Universality of intermittent fluctuations in the Alcator C-Mod scrape-off layer

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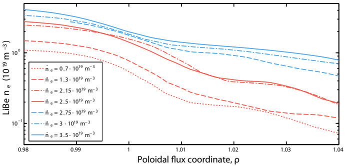
October 27, 2017

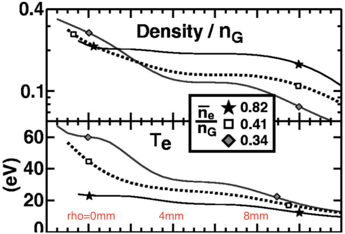
Universality: Density profiles in the SOL broaden with increasing plasma line-averaged density

ASDEX Upgrade



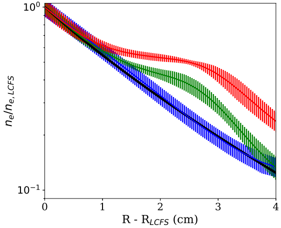
D. Carralero et al. NF **54** 123005 (2015)

Alcator C-Mod



B. LaBombard et al. PoP **8** 2107 (2001)

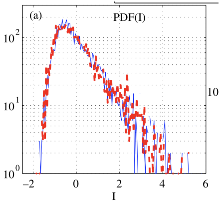
MAST



N.R. Walkden et al. PPCF **59** 085009 (2017)

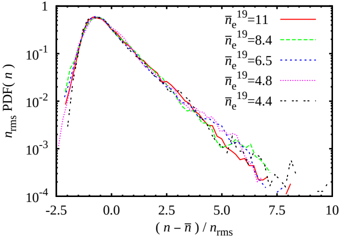
Universality: Histograms of density time series have similar shape across devices and plasma parameters

ASDEX Upgrade



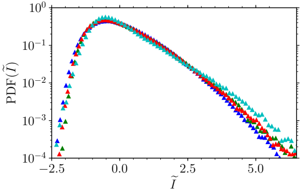
G. Y. Antar et al. PPCF **50** 095012 (2008)

TCV



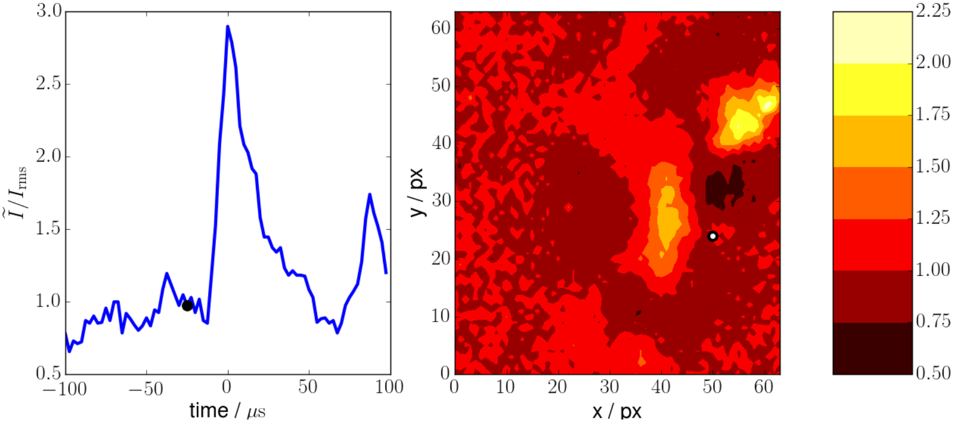
O.E. Garcia et al. NF **47** 667 (2007)

Alcator C-Mod



R. Kube et al. PPCF **57** 054001 (2016)

SOL plasmas show large fluctuation levels associated with blob propagation



Open questions

1. Do the fluctuations exhibit universal statistical properties?
2. Can the SOL fluctuation statistics be understood from a fundamental model?
3. Are the plasma fluctuation statistics the same in all confinement modes?
4. Do plasma blobs contribute to heat transport onto the vessel wall?

Outline

Stochastic model for SOL fluctuations

Gas-puff imaging measurements

Mirror Langmuir Probe measurements

Conclusion and take-away messages

Model data time series as superposition of uncorrelated pulses

$$\Phi_K(t) = \sum_{k=1}^{K} A_k \phi\left(\frac{t - t_k}{\tau_\mathrm{d}}\right)$$

k labels the pulses and

- K: number of pulses arriving in time interval $[0 : T]$
- A_k: amplitude of pulse k
- t_k: arrival time of pulse k
- ϕ Pulse shape

Intermittency parameter: $\gamma = \tau_\mathrm{d}/\tau_\mathrm{w}$ governs pulse overlap
Weak pulse overlap $\gamma \ll 1$
Strong pulse overlap $\gamma \gg 1$

O.E. Garcia, PRL **108** 265001 (2012); O.E. Garcia et al. PoP **23** 052308 (2016).

Empirically motivated distributions for random variables and pulse shape

Choosing distributions for all random variables and defining the pulse shape allows to calculate the statistics of the process $\Phi(t)$.

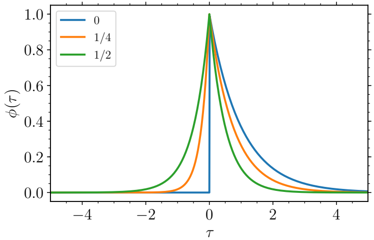
- Number of pulses is Poisson distributed
- Pulse amplitude is exponentially distributed

$$P_K(K|T) = \frac{1}{K!} \exp\left(-\frac{T}{\tau_\mathrm{w}}\right)\left(\frac{T}{\tau_\mathrm{w}}\right)^K$$

$$P_A(A_k) = \frac{1}{\langle A\rangle} \exp\left(\frac{A_k}{\langle A\rangle}\right)$$

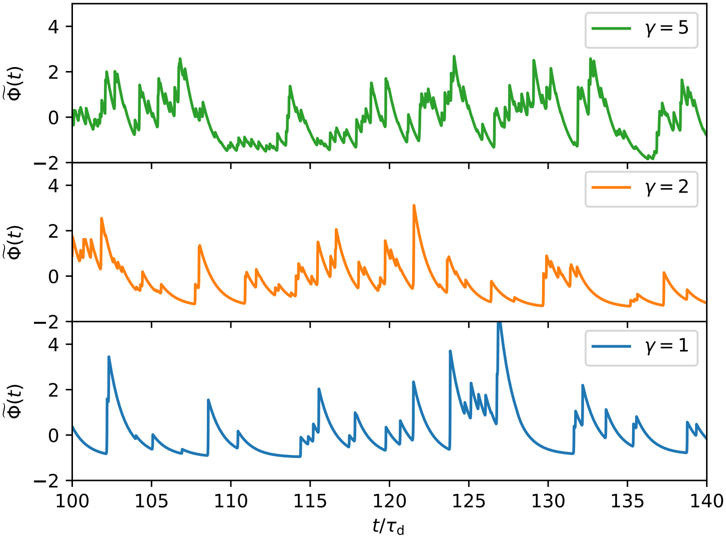
Two-sided exponential pulse shape:

$$\phi(\tau) = \Theta(-\tau)\exp\left(\frac{\tau}{\lambda}\right) + \Theta(\tau)\exp\left(-\frac{\tau}{1-\lambda}\right)$$



O.E. Garcia et al. PoP **20** 055901 (2013); O.E. Garcia et al. PoP **24** 032309 (2017).

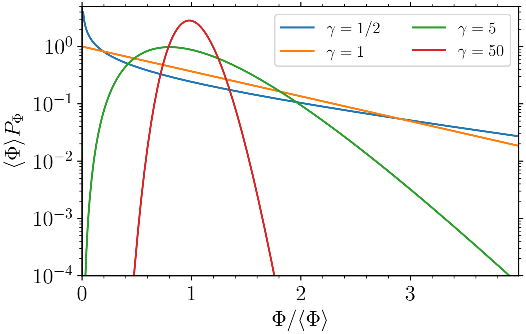
Intermittency parameter $\gamma = \tau_\mathrm{d}/\tau_\mathrm{w}$ determines degree of pulse overlap



The process is Gamma distributed with shape parameter γ

$$\langle\Phi\rangle P_\Phi(\Phi) = \frac{\gamma}{\Gamma(\gamma)}\left(\frac{\gamma\Phi}{\langle\Phi\rangle}\right)^{\gamma-1}\exp\left(-\frac{\gamma\Phi}{\langle\Phi\rangle}\right)$$

- $\gamma = 1/2$: convex shape
- $\gamma = 1$: exponential distribution
- $\gamma = 5$: elevated tails for large amplitude events
- $\gamma = 50$: near normal distribution

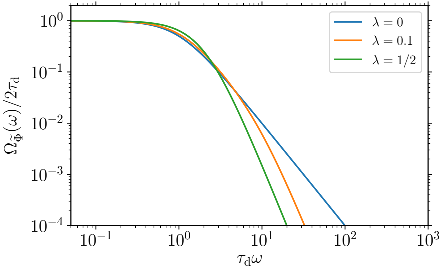


O.E. Garcia, PRL **108** 265001 (2012); A. Theodorsen et al., Phys. Scr. **92** 054002 (2017)

Power spectral density is determined solely by the pulse function

For exponential pulse shapes, the power spectral density of the process is given by

$$\Omega_{\widetilde{\Phi}}(\omega) = \frac{2\tau_\mathrm{d}}{\left[1 + (1-\lambda)^2\omega^2\tau_\mathrm{d}^2\right]\left[1 + \lambda^2\omega^2\tau_\mathrm{d}^2\right]}$$



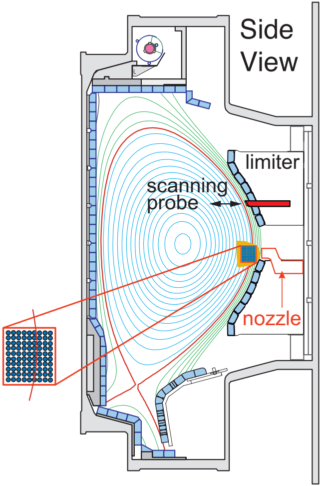
Spectrum shape is

- determined only by λ
- independent of pulse duration τ_d.
- independent of intermittency parameter γ.

O.E. Garcia et al., PoP **24** 032309 (2017); A. Theodorsen et al., Phys. Scr. **92** 054002 (2017)

Gas-puff imaging measurements

GPI diagnostic samples fluctuations in the radial-poloidal domain



I. Cziegler, PoP **17** 056120 (2010)

- 9 × 10 Avalanche Photo Diodes
- 2 MHz sampling frequency
- Light emission intensity, $I(n_\mathrm{e}, T_\mathrm{e})$
- $R \in [88 : 92]\mathrm{cm}$, $Z \in [-5.4 : -1]\mathrm{cm}$
- In-focus spot diameter $3.8\mathrm{mm}$

Data sampled under stationary plasma conditions in various confinement modes

Description	B_T/T	I_p/MA	$\overline{n}_\mathrm{e}/n_\mathrm{G}$	duration / s
Ohmic, low density	4.0	0.8	0.2	0.25
Ohmic, high density	4.0	0.6	0.7	0.45
Quiescent H-Mode	5.4	1.2	0.6	0.10
EDA H-Mode	5.4	0.9	0.6	0.23

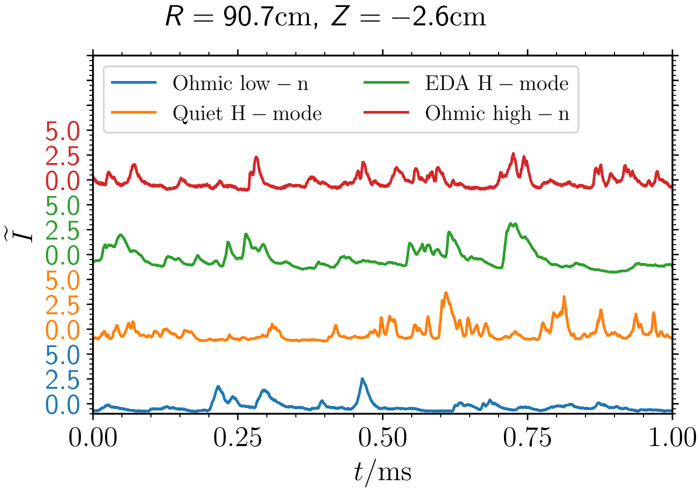
L-Modes

- Low-density: SOL is sheath-limited
- High-density: Detached divertor

H-Modes

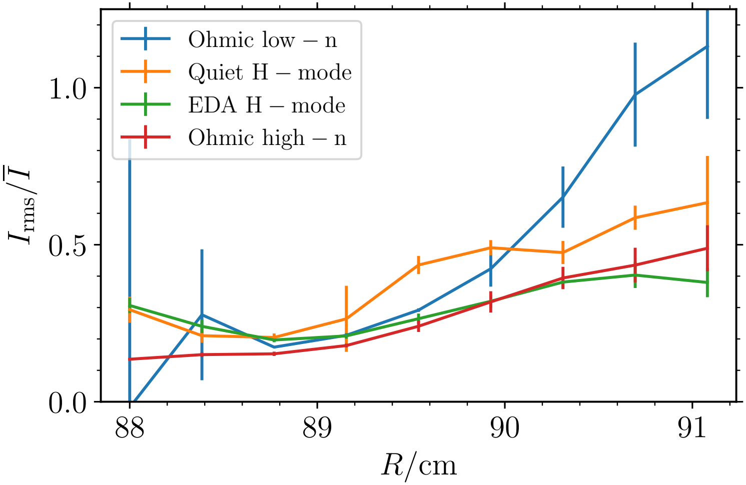
- EDA H-Mode: Quasi-coherent mode regulates transport, expels particles and heat.
- Quiescent: No transport regulation, particles and heat accumulate

Fluctuations time series feature large amplitude, intermittent bursts



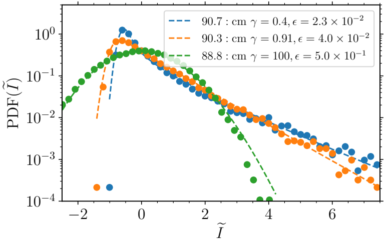
Normalization: $\widetilde{\Phi} = \dfrac{\Phi - \overline{\Phi}}{\Phi_{\mathrm{rms}}}$

Relative fluctuation level increases with radial coordinate - $\mathcal{O}(1)$ in far-SOL

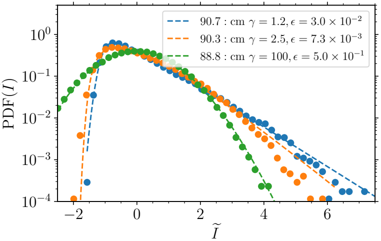


Universal behavior: Gaussian statistics in the edge, Gamma in the SOL

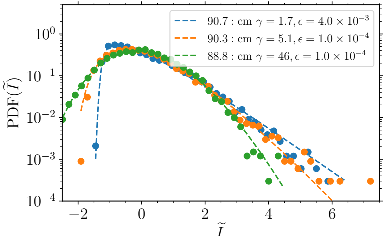
Ohmic low-density



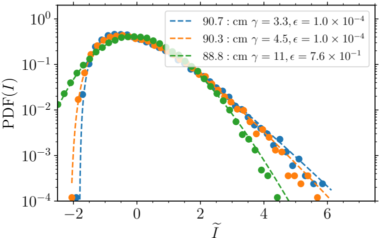
Ohmic high-density



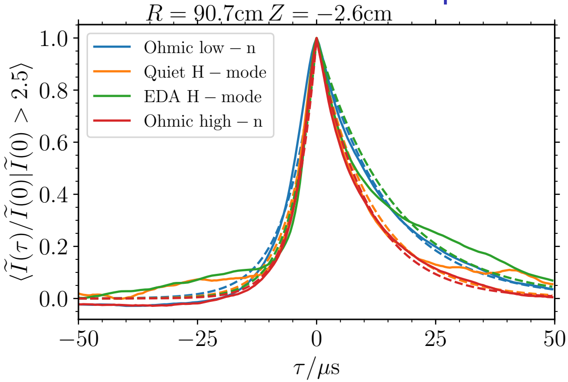
Quiescent H-Mode



EDA H-Mode



Universal behavior: two-sided exponential conditional waveform



Conditional waveform fitted to

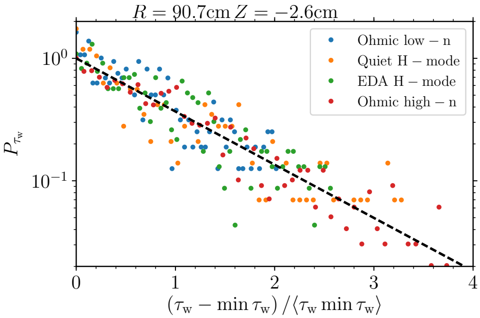
$$\phi(\tau) = \phi(0) \times \begin{cases} \exp\left(\frac{\tau}{\tau_\mathrm{r}}\right) & \text{if } \tau \leq 0 \\ \exp\left(-\frac{\tau}{\tau_\mathrm{f}}\right) & \text{if } \tau > 0 \end{cases}$$

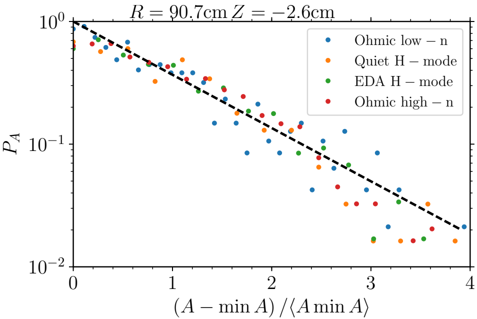
$$\tau_\mathrm{d} = \tau_\mathrm{r} + \tau_\mathrm{f}$$

Discharge	events	$\tau_\mathrm{r}/\mu\mathrm{s}$	$\tau_\mathrm{f}/\mu\mathrm{s}$	$\tau_\mathrm{d}/\mu\mathrm{s}$	λ
Ohmic low-n	432	6.3	15	21	0.30
Quiescent H-mode	2240	5.3	11	16	0.33
EDA H-mode	469	5.0	16	21	0.24
Ohmic high-n	1289	4.6	11	16	0.18

CO4.00012: Theodorsen et al., Monday 2pm

Universal behavior: exponential waiting time and amplitude PDFs

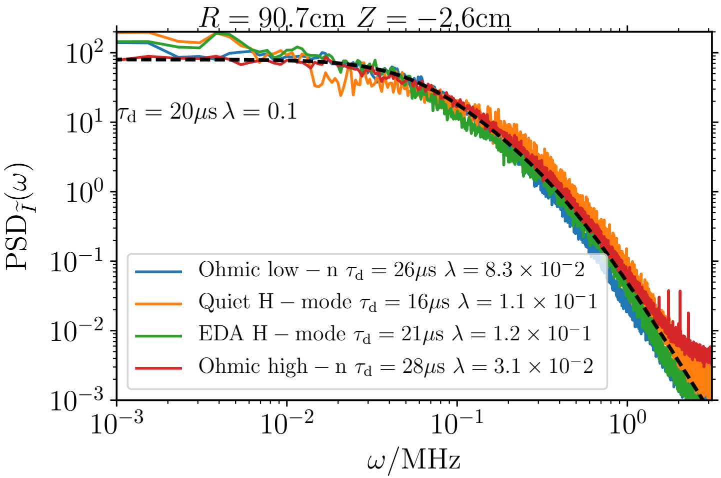




Large amplitude maxima and waiting times are exponentially distributed.

- Consistent with a Poisson process
- No clustering or periodicity of events
- Agrees with assumption of the stochastic model

Universal behavior: Similar power spectral density in all confinement regimes



O.E. Garcia, Phys. Plasmas **24** 032309 (2017).

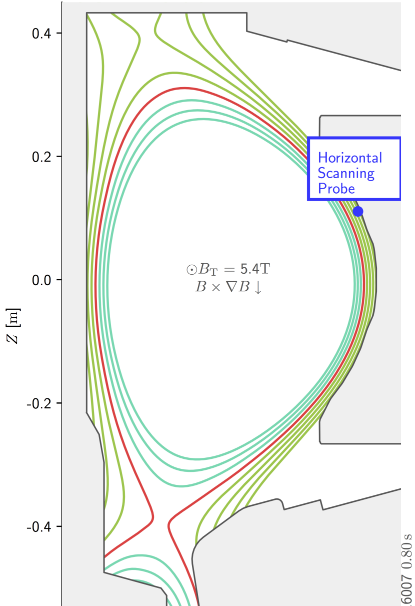
Universal statistical properties of SOL plasma fluctuations

- Fluctuation time series dominated intermittent, large amplitude bursts
- Relative fluctuation level in the SOL increases with radial coordinate
- Gaussian statistics close to the LCFS, skewed Gamma in the SOL
- Conditional waveform is well described by two-sided exponential function
- Arrival of large amplitude bursts described by a Poisson process
- Power spectral densities have a Lorentzian shape
- Fluctuation statistics are similar among all confinement modes
- Fluctuation statistics in excellent agreement with stochastic model

Mirror Langmuir Probe measurements

GPI measures $I(n_\mathrm{e}, T_\mathrm{e})$
MLP measures n_e and T_e

Mirror Langmuir Probes samples plasma parameters in real time



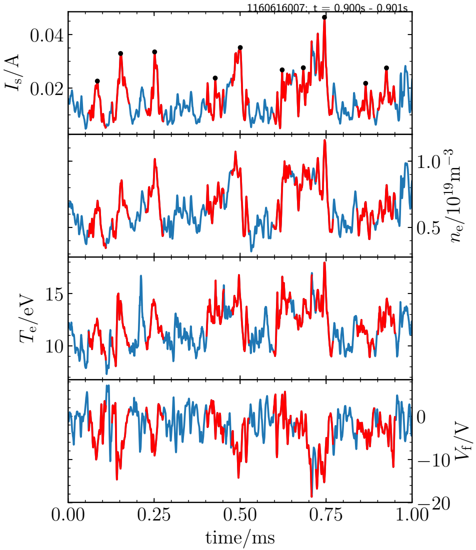
- MLPs sample I_s, T_e, V_f with 1 MHz sampling frequency

Analyzed discharge:

- Ohmic L-mode plasma, $\overline{n}_\mathrm{e}/n_\mathrm{G} = 0.12$.
- Probe head dwells at limiter radius
- $1\mathrm{s}$ long data time series in stationary plasma conditions

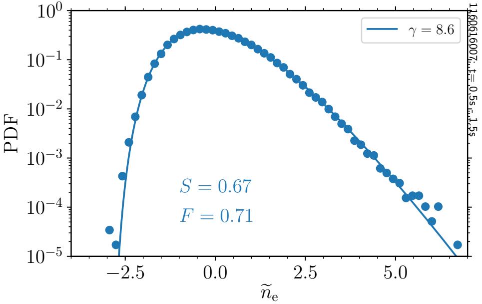
B. LaBombard et al., Rev. Sci. Instr. **78** 073501 (2007)

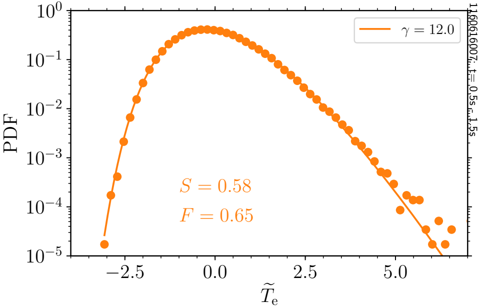
Data time series are dominated by intermittent, large amplitude bursts



- I_s shows stronger degree of pulse overlap than GPI time series.
- Large, intermittent bursts in I_s, n_e, T_e appear correlated.
- Bursts appear on a similar time scale
- Fluctuations in V_f appear on a similar time scale.

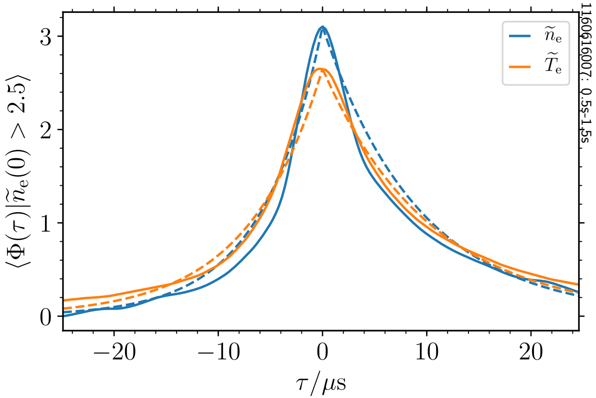
Electron density and temperature fluctuations are Gamma distributed





- Positively skewed and flattened histograms in the far-SOL
- Data time series well described by Gamma distribution over 4 decades
- $\widetilde{n}_\mathrm{e}$ and $\widetilde{T}_\mathrm{e}$ feature stronger degree of pulse overlap than GPI data

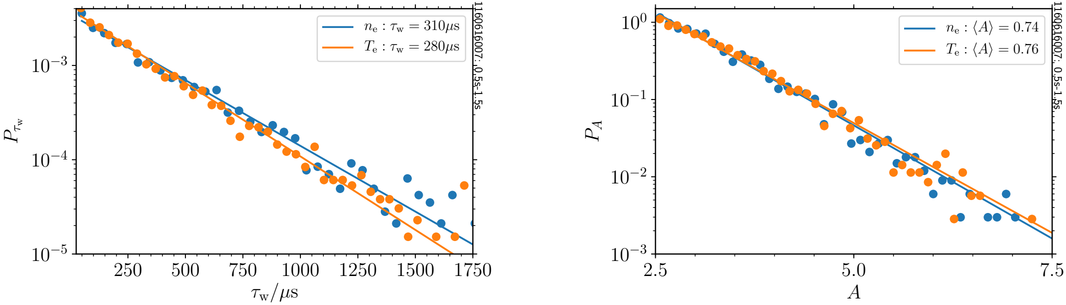
Conditional waveform described by a two-sided exponential function



- Duration time comparable to GPI data
- Weaker pulse asymmetry due to larger degree of pulse overlap

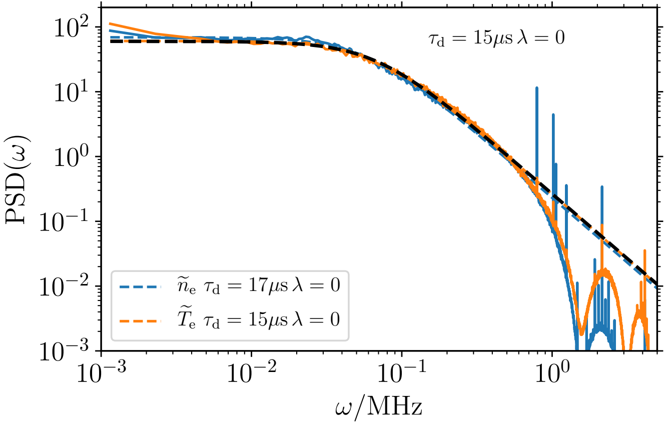
Quantity	events	$\tau_\mathrm{r}/\mu\mathrm{s}$	$\tau_\mathrm{f}/\mu\mathrm{s}$	$\tau_\mathrm{d}/\mu\mathrm{s}$	λ
$\widetilde{n}_\mathrm{e}$	2628	5.8	9.3	15	0.39
$\widetilde{T}_\mathrm{e}$		7.1	10	17	0.42

Exponential waiting time and amplitude PDFs for $\widetilde{n}_\mathrm{e}$ and $\widetilde{T}_\mathrm{e}$



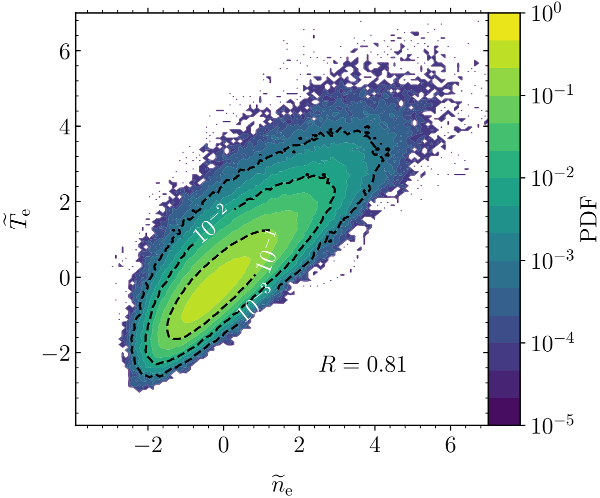
- Large amplitude maxima arrive on similar time scales for $\widetilde{n}_\mathrm{e}$ and $\widetilde{T}_\mathrm{e}$.
- $\widetilde{n}_\mathrm{e}$ and $\widetilde{T}_\mathrm{e}$ present same average burst amplitude.
- Data time series consistent with a Poisson process

Power spectral densities of $\widetilde{n}_\mathrm{e}$ and $\widetilde{T}_\mathrm{e}$ identical and in agreement with stochastic model



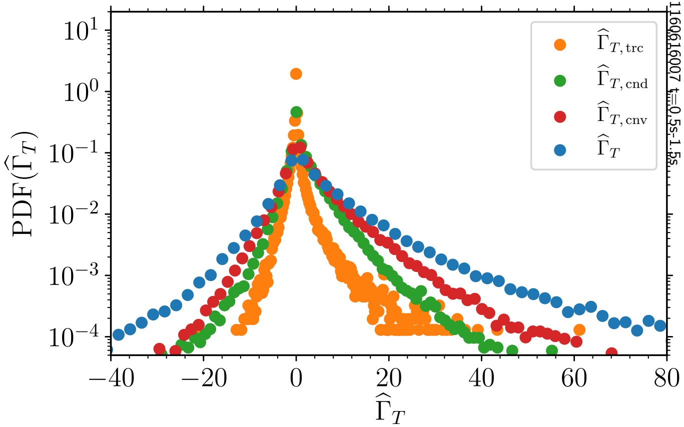
- Pulse asymmetry from conditional averaging over-estimated, possibly due to large pulse overlap.
- Noise at large frequencies due to data processing

Electron density and temperature fluctuations are strongly correlated



- Correlation of large amplitude values demonstrates large amplitude density blobs carry excess heat

Fluctuations of n_e and T_e contribute equally to total radial heat flux



$$\widehat{\Gamma}_{T,\mathrm{cond}} = \widetilde{U}\,\frac{\langle n_\mathrm{e}\rangle}{n_{e,\mathrm{rms}}}\,\widetilde{T}_\mathrm{e}$$

$$\widehat{\Gamma}_{T,\mathrm{conv}} = \widetilde{U}\,\frac{\langle T_\mathrm{e}\rangle}{T_{e,\mathrm{rms}}}\,\widetilde{n}_\mathrm{e}$$

$$\widehat{\Gamma}_{T,\mathrm{trc}} = \widetilde{U}\widetilde{n}_\mathrm{e}\,\widetilde{T}_\mathrm{e}$$

Statistical properties of $\widetilde{n}_\mathrm{e}$ and $\widetilde{T}_\mathrm{e}$ are in agreement with GPI measurements

- Superposition of uncorrelated exponential pulses describes fluctuation statistics of both electron density and temperature
- Both electron density and temperature are Gamma distributed
- Power spectral densities agree with stochastic model
- Plasma blob filaments contain both excess particles and heat
- Frequent large amplitude heat flux events onto the vacuum vessel wall

Take-away messages:

1. Do the fluctuations exhibit universal properties?
   Yes - Fluctuations in the SOL are Gamma distributed and have a Lorentzian power spectral density for all confinement modes investigated
2. Can the SOL fluctuation statistics be understood from a fundamental model?
   Yes - A superposition of uncorrelated, exponential pulses describes PDFs, power spectral densities, and many other statistical properties
3. Are the plasma fluctuation statistics the same in all confinement modes?
   Yes - PDFs and power spectral densities have the same shape in all investigated confinement modes – low- and high-density ohmic L-modes, EDA H-mode and quiescent H-mode
4. Do plasma blobs contribute to heat transport onto the vessel wall?
   Yes - Novel Mirror Langmuir Probe diagnostic demonstrates significant heat flux events associated with blob propagation
5. First principle models of SOL turbulence must reproduce the statistical properties demonstrated here