



SHO 6267

Master of Science in Technology

Modeling and simulation of space debris distribution

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July 24, 2013

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Title:
Modeling and simulation of space debris distribution

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Keywords:
Space Debris, Orbital Mechanics, Orbit Perturbations, Kessler Syndrome, Monte Carlo simulations, LEO

Abstract (English):
This thesis does an analysis of the space environment today, an estimation of the number of space debris at altitude 800 to 950 km, and presents a simulator that generates random space debris and a system that can remove them.

Abstract (Norwegian):
Denne master-oppgaven gjør en analyse av dagens rommiljø, en estimering av romsøppel ved høyde mellom 800 og 950 km, og presenterer en simulator som genererer tilfeldige romsøppel og et system som kan fjerne dem.

Date:
24.07.2013
Classification:
Open
Pages:

Attachments:
10

Master thesis project 2013

Modeling and simulation of space debris distribution

After more than 50 years of ventures into space, mankind has produced an enormous amount of space debris which currently is orbiting the Earth. Such space debris is currently counting approximately 22.000 traceable objects such as used launcher rockets, dead satellites blown apart, and various wreckage particles, and is now starting to represent a problem for the international space station and live satellites in orbit, which may be seriously damaged upon collision with debris particles. It is therefore necessary to develop methods to collect and remove space debris.

Some previous work has been done at NUC on review of approaches for space debris collection [1], and to analyze the collection capabilities of proposed solutions it is necessary to develop simulation tools.

Main task

The main task of this project is to develop possible mathematical models and a Matlab/Simulink toolbox with various simulation tools for analysis of space debris distribution.

Subtasks

1. Study the problem of space debris as well as suggested collection and removal methods, and present a summary of this work. Use [1] as a starting point.
2. Propose a mathematical model for space debris distribution in low Earth orbits.
3. Develop a Space debris toolbox in Matlab/Simulink which enables the possibility to analyze collection capabilities of debris collection methods.
4. Propose a method for debris collection using a picosatellite, and test this method using the simulation toolbox.

Preface

This is a rapport that is the requiring for the diploma thesis, SHO6267, for master degree in technology for Satellite Engineering at Narvik University College.

I would like to thank my supervisor Raymond Kristiansen for being supporting and always having time to discuss about my thesis, and my co-supervisor Rune Schlanbusch for being available for questions and discussions. I must also thank Espen Oland for advising and good ideas, and my classmates Tom Stian Andersen and Peter Smirnov for good discussions and helping with MATLAB and etc. I wish to thank the library at NUC for helping to find relevant literature to my thesis.

Summary

This thesis does an analysis of the space environment and tries to make simulator that can generate space debris and then catch as many as possible with removing system.

First it make an analysis over the present space environment where cataloged orbital objects for a period of 119 days through this project have been observed. This show that the total amount of space decays slowly but is at the same time very vulnerable when more space debris creates.

There become also done an estimation of how much space debris there are in the region with altitude between 800 and 950 km based on cataloged space object. This gives us good estimates but if we compare it with estimates from the DAS model we see that there are some differences. This can maybe come from that the model is buildup of other principles than our model.

The mathematical model for simulate and remove space debris builds up of three parts: a space debris generator, a satellite and a removing system. The space debris generator generates a fixed number of space debris with random position, mass and size, which the removing system based on a set of parameters tries to see if it can catch any of them. There are two removing methods that collect the space debris at the same time, so we can see who of them that collects most space debris. There do 10 Monte Carlo simulations to give the best estimations. The mass of the collected space debris from one of the removing system become added to the mass of the satellite which try together with perturbing to see if they can disturb its orbit. The simulations give good results which shows that both removing system is able their each amount of space debris, but because of a short simulation time there the perturbing forces shows no effect on the satellite orbit.

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1 Introduction

4. October 1957 is a historic day, [2]. At that day the first manmade object was set in orbit around Earth: the satellite Sputnik 1 launched by the Soviet Union. But Sputnik 1 with its rocket body SL-1 is also the two first manmade space debris. Already early next year was USA out with their first satellite: Explorer 1. After that the number of satellites and manmade space objects have exploded. Today (21. July 2013) there are manmade 3731 payloads in space of which are 1170 active, [2]. Additionally there are 13 075 cataloged manmade space debris around Earth. There are estimated to be 135 000 000 space objects over 1 mm, [3].

In earlier time space debris was not seen as any big problem. USA launched for example a set of the three satellites Vanguard 1 to 3 in the period 1958-59, [2]. They were built with expected lifetimes to be 240 - 300 years. Today over 50 years later the three satellites with their rocket bodies are still out there which gives them the status as the oldest manmade objects in orbit around Earth. Last year (2012) returned finally satellite Explorer 8 down to Earth. It has then been in orbit since 1960 and with its over 51 years in orbit it is the manmade object that has been longest in orbit and returned. Although all of the three Vanguard satellites and Explorer 8 have been long time in orbit they were still not operational for a very long time. Only three months after launch none of them was operational anymore.

During only the last ten years there have happened several bad incidents which have been involving space debris:

- This year (2013) there is 10 years since the accident of the space shuttle Columbia. Then it was a piece of foam that damaged the thermal protection to one of the wings that gets the space shuttle to breakup, [4]. 7 people died. This shows how much problem a little debris can make if one is not protected.
- In 2007 China destroyed one of their old satellites: Fengyun-1C, [5]. Today (21. July 2013) this is reasonable for 3046 cataloged space debris that are still in orbit, [2].
- In 2009 there was a collision between the old Russian satellite Cosmos 2251 and the then still operating American satellite Iridium 33, [6]. It was predicted they would pass by a range of 584 m but this was obviously not enough. Today (21. July 2013) there are still 1301 space debris from Cosmos 2251 and 449 space debris from Iridium 33 in orbit, [2].
- In 2011 the spacecraft GOES 10 suddenly decreased its perigee (lowest altitude in its orbit) by 20 km. The reason is unclear but space debris can be a possibility, [7].
- In 2012 was the happening of the breakup of the Breeze-M rocket body, [8]. It most probably exploded and was near its perigee at altitude 268 km.

In 1978 was the first time the term "Kessler syndrome" was presented, [9]. That says that every collision with space debris will make more space debris that again can collides with more objects to make even more space debris. It has since then been seen as frightening scenario where the space around Earth is so filled up of space debris that it becomes impossible to have neither satellites nor other spacecraft there.

The work to reduce space debris has continued since then. In the beginning of 1980's the design of rocket bodies was changed to reduce their explosion rate, [9]. Today is debris from explosion decreasing and it is no longer seen as a big source for space debris. Today are collision fragments the biggest source for the increasing of space debris.

Active collision avoidance is also an initiative to reduce the effect "Kessler syndrome". In 2012 the International Space Station (ISS) had done 15 avoidance maneuvers to avoid collision from space debris, [7]. Among these are space debris from Fengyun-1C, Iridium 33 and Cosmos 2251.

All the big space organizations (NASA, ESA, etc.) have now decided through standards (ISO 24113, etc.) that every manmade space object in Low Earth Orbits, LEO, should be designed to reentry to Earth within 25 years after end of operational time, [10]. In Geosynchronous Orbits (GEO) there will be enough to remove out of region after 25 years after operational time. But the only problem is that not many space objects have that capability yet. There is estimated that even if 100 % of all object could do this the population of space debris will still increase in LEO, [9]. The LEGEND (LEO-to-GEO Environmental Debris model) from NASA have calculated there are needing for removing at least 5 objects/year if there 90 % of the objects that are capable to remove their self within 25 years. If not there must be removed even more objects per year. This was confirmed this year (2013) at The Sixth European Conference on Space Debris in Darmstadt Germany where study space organizations ASI, ESA, ISRO, JAXA, NASA and UKSpace where all of them was had come to this same conclusion, [11].

1.1 Previous work

Since Kessler and Cour-Palais in 1978 presented the danger of the increasing population of space debris, later known as the "Kessler syndrome", there has been working to find methods to try stopping the increasing before it reach a critical level (the collision rate too high for spacecrafts to moving out there), [9]. There have been given stricter restriction of design of space object through international standards: ISO24113 [10], United Nations [12], IADC [13] and etc. Of the measures that is already running is the making of better shielding against space debris collision and better avoiding maneuvers for they of them that are too big. The spacecraft's that launches today have also much lower explosion rates than earlier and there are very strict rules against letting loose object from spacecraft without good reason.

As a part of the work to avoid big space debris all big space object is catalogued in databases, SATCAT [2] for instance, so there are possibility to detect and prepare for close passages between two objects and to see if avoiding maneuvers is necessary. Today it is only possible to see objects that are bigger than 10 cm in Low Earth Orbits and bigger than 70 cm in Geostationary orbits, [3].

In the work to try to understand the space environment better and in the attempt to predict how it will evolve in the future there are made a lot of simulation models:

- DAS 2.0.2 (Debris Assessment Software) is a simulation program from NASA that is designed to work with orbital debris assessment, [14];
- MASTER-2001 (Meteoroid and Space Debris Terrestrial Environment Reference) is a model from ESA that simulates the space environment, [15];
- LEGEND (LEO-to-GEO Environmental Debris model) is a model from NASA that simulate future collision based on databases of space object from today, [9];
- ORDEM2000 (Orbital Debris Engineering Model) is a model from NASA that describes the orbital environment in Low Earth Orbit (at the altitude 200 to 2000 km), [16].

As concluding in earlier work by T. I. Bredeli, [1] there will be need for designing a mathematical model that can simulate space debris and at the same time give the opportunity could test with a removing system to see how many space debris that could remove. There was discussing two methods about removing space debris: using a web or magnets. Both these methods was meant for picosatellites (10*10*10 cm units satellites,) but since it yet was not done any research on removing space debris with small satellites they will be ideas that need research.

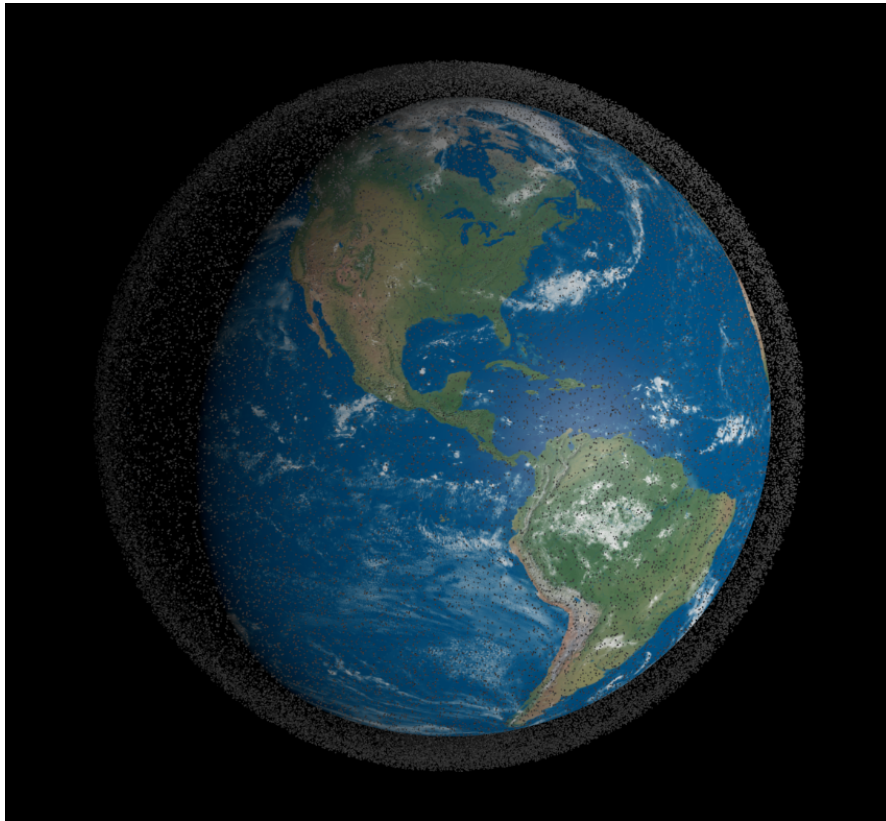


Figure 1: Illustration of space debris in Low Earth Orbit (the space debris are not in correct scale). Reproduced with courtesy to Tom Stian Andersen.

1.2 Contribution

My contribution of this thesis is to make the first design of a space debris removing simulator. I have made a space debris generator that generates space debris with random position and velocity in Low Earth Orbit (see Figure 1) and given them random size and mass. There is designed a simulation of a satellite that should be able to be affected by perturbations in its orbit and the effect of the masses from collected space debris.

I have also done an analysis of how the number of the observable space debris has changed in a period of 119 days.

An attempt to try to find how many space debris the satellite statistically will be able to catch over a set time is also done.

1.2.1 Delimitations

In this report it is not looked at any concrete method on active removing space debris since there today do not exists any plan or ideas of using picosatellites to remove space debris. I have instead tried to make a simulator that is based on a set of criterions of the removing system can catch space debris. This will make it easier for future design of removing system since this will works for more general design and not necessarily for methods only for picosatellites.

1.3 Report outline

This is the outline of the report:

Chapter 2 will look at all preliminary theoretical background that is essential for understanding the orbital mechanics in the simulation of the space debris and other orbital objects. The process of the whole simulator is also described in this section.

Chapter 3 presents all the analysis results. There will also be the results of calculating the probability of catching space debris.

Chapter 4 is all the result from the simulations.

Chapter 5 will give the main conclusion of the thesis and what needs for future work.

2 Theoretical backgrounds

In this chapter we will start with defining the notation in Section 2.1, before we look at reference frame in Section 2.2 and the laws of Kepler and Newton in Section 2.3. In Section 2.4 we will look at orbital parameters we will use, and in 2.5 we will define what a space debris is. We will get a description of the Low Earth Orbit in Section 2.6. Section 2.7 analyses the population of space debris over time period and Section 2.8 will define the mathematical model of the simulator and which altitude we will simulate for. In Section 2.9 we will look at the number and probability for meeting space debris around the altitude we will simulate in. Section 2.10, 2.11 and 4.2 describes each part of the simulator.

2.1 Notation

In this section we look at notation that will be used in the rest of the report.

The set of all real numbers is denoted as \mathbb{R} .

We will denote vectors as small bold letters as i.e. \mathbf{x} . A vector \mathbf{x} in n dimension is defined as:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n, \quad (1)$$

where \mathbb{R}^n is the Euclidian space in n dimensions and $x_1, x_2, \dots, x_n \in \mathbb{R}$.

The magnitude for a vector \mathbf{x} is defined as:

$$x = \sqrt{\mathbf{x} \cdot \mathbf{x}}. \quad (2)$$

A vector of a coordinate system a is denoted as \mathbf{x}^a .

The unit vector \mathbf{i}_x for a vector \mathbf{x} is defined as:

$$\mathbf{i}_x = \frac{\mathbf{x}}{x}. \quad (3)$$

The time derivative of the vector \mathbf{x} is denoted as $\dot{\mathbf{x}}$ and is defined as:

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \left[\frac{dx_1}{dt}, \frac{dx_2}{dt}, \dots, \frac{dx_n}{dt} \right]^T. \quad (4)$$

The second time derivative is denoted as $\ddot{\mathbf{x}}$ and is defined as:

$$\ddot{\mathbf{x}} = \frac{d^2\mathbf{x}}{dt^2}. \quad (5)$$

The cross-product \mathbf{z} between the two vectors \mathbf{x} and \mathbf{y} is denoted as:

$$\mathbf{z} = \mathbf{x} \times \mathbf{y}. \quad (6)$$

We will denote the matrices with capitalized bold letter and define them as:

$$\mathbf{A} = \{a_{ij}\} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \in \mathbb{R}^{n \times m}, \quad (7)$$

where n is the rows and m the columns of the matrix.

To get the transposed of matrix \mathbf{A} , $\mathbf{A}^\top = \{a_{ji}\} \in \mathbb{R}^{m \times n}$, we change places of the rows with the columns.

The reference frame for a coordinate system a is denoted as \mathcal{F}^a . The rotation matrix for transforming from reference frame \mathcal{F}^a to \mathcal{F}^b is denoted as \mathbf{R}_a^b .

In this report is only SI units (International Systems of Units) used.

2.2 Reference frames

To be able to could determine the position and velocity of a space object, we need to use reference frames. Reference frames have a defined origin, the center of the system, and three orthonormal axes, x , y and z . In our system we will use two reference frames: Earth Centered Inertial frame (ECI), \mathcal{F}^i , and orbit frame, \mathcal{F}^o .

The ECI, [17], have the center of mass to Earth as origin and the z -axis, \mathbf{z}^i , pointing in the direction of the North Pole along the spin axis of Earth. The x -axis, \mathbf{x}^i , is pointing in direction of vernal equinox line, γ . Vernal equinox line is the direction from the equator of Earth to the center of Sun when the Sun crosses the equator from south to north at noontime the first day in spring (≈ 21 March). Today vernal equinox is pointed at the constellation of Pisces. The y -axis, \mathbf{y}^i is orthonormal to \mathbf{x}^i and \mathbf{z}^i and complete the right-handed system. Both \mathbf{x}^i and \mathbf{y}^i lay in the equatorial plane of Earth. The ECI frame will look like Figure 2

The orbit frame, also known as Local-Vertical-Local-Horizontal (LVLH), use the mass center of the space object as origin and have the z -axis, \mathbf{z}^o , pointing at the center of mass of Earth along the r -axis (r is the magnitude of \mathbf{r} , which is the position vector of the space object seen from center of mass to Earth). The y -axis, \mathbf{y}^o , lies in opposite direction of the spin axis, \mathbf{h} , and the x -axis, \mathbf{x}^o , is the cross-product of y and z to fulfill the right-handed system. The LVLH can be defined as:

$$\mathbf{x}^o = \mathbf{y}^o \times \mathbf{z}^o, \quad \mathbf{y}^o = -\frac{\mathbf{h}}{h} \quad \text{and} \quad \mathbf{z} = -\frac{\mathbf{r}}{r}. \quad (8)$$

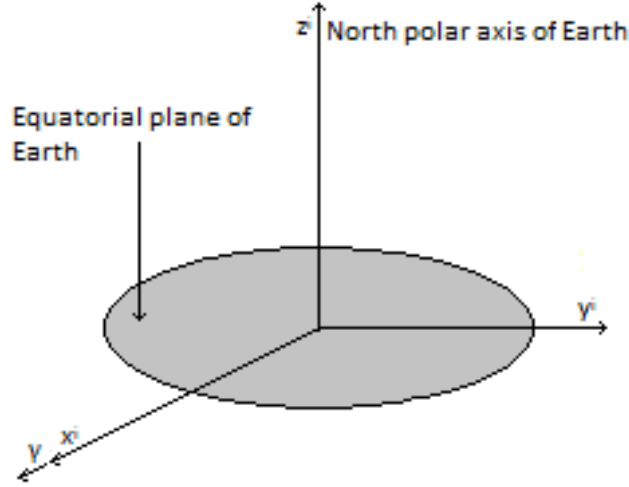


Figure 2: Earth Centered Inertial frame

2.2.1 Rotation between frames

Very often can we have a vector, \mathbf{x} , in coordinates of one frame, i.e. a , but wish to have it in coordinates of another frame, b . One opportunity we then have is to use a rotation matrix. If we use the rotation matrix \mathbf{R}_a^b we can use it to rotate \mathbf{x} from frame a to frame b :

$$\mathbf{x}^b = \mathbf{R}_a^b \mathbf{x}^a. \quad (9)$$

Not always have we the right rotation matrix available, we may have the rotation matrix to rotate from frame b to a instead of from a to b . Then we can use the fact that a rotation matrix is orthonormal. The inverse of an orthonormal matrix is equal to its transposed. By knowing this we can get the rotation matrix \mathbf{R}_a^b with replacing it with \mathbf{R}_b^a :

$$(\mathbf{R}_b^a)^{-1} = (\mathbf{R}_b^a)^\top = \mathbf{R}_a^b. \quad (10)$$

We can now rewrite Equation 9:

$$\mathbf{x}^b = (\mathbf{R}_b^a)^\top \mathbf{x}^a. \quad (11)$$

In some cases when we only need the magnitude of the vector we can drop the rotation matrix and say that the magnitude is equal for every reference system:

$$x^b = x^a. \quad (12)$$

2.3 The laws of Kepler and Newton

Most of the orbital mechanics can be related to a set of laws: Kepler's three laws for planetary motion and Newton's three laws of mechanics plus one for gravitational

attractions, [18]. Kepler's laws is originally defined for a planet in orbit around the sun, but can also be used for a space object in orbit around the Earth.

Kepler's laws:

1. the planet goes in an elliptic orbit with the sun as one of the focal points;
2. the planet sweeps with a radius, r , from the sun to itself over an equal area to an equal time interval;
3. the orbit time, T , of the planet is proportional with the powers of $\frac{3}{2}$ to the mean distance to Sun (\approx semimajor axis, a (the distance from origin to one of the farthest point of the ellipse)).

Newton's laws:

1. an object which is not affected by any forces will moves with constant velocity (or zero if the object was originally in rest) and no acceleration;
2. the rate of change of linear momentum, $m\mathbf{v}$ (m is the mass and \mathbf{v} the velocity vector of the object), of an object is equal to the force \mathbf{f} added on the object:

$$\mathbf{f} = \frac{d(m\mathbf{v})}{dt}, \quad (13)$$

3. if an object A is added a force from an object B it exerts a force with an equal magnitude but in opposite direction, which we can write as:

$$\mathbf{f}_{B \text{ on } A} = -\mathbf{f}_{A \text{ on } B}, \quad (14)$$

4. the gravitational forces between two object, A and B, can be described as

$$\mathbf{f} = \frac{Gm_A m_B \mathbf{r}}{r^3}, \quad (15)$$

where G is the gravitational constant ($6.6720 \times 10^{-20} \text{ km}^3/(\text{kg}\cdot\text{s}^2)$), M_A and M_B the mass of the objects A and B, and \mathbf{r} is the position vector between A and B with its magnitude r .

2.4 Orbit parameters

The orbit of an object around Earth is normal circular or elliptic. There exists also parabolic and hyperbolic orbits but we will not simulate objects in orbits like that since they are not continuously. A space object in these orbits will have so high velocity that they will escape the gravity of Earth. An object in circular orbit is easiest to do calculations on. It has constant distance to Earth and unchanged total velocity in orbit. In our model we will mostly look on elliptic or near-circular orbits. Elliptic orbits are some different from circular orbits but there are also many similarities with them. We will define some of the parameters for an elliptic orbit with Figure 3.

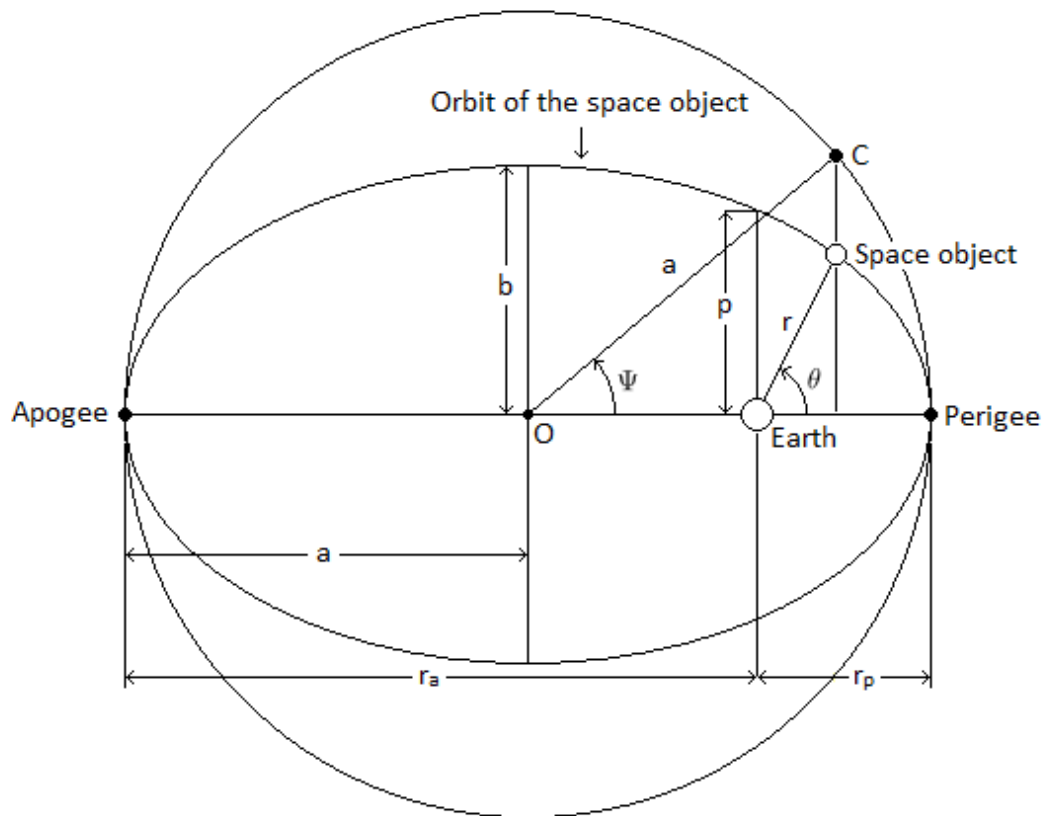


Figure 3: Parameters for an object in an elliptic orbit

Perigee is the point in the orbit that is closest to Earth, which is at one of the focal point of the ellipse. In other situations where Earth is not a focal point it is more common to use periapsis instead of perigee. The distance from center of the Earth to perigee is called the perigee radius, r_p .

Apogee is the opposite of perigee where it is the point in the orbit that is farthest from Earth. The more common name of apogee is apoapsis and the distance from center of Earth to apogee is the apogee radius, r_a .

\mathbf{r} is the position vector of the space object and has the magnitude r .

The semimajor axis, a , is the longest distance from the origin of the ellipse, O , to the elliptic orbit. If this have been a circular orbit a had been equal to the magnitude of \mathbf{r} . a can be defined by Equation 16 from [19]:

$$a = \frac{r_a + r_p}{2}. \quad (16)$$

The semiminor axis, b , is the shortest distance from O to the elliptic orbit. This parameter will not be used in our model.

The true anomaly, θ , is the angle between perigee radius and \mathbf{r} . This can help us to find the position of a space object in orbit.

p is the magnitude of \mathbf{r} when θ is equal 90° . With [15] it can be defined as:

$$p = a_0(1 - e^2). \quad (17)$$

Here is a_0 the semimajor axis at the start of orbit/simulation.

The eccentric anomaly, Ψ , is the angle between the apsis line from O to perigee and the radius from O to a point C . C is perpendicular to the semimajor axis, a , and is going through the position of the space object to a point on an auxiliary circle, which has O as center and the semimajor axis, a , as radius. Ψ is an auxiliary angle that let us transform the elliptic orbit into a circular orbit so we can use it instead of θ .

The six Kepler elements

To define the position and velocity for the space object it is normal to use set of orbital parameters. We will use the six Kepler elements in our simulation:

- a : the semimajor axis;
- e : the eccentricity;

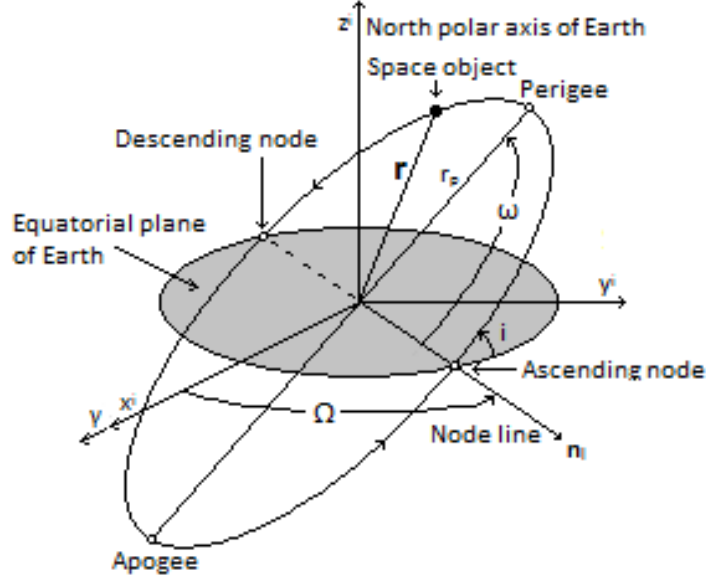


Figure 4: Orbital elements

- i : the inclination;
- Ω : the right ascension of ascending node;
- ω : the argument of perigee;
- M : the mean anomaly.

The eccentricity, e , is a parameter that describes the shapes of a conic orbit. e can with [19] be defined as:

$$e = \frac{r_a - r_p}{r_a + r_p}. \quad (18)$$

If e is equal to 0, $r_p = r_a$, the orbit is circular. The more e goes against 1, $r_p < r_a$, the more elliptic becomes the orbit. If e is equal to 1, $r_a = \infty$, the orbit is parabolic. All values over 1 give a hyperbolic orbit.

To defining the rotation angles i , Ω and ω we will use Figure 4. Here we have taken the orbit of the space objects and set it in \mathcal{F}^i frame (ECI). As we can see the orbit intersect with the equatorial plane at the ascending and the descending node. The line with center of mass to Earth as origin and goes through the ascending node is called the node line, \mathbf{n}_l .

The inclination i is the angle between equatorial plane of Earth and ascending orbit of the space object. It is positive with a range from 0° to 180° .

The right ascension of the ascending node, Ω , is the angle between the vernal equinox, γ , and \mathbf{n}_l . This angle is positive and has a range from 0° to 360° .

The argument of perigee, ω , is the angle between the equatorial plane of Earth and the radius of perigee, r_p . This angle is positive and has the same range as Ω .

The mean anomaly, M , is similar to the eccentric anomaly, Ψ , but instead of to be based on the position of the space object M is based on the total orbit time, T . Seen from O it is the mean angle of the auxiliary circle which is a transformation of the elliptic orbit. M can be expressed with help of integrating the mean motion, n , over a time interval from the time at last perigee, t_p , to the present time in orbit, t . In our simulation t_p always be 0, but we have in Appendix A.1 analyzed the case if it is not 0. Since we consider the mean motion to be constant we can define M as we know from [19]:

$$M = n(t - t_p). \quad (19)$$

One important fact to mention is that we are looking only at the definition of mean anomaly for an elliptic orbit. There is another definition for the mean anomaly for hyperbolic orbit, but this mean anomaly will we not need in our simulation.

The mean motion is as we can see the derivative of M and can as M also be defined out from the orbit time, T . T can be defined as the angular distance divided by the angular speed, [20]:

$$T = 2\pi \sqrt{\frac{a^3}{\mu}} = \frac{2\pi}{\sqrt{\frac{\mu}{a^3}}} \quad (20)$$

where a is the semimajor axis and μ the gravitational parameter. μ is defined, [19], as

$$\mu = G(M_e m_s) \cong GM_e \quad (21)$$

where G is the gravitational constant ($6,6720 \times 10^{-20} \text{ km}^3/(\text{kg} \cdot \text{s}^2)$), M_e the mass of Earth ($5.9742 \times 10^{24} \text{ kg}$) and m_s is the mass of the spacecraft/satellite. But since m_s will be incredible small compared to the M_e we can drop m_s from the calculations.

We can define n as the angular speed and with [19] we can get it to be:

$$n = \sqrt{\frac{\mu}{a^3}} \quad (22)$$

With all these orbital parameter we will later use they to calculate the position and velocity of the space debris and the satellite, see Section 2.10.2.

2.5 Definition of space debris

In orbits around Earth there are many manmade objects. Many of them are payloads although most of them are space debris. Debris in space around Earth can sort space debris in two groups: micro-meteorites and manmade space debris, [21]. In this thesis we will only look at manmade space debris when we are talking about space debris. We will also look at inactive payloads which are payloads that have the operational status as either nonoperational or unknown, [2]. They are no longer active and do the same as space debris: no nothing and to danger for other object in the same area.

2.5.1 The size of space debris

The size is something that is very important to know about the space debris. The bigger the space debris is the more damage they can do. The smallest debris can not be detected and the only way to avoid them is to use some kind of shielding. Big space debris can be impossible to shield against and the only way to save the spacecraft is to avoid them with good range. We will use the definition from [22] to define the size of the space debris:

- large space debris, have a diameter bigger than 10 cm;
- medium-sized debris: have a diameter from 1 mm to 10 cm;
- small debris: have diameter smaller than 1 mm.

2.6 Low Earth Orbit (LEO)

Low Earth Orbit (LEO) is a region that is closest to the surface of Earth and have the biggest density of orbital object. An object is defined to be in LEO if the mean altitude (\cong semimajor axis, a) is not more than 2000 km and the orbit time (T) not more than 127 minutes, [23]. In this region have the objects normally very circular orbits (low eccentricity).

2.7 Analysis of space environment

Through period of 119 days we will analyses the population of payloads (manmade space objects) and manmade space debris. We will base us on data from SATCAT (Satellite Catalog) from CelesTrak, [24]. SATCAT is a database where every known manmade space object is registered. We use DOY, Day Of Year, to note each sample. The period will begin at 4. Mars 2013 (63 DOY 2013) and end at 1. July 2013 (182 DOY 2013).

2.8 Definition of mission

In this section we will set the delimitation for the mission and try to define the mission and how the final product will look like.

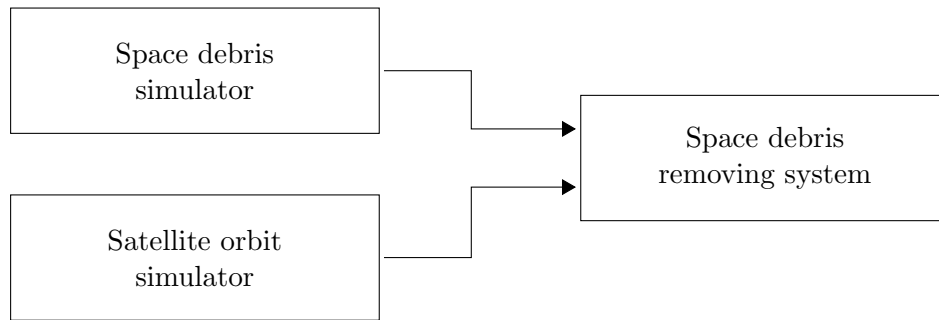


Figure 5: The total system

2.8.1 The mathematical model

We want to make a mathematical model that can simulate a number of space debris and check if a satellite in orbit around Earth can remove some of them. The mathematical model will look like in the Figure 5.

There will be three subsystem of the model:

- Space debris generator;
- Satellite orbit simulator;
- Space debris removing system.

These will work in the sequence as in Figure 6 which is the flowchart for the total model.

2.8.2 Definition of altitude

If we should define the altitude of the mission we must first define which type of orbit we will use. We can use either a circular or an elliptic orbit.

With a circular orbit we can use easier calculation since the eccentricity, e , will be zero, but then will also the altitude be constant. This means that it is very important that the altitude we choose in this case must be in a orbit with much space debris.

If we choose to use an elliptic orbit there is important to lock at the eccentricity, e . If e is big it will also be big difference between apogee and perigee. Then we will know from Kepler's second law, see Section 2.3 that the satellite will use more time at higher altitude than lower altitude since it will sweep over an smaller area in perigee than in apogee. The bigger the difference between apogee and perigee the more time will the satellite use close to the apogee than the perigee.

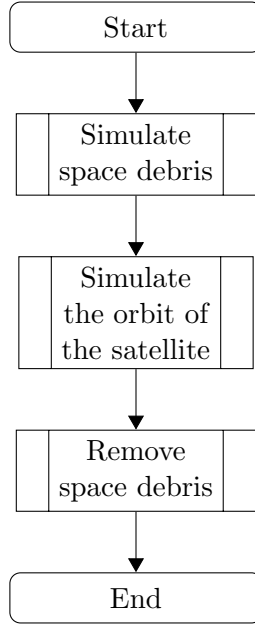


Figure 6: Flowchart for the total system

Earlier study done by Liou and Johnson in 2006, [25], have showed that close to 60 % of the space debris population in LEO lay on an altitude between 900 km and 1000 km. On lower altitude down to 750 km the population is tolerable high (totally 20 %). Over 1000 km is it not much noticeable space debris except for between 1400 - 1550 km (18 %).

After this study there have come some important changes. It have later been two big breakups in LEO: the destroying of the Chinese satellite Fengyun-1C at the altitude on 850 km in 2007 [5] and the collision between the American satellite Iridium 33 and the old Russian satellite Cosmos 2251 at the altitude 790 km in 2009 [6]. These two breakups had so big effect of the total amount of space debris which the population at altitude under 1000 km increased with 50 %.

We have then chosen the satellite to have an altitude between 800 and 950 km. The apogee, r_a , and perigee, r_p , is defined as the highest and lowest altitude added with the radius of Earth at equator, $R_e = 6378$ km:

$$r_a = 950km + R_e = 950km + 6378km = 7328km; \quad (23)$$

$$r_p = 800km + R_e = 800km + 6378km = 7178km. \quad (24)$$

Now we can use Equation 18 from Section 2.4 to find the eccentricity, e :

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{7328km - 7178km}{7378km + 7178km} = 0.0103 \quad (25)$$

This eccentricity is very small which will make the time the satellite is near apogee and very similar. With these values we can find the semimajor axis, a . We can use Equation 16 from Section 2.4:

$$a = \frac{r_a + r_p}{2} = \frac{7328km - 7178km}{2} = 7253km \quad (26)$$

Instead of using a we can write it as its altitude, a_{alt} . a_{alt} is the semimajor axis minus the radius of Earth, R_e . It can be express in the form of the altitude of the radius of apogee and perigee, $r_{a.alt}(=r_a - R_e)$ and $r_{p.alt}(=r_p - R_e)$, with help of Equation 16 from Section 2.4:

$$\begin{aligned} a &= \frac{r_a + r_p}{2} = \frac{(r_{a.alt} + R_e) + (r_{p.alt} + R_e)}{2} = \frac{2R_e + r_{a.alt} + r_{p.alt}}{2}, \\ &\Rightarrow a - R_e = \frac{r_{a.alt} + r_{p.alt}}{2}, \\ &= a_{alt} = \frac{r_{a.alt} + r_{p.alt}}{2}. \end{aligned} \quad (27)$$

This gives us a_{alt} to be 875 km which we will base our system on. The system should be able to simulate for altitudes over and under a_{alt} . We will set a range of the altitudes for simulator to be from 500 to 1000 km.

2.9 Number of space debris

There are today (1. July 2013) cataloged 16 820 manmade objects in space outside Earth, [24]. Of these are 13 092 registered as space debris and if we add the inactive payloads (=total number of payloads - number of active payloads) we get an amount of 15 650 objects.

2.9.1 Number of space debris in LEO and specific region

We have from CelesTrak, [24] only the total numbers of manmade object in space outside the Earth but not number of objects in Low Earth Orbit, LEO. To find that we will use their list of over the cataloged manmade objects in space: SATCAT. satcat.txt, see Appendix B5, is a list where all manmade objects in space since the satellite Sputnik 1 and his rocket body was shot up in the end of 1957 is catalogued with orbit data, operational status, and launch and, if it already decayed, reenter dates. We will make a program in C++ code that can sort out which of the object that occur in LEO and have not decayed yet. The program shall also count every object so we always know how many object we have to deal with. We will also look at the region where we will simulate the satellite: 800 to 950 km over the surface of Earth. Since we are most interesting in objects that are no longer active we will see most on space debris and inactive payloads. The program will in total make four lists:

1. orbital objects that are within LEO in their orbit;
2. space debris and inactive payloads that are within LEO in their orbit;

3. space debris and inactive payloads that will be all time within the region (altitude 800 - 950 km);
4. space debris and inactive payloads that are within the region (altitude 800 - 950 km) in their orbit.

To make these lists it will use the known apogee and perigee of each objects to see if they are within the region, their status to see if they are payloads or debris, and their operational status: decayed, active, inactive. All the lists will be based on data from SATCAT at 01.07.2013, [2].

Now we will have the number of how many space debris and inactive payloads the will be within the region (altitude 800 - 950 km), but we will also like to know how many of them will statistically be there all time. To do that we will only need to know their apogee, perigee and orbit time which we can read from the fourth list we made. We will sort these objects in five groups based on conditions of their apogee and perigee, r_a and r_p , according to high and low altitude, r_h (= 800 km) and r_l (= 950 km):

1. $r_a \leq r_h$ and $r_p \geq r_l$;
2. $r_a = r_l$ or $r_p = r_h$;
3. $r_a \leq r_h$ and $r_p \leq r_l$;
4. $r_a \geq r_h$ and $r_p \geq r_l$;
5. $r_a \geq r_h$ and $r_p \leq r_l$.

These five groups will actually do the same; calculate how much of the orbit time each object will use in the region. This ratio will we denote as t_s . The difference between the groups will be the complexity of the further calculations.

In the first two groups it is easiest to find t_s since we actually do not need any calculation at all. When the apogee is lower than the highest altitude and perigee higher than lowest altitude we know that the object will always be inside the region and we can set t_s to be 1. In the second group we have objects that will only touch the limits of the region and never use time to be inside it, so then we can set t_s to be 0.

The three next are some more complicated and we will have need some calculation to find t_s . We can define t_s as:

$$t_s = \frac{2(t_h - t_l)}{T}, \quad (28)$$

where T is the orbit time, t_h time since perigee when the object is in that's highest point in the region, and t_l is the same as t_h but is instead at the lowest point the object appears in the region. The equation can be explained in that way we calculate the time from the lowest point in the region to the highest before we multiplies with two since we calculate the time both ways in the orbit. To the end we divide it all with the orbit time to get the average.

The next step is to find t_h and t_l . To find them we will first need to find three parameters: the eccentricity, e ; the semimajor axis, a ; and the mean motion, n . These can we find with use of the equations from Section 2.4: Equation 18, 16 and 22:

$$e = \frac{r_a - r_p}{r_a + r_p}; \quad a = \frac{r_a + r_p}{2}; \quad n = \sqrt{\frac{\mu}{a^3}}.$$

The gravitational parameter, μ , can we write as $398\,600 \text{ km}^3/\text{s}^2$, [17]. We can now find t , time since perigee, with the corresponding r , distance from mass center of Earth, with help of some equations from [19]:

$$\cos \theta = \frac{r_p(1+e)}{re} - \frac{1}{e}; \quad (29)$$

$$\cos \psi = \frac{e + \cos \theta}{1 + e \cos \theta}; \quad (30)$$

$$t = \frac{\psi - e \sin \psi}{n}, \quad (31)$$

where θ and ψ is the true and the eccentric anomaly. When we want to find t_h we must only replace t with t_h and r with r_h . The same is for t_l where we replace t with t_l and r with r_l .

If we now look back at the groups 3 and 4 we will see we can do some simplifications. In group 3 we have $r_a \leq r_h$. This means that r_a is inside the region and we can define it as r_h . Since an orbital object use the half of his orbital time, T , from perigee to apogee (Kepler's second law) we get:

$$t_h = \frac{T}{2} \quad (32)$$

To get t_l we must use Equation 31 since r_p lies outside the region.

The group 4 is the opposite of group 3. Here lies r_p inside and r_a outside the region. We can then define r_p as r_l and set t_l to be zero by definition of t_p . To find t_h we use Equation 31.

At last in group 5 we have both r_a and r_p outside the region which means we must use Equation 31 to calculate both t_h and t_l .

Now we can use Equation 28 to calculate t_s . If we then sum up t_s for all objects we will know how much objects there will average all time be in the region.

This is of course only an estimate of the observed space debris. From [3] we know that can observe objects as small as 10 cm in the LEO-region so we know then that this estimate will for objects as small as that.

2.10 Mathematical model of space debris

In this section we will simulate the space debris. We will simulate amount of j (defined by user) space debris where each of them will get their own status vector which include their position, velocity, mass and size. These vectors will further be able to be expanded in future works. All these vectors are elements of a bigger matrix, SD_{matrix} that include all the these vectors. This matrix will be used in further parts of the model. The matrix will look like this:

$$SD_{matrix} = \begin{bmatrix} r_{x1} & r_{x2} & \cdots & r_{xj} \\ r_{y1} & r_{y2} & \cdots & r_{yj} \\ r_{z1} & r_{z2} & \cdots & r_{zj} \\ v_{x1} & v_{x2} & \cdots & v_{xj} \\ v_{y1} & v_{y2} & \cdots & v_{yj} \\ v_{z1} & v_{z2} & \cdots & v_{zj} \\ mass_1 & mass_2 & \cdots & mass_j \\ size_1 & size_2 & \cdots & size_j \end{bmatrix}, \quad (33)$$

where $[r_x, r_y, r_z]^T = \mathbf{r}^i$ and $[v_x, v_y, v_z]^T = \mathbf{v}^i$ the position and the velocity vector in inertial frame, and mass and size the mass and size for each space debris.

We can see a flowchart in Figure 7 where this process is illustrated.

2.10.1 Defining of orbital parameters

We must first define the orbital parameters if we want to find the position and the velocity of the space debris. We will use the definitions from Section 2.4.

In our system the user will only be able to choose the altitude of the semimajor axis, a_{alt} . The range of a_{alt} is from an altitude 500 to 1000 km (ideal 800 - 950 km), see Section 2.8.2. The rest of the parameters are determined by random function in the model.

The eccentricity, e , is set to be random between 0 and 0.01. This is because we simulate space debris in LEO so we let them have very circular orbits, [15].

We know from [26] most of the collisions, over 50 %, with the space debris happens when the inclination, i , is between 80° and 100° . To make a more correct model we will therefore set the inclination to be 50 % random between 80° and 100° and total 50 % random for 0° - 80° and 100° - 180° . The rest of the rotation angles, the right ascension of the ascending node, Ω , and the argument of perigee, ω , have we set to be random between 0° and 360° .

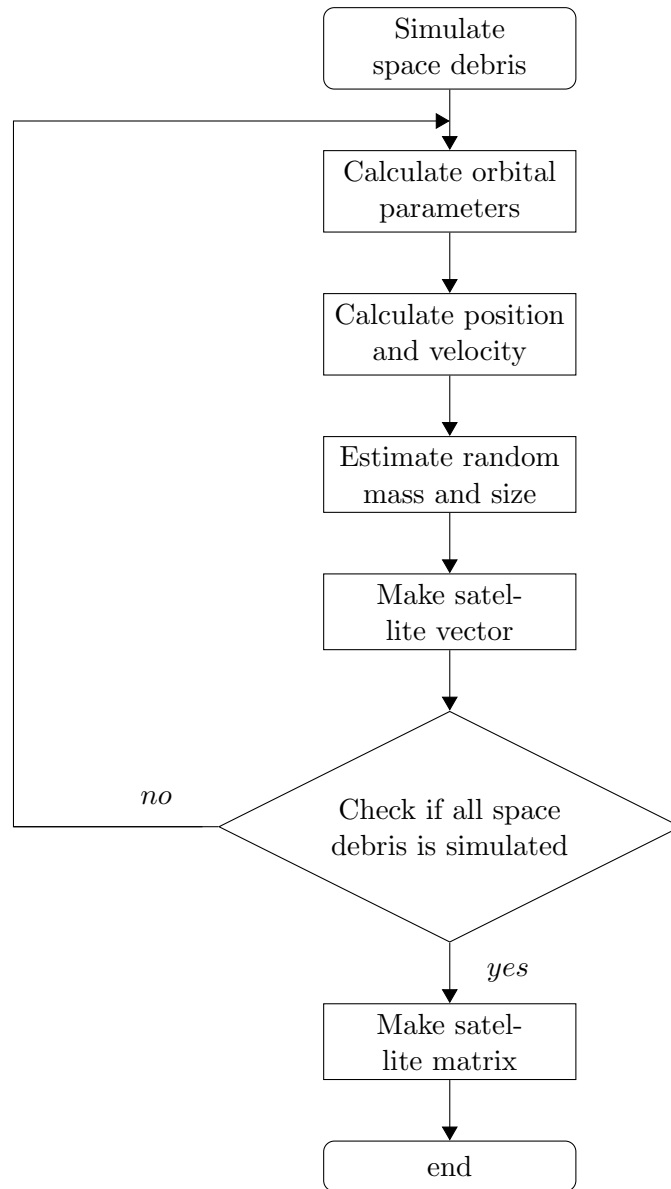


Figure 7: Flowchart for the Space debris subsystem

The mean anomaly have we defined with help of the total orbit time, T , which is defined with [19] as:

$$T = \frac{2\pi}{n}, \quad (34)$$

where n is the mean motion. We want the space debris to could be anywhere in its orbit so the space debris must set to be between the perigee, t_p (see Equation 19 in Section 2.4), and t_p plus one orbit time, T .

All these six parameters be placed in a table SD_{tab} before late use. SD_{tab} will look like this:

$$SD_{tab} = \begin{bmatrix} a_1 & a_2 & \cdots & a_j \\ e_1 & e_2 & \cdots & e_j \\ i_1 & i_2 & \cdots & i_j \\ \Omega_1 & \Omega_2 & \cdots & \Omega_j \\ \omega_1 & \omega_2 & \cdots & \omega_j \\ M_1 & M_2 & \cdots & M_j \end{bmatrix}, \quad (35)$$

where a , e , i , Ω , ω and M is the orbital parameters, and j the number of space debris.

2.10.2 The position and the velocity

We can use the orbital parameters defined in SD_{tab} and the mean motion, n , to find the position and velocity of each simulated space debris. The only parameter we must maybe change is the mean anomaly, M , which is not time-invariant. It was originally meant that M should be updated for every change in time but this was changed to reducing the complexity of the model. In this model is M therefore constant. An opportunity to make M dependent on time is included in Appendix A.1. The reason this is not included in the final version is because of the wish to make a faster model. If M is constant that means that also the position and velocity will be constant and they will only be need for calculating one time per simulation for each space debris instead for every time step.

With M defined we can then use it to get the eccentric anomaly, Ψ . We can get Ψ with help of M and the eccentricity, e , with a numeric method from [18]:

$$\begin{aligned} \Psi_0 &= M; \\ \Psi_1 &= M + e \sin(\Psi_0); \\ \Psi_2 &= M + e \sin(\Psi_1); \\ &\dots; \\ \Psi_{n+1} &= M + e \sin(\Psi_n). \end{aligned} \quad (36)$$

This method will continuous until $\Psi_{n+1} - \Psi_n > 1 \cdot 10^{-6}$ to get best possible result in our model. One important fact to mark is that this method only exist if $e < 1$. This will not be a problem in our model since we will only simulate space debris in elliptic and circular orbits.

With all parameters calculated and defined (semimajor axis (a), Ψ and e) we can calculate the position in orbit frame, \mathbf{r}^o with [18]:

$$\mathbf{r}^o = \begin{bmatrix} a(\cos \Psi - e) \\ a\sqrt{1 - e^2} \sin \Psi \\ 0 \end{bmatrix}. \quad (37)$$

Now we have the position in \mathcal{F}^o (orbit frame) but we need to have them in \mathcal{F}^i (inertial frame). To do that we need a rotation matrix from Equation 38 from [18] where i is the inclination, Ω is the right ascension node of ascending node, ω is argument of perigee, and c_{\cdot} and s_{\cdot} is \cos_{\cdot} and \sin_{\cdot} .

$$\begin{aligned} \mathbf{R}_i^o &= \begin{bmatrix} c\omega & s\omega & 0 \\ -s\omega & c\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_i & s_i \\ 0 & -s_i & c_i \end{bmatrix} \begin{bmatrix} c\Omega & s\Omega & 0 \\ -s\Omega & c\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c\omega c\Omega - c_i s\omega s\Omega & c\omega s\Omega + s\omega c_i s\Omega & s\omega s_i \\ -s\omega c\Omega - c_i s\Omega c\omega & -s\omega s\Omega + c\omega c_i c\Omega & c\omega s_i \\ s_i s\Omega & -s_i c\Omega & c_i \end{bmatrix} \end{aligned} \quad (38)$$

This is actually a rotation matrix to rotate from inertial frame to orbit frame, so we must remember to transpose it before we can use it. Finally we get the position in \mathcal{F}^i , \mathbf{r}^i :

$$\mathbf{r}^i = (\mathbf{R}_i^o)^\top \mathbf{r}^o. \quad (39)$$

To find the velocity vector we can use the equation for position, Equation 37. We know that the velocity is the derivative of the position so we can get:

$$\mathbf{v}^o = \frac{d\mathbf{r}^o}{dt} = \frac{d\mathbf{r}^o}{d\Psi} \frac{d\Psi}{dt}. \quad (40)$$

With the definition of $d\Psi/dt$ from [18] we have:

$$\frac{d\Psi}{dt} = \frac{n}{1 - \cos \Psi} = \frac{an}{r}. \quad (41)$$

This gives us finally the velocity vector in the orbit frame:

$$\mathbf{v}^o = \frac{an}{r} \frac{d\mathbf{r}^o}{d\Psi} = \begin{bmatrix} -\frac{a^2 n \sin \Psi}{r} \\ \frac{a^2 n \sqrt{1 - e^2} \cos \Psi}{r} \\ 0 \end{bmatrix}. \quad (42)$$

Now when we have the velocity in \mathcal{F}^o we only need to rotate it to \mathcal{F}^i . We can use the exact same rotation matrix as for the position to do that:

$$\mathbf{v}^i = (\mathbf{R}_i^o)^\top \mathbf{v}^o. \quad (43)$$

The position and velocity of each space debris will be placed in a table SD_{tab2} :

$$SD_{tab2} = \begin{bmatrix} r_{x1} & r_{x2} & \cdots & r_{xj} \\ r_{y1} & r_{y2} & \cdots & r_{yj} \\ r_{z1} & r_{z2} & \cdots & r_{zj} \\ v_{x1} & v_{x2} & \cdots & v_{xj} \\ v_{y1} & v_{y2} & \cdots & v_{yj} \\ v_{z1} & v_{z2} & \cdots & v_{zj} \end{bmatrix}, \quad (44)$$

where $[r_x, r_y, r_z]^\top = \mathbf{r}^i$, $[v_x, v_y, v_z]^\top = \mathbf{v}^i$, and j the number of space debris.

2.10.3 Size and mass

Besides the position and velocity there is also important to know the size and mass of the space debris.

To define the size of space debris we will use the numbers that N. L. Johnson from NASA presented in 2012, [3]:

- Objects larger than 10 cm: 20 000 +;
- Objects larger than 1 cm: $\sim 500\,000$;
- Objects larger than 1 mm: $\sim 135\,000\,000$.

As also mentioned in [1] we have only control over the biggest space debris since (> 10 cm in LEO and > 70 cm in GEO) that can be observed from Earth, [3]. The smaller the debris is the more inaccurate will the estimate become. That is also the reason why we not look at space debris smaller than 1 mm.

With these numbers we can make a probability model of the size. We will set the probability for the space debris over 1 mm to be equal the total number of space debris, 1. Then we get the probability for the debris over 1 cm to be $1/270$, or $25/6750$, and for debris over 10 cm to be $1/6750$. This will look like the Venn diagram in Figure 8. If we now make an random generator to generate random integers in a range from 1 to 6750 we can use Table 1 to define the size of the space debris.

To find the mass for the space debris we will base us on the density. The density of orbital debris it is normal to set to 2.8 g/cm^3 , [27], which is similar to the density of aluminum (2.7 g/cm^3). This can we implement with the result from the random size generator. We use the minimum diameter of each size and say that each space debris is a perfect cube and then get the volume. With using the product of the density and the volume we get the mass of each space debris. The mass of each size is added to Table 1.

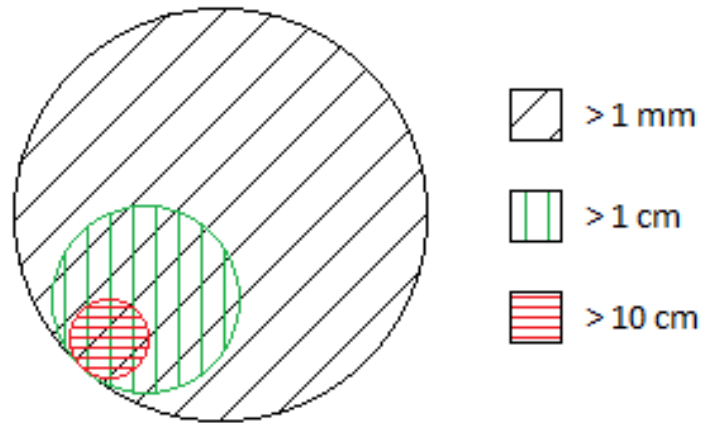


Figure 8: Venn diagram of size of space debris (not in correct scale)

Table 1: Probability distribution of space debris and the corresponding mass

Integer	Size	Mass
1	≥ 10 cm	2.8 kg
2 - 25	≥ 1 cm & < 10 cm	2.8 g
26 - 6750	≥ 1 mm & < 1 cm	2.8 mg

The model for generate the size and mass for the space debris not perfect yet. The size gives only an interval of the diameter and the mass is specific for only a few interval of size. This model will not give exact result but since it will be done simulation of many space debris, see Section 2.12.1, the result will give a better estimate for a bigger quantum. One thing that the model not will be good on is to simulate extreme occurrence. Extreme occurrence happen seldom but when they do they have a massive effect. An algorithm for this can maybe be implemented in future work.

2.11 The simulation the satellite orbit

In this system we will calculate the position and the velocity of the satellite and look especially at perturbing forces that can have an effect on the orbit on the system over time. The flowchart for the system can we see in Figure 9.

In this subsystem the user has bigger opportunity to define the orbital parameter and other parameters of the system. The parameters that must be defined by user is:

- a_{alt} : The altitude of the semimajor axis (km);
- e : The eccentricity;
- i : The inclination ($^{\circ}$);
- Ω : The right ascension of ascending node ($^{\circ}$);
- ω : The argument of perigee ($^{\circ}$);
- m_0 : Start mass of the satellite [kg];
- A : Effective area (m^2).

2.11.1 The position and velocity for the satellite

We will calculate the position and the velocity for the satellite in the same way as for the space debris model, see Section 2.10.1 and Section 2.10.2. The biggest difference between this model and the space debris model is that we want to not see the mean anomaly as a constant. We wish the satellite to move the whole orbit so we later can see if the orbit alters over time. We will use Equation 19 from Section 2.4:

$$M = n(t - t_p).$$

where n is the mean motion, t_p is the start time ($=0$), and t is the simulation time.

Another difference is that this simulates the orbit for only one element instead of many. An opportunity to include a constellation of more than one satellites is not included in this model but is discussed more in Appendix A.2.

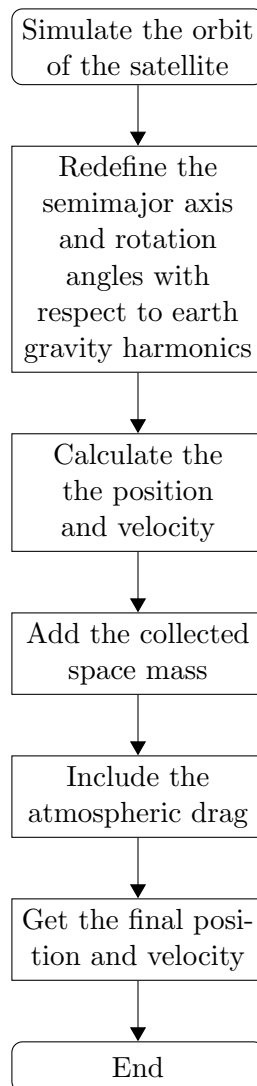


Figure 9: Flowchart for the Satellite system

2.11.2 Perturbations

To the satellite we want to include perturbations to see how the satellites will change his position and velocity over a longer time. It will also be an idea to see if the masses from the collected space debris will affect the long duration of the orbit to the satellite. We have looked at the perturbations and tried to find out which of them that disturb our system so we in that case must include them in our system. In our assumptions we have only based us on perturbations at an altitude on 500 - 1000 km and will in the model only include perturbations that affect this region.

To include the perturbing forces to the rest of the system, we must know the acceleration of the system. This can we find with help of the two-body problem equation form [18]:

$$\ddot{\mathbf{r}} = -\frac{G(M_e + m_s)}{r^3}\mathbf{r} = -\frac{\mu}{r^3}\mathbf{r}, \quad (45)$$

where

- \mathbf{r} = the position vector of the satellite;
- G = the universal constant of gravitation = $6.669 \times 10^{-20} \text{ km}^3/(\text{kg s}^2)$;
- M_e = the mass of Earth = $5.9742 \times 10^{24} \text{ kg}$;
- m_s = the total mass of the satellite [kg];
- μ = the gravitational constant = $G(m_1 + m_2)$.

The perturbation acceleration, \mathbf{a}_p , can then be added to the system, [21]:

$$\ddot{\mathbf{r}}_{new} = -\frac{\mu}{r^3}\mathbf{r} + \mathbf{a}_p. \quad (46)$$

\mathbf{a}_p will be the sum of all perturbing forces and will be used to give an better and more precise position and velocity vector.

With the new acceleration vector ($\ddot{\mathbf{r}}_{new}$) we can integrate it to get the new position and velocity vector. This is called the "Cowell Method", [15].

Earth gravity harmonics

The Earth is in fact not a perfect globe, the masses is not distributed uniform over it, [17]. Because of the centrifugal force there are more masses around the equator than the poles which cause the radius of Earth to vary from 6357 km at the poles to 6378 km around equator, a difference of 21 km. This means Earth is an ellipsoid, or more correct an oblate spheroid. It has an oblateness that makes variation of the gravity force that works on objects in orbits around Earth. The effect of this oblateness is called second zonal harmonics, J_2 . For the Earth specific defined to be 1.08263×10^{-3} .

We can now with help of J_2 redefine the mean motion, n , and get with [21] a more accurate value of it, \bar{n} :

$$\bar{n} = \sqrt{\frac{\mu}{a_0^3}} \left[1 + \frac{3 J_2 R_e^2}{2 p^2} \left(1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2} \right], \quad (47)$$

where

- μ : the gravitational constant;
- a_0 : the semimajor axis at the start of simulation;
- R_e : the equatorial radius of the Earth;
- e : the eccentricity of the satellite;
- i : the inclination of the satellite;
- p : the magnitude of the radius, r , when the true anomaly, θ is 90° , ($= a_0(1 - e^2)$, [21]).

With a new mean motion we can get a correction to the two rotation angles, right ascension of ascending node, Ω , and argument of perigee, ω . With [21] we get the rates $\dot{\Omega}$ and $\dot{\omega}$ which we can use to get new values for Ω and ω ;

$$\dot{\Omega} = -\frac{3 J_2 R_e^2}{2 p^2} \bar{n} \cos i, \quad (48)$$

$$\dot{\omega} = \frac{3 J_2 R_e^2}{2 p^2} \bar{n} \left(2 - \frac{5}{2} \sin^2 i \right). \quad (49)$$

There is also an other gravity harmonic we could looked on, the second tesseral harmonic, J_{22} . This have an effect on the geosynchronous orbits because of the ellipticity of equatorial plane to Earth. Since we in this model only will look at satellites in low earth orbit we will not include this effect in our model.

Atmospheric drag

The atmospheric drag is an effect that affects especially objects with an altitude under 1000 km, which is right in our zone. We can find the atmospheric drag acceleration, \mathbf{a}_D , with [21]:

$$\mathbf{a}_D = -\frac{1}{2} \rho V^2 \frac{C_D A}{M} \mathbf{i}_v, \quad (50)$$

where

- ρ = the density of the atmosphere;
- V = the speed of the satellite;

- C_D = the drag coefficient;
- A = the effective area of the satellite;
- M = the total mass of the satellite;
- \mathbf{i}_v = the unit vector for the velocity.

ρ is maybe the hardest parameter here to determine. It is dependent on, among other factors, the temperature, which is very varying. The temperature is depending on the altitude, day/night, seasons, solar year (solar flux variation with 11 years cycle), [21] and [4]. Therefore to do it easy we can use the 2001 United States Naval Research Laboratory Mass Spectrometer and Incoherent Scatter Radar Exosphere model who is an already model in Simulink/MATLAB called NRLMSISE-00 Atmosphere Model, see Appendix B9. This model is designed for objects with an altitude between 0 and 1000 km so it is perfect for our model. The only problem with this is the inputs. In addition to the altitude we must also know the latitude and longitude of the satellite, and the exact time (year, day of year and seconds on day (in universal time, UT)). To reduce the numbers of calculations we have set ρ to be a constant. With these inputs:

- a_{alt} : 875 km;
- latitude and longitude: 68° and -17° , (ca the position over Narvik);
- year: 2013;
- day of year: 203 (22. July (2013 is not a leap year));
- seconds in UT: 43 200 (12.00 am).

we get $\rho = 6.487 \times 10^{-15}$ kg/m³ as total output from the model.

C_D can vary from 1.9 to 2.6 and is usually determined by flight test. Since we not have that opportunity we will choose to set it to be 2.20, which is the normal, [4].

If the orbit is near-circular the atmospheric drag can affect the semimajor axis, [21]. Our satellite will not go in a circular orbit, but because of the low eccentricity we want to try to see if this can affect our system too. We will implement the semimajor axis decay rate, \dot{a} , so we can test our system:

$$\dot{a} = -na^2\rho\frac{C_DA}{M}. \quad (51)$$

In this equation is the parameters ρ, C_D, A and M exact as the same for Equation 50. The mean motion, n , is the n is new n we calculated in Equation 47 and a is the originally semimajor axis. With this rate we only need to integrate it one time get the new semimajor axis.

The lunisolar gravitational effect

The gravitation from the sun and the moon can also affects the objects in orbits around the Earth, [21]. They can have effect on the rotation angles right ascension of ascending node, Ω , and argument of perigee, ω . These angles will be affected if the orbit period, T , is longer than 12 hours (= 43 200 seconds). We will use this to check if we need to include this perturbation in our model. We will start with using Equation 34 from Section 2.10.1 and Equation 22 and 21 from Section 2.4 to find an expression for the semimajor axis, a , with help the orbit time:

$$\begin{aligned} T &= \frac{2\pi}{n} = \frac{2\pi}{\sqrt{\frac{\mu}{a^3}}} = \frac{2\pi}{\sqrt{\frac{GM_e}{a^3}}}, \\ \Rightarrow a &= \sqrt[3]{\frac{GM_e T^2}{4\pi^2}}, \end{aligned} \quad (52)$$

where G the gravitational constant ($6,6720 \times 10^{-20} \text{ km}^3/(\text{kg}\cdot\text{s}^2)$) and M_e the mass of Earth ($5.9742 \times 10^{24} \text{ kg}$). If we now set in number for all the parameters and set T to be 43 200 seconds we get the semimajor axis to be 26 610 km. This is a very high semimajor axis for our model. The perigee must not be bigger than 1000 km over the surface of Earth (total from center of Earth = 1000km + 6378km), which with the Equation 53 from [19] we the eccentricity, e :

$$\begin{aligned} r_p &= a(1 - e), \\ \Rightarrow e &= 1 - \frac{r_p}{a}, \\ e &= 1 - \frac{(1000 + 6378)km}{26610km} = 0.7227. \end{aligned} \quad (53)$$

This is a very big eccentricity and means that the orbit must be very elliptic and follow to the law of Kepler (see Section 2.10.1) the satellite will then be most of time at the highest altitude. We do not want satellites so high since we only should remove space debris so therefore we will ignore lunisolar gravitational effect in our model.

2.12 The removing model

The removing model will consist of more than one system to remove the space debris. This will give the opportunity to compare different methods over the same distribution of space debris over time. Since the distribution of space debris will vary for each simulation it will be need for testing the methods paralleled. In our model it will only be possible to simulate two system at the same time, but it shall be no problem to expend it for more system in later work since each system will be built at the same way.

The space debris will in each removing system go through a control system that check if the removing system is able to catch the space debris. They will be checked with some statement and if they passes all they will be removed. In our system we will check if

the space debris are close enough to the satellite and that they are not too heavy or big to be collected by the removing system. Every removed space debris will be noted as removed in a table, rem_{tab} , that will also be checked each time the space debris passes the system. The removing system will count each removed space debris and will also be able to add the mass from it to the mass of the satellite, but this will only work for one removing system for each simulation. The flowchart for the removing system is described in Figure 10.

To find out when the satellite is close enough to the space debris we need to find the distance between them, r_{dis}^o :

$$\begin{aligned} \mathbf{r}_{dis}^i &= \mathbf{r}_{SD}^i - \mathbf{r}_{sat}^i, \\ \mathbf{r}_{dis}^o &= \mathbf{R}_{i,sat}^o \mathbf{r}_{dis}^i, \\ r_{dis}^o &= \sqrt{\mathbf{r}_{dis}^o \cdot \mathbf{r}_{dis}^o}, \end{aligned} \quad (54)$$

where

- \mathbf{r}_{dis}^i is the position vector between the satellite and the space debris in \mathcal{F}^i (inertial frame);
- \mathbf{r}_{SD}^i is the position vector to the space debris in \mathcal{F}^i ;
- \mathbf{r}_{sat}^i is the position vector to the satellite in \mathcal{F}^i ;
- \mathbf{r}_{dis}^o is the position vector between the satellite and the space debris in \mathcal{F}^o (orbit frame);
- $\mathbf{R}_{i,sat}^o$ the rotation matrix to rotate the satellite from \mathcal{F}^i to \mathcal{F}^o (see Equation 38).

But since we know from Equation 12 from Section 2.2.1 that we do not need the $\mathbf{R}_{i,sat}^o$ when we only need the magnitude of the positions vector we can rewrite the equation of r_{dis}^o to:

$$r_{dis}^o = \sqrt{\mathbf{r}_{dis}^i \cdot \mathbf{r}_{dis}^i}. \quad (55)$$

There are also an opportunity to get speed between the space debris and the satellite, v_{pas}^o . In some case it can be interesting to know which speed a space debris has when it pass/been collected by the satellite/removing system. We can use same equation as for r_{dis}^o :

$$\begin{aligned} \mathbf{v}_{pas}^i &= \mathbf{v}_{SD}^i - \mathbf{v}_{sat}^i, \\ v_{pas}^o &= \sqrt{\mathbf{v}_{pas}^i \cdot \mathbf{v}_{pas}^i} \end{aligned} \quad (56)$$

where

- \mathbf{v}_{pas}^i is the velocity vector between the satellite and the space debris in \mathcal{F}^i ;

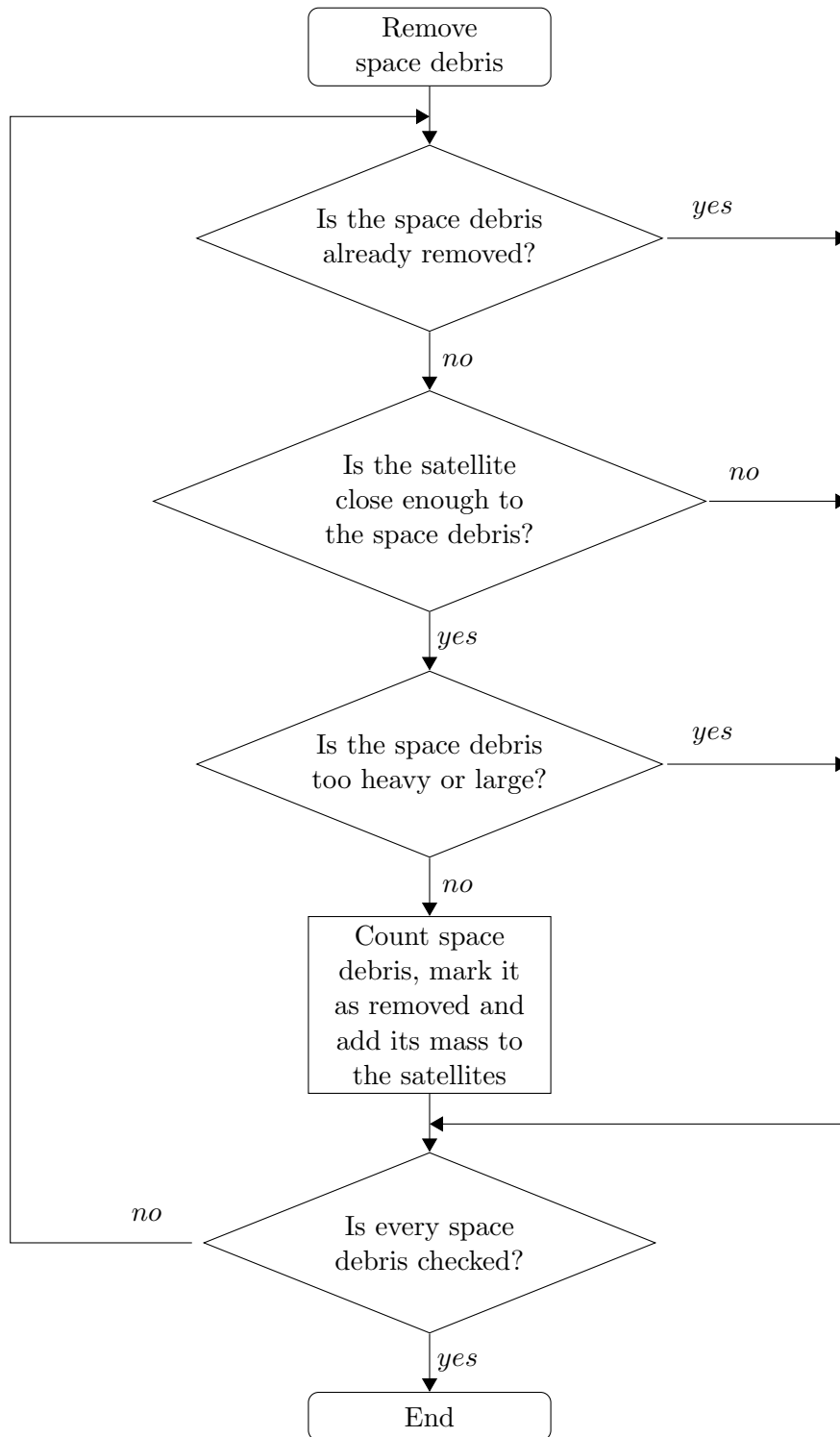


Figure 10: Flowchart for the Space debris removing subsystem

- \mathbf{v}_{SD}^i is the velocity vector to the space debris in \mathcal{F}^i ;
- \mathbf{v}_{sat}^i is the velocity vector to the satellite in \mathcal{F}^i ;
- \mathbf{v}_{pas}^o is the velocity vector between the satellite and the space debris in \mathcal{F}^o ;

In our system we have also given the opportunity to see how much energy the satellite will be inflicted by the space debris. We will use the classical equation for kinetic energy:

$$E_k = \frac{1}{2}m_{SD}v_{pas}^2 \quad (57)$$

where E_k [J] is the kinetic energy, m_{SD} the mass of the space debris and v_{pas}^i the speed between the satellite and the space debris (the frame of v_{pas} is not so important here since it is equal for each frame).

2.12.1 Monte Carlo simulation

To test the removing systems the best way we will do some Monte Carlo simulations on them. To do Monte Carlo simulations is to repeat a simulation with random parameters to get the most correct result over a set with random of data. We will do 10 simulations with the removing systems to try to find how many space debris each removing system in average will be able to catch. With using 10 simulations we will be more sure on the capacity of each removing system since the whole system is based on random generated space debris. Each simulation will go parallel with the two removing system A and B so if there should be any big variations in the simulations both of the removing system will be affected.

2.12.2 Avoiding active payloads

That had been good if the system could avoid space object that are still operational. In our system we will not include any detection of the debris is actual an operational payloads but that will maybe be possible to include in future work. The orbit of every payloads is known through databases

2.13 Probability of collision with space debris

Something that can be interesting to look at is the probability to be hit by a space debris, or more precise in our case: the probability to come close enough to a space debris to be able to catch it.

We will start with to look at the space we will try to remove the space debris. In Section 2.8.2 we defined that the best space to collect space debris was in the area 800 to 950 km from the surface of Earth. Since we are thinking that the satellite will cross this whole area (with perigee at 800 km and apogee at 950 km) we want to find the total volume of the area. To do it the easiest way we will find the total volume, V_{tot} ,

with only calculating the volume V_a of a globe with radius 7328 km (=apogee + Earth's equator radius = 950 km + 6378 km) and subtract it with the volume V_p of a globe with the radius 7178 km (=perigee + Earth's equator radius = 800 km + 6378 km):

$$V_{tot} = V_a - V_p = \frac{4\pi r_a^3}{3} - \frac{4\pi r_p^3}{3} = \frac{4\pi(r_a^3 - r_p^3)}{3}, \quad (58)$$

$$V_{tot} = \frac{4\pi((7328 \text{ km})^3 - (7178 \text{ km})^3)}{3} = 9.916 \cdot 10^{10} \text{ km}^3. \quad (59)$$

In Section 2.9.1 we was able to make a system to estimate the number of space debris and inactive payloads in a size on over 10 cm. Now we will use this result to estimate the number of objects size > 1 cm and >1 mm. In Section 2.10.3 we found out that there was 270 times more space debris with size > 1 mm than space debris with size > 10 cm. To find the average number of space > 1 mm we must multiply the average number of space debris > 10 cm with 6750. To get the average number of space debris > 1 cm we must multiply the average number of space debris > 10 cm with 25 (or first multiply with 6750 to get for > 1 mm and then divide by 270 to get for space debris > 1 cm, = 25).

If we then divide the number of space debris and inactive payloads in the region with the volume of the region we will get an estimate to how many space debris and inactive payloads there will be per cubic meter. This is the spatial density, S.

With S we will be able to find the number of space debris statistically we will hit: N. We can find N with [3]:

$$N = V A S T, \quad (60)$$

where V is average collision speed, A the cross-sectional area and T the time. In our calculation we will set V to be 10 km/s since that is the average speed of space debris, [21].

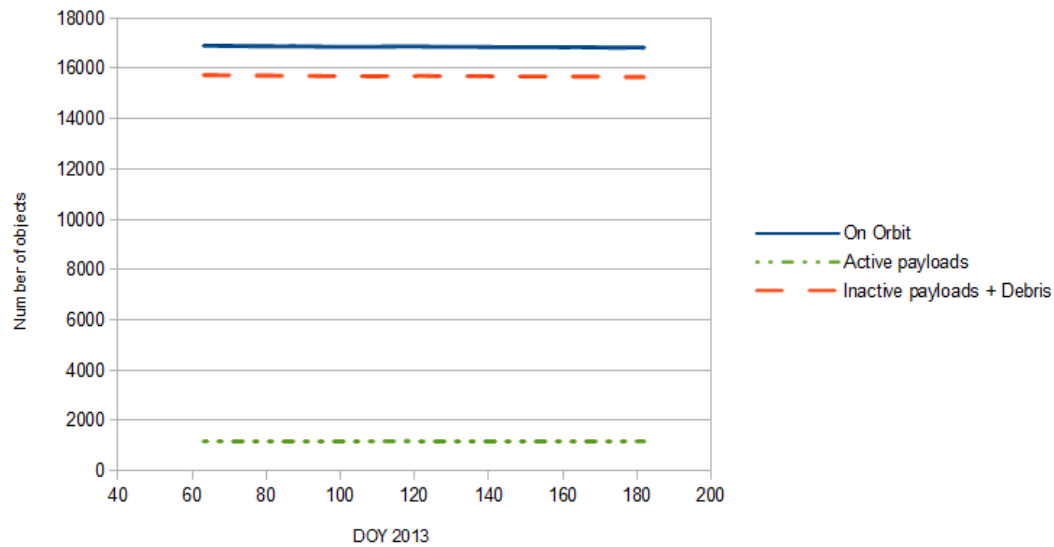


Figure 11: Known objects in orbit around Earth

3 Main Results

In this chapter we will look at the result of the analysis of the space environment after the period of 119 days, and we will try to estimate number of space debris at altitude 800 to 950 km.

3.1 The analysis of space the space environment

After period of 119 days (4. March (63 DOY) - 1. July (182 DOY)) we have done the analyses of the growth of the of the space object based on [24], see Appendix B6. We can see from Figure 11 that the number known objects in orbit around Earth have been very stable. No big outbreak of space debris have occurred in this period so the number of the space debris have been very stable. We can see from figure that the inactive payloads and space debris is well dominated over the active payloads. With respect to that all the space debris are bigger than 10 cm (cannot observe objects bigger than that, [3]) we see a big part of the object out there has no function, 93.0 % if we use the numbers from the last day, 1. July 2013.

If we only look at the payloads, see Figure 12, we will see that the inactive payloads alone are more dominating than the active payloads. At 1. July 2013 are 68.6 % of the payloads inactive.

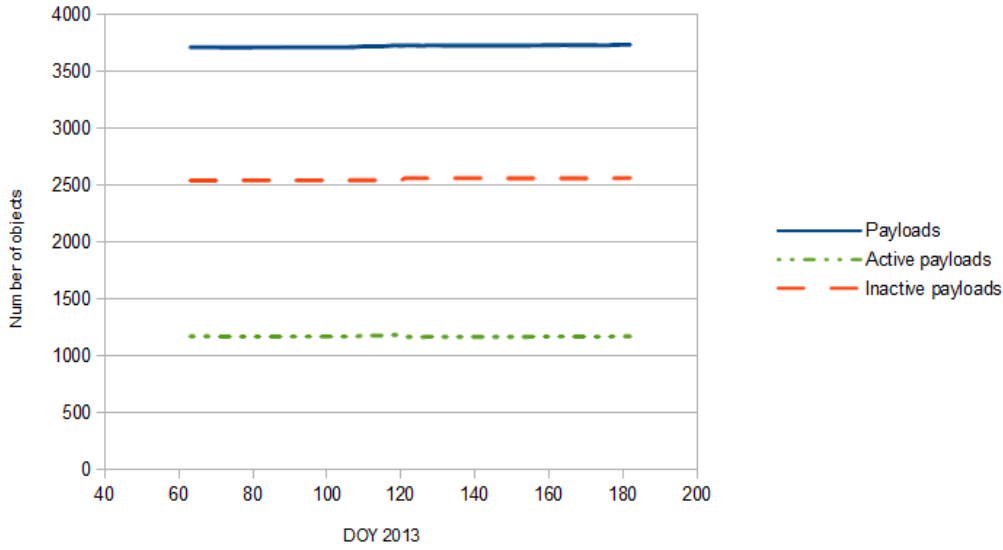


Figure 12: Payloads in orbit around Earth

Of the inactive payloads and the space debris, see Figure 13, we can see that the space debris is well dominating, 83.7 % at 1. July 2013. And with respect to that this are only the biggest space debris (over 10 cm), we can conclude with that the inactive payloads is in number not the biggest problem.

In Figure 14 and 15 we can see how many orbital objects that have decayed and how the total population has increased/decrease. We can see that the number of decaying objects is nearly linearly. In the period has 156 objects decayed, which means about 1.31 objects per day. If we look at the number of orbital objects we can see that it has been a more rough evolution but a clear decreasing. For the period the number of orbital objects have decreased by 76 or by average 0.64 per day. This is a good evolution but not good enough. The decay-rate is stable but slow. We can see in Figure 15 that even the number of objects decreases there have been small periods where the number of objects suddenly has jumped up again, e.g. at around 120 DOY. Since there has not been any big incidents in this periods we may see not how little robust this system is.

The reason for the jump in orbital objects around 120 DOY can we may see in other figures. Figure 16 shows the evolution of the space debris. Here we can see that the space debris decreases slowly but stable. Totally they have decreased by 98, or 0.82 per day. But also in this figure we have the little jump around 120 DOY. This is not so big as in Figure 15 so there must be more than only space debris that cause this.

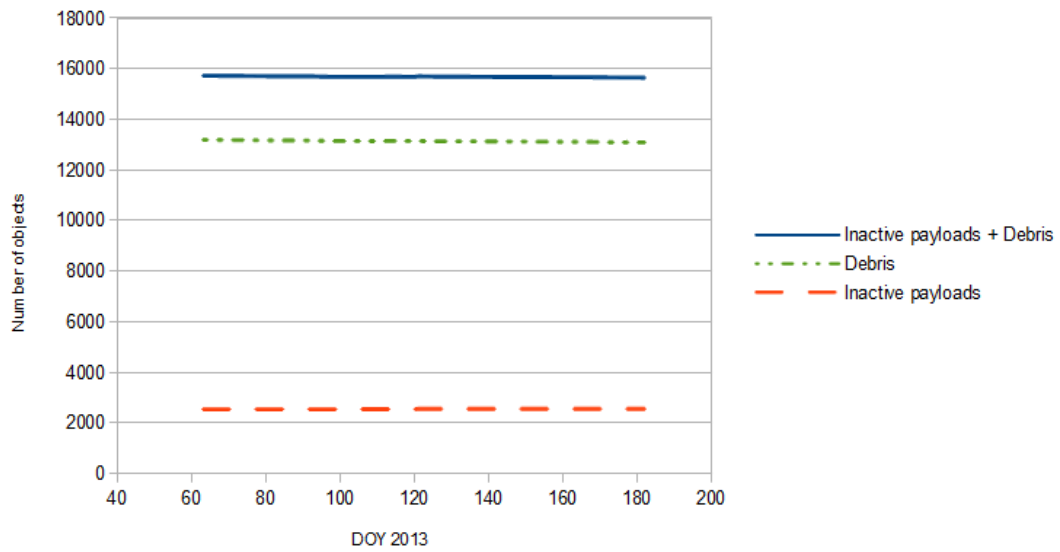


Figure 13: Space debris and inactive payloads in orbit around Earth

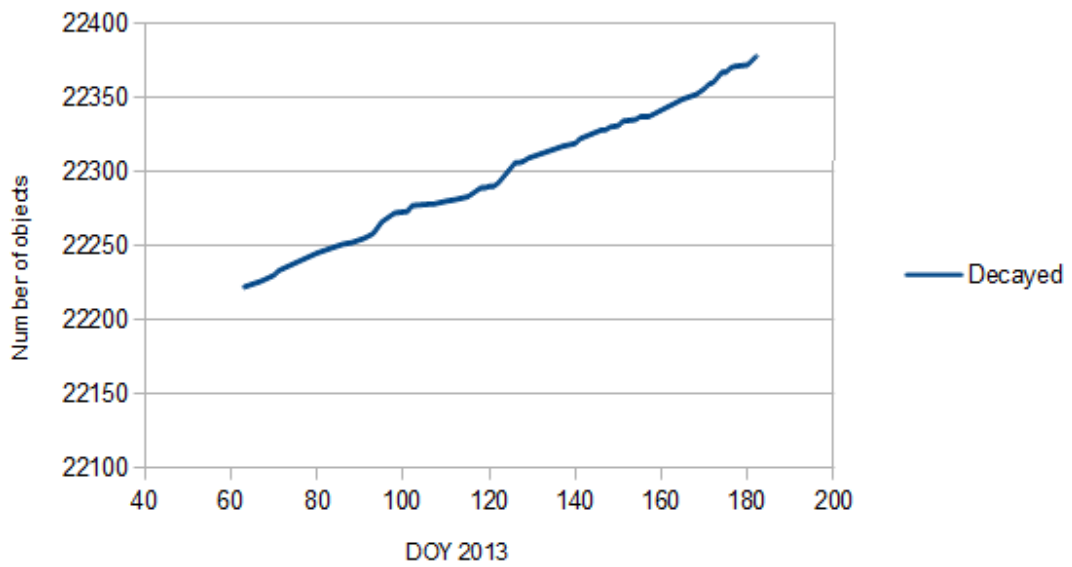


Figure 14: Number of decayed orbital objects in orbit around Earth

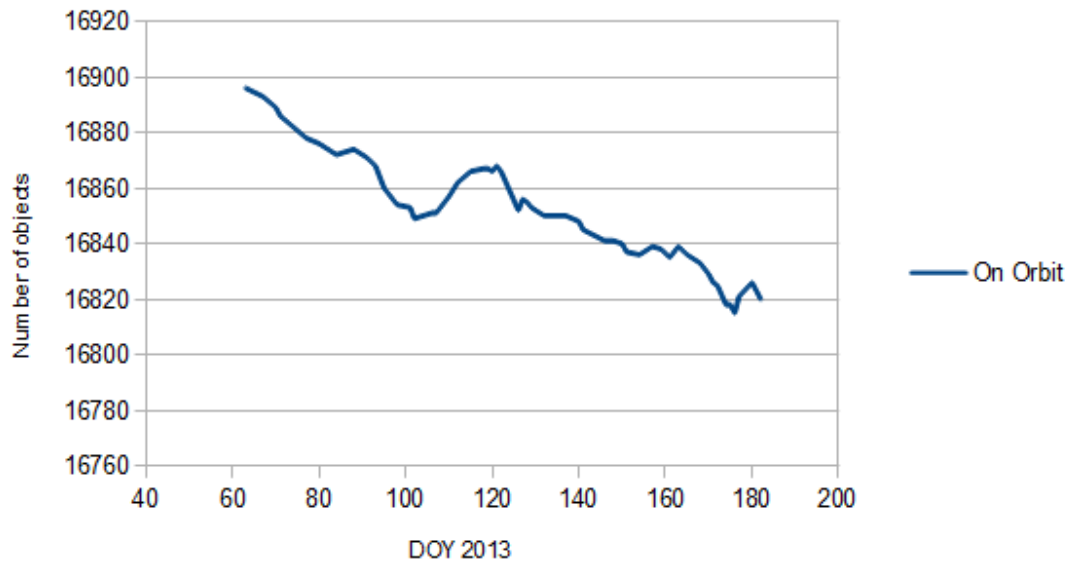


Figure 15: Evolution of the orbital objects in orbit around Earth

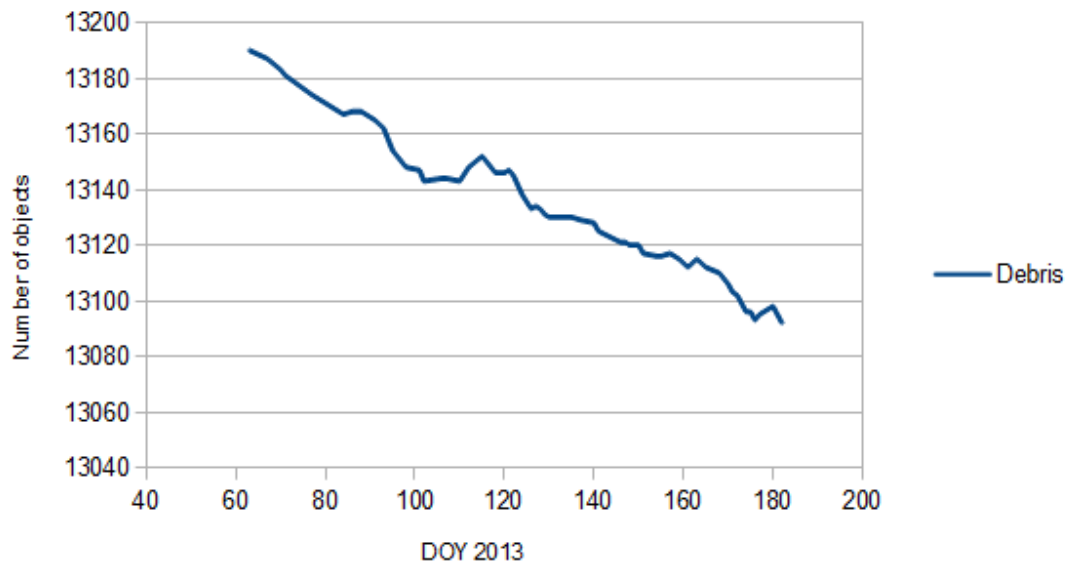


Figure 16: Evolution of space debris in orbit around Earth

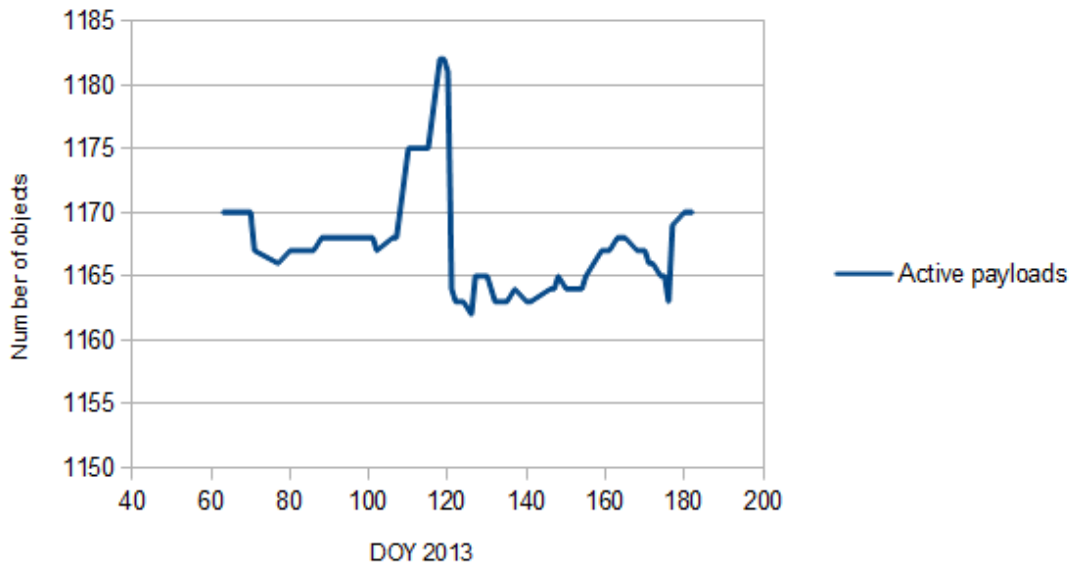


Figure 17: Evolution of active payloads in orbit around Earth

If we look at the payloads we can maybe get a little more explanation of the jump around 120 DOY. In Figure 17 we can see the evolution of the active payloads, and the evolution of the inactive payloads in Figure 18. We can see that the active payloads have a jump at 110 - 120 DOY just before it fell back right after. If we look at the inactive payloads we will see that they increase at the same time as the active payloads decrease. This can show that there have been replacements of active satellite at this time, but since the old payloads not become removed but instead become set as inactive we will see as in Figure 19 that the total amount of payloads increases, and especially after 120 DOY.

If we now add the inactive payloads to the space debris, see Figure 20, we will see a graph very close to the graph to the space debris in Figure 16. The difference here is that the inactive payloads raises the graph to the space debris. A little peak at 120 DOY indicates the jump from Figure 18.

3.2 Probability of collision with space debris result

In Section 2.9.1 we made a program in C++ code to estimate number of orbital objects, space debris and inactive payloads in the LEO-region and in our region (altitude 800 - 950 km). These results are noted in Table 2.

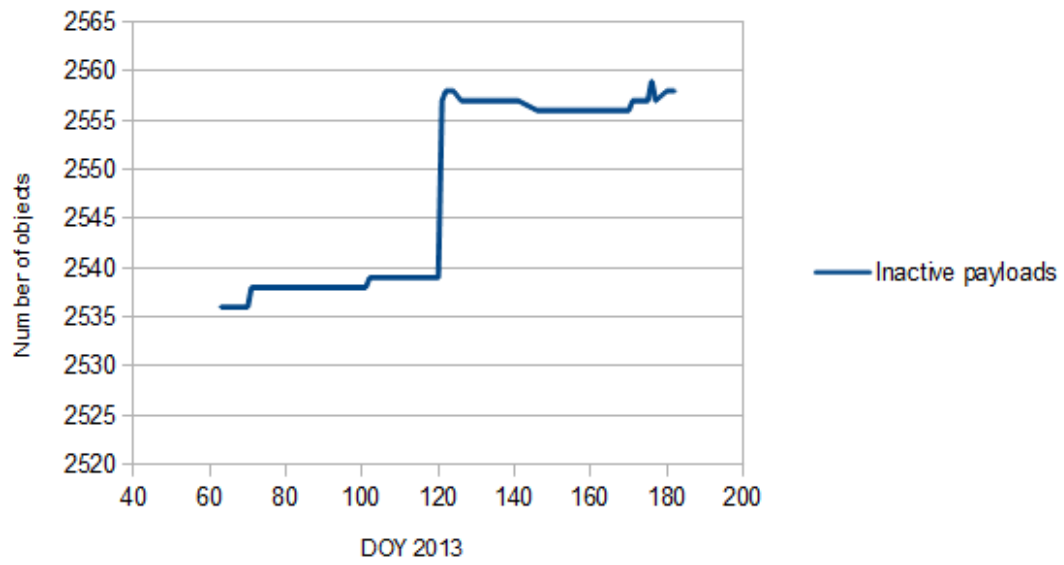


Figure 18: Evolution of inactive payloads in orbit around Earth

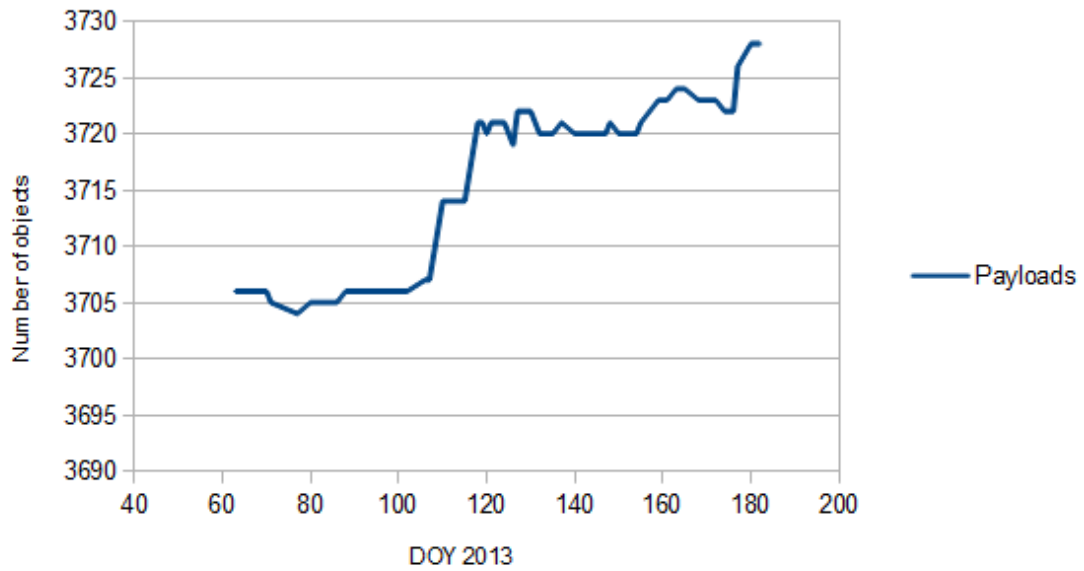


Figure 19: Evolution of the payloads in orbit around Earth

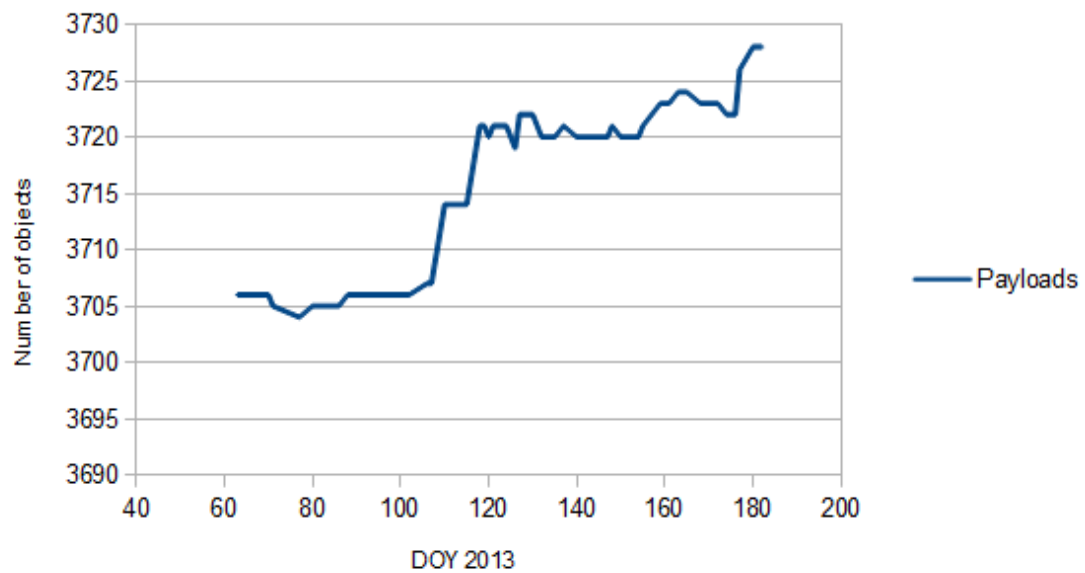


Figure 20: Evolution of the space debris and inactive payloads in orbit around Earth

Table 2: Result of number in LEO and the specific region

Orbital objects in LEO	13 961
Space debris and inactive payloads in LEO:	13 415
Space debris and inactive payloads in the region:	3034
Space debris and inactive payloads which passes the region:	7713
Average number of space debris and inactive payloads in the region:	3317.7
Average time used of each objects:	0.430144 s
Total calculated objects:	39 198

Table 3: Number and spatial density of space debris an inactive payload between 800 and 950 km

Diameter	Average number	Spatial density [number/m ³]
>= 10 cm	3317.7	$3.3458 \cdot 10^{-17}$
>= 1 cm	82 942.5	$8.3645 \cdot 10^{-16}$
>= 1 mm	22 394 475	$2.2584 \cdot 10^{-13}$

Table 4: Number space debris an inactive payload hit by over time

Diameter	Average number by one orbit	Average number by one year
>= 10 cm	$2.0547 \cdot 10^{-9}$	$1.0559 \cdot 10^{-5}$
>= 1 cm	$5.1366 \cdot 10^{-8}$	$2.6397 \cdot 10^{-4}$
>= 1 mm	$1.3869 \cdot 10^{-5}$	$7.1272 \cdot 10^{-2}$

Now we can use the average number of space debris and inactive payloads in the region to find the number of space debris with diameter bigger than 10 cm, 1 cm and 1 mm with using the method from Section 2.13. We will also find their spatial density, S, in the region with using a total volume of the region ($=9.916 \cdot 10^{10}$ km³ from the same section). The total result is noted in Table 3.

With S we could with help of Equation 60 from 2.13 find how many space debris we would be hit by for one orbit (6141 s) and one year (31 558 464 s). Here we set the cross-sectional area, A, to be equal the effective area of the satellite, see Table 6. These results is noted in Table 4.

3.2.1 Comparing with DAS 2.0.2

We want to control the results from Table 4 so we have used the program DAS 2.0.2, [14], to compare with our result. In the program we set these start parameters:

- start year: 2013.499 (1. July 2013);
- duration: 6141 s and 1 year;
- cross-sectional area: 1 m² ;
- inclination: 83°;
- type of environment: orbital debris only;
- impactor diameter: 1 mm, 1 cm and 10 cm.

These parameters are set be the same as for the satellite, see Table 6 where start year is the same as the date the data from CelesTrak, [2], is collected. These result is noted in Table 5.

Table 5: Number space debris an inactive payload hit by over time (DAS 2.0.2)

Diameter	Average number by one orbit	Average number by one year
≥ 10 cm	$1.0000 \cdot 10^{-9}$	$6.3096 \cdot 10^{-6}$
≥ 1 cm	$2.5119 \cdot 10^{-8}$	$1.2589 \cdot 10^{-4}$
≥ 1 mm	$2.5119 \cdot 10^{-5}$	$1.2589 \cdot 10^{-1}$

We can see here that our estimation has not quite the same result as from DAS 2.0.2. This can have several reasons:

1. In our estimation we counted with inactive payloads in the estimate for space debris. There are certainly not much inactive payloads among the smallest space debris. We know from Figure 13 in Section 2.7 that there are much fewer inactive payloads than space debris but is still a small source of error;
2. Another reason is that we have assumed that the distribution of space debris based on size is equal for each altitude. This is necessary not true even though it is a good estimation;
3. A third reason is the way the probability is estimated. In our estimated we have based us on the latest update of number of space debris while DAS 2.0.2 is based on earlier prediction and calculation about how much space debris there will be right now.

Table 6: Input parameters to the satellite system

Parameters	Values
Altitude of semimajor axis [km]	875
Eccentricity (< 1)	0.0103
Inclination [$^{\circ}$]	83
Right ascension of ascending node [$^{\circ}$]	0
Argument of perigee [$^{\circ}$]	0
Mass [kg]	2
Effective area [m^2]	1

4 Simulation Results

In this chapter we will look at the results from space debris generator and the removing systems. We will also see if the satellite orbit is affected by the perturbing forces and/or the masses from collected space debris.

4.1 The satellite and space debris model

The input parameters for simulating the orbit of the satellite are denoted in Table 6. The semimajor axis and the eccentricity is determined by the region the satellite should collect space debris in, see Section 2.8.2. The inclination is set to be 83° since that is one of two inclination with highest collision rate, [25], and the other two angles, right ascension of ascending node and argument of perigee, are set just to be zero since their value have nothing to say on the space debris collecting. The mass of the satellite is set to be 2 kg since that the normal mass of a picosatellite (2 unit) and the effective area is set only to be 1 m^2 since this can be an approximately size of the total surface of the satellite. The simulation time is set to be the orbit time of the satellite: 6141 s, from Equation 20.

The final orbit of the satellite can we see in Figure 21. Here we can see that the satellite goes in a nearly circular orbit with a high inclination exact as we wanted it to.

In Table 7 we have the input parameters of the simulated space debris. We have chosen to simulate only 1000 space debris to make the simulation faster, especially with respect to that there will be done several Monte Carlo simulation. The semimajor axis is the same as for the satellite. In this simulation there are no opportunity to choose many orbital parameters since most of them are random distributed in the simulator, see Section 2.10.1.

In Figure 22 we can see the space debris spread over the sphere. There should be most space debris around the poles because of the high inclination which look to be true.

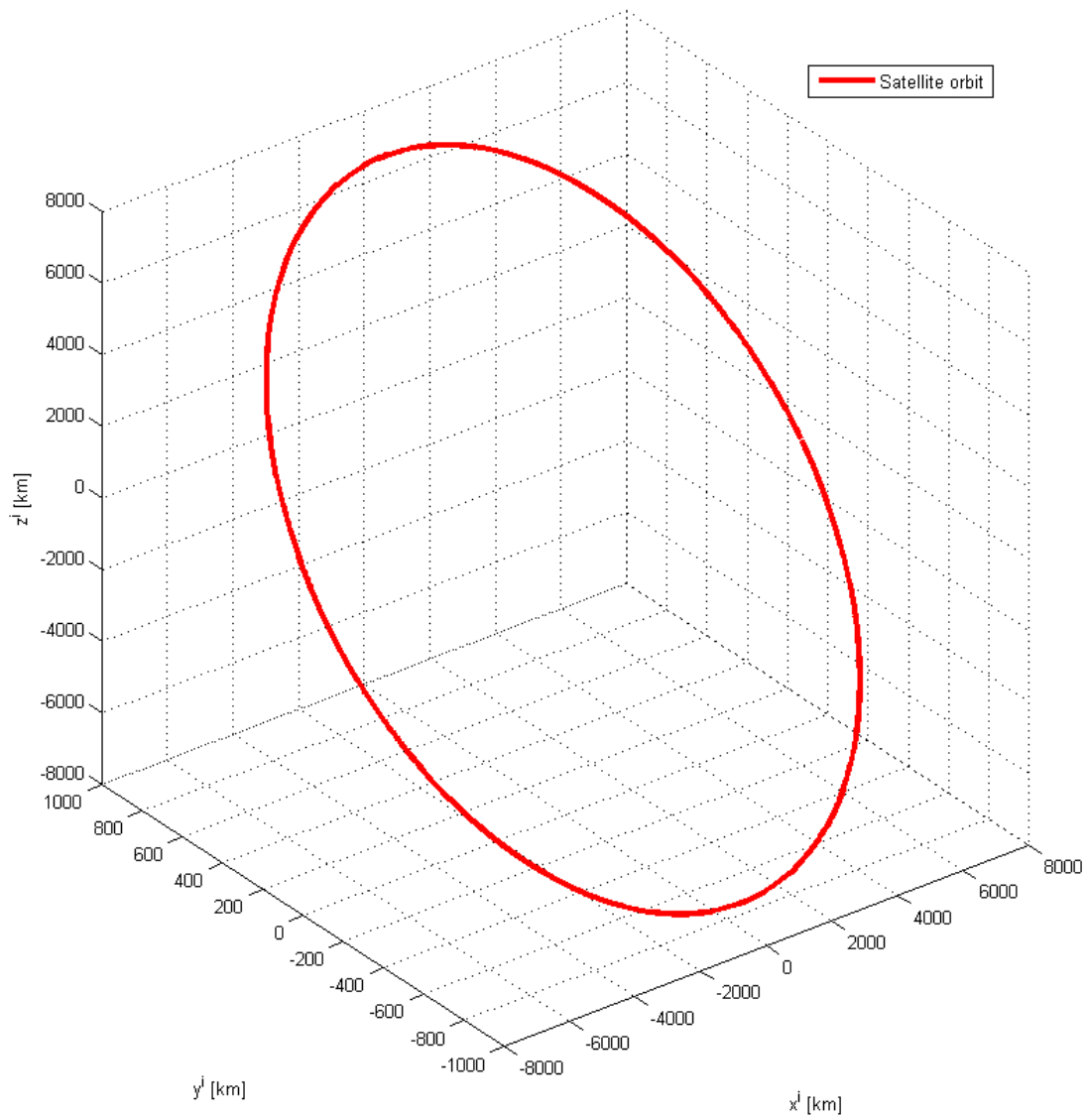


Figure 21: Orbit of the satellite

Table 7: Input parameters to the space debris system

Parameters	Values
Altitude of semimajor axis [km]	875
Number of space debris	1000

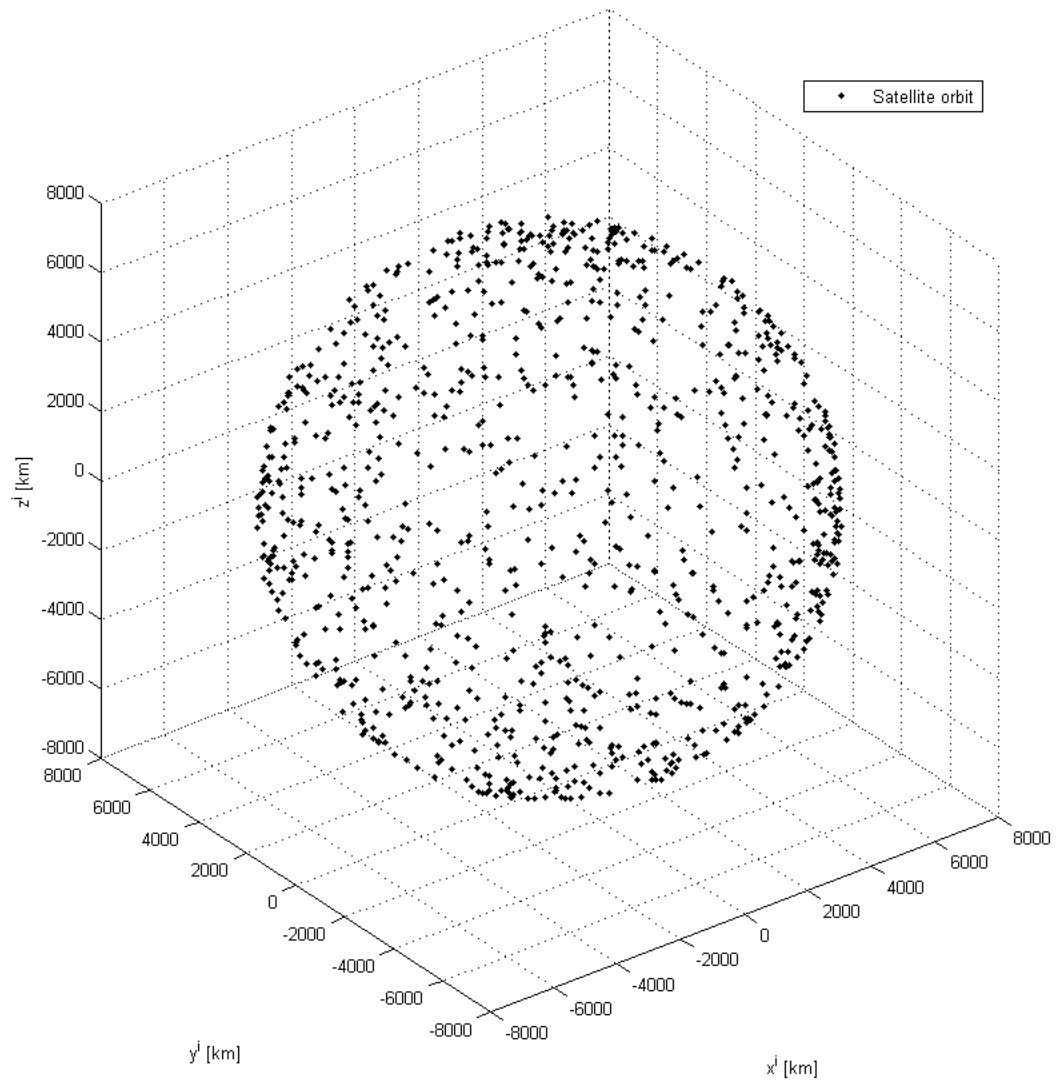


Figure 22: The simulated space debris

Table 8: Input parameters to the removing system

Parameters	Values
Which removing system should the satellite use?	Removing system A
Range [km], removing system A	100
Maximum mass [kg], removing system A	20
Size range, removing system A	< 10 cm
Range [km], removing system B	1000
Maximum mass [kg], removing system B	200
Size range, removing system B	< 10 cm

In Figure 23 we have compressed Figure 21 and 22 so we can see how the satellite orbit fits with the cloud of space debris. This seems to correspond well. The satellite look to be able to collect a lot of them.

4.2 The removing models

We have done a simulation test on the removing systems A and B. The input parameters of the test is listed in Table 8. One important thing to remark here is that the range for the distance to the space debris here is set to be unrealistic high. That is done because we want to get faster result so we can check if the systems work. Since we also simulate only 1000 unmoving space debris it is important to get much information as possible from only one orbit. A fast simulation makes it also easier to make several Monte Carlo simulations.

4.2.1 The two removing system

In Figure 24 we can see how much space debris removing system A have collected after one orbit. Here we can see that the system have gradually collected the space debris while the satellite have gone through its orbit.

We can see how much space debris the removing system B have collected in Figure 25. Removing system B collects space debris much better than removing system A something that corresponds well with the fact that the removing system B has much higher ranges than removing system A. Something we can see in this figure is that the system is able to collect more than one space debris at the same time. This is easier to see in Figure 25 than Figure 24 because of the higher range of removing system B.

In Figure 26 we have the result of ten Monte Carlo simulations of removing system A and B, see Appendix B8. Here we can see that there are big variations between each simulation. Removing system A varies from 8 to 17 space debris and has an average at 11.9 while removing system B varies from 136 to 176 space debris with an average at

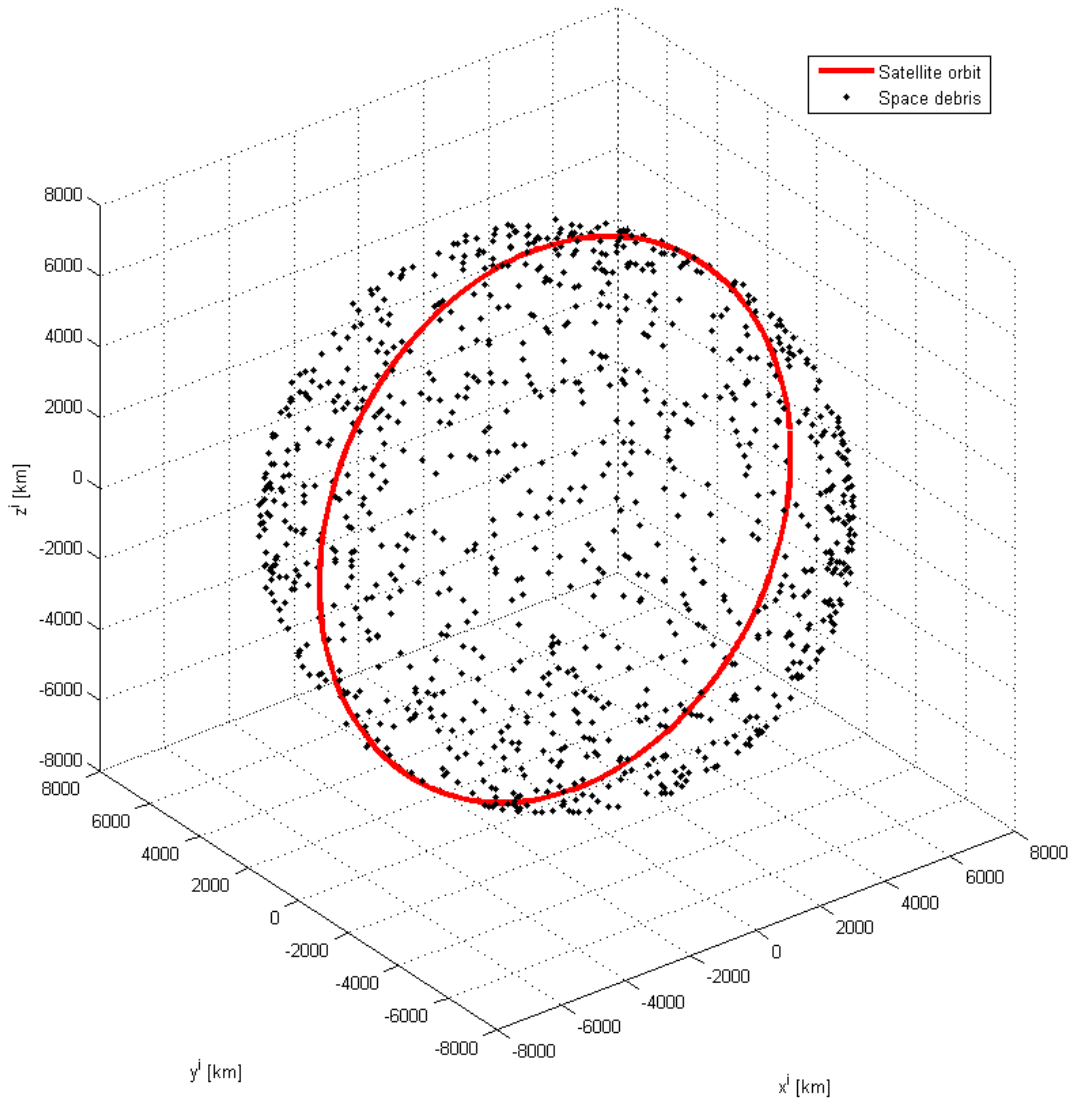


Figure 23: Orbit of the satellite and space debris

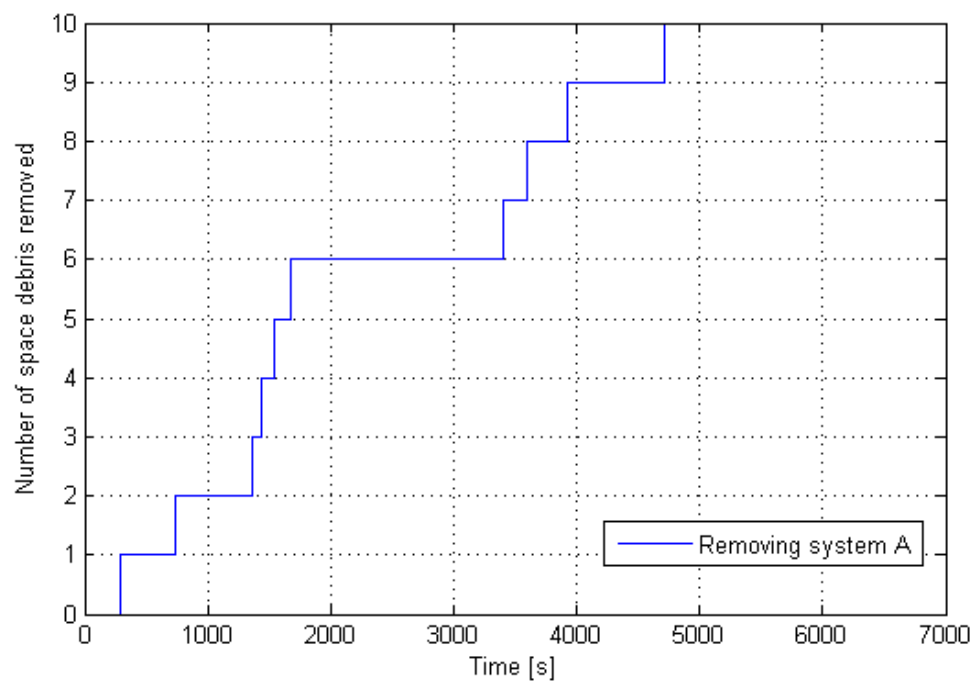


Figure 24: Removed space debris from removing system A

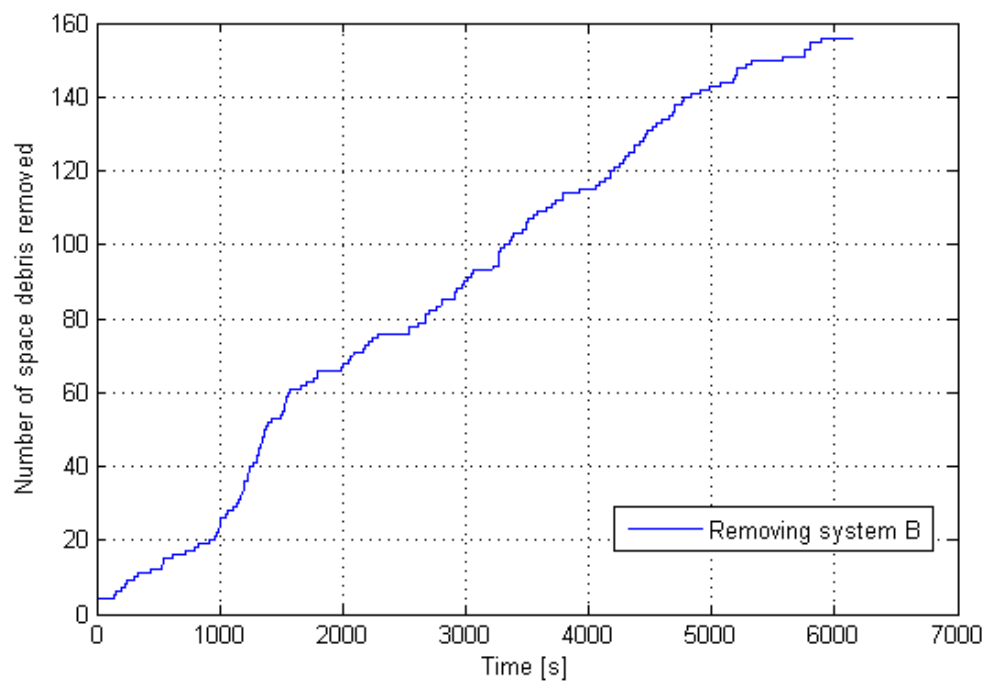


Figure 25: Removed space debris from removing system B

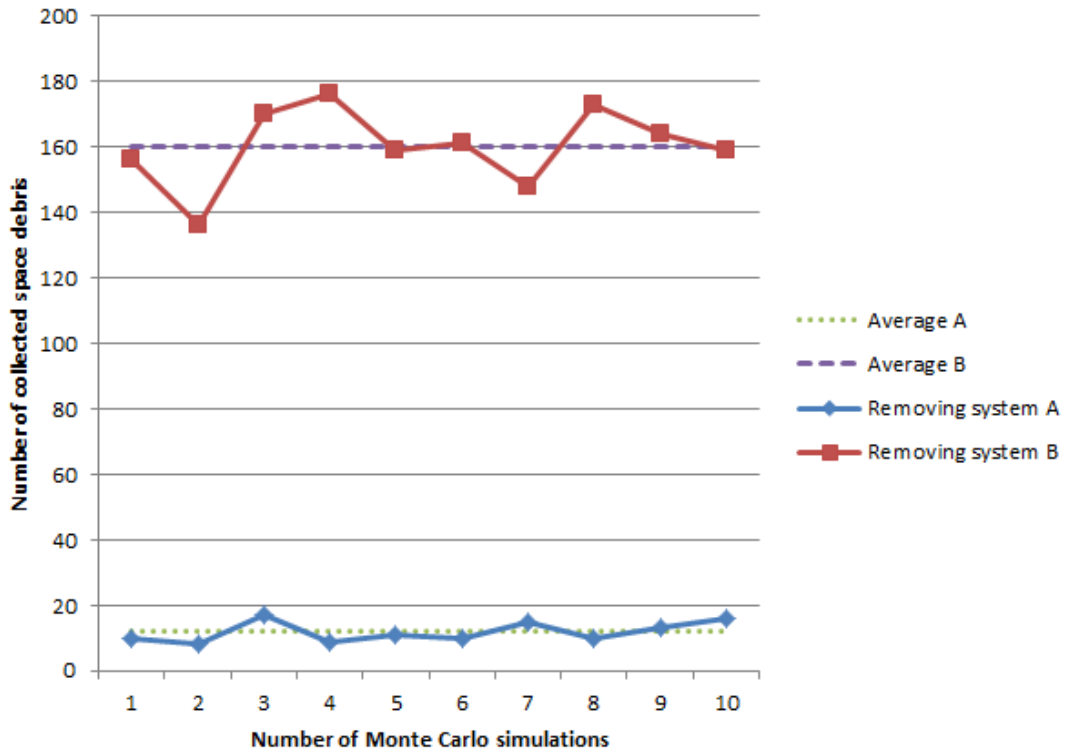


Figure 26: Monte Carlo simulation of removing system A and B

160.2. Due all these variation we see that more Monte Carlo simulation had been good for making the average values more precise.

We can see the total kinetic energy that is added from the space debris to the satellite with removing system A in Figure 27. About 1.31 kJ have the 10 collect space total inflicted the satellite. There have only been coincidences with small low energy debris so it should not be any problem to make a removing system that should be able to manage objects like this.

If we look at the removing system B in Figure 28 we can see that the kinetic energy from space debris can have much bigger effect. At one second the whole system is inflicted by nearly 8 kJ. This is something the designers of the removing system must be prepared on. There can always come object with high mass and/or high velocity. If the removing system is not strong enough it can be damaged/destroyed by the space debris it try to remove. Another fact is that these objects can be harder to remove because of their high kinetic energy that do it hard to change their orbit.

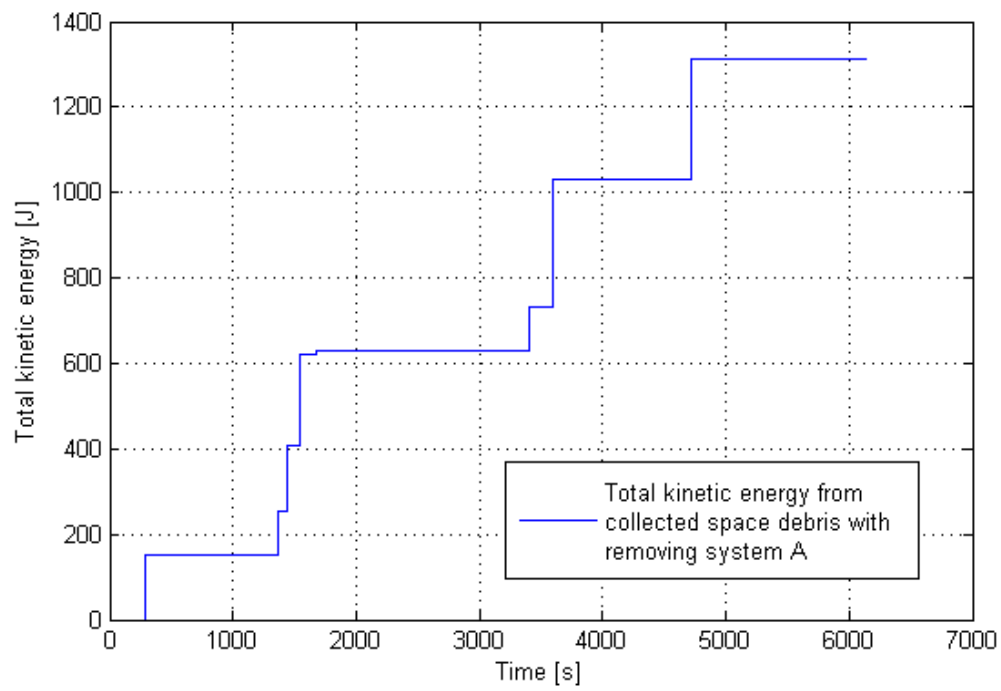


Figure 27: Total kinetic energy from collected space debris with removing system A

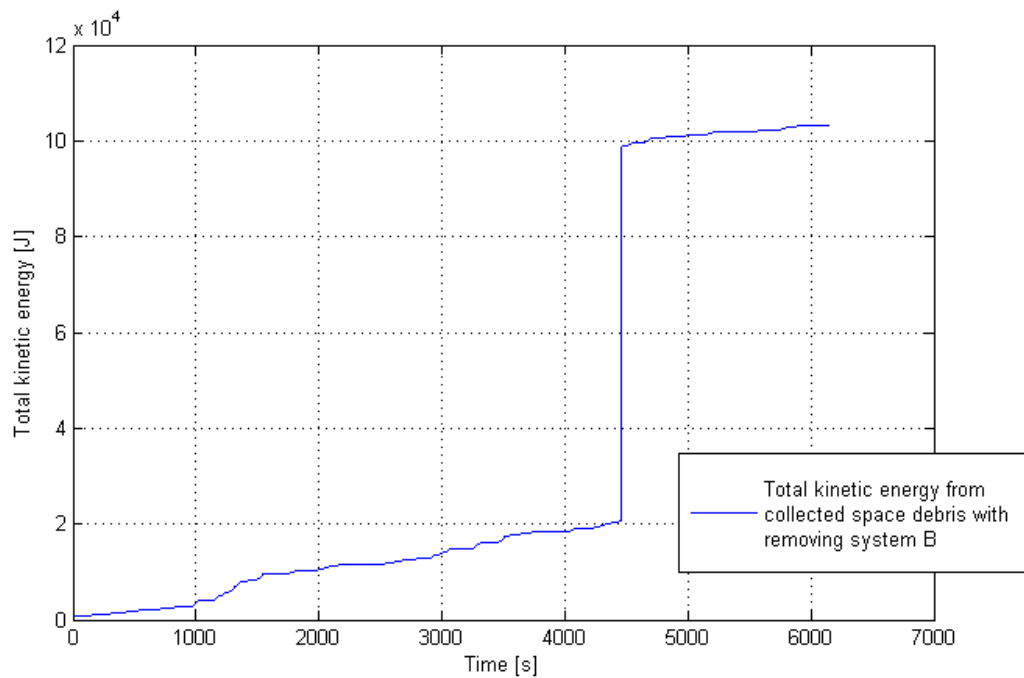


Figure 28: Total kinetic energy from collected space debris with removing system B

4.2.2 Effects of the collected space debris masses

One of our goal was to see if the masses from the space debris collected with the removing system had effect on orbit of the satellite. With the space debris collected with the removing system A, see the Figure 24, we get the new mass of the satellite in Figure 29. Here we can see that the masses from the space debris had very little effect on the total mass of the satellite. About 28 mg was added to the satellite mass by the ten objects. This itself can we correspond back to Table 1 in Section 2.10.3 where we see that a space debris with diameter smaller than 1 cm have a mass at 2.8 mg. Because this mass distribution is very limited we will know exact which space debris removing system A have collected. This corresponds also good with the low kinetic energy in Figure 27. If we had kinetic energy from removing system B, See Figure 28, we can guess there had come also a mass from a much bigger space debris.

When we look at the semimajor axis decay rate in Figure 30 we can see that the masses from the collected space debris from removing system have very little effect. The mass is in this rate the only variable factor here and we can see that it in the fact reduce this decay rate when it gets bigger, which also corresponds with Equation 51 from Section 2.11.2. The semimajor axis decay rate itself has very little effect on the semimajor axis. After one orbit there was no visible change of the semimajor axis. There are possible that we do not will see any changes of it before a much longer simulation. A big semimajor

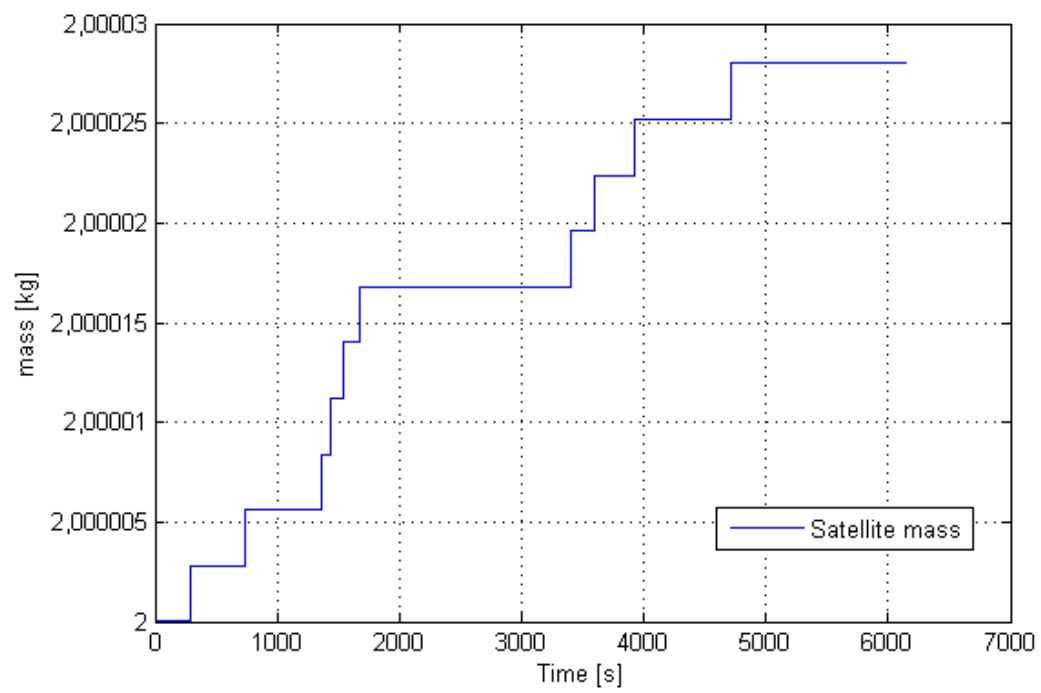


Figure 29: The mass of the satellite in orbit

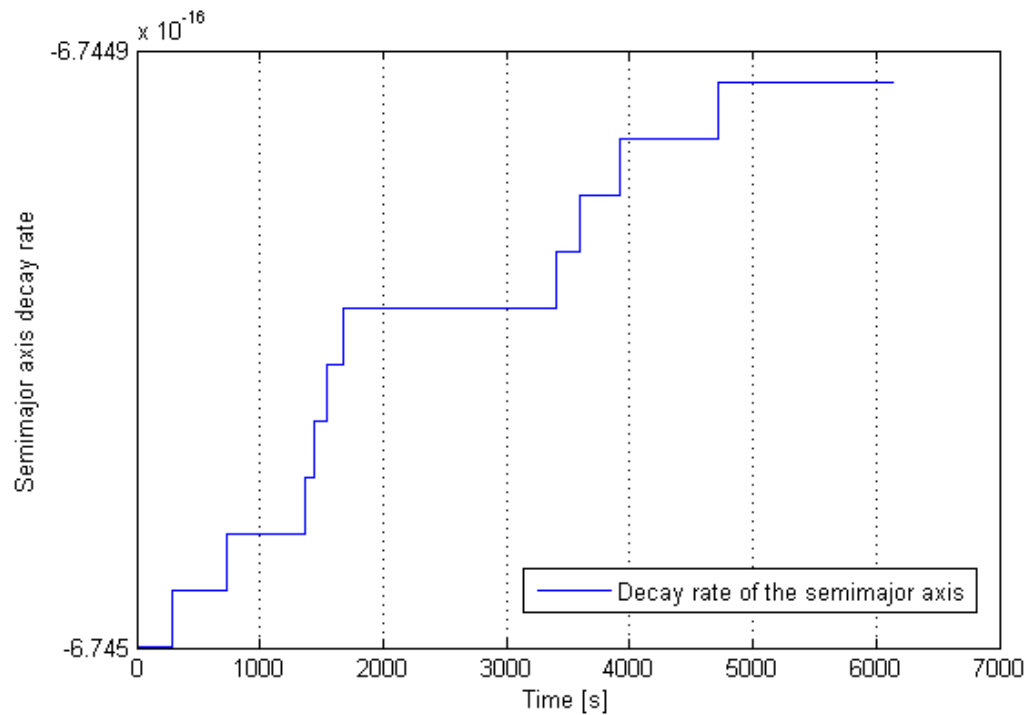


Figure 30: Semimajor axis decay rate

axis decay rate would anyway give a short lifetime.

The change of the satellite mass has also effect to the atmospheric drag in Figure 31. But as for the semimajor decay rate the mass have also negative effect on the atmospheric drag follow to Equation 50 in Section 2.11.2. But we can see from the figure that it is very hard to see the effect of the mass since another variable factor is much more dominating: the velocity of the satellite.

If we compare the atmospheric drag with the total acceleration in Figure 32 we can see that the atmospheric drag become very small on the total acceleration. The total acceleration looks like a very perfect sinus-function without any visible perturbations, as we also not can see in Figure 21. To see more perturbing forces we must maybe have simulated at least for a year instead of an orbit for be able to see any changes.

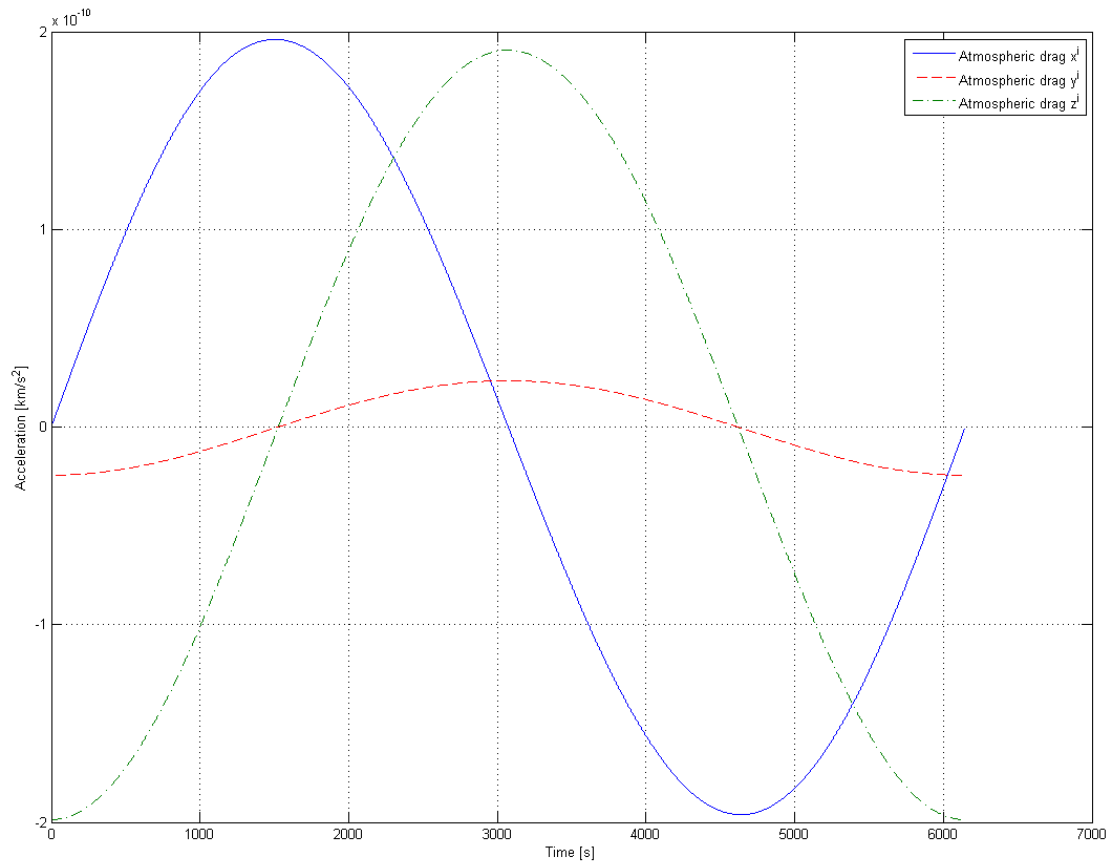


Figure 31: Atmospheric drag

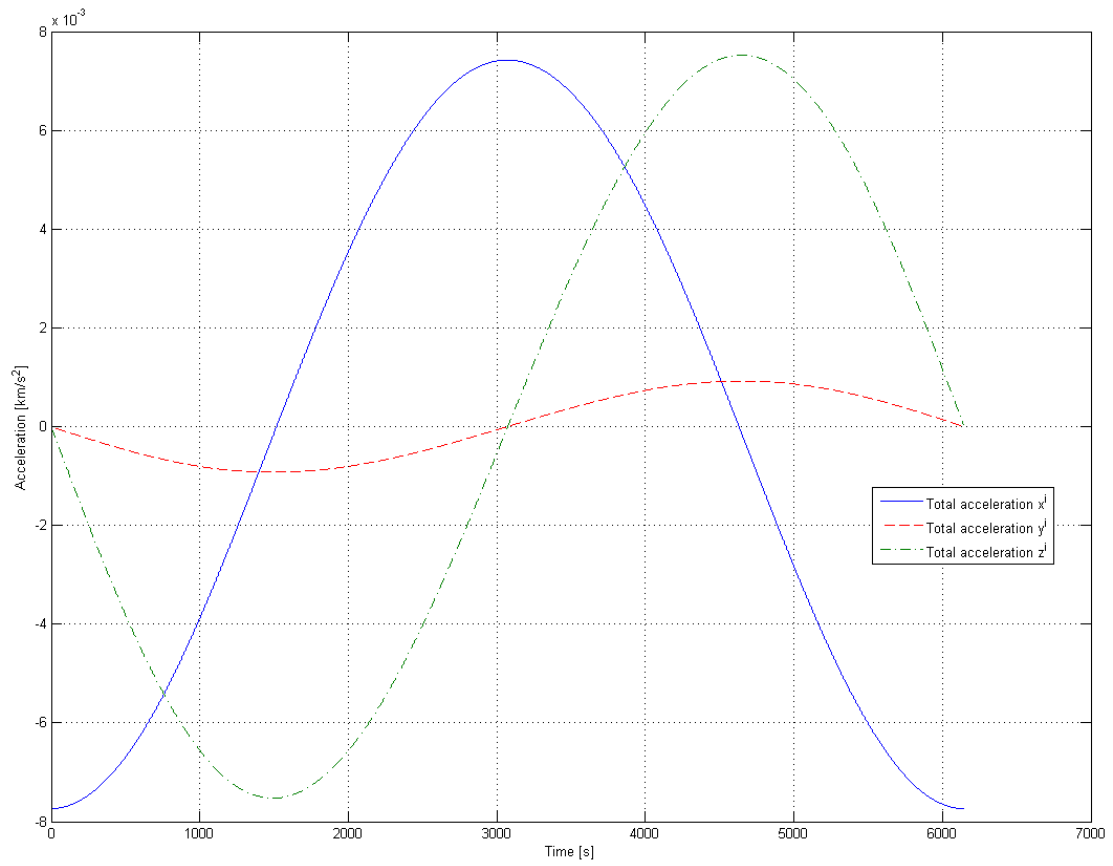


Figure 32: Total acceleration

5 Concluding remarks

In this chapter we will look at the conclusion of this report and see what can be done in future work.

5.1 Conclusions

We have seen through the analysis that the space debris decreases slowly, but this is only in periods where the creation of space debris is low and no big outbreak of space debris happens. We have also observed that big space debris, bigger than 10 cm, is, especially together with inactive payloads, dominating the space outside Earth.

Of our estimation to get the probability to meet space debris in the specific region (800 - 950 km) we got approximately good rates. With these rates there were possibilities to analyze the region better. It did not compare quite perfect with the model DAS 2.0.2 but was good enough to be only an estimate of number of space debris (and inactive payloads) from a database.

The mathematical model for space debris was able to generate space debris with random position in orbits around Earth. The two removing systems were both able to catch space debris and we could see that they with different ranges collected unequal amount of space debris. Of the satellite orbit we could see that the perturbing forces and the additional masses from collected space debris had little effect when we were looking at only small periods.

5.2 Recommendations for future work

The mathematical model of space debris must continuous become updated after how the population of space debris changes. We know that only one big breakup is needed to do massive changes, see Section 2.8.2, so every big breakup must be controlled and updated. It could eventually then be need for changing the altitude for the removing the system to increase the efficiency.

The analysis of space environment can be expanded to a much longer period, i.e. a year or longer. This can give more interesting results, especially if big breakups occur.

The space debris simulator is only the beginning design of a simulator that can do much more in later work. Through the process of this thesis there were some ideas that either was removed because they made the simulator slow or because they were outside the thesis's description. Some of these ideas are discussed in Appendix A so they may be able to be implemented in future versions of this model.

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A Concepts for future work

In this section we will discuss some concepts and ideas about how to improve the model to be even better. Many of these concepts are already tested in the model, but was removed because of complexity in earlier phases. This is mostly for they who want work further and improve the model.

A.1 Mean anomaly of space debris

In Section 2.10.2 we defined the mean anomaly, M , for space debris to be constant. This means that the space debris will not be moving in time. If we want they to do that, we need first to get the orbit time, T . The orbit time can we get with help of the known mean motion, n , [18]:

$$T = \frac{2\pi}{n}. \quad (61)$$

Since we must consider that this should exist for n orbits we will exchange 2π with t_{count} , which is 2π multiplied with n orbits, to calculate T_n . We will use 2π as the initialization value for t_{count} :

$$T_n = \frac{t_{count}}{n}. \quad (62)$$

We can use the old M to find the first time from last perigee, t_0 :

$$M_0 = nt_0, \quad (63)$$

$$t_0 = \frac{M_{old}}{n}. \quad (64)$$

If t_0 added by the present time, t_p , is smaller than T_n we can use Equation 63 where we add t_0 by t_p . From this we get the new M :

$$\begin{aligned} M_{new} &= n(t_0 + t_p), \\ M_{new} &= n\left(\frac{M_{old}}{n} + t_p\right), \\ M_{new} &= M_{old} + nt_p. \end{aligned} \quad (65)$$

If $t_{old} + t_p \geq T_n$ we must include t_{count} in Equation 65 which we subtract from the rest:

$$M_{new} = M_{old} + nt_p - t_{count}. \quad (66)$$

In this case we must also remember to increase t_{count} . We increase it with 2π for every new orbit.

This method has been tested in the originally model and we observed that it works, see Appendix B10.

A.2 Constellation of satellites

There are many good reason to could expand the system to simulate a constellation of satellites instead of just one, [1]. Then we could increase effective area, A , and collect more space debris on a shorter time. Each satellite in the constellation will together collect more space debris before returning to Earth. The constellation will also be more robust if one satellite should fail.

If we simulate more satellites in the same orbit we cannot let the space debris be passive (not moving in time) since they then will collect the same space debris. Then the total amount of collecting space debris will become the same for the constellation as for only one satellite. We can implement the system from Appendix A.1.

A.3 Changing of the limitations

Our system has many limitation due to the low altitude and the small size of the satellites, which is the reason why some was concepts ignored in the final product.

The region 800 - 950 km was the region with most space debris but there was also other regions that was good candidates:

- Low Earth Orbit: 1400-1550 km, [25]. This is a region with much space debris that lies on bigger altitude than the other bigger concentration of space debris. Because of that objects here will be lesser affected by perturbation from lower altitude and stay longer in their before deorbiting, [21].
- Geostationary Orbit (GEO) is a popular region for many satellites. It is defined together with the Low Earth Orbit as a protected and all object here have to be removed from this region within 25 year after their end of operational time, [13]. This is an region that is in good distance from Earth which means the objects here will much highly be affected by perturbation from the sun and the moon, [21].

Later use of the simulator can also have interest to change some of the other parameters:

- Increasing the eccentricity, e , will making more elliptic orbits. This let the satellite use more time at apogee (highest point in orbit) and then use more time to collect space debris there, see Section 2.3;
- Bigger satellites, will not make any change in our model since the size of the satellite has no limit.

B CD index

Here is a list for files on the CD.

1. This report;
2. T. I. Bredeli, "Space debris collection using picosatellites," Narvik University College, Tech. Rep., 2012;
3. Simulink file for simulator: SpaceDebrisRemovingSystem_verFinal.mdl;
4. C++-program for counting space debris;
5. Text files for C++: satcat.txt, SATCATdebris1-5.txt;
6. Analysis of space environment: SD_evolution.ods;
7. Pictures from this report;
8. Monte Carlo simulating: MC_RemSys.xlsx;
9. Atmosphere model in Simulink: AtmosphereModel.mdl;
10. Old version of Simulink file: SpaceDebrisRemovingSystem_ver4b.mdl.