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Partial information disclosure in a contest

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ABSTRACT

Zhang and Zhou (2016) use the concept of Bayesian persuasion due to Kamenica and Gentzkow (2011) to analyze information disclosure in a contest with one-sided asymmetric information. They show that an effort-maximizing designer can manipulate information disclosure to increase expected efforts in the contest, based upon active contest participation by all types of the informed player. We allow some informed types to exert no effort in the contest, showing how this (i) can increase the applicability of the previous results, and (ii) in some cases, can change the type of information disclosure.

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1. Introduction

Contests in which resources are sunk to win a prize capture competition in social, political and economic spheres. A common theme is how a designer (principal) can maximize the resources expended in the contest. Recently Zhang and Zhou (2016) introduced information disclosure as an instrument at the disposal of the principal, using the Bayesian Persuasion framework of Kamenica and Gentzkow (2011). In a two-player contest, Zhang and Zhou (2016) focus on one-sided informational asymmetry, where one player has better information than the competitor and the principal. The effort-maximizing, but uninformed, principal initially commits to a set of state-conditional distributions of signals before realization of the state, which is the value of the prize to the player with private information; the signals disclose all or no information at two extremes, but can also impart a particular posterior belief to the uninformed. The optimal distribution of signals raises the principal's payoff to the concavification of the total expected effort function.

Zhang and Zhou (2016) show first that binary values for the state yields an expected effort function that is either globally convex or concave; in the former case, full information disclosure is optimal, and in the latter there is no disclosure.[1] Only when there are more than two possible valuations can partial disclosure appear, in which the signal reveals the true value of the prize imperfectly to the uninformed player. Zhang and Zhou (2016) consider only fully internal solutions in which all types of the informed player have an effort level above zero. Epstein and Mealem (2013) show with two types for the informed player that an equilibrium exists in which the lower value type will not exert effort in the contest. We extend the results of Zhang and Zhou (2016) by considering equilibria in which some types exert no effort, and we fully characterize optimal information disclosure in the two-type case. Furthermore, we show how these results have consequences for deriving the optimal disclosure policy when there are more types.

2. Analysis

In Zhang and Zhou (2016), there are two risk-neutral players, A and B. Player A's value of winning the contest is v_A and this is common knowledge. Player B's value v_B (the state) is private information, but it is commonly known that it takes $N \geq 2$ values, $v_1 < v_2 < \cdots < v_N$, with prior $\mu^0 = \left(\mu_1^0, \ldots, \mu_N^0\right) \in P^N = \left\{(p_1, \ldots, p_N) : p_j \geq 0, \sum_{j=1}^N p_j = 1\right\}$. Before the state is realized, the contest designer commits to a signaling mechanism, which consists of a family of state-conditional distributions $\{\Pr\left[m_s \mid v_j\right] \geq 0 : m_s \in S, \sum_{m_s \in S} \Pr\left[m_s \mid v_j\right] = 1\}, j \in \{1, \ldots, N\}$ over a finite set of messages S. We denote the Bayesian posterior after observing message $m_s \in S$ by $\mu^s = \left(\mu_1^s, \ldots, \mu_N^s\right) \in P^N$. We use the notation $\mu \in P^N$ to represent any generic distribution over the state space.

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[1] This follows Kamenica and Gentzkow (2011), and is explained later.

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In the posterior contest, players exert non-recoverable effort (x_A, x_B), which gives player $i \in \{A, B\}$ a success probability of

$$p_i(x_A, x_B) = \frac{x_i}{x_A + x_B}.$$

Denote the pure strategy Bayes–Nash equilibrium by $\left[x_A^*, x_B^*\left(v_j\right)\right]$.

Observe that the effort of B maximizes

$$\frac{x_B}{x_B + x_A^*} v_j - x_B.$$

The first-order condition gives

$$x_B\left(v_j\right) = \begin{cases} \sqrt{v_j}\sqrt{x_A^*} - x_A^* & \text{for } \sqrt{v_j} - \sqrt{x_A^*} > 0 \\ 0 & \text{for } \sqrt{v_j} - \sqrt{x_A^*} \leq 0, \end{cases} \tag{1}$$

from which it is apparent that some low v_B types may not participate actively in the contest.

For now, fix a distribution $\mu \in P^N$ of player B types and a set of inactive types $1, \ldots, k$ (i.e., $x_B\left(v_j\right) = 0$ for $j = 1, \ldots, k$), whilst $k+1, \ldots, N$ participate actively (i.e., $x_B\left(v_j\right) > 0$ for $j = k+1, \ldots, N$); if $k = 0$, then all player B types exert effort. When $k > 0$, player A wins with certainty if he meets types $1, \ldots, k$, making his expected payoff

$$\left(\sum_{h=1}^{k} \mu_h + \sum_{m=k+1}^{N} \frac{\mu_m x_A}{x_A + x_B(v_m)}\right) v_A - x_A.$$

The first-order condition is

$$\left(\sum_{m=k+1}^{N} \frac{\mu_m x_B(v_m)}{\left(x_A + x_B(v_m)\right)^2}\right) v_A = 1. \tag{2}$$

Solving (1) and (2) gives a solution for x_A when k types are inactive as

$$x_A^*(k) = \left(\frac{\sum_{m=k+1}^{N}\left(\frac{\mu_m}{\sqrt{v_m}}\right)}{\frac{1}{v_A} + \sum_{m=k+1}^{N}\left(\frac{\mu_m}{v_m}\right)}\right)^2. \tag{3}$$

Replacing x_A^* in (1) by (3) gives

$$x_B^*\left(v_j\right) = \sqrt{v_j}\left(\frac{\sum_{m=k+1}^{N}\left(\frac{\mu_m}{\sqrt{v_m}}\right)}{\frac{1}{v_A} + \sum_{m=k+1}^{N}\left(\frac{\mu_m}{v_m}\right)}\right) - \left(\frac{\sum_{m=k+1}^{N}\left(\frac{\mu_m}{\sqrt{v_m}}\right)}{\frac{1}{v_A} + \sum_{m=k+1}^{N}\left(\frac{\mu_m}{v_m}\right)}\right)^2, j = k+1, \ldots, N. \tag{4}$$

None of the inactive player B types will want to exert positive effort as long as $\sqrt{v_k} - \sqrt{x_A^*(k)} \leq 0$. Using (3) and (4) yields total effort with k inactive types, $TE(\mu, k)$, as

$$TE(\mu, k) = x_A^*(k) + \sum_{m=k+1}^{N} \mu_m x_B^*(v_m). \tag{5}$$

Zhang and Zhou (2016) consider an internal solution, in which case $k = 0$ and the total expected effort is

$$TE(\mu, 0) = \frac{E_\mu\left[\sqrt{v_B}\right] E_\mu\left[\frac{1}{\sqrt{v_B}}\right]}{\frac{1}{v_A} + E_\mu\left[\frac{1}{v_B}\right]}. \tag{6}$$

The expression in (1) makes it clear that low v_B types may not find it profitable to exert effort. This implies that participation has to be checked for player B of lowest type v_1 first, given that the other players exert positive effort. Only if type v_1 makes a positive contest effort do we have the internal equilibrium of Zhang and Zhou (2016); if type v_1 does not exert effort, then active participation is checked for v_2 given that all types with a higher valuation participate. This proceeds in sequence until two adjacent types are identified such that $x_B^*(v_k) = 0, x_B^*(v_{k+1}) > 0$. Lemma 1 determines the set of active types for a given $\mu \in P^N$.

Lemma 1. *Consider $\mu \in P^N$. Thresholds $\theta_k(\mu) > 0$, $k \in \{1, \ldots, N-1\}$ exist where $\theta_k(\mu) \leq \theta_{k+1}(\mu)$ for all k and with strict inequality if $\max\{\mu_{k+1}, \ldots, \mu_N\} > 0$, such that $\theta_k(\mu) \leq v_A < \theta_{k+1}(\mu)$ yields $x_B^*\left(v_j\right) = 0$, for $j \in \{1, \ldots, k\}$ and $x_B^*\left(v_j\right) > 0$, for $j \in \{k+1, \ldots, N\}$.*

Proof. Suppose that player B types $j = 1, \ldots, k$ set $x_B^*\left(v_j\right) = 0$. From (1), type k will not want to change action if $\sqrt{v_k} \leq \sqrt{x_A^*(k)}$, i.e.,

$$\sqrt{v_k} \leq \frac{\sum_{m=k+1}^{N}\left(\frac{\mu_m}{\sqrt{v_m}}\right)}{\frac{1}{v_A} + \sum_{m=k+1}^{N}\left(\frac{\mu_m}{v_m}\right)}, \tag{7}$$

which reduces to

$$v_A \geq \frac{\sqrt{v_k}}{\sum_{m=k+1}^{N} \frac{\mu_m\left(\sqrt{v_m} - \sqrt{v_k}\right)}{v_m}} := \theta_k(\mu). \tag{8}$$

Type k being inactive, it follows from (7) that player B types with $v_j < v_k$ will not participate if $x_B^*(v_k) = 0$. By construction, player B types with $v_j > v_k$ will participate if $v_A < \theta_{k+1}(\mu)$. To see $\theta_k(\mu) \leq \theta_{k+1}(\mu)$, note that for any $m > k$,

$$v_k < v_{k+1} \Rightarrow \frac{\mu_m\left(\sqrt{v_m} - \sqrt{v_{k+1}}\right)}{v_m\sqrt{v_{k+1}}} \leq \frac{\mu_m\left(\sqrt{v_m} - \sqrt{v_k}\right)}{v_m\sqrt{v_k}}. \tag{9}$$

Summing (9) over $m \in \{k+1, \ldots, N\}$,

$$\sum_{m=k+2}^{N} \frac{\mu_m\left(\sqrt{v_m} - \sqrt{v_{k+1}}\right)}{v_m\sqrt{v_{k+1}}} \leq \sum_{m=k+1}^{N} \frac{\mu_m\left(\sqrt{v_m} - \sqrt{v_k}\right)}{v_m\sqrt{v_k}} \tag{10}$$
$$\Rightarrow \frac{1}{\theta_{k+1}(\mu)} \leq \frac{1}{\theta_k(\mu)} \Rightarrow \theta_k(\mu) \leq \theta_{k+1}(\mu).$$

The inequality in (10) holds strictly if $\max\{\mu_{k+1}, \ldots, \mu_N\} > 0$, in which case, $\theta_k(\mu) < \theta_{k+1}(\mu)$. □

Setting $\theta_0(\mu) = 0$ and $\theta_N(\mu) = \infty$, by Lemma 1, we can express the equilibrium total effort for a given belief $\mu \in P^N$ as

$$TE^e(\mu) = TE(\mu, k) \text{ if } v_A \in [\theta_k(\mu), \theta_{k+1}(\mu)), k = 0, 1, \ldots, N-1. \tag{11}$$

Lemma 1 includes two main results: (i) it characterizes the precise condition $(v_A < \theta_1(\mu))$ under which the Zhang and Zhou (2016) analysis holds in which all player B types actively participate in the contest for a given belief μ and a set of prize values $v_1, v_2, \ldots, v_N$, (ii) it gives conditions under which a subset of types $\{1, \ldots, k\}$ does not exert effort in the contest, A sufficient condition for full type participation can be derived by considering belief-free thresholds; these are outlined in Lemma 2.

Lemma 2. *Fix $k \in \{1, \ldots, N-1\}$. Denote $\min_{\mu \in P^N} \theta_k(\mu)$ by θ_k^{min}. Then,*

$$\theta_k^{min} = \min_{v_m \in \{v_{k+1}, \ldots, v_N\}} \frac{v_m\sqrt{v_k}}{\sqrt{v_m} - \sqrt{v_k}}. \tag{12}$$

Further, $4v_k \leq \theta_k^{min} < \theta_{k+1}^{min}$.

Proof. $\theta_k(\mu)$ is minimized by identifying the largest value of $\frac{\left(\sqrt{v_m} - \sqrt{v_k}\right)}{v_m}$ for $m = k+1, \ldots, N$, and attaching belief 1 to this particular v_m and zero to all others. To show that $\theta_k^{min} < \theta_{k+1}^{min}$, first note that $\frac{v_m\sqrt{v_k}}{\sqrt{v_m}-\sqrt{v_k}}$ is increasing in v_k. Therefore, for any $m \in \{k+2,\ldots,N\}$, $\frac{v_m\sqrt{v_{k+1}}}{\sqrt{v_m}-\sqrt{v_{k+1}}} > \frac{v_m\sqrt{v_k}}{\sqrt{v_m}-\sqrt{v_k}}$ for a common v_m. Suppose that $v_M \in \{v_{k+2},\ldots,v_N\}$ minimizes $\theta_{k+1}^{min} = \frac{v_M\sqrt{v_{k+1}}}{\sqrt{v_M}-\sqrt{v_{k+1}}}$. Then it is possible to choose the same v_M and reach a lower value of θ_k^{min}. Hence $\theta_k^{min} < \theta_{k+1}^{min}$ for $k \in \{1,\ldots,N-1\}$. Further, note that $\frac{v_m\sqrt{v_k}}{\sqrt{v_m}-\sqrt{v_k}}$ is decreasing in v_m for $v_m < 4v_k$ and increasing in v_m for $v_m > 4v_k$, which gives $\frac{v_m\sqrt{v_k}}{\sqrt{v_m}-\sqrt{v_k}} \geq \frac{v_m\sqrt{v_k}}{\sqrt{v_m}-\sqrt{v_k}}|_{v_m=4v_k} = 4v_k$ for any $v_m \in \{v_{k+1},\ldots,v_N\}$, and therefore, $\theta_k^{min} \geq 4v_k$. □

Lemma 2 makes two important observations regarding the validity of the internal solution considered in Zhang and Zhou (2016). First, we see that for $v_A < \theta_1^{min}$, all player B types participate actively in the contest for any prior μ and the internal solution of Zhang and Zhou (2016) is valid. However, the exact value of θ_1^{min} depends on the parameters $v_2,\ldots,v_N$. Lemma 2 further implies that if $v_A \leq 4v_1$, then $v_A < \theta_1^{min}$ for any $v_2,\ldots,v_N$ and the internal solution remains valid. This links to the analysis of Zhang and Zhou (2016, footnote 5) who state that a sufficient condition for the interior equilibrium is $v_A \leq 4v_1$. Our statement of the sufficient condition extends the parameter range for which Zhang and Zhou (2016) is valid.

Following Kamenica and Gentzkow (2011), we can determine the optimal information disclosure from the concave closure of $TE^e(\mu)$. The principal increases her expected payoff to the concavification of $TE^e(\mu)$ by optimally choosing a distribution of *Bayes-plausible* posteriors generated from the signal distributions $\{\Pr[m_s \mid v_j], m_s \in S\}, j \in \{1,\ldots,N\}$. If $TE^e(\mu)$ is globally concave (convex), then no- (full-) information disclosure yields the principal a payoff equal to the concavification of $TE^e(\mu)$. The principal's preferred signaling mechanism can partially disclose information only if $TE^e(\mu)$ has both concave or convex parts. To highlight the role of information disclosure in the case of $k=0$ (all types participate actively), and $k>0$ (some inactive types), we first present the binary-type case and then look at the case of more types.

2.1. $N = 2$

Consider a posterior $\mu = (\mu_1,\mu_2) \in P^2$ over player B types (v_1, v_2). Since $N=2$, the posterior μ can be identified with a scalar $\mu_2 = \Pr[v_B = v_2] \in [0,1]$. Both types exert effort in the contest for any μ_2 if $v_A < \theta_1^{min} = \frac{v_2\sqrt{v_1}}{(\sqrt{v_2}-\sqrt{v_1})}$. Zhang and Zhou (2016, Lemma 1 and Proposition 3) show that the total effort $TE(\mu,0)$ with both player B types active is strictly concave in $\mu_2 \in [0,1]$ for $v_A < \sqrt{v_2v_1}$ and therefore no disclosure is optimal; and $TE(\mu,0)$ is strictly convex in $\mu_2 \in [0,1]$ for $v_A > \sqrt{v_2v_1}$ and therefore full disclosure is optimal.[2] Note that $\theta_1^{min} > \sqrt{v_2v_1}$, and so the full-information disclosure finding of Zhang and Zhou (2016) holds for $\sqrt{v_2v_1} < v_A < \theta_1^{min}$.

Fact 1 (*Zhang and Zhou (2016, Proposition 3, modified)*). *For $N=2$, consider $v_A < \theta_1^{min}$. Then, both types of player B exert non-zero effort in the contest under asymmetric information for any posterior μ. Further, for $v_A < \sqrt{v_2v_1}$, no disclosure is optimal and for $\sqrt{v_2v_1} < v_A < \theta_1^{min}$, full disclosure is optimal.*

This is an important result since Zhang and Zhou (2016) show that the general case with $N>2$ can be reduced to that of $N=2$. For our extended parameter space, even the case $N=2$ is not so clear cut; we show below that partial information disclosure can be optimal.

Consider $v_A \geq \theta_1^{min}$. By Lemma 1 and the fact that $\theta_1(\mu_2)$ is decreasing in μ_2, there exists a unique $\widetilde{\mu}_2$ satisfying $v_A = \theta_1(\widetilde{\mu}_2)$ such that both types exert effort for $\mu_2 \in [0,\widetilde{\mu}_2)$. Direct calculation gives

$$\widetilde{\mu}_2 = \frac{v_2\sqrt{v_1}}{v_A\left(\sqrt{v_2}-\sqrt{v_1}\right)}.$$

For $\mu_2 \in [\widetilde{\mu}_2, 1]$, type 1 is inactive and $TE^e(\mu_2) = TE(\mu_2,1)$. We can calculate the derivatives as

$$\frac{\partial TE(\mu_2,1)}{\partial \mu_2} = \frac{2\mu_2 v_A v_2^2 (v_A+v_2)}{(\mu_2 v_A + v_2)^3} > 0, \tag{13}$$

$$\frac{\partial^2 TE(\mu_2,1)}{\partial \mu_2^2} = \frac{2v_A v_2^2 (v_2 - 2v_A\mu_2)}{(\mu_2 v_A + v_2)^4}. \tag{14}$$

Define $\widehat{\mu}_2 := \frac{v_2}{2v_A}$. From (13) and (14), it follows that $TE(\mu_2,1)$ is always increasing in μ_2, strictly concave (convex) for $\mu_2 > (<)\widehat{\mu}_2$. When $\widehat{\mu}_2 \geq 1$, which occurs if $v_A \leq \frac{v_2}{2}$, the total expected effort is piecewise convex in μ_2. Lemma 3 shows that full information disclosure remains optimal.

Lemma 3. *Suppose $\theta_1^{min} < \frac{v_2}{2}$ and consider $v_A \in \left[\theta_1^{min}, \frac{v_2}{2}\right]$. Then, full information disclosure is optimal.*

Proof. Note that $TE^e(\mu_2)$ is given by $TE(\mu_2,0)$ for $\mu_2 \in [0,\widetilde{\mu}_2)$, and $TE(\mu_2,1)$ otherwise; both functions are convex in μ_2 and $TE^e(\mu_2)$ is continuous at $\widetilde{\mu}_2$. Therefore, $TE^e(\mu_2)$ is continuous and piecewise convex in $\mu_2 \in [0,1]$. Further,

$$\begin{aligned} TE^e(\widetilde{\mu}_2) = TE(\widetilde{\mu}_2, k=0) &\leq (1-\widetilde{\mu}_2)\,TE(\mu_2=0,k=0) \\ &\quad + \widetilde{\mu}_2 TE(\mu_2=1,k=0) \\ &= (1-\widetilde{\mu}_2)\,TE(\mu_2=0,k=0) \\ &\quad + \widetilde{\mu}_2 TE(\mu_2=1,k=1) \\ &= (1-\widetilde{\mu}_2)\,TE^e(0) + \widetilde{\mu}_2 TE^e(1), \end{aligned}$$

which follows from convexity of $TE(\mu_2,0)$ and the fact that $TE(\mu_2=1,k=0) = TE(\mu_2=1,k=1) = \frac{v_Av_2}{v_A+v_2}$. Therefore, the graph of $TE^e(\mu_2)$ will always be lower than the straight line joining $TE^e(0)$ and $TE^e(1)$, implying that full disclosure is optimal. □

When $\widehat{\mu}_2 < 1$, which occurs if $v_A > \frac{v_2}{2}$, total expected effort is concave for $\mu_2 \geq \max\{\widehat{\mu}_2,\widetilde{\mu}_2\}$ and either convex or piecewise convex for $\mu_2 < \max\{\widehat{\mu}_2,\widetilde{\mu}_2\}$. Proposition 1 shows that partial information disclosure is optimal for sufficiently large values of v_A.

Proposition 1. *Consider $v_A > \max\left\{\theta_1^{min}, \frac{v_2}{2}\right\}$. Then, there exists $\overline{v}_A > \max\left\{\theta_1^{min}, \frac{v_2}{2}\right\}$ such that $\max\left\{\theta_1^{min}, \frac{v_2}{2}\right\} < v_A < \overline{v}_A$, full information disclosure is optimal and for $\overline{v}_A \leq v_A$, partial information disclosure is optimal.*

Proof. $TE^e(\mu_2)$ is given by $TE(\mu_2,0)$ for $\mu_2 \in [0,\widetilde{\mu}_2)$, and $TE(\mu_2,1)$ for $\mu_2 \in [\widetilde{\mu}_2,1]$; the former is convex, whilst the latter is either concave for $\mu_2 \in [\widetilde{\mu}_2,1]$ if $\widehat{\mu}_2 \leq \widetilde{\mu}_2$, or, first convex for $\mu_2 \in [\widetilde{\mu}_2,\widehat{\mu}_2]$ and then concave for $\mu_2 \in [\widehat{\mu}_2,1]$ if $\widetilde{\mu}_2 < \widehat{\mu}_2$. Full information disclosure is optimal if $(1-\mu_2)\,TE^e(0) + \mu_2 TE^e(1) = (1-\mu_2)\frac{v_Av_1}{v_A+v_1} + \mu_2\frac{v_Av_2}{v_A+v_2} > TE^e(\mu_2)$ for all $\mu_2 \in (0,1)$; necessary and sufficient for this is that the slope of the straight line is greater than the slope of $TE^e(\mu_2)$ measured at $\mu_2 = 1$, which requires

$$\frac{v_Av_2}{v_A+v_2} - \frac{v_Av_1}{v_A+v_1} > \frac{2v_Av_2^2}{(v_A+v_2)^2}$$

[2] Unlike us, Zhang and Zhou (2016) describe the concavity/convexity property of (6) in terms of $\mu_1 = \Pr[v_B = v_1]$. However, the findings are comparable since the second-order derivatives of TE^e with respect to μ_1 and $\mu_2 = (1-\mu_1)$ have the same sign.

$$\Leftrightarrow v_A^2 (v_2 - v_1) - v_A v_2 (v_1 + v_2) - 2v_1 v_2^2 < 0 \Leftrightarrow v_A < \overline{v}_A,$$

where $\overline{v}_A = \frac{v_2}{2}\left[\frac{v_1+v_2+\sqrt{v_2^2+10v_1v_2-7v_1^2}}{v_2-v_1}\right]$. When $v_A > \overline{v}_A$, define $\overline{\mu}_2$ that solves $\frac{TE(\overline{\mu}_2,1)-TE(0,0)}{\overline{\mu}_2} = \frac{\partial TE(\mu_2,1)}{\partial \mu_2}|\overline{\mu}_2$. The concavification of $TE^e(\mu_2)$ consists of the line $\left(\frac{\overline{\mu}_2-\mu_2}{\overline{\mu}_2}\right) TE(0,0) + \frac{\mu_2}{\overline{\mu}_2} TE(\overline{\mu}_2, 1)$ for $\mu_2 \in [0, \overline{\mu}_2]$ and $TE(\mu_2, 1)$ for $\mu_2 \in [\overline{\mu}_2, 1]$. Then the principal uses partial information disclosure for $\mu_2 \in [0, \overline{\mu}_2]$ and no disclosure otherwise. □

Example 1 illustrates the relationship between our results and those of Zhang and Zhou (2016).

Example 1. Consider $N = 2$, $v_1 = 1$, $v_2 = 4$. In this case, $\theta_1^{min} = \frac{v_2\sqrt{v_1}}{\sqrt{v_2}-\sqrt{v_1}} = 4$, and $\overline{v}_A = 8$. Combining Fact 1 and Proposition 1 gives the optimal policy for information disclosure:

$$\textit{Optimal disclosure} = \begin{cases} \text{no disclosure (ND)} & \text{if } v_A < 2 \\ \text{full disclosure (FD)} & \text{if } 2 < v_A < 4 \\ \text{full disclosure (FD)} & \text{if } 4 \le v_A < 8 \\ \text{partial disclosure (PD)} & \text{if } 8 \le v_A \end{cases} \tag{15}$$

The first two lines in (15) reflect the results of Zhang and Zhou (2016), and the last two are our extension.[3] Thus, we extend the parameter range for which full disclosure is the optimal policy, and after this the principal implements partial disclosure. To see how this is implemented, suppose that $v_A = 16$, and calculate $\widehat{\mu}_2 = \frac{v_2}{2v_A} = 0.125 < \widetilde{\mu}_2 = \frac{v_2\sqrt{v_1}}{v_A(\sqrt{v_2}-\sqrt{v_1})} = 0.25$. Therefore, for $\mu_2 < \widetilde{\mu}_2$, both types are active and $TE^e(\mu)$ is convex; For $\mu_2 \ge \widetilde{\mu}_2$, only type v_2 is active and $TE^e(\mu)$ is concave. Fig. 1 plots $TE^e(\mu)$ against $\mu_2 \in [0, 1]$. For $\mu_2 = 0.3$, the principal's payoffs from no disclosure and from full disclosure are 1.4876 and 1.61882, respectively. Consider a distribution of Bayes-plausible posteriors: $\mu^1 = (1, 0)$, $\mu^2 = (0.4, 0.6)$ with probabilities $\beta_1 = 1/2$, $\beta_2 = 1/2$, which can be generated with two messages m_1 and m_2 and the signal distributions matrix:

$$S = \begin{bmatrix} 5/7 & 2/7 \\ 0 & 1 \end{bmatrix},$$

where $S_{(ij)}$ denotes $\Pr[m_j \mid v_i], i \in \{1, 2\}, j \in \{1, 2\}$. From Kamenica and Gentzkow (2011), we know that the principal's payoff from partial disclosure of the above kind is $\beta_1 TE^e(\mu^1) + \beta_2 TE^e(\mu^2) = 1.71626$, which is higher than her payoffs from full or no disclosure.

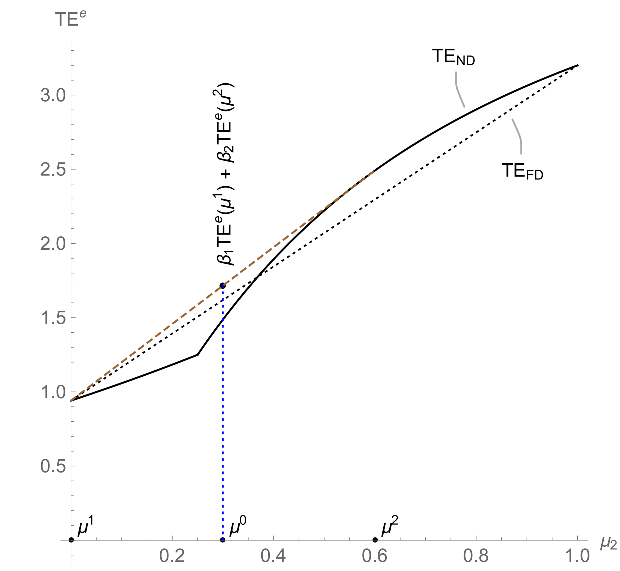

Fig. 1. TE^e against μ_2, $N = 2$.

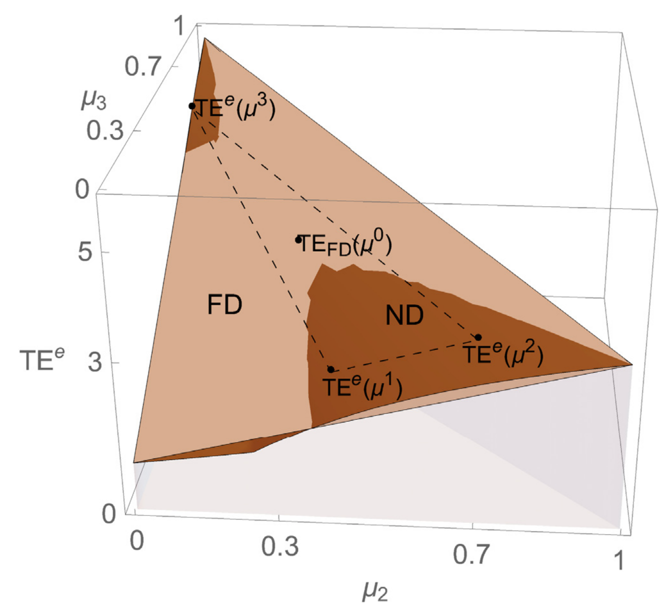

Fig. 2. TE^e against (μ_2, μ_3), $N = 3$.

2.2. $N \ge 3$

For $N \ge 3$, Zhang and Zhou (2016, Corollary 2) show that full disclosure is optimal for sufficiently high v_A (i.e., $v_A \ge \sqrt{v_{N-1}v_N}$), and partial disclosure can arise otherwise. For our extended parameter space, partial disclosure can be optimal even for high values of v_A. To understand why, recall the underlying mechanism in Zhang and Zhou (2016): For $\mu \in int(P^N)$, there always exists a direction along which $TE(\mu, 0)$ is convex, and therefore, the principal can obtain a higher expected payoff from a distribution over two Bayes-plausible posteriors on $Edge(P^N)$ where the directional vector intersects $Edge(P^N)$. This reduces the dimension of the problem by one, and gradually optimal posteriors can be found on pairwise edges. The analysis of the $N = 2$ case shows that these edges are fully convex (concave) for high (low) values of v_A when only interior solutions are considered. However, as we have shown, the possibility of a corner solution implies that pairwise edges will not always be convex for high v_A, because of which the findings of Zhang and Zhou (2016) will not hold.[4] Example 2 illustrates how partial disclosure can dominate full or no disclosure.

Example 2. Consider $N = 3$, $v_1 = 1$, $v_2 = 4$, $v_3 = 9$, and $v_A = 16$. We have $\theta_1^{min} = \min\left\{\frac{v_2\sqrt{v_1}}{\sqrt{v_2}-\sqrt{v_1}}, \frac{v_3\sqrt{v_1}}{\sqrt{v_3}-\sqrt{v_1}}\right\} = 4$, $\theta_2^{min} = \frac{v_3\sqrt{v_2}}{\sqrt{v_3}-\sqrt{v_2}} = 18$, and $\theta_1^{min} < v_A < \theta_2^{min}$. Further, $\theta_1(\mu) = \frac{36}{9\mu_2+8\mu_3}$ and $v_A < \theta_1(\mu) \Leftrightarrow 36\mu_2 + 32\mu_3 < 9$.

[3] When $v_A = 2$, total expected effort is independent of information disclosure.

[4] In addition, we conjecture that the finding that TE^e is convex along *some* directional vector for $\mu \in int(P^N)$, which holds when all types are active, is not robust when some types choose to remain inactive. Therefore, the optimal posteriors may not necessarily be found on $Edge(P^N)$.

Therefore, for prior μ^0 with $36\mu_2^0 + 32\mu_3^0 < 9$, all three types are active and for μ^0 with $36\mu_2^0 + 32\mu_3^0 \geq 9$, type v_1 will be inactive. Fig. 2 plots TE^e against $(\mu_2, \mu_3), 0 \leq \mu_2 + \mu_3 \leq 1$. TE^e is neither globally concave or convex. The darker region at the top of the graph represents the area where the principal's payoffs from no disclosure is higher than that from full disclosure. For $\mu^0 = (0.3, 0.3, 0.4)$, her payoffs from full and no disclosure are $TE_{FD}\left(\mu^0\right) = 3.54635$ and $TE^e\left(\mu^0\right) = 3.53056$, respectively. Consider a distribution of Bayes-plausible posteriors: $\mu^1 = (0.5, 0.4, 0.1)$, $\mu^2 = (0.2, 0.7, 0.1)$, $\mu^3 = (0.2, 0, 0.8)$ with probabilities $\beta_1 = 1/3$, $\beta_2 = 5/21$, $\beta_3 = 3/7$, which can be generated with three messages m_1, m_2, m_3, and the signal distributions matrix:

$$S = \begin{bmatrix} 5/9 & 10/63 & 2/7 \\ 4/9 & 5/9 & 0 \\ 1/12 & 5/84 & 6/7 \end{bmatrix},$$

where $S_{(i,j)} = \Pr\left[m_j \mid v_i\right]$. The principal's payoff from partial disclosure is $\beta_1 TE^e\left(\mu^1\right) + \beta_2 TE^e\left(\mu^2\right) + \beta_3 TE^e\left(\mu^3\right) = 3.60892$, which is higher than her payoffs from full or no disclosure. Although the posteriors considered here are not necessarily optimal, the exercise shows that the payoff from partial disclosure can dominate that from full or no disclosure.

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References

Epstein, G.S., Mealem, Y., 2013. Who gains from information asymmetry? Theory Decis. 75 (3), 305–337.

Kamenica, E., Gentzkow, M., 2011. Bayesian persuasion. Amer. Econ. Rev. 101 (6), 2590–2615.

Zhang, J., Zhou, J., 2016. Information disclosure in contests: A Bayesian persuasion approach. Econ. J. 126 (597), 2197–2217.