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A simple model for the velocity distribution of particles in a plasma with temperature gradient

P. Guio

The Auroral Observatory, Institute of Physics, University of Troms, N-9037 Troms, Norway, Fax: +47 77 64 62 80

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Abstract. In a plasma with a temperature gradient, the particle velocity distribution function deviates from the Maxwellian. A new simple analytic model for such a plasma, the two-temperature Maxwellian is introduced, hereafter referred to as the 2-T Maxwellian, and not to be mistaken for the purely anisotropic bi-Maxwellian with parallel and perpendicular temperatures for a magnetised plasma. The velocity moments of the 2-T Maxwellian are presented and compared with the moments from the classical transport theory of Spitzer. Furthermore a closed form of the dielectric response function for the 2-T Maxwellian is derived. The dielectric response function is used to calculate the Doppler frequency of the plasma lines in an incoherent scatter experiment. The result is compared with the Doppler frequency given by the heat flow approximation of the dispersion relation. While a good qualitative agreement is seen between the heat flow approximation and the exact estimation of the dielectric response, it is shown that for accurate calculation of the Doppler frequency of the plasma lines an exact estimation of the dielectric response is important, especially for plasma lines observation corresponding to Langmuir waves with large wave vector and small resonance frequency.

Key words. Non-Maxwellian electron velocity distribution · Temperature gradient

1 Introduction

It is interesting in several contexts to take into account the local gradient of temperature in the velocity distribution function of particles in a plasma. Forslund (1970) and Singer (1977) used the theory developed by Spitzer and Härm (1953) while Lundin *et al.* (1996) used a linear combination of three Maxwellians to simulate a velocity distribution function that reproduces the downward flow of a thermal component in order to study instabilities due to heat conduction in a moderately inhomogeneous plasma. Kofman *et al.* (1993) and Guio *et al.* (1998) studied the dispersion relation for Langmuir waves in a plasma in the presence of a temperature gradient in the frame of plasma lines observation using the incoherent scatter technique. Implicitly associated with the heat conduction is a skewing of the particle velocity distribution function. This skewing has been directly observed by satellite measurements (Hundhausen, 1968) or inferred through heat flow estimation using incoherent scatter measurements (Blelly and Alcaydé, 1994). Theoretically Cohen *et al.* (1950) and Spitzer and Härm (1953) solved directly a kinetic equation. The solution of this kinetic equation, the Spitzer function, is restricted only to velocities not larger than a few times the thermal velocity of the electron population, introducing a discontinuity in the distribution function (Guio *et al.*, 1998). Moreover the kinetic equation presents the inconvenience to be numerically unstable.

In this paper, it is first described the two-dimensional inhomogeneous and anisotropic 2-T Maxwellian. Expressions for the velocity moments of the 2-T Maxwellian are given and compared with the moments given by the Spitzer theory. In the second part, a closed form for the dielectric response function associated to this distribution function is described. In the third part, the dielectric response function is used in the frame of incoherent scatter plasma line. The plasma lines are a pair of spectral lines produced by scattering of a radio wave by Langmuir waves of the ionospheric plasma. They are Doppler shifted up and down with respect to the transmitted frequency by an amount that corresponds to two waves travelling towards and away from the transmitter. By measuring the Doppler frequency of these spectral lines, one would be able to infer the mean Doppler velocity of the electrons by solving the dispersion relation with the dielectric response function associated to the electron velocity distribution (Bauer *et al.*, 1976; Showen, 1979) and in theory to estimate the ionospheric field-aligned current when combined with parameters obtained from the incoherent scatter ion line. A deviation of the velocity distribution function from the Maxwellian modifies the dispersion relation and thus the estimated mean Doppler velocity of the electron population. We apply the 2-T Maxwellian to the estimation of the Doppler frequency of plasma lines in a plasma with temperature gradient and compare the result with the heat flow approximation of Kofman *et al.* (1993) which takes into account a temperature gradient through a corrective heat flow term. Finally, we discuss the results of our simulation.

Correspondence to: P. Guio
e-mail: patrick@phys.uit.no

2 The 2-T Maxwellian

The 2-T Maxwellian, denoted $f_{T_\pm}$, is defined as two half-Maxwellians with temperature T_+ and T_- over the two half-spaces where, respectively $v_\| < 0$ and $v_\| \geq 0$ and a Maxwellian with temperature $T_\perp$ over the perpendicular velocity space $v_\perp$. The two half-Maxwellians along $v_\|$ are joined continuously at $v_\| = 0$ and are normalised such that the integral over the velocity space is equal to the particle density n. Thus the 2-T Maxwellian can be seen as a modified bi-Maxwellian with a temperature inhomogeneity along the parallel velocity $v_\|$. The 2-T Maxwellian is written

$$f_{T_\pm}(v_\|, v_\perp) = \begin{cases} \frac{n}{(2\pi)^{\frac{3}{2}}} \frac{1}{\theta_\| \theta_\perp^2} \exp - \left(\frac{v_\|^2}{2\theta_-^2} + \frac{v_\perp^2}{2\theta_\perp^2} \right), \; v_\| \geq 0 \\ \frac{n}{(2\pi)^{\frac{3}{2}}} \frac{1}{\theta_\| \theta_\perp^2} \exp - \left(\frac{v_\|^2}{2\theta_+^2} + \frac{v_\perp^2}{2\theta_\perp^2} \right), \; v_\| < 0 \end{cases} \quad (1)$$

where $\theta_\perp^2 = T_\perp/m$ is the square of the thermal velocity along the perpendicular direction, $\theta_\pm^2 = T_\pm/m$ are the square of the mean velocities in the parallel direction, $\theta_\| = (\theta_+ + \theta_-)/2$ is the normalisation constant such that the two half-Maxwellians are continuous at $v_\| = 0$ and m represents the particle mass.

This velocity distribution function is both inhomogeneous and anisotropic and sketches the velocity distribution of particle at the particular point of space $r = 0$ between two regions of different temperature. Figure 1 shows the 2-T Maxwellian between these two regions, and the two bi-Maxwellians with hot temperature T_+ (at $r > 0$) and cold temperature T_- (at $r < 0$). This model mimics the situation where the hot plasma of temperature T_+ is diffusing toward the region of cold plasma of temperature T_- and vice-versa.

The velocity moments of a species distribution function f are expressed in the following way (Barakat and Schunk, 1982)

$$n\boldsymbol{u} = \langle \boldsymbol{v} \rangle, \quad (2)$$
$$\frac{3}{2} nT = \frac{1}{2} m \langle |\boldsymbol{v} - \boldsymbol{u}|^2 \rangle, \quad (3)$$
$$\frac{1}{2} nT_\| = \frac{1}{2} m \langle (v_\| - u_\|)^2 \rangle, \quad (4)$$
$$\frac{2}{2} nT_\perp = \frac{1}{2} m \langle (v_\perp - u_\perp)^2 \rangle, \quad (5)$$
$$\boldsymbol{q} = \frac{1}{2} m \langle |\boldsymbol{v} - \boldsymbol{u}|^2 (\boldsymbol{v} - \boldsymbol{u}) \rangle, \quad (6)$$
$$\boldsymbol{q}^\| = m \langle (v_\| - u_\|)^2 (\boldsymbol{v} - \boldsymbol{u}) \rangle, \quad (7)$$
$$\boldsymbol{q}^\perp = \frac{1}{2} m \langle (v_\perp - u_\perp)^2 (\boldsymbol{v} - \boldsymbol{u}) \rangle. \quad (8)$$

where the angle brackets denote the average

$$\langle \mathrm{A} \rangle = \int \mathrm{A} f(\boldsymbol{v}) d\boldsymbol{v} \quad (9)$$

Because of the symmetry around $v_\|$ of the 2-T Maxwellian, the Doppler velocity $\boldsymbol{u}$, the heat flow $\boldsymbol{q}$, the heat flow for parallel energy $\boldsymbol{q}^\|$ and the heat flow for perpendicular energy $\boldsymbol{q}^\perp$ are parallel to the $v_\|$-axis and have components $u_\|$, $q_\|$, $q_\|^\|$ and $q_\|^\perp$ respectively and are given by

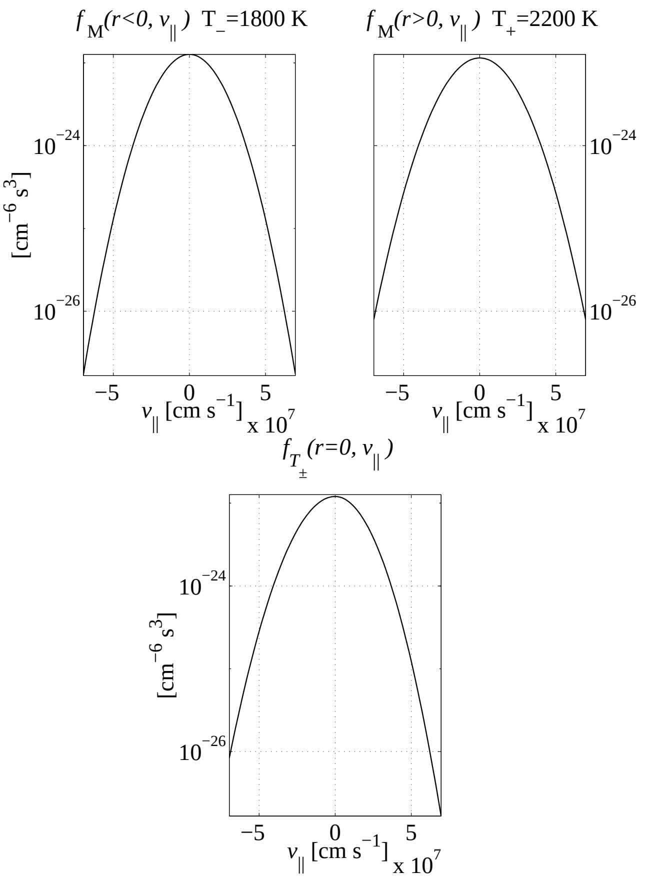

Fig. 1. The 2-T Maxwellian with $T_- = 1800\,\mathrm{K}$ and $T_+ = 2200\,\mathrm{K}$ (*lower* plate) and the two bi-Maxwellians of the cold region (*upper left* plate) and the hot region (*upper right* plate) plotted as a function of $v_\|$ and for $v_\perp = 0$. The perpendicular temperature of the 2-T Maxwellian and the two bi-Maxwellian was taken to be $T_\perp = 2000\,\mathrm{K}$

$$u_\| = -\frac{1}{\sqrt{2\pi}} \frac{\theta_+^2 - \theta_-^2}{\theta_\|} = -\frac{1}{\sqrt{2\pi}} \frac{1}{m\theta_\|} \delta T_\pm, \quad (10)$$
$$\frac{3T}{2} = \frac{1}{2} m \left(\frac{\theta_+^3 + \theta_-^3}{2\theta_\|} + 2\theta_\perp^2 \right) - \frac{1}{2} m u_\|^2 \quad (11)$$
$$T_\| = m \frac{\theta_+^3 + \theta_-^3}{2\theta_\|} - m u_\|^2, \quad (12)$$
$$T_\perp = m\theta_\perp^2, \quad (13)$$
$$q_\| = -\frac{n}{\sqrt{2\pi}} \frac{\theta_+^2 + \theta_-^2}{\theta_\|} \delta T_\pm - \frac{3}{2} n T_\| u_\| - \frac{1}{2} m n u_\|^3, \quad (14)$$
$$q_\|^\| = -\frac{2n}{\sqrt{2\pi}} \frac{\theta_+^2 + \theta_-^2}{\theta_\|} \delta T_\pm - 3nT_\| u_\| - m n u_\|^3, \quad (15)$$
$$q_\|^\perp = 0, \quad (16)$$

where $\delta T_\pm$ represents the difference between the temperatures of the hot and the cold region $\delta T_\pm = T_+ - T_-$.

Assuming the plasma to be an electron gas, the velocity moments of the 2-T Maxwellian can be compared with the velocity moments of the Spitzer distribution with electron temperature T_e, thermal velocity denoted θ_e and Knudsen number $\epsilon_T = 2\lambda_e \nabla T_e / T_e$ where λ_e is the electron mean free path. The Knudsen number represents the ratio of the microscopic length scale λ_e to the macroscopic length scale $T_e/\nabla T_e$ (Guio *et al.*, 1998). The velocity moments of the Spitzer distribution are written

$$u_\parallel = -\frac{6}{\sqrt{2\pi}}\theta_e \gamma_T \epsilon_T, \tag{17}$$

$$\frac{3T}{2} = \frac{1}{2}m_e(\theta_e^2 + 2\theta_e^2) - \frac{1}{2}m_e u_\parallel^2, \tag{18}$$

$$T_\parallel = m_e\theta_e^2 - m_e u_\parallel^2, \tag{19}$$

$$T_\perp = m_e \theta_e^2, \tag{20}$$

$$q_\parallel = -\frac{40n}{\sqrt{2\pi}}\theta_e^3 \delta_T \epsilon_T - \frac{5}{2}nTu_\parallel + \frac{1}{6}m_e n u_\parallel^3, \tag{21}$$

$$q_\parallel^\parallel = -\frac{48n}{\sqrt{2\pi}}\theta_e^3 \delta_T \epsilon_T - 3nTu_\parallel + m_e n u_\parallel^3, \tag{22}$$

$$q_\parallel^\perp = -\frac{16n}{\sqrt{2\pi}}\theta_e^3 \delta_T \epsilon_T - nTu_\parallel - \frac{1}{3}m_e n u_\parallel^3, \tag{23}$$

where γ_T and δ_T are the normalised transport coefficients defined in Spitzer and Härm (1953).

There is a formal analogy between Eqs. (10)–(15) and Eqs. (17)–(22). It can be pointed out how the temperature difference $\delta T_\pm$ mimics ∇T_e which appears in the Knudsen number ϵ_T. The temperature difference $\delta T_\pm$ can be thought as a temperature gradient between the two regions of different temperatures, and thus the Doppler velocity can be interpreted as a thermal diffusion process while the heat flow can be seen as a thermal conductivity process (Banks, 1966).

It is possible, for any value of the electron density n_e and the electron temperature T_e, to determine values of T_+ and T_- in order to get identical heat flow $q_\parallel$ for the 2-T Maxwellian and the Spitzer distribution function and at the same time keeping the respective temperatures T equal. The first term in the heat flow $q_\parallel$ of Eqs. (14) and (21) represents the thermal heat flow without Doppler velocity, we therefore require that these two terms should be equal. Moreover, if we take $\theta_\perp$ equal to θ_e, we just have to require that the first term of the parallel temperature $T_\parallel$ of Eqs. (12) and (19) should be equal. The temperatures T_- and T_+ are then uniquely determined by solving the following system of equations:

$$\begin{cases} (x-y)(x^2+y^2) & = 20\delta_T\epsilon_T \\ (x^3+y^3) \quad -(x+y) & = 0 \end{cases} \tag{24}$$

where $T_+ = x^2 T_e$ and $T_- = y^2 T_e$. (x, y) are the real solutions of Eqs. (24) such that $x > 1$ and $y < 1$. The first equation represents the condition on the first term of the heat flow and the second equation is the condition on the temperature. If we want the heat flows for parallel energy $q_\parallel^\parallel$ to be equal instead (as we will require in the last section) we simply replace the right hand side term of the first equation $20\delta_T\epsilon_T$ by $12\delta_T\epsilon_T$.

Figure 2 shows the parameters $u_\parallel$, T and $q_\parallel$ for the 2-T Maxwellian and the Spitzer distribution. The heat flow $q_\parallel$ of the two distribution functions will be equal by shifting the parallel velocity $v_\parallel$ of the distribution functions by a Doppler velocity of the same values as the one of the upper left plate of Figure 2 but of opposite sign. Note however that while

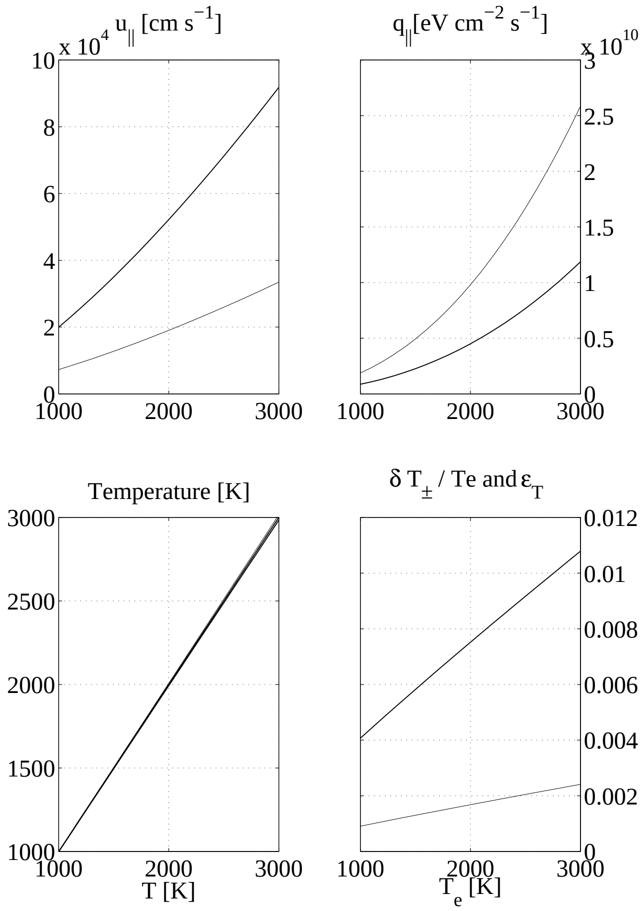

Fig. 2. The mean Doppler velocity $u_\parallel$ (upper left plate), temperatures T (lower left plate) and heat flow $q_\parallel$ (upper right plate) of Eqs. (10)–(14) (*thick* line) and Eqs. (17)–(21) (*thin* line) as a function of the electron temperature T_e and for an electron density $n_e = 10^6\,\mathrm{cm}^{-3}$ and an electron temperature gradient $\nabla T_e = 5\,\mathrm{K\,km}^{-1}$. The lower right plate shows the corresponding Knudsen number ϵ_T (*thin* line) and the ratio $\delta T_\pm/T$ (*thick* line)

the heat flows $q_\parallel$ will be equal, the heat flow of parallel energy $q_\parallel^\parallel$ will remain different since the anisotropy factor $\rho_e = q_\parallel^\parallel / q_\parallel^\perp$ for the 2-T Maxwellian is different from the one of the Spitzer function. In the Spitzer theory, $\rho_e = 3$ while for the 2-T Maxwellian $\rho_e = \infty$, which clearly means that for the 2-T Maxwellian, the energy is only transported along the direction of the temperature gradient.

3 Dielectric response function

To calculate the dielectric response function of an unmagnetised and non-collisional plasma, the following integral of the normalised velocity probability distribution needs to be calculated

$$I_f(\boldsymbol{k}, \omega) = -\int \frac{\boldsymbol{k}\cdot\nabla_{\boldsymbol{v}} f(\boldsymbol{v})}{\boldsymbol{k}\cdot\boldsymbol{v} - \omega} d^3\boldsymbol{v}. \tag{25}$$

In the geometry of a wave vector $\boldsymbol{k}$ parallel to the $v_\parallel$-axis oriented toward the cold region and in the convention that a positive velocity $v_\parallel$ gives a positive Doppler frequency, the temperatures are swapped. Then the integration over $v_\perp$ is carried out independently and we define the one-dimensional reduced 2-T Maxwellian $F_{T_\pm}$ by

$$F_{T_\pm}(v_\parallel) = \begin{cases} \frac{1}{\sqrt{2\pi}} \frac{1}{\theta_\parallel} \exp\left(-\frac{v_\parallel^2}{2\theta_+^2}\right), \; v_\parallel \geq 0 \\ \frac{1}{\sqrt{2\pi}} \frac{1}{\theta_\parallel} \exp\left(-\frac{v_\parallel^2}{2\theta_-^2}\right), \; v_\parallel < 0 \end{cases} \quad (26)$$

The integral $I_{f_{T_\pm}}(\boldsymbol{k}, \omega)$ is then written as a one-dimensional integral function of the derivative of the reduced 2-T Maxwellian $F'_{T_\pm}$

$$I_{f_{T_\pm}}(k,\omega) = -\int_{-\infty}^{\infty} \frac{kF'_{T_\pm}(v_\parallel)}{kv_\parallel - \omega} dv_\parallel \quad (27)$$

This integral has the following analytic form

$$I_{f_{T_\pm}}(k,\omega) = \begin{cases} \frac{1}{\theta_\parallel \theta_-} W_\pm\left(\frac{\omega}{k\theta_-}\right) + \frac{1}{\theta_\parallel \theta_+} W^c_\pm\left(\frac{\omega}{k\theta_+}\right), \; \mathrm{Re}\,\omega > 0 \\ \frac{1}{\theta_\parallel \theta_-} W^c_\pm\left(\frac{\omega}{k\theta_-}\right) + \frac{1}{\theta_\parallel \theta_+} W_\pm\left(\frac{\omega}{k\theta_+}\right), \; \mathrm{Re}\,\omega < 0 \end{cases} \quad (28)$$

$W_\pm$ and $W^c_\pm$ are defined for complex argument $\xi = x + iy$ such that $y \geq 0$ and are written

$$W_\pm(\xi) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{t\exp(-t^2/2)}{t+\xi^\dagger} dt, \quad (29)$$
$$W^c_\pm(\xi) = W(\xi) - W_\pm(\xi), \quad (30)$$

where

$$W(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty \frac{t\exp(-t^2/2)}{t-\xi} dt \quad (31)$$

is the classical dispersion function for a Maxwellian that can be found for example in Ichimaru (1992) and $\xi^\dagger = |x| + iy \,\mathrm{sgn}\, x$. $W_\pm$ is related to the function

$$Z_\pm(\xi) = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{\exp(-t^2/2)}{t+\xi} dt, \quad (32)$$

through the relation

$$W_\pm(\xi) = \frac{1}{2} - \xi^\dagger Z_\pm(\xi^\dagger). \quad (33)$$

Finally, $Z_\pm$ is written as a function of Dawson's integral daw and the exponential integral Ei; Dawson's integral can be expressed as a function of the modified complex error function erfc (Abramowitz and Stegun, 1972):

$$Z_\pm(\xi) = \frac{1}{\sqrt{2}} \mathrm{daw}\left(\frac{\xi}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2\pi}} \exp\Big(-\frac{\xi^2}{2}\Big) \mathrm{Ei}\Big(\frac{\xi^2}{2}\Big) \quad (34)$$
$$\mathrm{daw}(\xi) = -i\frac{\sqrt{\pi}}{2} e^{-\xi^2} \big(\mathrm{erfc}(-i\xi) - 1\big). \quad (35)$$

4 Plasma lines Doppler frequency

In an incoherent scatter plasma lines experiment, one measures two sharp and narrow spectral lines, the down- and up-shifted plasma lines corresponding to two Langmuir waves $(\boldsymbol{k}_-, \omega_-)$ and $(\boldsymbol{k}_+, \omega_+)$ travelling away from and toward the radar. The frequency of the two plasma lines are solutions of the following dispersion relation

$$k_\pm^2 + \omega_e^2 I_f(\boldsymbol{k}_\pm, \omega_\pm) = 0, \quad (36)$$

where ω_e is the electron plasma frequency.

We investigate the two solutions $(\boldsymbol{k}_-, \omega_-)$ and $(\boldsymbol{k}_+, \omega_+)$ of Eq. (36) for the 2-T Maxwellian and we define the Doppler frequency $\Delta F_\pm$ as

$$\Delta F_\pm = \frac{\omega_+ + \omega_-}{2\pi} \quad (37)$$

The Doppler frequency $\Delta F_\pm$ is then compared with the Doppler frequency given by solving the heat flow approximation of the dispersion relation of Kofman *et al.* (1993). To derive the heat flow approximation, the denominator of the integrand of Eq. (25) is expanded in power series of $\boldsymbol{k}\cdot\boldsymbol{v}/(\omega - \boldsymbol{k}\cdot\boldsymbol{u})$, then integrated by parts, each term containing an average – defined in Eq. (9) – of a power of the velocity of the probability distribution.

For $\boldsymbol{k}$ along the $v_\parallel$-axis, $I_f(\boldsymbol{k}, \omega)$ takes the following form

$$I_f(k,\omega) = -\frac{k^2}{(\omega - ku_\parallel)^2}\Bigg(1 + 3\frac{k^2\langle(v_\parallel - u_\parallel)^2\rangle}{(\omega - ku_\parallel)^2} + 4\frac{k^3\langle(v_\parallel - u_\parallel)^3\rangle}{(\omega - ku_\parallel)^3} + \cdots + (n+1)\frac{k^n\langle(v_\parallel - u_\parallel)^n\rangle}{(\omega - ku_\parallel)^n}\Bigg) \quad (38)$$

Assuming in addition that $|\omega - ku_\parallel| \gg kv_{T_\parallel}$ where $v_{T_\parallel}^2 = T_\parallel/m_e$ and that the distribution does not deviate dramatically from a Maxwellian, the even order moments are lumped into the W function of Eq. (31) and the odd order moments are truncated at the third order, which gives the heat flow approximation

$$\tilde{I}_f(k,\omega) = \frac{1}{v_{T_\parallel}^2} W\left(\frac{\omega - ku_\parallel}{kv_{T_\parallel}}\right) - 4\frac{k^5 q_\parallel^\parallel/(m_e n)}{(\omega - ku_\parallel)^5}. \quad (39)$$

Results of the computation of $\Delta F_\pm$ using the analytic form of Eq. (28) and the heat flow approximation of Eq. (39) for the 2-T Maxwellian as well as using a numerical code of the dielectric function with the Spitzer function (Guio *et al.* (1998)) are shown in Figure 3 for the EISCAT VHF radar (224 MHz) and in Figure 4 for the EISCAT UHF radar (931 MHz). The effect of the Doppler velocity $u_\parallel$ has been eliminated by subtracting from the parallel velocity $v_\parallel$ of the 2-T Maxwellian and the Spitzer function the mean Doppler velocity $u_\parallel$ of Eqs (10) and (17) respectively. The difference from the Maxwellian when it comes to evaluate $\Delta F_\pm$ is therefore only the effect of the skewness of the velocity distribution function.

For the VHF radar, there is a good qualitative agreement of the Doppler frequency $\Delta F_\pm$ as a function of the electron density n_e using: the exact expression of the dielectric response function for the 2-T Maxwellian (Eq. 28), the heat

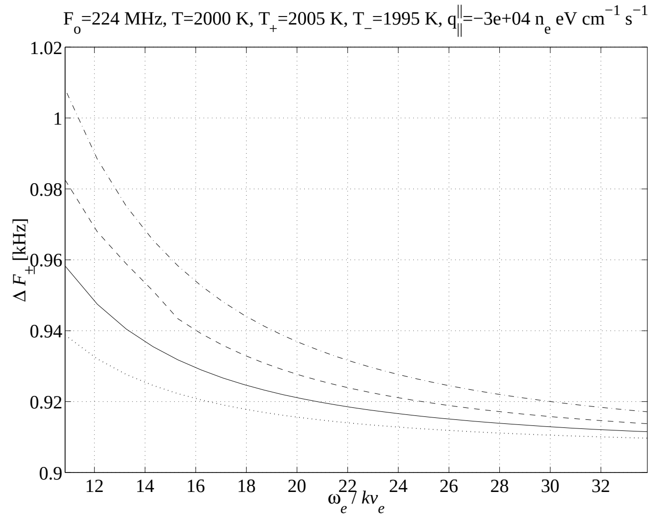

Fig. 3. EISCAT VHF radar. The calculated Doppler frequency $\Delta F_\pm$ using Eq. (28) (*solid* line) and Eq. (39) (*dashed dot* line), using a numerical code (Guio *et al.* (1998)) to calculate Eq. (25) for the Spitzer distribution (*dashed* line) and for a Maxwellian (*dotted* line). The Doppler frequency is plotted as a function of the ratio $\omega_e/kv_{T_\parallel}$, the electron temperature T_e is $2000\,\mathrm{K}$ and the Knudsen number ϵ_T is $2 \cdot 10^{-3}$. The frequency f_e that corresponds to $\omega_e/k\theta_\parallel$ varies from about $3\,\mathrm{MHz}$ to nearly $9\,\mathrm{MHz}$ and corresponds to an electron density n_e varying from 10^5 to $10^6\,\mathrm{cm}^{-3}$

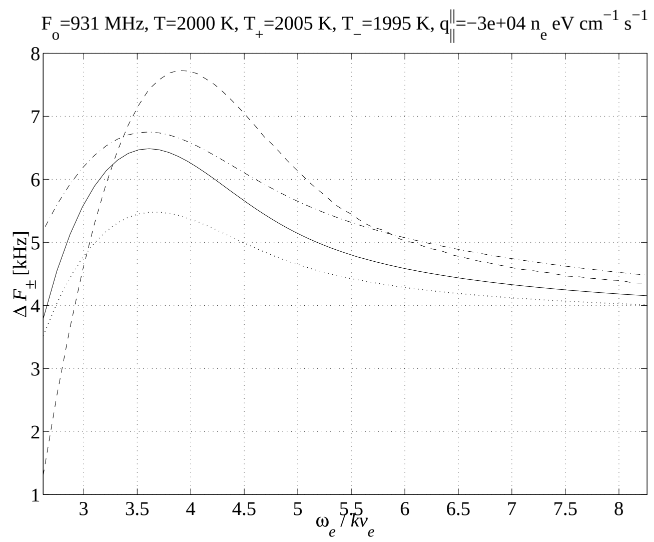

Fig. 4. EISCAT UHF radar. The calculated Doppler frequency $\Delta F_\pm$ using Eq. (28) (*solid* line) and Eq. (39) (*dashed dot* line), using the same numerical code to calculate Eq. (25) for the Spitzer distribution (*dashed* line) and for a Maxwellian (*dotted* line). The Doppler frequency is plotted as a function of the ratio $\omega_e/kv_{T_\parallel}$ for the same temperature T_e and Knudsen number ϵ_T as in Figure 3. The corresponding frequency f_e varies also from about $3\,\mathrm{MHz}$ to nearly $9\,\mathrm{MHz}$ which corresponds to an electron density n_e varying from 10^5 to $10^6\,\mathrm{cm}^{-3}$

flow approximation (Eq. 39) and the numerical calculation of the dielectric response function for the Spitzer distribution (Guio *et al.* (1998))). For such large values of the phase velocity $v_\pm = \omega_\pm/k_\pm$, i.e. when $|\omega_\pm| \gg k_\pm v_{T_\parallel}$, the three calculations predict a moderate increase of the observed Doppler frequency compared to the Maxwellian case (less than $100\,\mathrm{Hz}$) since the terms in the expansion of Eq. (38) are small. As a consequence, the effect of the skewness of the distribution function on the Doppler frequency is, as the heat flow approximation shows, to shift the Doppler frequency in the same direction as the heat flow and the heat flow behaves like a mean Doppler velocity.

For the UHF radar there is also a good qualitative agreement of the behaviour of the Doppler frequency $\Delta F_\pm$ as a function of the electron density at large plasma frequency ω_e between the three calculations. The three calculations of the dielectric response function predict an increase of the measured Doppler frequency compared to the Maxwellian which can be rather important. At low plasma frequency, the Doppler frequency calculated using the exact calculation of the dielectric response function differs from the one given by the heat flow approximation. While the heat flow approximation gives a relatively constant shift in the Doppler frequency compared to the Maxwellian, independent of the electron density, the exact calculations of the dielectric function for the 2-T Maxwellian and the Spitzer function tend to give smaller Doppler frequency. The discrepancy is getting larger the smaller the plasma frequency is, i.e. when the condition $|\omega| \gg kv_{T_\parallel}$ is not well fulfilled.

This shows that the truncation done for the heat flow approximation has to be done very carefully and that the approximation breaks for ratio $|\omega|/kv_{T_\parallel}$ smaller than 5–6. Moreover it is seen that even though the two distribution functions considered have the same temperature and the same heat flow for parallel energy, the dielectric response behaves qualitatively in an identical way but quantitative differences are noteworthy. These differences have to be accounted to the differences in higher order moments of the distribution function.

5 Conclusion

We have presented a new tool, the 2-T Maxwellian, to model the particle velocity distribution in a plasma with a temperature gradient and have compared the properties of the velocity moments to the results of the classical Spitzer distribution function. We have seen that it is possible to parametrise the 2-T Maxwellian to get an equal heat flow to the Spitzer result. An analytic form of the dielectric response function has been presented for this new distribution, and has been used to calculate the Doppler frequency of plasma lines in an incoherent scatter experiment. The result has been compared to the Doppler frequency given by the heat flow approximation. It has been shown that good qualitative agreement is obtained between the heat flow approximation of the dielectric function and the exact calculation for low-frequency radars also for high-frequency radars if the plasma frequency is high. However for accurate calculations such as the calculation of the plasma line Doppler frequency, it is seen that the exact calculation of the dielectric function is important together with a good representation of the distribution function, especially for high-frequency radars and at low plasma frequency, i.e when the ratio $|\omega|/k\theta_e$ is smaller than 5–6.

The 2-T Maxwellian is not expected to represent a true physical model of the distribution function in the presence of a gradient of temperature but nevertheless is a realistic tool for investigating this type of plasma. We expect that the 2-T

Maxwellian should be useful in the qualitative study of instabilities due to heat conduction in a plasma, especially in ionospheric studies where temperature gradients are present. The 2-T Maxwellian could also be a good investigation tool to study the effect of an angle with the magnetic field on the Doppler frequency in incoherent scatter plasma lines observations.

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