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Working Paper Series in Economics and Management
No. 05/05, August 2005

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Acknowledgement: We should like to thank seminar participants at the University of Tromsø for comments on an early draft, and especially Espen Sirnes for performing the numerical simulations. Errors are our own.

Abstract

The standard contest model in which participants compete in a single dimension is well understood and documented. In this paper we propose an extension in which competition ensues in several dimensions and a competitor that wins a certain number of these is awarded a prize. We look at the design of this contest from the point of view of maximizing effort in the contest (per dimension and totally), and reducing the cost to the designer of providing effort incentives. The standard Tullock model and its results are encompassed by our framework.

Keywords: tournament design, multi-tasking, effort incentives

JEL Classification: D72

1. Introduction

In many areas, including lobbying, internal labor markets, promotional competition, political campaigns, sports, litigation, international conflict and war and in many areas of biology, situations are characterized by one or several prizes that are allocated among a set of players as a function of the players' costly efforts. Such games have been called conflicts, tournaments, all-pay auctions, or wars, and the common underlying structure has been studied intensively.¹ Some more recent contributions focus on the problem of how such games should be designed if the designer pursues certain objectives. The role of the different types of contest success functions and reward functions (Kräkel 2003), the size of the prize, multiple prizes and their optimal structure (Clark and Riis 1998, Moldovanu and Sela 2001, 2005), resource constraints (Che and Gale 1997), spending limits (Che and Gale 1998), the contestants' choice of points of aspiration and the incentive to moderate their conflicting demands (Epstein and Nitzan 2004), timing (Baik and Shogren 1992, Leininger 1993), the role of fee-shifting rules in litigation contests (Farmer and Pecorino 1999, Baye, Kovenock and de Vries 2005), the role of tournaments with multiple, more complex structures (Amegashie 1999 and Gradstein and Konrad 1999) have all been analysed. Related to this, researchers in industrial organization and in political economy consider questions of sequential contests in which the same contestants interact repeatedly. In the industrial organization context, early influential work is by Harris and Vickers (1985, 1987) and Budd, Harris and Vickers (1993) who considered two structures of repeated contests, the race, and the tug-of-war. In the first type, the number of single-stage contests is finite and the prize is awarded to the contestant who

¹ In the context of lobbying, some selective surveys are Nitzan (1994) and Lockard and Tullock (2001).

first wins a certain threshold of single-stage contests. In the tug-of-war, the number of single-stage contests is possibly infinite, and the tug-of-war ends if one contestant has gained a sufficiently large advantage. Konrad and Kovenock (2005) consider the tug-of-war in the context of all-pay auctions and offer complete solutions to the problem. Klumpp and Polborn (2005) consider the primaries in the U.S. presidential elections. They focus on the sequential nature of this process, and compare it with simultaneous contests in several states. The sequential nature causes considerable dynamics in terms of spending levels in the sequence of elections.

Our paper considers a particular aspect of contest design: in many situations the number of prizes that can be awarded is much smaller than the number of dimensions along which contestants compete with each other. The prize(s) must be awarded as a function of the outcomes of a larger number of contests or tournaments, one for each of several tasks of similar importance, such that the contest designer or principal knows in how many tasks each player performed better than his competitor.

To be more specific, consider tournaments in the labor market as an example. Such tournaments are frequently used and their efficiency properties have been intensely studied since they were first considered by Lazear and Rosen (1981). In a standard tournament, participants are invited to try to win a prize of fixed value by making an irretrievable effort or outlay, with the winner being determined by a contest success function. The advantages and disadvantages of tournaments compared to a standard principal-agent contract are well understood in the context in which agents spend effort

along one dimension only, and are surveyed by Kräkel (2004) who introduces limited liability as a further dimension along which standard contracts and tournaments should be compared. Generally, non-verifiability of output and the contractual problems for the principal that this may generate, systematic noise, or limits to the comparability of outputs on an ordinal scale are known to be main reasons that may make tournaments, or relative reward schemes more generally, superior to other, standard incentive mechanisms. Their wide use (see Lazear 1996 and Kräkel 2004 for discussions of examples) in firms, sports and other contexts suggests that the conditions for their superiority are often met.

This tournament literature concentrates on problems in which the agents perform a single task, or, where performance is measured along one dimension. In many organizational problems, agents have to decide about their overall effort, and how to allocate this effort between different tasks. For instance, members of a university faculty spend time on teaching, supervising Ph.D.-students, research, applying for grants, evaluation of the work or applications of others, committee work and other administrative duties. The importance of multi-tasking, and the problems this causes in the context of principal-agent theory was first formally studied by Holmstrom and Milgrom (1991). Principals may want agents to spend effort along several dimensions and to pursue several goals, whereas the correlation between input or output along these different dimensions need not be equal, and they need not be equally well observable or contractible.

The problem of multi-tasking may also come together with one or several of the contractual problems that make a contest or tournament the appropriate incentive tool. The agents may compete along several dimensions, providing several types of effort that generate several types of output, and the contest designer may need to decide about the structure of prizes as a function of relative performance along this set of outputs. If each type of effort leads to one different type of output, compared to the problem of the choice of the structure of prizes when contestants compete along one dimension, a tournament designer can essentially choose the number of tournament dimensions along which an agent needs to win in order to win a prize. This is the framework analysed in this paper.

In the analysis we focus on symmetric equilibria with the particular property that the same effort is spread out along all dimensions of the contest.² If the principal values the sum of agents' effort along each dimension equally and independently, but has decreasing marginal benefits from the agents' sum of efforts in each dimension, holding everything else equal, the principal would like agents to attribute equal effort to each dimension. We focus on the type of equilibria which this principal, loosely speaking, likes most.

In a contest with multiple dimensions, the principal may simply want to award one prize in each dimension, instead of one big prize. However, in some contexts, such as tournaments for promotion in internal labor markets, the prizes are indivisible and absolutely limited in number. Multi-tasking, compared to single-tasking, is then costly for

² This is analogous to the procedure adopted by Klumpp and Polborn (2005) that they call Symmetric Uniform Campaign Equilibrium.

the principal in the tournament context as well. We consider optimal tournament design in such a framework, where the principal can optimize only along two design variables. Suppose that participants compete in n symmetric, mutually independent dimensions of output and effort, with a winner in each dimension being determined as in Tullock's (1980) lottery contest. In the tournament that we present in this paper, participants compete in a number of dimensions, and the winner must beat the opponent in at least some pre-specified number. The problem is tractable for the case of identical players and equally important dimensions, for which we derive the symmetric Nash equilibrium. We present existence conditions for this equilibrium, and look at how the specification of the rule affects incentives to exert effort. We look at the optimal contest design from the point of view of maximizing effort (per dimension or totally), and from the point of view of minimizing the cost to the designer of inducing this effort. The standard Tullock contest is encompassed in our framework, and provides a natural point of comparison. The principal may specify a rule in which the winner must beat the opponent in a least k of these dimensions and may choose k to maximize his objective function. If $2k > n$ then this rule establishes that the winner must beat the opponent in the majority of the dimensions to win, with the size of the required majority increasing as k gets closer to n . In these cases there can be at most one winner, but there may be no winner at all. If the principal chooses k with $n \geq 2k$ then a majority is not needed to win, and situations can arise in which one or both of the participants win a prize.

Internal labor markets have been used as an illustrative example here. The multi-tasking tournament is a structure that is relevant in many contexts, where the choice of k or n

may be a design problem in some context, like in sports tournaments, or in deliberately designed research tournaments, but may also be exogenously given in other contexts in which the decision is based on a whole set of relative performance measures, which have been discussed in the theory of contests more generally.³

2. The model

There are two risk neutral players who compete for a prize of value V by making irretrievable efforts at constant unit marginal cost. The effort of player $i = 1, 2$ is spread over n dimensions, and a player must win at least k of these dimensions in order to win the prize. Thus there are feasible contest designs in which both players may win the prize ($n \geq 2k$), or in which there is a single winner or no winner. The effort of player 1 (2) in dimension j is given by x_j (y_j), $j = 1, 2, \dots, n$, and the probability that player 1 wins dimension j takes a common form:

$$(1) \quad p_j = \frac{x_j}{x_j + y_j}$$

if at least one effort is positive, and $p_j = (1 - p_j) = 1/2$ if both players choose zero effort along this dimension. From (1) it is clear that the competitions to secure each dimension are independent of each other. Players decide upon their efforts simultaneously at the start of the game, and the outcome of each dimension is then determined by (1) and payoffs are then awarded accordingly.

³ Examples are litigation (Farmer and Pecorino 1999, Baye, Kovenock and deVries 2005), campaigning (Skaperdas and Grofman 1995), lobbying (Tullock 1980, 1988), or bribing games (Clark and Riis 2000).

A key assumption of the analysis here is that the number of prizes is not a choice variable: a contestant can win at most one prize, as a function of the number of dimensions in which he wins the contest. In particular, this rules out that the contest designer simply awards contestants with one prize in each dimension, which would generally be superior from the perspective of contest design.

In the model we focus on a symmetric equilibrium in pure strategies. Let this symmetric pure strategy equilibrium be denoted by $\mathbf{x}^* = \mathbf{y}^* = \chi = (\chi_1, \chi_2, \dots, \chi_n)$. Then, as is shown in the Appendix, $\chi_1 = \chi_2 = \dots = \chi_n$ must hold. This makes room for a simplification of the expected payoff function that can be used to characterize the equilibrium.⁴ We set up the dependence of the payoff for player 1 given that player 2 has a symmetric effort in all of the dimensions, and that player 1 has the same outlay in all dimensions but the first. We then differentiate this expression with respect to x_1 and set this equal to the common effort in each dimension to find the equilibrium. Given an expenditure x_1 for player 1 in the first dimension, and a symmetric outlay χ for player 2 in all dimensions, and player 1 in all but the first, the probability that player 1 wins none of the competitions is:

$$(2) \quad P(0) = \frac{\prod_{j=1}^n y_j}{\prod_{j=1}^n (x_j + y_j)} = \frac{\chi^n}{2^{n-1} \chi^{n-1} (x_1 + \chi)} = \frac{\chi}{2^{n-1} (x_1 + \chi)}.$$

⁴ The general form of the expected payoff function would involve a specification of all of the combinations of winning at least k of n trials, and the corresponding probabilities.

If player 1 wins exactly one of the dimensions, then it could either be dimension 1 (with

probability $\frac{x_1 \chi^{n-1}}{2^{n-1} \chi^{n-1} (x_1 + \chi)}$ or it could be one of the other $n-1$ dimensions with

probability $\frac{(n-1)\chi^n}{2^{n-1} \chi^{n-1} (x_1 + \chi)}$. Hence the probability of winning exactly one of the

dimensions is

$$(3) \quad P(1) = \frac{x_1 \chi^{n-1} + (n-1)\chi^n}{2^{n-1} \chi^{n-1} (x_1 + \chi)} = \frac{x_1 + (n-1)\chi}{2^{n-1} (x_1 + \chi)}.$$

Proceeding in this way, one can write the total probability of winning exactly $j \geq 2$

dimensions as:

$$(4) \quad P(j) = \frac{{}_{n-1}C_{j-1} x_1 \chi^{n-1} + {}_{n-1}C_j \chi^n}{2^{n-1} \chi^{n-1} (x_1 + \chi)}$$

where ${}_n C_j = \frac{n!}{j!(n-j)!}$ is the formula for the number of combinations of j from n . There

are ${}_{n-1}C_{j-1}$ combinations of j from n that involve winning dimension 1, and ${}_{n-1}C_j$ that do

not. Let $P \equiv \sum_{j=0}^{k-1} P(j)$ be the probability that player 1 wins less than k of the dimensions.

Then the dependence of 1's payoff on his effort in dimension 1 is captured by:

$$(5) \quad \pi_1(x_1, \chi) = (1 - P(x_1, \chi))V - x_1 - (n-1)\chi$$

Maximization of this expression by choice of x_i yields the following result:

Proposition 1

Suppose that the contest design (n, k) satisfies the following condition:

$$(6) \quad \sum_{j=k+1}^n {}_n C_j - \frac{(k-2)}{2} {}_n C_k \geq 0.$$

Then contest design (n, k) possesses a symmetric equilibrium in which each player's effort in each dimension is

$$(7) \quad \chi(n, k) = \frac{(n-1)!V}{2^{n+1}(k-1)!(n-k)!} \equiv \frac{V {}_{n-1} C_{k-1}}{2^{n+1}}.$$

The total effort for each player is $n\chi(n, k)$ which can be written as

$$n\chi(n, k) = \frac{kV}{2^{n+1}} {}_n C_k \equiv X(n, k).$$

The equilibrium probability of winning is given by

$$P(n, k) = 1 - \frac{\sum_{j=0}^{k-1} {}_n C_j}{2^n}$$

and the equilibrium payoff is $\pi(n, k) = P(n, k)V - n\chi(n, k)$.

Proof

Differentiating (5) with respect to x_i yields:

$$(8) \quad \frac{\partial \pi_1}{\partial x_1} = - \left[\frac{(x_1 + \chi) \left[1 + \sum_{j=1}^{k-2} C_j \right] - [x_1 + \chi] \left[1 + \sum_{j=1}^{k-2} C_j \right] - C_{k-1} \chi}{2^{n-1} (x_1 + \chi)^2} \right] V - 1$$

Setting (8) equal to zero and evaluating at a symmetric situation ($x_i = \chi$) yields the effort indicated in the Proposition. Differentiating (8) with respect to x_i and evaluating at the symmetric situation reveals that this effort maximizes 1's expected profit (5). The expressions for the equilibrium probability of winning and the expected payoff follow directly. The existence condition in (6) secures a non-negative profit in equilibrium.

QED

Notice that the contest design $(1, 1)$ is the usual Tullock case, yielding an effort per player of $\chi(1, 1) = V/4$. This contest design naturally satisfies the existence condition in (6) which guarantees a non-negative payoff in equilibrium. Intuitively, the probability of winning a prize in equilibrium must be large enough when weighed up against the total cost of effort; this means that n must be sufficiently large in relation to k . There does not seem to be a single relationship between n and k that satisfies (6), but Table 1 presents results of simulations of the existence condition for $20 \geq n \geq 1$. Figure 1 presents numerical results for larger values of n . For a given n , $k^*(n)$ represents the maximum number of dimensions that must be won that is consistent with existence of the symmetric equilibrium. In Figure 1, contest designs on and below the line are commensurate with the equilibrium in Proposition 1

Table 1

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
k*(n)	1	2	2	2	3	3	3	4	4	5	5	5	6	6	7	7	7	8	8	9

Figure 1

About here

3. The optimal design

Suppose that the contest designer has specified a rule for how many dimensions a player must succeed in to win a prize. Given this k , we can calculate the number of dimensions that maximizes outlay per dimension, and the total outlay. Holding k fixed and increasing the number of dimensions from n to $n+1$ has the following effect on effort per dimension, and the total effort:

$$(9) \quad \chi(n+1, k) - \chi(n, k) = \frac{V(n-1)!(2k-2-n)}{2^{n+2}(n-k)!(k-1)!(n-k+1)}$$

$$(10) \quad (n+1)\chi(n+1, k) - n\chi(n, k) = \frac{V(n-1)!n(2k-1-n)}{2^{n+2}(n-k)!(k-1)!(n-k+1)}$$

From (9) it is clear that the change in effort per dimension is positive until $n=2k-2$ upon which the addition of another dimension (to $2k-1$) does not change the effort, but further additions reduce the effort in each dimension. Equation (10) indicates that total effort

increases in n until $n=2k-1$ upon which the addition of another dimension (to $2k$) has no effect on the total effort; adding more dimensions than this reduces the total effort.

We must, however, check whether these maxima actually exist, i.e. if the efforts are consistent with equilibrium. Define $n^*(k)$ as the minimum number of dimensions that must be present for the equilibrium to exist. For $k=1$, equilibrium exists for $n \geq 1$ so that $n=2k-1$ indeed maximizes effort per dimension, and $n=2k-1$ and $n=2k$ maximizes their sum. For $k=2$ we have $n^*(2)=2$ so that the designs that maximize efforts are feasible. For $k=3$ it is the case that $n^*(3)=2k-1$ so that this is the design that maximizes effort per dimension; the maximal efforts for $k=3$ are consistent with equilibrium. For $k \geq 4$, the existence condition in (6) is not fulfilled at the n that maximizes efforts; hence n must be increased to the lowest level that is consistent with equilibrium (since efforts are decreasing in n in this region). The required n is thus $n^*(k)$.

These results are summed up in Proposition 2.

Proposition 2

Fix the number of dimensions that must be won to secure a prize at k . If $k=1$ then the effort per dimension is maximal for $n=1$, and the total effort is maximal for $n=1$ and $n=2$. If $k=2$, the effort per dimension is maximized for $n=2$ and $n=3$, and the total effort per player is maximized for $n=3$ and $n=4$. For $k=3$, then the effort per dimension is maximized for $n=5$, and the total effort per player is maximized for $n=5$ and $n=6$. For $k \geq 4$, total effort and the effort per dimension is maximized for $n=n^(k)$.*

As an application of Proposition 2, consider two R&D laboratories that compete to innovate a new product that comprises k components. In order for both to want to compete, there must be a large enough potential number of components available. Indeed if the required number of components is at least four then maximal effort per component and the total effort is maximized at $n^*(k)$, and this is a maximum that is constrained by the existence condition for equilibrium. For lower k it is possible to achieve the maximum efforts by use of a simple rule as indicated in Proposition 2. The fact that there may be two n that maximize efforts comes from the fact that this variable is an integer (and the true maximum occurs between these values).

If only a single dimension must be won, then efforts per dimension and total effort are maximized for the standard Tullock contest design with competition in a single dimension. Adding further dimensions increases the chances of winning for a given effort and this makes the competitors reduce their effort and cost in each dimension. Adding a second dimension reduces output in each but does not affect the total. Hence the Tullock contest yields an equivalent total effort to the design $(2, 1)$; the former only permits one winner, while the latter permits up to two. If a prize is awarded to a contestant that wins at least two dimensions, then the designs $(2, 2)$ and $(3, 2)$ yield the maximal effort per dimension and the designs $(3, 2)$ and $(4, 2)$ maximize the total effort. For $k=2$ then one can conclude that the design $(3, 2)$ is optimal in the sense that it maximizes both definitions of effort and secures that only a single prize is awarded. When $k=3$ the corresponding optimal design is $(5, 3)$ for the same reason.

For larger values of k , the existence condition constrains the maximum, so that the lowest n that is consistent with equilibrium maximizes efforts. Numerical simulations have shown that for $k \geq 4$, $n^*(k)$ can be written in the form $n^*(k) = 2k + i$ where $i \geq 0$ and i increases periodically as k increases.⁵ This means that in order to maximize effort in the contest requires contest designs that potentially permit two winners. As the number of successes needed to win gets larger, the probability of winning for a given effort gets reduced; to balance this, the number of dimensions in the contest must be increased. To ensure existence it must be potentially possible for both players to win a prize.

In many applications it is the number of dimensions in the competition that will be fixed at n ; how should the contest designer set the number of successes on which to base the awarding of the prize? Treating k as a continuous variable, the sign of $\frac{\partial \chi(n, k)}{\partial k}$ is the same as the sign of $(2k - 1 - n)$; hence this function reaches its maximum at $k = 0.5(n + 1)$. When n is odd then this maximum can be reached exactly if the equilibrium exists, but when n is even, the efforts to either side of this k are maximal since k is an integer. This can also be seen directly since the sign of the change in effort (and total effort since n is fixed here) is given by

$$(11) \quad \chi(n, k + 1) - \chi(n, k) = \frac{V(n-1)!(n-2k)}{2^{n+1}(n-k)!k!}$$

⁵ It does not appear that the period is constant, nor does it follow a specific pattern, however. It is for this reason that we present numerical results.

so that effort is increased by adding more dimensions if $k < 0.5n$. The change is zero between $k = 0.5n$ and $k = 0.5n + 1$, and negative thereafter. For $n = 1, 3$ and 5 , the choice of k that maximizes effort is consistent with equilibrium. When $n = 2$, both of the k that maximize effort are consistent with equilibrium, but for $n = 4$ the design $k = 0.5n + 1$ is not feasible hence there is a single design that maximizes effort for this n . When $n \geq 6$, the designs that maximize effort are not feasible. Since effort is increasing in k in this region, the largest k that is consistent with equilibrium should be chosen in order to maximize effort. This is precisely the definition of $k^*(n)$.

Proposition 3 sums up this set of results.

Proposition 3

Suppose that n is fixed. Effort per dimension and aggregate effort are maximized for $k = k^(n)$; for $n = 2$ the maximum can also be achieved for $k = k^*(2) - 1 (= 1)$.*

For some small contests (i.e. where the number of dimensions is 1, 2, 3, or 5), it is possible to design a contest that achieves maximum effort within the constraint of the existence condition. When n is odd in this region, there is a single k that yields this maximum and it is such that there can only be a single winner of the contest. At $n = 2$ there are two values of k that maximize effort (each side of the “true” maximum). For $n = 4$ the existence condition constrains the maximal effort that can be achieved as is the case for larger contests with $n \geq 6$. Numerical simulations suggest that all of these contests allow the possibility that there are two winners. The form of $k^*(n)$ that is suggested by

the simulation is $k^*(n)=0.5n-i$ where $i \geq 0$, and increases with n . For $n=6$, for instance, efforts are maximized for $k^*(6)=3$ so that two winners occur if each wins half of the dimensions. As n grows, the potential for two winners must increase for the constrained maximum for effort to be consistent with equilibrium; $k^*(50)=21$ for example, so that there are many outcomes of the 50 dimensions in which both players will receive a prize.

Up to this point we have focused attention on contest designs that maximize effort per dimension or in aggregate. Suppose now that the prize is of value to the designer. For instance we could think of V as being a bonus awarded to salespeople if they capture at least k of the potential n sales in a period; the firm wishes to give its employees an effort incentive and this has a cost of V (or $2V$ in some outcomes of some designs). In equilibrium the expected payoff of the contest designer will depend upon the total outlays from each player $2X(n,k)$, the value of this effort, and the value and number of prizes that are expected to be awarded if he attaches value to these. Assuming risk neutrality on the part of the contest designer, this expression is

$$(12) \quad \pi^e(n,k,\alpha) = 2\alpha X(n,k) - 2P^e(n,k)(1-P^e(n,k))V - P^e(n,k)^2 2V$$

where an e indicates equilibrium values, and $\alpha \geq 1$ is the weight that the designer attaches to the efforts of the players. The first element represents the value of total efforts from both players and the second element is the probability that one prize is awarded multiplied by its value and the third element is the probability that two prizes are awarded multiplied by their value. If the contest design does not permit the awarding of two

prizes then this final element is zero. Notice that for small α , the expected payoff in (12) must be negative (for $\alpha=1$, the expected loss to the designer is of course equal to the expected gain to the participants)

Using the equilibrium probabilities and efforts from Proposition 1 gives the following expression:

$$(13) \quad \pi^e(n, k, \alpha) = \frac{V}{2^{n-1}} \left[\frac{(\alpha k - 2)}{2} {}_n C_k - \sum_{j=k+1}^n {}_n C_j \right].$$

For a given n , the effect of increasing k by one can be calculated to be:

$$(14) \quad \pi^e(n, k+1, \alpha) - \pi^e(n, k, \alpha) = \frac{V {}_n C_k (\alpha n + 2 - 2\alpha k)}{2^n}$$

So that the designer's expected payoff increases in k as long as $\hat{k}(\alpha, n) \equiv \frac{\alpha n + 2}{2\alpha} > k$,

given $k+1 < k^*(n)$, i.e. that the equilibrium exists after k is increased by one. Hence it is optimal to increase k up to this point, unless such a change makes the equilibrium cease to exist. This is summed up in the final proposition.

Proposition 4

The designer's expected profit $\pi^e(n, k, \alpha)$ is maximized by the contest design $(n, k(\alpha, n))$

where $k(\alpha, n) = \min[k^(n), \hat{k}(\alpha, n)]$.*

4. Conclusion

This paper has presented a simple model of a tournament in which participants compete by performing tasks in several equally important dimensions for a prize of fixed value. The award of a prize is secured when a participant surpasses some pre-specified threshold level. We have investigated the consequences of different contest designs on the effort supplied in the contest, and have found optimal designs for the cases in which the designer does and does not attach intrinsic value to the prize. The standard Tullock contest is a special case of the model, and has provided a useful point of comparison. Indeed our results can be used to help explain the conundrum posed by Tullock (1988) that theoretical models predict more rent-seeking activity than is observed in practice. Many of the contest designs that we consider in this paper predict less rent-seeking than the Tullock contest. Starting from the design (1,1) which is the Tullock case, total rent-seeking is unaffected by introducing a second dimension of competition, but gets reduced upon the addition of yet more dimensions. This is intuitively reasonable as the number of chances to gain a single success increases so participants save cost.

The model can be applied to analyse effort incentives and the lobbying of voters or committee members. Of relevance to the latter application is the fact that the symmetric equilibrium does not exist for the majority rule if the number of committee members is sufficiently large (from Table 1 it is clear that “sufficiently large” means above 5). In these cases, it is not equilibrium behaviour to target each member equally, suggesting that

each lobby group would target a subset of the committee members. Characterising this equilibrium is a topic for our future research.

Appendix

We show the following

Lemma *If \mathbf{x} and \mathbf{y} with $\mathbf{x} = \mathbf{y}$ are the effort vectors that characterize a symmetric equilibrium in pure strategies, then $x_i = y_i = y_j = x_j$ for all i, j in $\{1, 2, \dots, n\}$.*

For a proof, note first that a symmetric equilibrium in pure strategies cannot have zero effort along all dimensions if the prize has a strictly positive value, as each contestant can increase the probability for winning the prize from some probability strictly smaller than 1 to 1 by an infinitely small amount of effort.

Note further that different effort along different dimensions can also not be an equilibrium. We show this by way of a contradiction. Consider a candidate equilibrium $\mathbf{x} = \mathbf{y} = \boldsymbol{\chi}$ with $x_1 = y_1 < y_2 = x_2$ and $x_1 > 0$ (the case $x_1 = 0$ can be treated along similar lines) in a contest with n dimensions and a requirement to win at least $k \geq 2$ dimensions in order to win a prize. Let $\bar{p}(k-i)$ be the probability that player 1 wins $k-i$ of the contests along the dimensions $3, 4, \dots, n$. The outcome in the contests along dimensions 1 and 2 are payoff relevant for player 1 only if the outcome can increase the number of dimensions which the player wins from below k to k or above. Accordingly, the candidate equilibrium can be an equilibrium only if there is no reallocation of the effort $x_1 + x_2$ between the two dimensions 1 and 2 by which the player can increase his overall probability of winning at least k contests. Hence, a necessary condition for the candidate to constitute an equilibrium is that

$$\frac{x_1}{x_1 + y_1} \frac{x_2}{x_2 + y_2} \bar{p}(k-2) + \left(1 - \frac{y_1}{x_1 + y_1} \frac{y_2}{x_2 + y_2}\right) \bar{p}(k-1)$$

cannot be increased, for instance, by a small increase $dx_1 = -dx_2 > 0$. The impact of such a shift of effort is

$$\begin{aligned} & \left(\frac{y_1}{(x_1 + y_1)^2} \frac{x_2}{x_2 + y_2} - \frac{x_1}{x_1 + y_1} \frac{y_2}{(x_2 + y_2)^2} \right) \bar{p}(k-2) \\ & - \left(\frac{-y_1}{(x_1 + y_1)^2} \frac{y_2}{x_2 + y_2} - \frac{y_1}{x_1 + y_1} \frac{-y_2}{(x_2 + y_2)^2} \right) \bar{p}(k-1). \end{aligned}$$

Evaluating this term at $x_1 = y_1$ and $x_2 = y_2$ reduces it to

$$\begin{aligned} & \left(\frac{1}{4x_1} \frac{1}{2} - \frac{1}{2} \frac{1}{4x_2} \right) \bar{p}(k-2) + \\ & + \left(\frac{1}{4x_1} \frac{1}{2} - \frac{1}{2} \frac{1}{4x_2} \right) \bar{p}(k-1). \end{aligned}$$

Accordingly, each of these terms is positive if $x_1 < x_2$. As this contradicts the optimality of the strategy x in the candidate equilibrium, the efforts must be distributed uniformly across all dimensions in a symmetric equilibrium in pure strategies.

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