Paper II

Anne Birgitte Fyhn: *A climbing girl's reflections about angles,* in Journal of Mathematical Behavior 25 (2006) 91-102.

ELSEVIER

Journal of Mathematical Behavior 25 (2006) 91–102

Mathematical Behavior

www.elsevier.com/locate/jmathb

A climbing girl's reflections about angles

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Abstract

The main research question in this paper is whether a climbing discourse can be a resource for a school-geometry discourse. The text is based on a 12-year old girl's story from an exciting climbing trip during her summer holiday. The girl uncovers some of her knowledge that had been invisible to her; she is guided to see some relations between her climbing and her understanding of angles. In the beginning, this girl believes her story does not concern angles at all. The tools for uncovering angles in her story are based on different levels of visibility and objects of the climbing discourse combined with different conceptions of space. The girl develops her consciousness about *angles* as natural elements in her climbing activity and she is guided to see the *angle* as an object of her climbing discourse.

Keywords: Angle; Boring; Climbing; Discourse; Exciting; Geometry; Visibility

> An angle is the inclination to one another of two lines in a plane which meet on another and do not lie in a straight line.
>
> Euclid. *Elements*, Definition[1]

This text focuses on geometry as an integrated part of a young girl's experiences from one particular climbing trip. Could it be that these bodily experiences are able to influence the girl's geometrical thinking?

In the summer of 2004, 12-year old M, the girl used as my reference, climbed the pinnacle Svolværgeita (shortened: Geita/the Goat) in Northern Norway together with her family and some local friends. I visited them the weekend after this particular climb. At that time, I studied how pupils at M's age learned geometry. M was quite a successful pupil at school mathematics, so I expected her to have a good understanding of the basic concepts of geometry.

I wondered if a 12-year-old girl's understanding of geometry would be influenced by a growing consciousness about how she moves her body in space: This led to the main question of this research:

Is a climbing discourse a possible resource for a school-geometry discourse?

1. Background

According to Lakoff and Núñez (2000), space has been conceptualized in two different ways in the history of mathematics. Most people normally think of it as a naturally continuous space. "It arises because we have a body and

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[1] Euclid. Euclid's Elements (translator and editor T.L. Heath). New York: Dover, 1956. Henderson and Taimina (2005, p. 364). "*What Euclid meant by the term "inclination" is not clear to us or, apparently, to Heath*" (ibid., p. 364).

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doi:10.1016/j.jmathb.2006.02.004

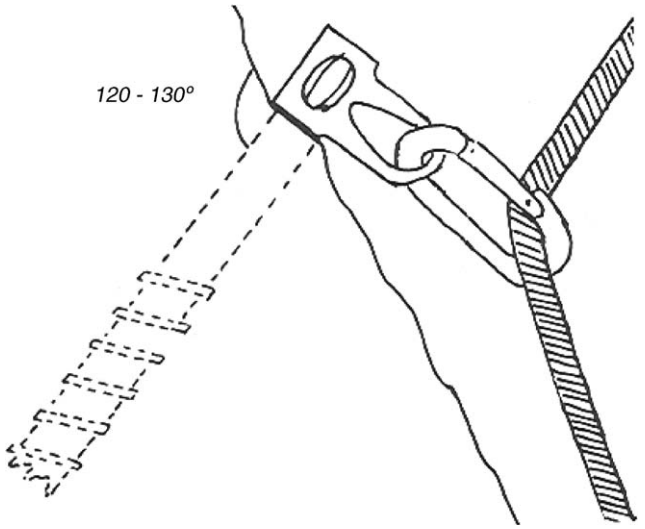

Fig. 1. The ice-screw's eyelet *has* to point in the direction where the strain will come (from "Skandinavisk Høyfjellsutstyr A/S katalog 1984–85" Printed by permission of T. Carlstrøm).

a brain and we function in the everyday world. It is unconscious and automatic" (ibid., p. 265). Climbing takes place in this naturally continuous space where coordinates and axes do not exist.

Descartes' invented metaphor "Numbers are points on a line" (ibid., p. 260) leads to the conceptualization of space as a set of points.

> It takes special training to think in terms of the Set-of-Points metaphor. Moreover, one must learn which kinds of mathematical problems require which metaphors.
>
> The Set-of-Points conception is the one taken for granted throughout contemporary mathematics. (ibid., p. 265)

Years ago when I started climbing, I had done some mathematics at the university. Gradually I found myself reflecting upon what I had learned about vectors. I discovered that my different climbing problems such as: "*How to place your foot on a slab*,[2] *how to place an ice-screw* (Fig. 1) *and so on*" were all about vectors. In trying to get the best solutions to these climbing problems, I used my new "mathematics tools"; the rediscovered vectors, as well as trial and error.

2. Why use a single case study?

Based on my knowledge and experiences from climbing as well as from teaching geometry, I realized that M's climbing trip was a case with possibilities of generalization. The case has some specific and limited data: My written version of M's story.

The above research question will be answered by analyzing the geometry in M's story by some special tools (theory). Before that, the climbing discourse will be described: "*How do I interpret a twelve-year-old girl's climbing discourse?*" Then this climbing discourse will be analyzed by answering the question "*What kind of geometry is to be found in this discourse?*" In accordance with Yin's belief, a case study seemed to be an appropriate approach to my questions:

> The need to use a case study arises when the empirical inquiry must examine a contemporary phenomenon in its real-life context, especially when the boundaries between phenomenon and context are not clearly evident. (Yin, 1981, p. 98)

[2] A slab is the climbers' word for a slope rock. The friction between the rock and the shoe's rubber sole is the reason why the climber can stand on quite steep slabs. If you get your heel low you increase the area of shoe sole that is in contact with the rock. If you decrease the angle between your leg and the rock you decrease the friction between your shoe sole and the rock.

My interpretation of the climbing discourse was made while I was fully aware that I had my school-geometry glasses on. Yin (ibid.) further claims that case studies are relevant for studying knowledge utilization. Describing the geometry in the climbing discourse involves knowledge utilization too.

A central problem for quantitative research in mathematics education is control of unwanted variation. However,

> There is no given procedure that can guarantee everything. . . . On the other hand, case studies often give broader insight into details. (Andersen, 2003, p. 28, my translation)

Lakoff and Núñez (2000) claim that human ideas, to a large extent are, based on sensory-motor experiences. One aim of this study was to get a wide insight into details about how bodily experiences can be related to the understanding of geometry. Therefore, to ensure that as few details as possible are overlooked, a single case study has been chosen.

3. To describe versus to analyze

I informed M that I would write down what she said so she could concentrate on telling her story. I felt that this method would suit me better than using a tape recorder. The informant had to wait for me to write down her words. It was important for me not to become eager and push her through her story; I wanted her to have the possibility of reflecting about what she said during the interview.

At regular intervals I stopped and asked her if what I had written was correct; I invited her to adjust my written version of her story. I wanted her to take part in the process of interpreting her story into a written text. Afterwards she looked at all the pages and said her arm would have ached if she had written that much.

When you write down a recorded dialogue, you immediately start to interpret what you hear; you wonder what is actually going on. In this interview situation, the social interaction between the girl and me was not part of the data to become analyzed, I just needed her version of the story. If my data were a recorded interview, then my written version of her story definitely would have been my interpretation of the data. Alvesson and Sköldberg (2003, p. 22) support my choice about writing the girl's story as she told it: "*the closer the material is to the empirical data, the better*." I would probably have come closer to what really happened, if I had joined M and her family on this trip.

What are the limits between data and theory? Hanson (1958) claims that all facts are theory-laden, referring to one specific example: Imagine Tycho Brahe and Johannes Kepler sitting together watching the sunrise. Do they really see the same thing in the east at dawn? They can both see a round thing slowly moving up above the horizon, but their own views of the world and their understandings of what happens are far from equal. This leads us to realize that these two persons really have different interpretations of what they see.

So was for M and me, too: We were watching two identical copies of the same written text. Did we really see the same thing? We both could see papers with the same letters in Times New Roman size 12. I had done the same climbing trip myself. However, I am doing research in how to teach and learn mathematics, while M is thinking about an exciting event from her holiday. Our understandings of the text's content were far from equal. There are reasons to believe that we really had different interpretations of what we could see.

By the time M had finished telling me her story, I had a written version of it. This story constitutes my data; the theory-laden facts that I have. From the moment I started listening to M, I started to interpret what she told me. This supports Andersen (2003) who claims that clean descriptive case studies almost do not exist.

4. The climbing discourse and its practice

There are many different definitions of "*discourse*." According to Alvesson and Sköldberg (2003, p. 203) it is "*a social text*," while "*Foucault's interest was more on how discourse constitutes objects and subjects than in the details of language use in social interaction*" (ibid., p. 224). In Foucault's (2004, p. 53) own words, ". . . *discursive practice is a place in which . . . objects is formed and deformed*."

Lakoff and Núñez (2000, p. xiv) support this by claiming:

> One of the great findings of cognitive science is that our ideas are shaped by our bodily experiences – not only in any simpleminded one-to-one way but indirectly, through the grounding of our entire conceptual system in everyday life.

Thus, to know a discourse implies knowledge about the discursive practice. I have climbed Svolværgeita several times. This means that my own personal experiences have given me knowledge about this discursive practice. These experiences are the basis for my interpretations of the girl's story. A person unfamiliar with climbing contexts might quite reasonably misunderstand the girl and miss some important points in her story.

5. The delimitation of the climbing discourse

Ascending the granite pinnacle Svolværgeita is a famous, classic climb in northern Norway.

Once you have reached the top of the Storhornet you are facing a jump over to the lower horn, the Lillehornet. Fig. 2 shows the jump and Webster (1994, p. 195) describes it this way:

> The climb's climax, the exhilarating, 1.5 meter jump between the two summit horns, must be made, quite ironically (at least from a climber's point of view) while directly above the town cemetery! The 300 meter of air below you is guaranteed to produce plenty of adrenalin, but luckily the landing on top of the Lillehornet is relatively safe.

My discursive practice mainly took place in the mountains and only sporadically on indoor artificial climbing walls. Whereas M's discursive practice, which takes place about 20 years later than mine, mainly consists of indoor climbing. Thus, the climbing discourse is not the same for M as it is for me.

I delimit the climbing discourse at first to the sun-warm rock climbing discourse and then further to the Svolværgeita-in-nice-summer-weather-young-female climbing discourse. This is what is meant by the term "climbing discourse" further throughout this text.

6. The objects of the discourse

To characterize the discourse's practice, its objects will be paid some attention:

> When one describes the formation of the objects of a discourse, one tries to locate the relations that characterize a discursive practice (Foucault, 2004, p. 53)

The objects of the climbing discourse are connected to the actual climbing situation: the use of ropes, the names of the climbing routes, the crux of a route, the problem solving, the psyches of different climbers, the descent of a route and so on. *Climbing* is the main object of this climbing discourse.

One more object of this discourse is *the rock formations*. They are important for how the climbing is being planned and carried out, but will be treated as a separate object of the discourse. The granite rock formations themselves are static, while their relations to the discourse's practice are dynamic. Both the development of new equipment and special indoor training possibilities will influence the climbers' relation to the rock formations.

Fig. 2. The Svolværgeita jump (photo: Kjell Ove Storvik).

Due to safety, young people like M are not allowed to lead this climb. That is the reason why the leading of the team is not an object of this climbing discourse. The rope connects the leader to the rest of the team and is important for the climbing safety. Thus, *the rope* is an object of the discourse.

Boring versus exciting is one more object that will be identified. This object is closely related to the attitude towards the activity, while the other three objects are closely related to the body movements.

7. What is an angle

The geometry in this research is delimited to angles; the understanding of angles is a central part of the fundament for understanding what a vector is. The angle concept is really difficult to understand for young Norwegians (Fyhn, 2004).

According to Henderson and Taimina (2005), an *angle* can be defined from three different perspectives

angle as geometric *shape*

a *dynamic* notion – angle as movement

angle as *measure*

Angle as shape refers to what the angle looks like; angle as a visual gestalt. A dynamic notion includes an action, a turning point and a rotation. A dynamic notion can refer to change in direction; it does not necessarily involve any reference to the angle's measured size. Both angle as shape as well as the dynamic notion can be referred to without using any numbers. Both these perspectives can make sense for pupils who are not very familiar with the decimal number system. You can tell a small child to let the door be wide open or almost shut without referring to any angles between the door and the wall. Thus, angle as shape and angle as turn can make sense in the "*Naturally Continuous Space*."

When measuring an angle there is no correlation between the angle Φ and the length of the arc subtended by Φ. According to Lakoff and Núñez (2000, p. 387).

> Angles themselves are nonnumerical. In plane geometry there are angles but no numbers. If we are going to do arithmetic calculation on angles, we have to get angles to *be* numbers. That is a job for a metaphor – what we call the Trigonometry metaphor.

The post-Cartesian version of the Trigonometry metaphor is based on the unit circle (ibid.). Referring to an angle by its measure in degrees involves, at first, the unit circle and then the numbers are "*Points-on-a-Line*" metaphor. Thus, one way to have angles as measure make sense is by use of the metaphor: numbers are "*Points-on-a-Line*." According to Lakoff and Núñez there are no points on a line in the "*Naturally Continuous Space*" metaphor; points on a line belong in the "*Space as a Set-of-Points*" metaphor.

All three angle perspectives are useful for mathematics: "*Each of these perspectives carries with it methods for checking angle congruency*" (Henderson & Taimina, 2005, p. 39).

The Norwegian word for angle is *vinkel*, which is similar to the German word for angle, *winkel*. According to Fowler and Fowler (1964) the English word *angle* refers to the Latin *angulus*, a corner, which, in turn, is related to the verb *to bend*. A fishing hook is called "*angel*" in Norwegian. The name angle refers to the bent *shape* of an object.

In geometry the dihedral angle means the angle between two intersecting planes (Weisstein, 2005). In climbing terminology the word dihedral refers to a *shape*; a rock formation. The dihedral formation has special properties; it is a corner with two walls. Where the walls meet, you can expect to find some kind of crack.

8. School geometry and climbing — two different discourses

Angles are objects in school-geometry discourses. According to Hanson (1958) people with different experiences will not necessarily see the same angles in their surroundings. So is for the girl and for me. According to Niss (1994) some mathematics in the society is invisible because it is hidden, not because it is missing. So is for the angles in the girl's story.

Table 1
"*The visibility tool*"

Angles	Conceptions of space	
	Naturally Continuous Space, *NCS*	Space as a Set-of-Points, *SSP*
Visibility level		
1.Obviously visible		
2. Unconsciously used		
3. Visible through discussion		
4. Invisible at the moment		

The four visibility levels related to the different conceptions of space.

Foucault (2004) refers to the discourse object's *surface of emergence*. The object angle will appear with a larger *surface of emergence* in my school-geometry discourse than in the girl's. This is because I can find angles as element in many contexts where she cannot find them. Foucault (ibid., p. 45) supports this: "*The surface of emergence is not the same for different societies, at different periods, and in different forms of discourse*."

A hidden object has hardly any surface of emergence at all. Uncovering a hidden object increases its surface of emergence. This is true for the climbing discourse as well as for the school-geometry discourse.

9. *Angle* as discourse objects

When I had completed the written version of M's story, I asked if she could help me search for geometry in that text. At first M pointed out that she knew absolutely nothing about what geometry had to do with climbing. Her mother even carefully told me on the phone that M did not know what the word geometry meant. When I told M that we would be searching for angles in her story, she expressed doubt that her story had anything to do with angles.

This shows that when M and I first met, angle was not an object of her climbing discourse. However, among other things, M was able to both describe some angles and to refer to angles by their approximate size in degrees. This I interpreted to mean that angle was an object of her school-geometry discourse.

10. Tools for analyzing the identified geometry in the climbing discourse

The "*Natural Continuous Space*" – the *NCS* and the "*Space as a Set-of-Points*" – the *SSP* are treated as different conceptions of space in the analyses of the girl's story. Lakoff and Núñez (ibid.) treat the two conceptions of space as metaphors. This is not a matter of importance in this text.

When I used vectors as a tool in solving climbing problems, I moved back and forth between the two conceptions of space. My climbing body was in the naturally continuous space. I used the *SSP* in analyzing the climbing problems and then returned to the *NCS* to investigate if my solutions were correct. I also tried to determine whether my experiences from *NCS* could be verified mathematically in the *SSP*. The awareness of these two different conceptions of space became useful for me in the analysis of my own mathematical thinking.

To describe whether an angle is acute, right or obtuse can be done in the naturally continuous space without use of any other mathematical knowledge than recognizing a shape. Referring to an angle by describing its size in degrees is a more abstract way of working on angles. M's use of angles will be categorized by whether she refers to an angle by its size in degrees or not. When she refers to an angle by its size in degrees, I interpret it as *SSP*, and when she refers to an angle without mentioning its size in degrees, I interpret it as *NCS*.

Some of the angles in M's story are visible, i.e., quite obvious to find. But some of the angles are invisible because they are hidden, not because they are missing (Niss, 1994). I will extend the antagonists visible and invisible into four visibility levels to focus on the transition from invisible to visible. The "*visibility tool*" shown in Table 1 uses the four distinctions in analyzing the angles in the climbing discourse. Some of these angles are interpreted as belonging to the *NCS* and some to the *SSP*.

The visibility levels show the object angle's surface of emergence in the climbing discourse. The analyze will show how M forms and deforms the object angle, supported by Foucault (2004, p. 53): "*. . . discursive practice is a place in which . . . objects is formed and deformed*."

Table 2
"*The object tool*"

Angles	Conceptions of space	
	Naturally Continuous Space, *NCS*	Space as a Set-of-Points, *SSP*
Object		
The formations of the rock		
The climbing		
The rope		

Different objects of the climbing discourse related to the different conceptions of space.

The Level 1, "obvious visible," is empty because M had told me beforehand that she had no idea about what angles had to do with climbing. This category stays, even though it is empty. I have left it as a possibility for M to be aware of some geometry in her daily life, as she became aware of this way of thinking. It would have been clearly unrealistic of me to expect M to start thinking in this way before she has been told about such possibilities.

The next step is to analyze the girl's reference to angles related to each of the objects of the discourse. Table 2 shows the tool used for this property. This analysis leads to a more profound description of the girl's geometrical thinking. The *boring versus exciting* is closely related to the attitude towards climbing and is therefore not part of this analytic tool.

11. "*You have to try until you succeed*"

At our first meeting I asked M for her opinion about climbing. She answered that it was amusing and continued:

> On Geita you have to get up. If you don't know how to continue, you have to keep trying until you succeed.

She told about some difficulties that she had met, "*I remember the details particularly well because I struggled with them a lot*." Furthermore she described how she proceeded to the top by using different climbing techniques. M even used the words "*square shaped*" and "*angle*," even though she claimed not to know what her story had to do with either geometry or angles.

> Discourse analysis is often interested in accounts or documents that have arisen in the natural course of events, rather than in interaction between participants and researchers. (Alvesson & Sköldberg, 2003, p. 206)

My discursive practice and my knowledge about climbing made me familiar with the details she was talking about. Once you have done a climb, it is not easily forgotten.

12. The "*boring versus exciting*" object of the climbing discourse

M said the trip consisted mostly of waiting; actually there was very little climbing, and the waiting periods were really boring. The last rope length[3] ends up at the sharp-pointed Storhornet. On arrival at the top of the pinnacle, M had to cling to the Storhornet and climb around to its front side where she met an enormous view of the area around.

> I was so exhausted that I was really shivering when I was on the top. I could see the whole of Svolvær and all the way to Henningsvær. I thought that if I jump here, I would fall down between the horns.

M experienced the climbing as really exciting and the breaks as boring.

13. Visibility Level 2 — unconscious use

Unconscious use of the word *angle* occurred twice in the written version of M's story; I categorized both of the descriptions to belong in the *NCS* — the "*Natural Continuous Space*." The first time she described how she was

[3] The leader of a climbing team is fastened in one end of the rope and another member of the team is fastened in the other end. This other person is belaying the climber. The rope goes through a breaking mechanism; a belay device that is fastened to the other person. As the climber climbs, the belayer hands out rope through this belay device. The distance the climber can climb before the complete rope is stretched out is called a rope length or a pitch.

standing on a steep slab: "*If the wall is too steep, then the angle between your leg and the sole of your foot becomes very small*." M showed what she meant. "*Then it probably hurts . . .*" Here she refers to the angle by its shape and not by its measured size and thus I interpret this to be in the *NCS*. Further in the text, she explained that the pain was caused by stretching her foot's tendons.

In the comments above, M showed that she was able to use the word angle as a tool for describing the use of her body, more precisely her feet, during the climb. This time she did not refer to angles as something being measured. In this particular situation, she used the word correctly without being aware of it; this knowledge was unconscious. Here the concept *angle* was shown to be an integrated part of the object *climbing* in the climbing discourse.

The second use of the word *angle* was when the route upwards was aslant:

> You get quite a pendulum when you fall if the rope is not fastened to a belay. It depends on the angle to the belay and me. If it is three metres across and far above me, then I get quite a lot of pendulum.

I asked if she could explain the pendulum-description once more. She answered:

> The angle from the wall where the rope is fastened and to me. A little slantwise. If there were quite a large slant, there would be a lot of pendulum.

Then she used her hand to show what this situation was like. It sounded as if an angle had only one side. M related the size of the pendulum to an angle, but my interpretation of this part of her story is that one of the angle's sides was hidden to her. There could be several reasons for this. The pendulum was the only place in her story where the angle was described by a *dynamic* notion. No matter, I categorized the second side of the pendulum's angle as "*invisible at the moment*" because she was not able to mention its existence.

14. Visibility Level 3 and different conceptions of space

M's remaining angle descriptions belonged in the category "*visible through discussion*." These descriptions were made during our second meeting. M could describe the shape of a dihedral, as well as how to climb such a formation. I interpret her descriptions as taking place in the *NCS* because she does not use any numbers in her descriptions. M even supplied her descriptions by relating the dihedral to a chimney[4]; the chimney with its two parallel walls lacks the dihedral's property of plane intersection. I asked her what a dihedral was:

> Dihedral, that is an angle. That is a small angle . . . chimney – it is not much angle on it . . . at the very bottom there was an angle of 30°.

According to Lakoff and Núñez (2000, p. 265).

> Euclid used the ordinary concept of a lack: A surface lacks thickness, a line lacks breadth and thickness . . .. In modern mathematics, the lack of a feature is conceptualised metaphorically as the presence of that feature with value zero.

M's description of a dihedral, to some extent, shows part of such thinking. So here the climbing discourse can function as a resource for a school-geometry discourse.

When M improved her description of the difference between a chimney and a dihedral, she referred to the angle between the walls in the chimney by its size approximately measured in degrees. This description includes no numbers before in the very end when she describes the small angle as about 30°.

Different angles made by her body as well as angles in the rock formations were described in nine dissimilar situations. I categorized these references both as the *NCS* and as the *SSP* according to whether she referred to an angle by its size in degrees or not.

Eight of the times M used the word angle during our talk, she furnished an angle's approximate value in degrees. Her main experiences with angles seemed to be in measuring them. This possible lack of experience in working with

[4] A rock formation that consists of two more or less parallel slabs is denoted as a chimney. You climb a chimney by stemming parts of your body towards each of the walls and then you are able to proceed upwards.

angles from other perspectives than as measure could be a possible explanation for why she failed when describing the pendulum. This could also be the reason she normally referred to angles by their approximate size in degrees. If so, her reference to angles by their sizes in degrees would not necessarily be related to *SSP* but rather to a narrow experience in work with the conceptions of angles.

15. Discourse objects and conceptions of space

The tool for analyzing relationships between a conception of space and an object showed that M used both conceptions related to the objects *rock formations* and *climbing*.

M's first thoughts about angles related to the climb, concerned the dihedral shape that often is found in granite. Showing the formation with her hands, she first hesitatingly mentioned the word dihedral. I nodded. Still doubtful about the formation's name, she knew exactly what it looked like as well as how to climb it. My interpretation of this is that M made a generalization in her description of a dihedral — the dihedral shape is independent of the angle's size in degrees. I categorize this dihedral shape to the *NCS*; the dihedral shape had nothing to do with measuring anything.

M listed a total of five *rock formation's* angles by their approximate measure in degrees, for instance, one slanting passage up to the Storhornet. She remembered the angle between the wall and the ledge to be about 120°. M said it was possible to climb round the Storhornet on that slanting ledge. One possible interpretation here is that she used the angles' approximate sizes in degrees to give a precise description of what the rock formations looked like. If so. I choose to categorize that use of angles in these descriptions as the *SSP*.

The *climbing* of a dihedral was based on her knowledge about properties of the rock formation. *Climbing* leads to angles being formed everywhere in the body, including knees, feet and arms. "*There are angles everywhere when you are climbing*," she claimed. She also added that the angle between the foot and the leg would be about 90° when a person was standing, and that it was harder to rise on one foot if there was a small angle between the leg and the thigh.

She continued:

> If you stand on your toe with your heel low, then it is harder to raise oneself than if you have the heel a little higher. The angle between the leg and the foot should be large.

Here M used geometric knowledge when explaining the use of her body during the climb. M's understanding of this was connected to the use of her feet, not her arms.

Most of the bodily angles referred to her legs. She referred to angles in connection with her arms only once and that was superficially. I asked her about angles in her arms: "*What about the angle between the upper arm and the forearm?*" She answered by explaining that it was hard to rise if your only handhold was an under cling. The information M gives about angles made by her body gave support to the idea that her climbing skills, to a large extent, concern the use of her feet. In fact she rarely mentioned the use of her arms in her story. Thus, the angle between upper arm and forearm is categorized as "invisible at the moment."

The object *rope* concerned one single situation. As mentioned above she was not able to define more than one of the sides of the angle related to the pendulum.

16. Dihedral climbing technique — to know *how* but not *why*

The last rope length started with a dihedral formation as shown on the Svolværgeita-batik in Fig. 3. M explained:

> Find good holds for both of the hands and climb up the wall. A[5] climbed with one foot on each wall. Then you mostly use your feet, while your hands are used as points of support so that you do not slide. I climbed with slightly less distance between my feet and that was a little less stable. It became more difficult to get support on the other side if both of my legs were on the same wall.

[5] M's family were with a local climber, A, and his 12-year son on this trip. A lead the climbing team.

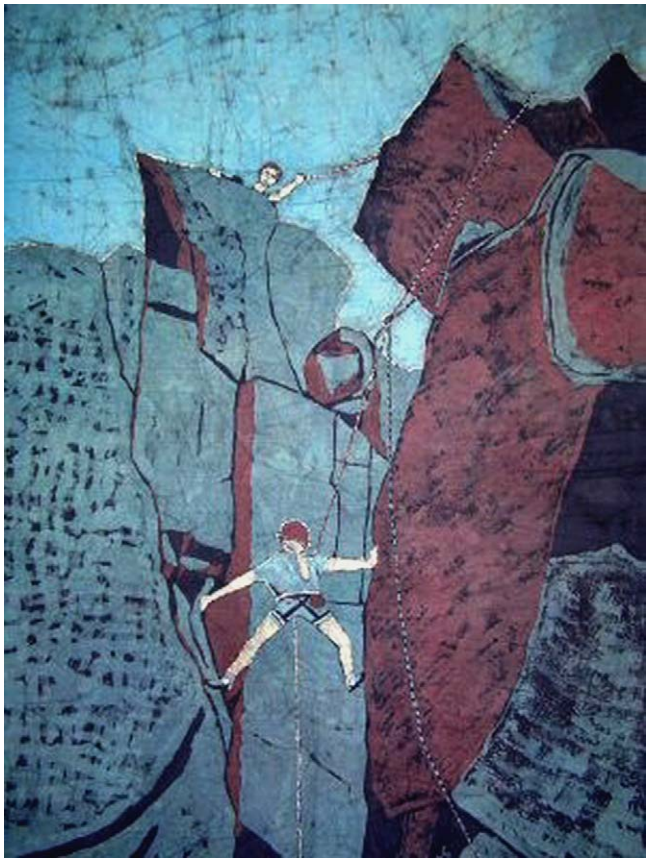

Fig. 3. Dihedral climbing. Svolværgeita-batik (printed by permission of the designer).

I asked what she meant by the word "stable" and she answered:

> Be able to let your hands go without being unsteady if you are stable. There is better support when there is larger space between your legs.

In the beginning of the first rope length, M experienced a situation where she used this kind of technique. She needed a stable position for her feet so that she could use her arms in releasing some protection gear from a deep and wet crack. "*I was standing on my feet. I had my feet straddled wide apart, and got a solid position*."

M showed that she knew quite a lot about how to place her feet so that she was in a stable position, but she could not explain why. I interpret what she said to be that her explanation is invisible at the moment; for instance it could be that she just had not thought about it yet. According to Lakoff and Núñez (2000), human ideas are to a large extent based on sensory-motor experiences. If school-geometry makes use of M's sensory-motor dihedral experiences to make this explanation visible to her, then the climbing discourse is a resource for a school-geometry discourse at this task.

17. Climbing = problem solving

Nowadays many exciting mathematical activities take place in school. One intention of these activities is to influence the pupils' attitude towards mathematics in a positive way. Many pupils regard such mathematical activities as just entertaining stunts, disconnected from school-mathematics. But if pupils learn to reflect about the mathematics of their experiences, they have the possibility of learning some mathematics from them.

The Norwegian Ministry of Education (UFD, 2002) claims that mathematics and science often appear as not very exciting subjects with a boring image. Analysis of the *boring versus exciting* object of the climbing discourse in this study shows that the climbing really was an exciting part of the trip. To succeed in problem solving is, to a large extent,

a question about attitude, "*you have to keep trying until you succeed*," as M claimed in her story. It could be that M's positive attitude towards climbing could support her attitude towards mathematics.

If climbing was introduced into ordinary mathematics lessons, then perhaps pupils' positive attitude towards problem solving in climbing could influence their attitude towards problem solving in mathematics. Perhaps the use of climbing as an element in the teaching about angles can support the pupils to make efforts in their attempts to grasp the complex angle concepts.

In this study, my focus has been on *reflection* and *concepts* when searching through a girl's story about her climbing experiences. M chose to tell about a topic that she had a positive attitude towards. This study leads to ideas about possible ways of integrating climbing into mathematics teaching. Many pupils experience climbing as an exciting activity.

One way of using these experiences in ordinary teaching can be to let groups of pupils go to their local indoor climbing walls. For safety reasons, there must be at least one person who is responsible for the security and the groups should not be too large. Parents and elder brothers and sisters with some climbing experiences could be invited to join and give a helping hand in the work with belaying[6] climbing pupils. After some climbing, the pupils will need breaks and then they might identify and describe bodily angles related to their own climbing, both orally and in written words. The pupils' reflections about their conceptions of angles could then be guided by their teachers. Such a day or two with climbing could function as an introduction to the work with written and drawn tasks that concern angles.

18. Final conclusions

This study questions whether the climbing discourse could be a possible resource for a school-geometry discourse. This question is being illuminated through the description and analysis of the occurrence of geometry in one girl's climbing discourse. The focus was on the girl's knowledge about angles, a knowledge that was invisible to her before this study took place. The discussion between the girl and the researcher lead to that she sees the *angle* as an object of her climbing discourse.

Climbing, as well as mathematics, concerns problem solving. According to Lakoff and Núñez (2000), human ideas are, to a large extent, based on sensory-motor experiences. Pupils who have an interest in climbing will be able to talk about climbing by expressing their experiences in words. Guided reflections about pupils own stories can cause that they observe these stories in a new way: "*But physical science is not just a systematic exposure of the senses to the world; it is also a way of thinking about the world, a way of forming conceptions*" (Hanson, 1958, p. 30).

There are reasons to claim that a climbing discourse could be a possible resource for a school-geometry discourse; pupils who enjoy climbing could work on *angles* in a corresponding way as in this study. Similar work on *vectors* among elder pupils should be worth trying, too. Pupils with interests in other physical activities could probably use experiences from their activities as basis for their reflections about geometry. However, climbing experiences will hardly serve as a proper basis for teaching pupils with a negative attitude towards climbing.

Acknowledgments

Two nice summer days together with M's family including discussions with M's mother led to this work. I would like to give special thanks to the 12-year old girl, M, for sharing her story with me and for spending some of her time with me to search for the geometry content of her story. I also would thank M's sister, S, who has supported her sister and helped me with the illustrations. I am very grateful to the batik designer I.J. Samuelsen, who offered me the use of her Svolværgeita batik for this purpose. Many thanks also to all those who supported me in this work, especially to Odd Valdermo and Therese Nøst for all their comments.

[6] In top-roping on indoor walls the rope goes from the climber, through a carabiner in top of the wall and down again. There the rope is closely connected to a breaking mechanism; a belay device that is fastened to another person. This other person is continuously pulling in the rope through the belay device. If the climber falls, then the rope is tightened and the climber will not be falling down to the ground.

Appendix A. The Norwegian version of the girl's utterances, chronologically

1. På Geita må man opp. Når man ikke vet hvordan man kommer videre, så må man prøve helt til man får det til.
2. Detaljene husker jeg spesielt godt fordi jeg strevde med dem.
3. Jeg var så sliten at jeg var helt skjelven da jeg var på toppen. Jeg kunne se hele Svolvær og helt til Henningsvær. Jeg tenkte at hvis jeg hopper her, kommer jeg til å falle ned mellom hornene.
4. Hvis veggen er for bratt så blir det veldig liten vinkel på foten mellom leggen og fotbladet. Da gjør det sikkert vondt.
5. Man får ganske mye pendel når man detter hvis tauet ikke sitter fast i en mellomforankring. Det kommer an på vinkelen til meg og mellomforankringa. Hvis det er tre meter bort og langt over meg, så får jeg mye pendel.
6. Vinkelen fra veggen der tauet sitter fast og til meg. Litt skrått. Hvis masse på skrå, ville det blitt masse pendel.
7. Dieder, det er jo en vinkel. Det er en liten vinkel . . . kamin — det er ikke mye vinkel på den . . . helt nederst var det 30° vinkel.
8. Står du på tåa med hælen lavt, er det tyngre å løfte seg enn hvis du har hælen høyere. Vinkelen mellom legg og fot bør være stor.
9. Finne godt tak til begge hendene og gå opp veggen. A gikk med en fot på hver vegg. Da bruker du mest føttene, mens hendene er støttepunkter så du ikke glir. Jeg gikk med litt mindre avstand mellom beina og det blei litt mindre stabilt. Det blei vanskeligere å få støtte på andre sida hvis begge beinan var i samme vegg.
10. Kunne slippe hendene uten å vingle hvis stabilt. Det blir bedre støtte med større mellomrom mellom beina.
11. Jeg sto med føttene. Jeg skreva med føttene, og fikk solid stilling.

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