Paper I

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Bridging outdoor Physical Activities with Written Work in Geometry

Rewritten version of

PAPER ICME 10 Topic Study Group 10

Original Title: How can experiences from physical activities in the snow influence geometry learning? (Fyhn, 2004)

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Abstract

This paper gives an example of how a student's eager motivation is not enough to guarantee her or his learning even though the teaching is based upon the curriculum's intentions. The Norwegian curriculum of 1997 (KUF, 1996a; KUF, 1996b) focuses on the turn approach to angles but this paper's conclusion is to not recommend any use of compass for students who are trying to grasp what it means that one angle is larger than another one.

The twelve-year-old Sindre and his classmates took part in some outdoor mathematics activities in wintertime; the work focused on angles and on circles. The students were given an introduction to the use of a compass and they did some exercises using this tool. One year later Sindre's class drew angles in the snow and compasses were used in measuring these angles' sizes in degrees. Shortly after this, Sindre and two of his four classmates failed in a task that concerned comparison of the size of some given angles.

1. Introduction

Students can remember what they experience but that does not mean that they automatically are able to transform their memories into abstract knowledge. This paper presents a case study of the twelve-year-old Sindre and his work in geometry; how Sindre's physical experiences are being used as basis for his written work in geometry. Sindre succeeds quite well in mathematics but he has some trouble with his short term memory. This gave rise to the idea that maybe Sindre would benefit from teaching that was specially designed for him. Thus the research question in this paper is

> ***How can students' experiences from physical activities in the snow influence their understanding of geometry?***

To enlighten this question a single case study was chosen.

2. Theoretical Framework

2.1. The Map and the Territory

Teachers and students may improve their understanding of each other through the use of physical activities in mathematics education. It is possible for non-verbal communication to support spoken words and words can support non-verbal communication, too. This can be an aid in helping to prevent double communication in the teaching and learning of geometrical concepts. "*Language bears to the objects which it denotes a relationship comparable to that which a map bears to a territory"* (Bateson, 1972).

Even the best available map will not be the same as the territory; it will still be a map (Hanson, 1958). In an attempt to help the student discover some connections between territory and map in this project, the words used in the teaching of mathematics are also used in connection with elements of bodily practice in Sindre's daily life. This research has two main aims. The first aim concerns the need for understanding of how students learn mathematics; to survey Sindre's learning of geometry from this work. The second aim concerns a need for educational change. This aim is to study mathematics teaching based on students' outdoor experiences; to investigate whether Sindre can discover some connections between outdoor activities and mathematics; whether he can discover some mathematics in elements of his daily life.

2.2. Rotation and Angles

According to Lakoff & Núñez (2000), numbers often are conceptualised as points on a line. In Norwegian primary school students are familiar with using one-dimensional scales to measure lengths, weight, time and so on. Angles, however, cannot spontaneously be measured this way, thus there is no immediate correlation between an angle, Φ, and the length of the arc subtended by Φ (ibid). Johnsen (1996) claims that measurement is the most frequently used way of working with angles in Norwegian primary school classrooms. Teacher's intuitive understanding (Fishbein, 1994) of the unit circle seems to underlie their instructions about how to use a protractor. The compass, however, does not refer to any unit circle.

Most Norwegian textbooks present angles as static objects, as two rays from a fixed point. This way of working with angles is less natural to children than working with dynamic angles (Johnsen, 1996). *"Rotation"* is a frequently used word by television sports reporters; rotation is an important concept in physical education. Students and teachers use many different words for this: rotate, turn, revolve, spin, circle, twist, whirl and pivot to mention just a few. It is far from obvious to students that the word rotation has an equal meaning in mathematics and in physical activities. According to the curriculum (KUF, 1996a; KUF 1996b), students will meet angles in this dynamic way several times during their years at school.

Some students have specific experiences with this, for example the *skaters,* students who participate in snowboard or skateboard activities. In the skaters' terminology, ordinary jumps are labelled three-sixty, five-forty, and so on. The code is: three-sixty is a jump where the board is rotated 360° horizontally. Even though they are familiar with using this terminology, many skaters do not know that a circle is 360°. They have been told that the name of this jump is three-sixty and when they refer to this jump they just use its name (Fyhn, 2005).

2.3. Six stages in the process of learning mathematics

According to Dienes (1973) there are six stages in the learning of mathematics. The first stage is "free play". All children's games represent a kind of exercise, which helps the child to adapt to situations that she or he is going to meet during their lifetime. At the second stage the child observes some restrictions; there are "rules in the games". "Abstraction" occurs at the third stage.

> How is the child to be able to extract from this set of games the underlying mathematical abstractions? The psychological means of doing this is to play some games which possess the same structure, but which appear very different to the child.... This is what we call the isomorphism game. (Ibid., p. 7)

Before the child can be fully aware of an abstraction, she or he needs a method of "representation", stage four. The representation can be visual; a figure, an icon, a graph - or the representation can be auditory as well. At stage five it becomes necessary to describe the representation, and for a description we must have a "language". *"Each part of the description may serve as an axiom or later even as a theorem."* (ibid., p. 9) At the sixth and last stage "rules for proving games" occur.

The importance of nonverbal communication is made quite clear when it comes to dogs. Dogs are able to reach, as a maximum, stage four. They have no verbal language, but body language supported by some sounds enables them to communicate at quite a high level. Dogs can understand some abstractions – they can understand that the word "sit" has the same meaning both inside their house and outside. They can learn that a raised arm represents exactly the same meaning as the word “sit”. However, dogs have no number sense. Dogs do not learn mathematics, but dogs are able to deduce some abstractions. In mathematics teaching and learning the conscious use of body language could be useful for teachers who are guiding students to learn how to deduce abstractions.

3. Method

It is not easy to find out how students think and reason while they are working with mathematics. A closer study of one student’s mathematical reasoning can lead to a more thoroughly description then a broad survey. But one single student is not necessarily representative for others than him- or herself.

A wide data collection based on one student can lead to some understanding of how this student reason in mathematics in her or his daily life. *“The case study is the method of choice when the phenomenon under study is not readily distinguishable from its context”* (Yin, 2003, p 4). Human beings are normally gregarious and not solitaries; thus the study of one individual includes its co-operation with other people.

3.1. The Informant

Sindre’s mother is a teacher for children with special needs. Therefore she is able to observe much more than an average mother can do. When Sindre was a baby she observed that her son had some specific difficulties, and she let him undergo some special training. The school, however, did not listen to his mother. In 5th grade teachers agreed with her in that Sindre had some reading and writing difficulties and that he had problems with his short-term memory, too.

When asked what type of mathematics he prefers, Sindre answers *“The most fun – addition. Multiplication, too, and subtraction. When first got started, it is fun.”* Sindre knew the multiplication table by heart. He had some understanding of proportionality as a multiplicative structure, too. At the end of 5th grade a diagnostic test in geometry showed that Sindre could recognise a right angle among other angles and he scored above average for sixth graders in the tasks about symmetry.

In his leisure time Sindre goes to ski practice regularly and he participates in some skiing competitions, too. He takes pleasure in outdoor life; when his class was going for an outdoor mathematics day last winter, Sindre was the only one who remembered to bring matches for the campfire.

Maybe Sindre had some skills that made him feel comfortable with these activities and caused him to take part in them, or maybe he had developed such skills due to participating in the activities. The point in relation to this research, however, was that Sindre enjoyed taking part in these activities. According to Niss (1994) mathematics is an essential but often ignored element in areas of practice in everyday life. The focus of this paper is on geometry as an element in some areas of practice in Sindre's everyday life.

3.2. A Two-Year Study

The field work of this research took place from Sindre's final part of the 5th grade and until his final part of the 7th grade. During this period the Special Needs education office was investigating whether the background of Sindre's reading and writing difficulties was due to his squinting, to something in his brain or maybe both.

Parallel to those investigations this research project was investigating whether a specific kind of teaching based on some of Sindre's favourite leisure time activities would make him more successful in learning geometry. This 'specific kind of teaching' consisted of outdoor geometry activities in the snow which was followed up by some written tasks.

3.3. The Pre-Test and the Post Tests

One of the tasks in the diagnostic test in mathematics that Sindre's class performed at the end of 5th grade was the angle task in figure 1.

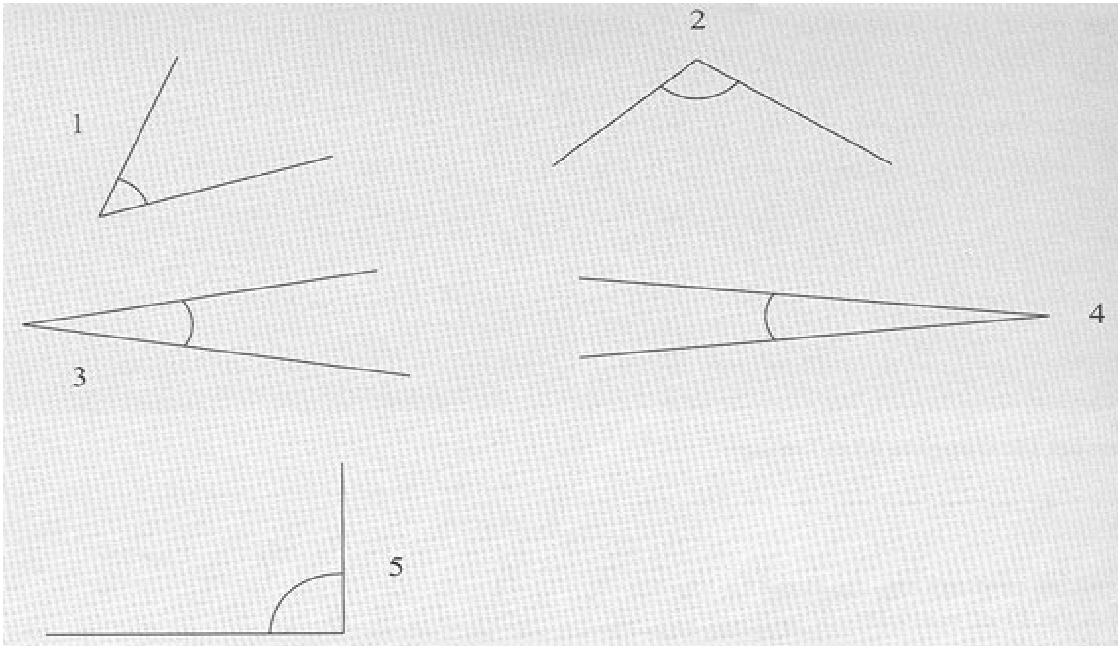

Figure 1. The angle task from the test in 5th grade and in 6th grade. Questions: a) Which one among the marked angles do you believe is the largest one? b) Which one among the marked angles do you believe is the smallest one? c) Are there any right angles (90°) among the marked angles?

After the outdoor activities and the follow-up period Sindre repeated this angle task. One year later, after one more period of outdoor geometry activities the five students in Sindre's performed a post-test that included the angle task in figure 2.

3.4. The Analyses

Sindre's answers to the angle tasks in figure 1 and figure 2 will be interpreted as an indication of whether he is able to distinguish between angles of different sizes. In addition Dienes' stages will be used as categories in analysing Sindre's answer to one particular task.

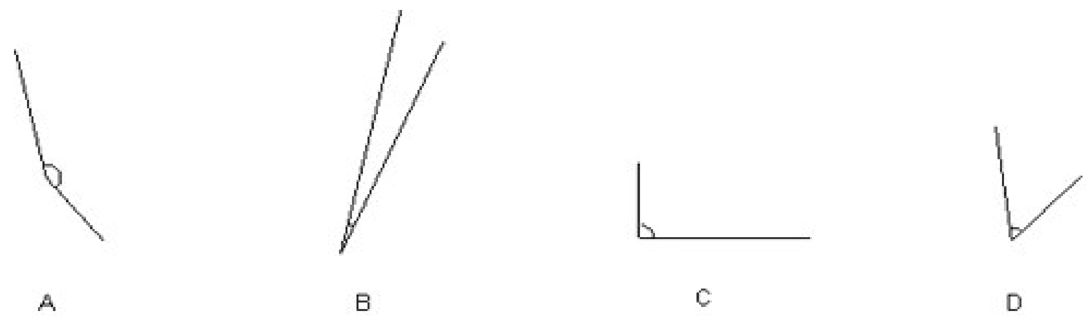

Figure 2. The angle test from the post-test in 7th grade. The task included the same questions as the task in figure 1.

4. Outdoor and Indoor Geometry with Sindre and his class

The curriculum (KUF, 1996a; KUF 1996b) pointed out that primary school students should experience angle as turn round a fixed point. This supported the use of the compass as an approach to angles. In 6th grade Sindre and his class had some outdoor geometry days. Most of the activities they performed included the use of skis or compasses; students used their bodies and skis to make lines, curves and figures in the snow. This work focused on the concepts of angles and circles. The students were given an introduction to the use of a compass and they did some exercises using this tool.

4.1. A Circle in the Snow

Norwegian students' solutions to tasks from the first two TIMSS studies indicate that the counting of paces could be a context where students' intuitive understanding of proportionality is well developed (Fyhn, 2000). This lead to an activity where the students were to make circles in the snow like the circles in figure 3.

In part two of this activity a segment was marked from the centre to the periphery in a northern direction as shown in figure 3. The students would then be placed on this segment with their faces in an eastward direction, holding hands. The next step was to walk 90 degrees and stop while holding hands. The final step was to walk 180 degrees from there and stop.

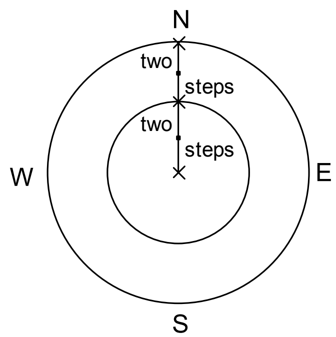

Figure 3. Circles which students will make when doing this activity: Three students stand in a line holding each other’s hands. The distance between each one of them is to be about two paces. Each student is marked as a cross on the figure. One of the students stands still at the central point while the others walk around her/him, making two circles. How many paces will the student furthest from the centre take around the whole periphery?

4.1.1. How Sindre’s Group Performed the Circle in Snow

Sindre’s group consisted of four students; the radius was six paces. The students were asked to guess how many paces they would use walking round the circumference. Afterwards each member of the group had to count the number of paces they needed.

Sindre suggested that there would be six radiuses round the circle. He suggested a correct way of solving this task, but the others disagreed, which resulted in Sindre becoming uncertain. This was not the only situation where Sindre made quite good qualified guesses, but his low self-confidence made him take more note of other students’ guesses than of his own suggestions. The guiding of students when doing mathematics reasoning activities like this, gives possibilities of strengthening their self-confidence in mathematics.

> Pupils’ experience and previous knowledge, and the assignments they are given, are important elements in the learning process (KUF, 1996b, Approaches to the Study of Mathematics; KUF, 1996a, p. 155).

4.1.2. The Letter Task

A week after the above activity each student in Sindre's class received a letter containing a photograph, some text and the following task, including the illustration showed in figure 4:

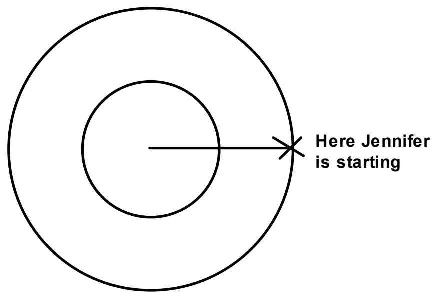

Figure 4. Illustration used in the task one week after the days of outdoor work: This figure represents the circles you drew in the snow. Imagine Jennifer standing at the mark shown in the figure. Jennifer has counted that she has taken five paces from where she stands on the outer circle to the centre following a straight line. How many paces do you think she will use to walk the entire outer circle?

This letters was not part of their actual lessons. Two weeks after receiving the letter Sindre was the only student who had worked on this task and thus he was asked to show his solution. While one of the girls was watching Sindre made a drawing similar to figure 5.

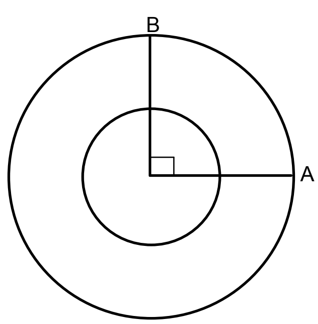

Figure 5. Sindre's first solution to the letter task.

Sindre claimed that when the marked angle was right the distance AB would equal the radius. He further claimed that the circumference had to be four times the radius; for example if the radius was five paces then the circumference had to be 20 paces.

The girl picked up her ruler and they measured the radius to be about 3,5 cm and AB to be about 4,7 cm. Then Sindre said the circumference could not be four times the radius and he remembered from the outdoor exercise that he measured the radius to be 6 paces and the circumference to be 36 paces. Before leaving the room the girl claimed that six times six makes thirty six.

Researcher:	What about the lavvu[1] that stands next to the school's front door? If you measure the distance from the centre of the fireplace to the doorway, what is the distance around the lavvu then?
Sindre:	It depends on how long the distance is. One cannot be sure that it is also six paces.
Researcher:	What if it is five paces?
Sindre:	Then there are 25 paces around, because five times five makes 25.

4.2. The Circle's Centre

The entire group of students then went for a two-day trip to the mountain. They went skiing for about seven kilometres to get to the camp area where they slept overnight in lavvus. When we arrived, we had to stamp the snow down in circles at the places we were to set up the lavvus. When one such circular area was stamped, the students were asked if anybody knew how to find the centre of a circle. There were no suggestions.

Two teachers were placed at different points of the circumference while the students were asked to place themselves an equal distance from both of the teachers. A ski stick was used in measuring the distances from each student to the two teachers. Having adjusted the placing of some of the students, one student marked where each one of the students was

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[1] A lavvu is a Sami tent. Its shape is conical like the Native Americans' tipi.

standing. Sindre volunteered to complete this work and on his own initiative he marked a diameter through these points. Then he suggested using a ski stick to measure the diameter's length and divide it into two to find the mid point. According to the Norwegian curriculum (KUF, 1996a; KUF, 1996b), the mathematics content of this task belongs to lower secondary school.

4.3. Sindre and the Compass

Sindre enjoys visiting his father who lives a several hundred kilometres away from him. The father is an officer on a pilot boat and thus he uses many different kinds of compasses in his daily work.

The students did several mathematical activities in the snow and several of these activities included the use of compass. After returning to school Sindre was asked if he had done anything during the trip that concerned mathematics. After some thought he said that using a compass was mathematics, but he was unable to say in what way. He still felt that using a compass was a bit difficult and he appreciated being offered a course in how to use it. Then the researcher gave a special compass lesson where Sindre and two of the five students in his class participated.

Afterwards Sindre repeated the tasks from the pre-test, this time his mother let him spend 30 minutes at the kitchen table working on the test. Sindre did still not know how to compare the sizes of different angles. When he was asked about his reasoning he said that it was difficult to measure the angle's sizes because he did not know what to measure; the sides of the angle, the length of the arc or the distance from the vertex to the arc.

One year later, Sindre's seventh grade class who consisted of five students, worked with angles in an outdoor compass context. The students started close to a tall mast in the middle of their schoolyard. First they marked the angle between north and northeast in the snow before compasses were used in measuring this angle's size in degrees. The next step was to make and measure some more angles with visible sides. This lesson was given shortly before a post-test on which Sindre and two of his classmates failed the angle task that is presented in figure 2.

4.4. The Further Use of Video-Clips

As an introduction to the second year of outdoor activities, Sindre's class was shown short videotapes from last year's activities. Then the students made improved videos from similar work this winter. These videos were edited by the researcher and collected on two different CDs together with some related tasks in mathematics. The students had to decorate their own CD covers, too. In some of the tasks, students were asked to describe both in words and in written texts what they saw on the videotapes. This enabled them to repeat tasks both at school and at home.

It seems quite clear that Sindre is able to reason in mathematics, but that he often forgets his own reasoning afterwards. Numbers, however, he often remembers. The intention was that maybe Sindre would find the video-clips helpful in remembering his own mathematical reasoning.

Because of the lack of equipment these films were presented on CDs and not on DVDs. The bad film quality lead to the students' self-made video-clips not becoming a success. But the mathematics tasks that were designed for this purpose was put together with some of the video-clips from the mountain trip in order to form an instruction DVD for teachers (Fyhn, 2007).

4.5. Dienes' Stages applied to the Letter Task

The students' outdoor play during the mountain trip was categorised as being free play; Dienes' (1973) stage one. The second stage which is constituted by rules of the game occurred when the students performed the practical activities. In the circle task the students were told what to do.

Students who have reached the third stage are able to play some other games that process the same structure (ibid.). Sindre's work with the letter task is interpreted to show how he tried just this. But because he was not quite sure of the structure of the outdoor game, he failed in his first attempt to apply this structure in the letter task.

Figure 5 shows Sindre's first representation of his solution to the letter task. Because he had not reached the third stage yet, he failed on stage four; the representation. According to Dienes (ibid.) language is not focused before the fifth stage. In the work on this particular task, the oral reasoning plays an important role from the second stage on. This indicates a weakness in Dienes' theory of stages.

However, one strongpoint in Dienes' theory comes to surface by his focus on stages number three and four. The crucial part of bridging the gap between students' experiences with physical activities and their reasoning in geometry is underlined here.

5. Conclusion

The introduction to this paper points out that students can remember what they experience but that does not mean that they are automatically able to transform their memories into abstract knowledge. Sindre enjoyed outdoor life and he had had a particular interest in learning how to use a compass. One year later the researcher returned to Sindre's school and after an attempt to include angles with visible sides into the teaching of compass both Sindre and two of his classmates still failed on the angle task from the post test. This seems to be a good example of that motivated students combined with an engaged teacher who intends to follow the curriculum is not necessarily enough for the students' learning.

5.1. Sindre's Measurement of Angles

It appears that students who regularly engage in snowboard and skateboard activities seem to succeed better than other students in recognising rotated geometrical figures, despite the fact that they may otherwise have less success than the others in mathematics (Fyhn, 2000). In the 7th grade Sindre could describe 180° as half a turn and 360° as a full turn and he related this to snowboard activities. Angles of less than 180°, howeveer, he regarded as static objects. He was able to recognise a right angle, but he was not sure about how to compare the size of angles.

Sindre said that he did not know what should be measured, the length of the sides of the angle, the distance between the end of the sides of the angle or maybe something else. This finding is interpreted to mean that his understanding of how to compare the size of different angles has not been improved from 5th to 7th grade. Sindre's answers to a task about angles in a diagnostic test in 7th grade, showed that his understanding seemed to have improved. But when asked about his reasoning on the actual angle task Sindre said that he just guessed something because he did not know what to answer.

5.2. The Teacher DVD

Sindre is a boy who knows quite a lot about mathematics, and he enjoys outdoor life. An investigation into approaches that could be used to improve his knowledge of geometry has resulted in the following: Students are asked to do some geometrical tasks including specific physical activities in the snow. These activities have been filmed, edited, and recorded onto a DVD together with follow-up tasks in mathematics (Fyhn, 2007).

The original idea was that the students could work with their DVDs at home as well as in school, and the parents as well could see what was going on at school. However, a teaching program that was made for one particular student with special needs seems to be suitable for the rest of the student's class. The idea of making a DVD for students developed into a DVD for teachers; the activities on the DVD can easily be replicated by teachers. If teachers use both the tasks in mathematics and the activities from the DVD then maybe they can help some of their students to decrease the distance between their physical experiences and their reasoning in geometry.

The Norwegian Version of the Students' Utterances, Chronologically

1. Artigst – pluss. Ganging også, og minus. Når først er begynt er det artig.
2. Seks ganger seks er trettiseks!
3. Enn lavvuen ved skolen, hvis du sjekker hvor langt det er fra midt i bålet og til døra, hvor langt blir det da rundt lavvuen?
4. Det kommer an på hvor langt det er, det er ikke sikkert det er seks skritt der også.
5. Enn hvis det er fem skritt?
6. Da er det 25 skritt rundt fordi fem ganger fem er 25.

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