Essays on exploitation of natural resources

Optimal control theory applied on multispecies fisheries and fossil fuel extraction

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Chapter 1

Synthesis

1.1 Introduction

The main scope of this thesis is modeling and optimizing motivated by real-world problems in fishery and environmental economics. The work is interdisciplinary and the emphasis is on combining economics with biology, mathematics and numerics. Key words are *the art of modeling* and *optimal control theory*.

Optimal control theory is a well-established concept for the solution of management problems where one has to maximize/minimize some objective or function subject to the state of a dynamic system dependent on the level we choose for the free variables, the *control variables*. Control theory formulations are used in a variety of areas, including process control, robotics, bioengineering, economics, finance and management science. In resource economics such formulations are very popular, e.g. in deciding maximum sustainable yield of specific fish resources. More curious applications can also be found which show that the concept of optimal control is suitable for a wide range of decision problems. An example is Metric and Weitzman (1998) who uses optimal control theory to decide which crane bird species one should put most resources into preserving in order to maintain maximum biological diversity. Optimal control theory formulations have limitations, however, to which we will return later in this section.

In the essays of this thesis the focus is somewhat phenomenological, but adaptations that rest on econometrics and employ multivariate regression analysis and parameter estimations are also made. Since solving the models demands extensive use of numerical methods, some attention is also paid to operational analysis and mathematical technicalities. All in all the thesis is a cross between bioeconomics and applied mathematical modeling.

There are several attributes of a good model. Perhaps the most important are realism/accuracy and usefulness. A famous quote by the industrial statistician George Box (1979) is straight to the point. "All models are wrong; some are useful". Between accuracy and usefulness, usefulness should come first, since only a model that brings light to some shade is worthwhile. For optimizing problems that need to be simplified in a model, incorporating the full complexity of the real world is rarely possible, but the model should be a window on reality and present a clear and concise view of that reality. In this sense one could say that the art of modeling is the art of simplifying. First, important characteristics must be identified. Next, one or more of these characteristics should be incorporated in the problem formulation. Which characteristics to choose is mainly a question about their contribution to the realism of the formulation, but if they add much complexity one must find out if they are absolutely necessary. Solvability is essential since a problem formulation that cannot be solved loses much of its purpose regardless of how realistic it might be.

When considering using optimal control theory formulations for the purpose of managing complex systems, one has to weight up the importance of an accurate system description against the need to find the exact, one and only, optimal solution to a formulated problem. Control theory formulations usually have an exact solution, but the concept of usefulness is not fulfilled if the system description has no realism. Even with the best computer technology available, system complexity very often is too overwhelming for a model to allow optimizing when acceptable accuracy and realism are to be maintained. In many cases it is impossible to tell which strategy is the best; either to relax model complexity so that one can use e.g. optimal control theory to find the optimal solution, or to raise complexity to gain more realism and search for an acceptable strategy rather than an optimal one.

1.2 Models

The problem formulations of the essays in this thesis consist of one or more ordinary differential equations expressing the current state of the system, $x(t)$, and of an objective function that is to be maximized over time. The system is controlled by the control variable(s), $u(t)$. Typically, in the case of a fishery $x(t)$ could be a vector of biomasses and $u(t)$ could be a vector of catches. If we assume current profit given by $\Pi(x(t), u(t))$, discount rate δ and an infinite time horizon, the problem is given by

$$\max_{u} \int_{0}^{\infty} \Pi(x(t), u(t))\, e^{-r\,t}\, dt \tag{1.1}$$

subject to

$$x'(t) = f(x(t), u(t)). \tag{1.2}$$

In other words, we have to find optimal controls that maximizes the sum of all future discounted profit on condition that the state vector changes according to eq. (1.2). A related problem also explored in this thesis (*essay I*) is when the time horizon is finite but unknown. Instead of maximizing eq. (1.1), one maximizes

$$\max_{u} \int_{0}^{T} \Pi(x(t), u(t))\, e^{-r\,t}\, dt + \psi(x(T), T), \tag{1.3}$$

where $\psi(x(T), T)$ is some kind of salvage value remaining when the period $[0, T]$ of optimizing is ended. Since T is unknown, however, it is part of the problem to find its size.

The rest of the model formulations are specific to each essay and are therefore not described here.

An important common characteristic of all the models in this thesis is that the objective functions (eq. (1.1)) are autonomous (independent of time) when one disregards the discount factor. In addition, the state equations (1.2) are not explicitly time-dependent. These properties are very important since they allow us to find an autonomous "feedback solution" to the problems. The autonomous feedback $u = u(x)$ is a class of solutions that defines the optimal control, u, for every admissible state, x. The autonomity simplifies the solution. It makes it possible to find the optimal control without worrying about the optimal time-path, and is essential in all the solution procedures of this thesis. Without it, the feedback would have had the form $u = u(x, t)$, which would have increased the complexity considerably.

1.3 Background and perspectives

Two of the papers in this thesis concern management of capelin and cod in the Barents Sea (*essay III* and *essay IV*) and a third *essay (II)* regards the techniques used to solve the models in these essays. The first essay is, however, somewhat different. Its theme is carbon taxes and management of the resources of fossil fuel. This is currently a much hotter topic than it was when the essay was written. The climate question rose to the top of the world media's agenda when the Intergovernmental Panel of Climate Change and Al Gore shared the Nobel Peace Price of 2007. In the UN climate conference on Bali the participants finally reached an agreement on negotiations for a new agreement in Copenhagen in 2009 to replace the existing Kyoto agreement. There is now a consensus that emissions must be reduced. The battle now is about how and how much.

Basically, there are two leading approaches to how emissions can be reduced. Carbon markets and carbon taxes (see *Essay I*). In January 2005 the European Union Greenhouse Gas Emission Trading Scheme (EU ETS) emerged, but it is not working very well because the supply of emission permits is too large. Most economists also favor carbon tax over other approaches (54% according to a Wall Street Journal Survey in February 2007). The advantage with carbon taxes is that the extra costs are passed downstream to all the consumers in a way that does not unreasonably favor particular companies or countries. Although they do not set an actual limit on emissions, they turn the market in a more environment-friendly direction.

The British government addressed the time scale and volume of an optimal reduction in emissions with their publication of the Stern Review (Nordhaus 2007 a). The Stern Review claims that the current situation demands an immediate response. If we do not act, the report estimates all risks and impacts will cost 20% of GDP (gross domestic product) or more. Moreover, it suggests global emission reductions of between 30% and 70% over the next two decades. The Stern Review is, however, strongly criticized by Nordhaus (2007 a), who points out that the results rely heavily on an unrealistic model parameterization. In his opinion, answers in the Stern report are too bombastic and too dramatic: "How much, how fast, and how costly - remain open" (Nordhaus 2007 b).

In many of the world's fisheries the need for immediate management and control is even more urgent than in the fossil fuel market. Typically, knowledge about the situation is lacking just as much as the willingness to reach agreements. Fortunately, in the case of the Barents Sea fisheries the situation is much better. Norwegian and Russian scientists know much about the present state of the ecosystem, and assessment data are available for the most important stocks. Although unreported unlicensed fishery has been a considerable problem, the two countries more or less agree on the management.

The Norwegian-Russian cooperation was prepared by the introduction of *Exclusive Economic Zones* (EEZ) in 1977, where Norway extended its geographical area by approximately six times its land territory (Hoel 2005). Since then, *The Mixed Norwegian-Russian Fisheries Commission* (established in 1976) has played a decisive role in the management of cod, haddock and capelin in the area. Their conclusions are largely based on statements from *The International Council for the Exploration of the Sea (ICES)*, which provides a "quality control" on the advices from Norwegian and Russian scientists.

In spite of the well-established management of the fish resources in the Barents Sea, improvements are still possible. Much more could be done in combining biologicals and economic aspects in a multi-species setting. The Barents Sea fisheries are of great economic importance, yet the management of the most important species, capelin and cod, is not founded on scientific economic advice. The total allowable catch (TAC) of capelin determines a fishery from January to February and is decided on the basis of the expected size of the spawning stock when predation by cod is taken into account. Similarly, the catch of cod is based on the stock size, but in a single-species context with an aim of not changing the TAC too much from year to year. The cod stock is shared evenly between Norway and Russia with a small allocation to third countries (Hannesson 2006), and the division key for capelin is 60-40 in favor of Norway.

Failure of previous management made it clear in the early 1980s that traditional single species models for many species were incapable of predicting future stock sizes adequately for management purposes (Mehl 1986). Analysis of stomach content gave a better understanding of the dynamics in the fisheries (Mehl 1991), and the collapse of the capelin stock in the middle of the 1980s was understood and explained by the dynamics between capelin, cod, herring (*Clupea harengus*) and man (Hopkins and Nilssen 1991).

Hjermann, Ottersen and Stenseth (2004) conclude that capelin, cod and herring should be managed at a multi-species/community level, and that the "one-way" multi-species approach currently used should be extended to jointly decide harvest levels for all three stocks. The need of a multi-species perspective, however, was already addressed in *Models for multi-species management* (Rødseth 1998).

In both *essay III* and *essay IV* the management has a multi-species perspective. Only capelin and cod are considered in the former, but in *essay IV* the perspective is extended to include juvenile herring. Such an extension is rather challenging from a computational perspective, and the methods used are the theme of *essay II*, which establishes a technical basis for the next two essays.

The economic model used in both *essay III* and *essay IV* acknowledges the fact that the Barents Sea is jointly managed by Norway and Russia. This model assumes that the TACs of both Norway and Russia can be controlled by a social planner, and the objective of the planner is to maximize Norwegian surplus. In spite of the promising cooperation between Norwegian and Russian scientists today, there have been several disputes between the two countries during the last thirty years. Owing to pressure from the Russian side, TACs have often been set far above the level recommended by scientists. Moreover, in the beginning of the 1990s overfishing by the Russian TACs was a threat to the joint fishery, but after 1993 cooperation between Norwegian and Russian coast guards has eased the problem. There are, however, still problems with unreported fishing on both sides. Nevertheless, p.t. (in 2009) the stock situation in the Barents Sea is very encouraging.

The common basis of all the essays is the optimizing problem stated in eq. (1.1) and (1.2). Moreover, all the essays solve it using different variants of optimal control theory. These variants are Pontryagin's maximum principle (see Kamien and Schwarz 1991), which we make use of in *essay I*, and dynamic programming (see Bertsekas 2005) that we employ in the other works. Unfortunately, these methods impose some limits on the degree of complexity one can choose for the system to be optimized. In e.g. the case of multi-species fishery management one has to moderate the number of interacting species in the growth model describing the state of the system. The reason for this moderation is that the numerical cost (throughput) grows very fast with the dimensionality of the system. This also limits the possibility of working with year classes within each species. Although a lot of work has been done in the field of operational analysis, there is still a long way to go before optimal control theory is well suited for general problems in more than four dimensions, such as a three-species management model with two year classes per species, which results in a six-dimensional state space.

For natural systems, like the marine ecosystem in the Barents Sea, it is impossible to model all attributes. One has to set a limit for the complexity. Where one sets this limit determines whether it is possible or impossible to find the optimal solution to the model.

If the model gets a much better assessment of the real system when more complexity is added to it, such an extension could also be evaluated when this means that it is impossible to find an exact solution to the model. In such cases scenario-modeling (see Schweder 2006), which we use in *essay III*, is a good method for evaluating different management strategies. One cannot expect to find the best strategy, but by evaluating a number of strategies one can choose the best strategy tested.

There is always room for fundamental stylized models that investigate new principles in management, but there is also much to be done in the field of applied research where real data analysis is given priority. The optimal control theory methods we make use of in *essay II*, *essay III* and *essay IV* can be employed in a wide range of applied studies as long as the dimension of the state space (fish species, year classes, geographical areas, etc.) is fairly restricted. A modest rise in dimensions is possible, but introduction of stochastics and applications in models with marine protected areas (MPAs) are more striking extensions of the work in this thesis.

1.4 Overview of the thesis

The thesis consists of four essays, all based on optimal control theory. The subject of *essay I* is optimal exploration of fossil fuel when negative externalities regarding CO_2 emissions are accounted for. In this essay, a Pontryagin approach is used to solve the optimal control problem described by eq. (1.1) and (1.2). The objective is to study how a tax on fossil fuel may shift total world exploitation of fossil fuel to a level that is optimal for society when negative externalities from exploitation and consumption are accounted for.

Essay II focuses on dynamic programming approaches to solving similar problems. This is most of all an essay about technicalities demonstrating the techniques employed in *essays III* and *IV*, but a few examples of concrete problems are also solved.

Essay III is a scenario-model. Optimal TACs of capelin and cod are found based on a top-down biological growth model from Agnarsson et. al. (2008).Consequences of this catch strategy are compared with regard to a bottom-up growth model from the Institutes of Marine Research in Bergen (IMR) as the real growth. Priority is largely given to stock scenarios resulting from the different strategies tested.

In *essay IV* a three-species top-down growth model for capelin, cod and herring in the Barents Sea is established on the basis of historical stock data. In accordance with this growth model optimal TACs for capelin and cod are found, and long-term effects of following this policy are investigated.

1.5 Summary of essays

1.5.1 Essay I

In this essay a principal model for optimal management of the world's resources of fossil fuel is constructed. The profit function is defined as the sum of consumer surplus and producer surplus, and the objective is to maximize discounted profit for the future. Negative externalities associated with emissions resulting from production and use of fossil fuel are, however, also accounted for. Atmospheric level of CO_2 and the remaining reserves of fossil fuel represent the environmental state of the system as a two-dimensional state vector, and a convex damage function damps down the profit.

Accumulation of CO_2 in the atmosphere is modeled to grow linearly with the world production of fossil fuel and decline non-linearly with a purification function $f(a)$ that has a skew normal distributed form and represents the self-cleaning ability of the ecosystem. Similar models have been made previously, but with a linear function representing this ability. With a linear $f(a)$ it is not possible to model scenarios with irreversible pollution. The coefficients of the function we use are calibrated from real historical data for emission and atmospheric CO_2 levels.

Since fossil fuel is a non-renewable resource in all practical interpretations of the term, there comes a time $t = T$ when either all fossil fuel is exhausted or a new technology replaces fossil fuel. Therefore the maximizing problem is given above by eq. (1.3), where the time of the technology shift T is decided endogenously.

We study the pure market solution and compare it with the optimal solution adjusted for externalities. The unregulated solution results in irreversible pollution since the CO_2-levels becomes so high that the modeled self-cleaning ability of the ecosystem collapses. When $f(a)$ is replaced by a linear purification function the optimal tax level is highly reduced compared with the non linear case. The reason is that linear purification over estimates the cleaning ability of the ecosystem for high levels of CO_2 in the atmosphere.

The influence of the discount rate factor is considerable in this model. Low discount rate gives higher tax and earlier shift in technology and, in the first years especially, tax is higher.

1.5.2 Essay II

This essay concentrates on a specific dynamic programming approach well suited for a lot of optimal control problems in fishery management. Since the term "curse of dimensionality" is frequently used in connection with dynamic programming, the approach was developed with the intention of solving optimal control problems with three- or four-dimensional state spaces within reasonable time.

Standard discretization of the state space and well-known fixed-point iterative methods are employed, but the main idea is to avoid discretization of the control space. Instead our method finds the optimal controls from first-order conditions on an approximated discrete version of the Hamilton-Jacobi-Bellman equation. This method can only be effectively used when the optimal value function is at least piece-wise differentiable. Other characteristics are that we shift between policy and value iteration and use interpolation to find approximated values of the value function to state values between the nodes of the state space grid. Such values are used in the iterative process when we calculate the optimal value function.

Interpolation is also used in the process of solving the first-order conditions. For many problems the first-order conditions cannot be solved algebraically, and then solving of the first order conditions is in itself a task deserving the attention of operational analysis experts. In stead of solving the exact first-order condition problem, we interpolate one or both of the functions in the equation to be able to find an algebraic solution to a related problem. This way an approximately optimal control can be found computationally cheaply.

At the end of the essay, three examples are solved with our methods. These are one investment problem and two problems for optimal management of fishery resources.

1.5.3 Essay III

The issue in this essay is optimal management of Barents Sea capelin (*Mallotus villosus*) and North East Arctic cod *Gadus morhua*). A two-species management model with pure economical objectives is presented, and its biological consequences are explored through growth simulations with an independent simulator, the Bifrost simulator. A growth model is also integrated in the management model, but the approach in the two growth models is very different. Although the growth of the management model is multi-species, it has a top-down approach, whereas the Bifrost simulator is much more bottom-up. The simulations are therefore a powerful validity test for the management model.

In order to find out how the management model performs compared with existing strategies, many different simulations are done. Four of them are based on the management model and two are inspired by existing management.

A simulation period of 565 years with 2006 as a starting year is chosen. The results show that the strategies revealed from the management model perform surprisingly well. First of all, none of the strategies leads to extinction of any of the stock. Second, with regard to long-term average of spawning stock biomasses and TACs of cod, the two best strategies are picked from the management model. It also emerges that when the herring assessment model Seastar runs concurrently with Bifrost to simulate a dynamical herring influence, the management model performs relatively better compared with existing strategies than it does when herring influence is constant.

The positive results from the simulations of this work indicate that further studies with both biologically and economically-based multi-species management should be performed. It is not a given that the biologically-based management is more fish-stock preserving and aware than economically-based management, and the multi-species approaches may contribute much.

1.5.4 Essay IV

In this essay we develop a three dimensional Barents Sea growth model for the key species capelin, cod and herring. This growth constitutes the state equations in an optimal control problem of the form defined in eq. (1.1) and (1.2). The profit function is derived from a Nordic ministry report (Agnarsson et. al. 2008), and represents the Norwegian fraction of profit from the Russian and Norwegian fishery of capelin and cod in the Barents Sea.

Logistic growth modified by predation is assumed as the overall structure of the growth model. Herring enters the model only for age groups one to three, since herring aged three to four years leave the Barents Sea area and join the mature part of the population further south-west (Huse 2002) and the zero group of herring is of little importance as far as predation relationships are concerned (Hjermann 2004). The inflow of herring is exogenous to the model, but the growth also depends on the size of the Barents Sea fraction.

Coefficients of the growth model are found through regression analysis based on data from 1973 to 2004. The statistical results are quite good, and both the equilibrium levels resulting from fishery closure and the equilibrium levels resulting from optimal harvest make sense. One should be careful, however, to extrapolating growth when stock sizes are very far from historical ones.

Optimal control problems are not usually considered suitable when the state space is three-dimensional because of high computational expenses, but with the methods presented in *essay II* this is no problem. As far as the concrete results are concerned, we find that according to this model optimal TACs of capelin are very dependent on the size of the capelin stock itself, the cod stock and also the herring stock. Cod is the top-predator in the model and therefore optimal TACs of cod are less dependent on the size of the capelin and herring stock. The most important finding in this work is that it indicates that historic TACs have been much too high for both capelin and cod. Starting with stock data from 1973 as initial condition, the biologic growth model and the feedback-catch solution would gives a cod stock on average more than 100 % bigger than actual historic average between the years 1973 to 2005. In spite of the huge cod average, the capelin stock given by the feedback solution is more also than 10 % bigger than historic average.

The feedback-solution implies that capelin to a greater extent should be left as food for cod, which is more valuable on the market of fish. That gives better growth conditions for cod, and a higher cod stock increase the cost effectivity for each unit of cod fished. The total harvested biomass of cod with the feedback solution is slightly lower than actual harvest, but the profit is much bigger due to lower costs. The conclusion is that from an economical point of view, the capelin and god stocks must be managed jointly.

From initial conditions with sensible combinations of stock sizes, optimal management leads to a single stable equilibrium with rather high stocks of all the species in the model. Another interesting finding is that a moderate presence of herring has a positive influence on the sum of yield from the capelin and cod fishery of the Barents Sea. According to the biological growth model a moderate herring stock stimulates cod growth without harming the capelin stock critically much. An obvious conclusion of this finding is that a focus on only the capelin-predator role of herring is too narrow.

1.6 The background of the essays

The four essays of this work are fully written by the main writer. However, parts of them are also read through and commented by one or both of the supervisors. As far as the models are concerned, there have been more contributions from the supervisors. A detailed description follows below. With regard to the amount of contribution by these essays to existing knowledge, the reader is referred to the introduction of each essay for details.

Essay I
The model was originally given by the second supervisor, Professor Leif K. Sandal, but some modeling details are provided by the main writer, who also is responsible for all the numerics and the writing. The mathematical principles behind the solution procedures are the end product of discussion between Sandal, Professor Emeritus Gerhard Berge and the undersigned. Whereas Sandal was the main contributor in the discussions, Berge was responsible for much of the proof readings.

Essay II
The essay is fully produced by the main author. However, the principals behind the solution procedure are a result of discussions with Sandal, who i.a. introduced the article by Grüne and Semmler (2004). All the examples are found by the main author,

who also is responsible for the numerics.

Essay III

The idea of working with a capelin and cod model was suggested from the main supervisor, Arne Eide. The starting point was, however, a bottum-up biological model, which turned out to be a poor basis for economical optimizing performed by dynamic programming. Discussions with Geir Huse at the Institute of Marine Resource in Bergen did for a while lead the work in the direction of so-called individual based modelling. This work was left when Sandal in a meeting introduced the undersigned with some of the work behind Agnarsson et.al (2008), where a two dimensional growth model is presented along with a utility/profit model and a feedback-solution for optimal catch. We agreed that the undersigned should start working to improve the numerical algorithms in order to form the basis for solutions of similar models in more than two dimensions. E.g models with a three- or four-dimensional state space.

In a meeting between Eide and the undersigned, the improved numerical method was introduced to Eide, and in the proceeding discussions we agreed on the final model structure for *Essay III*, which came up as an approach that combined the starting point of bottum-up biological modelling with the two-species top-down management model. A scenario model was to be made to test the flexibility of the feedback solution. The flexibility should be tested by simulating possible stock scenarios when an independent assessment model was used as input for the biomass development of capelin and cod. In the coming weeks investigations were done concerning the availability of several assessment models in use at the Institute of Marine Research. Finally, it turned out that researcher Sigurd Tjelmeland had two assessment models that fitted the project well, namely Bifrost and SeaStar. The feedback-solution was integrated into the assessment models, and Tjelmeland provided figures that showed simulated scenarios.

Essay IV

The model in *Essay IV*, with a three-dimensional state space (capelin, cod and herring), is a natural extension of the two-dimensional model presented in Agnarsson et. al. (2008). Several discussions with the working environment at the Institute of Marine Research convinced the undersigned that a three-dimensional growth model is a significant improvement to existing two-dimensional models, as juvenile herring demonstrable impacts the capelin-cod system considerably.

On the way, several growth models were made, tested and discussed. Early in the process Eide had some very useful critical comments and Sandal had some good advices concerning both software to use and statistical methods. However, the final growth model is fully made and discussed by the undersigned. Both Sandal and Eide has done a very good job in proof-reading parts of the text. In addition Sandal has provided some assistance concerning interpretation of the feedback solution and choice of figures to present.

1.7 References

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