compressional modes can be expected in magnetized plasmas.

There are no obvious reasons for turbulent spectra to follow a power law, but it has been found to be the case sometimes also for fully developed strong plasma turbulence. There are thus solid evidences that fully developed strong resistive electrostatic drift-wave turbulence in plasmas confined by strong magnetic fields develops a $\sim k^{-5}$ power-law spectral subrange for fluctuations in the electrostatic potential (Tchen et al., 1980; Pécseli, 2015; Pécseli, 2016). The continuation of this drift-wave spectrum to wavelengths shorter than the ion Larmor radius remains poorly understood.

The conditions in the Earth’s ionospheric *E*- and *F*-regions are special due to the importance of collisions between charged and neutral particles. It has been speculated that also some low-frequency ionospheric waves in these regions can develop strong turbulence with universal power law spectra (Ott and Farley, 1974; Sudan and Keskinen, 1977; Sudan, 1983; Hassam et al., 1986), but the conjecture has not been studied systematically in any detail (Pécseli, 2016), although some observations and numerical simulations seem to be consistent with the hypothesis (Kelley, 1989). Studies of rocket data, in particular, have shown power law spectra for both fluctuating plasma density and electrostatic potential (Basu et al., 1988; Krane et al., 2000; Dyrud et al., 2006) in reasonable agreement also with laboratory results (Mikkelsen and Pécseli, 1980). Intermittency is also found to be an issue for plasma turbulence (Dyrud et al., 2008). Parts of it has to do with coherent interactions of large amplitude waves in a background of a low level of turbulence (Vladimirov and Stenflo, 1997).

Fully developed strong plasma turbulence with a large β -value is mostly found in the solar wind as summarized by e.g., Bruno and Carbone (2005), where conditions are well described by magnetohydrodynamics (MHD). Turbulent plasma energy spectra (i.e., the sum of kinetic and magnetic energy), in the form $\sim (\epsilon V_A)^{1/2} k^{-3/2}$, have been predicted by Iroshnikov (1964) and Kraichnan (1965). It has been found

that large scale cavities can form in the Earth’s magnetosphere (Fritz et al., 2003) where the magnetic field strength is small, thus creating large regions with relatively large β -value where the Iroshnikov-Kraichnan results are also relevant. A worthwhile investigation would be to analyze the energy spectra for these conditions.

2.2.1 Plasma Turbulence as a ‘Soliton Gas’

Turbulence is associated with many degrees of freedom being excited. An intermediate scenario has been suggested, where a nonlinear evolution of waves saturate in an ensemble of solitons (Kingsep et al., 1973; Dysthe et al., 1986). Once excited, these solitons will collide but recover their original form after collisions (Drazin and Johnson, 1989). Each of these nonlinear structures are described by a wide band of phase coherent Fourier components, so that each soliton can be described by a reduced number of parameters, such as amplitude, width, position and velocity, where some of these parameters can be interrelated as for instance for Korteweg-de Vries solitons (Drazin and Johnson, 1989). This turbulence model can seem unrealistic since most soliton studies refer to spatially one dimensional problems. Several observations can, however, be discussed in terms of this framework. In these cases the wave field can be accounted for by a random distribution of slowly evolving semi-coherent structures, possibly in a background of random noise. A simple spectral analysis does not account for coherent phase relations and the diagnostic methods have to be supplemented by, for instance, triple correlations or bispectral methods (Kim and Powers, 1978). Large coherent structures are best identified by conditional sampling of the data (Johnsen et al., 1987), possibly supplemented by filtering methods, matched filters for instance (Teliban et al., 2007; Fredriksen et al., 2008).

2.3 Taylor’s Hypothesis

Measurements of wavenumber power spectra require a minimum of ‘two-point’ measurements with movable probes. Often, this is not possible, and only time series obtained at one probe position are available. An almost universally used approximation, the Taylor’s hypothesis or frozen turbulence approximation, is relates measured frequencies to wavenumbers (Taylor, 1938; Shkarofsky, 1969; Wyngaard and Clifford, 1977). Here it is implicitly argued that the wave field is swept rapidly past the observation point (onboard e.g., an instrumented rocket or a spacecraft), so that the time variation observed is actually due to a moving spatial variation i.e., $\omega \approx \mathbf{k} \cdot \mathbf{U} \equiv k_{\parallel} U$, where ω is the detected frequency, and \mathbf{U} the relative velocity between the turbulent medium and the observer. Since the frequency ω is only related to the wave vector component k_{\parallel} along \mathbf{U} , application of Taylor’s hypothesis may need to be combined with assumptions of homogeneity and isotropy in two or three dimensional space. Both scalar and vector spectra can be defined so that the total power is e.g., $\langle \phi^2 \rangle = \int_0^{\infty} F(k) dk$, or alternatively $\langle \phi^2 \rangle = \iiint G(\mathbf{k}) d^3\mathbf{k}$. The physical dimension of the spectra F and G are different, and care should be taken not to confuse them, in particular when comparing theoretical and experimental results (Mikkelsen and Pécseli, 1980). The

distinction of spectral definition is relevant also for power spectra in two spatial dimensions, e.g., in a plane $\perp \mathbf{B}$.

3 TURBULENT TRANSPORT

One of the most important properties associated with turbulence is anomalous transport. This is particularly true for confinement of fusion plasmas physics and in many industrial applications. In space, the anomalous transport of plasma across magnetic field lines is important for the spatial distribution of plasma (Horton, 1990). To illustrate this problem, we take a low- β plasma confined by a homogeneous magnetic field \mathbf{B} . The transport is often due to electrostatic fields, $\mathbf{E} = -\nabla\phi$, with frequencies well below the ion cyclotron frequency Ω_{ci} . The ion fluid velocity $\mathbf{E} \times \mathbf{B}/B^2 + \Omega_{ci}^{-1} (B^2 \partial \mathbf{E} / \partial t + (\mathbf{E} \times \mathbf{B}) \cdot \nabla_{\perp} \mathbf{E}) / B^3$ is taken to be perpendicular to the magnetic field. Nonlinearities enter through the ion polarization drifts, being the second bracketed term in the ion flow velocity. Finite ion Larmor radius effects (Knorr et al., 1988; Hansen et al., 1989; Chen, 2016) are usually not accounted for. Allowing for a slight deviation from strictly \mathbf{B} -normal wave propagation, the electrons can flow along the magnetic field lines to obtain an isothermal Boltzmann equilibrium, $n_e = n_0(\mathbf{r}_{\perp}) \exp(e\phi/T_e)$ where T_e is the electron temperature and $n_0(\mathbf{r}_{\perp})$ is a reference density allowed to vary across magnetic flux tubes (Chen, 2016) i.e., $\mathbf{r}_{\perp} \perp \mathbf{B}$. In the quasi-neutral limit, $n_e \approx n_i \equiv n$, keeping only the terms up to second order, the Hasegawa-Mima (HM) equation (Hasegawa and Mima, 1978; Gruzinov and Pogutse, 1986; Albert et al., 1990) is readily derived. It has been widely applied to study weakly nonlinear electrostatic drift waves in low- β plasmas. Also this equation can be expressed (Albert et al., 1990) in the form of Eq. 1. The HM-equation has linearly stable solutions, but it accounts for the interaction of many modes excited by a source. Linearly unstable resistive electrostatic drift waves are described by the Hasegawa-Wakatani (HW) set of equations (Hasegawa and Wakatani, 1983), which extend the HM-equation. Numerical solutions of the HW-equation have been found to evolve into a strongly turbulent state with a continuous power spectrum as summarized elsewhere (Pécseli, 2015). When the magnetic field is weaker, $\beta \sim m_e/m_i$, the electron motion can couple to Alfvén waves and the equations have to be modified, but the possibility for a strongly turbulent state remains.

The space-time variations of low frequency, large spatial scale electromagnetic fluctuations in high- β plasmas is well described by the MHD equations, and strongly turbulent states can develop (Goedbloed and Poedts, 2004; Bruno and Carbone, 2005; Galtier, 2009). Relevant space observations relates, for instance, to clouds in the interstellar medium composed of ionized hydrogen atoms (H II regions), regions with neutral atomic hydrogen (H I) and to diffuse nebula or emission nebula (Cox, 2005). On astronomical scales, we find H I and H II regions to be clearly separated. This can be seen as a paradox since on such large scales relevant for interstellar media we could expect correspondingly large Reynolds numbers. Such a turbulent state should result in a

strong mixing smearing out boundaries between the H I and H II regions in contradiction with observations. A possible explanation for the lack of turbulent mixing could be that there is not sufficient free energy available to drive a substantial high- β MHD-turbulence on those scales.

Anomalous transport can be particularly important for the plasma at the boundaries between the solar wind and the magnetosphere i.e., the polar cusps and the magnetopause. The cusps offer a particularly easy access between the two regions (D'Angelo, 1977; Fritz et al., 2000). Solar wind plasma particles can thus get direct access to the Earth's ionosphere through the northern and southern cusp points. The shear flow at the interface between open and closed magnetic field lines can give fluid-like Kelvin-Helmholtz (KH) instabilities that can mediate anomalous transport between the two regions. The nonlinear saturated stage of the KH instability plays a critical role for the solar wind interaction with the Earth's magnetosphere (Johnson et al., 2014; Ma et al., 2017).

The solar wind, supersonic and supersonic at 1 AU, is an important source of free energy driving turbulence on large spatial scales in the Earth's magnetosphere. The coupling between the solar wind and the plasma of the Earth magnetosphere can involve magnetic field reconnection, and the efficiency of these processes depends critically on the polarity of the interplanetary magnetic field (IMF) with respect to the Earth's magnetic field. The process will depend on the solar activity as evidenced by its correlation with the distribution and intensity of ionospheric plasma irregularities (Jin et al., 2020). The coupling process itself can be accounted by resistive MHD, but resistivity due to classical collisions is found to be insufficient for the relevant plasma parameters. Anomalous resistivity has been suggested as a remedy for explaining enhanced reconnection rates. Lower hybrid wave turbulence can be one source of such anomalous resistivity (Huba et al., 1977), and this wavetype is indeed observed near the Earth's magnetopause (Graham et al., 2017). There is, however, seemingly no consensus on the detailed nature of the relevant anomalous resistivity, nor agreement concerning the origin of the free energy driving the necessary plasma instabilities (Biskamp, 1997).

The discussion of turbulent transport, so far, dealt with 'absolute turbulent diffusion', a phenomenon that is adequately accounted by considering a single representative particle moving with respect to its origin of release. Alternatively a problem of 'relative diffusion' can be formulated, which can be described by the relative motion of two particles (Misguich et al., 1987). The two problems are substantially different: the first case refers to motion in a fixed frame, the later to the center of mass frame moving randomly itself. Relative motion is important for discussing the expansion of a cloud of contaminants instantly released. This could be a barium cloud released in a controlled experiment described by e.g., Haerendel (2019), or micro meteorites evaporating in the ionosphere. In a magnetized plasma the problems of anomalous transport are strongly anisotropic regarding the directions along and across magnetic field lines.

4 ANOMALOUS RESISTIVITY

Since plasma in space is a highly conductive medium, its dynamics is particularly sensitive to changes in current distributions, and the electric and magnetic fields being induced. In a magnetized plasma under normal conditions, the ions dominate the B -transverse current, while the electrons usually carry the current along the magnetic field lines. Obstacles to the free electron flow can be caused by fluid or by kinetic effects (Dupree, 1970; Papadopoulos, 1977; Büchner and Elkina, 2006). These obstacles can be generated by nonlinear wave phenomena (Davidson and Gladd, 1975; Guio and Forme, 2006). While classical resistivity is caused by particle collisions with known collision frequency, we can find an equivalent nonlinear source induced by kinetic wave-particle interactions giving rise to an anomalous collision frequency affecting the electron as well as the ion dynamics.

Together with other sources, the interaction between the solar wind flow and the Earth's magnetosphere can impose large scale steady state electric fields perpendicular to the magnetic field (Kelley, 1989). This electric field drives the equatorial and the auroral electrojets, predominantly in the ionospheric E -region. The classical resistivity is too small to give any significant energy deposition by this electric field. Low frequency plasma sound waves can, however, be excited in the collisional ionospheric E -region due to a modified two-stream instability, the Farley-Buneman (FB) instability found by Farley (1963) and Buneman (1963) independently. It can also develop into a turbulent spectrum (Mikkelsen and Pécseli, 1980; Krane et al., 2000; Pécseli, 2015; Young et al., 2020). An enhanced anomalous collision frequency induced by these fluctuations can give rise to a bulk heating of the plasma while saturating the instability (Schlegel and St.-Maurice, 1981; Primdahl, 1986; Oppenheim et al., 1996). It was anticipated (Pécseli, 2015) that the turbulence generated by the FB instability have similarities with the current driven ion sound instability (Kadomtsev, 1965; Machalek and Nielsen, 1973; Horton, 1985). The stability conditions in the ionospheric E - and F -regions are complicated by the possibility for two instabilities being present at the same time (Sudan, 1983), a gradient instability and the two-stream FB-instability mentioned before.

The nature of the obstacles inhibiting the free electron flow can be kinetic, while others can adequately be described by a simpler fluid model. Their excitation requires free energy and they can form as a result of a plasma instability, such as current driven instabilities (Büchner and Elkina, 2006), where the nonlinear saturated stage is often found to form phase space vortices, an ubiquitous nonlinear kinetic plasma phenomenon (Bernstein et al., 1957; Morse and Nielson, 1969; Lynov et al., 1979; Saeki et al., 1979; Pécseli et al., 1984; Schamel, 1986; Drake et al., 2003). These vortices are found also as three dimensional forms in magnetized plasmas (Børve et al., 2001; Daldorff et al., 2001), while they appear to be unstable for unmagnetized conditions in two or three spatial dimensions as observed in numerical simulations by Morse and Nielson (1969). Phase space structures are observed in space plasmas (Ergun et al., 1998; Tong et al., 2018; Wang et al., 2020), albeit often with relatively

small spatial scales. Laboratory experiments and particle simulations (Morse and Nielson, 1969; Saeki et al., 1979; Pécseli et al., 1984; Guio et al., 2003) demonstrate the existence of structures also at large spatial scales in units of the Debye length. Current driven instabilities can indirectly cause localized potential variations by, in a first step, exciting plasma waves, Langmuir waves for instance. By their nonlinear evolution these structures form localized coherent electrostatic structures. Vortices in ion as well as electron phase space can also be formed by particle beams, or particle bursts (Morse and Nielson, 1969; Sakanaka, 1972; Børve et al., 2001; Guio et al., 2003; Wang et al., 2020). Such beams can have their origin in electrostatic double layers (Sato et al., 1981; Jovanović et al., 1982; Schamel, 1986) separating high and low potential regions connected by magnetic field lines. Such regions are observed as 'inverted V-events' by instrumented space craft (Partamies et al., 2008). Significant plasma wave amplitudes can also be excited during ionospheric heating experiments (Hanssen et al., 1992; Dubois et al., 1993). Here we give a brief discussion of some wave types that can play a role in the nonlinear plasma dynamics.

4.1 Electron Plasma Waves

For unmagnetized plasmas the only high frequency electrostatic waves are the Langmuir waves. In magnetized plasmas, the wave dispersion relation includes upper-hybrid waves propagating at large angles to the magnetic field. The nonlinear features of these electron waves have been studied intensively (Thornhill and ter Haar, 1978; Briand, 2015) with particular attention to the wave-collapse phenomenon, where an initial spatial wave distribution collapses to a singularity in a finite time (Zakharov, 1972; Rasmussen and Rypdal, 1986; Robinson, 1997; Kono and Škorić, 2010). It has been amply demonstrated that Langmuir waves collapse as a physical phenomenon is realizable, but so far there seems to be no evidence that it plays any significant role in nature. In its classical form, the collapse phenomenon refers to an initial value problem where a localized large amplitude wave-field is excited. One element in the phenomenon, cavitation of Langmuir waves, is believed to be important under 'driven' conditions where a continuous external energy source is present. This can be in the form of naturally occurring particle beams (Forme, 1999; Guio and Forme, 2006; Isham et al., 2012; Akbari et al., 2016) or perturbations induced from the ground through ionospheric heating experiments (Hanssen et al., 1992; Dubois et al., 1993). Wave cavities can be described by a random distribution of slowly evolving and weakly interacting structures resembling wave-filled plasma density depletions.

4.2 Lower-Hybrid Waves

In magnetized plasmas, the lower-hybrid waves play a particular role by having an approximately equal distribution of the wave energy between the electron and the ion components. The nonlinear space-time evolution has been studied analytically (Musher and Sturman, 1975; Sotnikov et al., 1978; Shapiro et al., 1993) for these wave types. Such waves have indeed been observed in nature with solid indications of a nonlinear evolution (Kjus et al., 1998; Høyværk et al., 2000; Schuck et al., 2003; Schuck et al., 2004), but it remains questionable whether

this evolution can be attributed to wave-collapse phenomena (Pécseli et al., 1996; Pécseli and Trulsen, 2006). The scales of the local wave packets in the B -transverse direction are well known from satellite observations, but the magnetic field aligned length scales have not been studied in space. All we know is that these scale lengths are very large, probably limited by collisional mean free paths (Pécseli et al., 1996), and this makes also numerical simulations challenging. Lower-hybrid drift waves excited at steep plasma density gradients can also contribute to anomalous transport and plasma heating as suggested by Davidson and Gladd (1975). Lower-hybrid drift waves have been observed in space (Huba et al., 1978; Bale et al., 2002; Walker et al., 2008; Graham et al., 2017) but the associated transport properties have not been fully documented.

The soliton turbulence model described in **Section 2.1** can seem unrealistic, but observations (Pécseli et al., 1997) of an ensemble of saturated lower-hybrid wave structures show examples that can be interpreted by such a model. The properties of such individual structures have been studied as well, with results summarized by Schuck et al. (2003).

4.3 Whistlers

Electromagnetic waves, whistlers, with frequencies below the electron cyclotron resonance frequency can propagate in magnetized plasmas (Stenzel, 2016). In a limited frequency range, $\Omega_{ci} \ll \omega \ll \omega_{ce}$, whistler waves can be seen as electron equivalents to MHD waves (Kingsep et al., 1990; Biskamp et al., 1999; Stenzel, 1999; Galtier, 2009; Lyutikov, 2013). Such waves are frequently observed in the ionosphere (Yeh and Liu, 1972). Often whistlers appear in form of wide band electromagnetic modes, Chorus (Shawhan, 1985; Li et al., 2019; Aryan et al., 2020), or as Hiss in plasmaspheric plumes (Zhang et al., 2019). Seemingly, whistlers are often observed at modest amplitudes, but evidence for weakly nonlinear effects in form of self-ducting (Karpman et al., 1974, 1990) have been reported by Bell (1985). The basic features of whistlers are well explained by fluid models (Yeh and Liu, 1972) but details in the wave energy distributions need kinetic models to account for nonlinear wave-particle couplings (Dysthe, 1971; Nakamura et al., 2018). The evolution of wide band whistler wave spectra have been studied by fluid models (Biskamp et al., 1999; Cho and Lazarian, 2009) indicating that universal energy power spectral subranges with a $\sim k^{-7/3}$ wavenumber scaling can develop.

5 DISCUSSION

Weakly nonlinear wave phenomena include parametric decay and modulational instabilities, usually described in terms of three and four wave phenomena. Most studies refer to the stability of processes involving modulated plane waves. In nature this condition is unlikely to be found, broad band wave spectra are much more often observed. Simple physical arguments (Alber, 1978; Pécseli, 2014) give that a wave-decay or a modulational instability involving wavelengths longer than the correlation length (i.e., the inverse wavenumber spectral width) associated with the spectrum are stable (Alber, 1978; Pécseli, 2014), although details will differ for decay and modulational instabilities. For ocean waves this argument seems to hold promise (Alber, 1978; Dysthe et al., 2003). Heuristic arguments then imply

that, due to nonlinear effects, the wavenumber spectra should broaden until some quasi stationary conditions are established. Analytical studies of electron plasma waves in magnetized as well as unmagnetized plasmas (Bhakta and Majumder, 1983; Pécseli, 2014; Kono and Pécseli, 2016; Kono and Pécseli, 2017a; Kono and Pécseli, 2017b) demonstrated that with the given assumptions a wide band spectrum was indeed less unstable than a narrow band. The results did not, however, predict complete modulation- or parametric decay-stabilizations for wide band wavenumber spectra: this could be due to a simplifying quasi-normal (sometimes termed quasi-Gaussian) assumption (Leslie, 1973; Alber, 1978) made in the analysis. The scenario outlined here has not been systematically analyzed experimentally nor numerically for plasma media. For numerical tests it is possible that one dimensional simulations can suffice. The low frequency decay products of modulational as well as oscillating two-stream instabilities do not obey any dispersion relation (Thornhill and ter Haar, 1978; Kono and Škorić, 2010; Pécseli, 2014). This feature is common to BGK-type phase space structures (Bernstein et al., 1957), such as phase space vortices, making it difficult to distinguish the phenomena by remote sensing, e.g., radio wave scattering (Vierinen et al., 2017).

6 CONCLUSION

Turbulence in fluids and plasmas have been studied for over a century but is still considered to represent one of the major unresolved problems in nonlinear physics. Elements in the theory of classical fluid turbulence, intermittency in particular, are not fully understood, but the available understanding suffices to allow adequate predictions for central problems such as turbulent transport also under inhomogeneous conditions. The understanding of plasma turbulence, for magnetized plasmas in particular, has not reached the same level, although convincing agreement between analytical results and observations have been achieved for a number of specific questions. Theories for weak turbulence in plasmas are well established and tested, in particular for the special case of ‘quasi linear theory’ (Kadomtsev, 1965) which accounts for the spreading in velocity space of an electron beam exciting electron plasma waves (Kontar, 2001; Ratcliffe et al., 2014). Generalizations and extensions of the analysis have been suggested (Orszag and Kraichnan, 1967; Horton, 1985; Similon and Sudan, 1990), but it is not known how well these models perform in comparison with observations, or realistic numerical simulation results of plasma turbulence.

AUTHOR CONTRIBUTIONS

Both authors have made a substantial, and direct, contribution to the work and approved it for publication.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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